COALESCING COMPACT BINARIES: THE THEORY

INTERFACING NUMERICAL AND ANALYTICAL RELATIVITY

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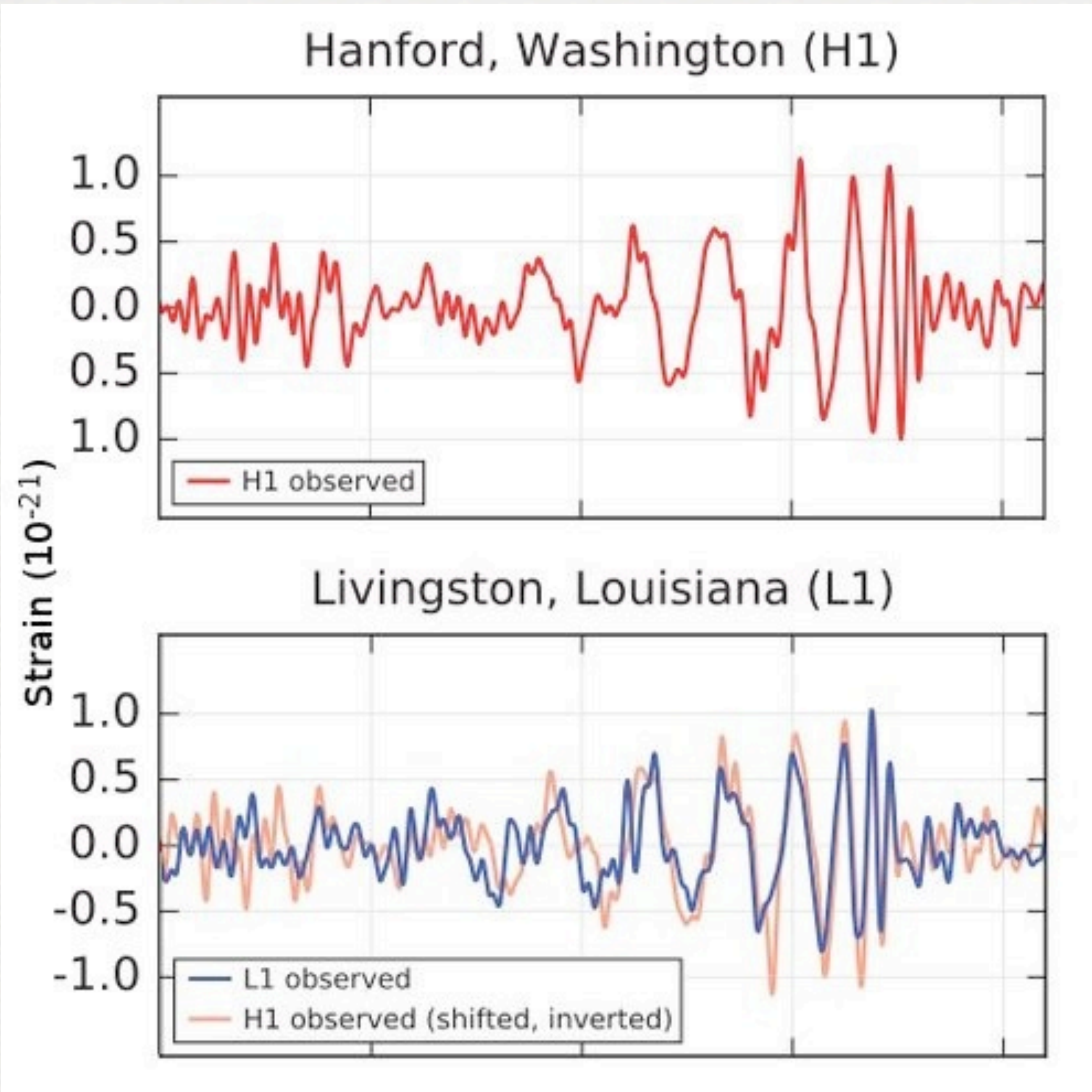
The IHES effective-one-body (EOB) code: eob.ihes.fr
T. Damour, AN,
S. Bernuzzi
D. Bini...

GW150914



$$\text{strain} = \frac{\delta L}{L}$$

GW150914 parameters:

$$\boxed{\begin{aligned} m_1 &= 35.7 M_\odot \\ m_2 &= 29.1 M_\odot \\ M_f &= 61.8 M_\odot \end{aligned}}$$

$$a_1 \equiv S_1/(m_1^2) = 0.31^{+0.48}_{-0.28}$$

$$a_2 \equiv S_2/(m_2^2) = 0.46^{+0.48}_{-0.42}$$

$$a_f \equiv \frac{J_f}{M_f^2} = 0.67$$

$$q \equiv \frac{m_1}{m_2} = 1.27$$

Symmetric mass ratio

$$\boxed{\nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2} = 0.2466}$$

HOW TO MEASURE: MATCHED FILTERING!

To extract/do parameter estimation of the GW signal from detector's output (lost in broadband noise $S_n(f)$)

$$\langle output | h_{\rm template}\rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\rm template}(f)$$

Detector's output

Template of expected GW signal

Need waveform templates!

THE THEORY...

Is needed to compute waveform templates for characterizing the source (GWs were detected...but WHAT was detected?)

Theory is needed to study the 2-body problem in General Relativity (dynamics & gravitational wave emission)

Theory: SYNERGY between

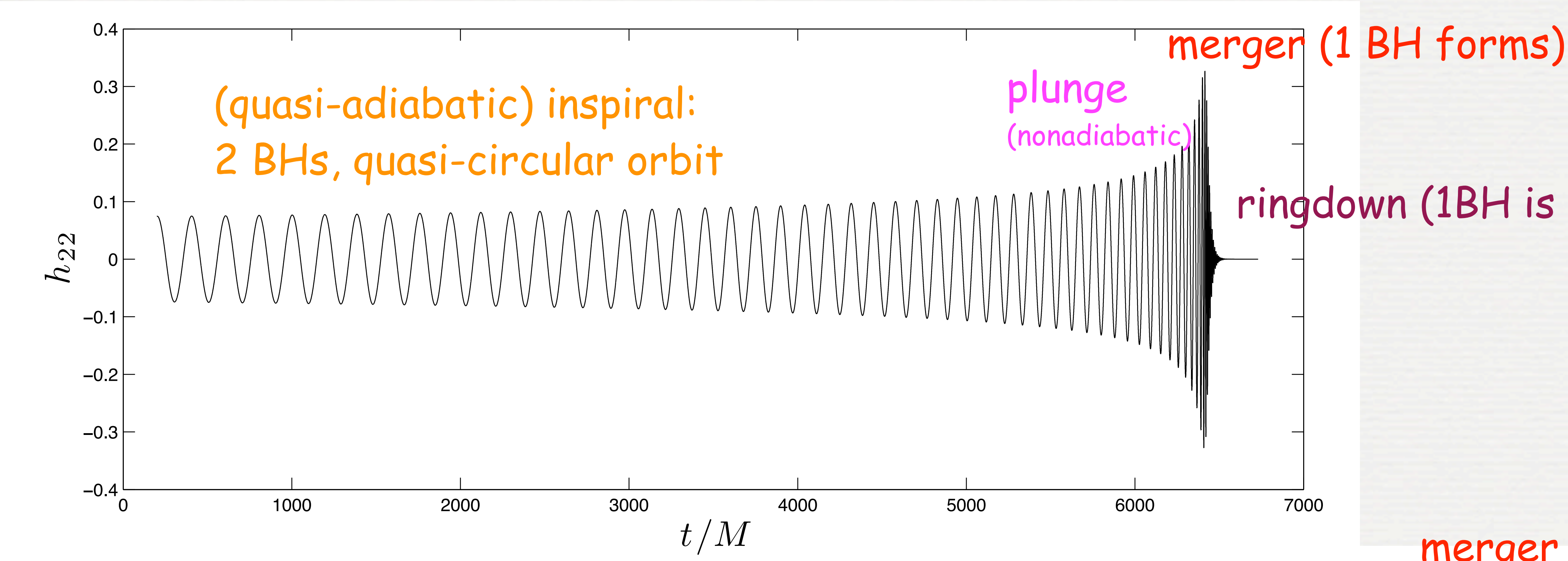
Analytical and Numerical General Relativity

(AR/NR)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

BBHS: WAVEFORM OVERWIEV

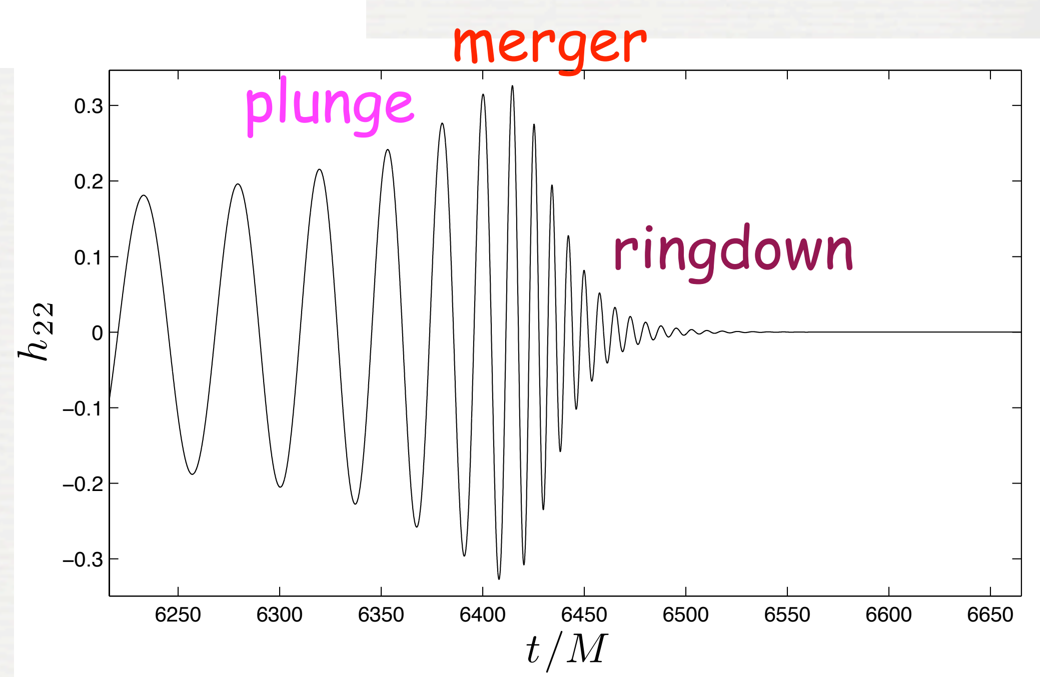
$$h_+ - ih_\times = \frac{1}{r}\sum_{\ell m} h_{\ell m}\ {}_{-2}Y_{\ell m}(\theta,\phi) \qquad h\left(m_1, m_2, \vec{S}_1, \vec{S}_2\right)$$



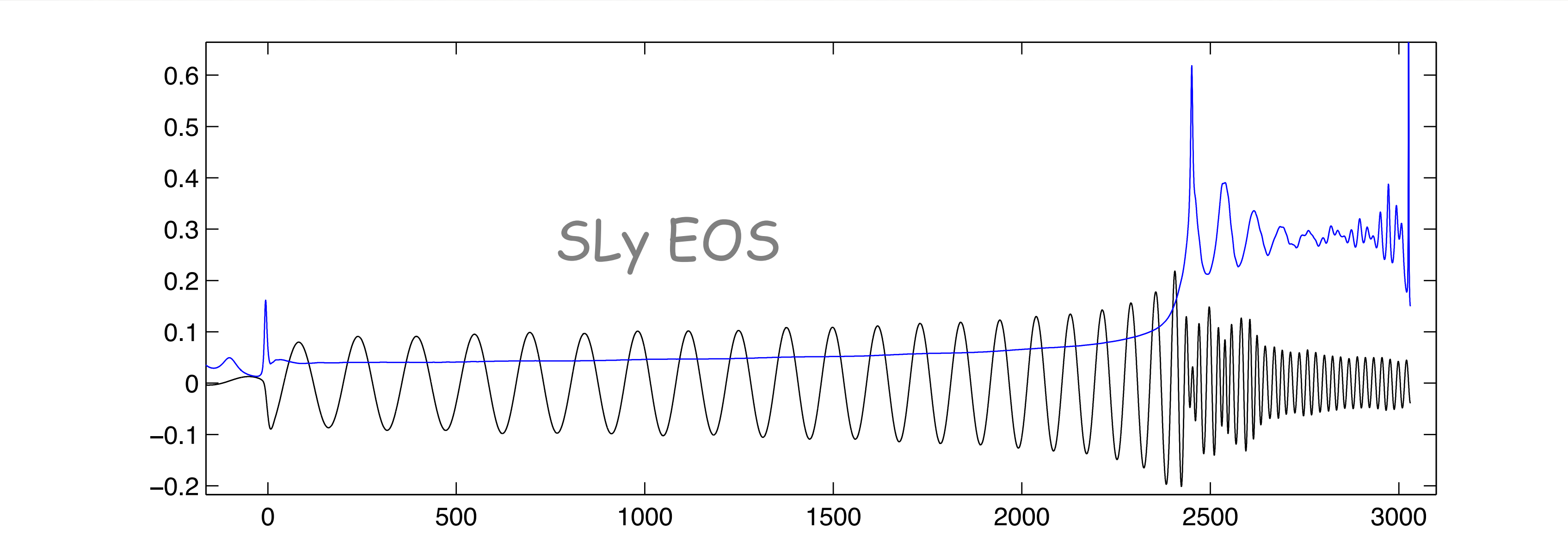
e.g: equal-mass BBH, aligned-spins

$$\chi_1 = \chi_2 = +0.98$$

- SXS (Simulating eXtreme Spacetimes) collaboration
- www.blackholes.org
- Free catalog of waveforms (downloadable)



BINARY NEUTRON STARS (BNS)

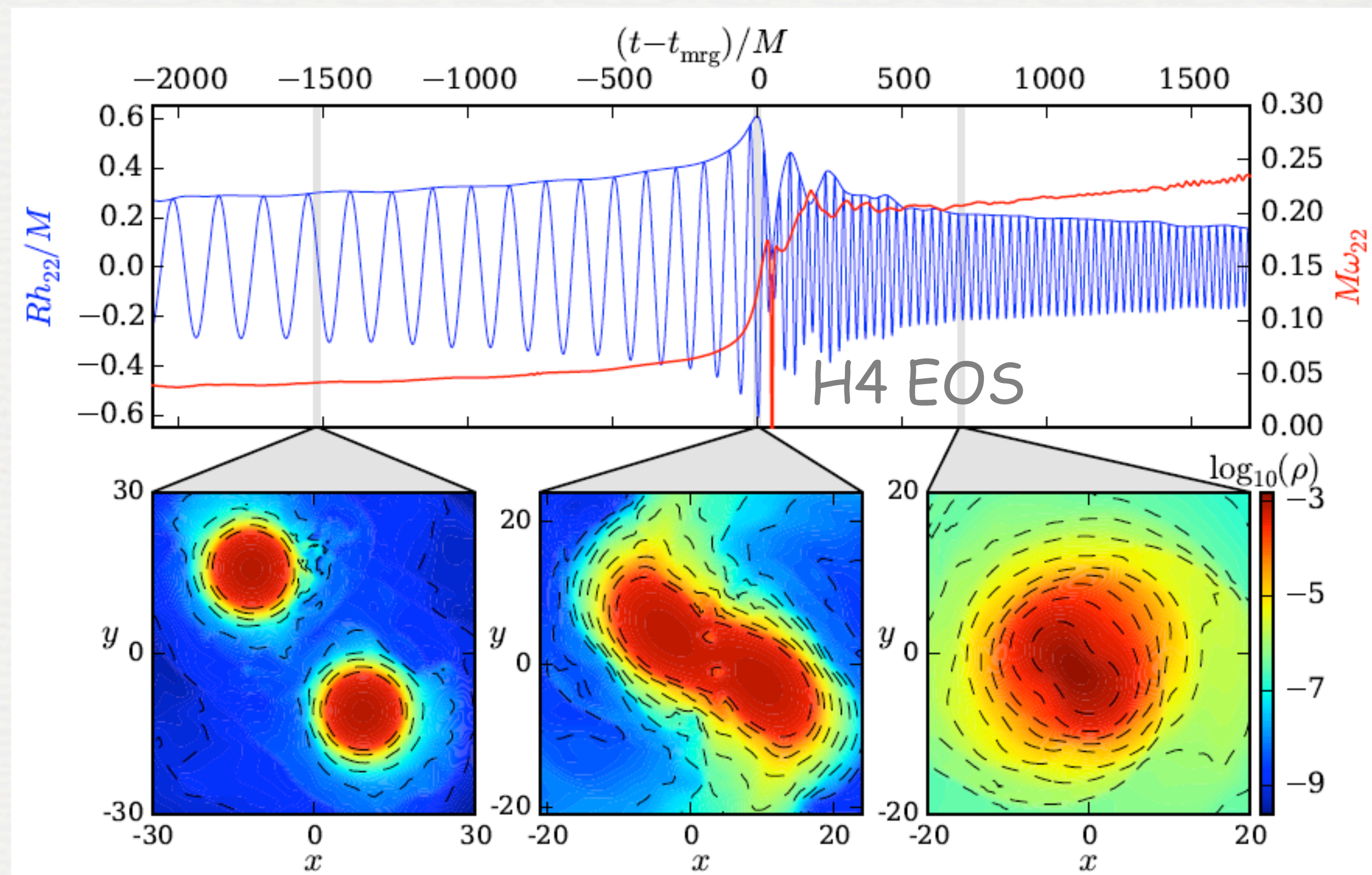


All BNS need is Love!

(also ECO need love...)

$$q = 1 \qquad M = 2.7 M_\odot$$

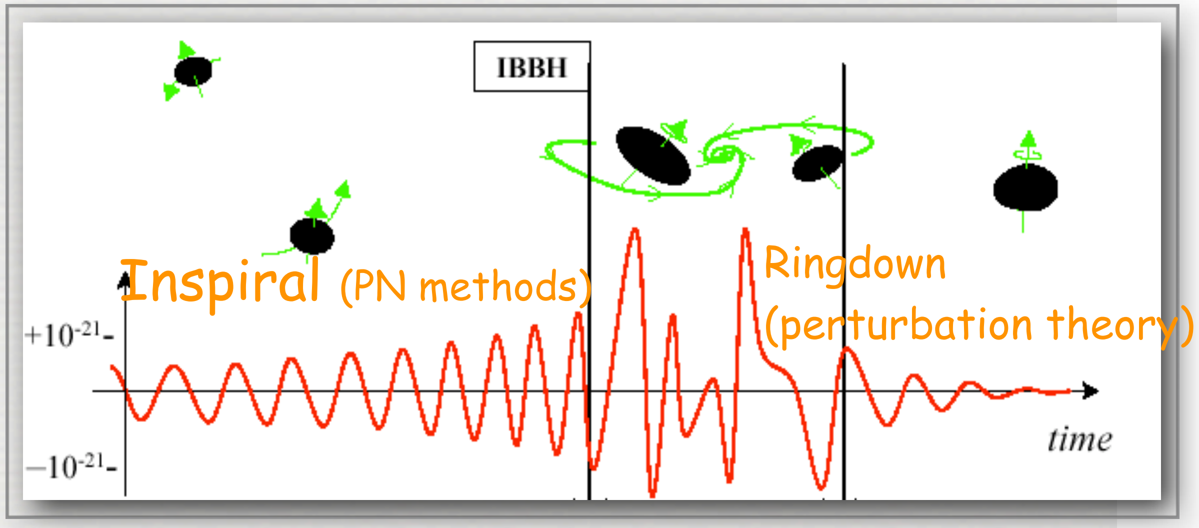
- Tidal effects
- Love numbers (tidal "polarization" constants)
- EOS dependence & "universality"



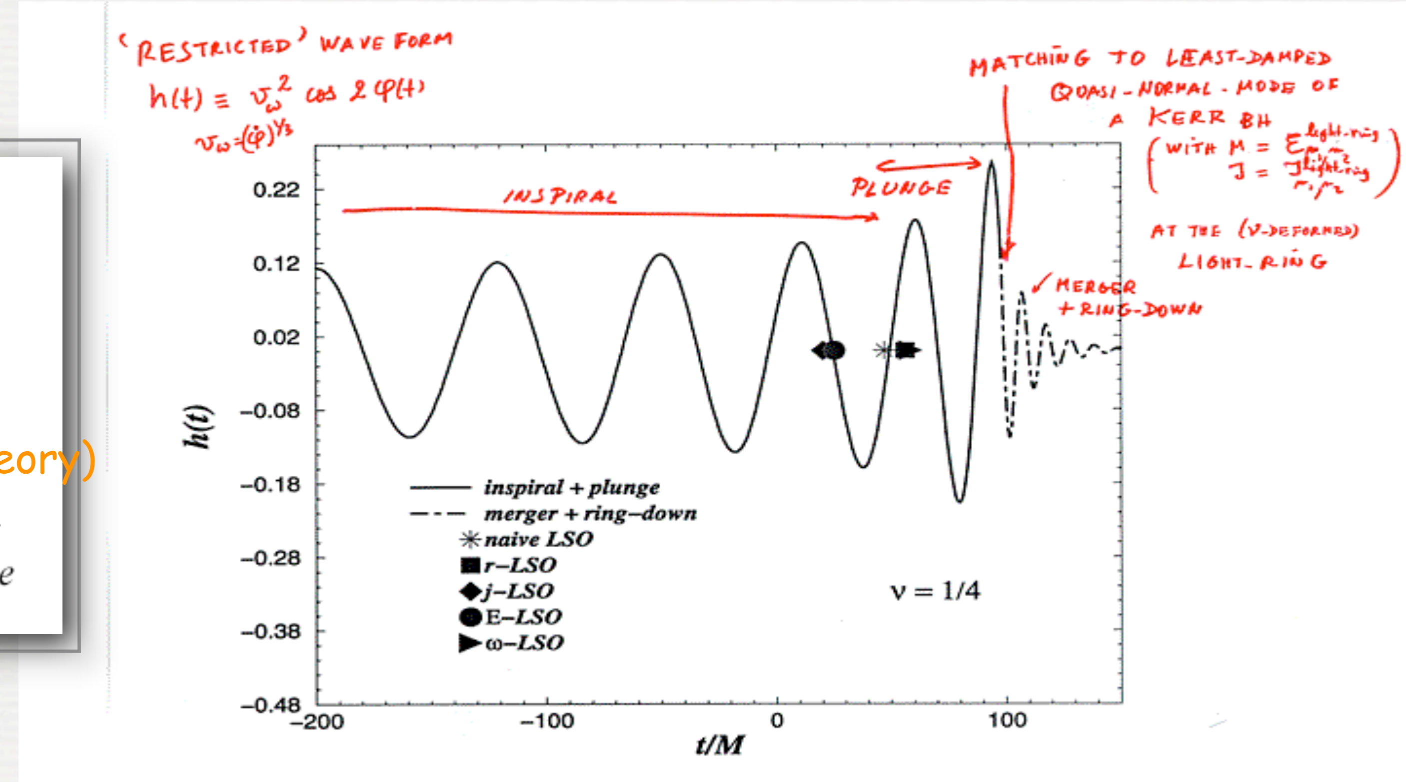
See:
Damour, 1983
Damour,Soffel,Xu, 1999-2001
Flanagan&Hinderer, PRD 2008
Damour&Nagar, PRD 2009
Damour&Nagar, PRD 2010
Damour,Nagar et al., PRL 2011
Bini,Damour&Faye, PRD2012
Bini&Damour, PRD 2014
Bernuzzi, Nagar, et al, PRL 2014
Bernuzzi, Nagar, Dietrich, PRL 2015
Bernuzzi, Nagar, Dietrich & Damour,PRL, 2015

TEMPLATES FOR GWS FROM BBH COALESCENCE

Brady, Craighton & Thorne, 1998



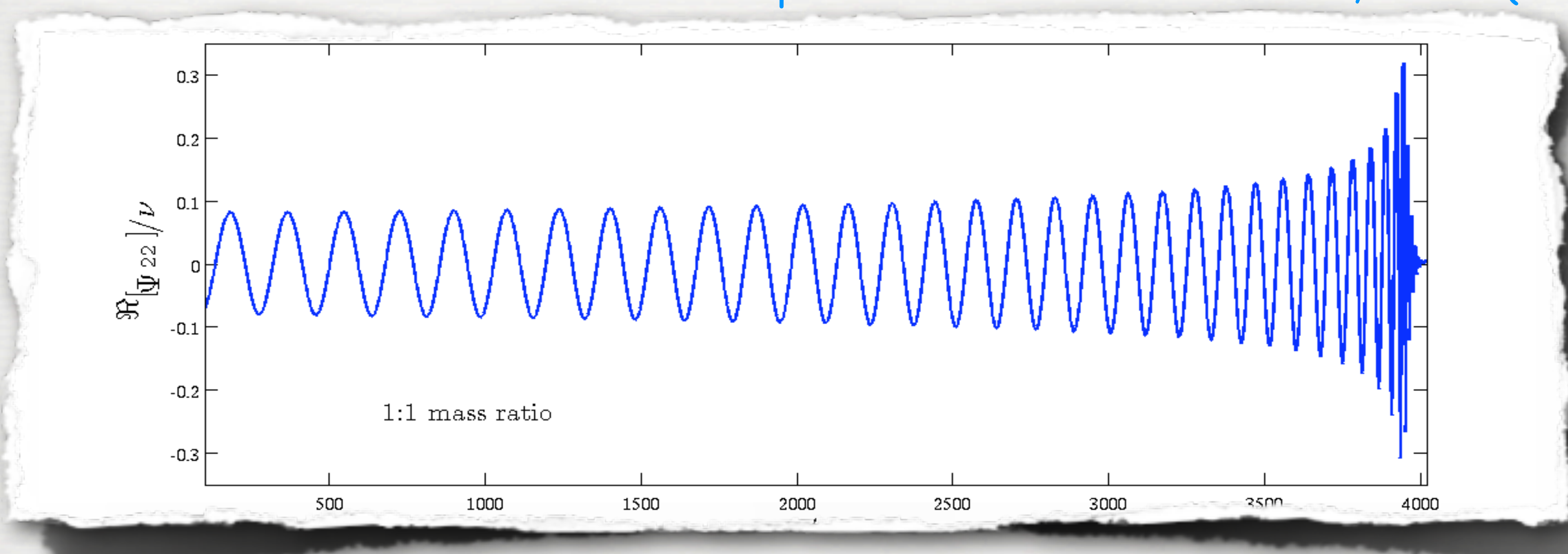
Merger:
Numerical Relativity



Effective-One-Body (Buonanno & Damour (2000))

Numerical Relativity: >= 2005 (F. Pretorius, Campanelli et al., Baker et al.)

