JET INTERFERENCE EFFECTS IN HIGHLY NON-DIPOLE RADIATION SPECTRA

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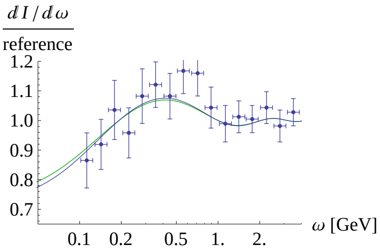
Outline

Interference effects – dipole and non-dipole

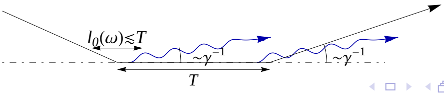
Interference effects in radiation are most celebrated in such processes as coherent bremsstrahlung and undulator or channeling radiation. But there, they are mostly studied in dipole regime, because that makes the interference pattern the sharpest, and accordingly, the use of the dipole regime is best suited for designing sources of monochromatic radiation.

However, interference can be marked also in strongly non-dipole radiation, when the target is thick and uniform, so that the transmitted electron does not oscillate but accumulates its deflection angle. The prerequisite for this kind of interference is the existence of sharp target edges, separated by a definite distance.

Radiation on two amorphous foils. Experiments



Studies of non-dipole radiation interference effects were triggered by the prediction of interference fringes in spectra of non-dipole radiation from electron scattering on two amorphous foils by Blankenbecler and Drell [1]. They were confirmed in subsequent CERN experiments [2], and eventually, explained by collinear-collinear interference (semi-bare electron resonances) [3].



Issuing from the standard representation for the angle-integral radiation spectrum

$$\frac{dI}{d\omega} = \omega^2 \int d^2 n \left| \frac{e}{2\pi} \int_{-\infty}^{\infty} dt [\vec{n}, \vec{v}(t)] e^{i\omega t - i\vec{k}\cdot\vec{r}(t)} \right|^2, \tag{1}$$

one observes that for bremsstrahlung on a pair of thin foils, the time integral can be taken exactly, and the spectrum be expressed for arbitrary strength of scattering. Its oscillations are most prominent in a highly non-dipole regime $\sqrt{1-v^2} = \gamma^{-1} \ll \chi \ll 1$. That is not surprising, inasmuch as in the dipole limit they are averaged to zero. The generic result reads

$$\left\langle \frac{dI}{d\omega} \right\rangle = \left\langle \frac{dI_{\mathrm{BH}}}{d\omega} \right\rangle_1 + \left\langle \frac{dI_{\mathrm{BH}}}{d\omega} \right\rangle_2 - \frac{2e^2\gamma^4}{\pi} \int_0^{\infty} d\theta^2 \theta^2 \left\langle G \right\rangle_1 \left\langle G \right\rangle_2 \cos \frac{\omega T}{2\gamma^2}(1+\gamma^2\theta^2), \tag{2}$$

with

$$\gamma^2 G \approx \frac{1}{\gamma^{-2}+\theta^2} - \frac{1}{2\theta^2}\left(1 + \frac{\theta^2 - \chi^2 - \gamma^{-2}}{\sqrt{[\gamma^{-2}+(\theta-\chi)^2][\gamma^{-2}+(\theta+\chi)^2]}}\right) \tag{3}$$

At that $\int_0^{\infty} d\omega$ from the last (interference) term in (2) equals zero.[1]

[1] That property (no interference in total radiative losses) is the consequence of locality of the radiative energy loss in classical electrodynamics [?]. In QED, respecting the additional restriction when $\hbar\omega < E$, it may be violated.

Why does the interference term only depend on $T/l_0(\omega)$, with $l_0(\omega) = 2\gamma^2/\omega$? Where is the scattering-modified coherence length $l_\chi = 2/\omega\chi^2$? It appears to be concealed by averaging. As we choose a problem without such averaging, there must appear other interference effects related to l_χ and located at low ω. To elucidate physical nature for all types of the interference effects, it is instructive to refer to different representations for the radiation spectrum in different spectral regions.

Low ω. Double time integral representation

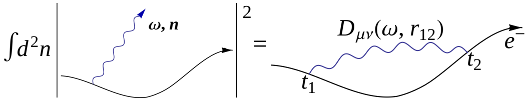


Figure: Graphical illustration of unitarity relation for the angle-integral spectrum.