Most accurate data: Caltech-Cornell spectral code: M. Scheel et al., 2008 (SXS collaboration)



Spectral code

Extrapolation (radius & resolution)

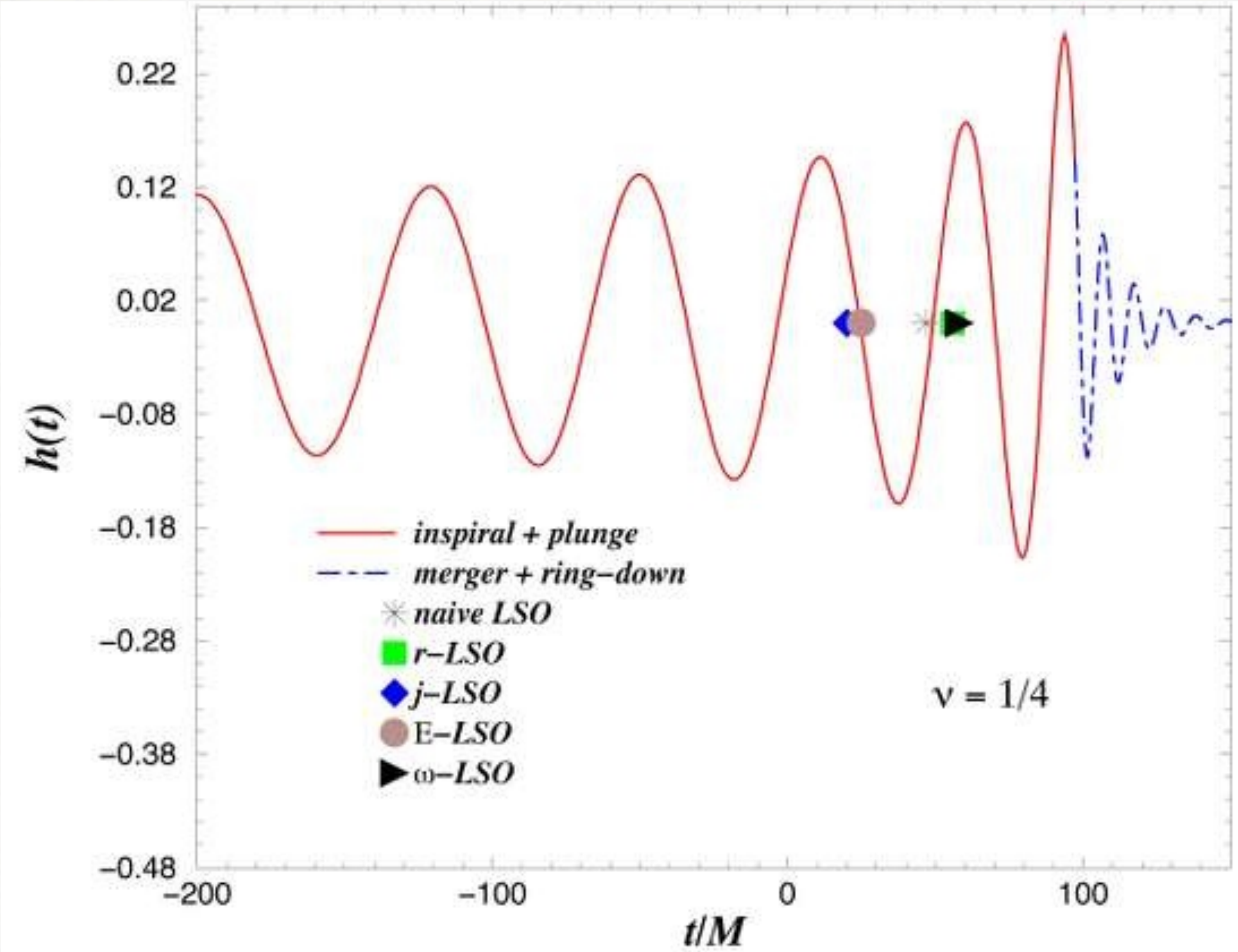
Phase error:

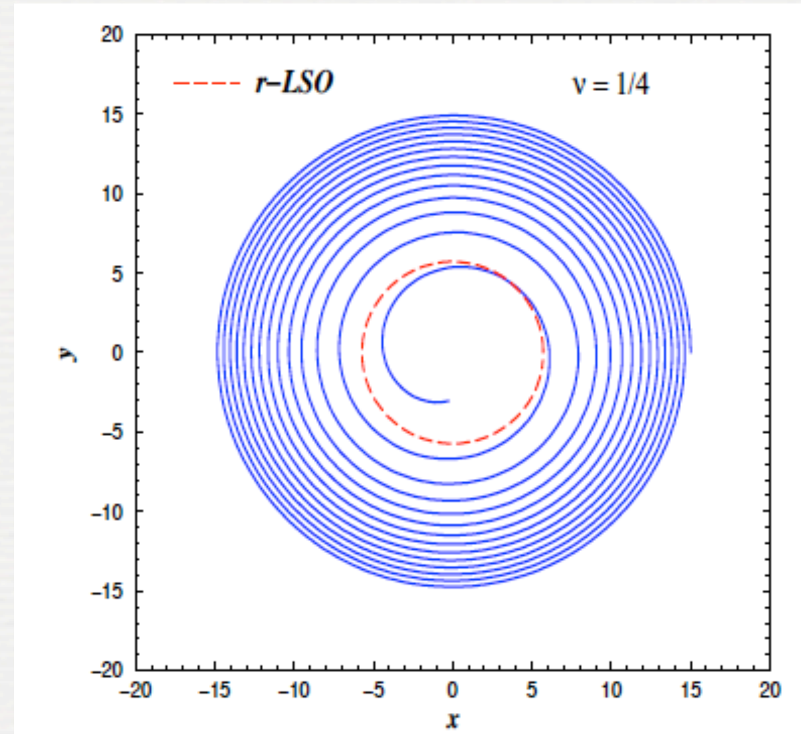
< 0.02 rad (inspiral)

<0.1 rad (ringdown)

EFFECTIVE ONE BODY (EOB): 2000

Numerical Relativity was not working (yet...)
EOB formalism was predictive, qualitatively and semi-quantitatively correct (10%)





A. Buonanno & T. Damour, PRD 59 (1999) 084006
A. Buonanno & T. Damour, PRD 62 (2000) 064015

- Blurred transition from inspiral to plunge
- Final black-hole mass
- Final black hole spin
- Complete waveform

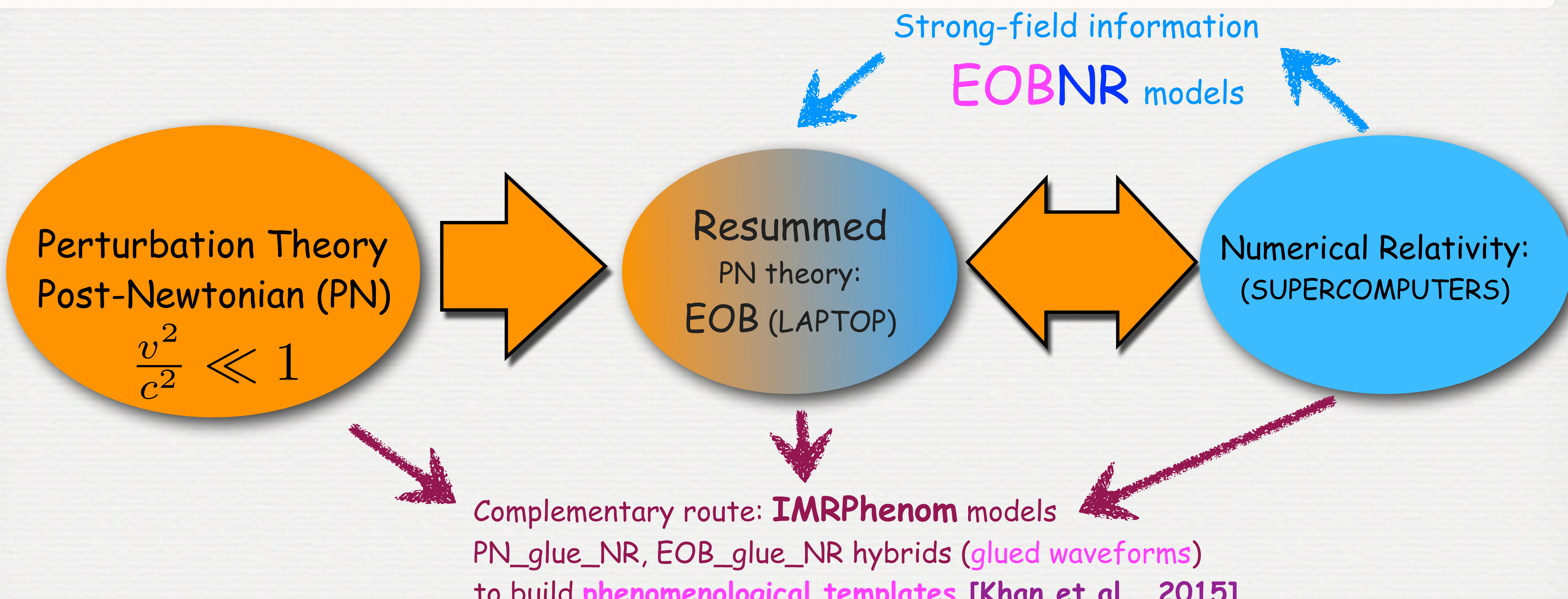
> 2005: Developing EOB & interfacing with NR
2 groups did (and do) it

- A.Buonanno et al. (AEI)
- T.Damour & AN + (>2005)

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2} = \frac{\mu}{M}$$

IMPORTANCE OF AN ANALYTICAL FORMALISM

- **Theoretical**: physical understanding of the coalescence process, especially in complicated situations (e.g., precessing spins).
- **Practical**: need many thousands of accurate GWs templates for detection and data analysis. Need analytical templates: $h\left(m_1, m_2, \vec{S}_1, \vec{S}_2\right)$
- **Solution**: synergy between analytical & numerical relativity



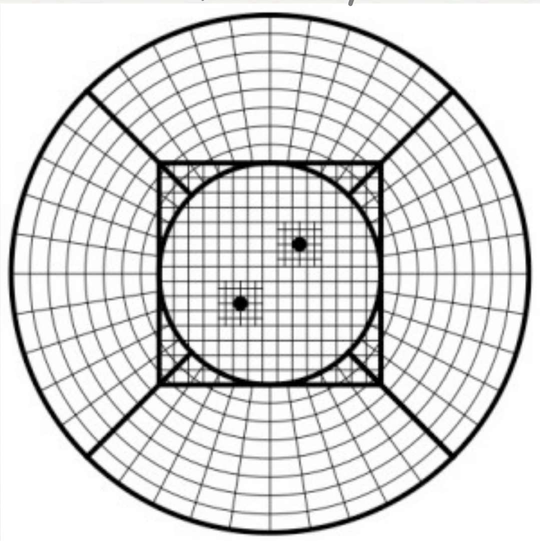
BBH & BNS COALESCENCE: NUMERICAL RELATIVITY

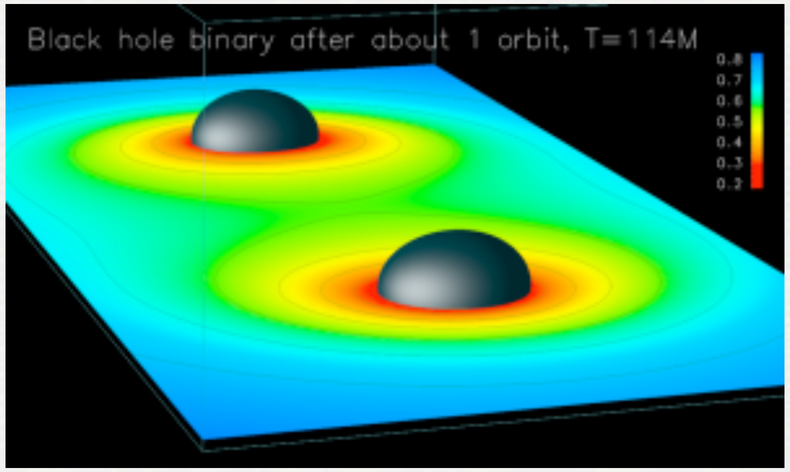
Numerical relativity is complicated & computationally expensive:

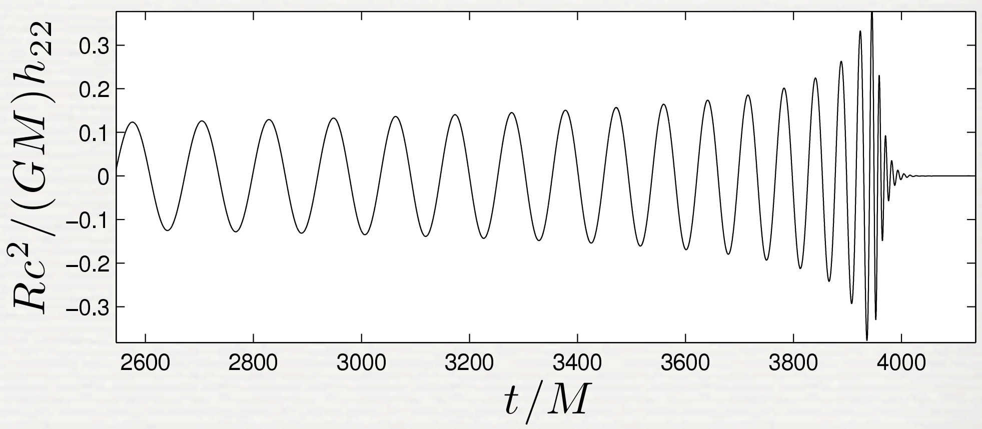
- Formulation of Einstein equations (BSSN, harmonic, Z4c,...)
- Setting up initial data (solution of the constraints)
- Gauge choice
- Numerical approach (finite-differencing (FD, e.g. Llama) vs spectral (SpEC,SXS))
- High-order FD operators
- Treatment of BH singularity (excision vs punctures)
- Wave extraction problem on finite-size grids (Cauchy-Characteristic vs extrapolation)
- Huge computational resources (mass-ratios 1:10; spin)
- Adaptive-mesh-refinement
- Error budget (convergence rates are far from clean...)
- For BNS: further complications due to GR-Hydrodynamics for matter
- Months of running/analysis to get one accurate waveform....

Multi-patch grid structure

(Llama FD code, Pollney & Reisswig)







A catalog of 171 high-quality binary black-hole simulations for gravitational-wave astronomy [PRL 111 (2013) 241104]

Abdul H. Mroué,[1] Mark A. Scheel,[2] Béla Szilágyi,[2] Harald P. Pfeiffer,[1] Michael Boyle,[3] Daniel A. Hemberger,[3] Lawrence E. Kidder,[3] Geoffrey Lovelace,[4,2] Sergei Ossokine,[1,5] Nicholas W. Taylor,[2] Anıl Zenginoğlu,[2] Luisa T. Buchman,[2] Tony Chu,[1] Evan Foley,[4] Matthew Giesler,[4] Robert Owen,[6] and Saul A. Teukolsky[3]

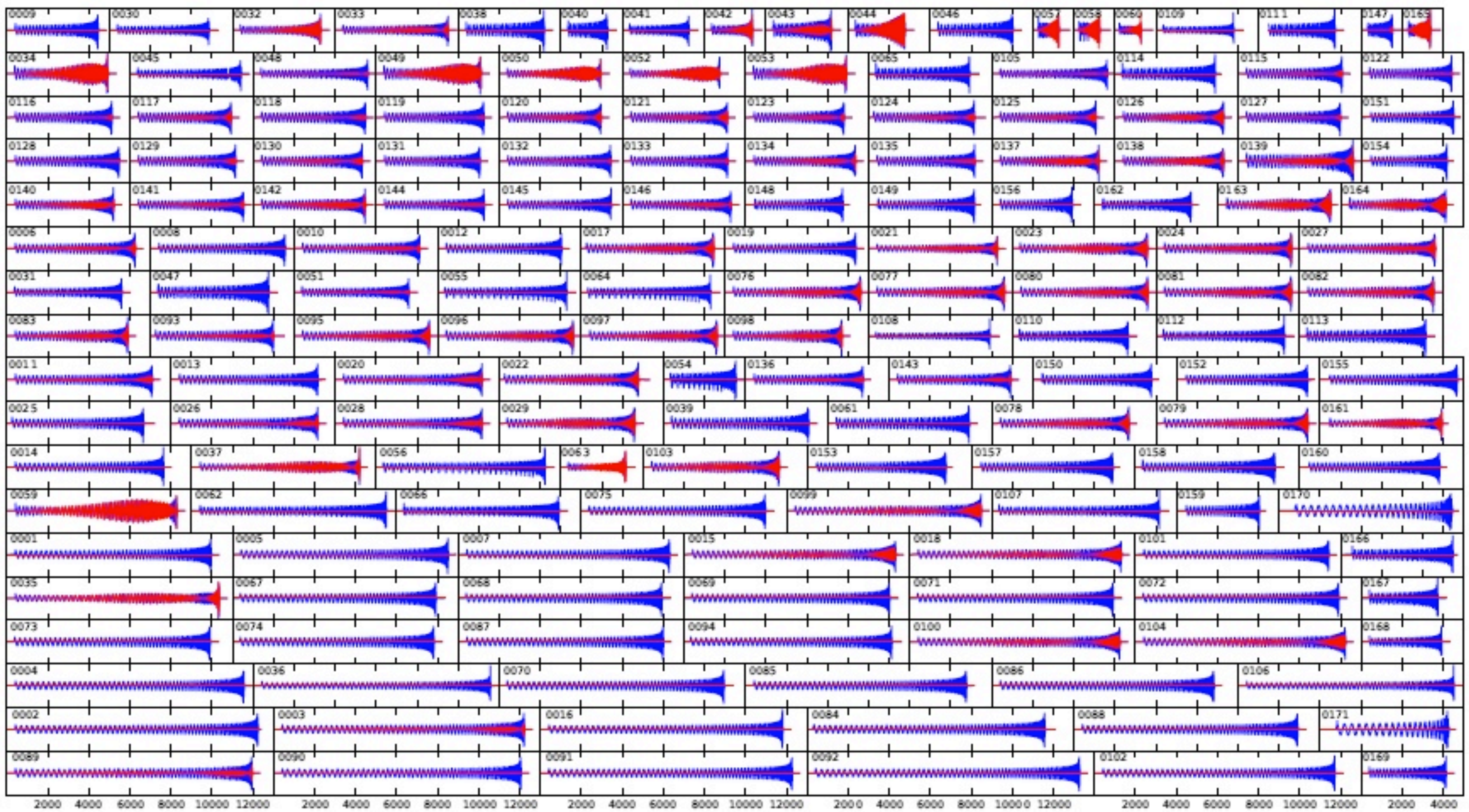


FIG. 3: Waveforms from all simulations in the catalog. Shown here are h_+ (blue) and $h_\times$ (red) in a sky direction parallel to the initial orbital plane of each simulation. All plots have the same horizontal scale, with each tick representing a time interval of $2000M$, where M is the total mass.

• www.blackholes.org

But (more than) 250.000 templates were used...

ANALYTICALLY: MOTION AND GW IN GR

Hamiltonian: conservative part of the dynamics

Radiation reaction: mechanical energy/angular momentum goes away in GWs and backreacts on the system.

The (closed) orbit CIRCULARIZES and SHRiNks with time

Waveform

General Relativity is NONLINEAR!

Post-Newtonian (PN) approximation: expansion in $\dfrac{v^2}{c^2}$

PROBLEM OF MOTION IN GENERAL RELATIVITY

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)\ ,\ \ h_{\mu\nu} \ll 1$$

$$h_{00} \sim h_{ij} \sim \frac{v^2}{c^2}\ ,\ \ h_{0i} \sim \frac{v^3}{c^3}\ ,\ \ \partial_0 h \sim \frac{v}{c}\partial_i h$$

Approximation methods

- post-Minkowskian (Einstein 1916)
- post-Newtonian (Droste 1916)
- Matching of asymptotic expansions: body zone/near zone/wave zone
- Numerical Relativity

One-chart versus Multi-chart approaches

Coupling between Einstein field equations and equations of motion

Strongly self-gravitating bodies: neutron stars or black holes

$$h_{\mu\nu}(x) \sim 1$$

Skeletonized: $T_{\mu\nu}$ point-masses ? delta-functions in GR

Multipolar Expansion

Need to go to very high-orders of approximation

QFT-like calculations

Use a "cocktail": PM, PN, MPM, MAE, EFT,an. reg., dim. reg.,...

POST-NEWTONIAN HAMILTONIAN (C.O.M)

$$\hat{H}^{\rm NR}_{\rm real}(\mathbf{q},\mathbf{p}) = \hat{H}_{\rm N}(\mathbf{q},\mathbf{p}) + \hat{H}_{\rm 1PN}(\mathbf{q},\mathbf{p}) + \hat{H}_{\rm 2PN}(\mathbf{q},\mathbf{p}) + \hat{H}_{\rm 3PN}(\mathbf{q},\mathbf{p}), \qquad (4.27)$$

where

$$\hat{H}_{\rm N}(\mathbf{q},\mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{q}, \qquad (4.28a)$$

Newton (0PN)

$$\hat{H}_{\rm 1PN}(\mathbf{q},\mathbf{p}) = \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\left[(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\right]\frac{1}{q} + \frac{1}{2q^2}, \qquad (4.28b)$$

(1PN, 1938) - [Einstein-Infeld-Hoffman]

$$\begin{aligned}\hat{H}_{\rm 2PN}(\mathbf{q},\mathbf{p}) ={}& \frac{1}{16}\left(1-5\nu+5\nu^2\right)(\mathbf{p}^2)^3 + \frac{1}{8}\left[\left(5-20\nu-3\nu^2\right)(\mathbf{p}^2)^2 - 2\nu^2(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - 3\nu^2(\mathbf{n}\cdot\mathbf{p})^4\right]\frac{1}{q} \\ &+ \frac{1}{2}\left[(5+8\nu)\mathbf{p}^2 + 3\nu(\mathbf{n}\cdot\mathbf{p})^2\right]\frac{1}{q^2} - \frac{1}{4}(1+3\nu)\frac{1}{q^3}, \qquad (4.28c)\end{aligned}$$

(2PN, 1982/83) - [Damour-Deruelle]

$$\begin{aligned}\hat{H}_{\rm 3PN}(\mathbf{q},\mathbf{p}) ={}& \frac{1}{128}\left(-5+35\nu-70\nu^2+35\nu^3\right)(\mathbf{p}^2)^4 \\ &+ \frac{1}{16}\left[\left(-7+42\nu-53\nu^2-5\nu^3\right)(\mathbf{p}^2)^3 + (2-3\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + 3(1-\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - 5\nu^3(\mathbf{n}\cdot\mathbf{p})^6\right]\frac{1}{q} \\ &+ \left[\frac{1}{16}\left(-27+136\nu+109\nu^2\right)(\mathbf{p}^2)^2 + \frac{1}{16}(17+30\nu)\nu(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \frac{1}{12}(5+43\nu)\nu(\mathbf{n}\cdot\mathbf{p})^4\right]\frac{1}{q^2} \\ &+ \left\{\left[-\frac{25}{8} + \left(\frac{1}{64}\pi^2 - \frac{335}{48}\right)\nu - \frac{23}{8}\nu^2\right]\mathbf{p}^2 + \left(-\frac{85}{16} - \frac{3}{64}\pi^2 - \frac{7}{4}\nu\right)\nu(\mathbf{n}\cdot\mathbf{p})^2\right\}\frac{1}{q^3} \\ &+ \left[\frac{1}{8} + \left(\frac{109}{12} - \frac{21}{32}\pi^2 + \omega_{\rm static}\right)\nu\right]\frac{1}{q^4}. \qquad (4.28d)\end{aligned}$$

(3PN, 2000) - [Damour, Jaranowski,Schaefer]

...and 4PN too, [Damour, Jaranowski&Schaefer 2014/2015] - 4 loop calculation

$$\mathbf{q} = \mathbf{q}_1 - \mathbf{q}_2$$

$$\mathbf{p} = \mathbf{p}_1 = -\mathbf{p}_2$$

FLUX & WAVEFORM (3.5PN)

$$\frac{dE}{dt} = -\mathcal{L}$$

balance equation

Mechanical loss   GW luminosity

$$\mathcal{L} = \underline{\frac{32c^5}{5G}\nu^2 x^5}\left\{1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right)x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right)x^2 + \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2} + \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16\,x) + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right]x^3 + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\right\}.$$

Newtonian quadrupole

$$h^{22} = -8\sqrt{\frac{\pi}{5}}\frac{G\nu m}{c^2 R}e^{-2i\phi}x\Big[1 - x\Big(\frac{107}{42} - \frac{55}{42}\nu\Big) + x^{3/2}\Big[2\pi + 6i\ln\Big(\frac{x}{x_0}\Big)\Big] - x^2\Big(\frac{2173}{1512} + \frac{1069}{216}\nu - \frac{2047}{1512}\nu^2\Big) - x^{5/2}\Big[\Big(\frac{107}{21} - \frac{34}{21}\nu\Big)\pi + 24i\nu + \Big(\frac{107i}{7} - \frac{34i}{7}\nu\Big)\ln\Big(\frac{x}{x_0}\Big)\Big] + x^3\Big[\frac{27\,027\,409}{646\,800} - \frac{856}{105}\gamma_E + \frac{2}{3}\pi^2 - \frac{1712}{105}\ln 2 - \frac{428}{105}\ln x - 18\Big[\ln\Big(\frac{x}{x_0}\Big)\Big]^2 - \Big(\frac{278\,185}{33\,264} - \frac{41}{96}\pi^2\Big)\nu - \frac{20\,261}{2772}\nu^2 + \frac{114\,635}{99\,792}\nu^3 + \frac{428i}{105}\pi + 12i\pi\ln\Big(\frac{x}{x_0}\Big)\Big] + O(\epsilon^{7/2})\Big\},$$

$C = \gamma_E = 0.5772156649...$

EOBNR

$x = (M\Omega)^{2/3} \sim v^2/c^2$

EFFECTIVE-ONE-BODY (EOB)

approach to the general relativistic two-body problem

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

(1) Replace two-body dynamics (m_1, m_2) by dynamics of a particle $(\mu \equiv m_1 m_2/(m_1 + m_2))$ in an effective metric $g_{\mu\nu}^{\rm eff}(u)$, with

$$u \equiv GM/c^2 R\,, \quad M \equiv m_1 + m_2$$

(2) Systematically use RESUMMATION of PN expressions (both $g_{\mu\nu}^{\rm eff}$ and $\mathcal{F}_{RR}$) based on various physical requirements

(3) Require continuous deformation w.r.t. $\nu \equiv \mu/M \equiv m_1 m_2/(m_1 + m_2)^2$ in the interval $0 \leq \nu \leq \frac{1}{4}$

STRUCTURE OF THE EOB FORMALISM

PN dynamics
(DD81,D82,DJS01,IF03,BDIF04)

PN rad losses
WW76, BDIWW95, BDEFI 05

PN waveform
BD89, B95&05,ABIQ04,

BH perturbations
RW57, Z70, Z72

Resummed (BD99)

Resummed (DIS98)

Resummed (DN07,DIN08)

QNMs spectrum
$\sigma_N = \alpha_N + i\omega_N$

EOB Hamiltonian
$H_{\rm EOB}$

EOB Rad. Reac. force
$\hat{\mathcal{F}}_\varphi$

Factorized waveform

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)} \hat{h}_{\ell m}^{(\epsilon)}$$

$$\hat{h}_{\ell m}^{(\epsilon)} = \hat{S}_{eff}^{(\epsilon)} T_{\ell m} e^{i\delta_{\ell m}} \rho_{\ell m}^{\ell}$$

Matching at merger time

$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}},$$

$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r},$$

$$\Omega \equiv \frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi},$$

$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi .$$

BNS: tides
(Love numbers)

$$h_{\ell m}^{\rm ringdown}(t) = \sum_N C_N^+ e^{-\sigma_N^+(t-t_m)}$$

EOB waveform

$$h_{\ell m}^{\rm EOB} = \theta(t_m - t) h_{\ell m}^{\rm insplunge}(t) + \theta(t - t_m) h_{\ell m}^{\rm ringdown}(t)$$

EXPLICIT FORM OF THE EOB HAMILTONIAN

EOB Hamiltonian

$$H_{\rm EOB} = M\sqrt{1+2\nu\left(\hat{H}_{\rm eff}-1\right)}$$

All Functions are a ν-dependent deformation of the Schwarzschild ones

$$\underline{A(r) = 1 - 2u + 2\nu u^3 + a_4 \nu u^4}$$

$$a_4 = \frac{94}{3} - \frac{41}{32}\pi^2 \simeq 18.6879027$$

$$A(r)B(r) = 1 - 6\nu u^2 + 2(3\nu - 26)\nu u^3$$

$$u = GM/(c^2 R)$$

Simple effective Hamiltonian:

$$\hat{H}_{\rm eff} \equiv \sqrt{p_{r_*}^2 + A(r)\left(1 + \frac{p_\varphi^2}{r^2} + z_3\frac{p_{r_*}^4}{r^2}\right)}$$

$$p_{r_*} = \left(\frac{A}{B}\right)^{1/2} p_r$$

Crucial EOB radial potential

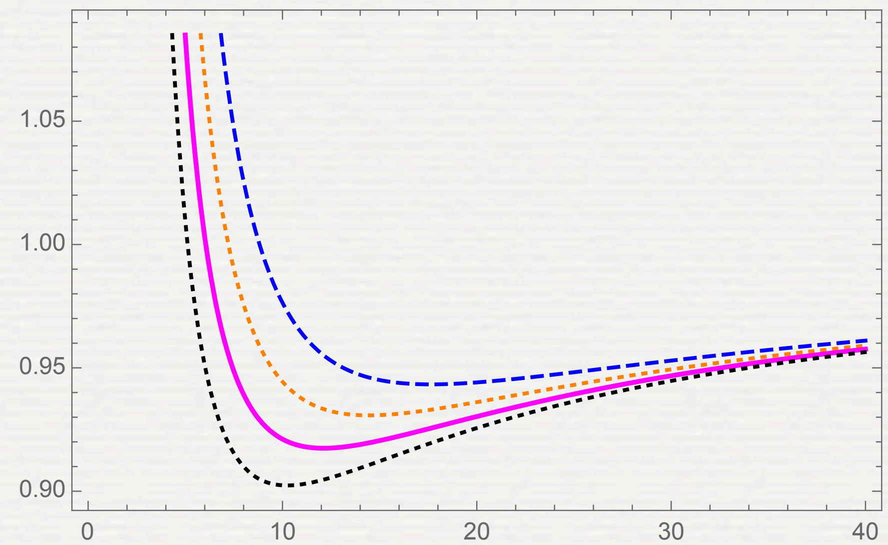
Contribution at 3PN

A. Nagar - 18 May 2016 - GGI

EFFECTIVE POTENTIALS

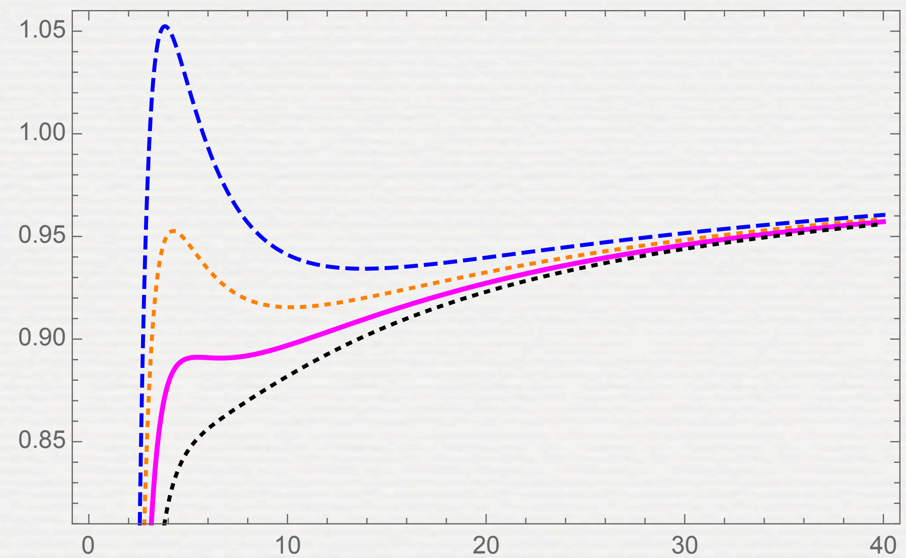
Newtonian gravity (any mass ratio):
circular orbits are always stable. No plunge.

$$W^{\rm eff}_{\rm Newt} = 1 - \frac{2}{r} + \frac{p_\varphi^2}{r^2}$$



Test-body on Schwarzschild black hole:
last stable orbit (LSO) at r=6M; plunge

$$W^{\rm eff}_{\rm Schwarzschild} = \left(1 - \frac{2}{r}\right)\left(1 + \frac{p_\varphi^2}{r^2}\right)$$



EOB, Black-hole binary, any mass ratio:
last stable orbit (LSO) at r<6M plunge

$$W^{\rm eff}_{\rm EOB} = A(r;\nu)\left(1 + \frac{p_\varphi^2}{r^2}\right)$$

ν -deformation of the Schwarzschild case!

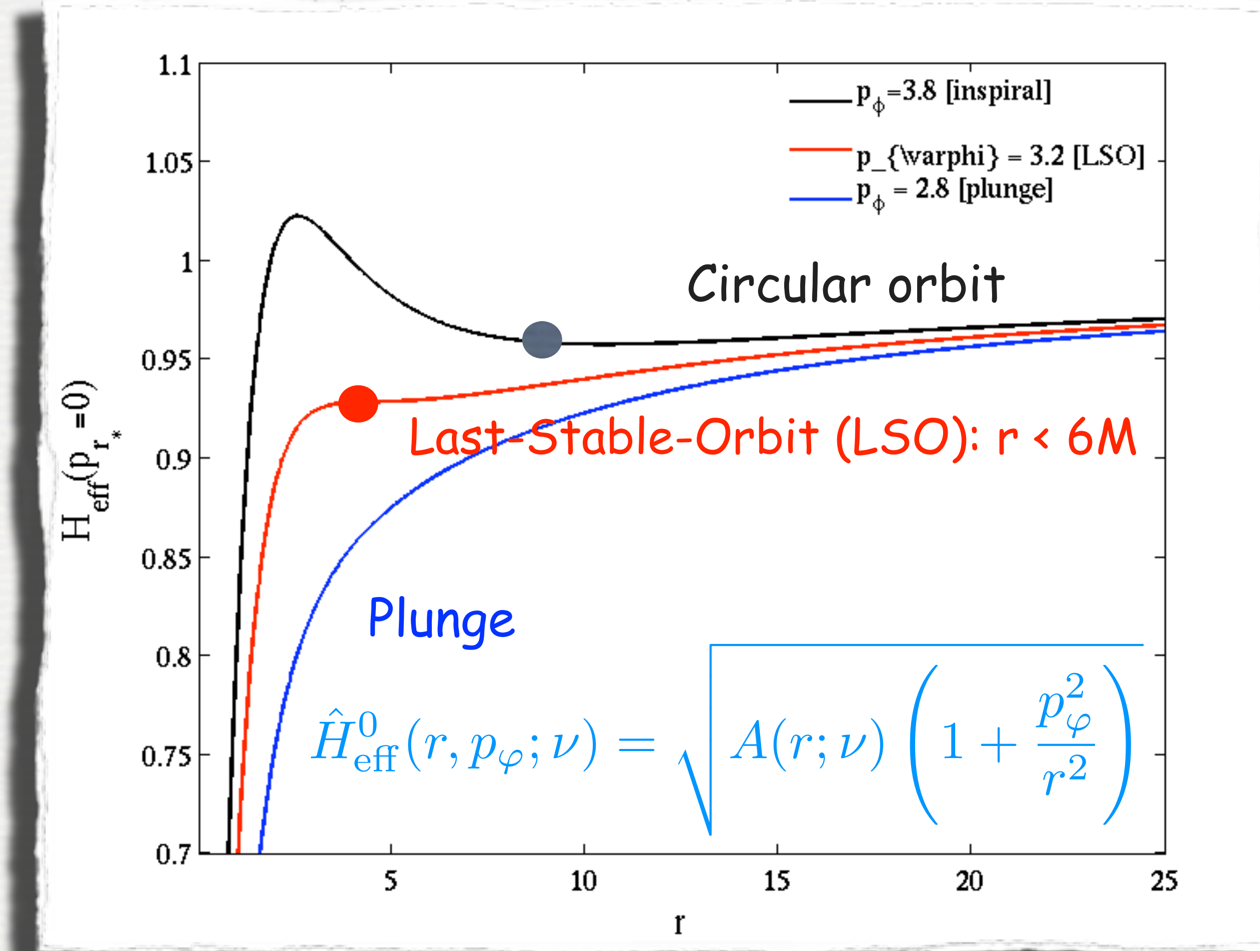
HAMILTON'S EQUATIONS & RADIATION REACTION

$$\dot{r} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}}$$

$$\dot{\varphi} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi} \equiv \Omega$$

$$\dot{p}_{r_*} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r} + \hat{\mathcal{F}}_{r_*}$$

$$\dot{p}_\varphi = \hat{\mathcal{F}}_\varphi$$



- The system must radiate angular momentum
- How?Use PN-based (Taylor-expanded) radiation reaction force (ang-mom flux)
- Need flux resummation

$$\hat{\mathcal{F}}_\varphi^{\rm Taylor} = -\frac{32}{5}\nu\Omega^5 r_\Omega^4 \hat{F}^{\rm Taylor}(v_\varphi)$$

Resummation multipole by multipole
(Damour&Nagar 2007,
Damour, Iyer & Nagar 2008,
Damour & Nagar, 2009)

Plus horizon contribution [AN&Akcay2012]

MULTIPOLAR WAVEFORM RESUMMATION

Resummation of the waveform (and flux) multipole by multipole (CRUCIAL!)

[Damour&Nagar 2007, Damour, Iyer, Nagar 2008, Pan et al. 2011 (spin)]

Next-to-quasi-circular correction

$$h_{\ell m} \equiv h^{(N,\epsilon)}_{\ell m} \hat{h}^{(\epsilon)}_{\ell m} \hat{h}^{\rm NQC}_{\ell m}$$

Newtonian x PN x NQC

PN-correction

$$\hat{h}^{(\epsilon)}_{\ell m} = \hat{S}^{(\epsilon)}_{\rm eff} T_{\ell m} e^{i\delta_{\ell m}} \rho^{\ell}_{\ell m}$$

Remnant phase and modulus corrections: "improved" PN series

The "Tail factor"

$$T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{\hat{k}})}{\Gamma(\ell+1)} e^{\pi \hat{k}} e^{2i\hat{\hat{k}} \ln(2kr_0)}$$

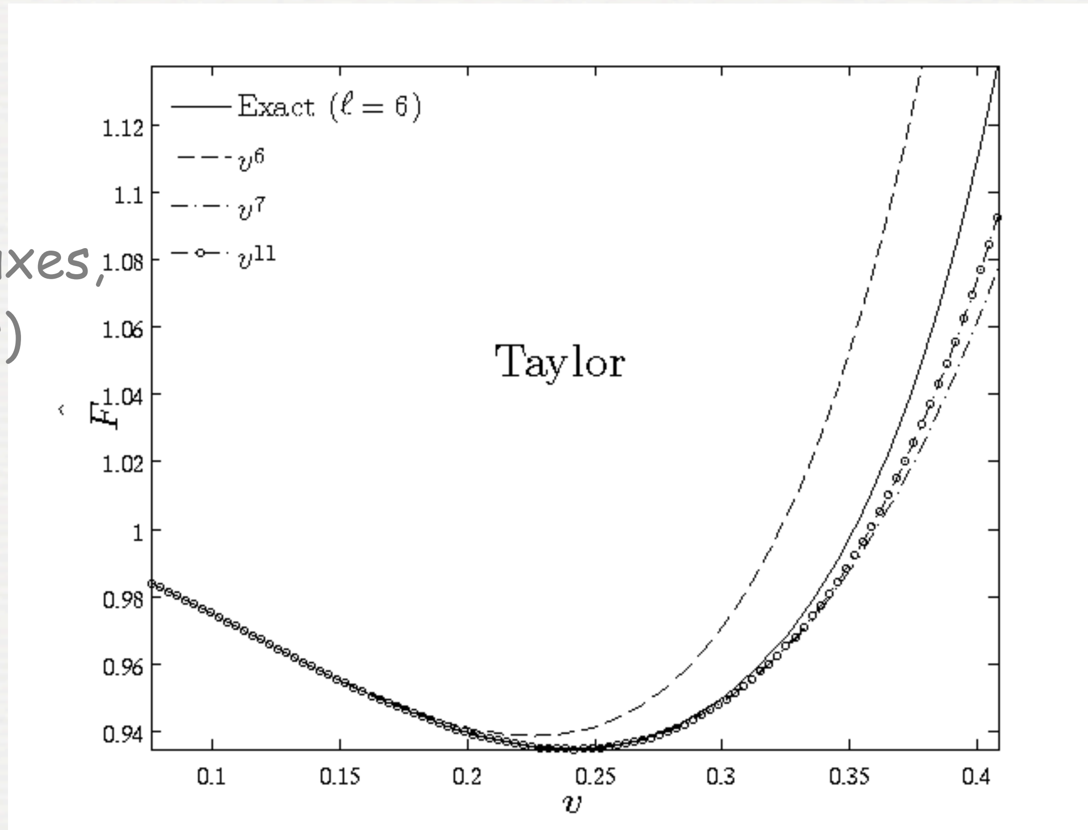
Resums an infinite number of leading logarithms in tail effects (hereditary contributions)

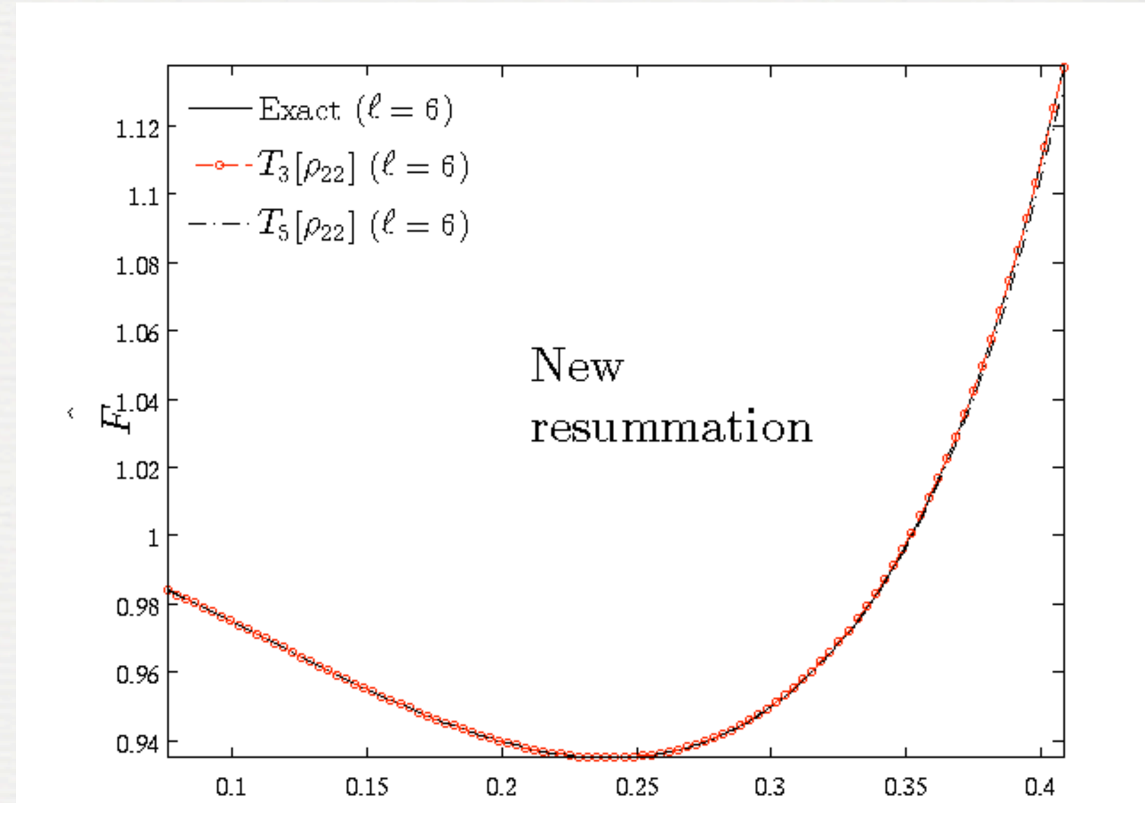
Effective source:
EOB (effective) energy (even-parity modes)
EOB angular momentum (odd-parity modes)