At low ω, some time scales, being reciprocal to ω, become long, while others, being determined by the target thickness, remain finite.
To treat the spectrum in terms of time scales, the common approach is to exactly integrate in (1) over photon emission angles prior to integration over times. In fact, that leads to a kind of unitarity relation connecting the total probability of decay into a 2-particle state to an imaginary part of a loop diagram (see Fig. 1):

$$\frac{1}{\omega}\frac{dI}{d\omega} = \frac{e^2}{\pi}\int_{-\infty}^{\infty} ds_2 \int_{-\infty}^{s_2} ds_1 u_\mu(t_1)u_\nu(t_2)\mathfrak{Im}\, e^{-i\omega(t_2-t_1)} D_{\mu\nu}\left(\omega, \left|\vec{r}(t_2)-\vec{r}(t_1)\right|\right), \tag{4}$$

where $s = t/\gamma$ is electron's proper time, u_μ its 4-velocity, and $D_{\mu\nu}$ the photon propagator. Specializing $D_{\mu\nu}$ in Feynman gauge and the frequency-position representation,[2]

$$D_{\mu\nu}(\omega, r) = -\frac{g_{\mu\nu}}{r - i0} e^{i\omega r}, \tag{5}$$

leads to the widely used formula [4]

$$\frac{dI}{d\omega} = -\omega\frac{e^2}{\pi}\int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{t_2} dt_1 \left\{\gamma^{-2} + \frac{1}{2}\left[\vec{v}(t_2) - \vec{v}(t_1)\right]^2\right\} \times \mathfrak{Im}\frac{1}{t_2 - t_1 - i0} e^{-i\omega\left[t_2 - t_1 - \left|\vec{r}(t_2) - \vec{r}(t_1)\right|\right]}, \tag{6}$$

or, taking the imaginary part and adjusting the instantaneous (a.k.a. “vacuum" [1]) term, to [1]

$$\frac{dI}{d\omega} = \omega\frac{e^2}{\pi}\int_{0}^{\infty} \frac{d\tau}{\tau} \int_{-\infty}^{\infty} dt_2 \Bigg(\left\{\gamma^{-2} + \frac{1}{2}\left[\vec{v}(t_2) - \vec{v}(t_2-\tau)\right]^2\right\} \times \sin\omega\left[\tau - \left|\vec{r}(t_2) - \vec{r}(t_2-\tau)\right|\right] - \gamma^{-2}\sin\omega(1-v)\tau\Bigg). \tag{7}$$

NLO

One of the advantages of representation (7) is that it allows one to derive a NLO correction to the low-ω approximation:

$$\frac{dI}{d\omega} \underset{\omega\to 0}{\simeq} I_{\mathrm{BH}}(\gamma\chi) + C_1\omega + \mathcal{O}(\omega^2). \tag{8}$$

where

$$I_{\mathrm{BH}}(\gamma\chi) \underset{\gamma\chi\gg 1}{\simeq} \frac{2e^2}{\pi}[\ln\gamma^2(\vec{\chi}_f - \vec{\chi}_i)^2 - 1] \tag{9}$$

is the well-known factorization limit, and

$$C_1 = -\frac{e^2}{2}\int_{-\infty}^{\infty} dt[\vec{\chi}(t) - \vec{\chi}_i]\cdot[\vec{\chi}_f - \vec{\chi}(t)]. \tag{10}$$

Note that for C_1, only one of the contributing times is large; that is why the NLO expansion for $dI/d\omega$ begins with $\mathcal{O}(\omega)$ [rather than $\mathcal{O}(\omega^2)$ as is the case for $dI/d\omega d^2n = |\mathcal{O}(1) + i\mathcal{O}(\omega)|^2$]. In most important cases, $C_1 \leq 0$: For double hard scattering, $C_1 = -\frac{e^2}{2}T\vec{\chi}_1\cdot\vec{\chi}_2$, for passage through a finite magnet, $C_1 = -\frac{e^2}{12}T\chi^2$, and for passage through an amorphous target, $C_1 = 0$.
Note, too, that C_1 is independent of γ. Can this analysis be continued to higher orders? Below we will answer in the affirmative.

Intermediate ω. Impact parameter representation

At intermediate ω, as the electron trajectory bending during the photon formation process becomes essential, there is need for analyzing transverse dimensions of the process. In some cases, it is straightforward to derive impact parameter representations.
For single scattering,

$$\frac{dI}{d\omega}=\left(\frac{e}{\pi}\right)^2\int d^2\xi\left[\frac{\partial}{\partial\vec{\xi}}K_0(\xi/\gamma)\right]^2\left|1-e^{i\vec{\chi}\cdot\vec{\xi}}\right|^2=I_{\mathrm{BH}}(\gamma\chi), \tag{11}$$

where $\vec{b}=\vec{\xi}/\omega$ is the impact parameter. Integration in (11) gives (9).
For double scattering [3],