EFFECTIVENESS OF FLUX RESUMMATION

Test-mass

(Comparing fluxes, circular orbits)

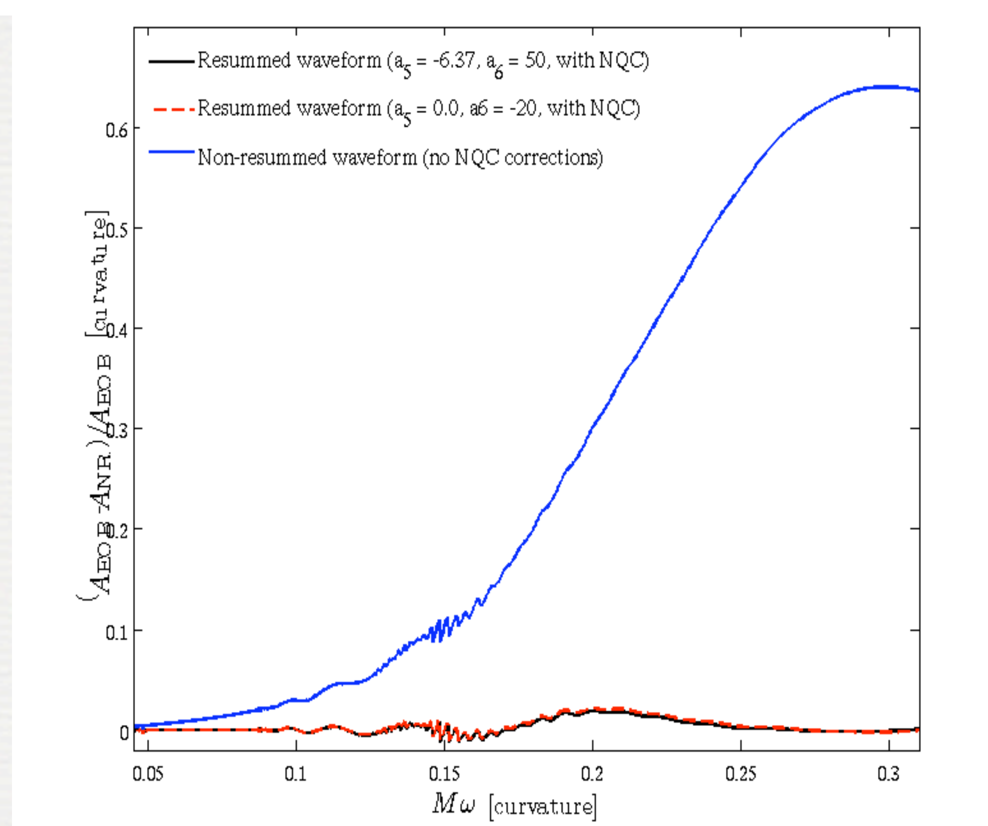




Equal-mass

(Comparing non-resummed & EOB-resummed amplitudes to Caltech-Cornell BBH data)

$$\mathcal{F}_\varphi \equiv -\frac{1}{8\pi\Omega}\sum_{\ell=2}^{\ell_{\max}}\sum_{m=1}^{\ell}(m\Omega)^2|Rh^{(\epsilon)}_{\ell m}|^2$$



4PN analytically complete + 5PN logarithmic term in the A(u) function:

[Damour 2009, Blanchet et al. 2010, Barack, Damour & Sago 2010, Le Tiec et al. 2011, Barausse et al. 2011,Akcay et al. 2012, Bini& Damour2013, DamourJaranowski&Schaefer 2014].

$$A_{5\mathrm{PN}}^{\mathrm{Taylor}} = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 + \nu[a_5^c(\nu) + a_5^{\ln}\ln u]u^5 + \nu[a_6^c(\nu) + a_6^{\ln}\ln u]u^6$$

1PN 2PN 3PN 4PN 5PN

$$\left.\begin{aligned}
a_5^{\log} &= \frac{64}{5} \\
a_5^c &= a_{5_0}^c + \nu a_{5_1}^c \\
a_{5_0}^c &= -\frac{4237}{60} + \frac{2275}{512}\pi^2 + \frac{256}{5}\log(2) + \frac{128}{5}\gamma \\
a_{5_1}^c &= -\frac{221}{6} + \frac{41}{32}\pi^2
\end{aligned}\right\}$$

4PN fully known ANALYTICALLY!

$$a_6^{\log} = -\frac{7004}{105} - \frac{144}{5}\nu$$

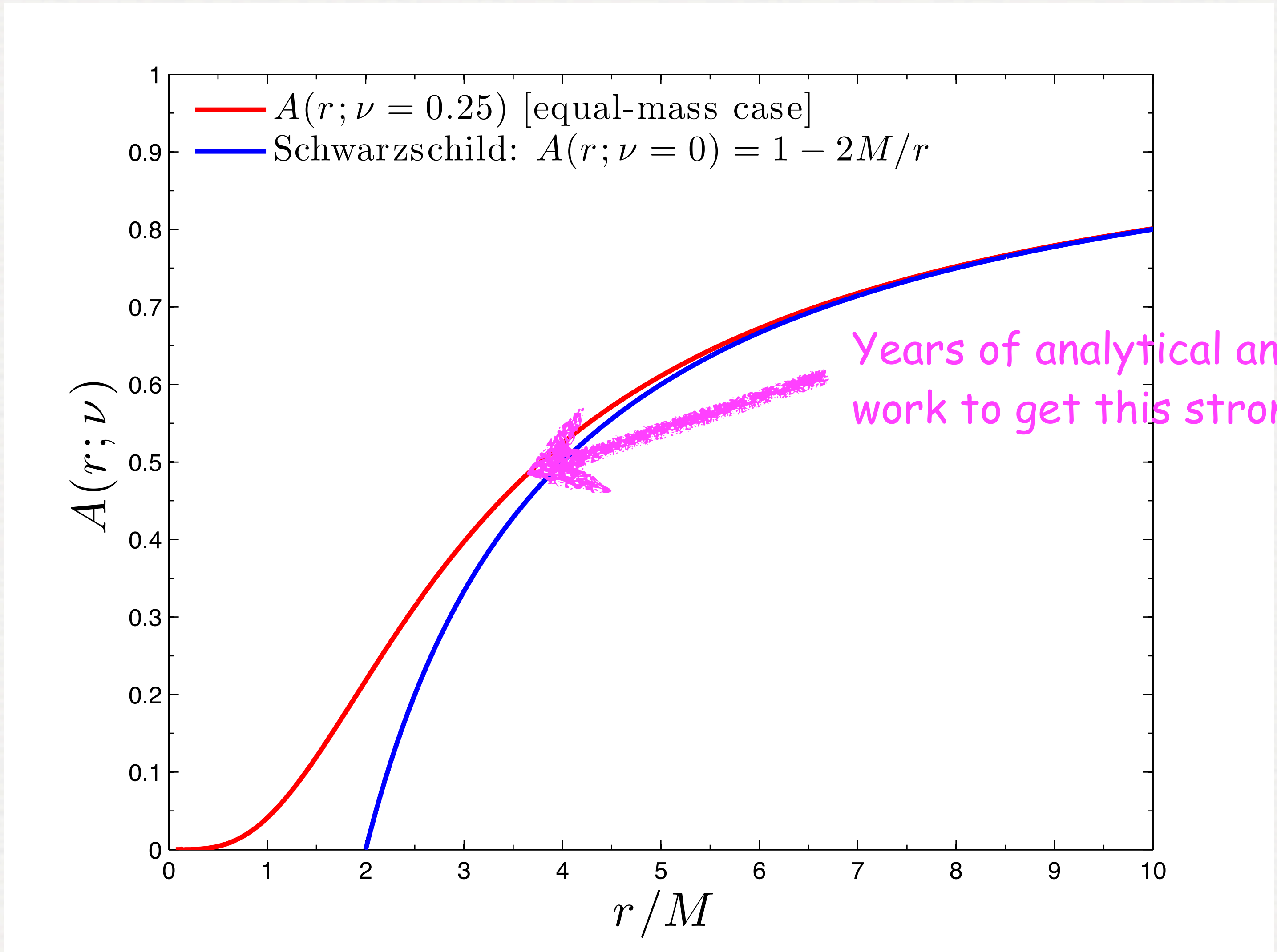
5PN logarithmic term (analytically known)

NEED ONE "effective" 5PN parameter from NR waveform data: $a_6^c(\nu)$

State-of-the-art EOB potential (5PN-resummed):

$$A(u;\nu,a_6^c) = P_5^1[A_{5\mathrm{PN}}^{\mathrm{Taylor}}(u;\nu,a_6^c)]$$

THE EOB[NR] POTENTIAL



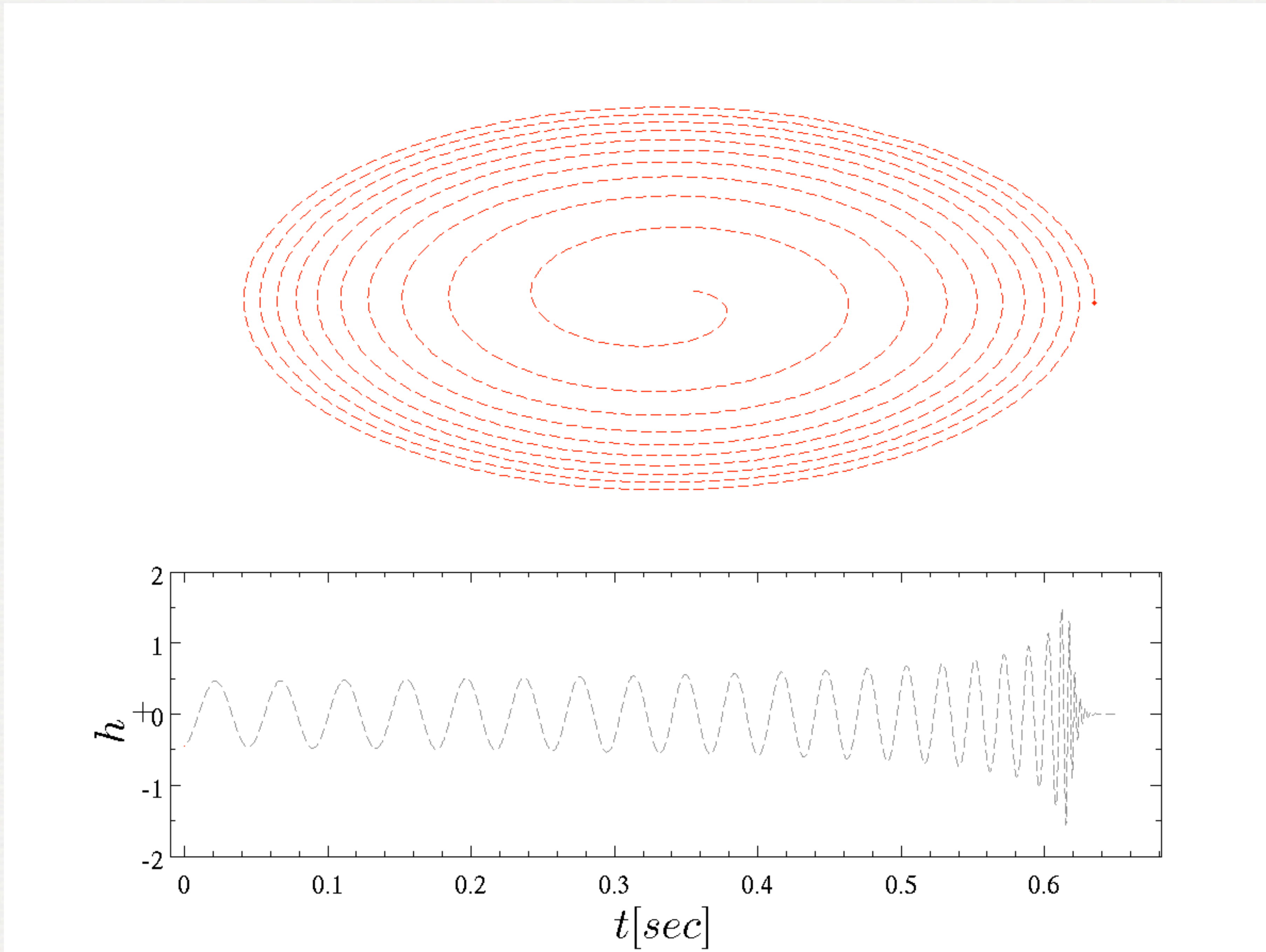
From EOB/NR-fitting: $a_6^c(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.3804$

TAKE AWAY:

system is more bound, smaller "separation" and higher frequencies!

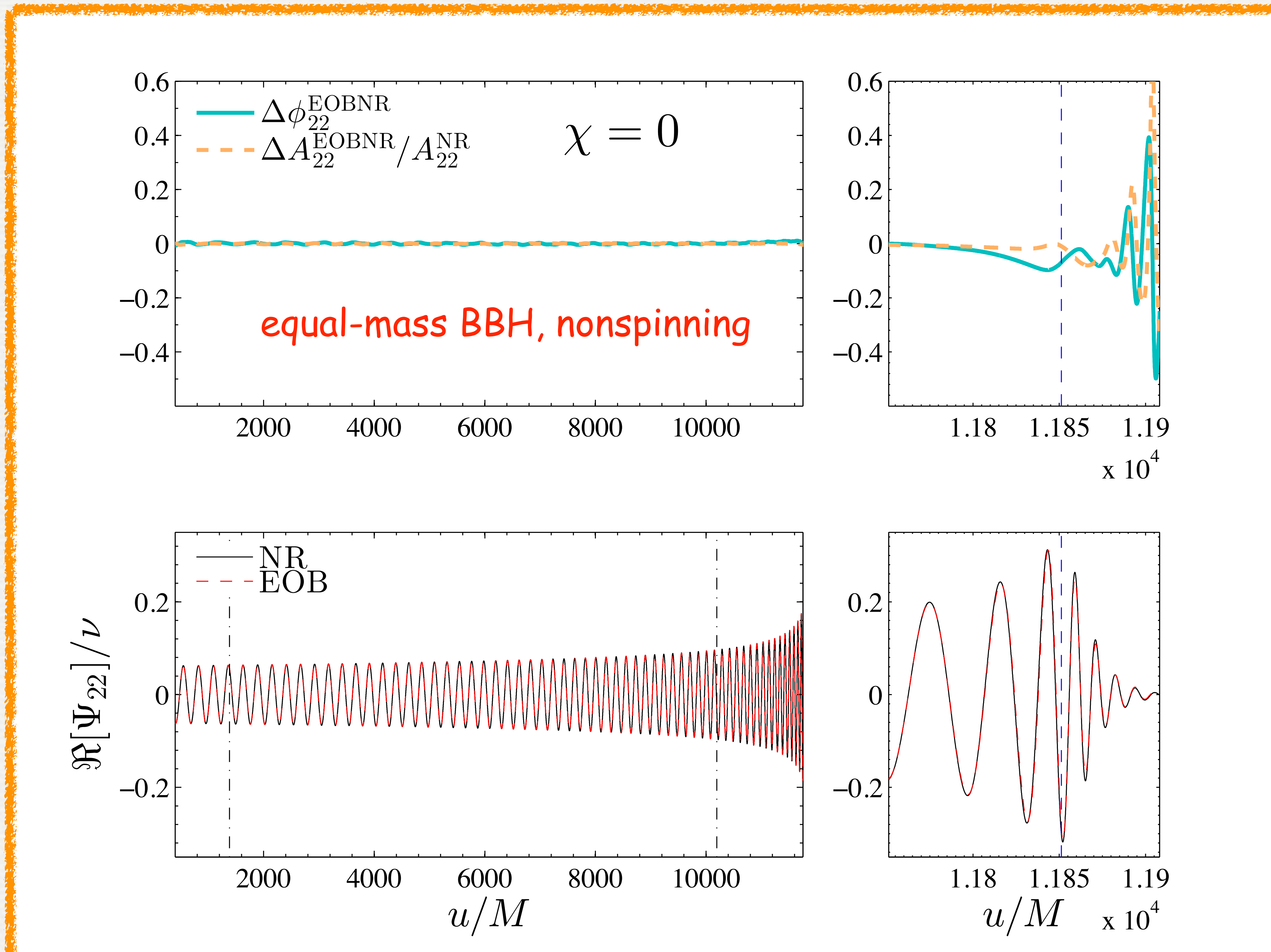
A. Nagar - 18 May 2016 - GGI

NDRP, arXiv:1506.08457



2
1
0
-1
-2
h_+
0
0.1
0.2
0.3
0.4
0.5
0.6
$t[sec]$

RESULTS: EOBNR/NR WAVEFORMS (NO SPIN)

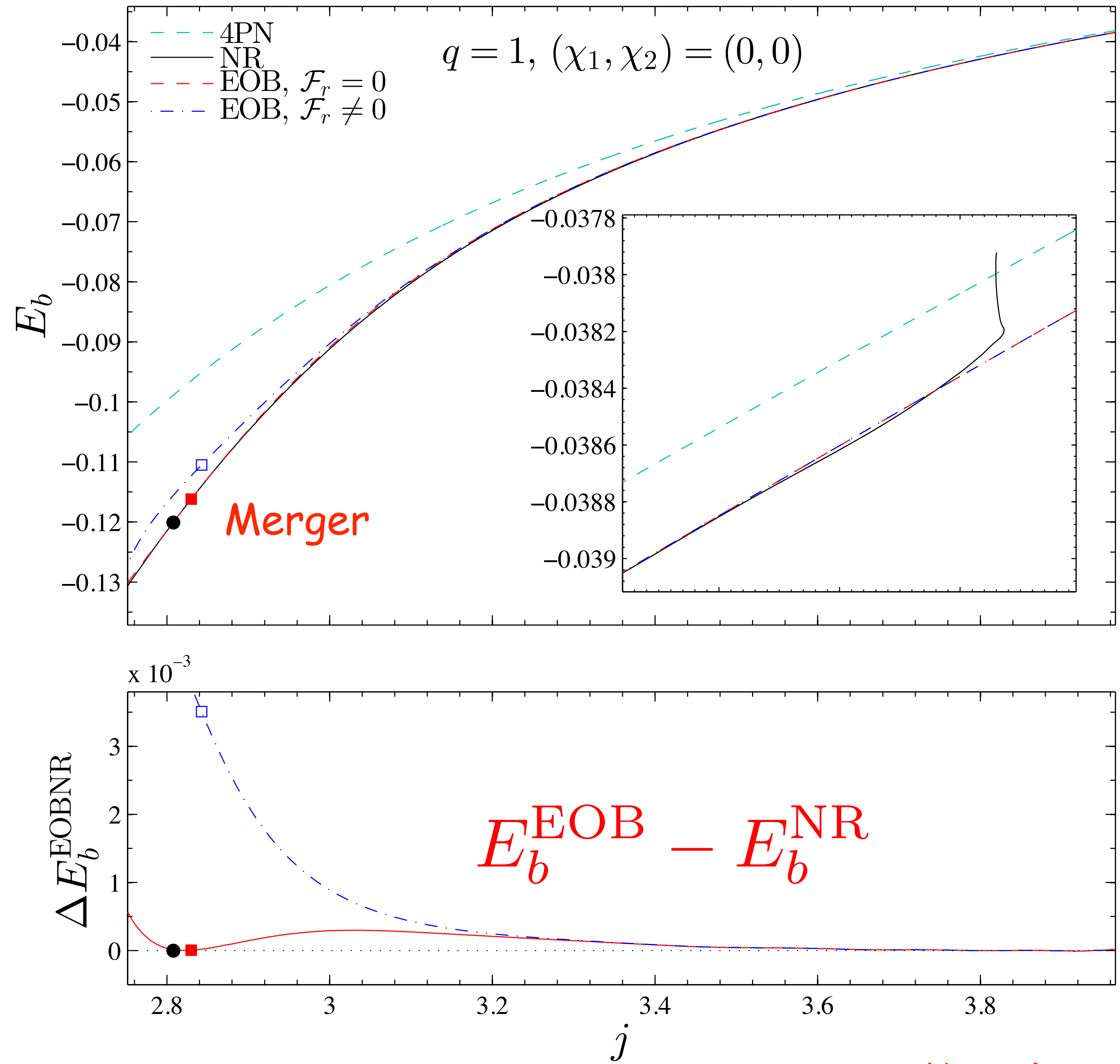


equal-mass case

Nagar, Damour, Reisswig & Pollney, arXiv:1506.08457

ENERGETICS - NONSPINNING

Binding energy vs angular momentum (Llama NR data)



$$E_b = \frac{E - Mc^2}{\mu}$$

Nagar, Damour, Reisswig & Pollney, PRD 93 (2016), 044046

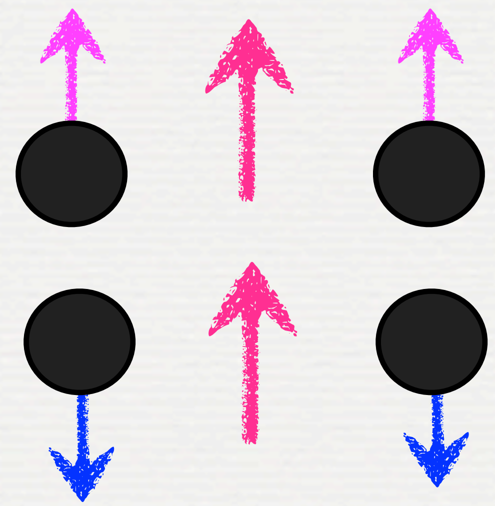
SPINNING BBHS

Spin-orbit & spin-spin couplings

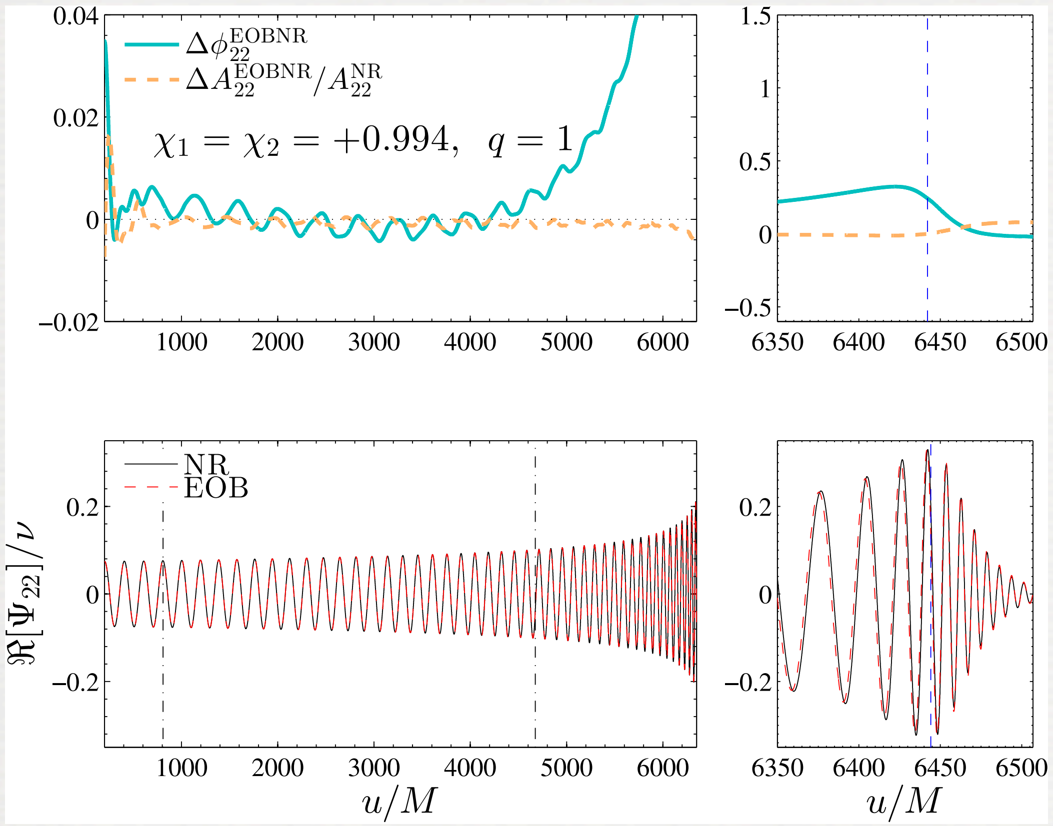
(i) Spins aligned with L: repulsive (slower) L-o-n-g-e-r INSPIRAL

(ii) Spins anti-aligned with L: attractive (faster) Shorter INSPIRAL

(iii) Misaligned spins: precession of the orbital plane (waveform modulation)



$$\chi_{1,2} = \frac{c\,\mathbf{S}_{1,2}}{G m_{1,2}^2}$$

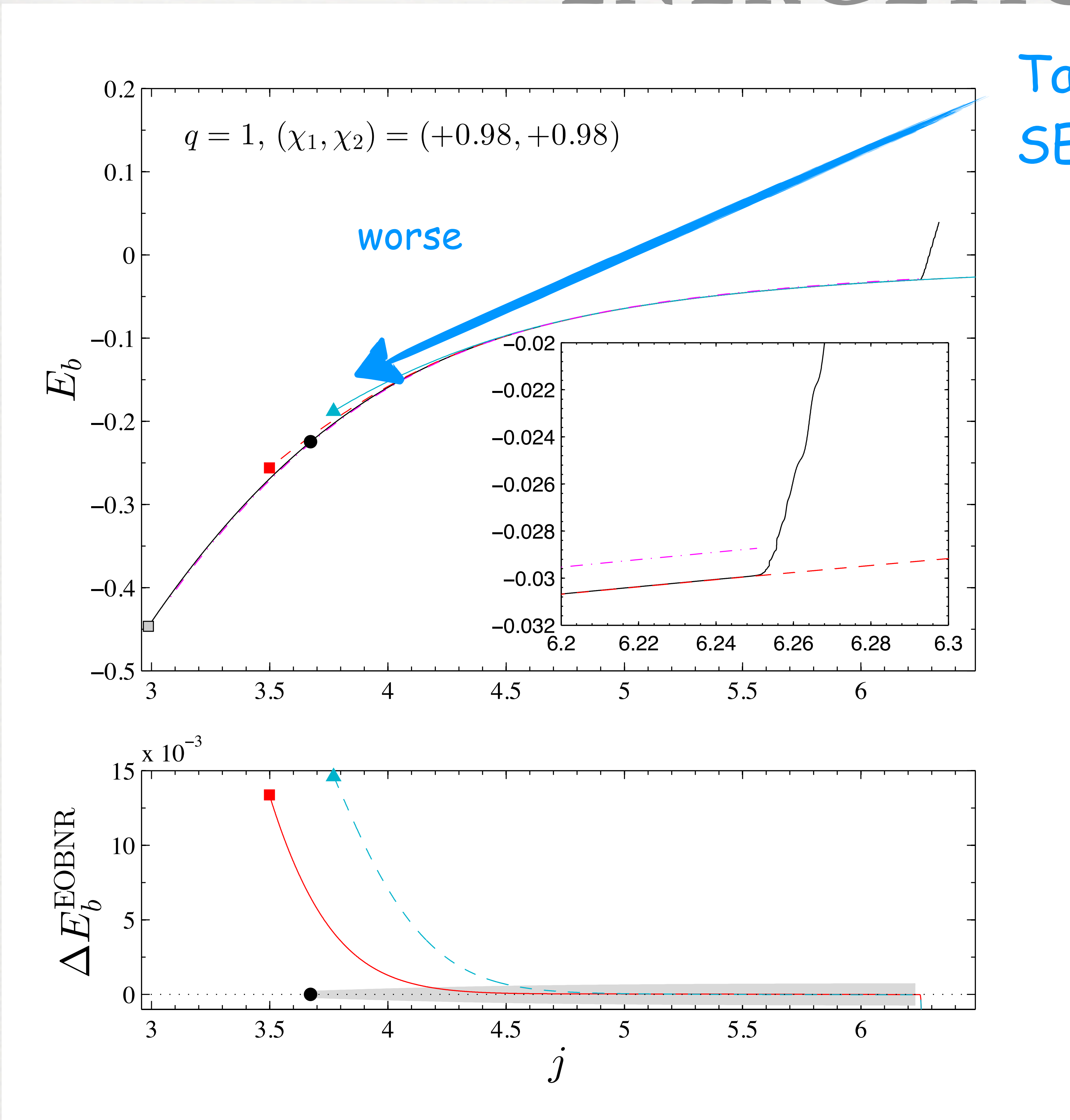


EOB/NR agreement: sophisticated (though rather simple) model for spin-aligned binaries

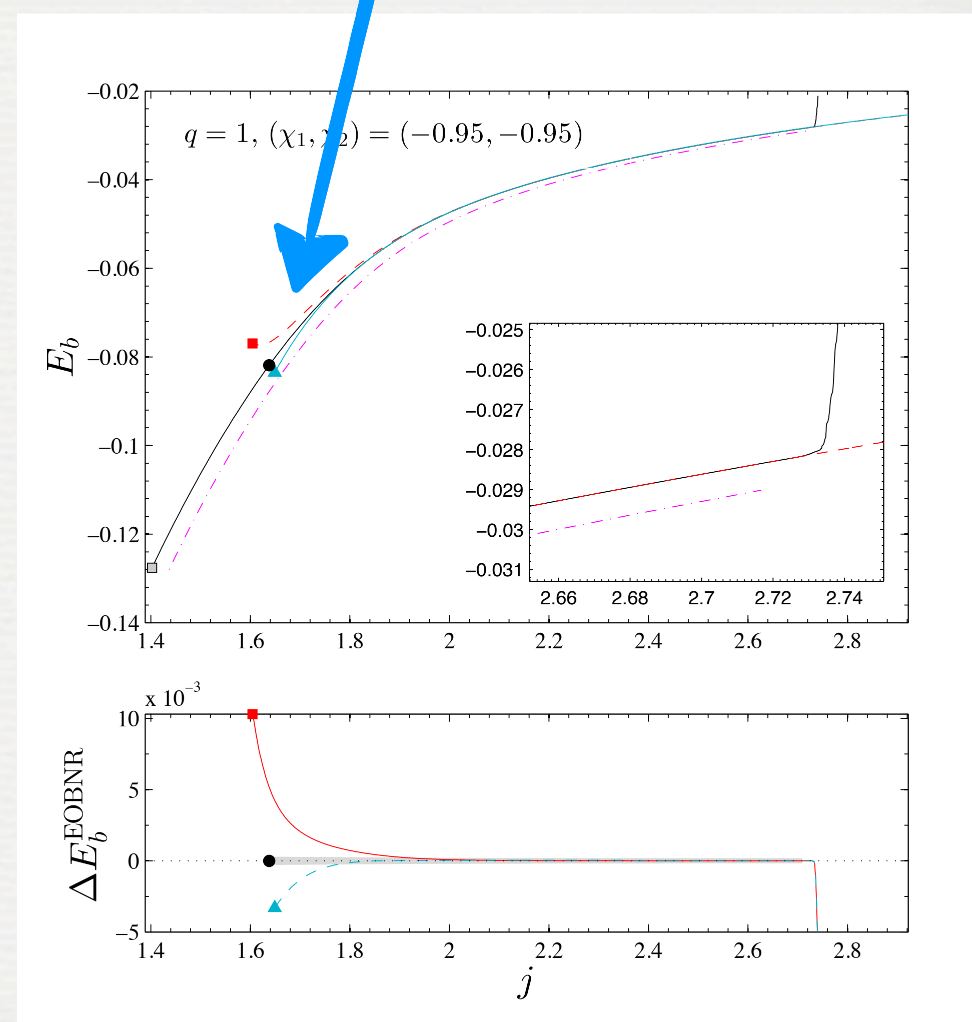
Damour&Nagar, PRD90 (2014), 024054
Damour&Nagar, PRD90 (2014), 044018
Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

ENERGETICS

Taracchini, et al., 2014
SEOBNRv2 (LAL library)



better

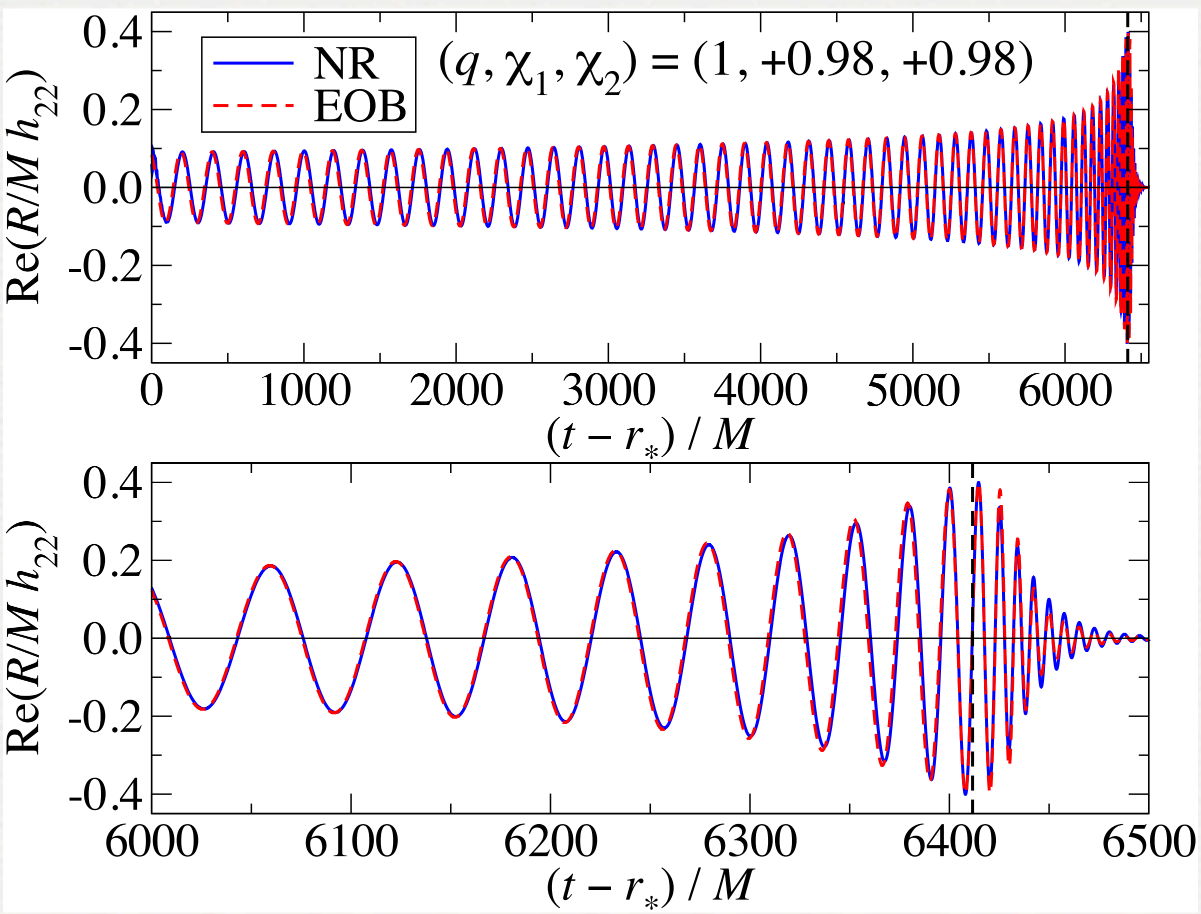


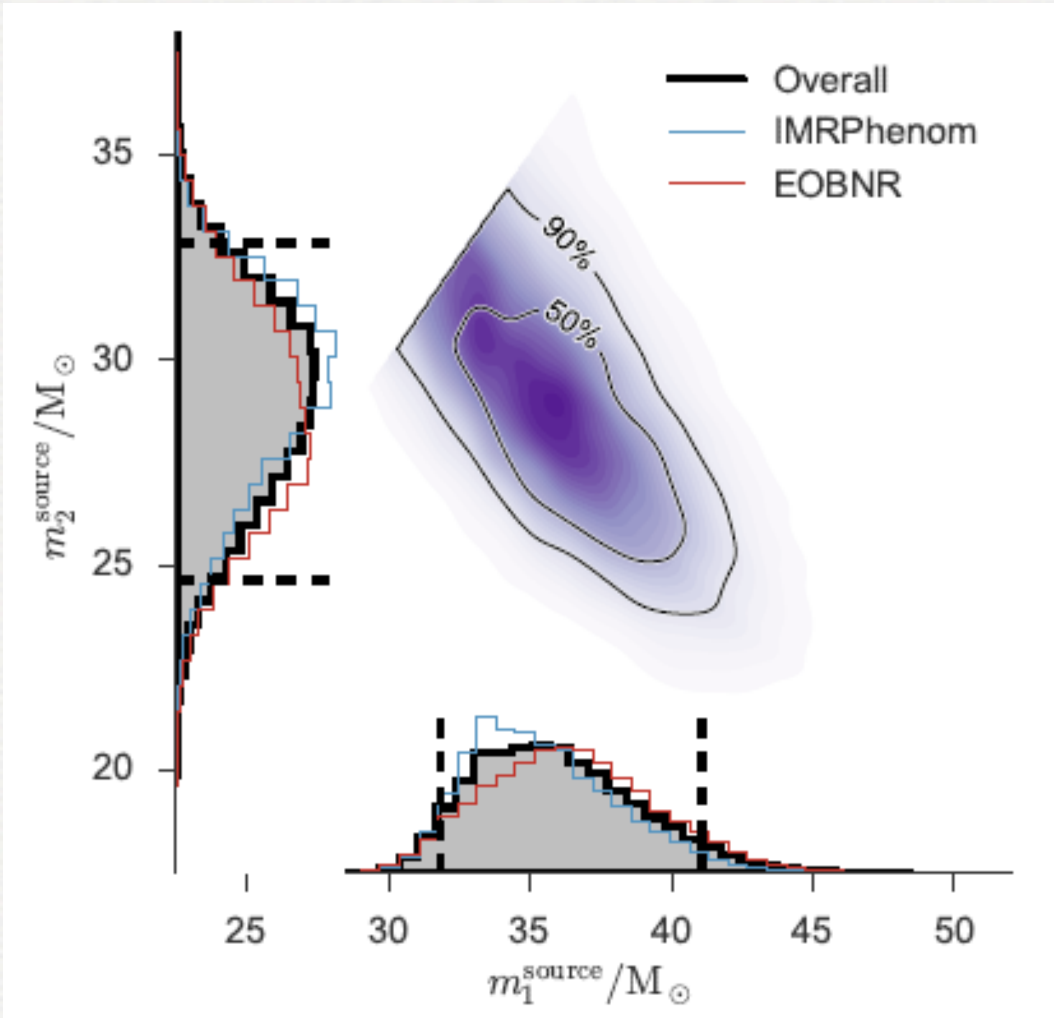
EOBNR MODEL USED FOR GW150914

Different EOB Hamiltonian [Barausse & Buonanno11, Taracchini et al.12]

SEOBNRv2: Taracchini, Buonanno et al., PRD 89, 061502 (R), 2014

SEOBNRv2_ROM_DoubleSpin: M. Puerrer, CQG 31, 195010 (2014)





Effectively used to get the masses:

SEOBNRv2_ROM_DoubleSpin

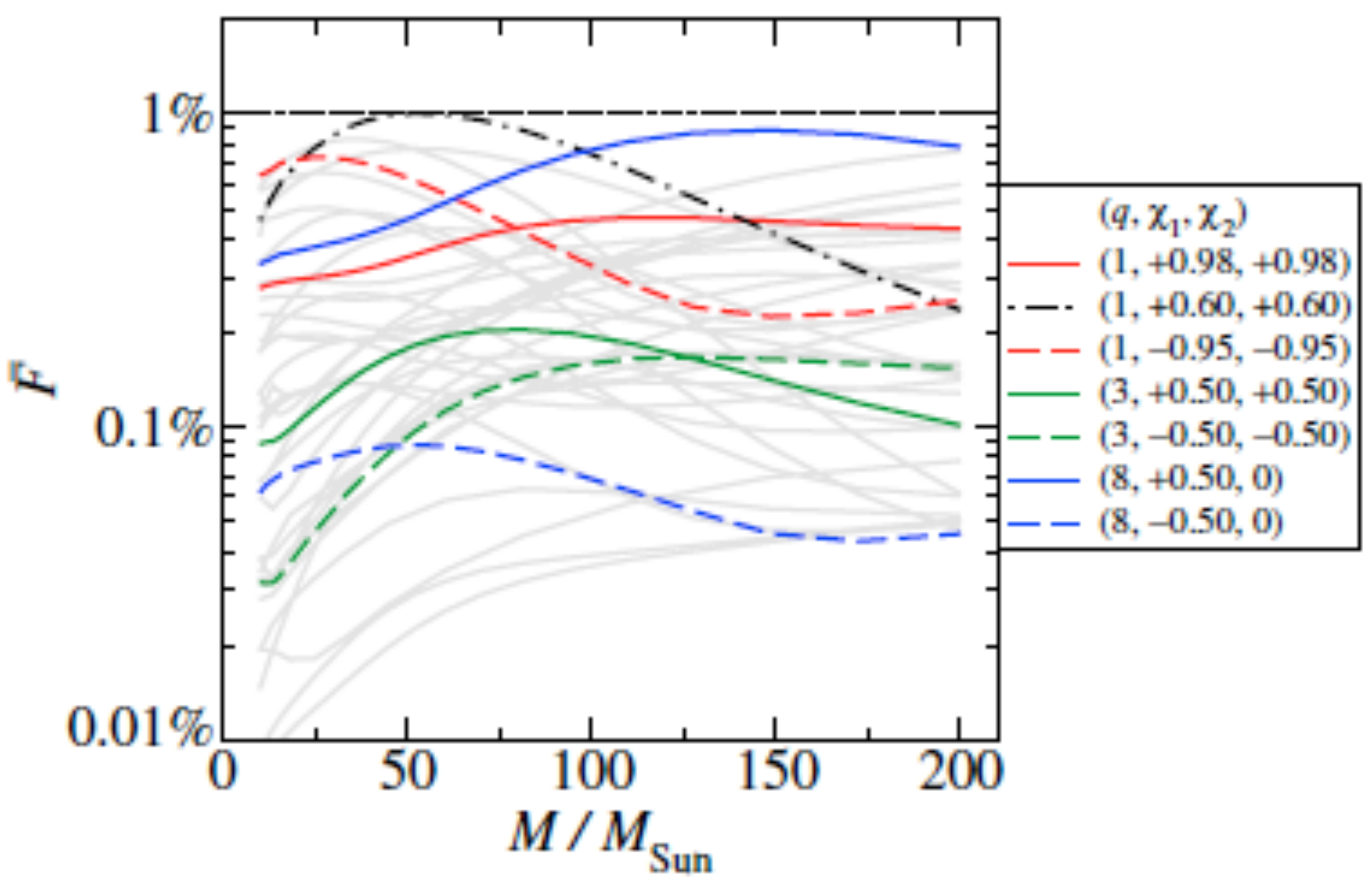
IMRPhenom (Khan et al., 2015)

just AFTER, the best choices
were cross checked with NR simulations!