$$\frac{dI}{d\omega}=I_{\mathrm{BH}}(\gamma\chi_1)+I_{\mathrm{BH}}(\gamma\chi_2)-\frac{e^2}{\pi^3\omega T}\iint d^2\xi_1 d^2\xi_2\frac{\partial}{\partial\vec{\xi}_1}K_0\left(\frac{\xi_1}{\gamma}\right)\cdot\frac{\partial}{\partial\vec{\xi}_2}K_0\left(\frac{\xi_2}{\gamma}\right)$$
$$\times\mathfrak{Im}\left(1-e^{-i\vec{\chi}_1\cdot\vec{\xi}_1}\right)\left(1-e^{-i\vec{\chi}_2\cdot\vec{\xi}_2}\right)e^{-i\frac{\omega T}{2\gamma^2}+i\frac{(\vec{\xi}_1-\vec{\xi}_2)^2}{2\omega T}}. \tag{12}$$

The latter representation elucidates the relevance of ray optic notions:

When $\chi \gg \gamma^{-1}$, exponentials $e^{-i\vec{x}_1\cdot\vec{\xi}_1}$, $e^{-i\vec{x}_2\cdot\vec{\xi}_2}$ are rapidly oscillating, and along with the Gaussian factor $e^{i\frac{(\vec{\xi}_1-\vec{\xi}_2)^2}{2\omega T}}$, they form stationary phase points, which define rays parallel to one of the external electron lines. The exponential decrease of impact-parameter-dependent photon distributions at these impact parameters give rise to jet formfactors – see Fig. 2.

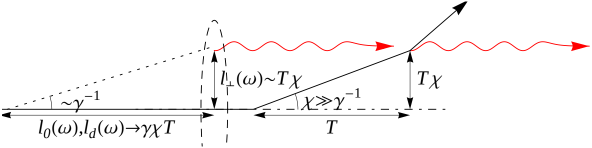


Figure: Diagram for collinear-loose radiation interference (intermediate ω).

The underlying reason for existence of interference effects in non-dipole radiation is manifestation of photon jets even when their angles are integrated over. One needs at least one jet for stability of the phase.
At that, one must distinguish intRA-jet radiation (narrowly collimated along parent electron lines) and intER-jet radiation (broadly distributed in between the jets). They both can take part in the interference – see previous figure.

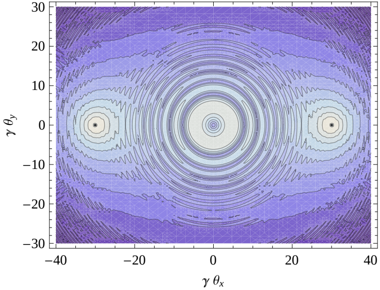
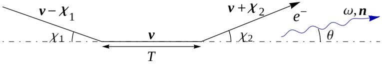


Figure: Angular distributions of radiation from a double scattering electron, with $\vec{\chi}_1 = \vec{\chi}_2$, $|\vec{\chi}_{1,2}| = 30\gamma^{-1}$, and $\frac{\omega T}{2\gamma^2} = 0.1$.

On the other hand, from the standpoint of representation (7), oscillations stem from endpoint contributions. Their slow exponential damping owes to decoherence – the spread of photon emission points along the external line, or the spread of radiation angles at emission from an external line.

Double hard scattering [Bondarenco, Shul'ga, to be published]



Let the electron undergo successive scattering through definite angles $\vec{\chi}_1$, $\vec{\chi}_2$, with $\chi_{1,2} \gg \gamma^{-1}$ separated by time interval T (see Fig.) (This case was already alluded to above). Doing all the relevant integrations (without averaging over $\vec{\chi}_{1,2}$ azimuths), one is led to the expression for the radiation spectrum

$$\frac{dI}{d\omega} \underset{\chi_{1,2}\gg\gamma^{-1}}{\simeq} I_{\mathrm{BH}}(\gamma\chi_1) + I_{\mathrm{BH}}(\gamma\chi_2) + \frac{2e^2}{\pi}\left[g\left(\frac{\omega T}{2\gamma^2}\right) + A_1\left(\frac{\omega T\chi_1^2}{2}, \frac{\omega T\chi_1\bar{\chi}_2}{2}\right) F_j\left(\frac{\omega T\chi_1}{\gamma}\right) \right.$$
$$\left. + A_1\left(\frac{\omega T\chi_2^2}{2}, \frac{\omega T\chi_1\bar{\chi}_2}{2}\right) F_j\left(\frac{\omega T\chi_2}{\gamma}\right) + A_2\left(\frac{\omega T\chi_1\bar{\chi}_2}{2}\right)\right]. \qquad (13)$$