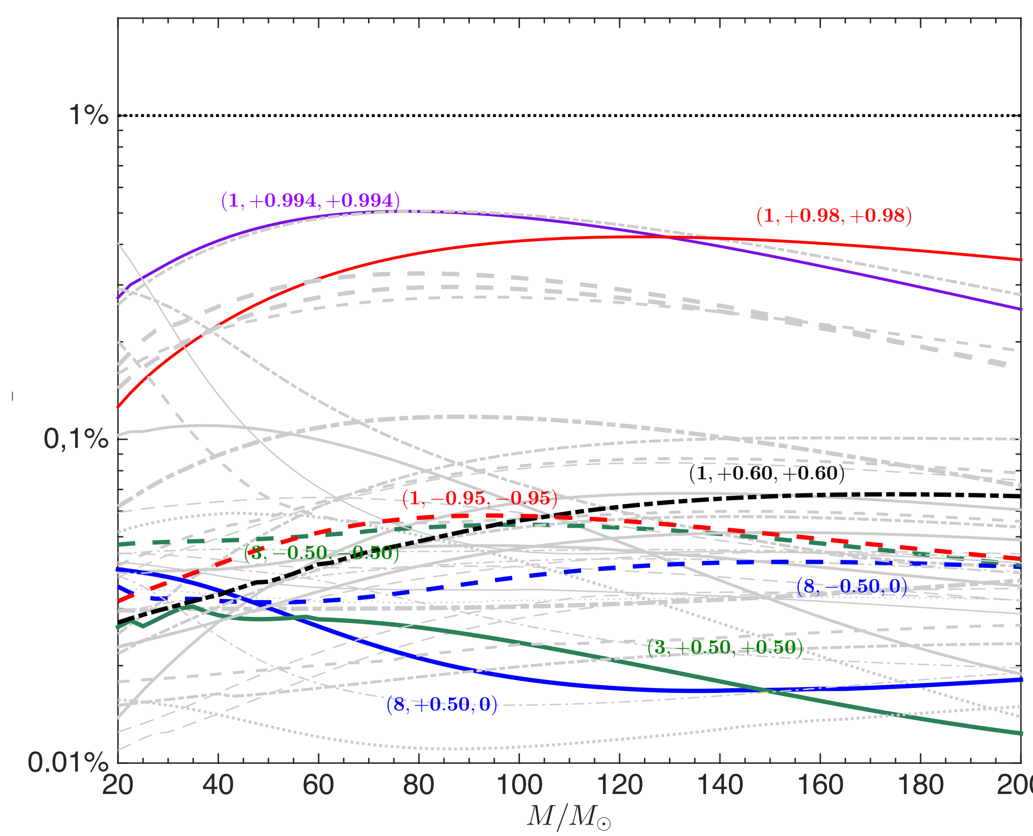
IHES EOBNR MODEL

SEOBNR_IHES model WAS NOT used for parameter estimation:
EOB/EOBNR UNFAITHFULNESS (40 NR SXS dataset)

SEOBNRv2



IHESEOB_spin

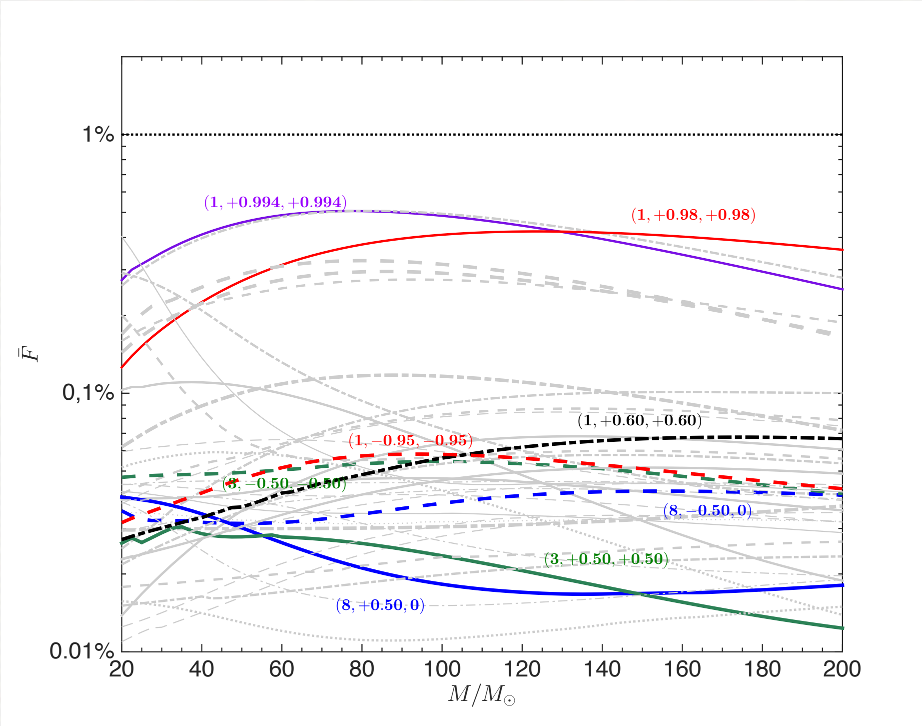


$$\bar{F} \equiv 1 - \max_{t_0,\phi_0} \frac{\langle h_{22}^{\rm EOB}, h_{22}^{\rm NR}\rangle}{||h_{22}^{\rm EOB}||\,||h_{22}^{\rm NR}||}$$

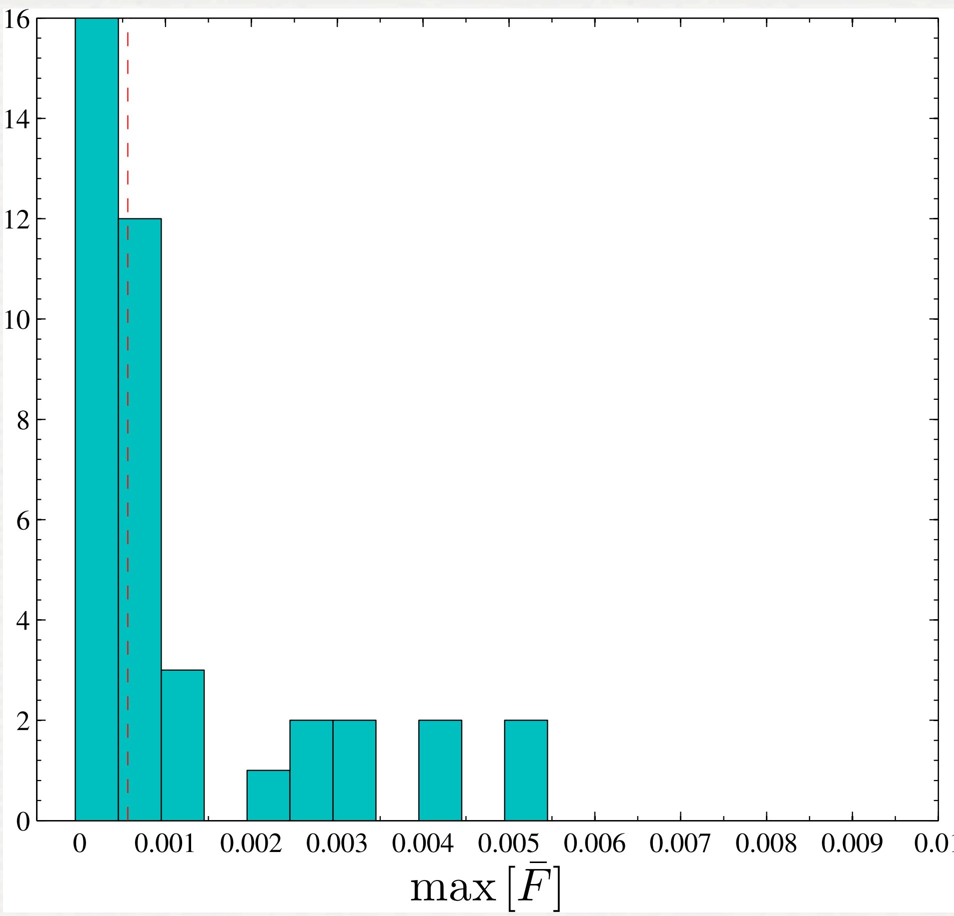
$$\langle h_1, h_2\rangle \equiv 4\Re \int_{f_{\rm min}}^{\infty} \tilde{h}_1(f)\tilde{h}_2^*(f)/S_n(f)\, df$$

Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

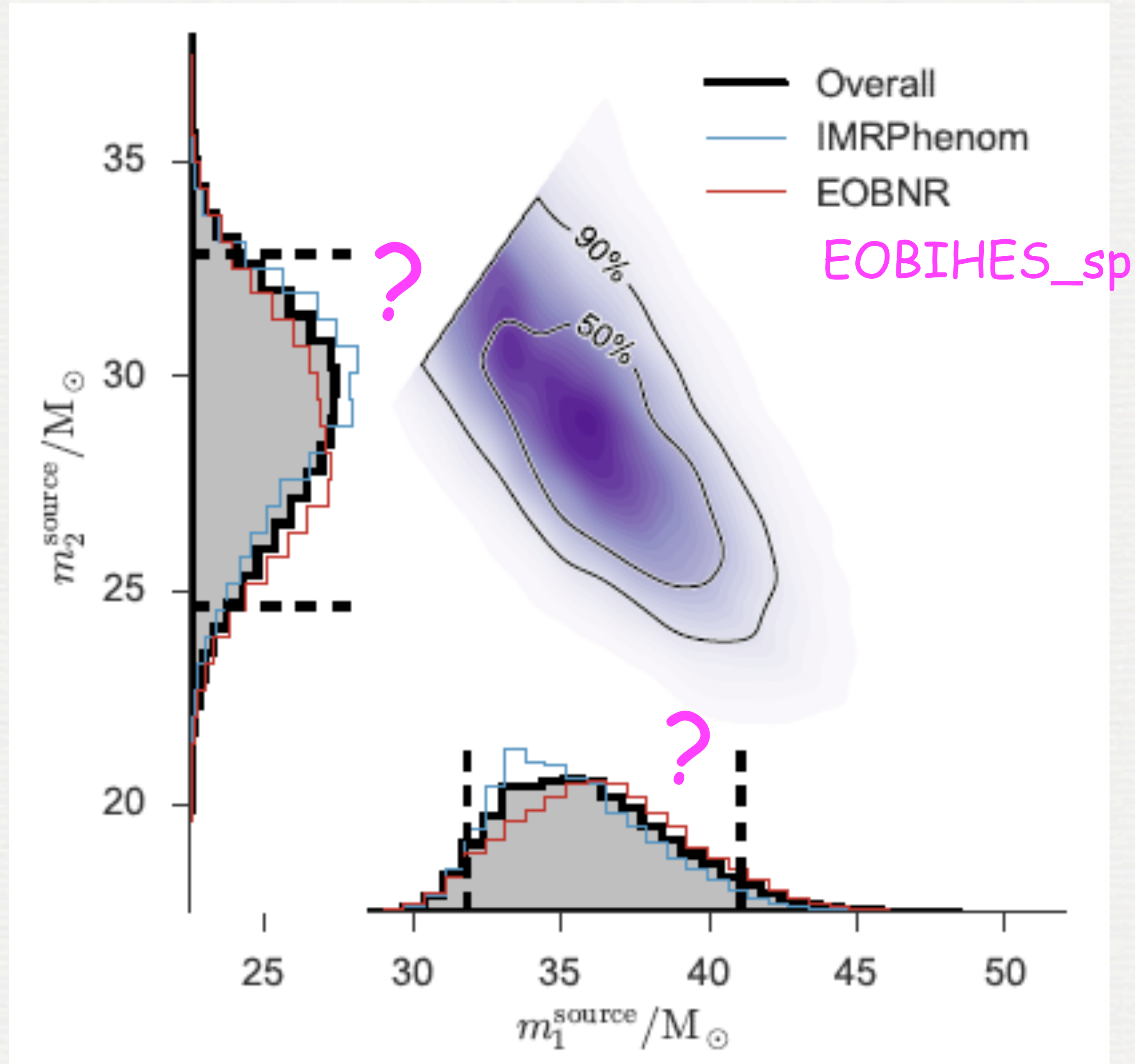
SO WHAT?



1%
0,1%
0.01%
$\bar{F}$
(1, +0.994, +0.994)
(1, +0.98, +0.98)
(1, +0.60, +0.60)
(1, −0.95, −0.95)
(8, −0.50, 0)
(3, +0.50, +0.50)
(8, +0.50, 0)
20 40 60 80 100 120 140 160 180 200
$M/M_\odot$

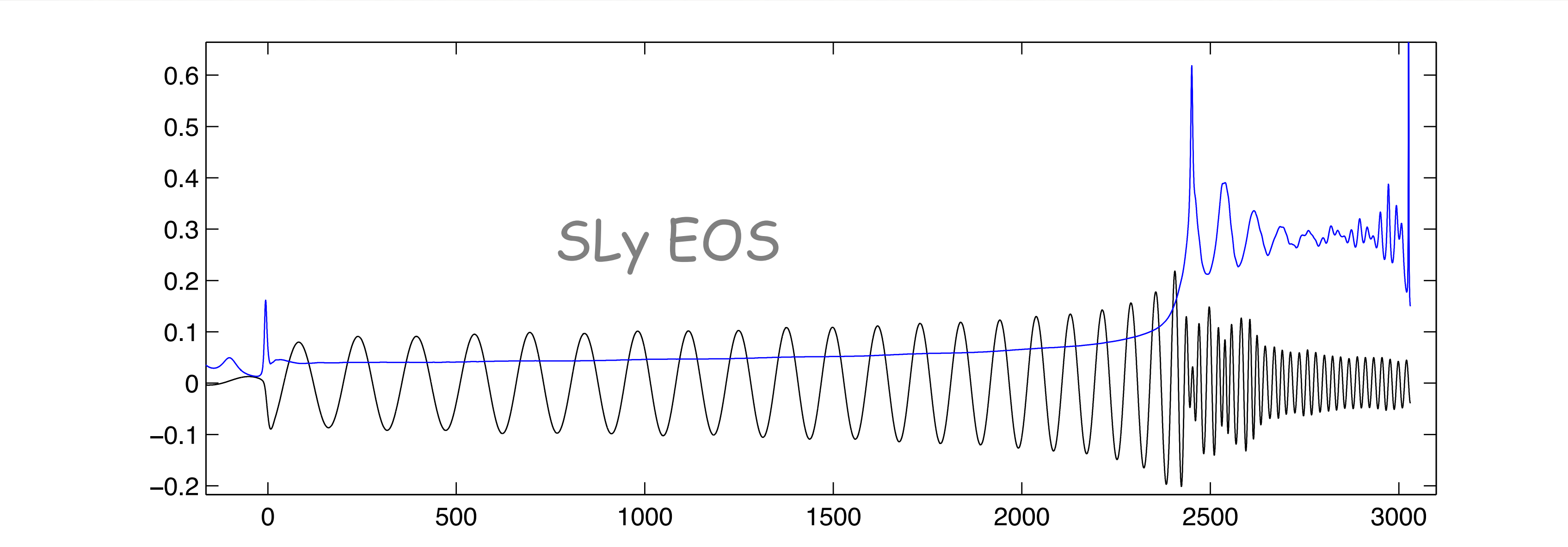


16 14 12 10 8 6 4 2 0
0 0.001 0.002 0.003 0.004 0.005 0.006 0.007 0.008 0.009 0.01
$\max\,[\bar{F}]$



Overall
IMRPhenom
EOBNR
EOBIHES_spin?
?
?
90%
50%
35 30 25 20
$m_2^{\rm source}/M_\odot$
25 30 35 40 45 50
$m_1^{\rm source}/M_\odot$

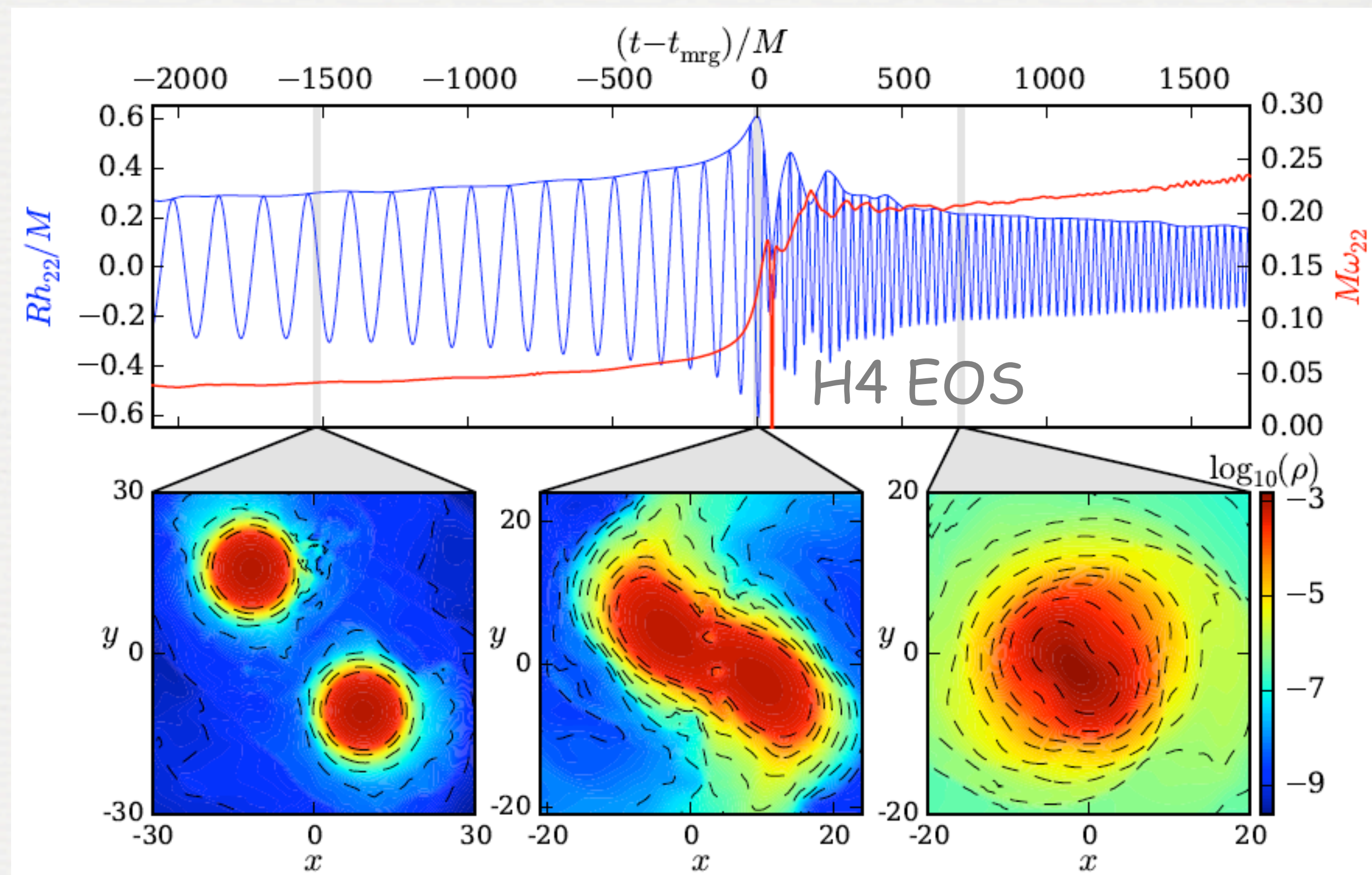
BINARY NEUTRON STARS (BNS)



All BNS need is Love!

$$q = 1 \qquad M = 2.7 M_{\odot}$$

- Tidal effects
- Love numbers (tidal "polarization" constants)
- EOS dependence & "universality"

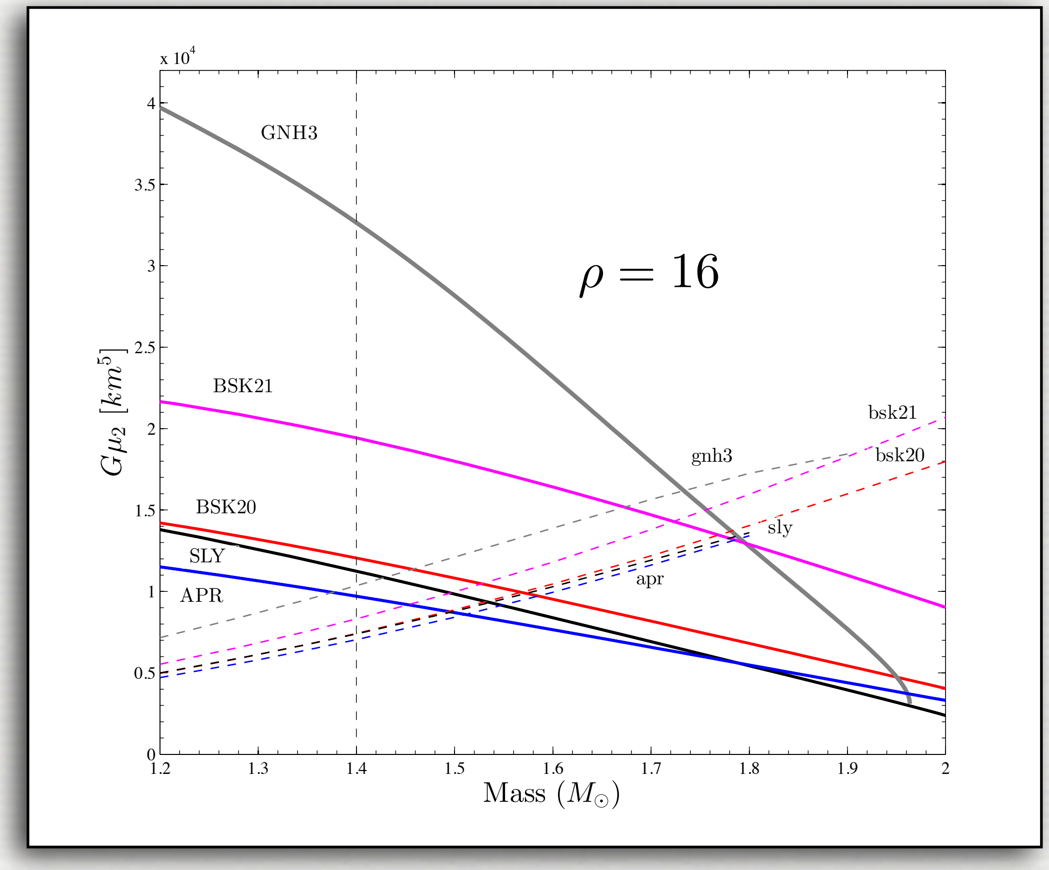


See:
Damour, 1983
Damour,Soffel,Xu, 1999-2001
Flanagan&Hinderer, PRD 2008
Damour&Nagar, PRD 2009
Damour&Nagar, PRD 2010
Damour,Nagar et al., PRL 2011
Bini,Damour&Faye, PRD2012
Bini&Damour, PRD 2014
Bernuzzi, Nagar, et al, PRL 2014
Bernuzzi, Nagar, Dietrich, PRL 2015
Bernuzzi, Nagar, Dietrich & Damour,PRL, 2015

MEASURING LOVE NUMBERS

<2012. Inspiral only; not very promising [Hinderer et al. + 2008]

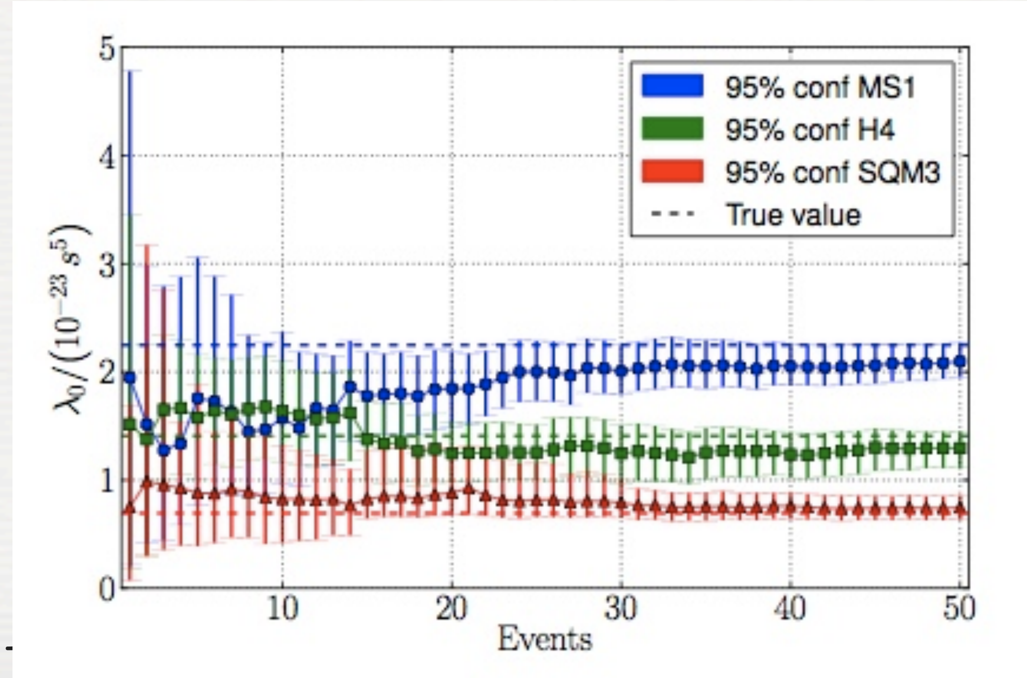
IMPORTANT RESULT (Damour ,Nagar, Villain 2012)



Tidal polarizability parameters *can actually be measured* by adv LIGO with a reasonable SNR=16

Use EOB controlled, accurate, description of the phasing up to BNS merger!

Confermed by Bayesian analysis: Del Pozzo+ 2013 Agathos+2015



THREE RESULTS

1. Numerical-relativity matches effective-one-body (EOB) analytical-relativity waveforms and dynamics essentially up to merger. Method to compute GW templates for LIGO/Virgo to measure EOS out of tidal effects
S. Bernuzzi, A. Nagar, T. Dietrich & T. Damour, PRL 114 (2015), 161103
"Modeling the Dynamics of Tidally Interacting Binary Neutron Stars up to Merger"
[Consistency with Hotokezaka et al., PRD 91 (2015) 6, 064060, notably with reduced eccentricity. With ourselves with improved simulations (unpublished) & Hinderer et al. 2016 (see AB talk)]

2. Quasi-universality in BNS merger (binding energy, angular momentum, GW frequency vs tidal coupling constant): explained using EOB theory
S. Bernuzzi, A. Nagar, S. Balmelli, T. Dietrich & M. Ujevic, PRL 112 (2014), 201101
"Quasiuniversal properties of neutron star mergers"

3. Quasi-universality of post-merger Mf_2 frequency vs tidal coupling constant
S. Bernuzzi, T. Dietrich & A. Nagar, PRL 115 (2015), 091101
"Towards a description of the complete gravitational wave spectrum of neutron star mergers"
Unifying description of inspiral, merger and post-merger phases

Relativistic star in an external gravito-electric & gravito-magnetic (multipolar) tidal field

The star acquires induced gravito-electric and gravito-magnetic multipole moments

Linear tidal polarization

Induced multipole moments

$$M_L^{(A)} = \mu_\ell^A G_L^{(A)}$$
$$S_L^{(A)} = \sigma_\ell^A H_L^{(A)}$$

External multipolar field

$$G\mu_\ell = [length]^{2\ell+1}$$
$$G\sigma_\ell = [length]^{2\ell+1}$$

$$2k_\ell \equiv (2\ell - 1)!! \frac{G\mu_\ell}{R^{2\ell+1}}$$

$$j_\ell \equiv (2\ell - 1)!! \frac{4(\ell + 2)}{\ell - 1} \frac{G\sigma_\ell}{R^{2\ell+1}}$$

Dimensionless relativistic "second" Love numbers

Actual calculation based on star perturbation theory: Love numbers are obtained as boundary conditions (matching interior to exterior perturbations)

RELATIVISTIC LOVE NUMBERS (POLYTROPIC EOS)

"rest-mass polytrope" (solid lines)

$$p = K\mu^{\gamma}$$
$$e = \mu + \frac{p}{\gamma - 1}$$

"energy polytrope" (dashed lines)

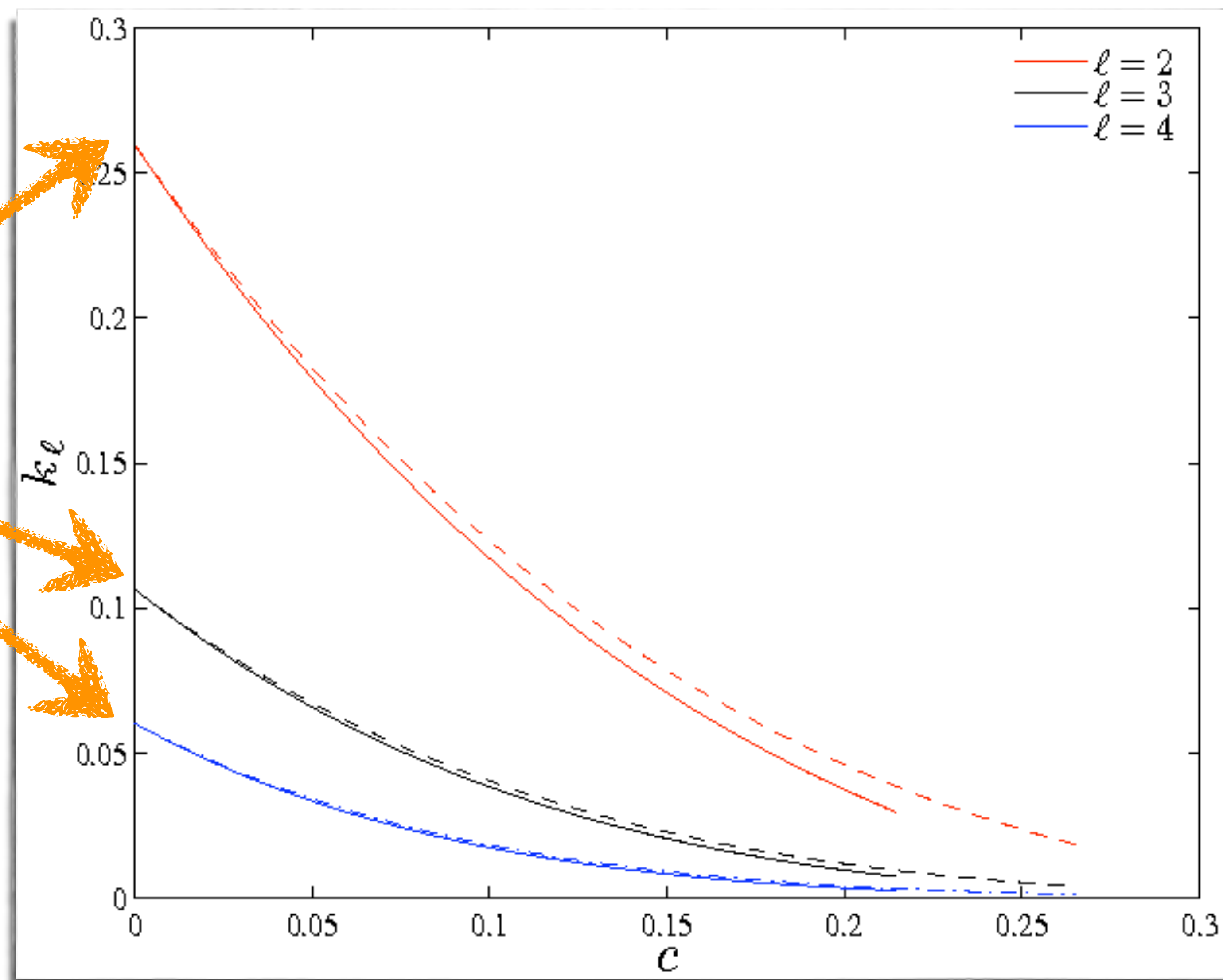
$$p = Ke^{\gamma}$$

Tidal polarization parameters

$$M_L^{(A)} = \mu_\ell^A G_L^{(A)}$$
$$2k_\ell \equiv (2\ell-1)!!\frac{G\mu_\ell}{R^{2\ell+1}}$$

Newtonian values

Newtonian values



Relativistic values

EOBNR

TIDAL EFFECTS IN EOB FORMALISM

Tidal extension of EOB formalism: nonminimal worldline couplings

$$\Delta S_{\rm nonminimal} = \sum_A \frac{1}{4}\mu_2^A \int ds_A \left(u^\mu u^\nu R_{\mu\alpha\nu\beta}\right)^2 + \dots$$

Damour&Esposito-Farèse96, Goldberger&Rothstein06, TD&AN09

Relativistic Love number

Modifications of the EOB effective metric...

$$\begin{aligned} A(r) &= A_r^0 + A^{\rm tidal}(r) \\ A^{\rm tidal}(r) &= -\kappa_2^T u^6 \left(1 + \bar{\alpha}_1 u + \bar{\alpha}_2 u^2 + \dots\right) + \dots \end{aligned}$$

And tidal modifications of GW waveform & radiation reaction

- Need analytical theory for computing $\mu_2,\ \kappa_2^T,\ \bar{\alpha}_1 \dots$
- (?)Need accurate NR simulations to "calibrate" the higher-order PN tidal contributions, that may be quite important during the late inspiral

TIDAL INTERACTION POTENTIAL

Tidal "coupling constant":

$$\kappa_\ell^T \equiv 2\left[\frac{1}{q}\left(\frac{X_A}{C_A}\right)^{2\ell+1} k_\ell^A + q\left(\frac{X_B}{C_B}\right)^{2\ell+1} k_\ell^B\right]$$

$X_{A,B} \equiv M_{A,B}/M$

$M_A = M_B$, $\ell = 2$, $k_2^A = k_2^B$ $\Rightarrow$ $\kappa_2^T = \frac{1}{8}\frac{k_2}{\mathcal{C}^5}$

Function of: masses, compactnesses and relativistic Love numbers

In the dynamics:

$\kappa_2^T \sim 100$

$$A(u) = A^0(u) + A^{\rm tidal}$$

$$A^{\rm tidal} = \sum_{\ell\geq 2} -\kappa_\ell^T u^{2\ell+2} \hat{A}_\ell^{\rm tidal}(u)$$

"Newtonian" (LO) part
+ PN corrections (NLO, NNLO, ...)

NLO & NNLO tidal PN corrections known analytically
[Bini, Damour& Faye 2011]

$$\hat{A}_2^{\rm tidal} = 1 + \frac{5}{4}u + \frac{85}{14}u^2$$

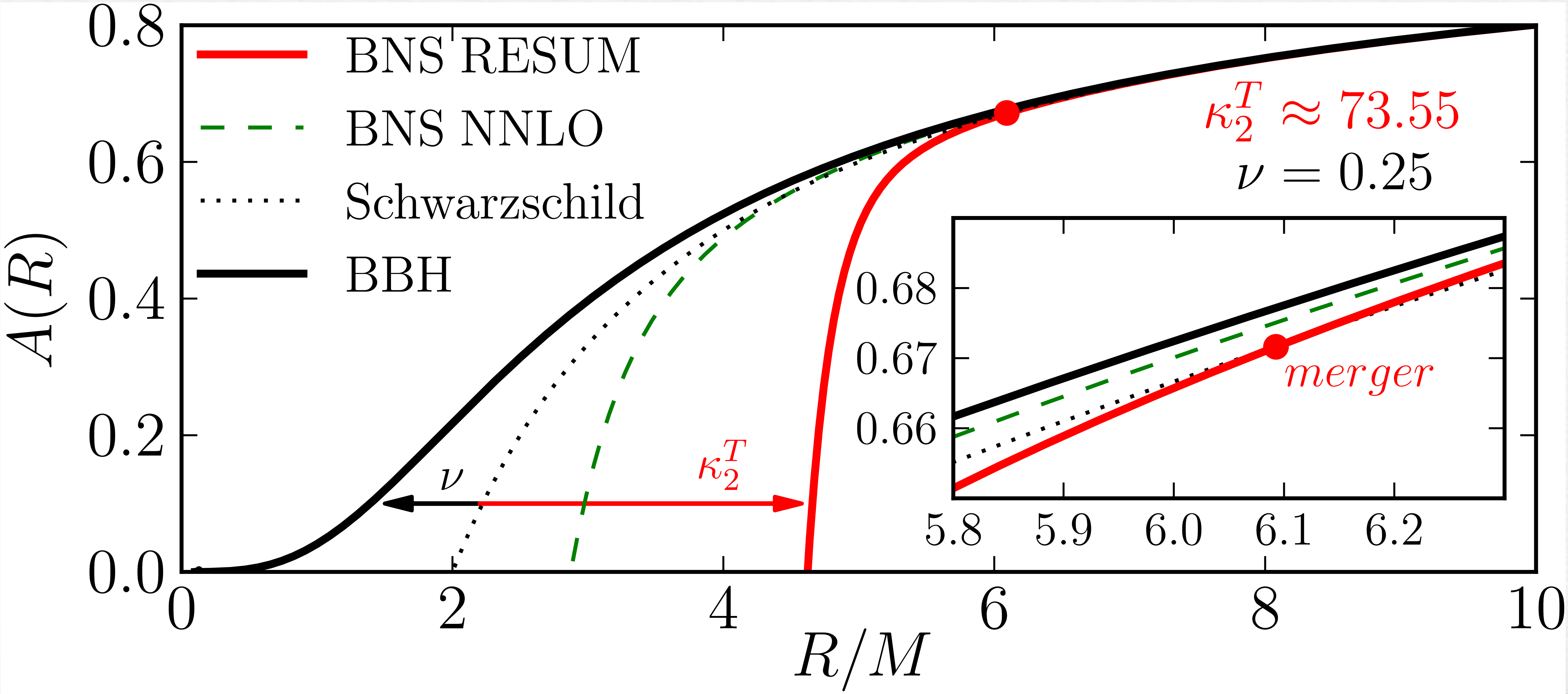
RESUMMED TIDAL INTERACTION

Bini&Damour (2015) resummed expression for $\hat{A}_\ell^{\rm tidal}$

Presence of a pole: potential strongly attractive @ mrg

$$A_T^{(+)}(u;\nu) \equiv -\sum_{\ell=2}^{4}\left[\kappa_A^{(\ell)} u^{2\ell+2} \hat{A}_A^{(\ell^+)} + (A \leftrightarrow B)\right]$$

$$\hat{A}_A^{(2^+)}(u) = 1 + \frac{3u^2}{1-r_{\rm LR}u} + \frac{X_A \tilde{A}_1^{(2^+)1{\rm SF}}}{(1-r_{\rm LR}u)^{7/2}} + \frac{X_A^2 \tilde{A}_2^{(2^+)2{\rm SF}}}{(1-r_{\rm LR}u)^p}$$



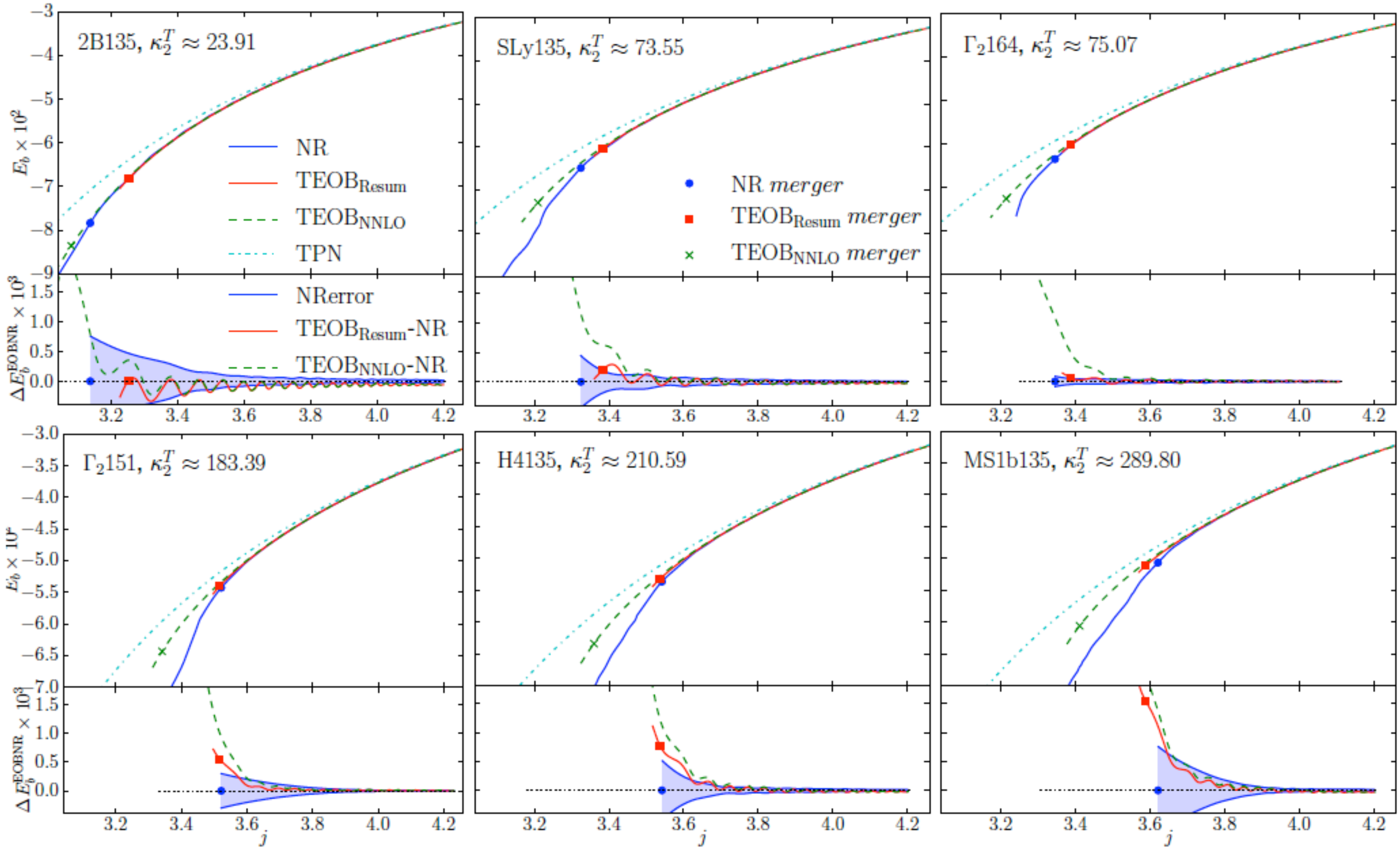


FIG. 2: Energetics: comparison between NR data, TEOB$_{\rm Resum}$, TEOB$_{\rm NNLO}$ and TPN. Each bottom panel shows the two EOB-NR differences. The filled circles locate the merger points (top) and the corresponding differences (bottom). The shaded area indicates the NR uncertainty. The TEOB$_{\rm Resum}$ model displays, globally, the smallest discrepancy with NR data (notably for merger quantities), supporting the theoretical, light-ring driven, amplification of the relativistic tidal factor.

S. Bernuzzi, A. Nagar, T. Dietrich & T. Damour, PRL 114 (2015), 161103

Waveform

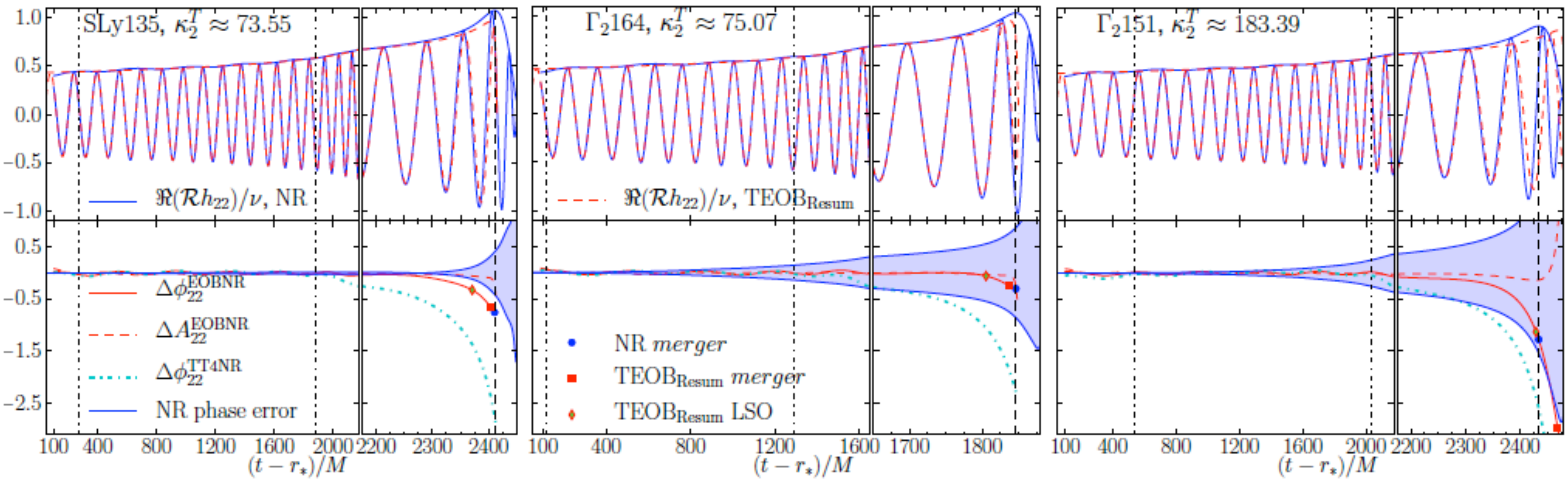
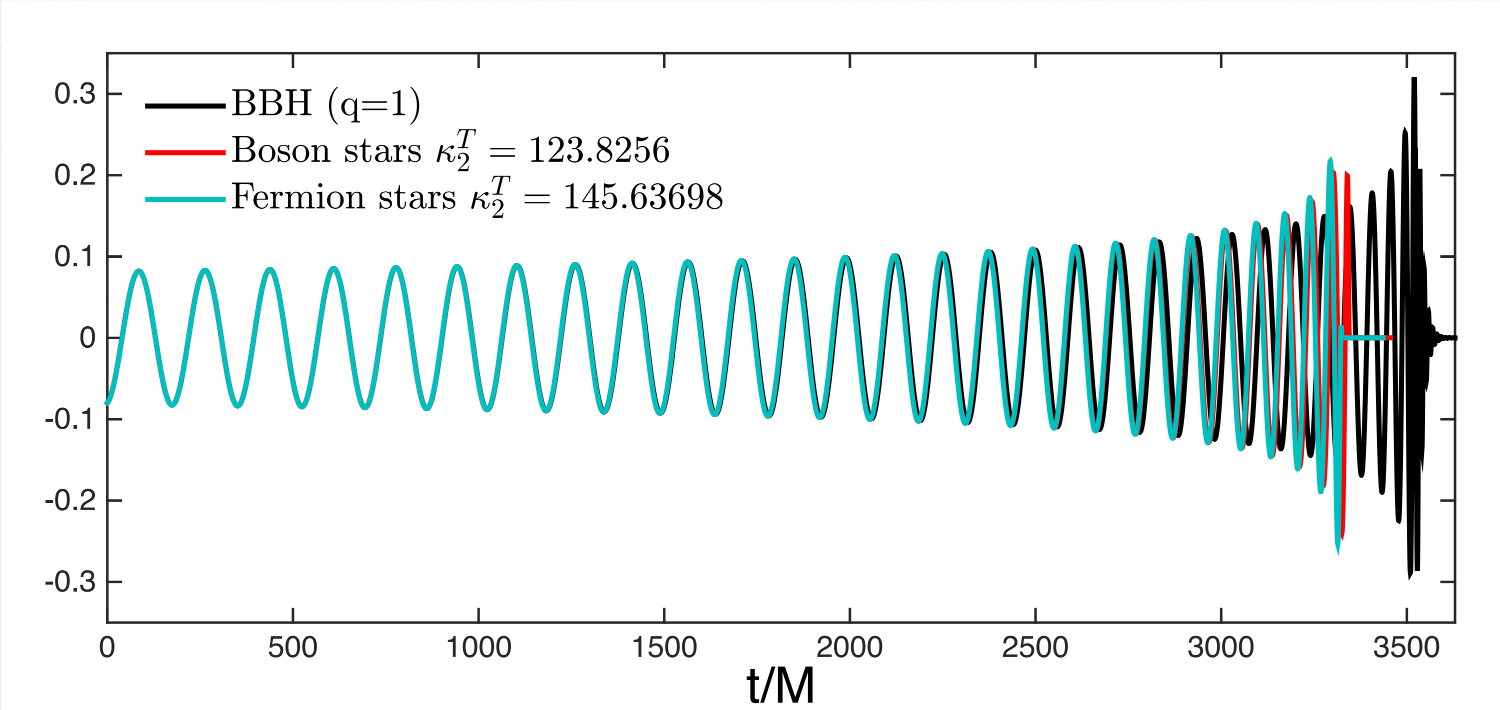


FIG. 3: Phasing and amplitude comparison (versus NR retarded time) between TEOB$_{\rm Resum}$, NR and the phasing of TT4 for three representative models. Waves are aligned on a time window (vertical dot-dashed lines) corresponding to $I_\omega \approx (0.04, 0.06)$. The markers in the bottom panels indicate: the crossing of the TEOB$_{\rm Resum}$ LSO radius; NR (also with a dashed vertical line) and EOB merger moments.

Name	EOS	κ_2^T	$r_{\rm LR}$	$\mathcal{C}_{A,B}$	$M_{A,B}[M_\odot]$	$M^0_{\rm ADM}[M_\odot]$	$\mathcal{J}^0_{\rm ADM}[M_\odot^2]$	$\Delta\phi^{\rm TT4}_{\rm NRmrg}$	$\Delta\phi^{\rm TEOB_{NNLO}}_{\rm NRmrg}$	$\Delta\phi^{\rm TEOB_{Resum}}_{\rm NRmrg}$	$\delta\phi^{\rm NR}_{\rm NRmrg}$
2B135	2B	23.9121	3.253	0.2049	1.34997	2.67762	7.66256	-1.25	-0.19	$+0.57^a$	± 4.20
SLy135	SLy	73.5450	3.701	0.17381	1.35000	2.67760	7.65780	-2.75	-1.79	-0.75	± 0.40
$\Gamma_2 164$	$\Gamma = 2$	75.0671	3.728	0.15999	1.64388	3.25902	11.11313	-2.29	-1.36	-0.31	± 0.90
$\Gamma_2 151$	$\Gamma = 2$	183.3911	4.160	0.13999	1.51484	3.00497	9.71561	-2.60	-1.92	-1.27	± 1.20
H4135	H4	210.5866	4.211	0.14710	1.35003	2.67768	7.66315	-3.02	-2.43	-1.88	± 1.04
MS1b135	MS1b	289.8034	4.381	0.14218	1.35001	2.67769	7.66517	-3.25	-2.84	-2.45	± 3.01

ECO?

Exotic Compact Objects (ECO) [why not BIO (B? I? Objects)]



Tides + objects more massive than NS

Effect of spins? ECO EOS? Whatever you want...

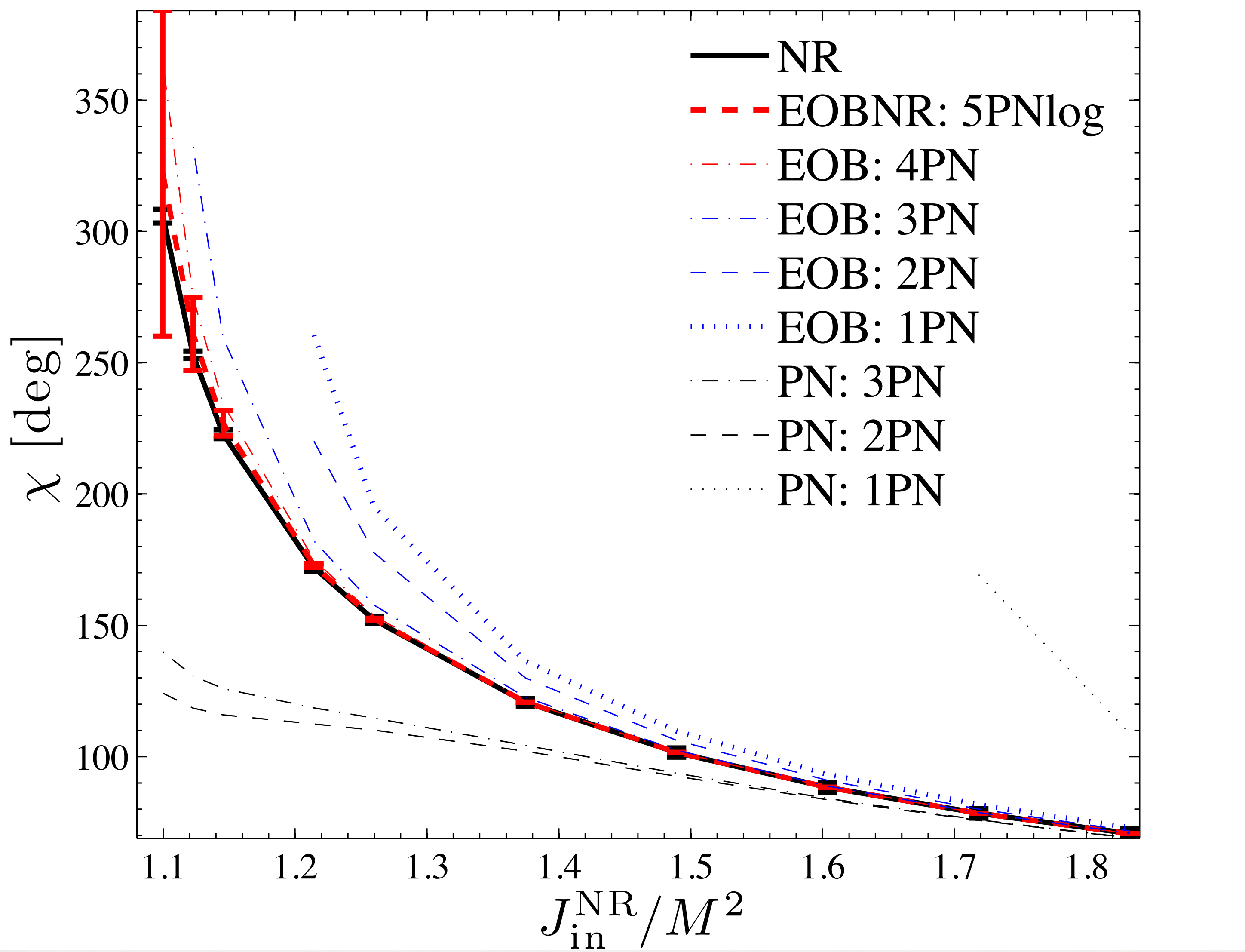
There are compensating effects during inspiral. No very evident and catchy "smoking guns"... Actual differences might be very small...("..subtle is the Lord...")

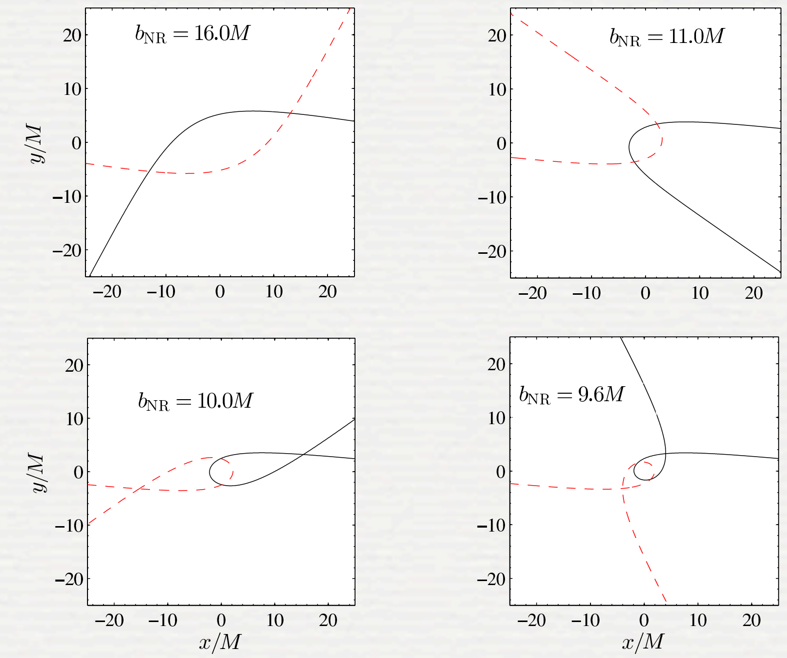
Post-merger might be different...(e.g. different post-merger and "QNMs")

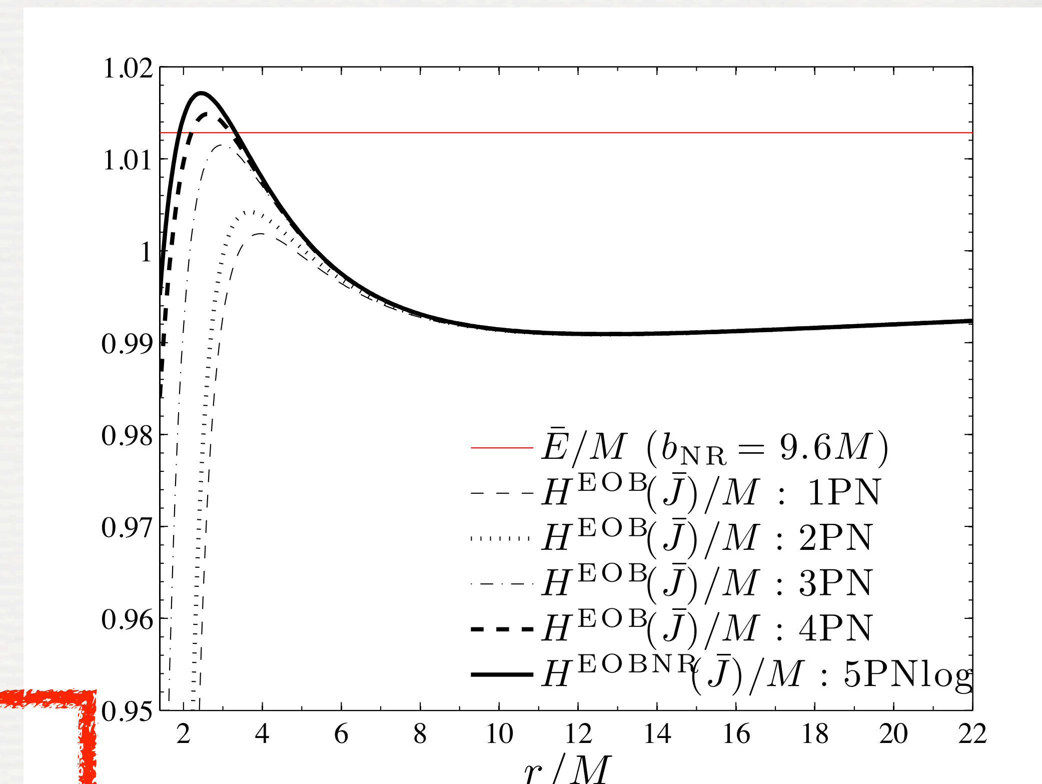
Insplunge: $(\nu, \kappa_2^T, S_1, S_2)$

STRONG FIELD: EOB/NR SCATTERING ANGLE

Damour, Guercilena, Hinder, Hopper, Nagar and Rezzolla, PRD **89**, 081503 (R), 2014





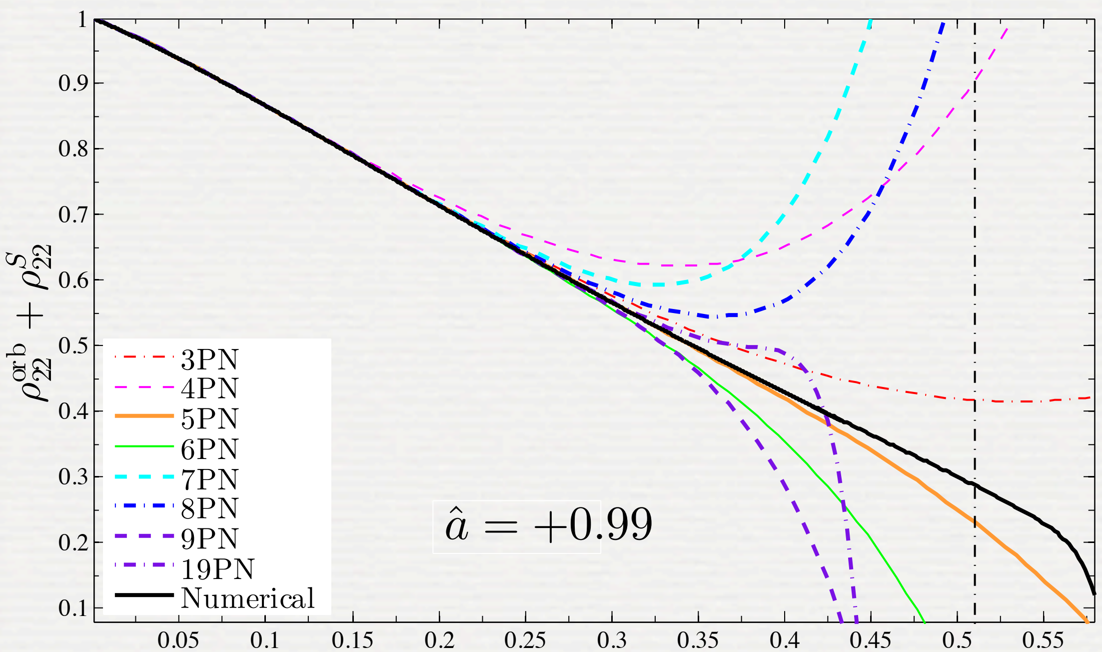


NR uncertainties on scattering angles are still large to firmly distinguish one A function to the other.

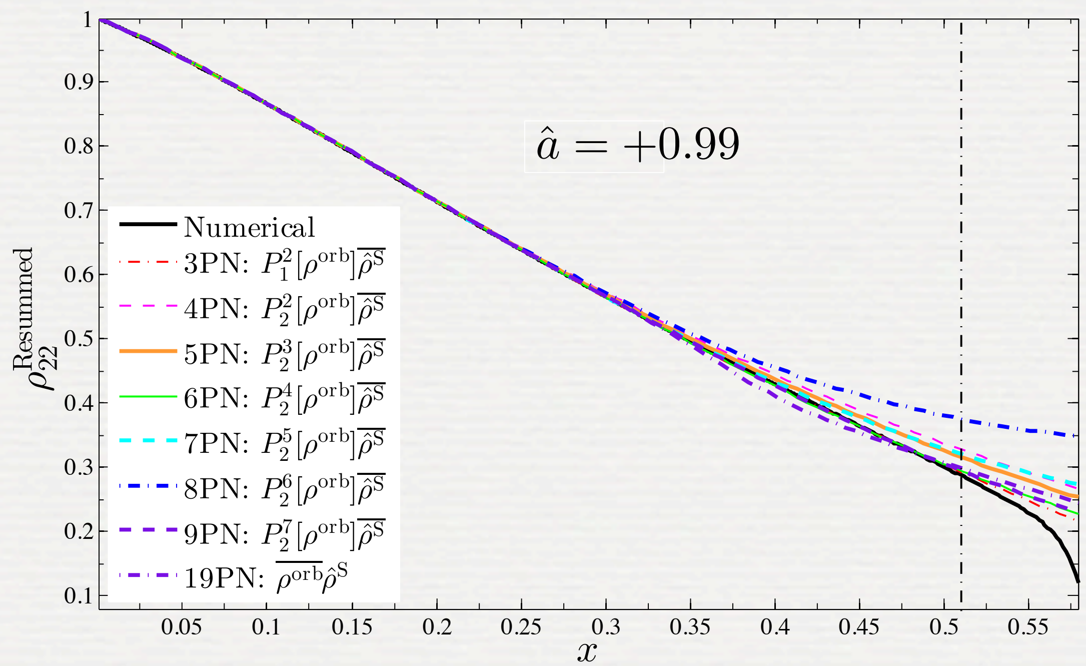
Damour&Nagar, wip

WHAT NEXT: FLUX (SPIN)

Nagar & Shah, in preparation. Test particle + Kerr black hole. Residual wave amplitudes.



Standard [Pan et al., 2011]

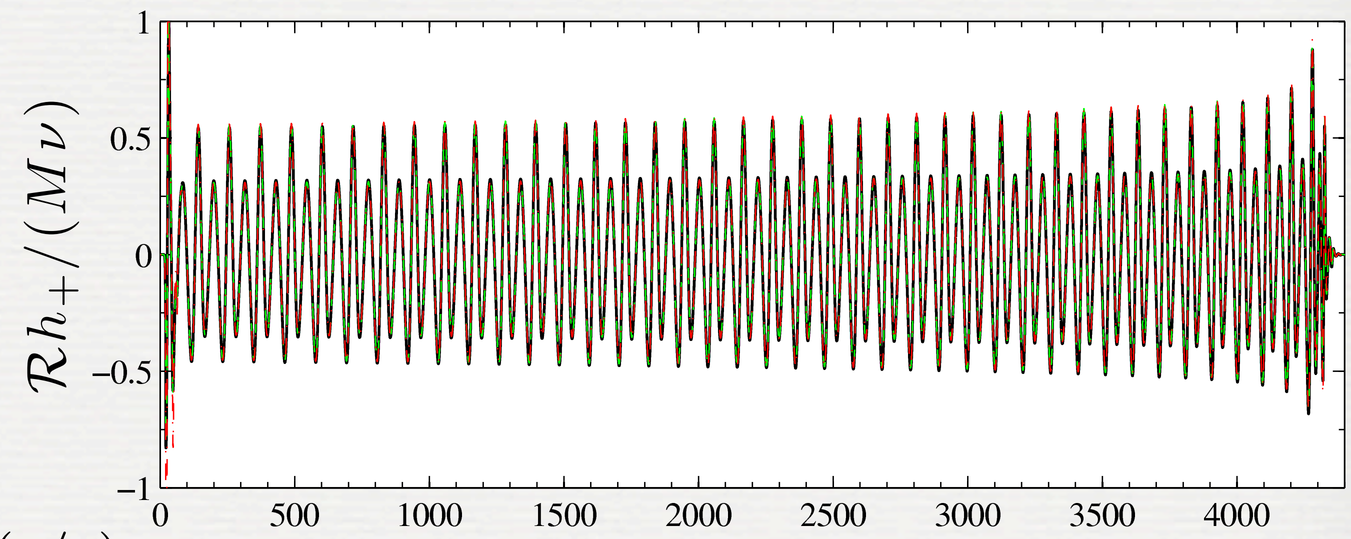


Orbital factorization + further resummation

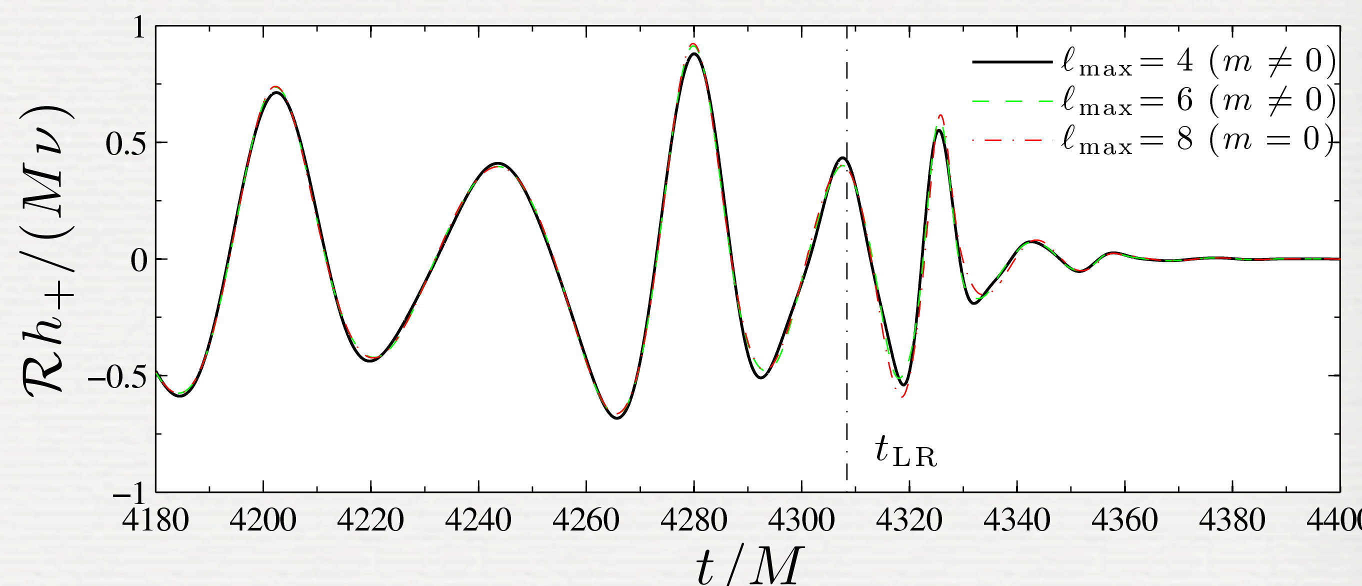
Take away: waveform & radiation reaction in current EOB[NR] models will need to be revisited/improved. WIP

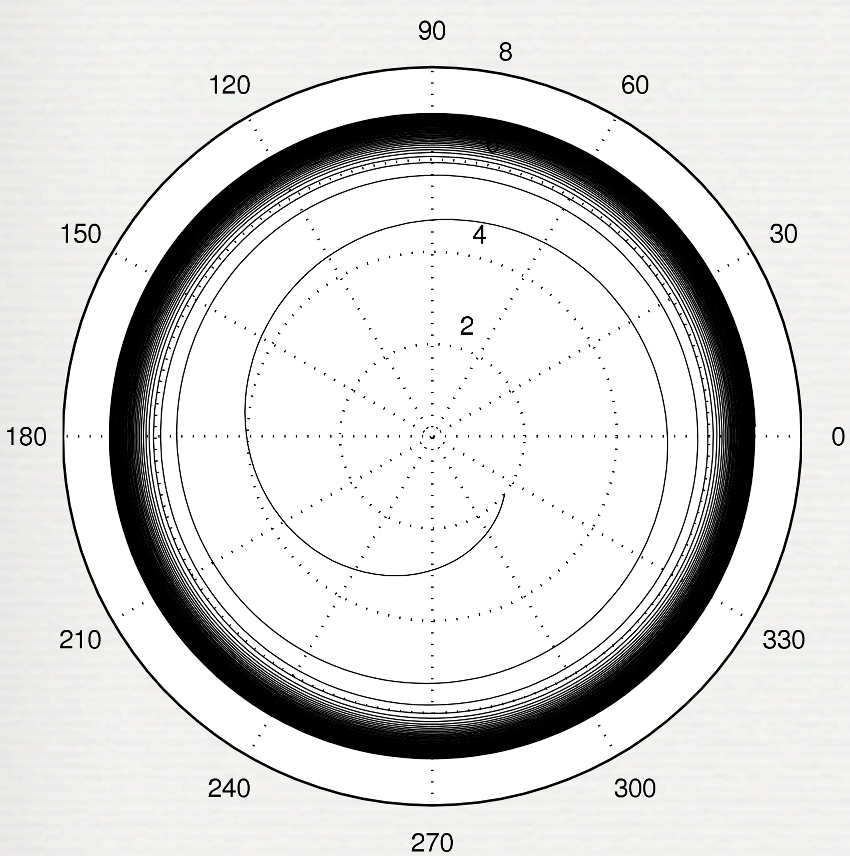
RWZ/TEUKOLSKY WAVEFORMS

T. Damour, AN, & A. Tartaglia, 2006
T. Damour & AN, 2007
S. Bernuzzi & AN, 2010
S. Bernuzzi, AN, A. Zenginoglu, 2010
S. Bernuzzi, AN, A. Zenginoglu, 2011
E. Harms, S. Bernuzzi, AN, Zenginoglu 2014 (spin)



$$\partial_t^2 \Psi_{\ell m}^{(e/o)} - \partial_{r_*}^2 \Psi_{\ell m}^{(e/o)} + V_\ell^{(e/o)} \Psi_{\ell m}^{(e/o)} = \mathcal{S}_{\ell m}^{(e/o)}$$





- Quasi-circular initial data
- EOB-resummed (5PN-accurate) radiation reaction
- 4th/6th-order numerical method
- High-order multipoles (l=8)
- Hyperboloidal-layer method & extraction at null-infinity

A. Nagar - 18 May 2016 - GGI

CONCLUSION

The wave(s) have passed....

...and we were (reasonably) prepared!

Though more work to improve modelization further is needed!

Matlab EOB code (working for BNS [& spin, C++] too...), free download: https://eob.ihes.fr.
More infos: https://gravitational_waves.ihes.fr/