Here $I_{\mathrm{BH}}(\gamma\chi)$ is given by Eq. (9), the "semi-bare electron" contribution

$$g\left(\frac{\omega T}{2\gamma^2}\right) = -\int_0^\infty \frac{d\theta^2\theta^2}{(\gamma^{-2}+\theta^2)^2}\cos\frac{\omega T}{2\gamma^2}\left(1+\gamma^2\theta^2\right) \qquad (14)$$

[similar to the non-dipole limit of the interference term in (2)] is independent of the scattering angles, while

$$A_1(z_1, z_2) = -\mathrm{Ci}(z_1) + \mathfrak{Re}\left\{\cos z_2 \mathrm{Ci}(z_1+z_2) + \sin z_2 \mathrm{si}(z_1+z_2)\right\}, \qquad (15)$$
$$A_2(z) = -\mathfrak{Re}\left\{\cos z \mathrm{Ci}(z) + \sin z \mathrm{si}(z)\right\} \qquad (16)$$

(independent of γ) may be interpreted as "antenna" form factors, with
$\chi_1\bar{\chi}_2 = (\chi_{1x}+i\chi_{1y})(\chi_{2x}-i\chi_{2y})$, and

$$F_j(z) = zK_1(z), \tag{17}$$

normalized by condition $F_j(0) = 1$, is a jet form factor, originating as shown in Fig. 2. Semi-bare electron (collinear-collinear interference) and antenna (collinear-loose interference) contributions, depend on different coherence lengths and exhibit oscillations in different spectral regions (see Fig. 4).

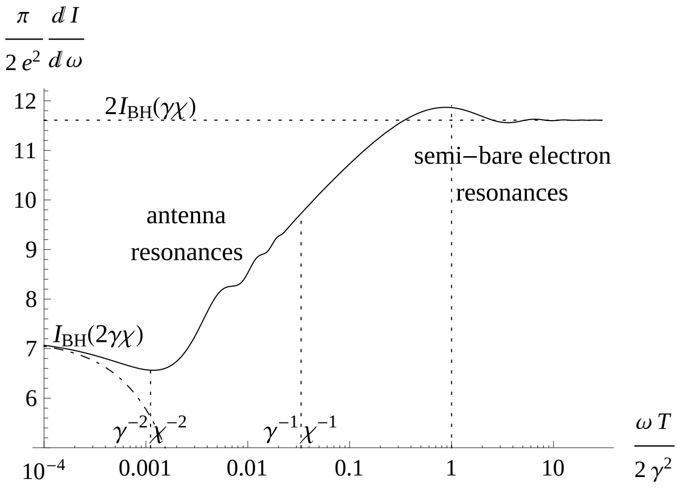
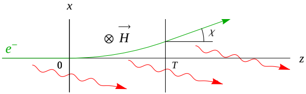
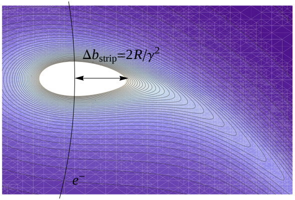


Figure: Spectrum of electromagnetic radiation from an electron scattering two times through equal co-planar angles $\chi = 30\gamma^{-1}$.

Radiation in a finite magnet





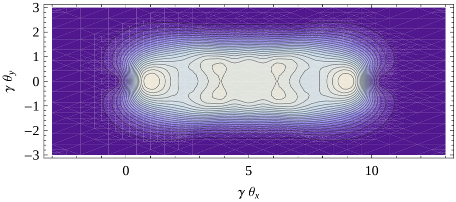
The problem of radiation from an electron passing through a finite domain of uniform magnetic field was studied in [8], but at that time not addressing issues of volume and edge contribution separation. Meanwhile, this is the benchmark case for such a separation.

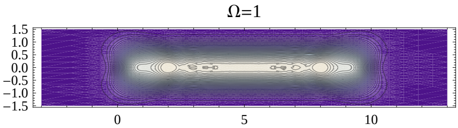
An appropriate impact parameter representation for this case may be unobvious, since there is a critical radius on the outer side of the electron's orbit, at which the radiation field is stripped from the electron's distorted proper field [6]. So, in a finite magnet the impact parameter analysis is generally more complicated than that for radiation at double scattering. But under highy non-dipole conditions,

$$\Delta b_{\text{strip}} \sim R/2\gamma^2 \ll \Delta b_{\text{interf}} \sim T\chi = R\chi^2.$$

Therefore, at intermediate ω the impact parameter relationships should be the same as for double scattering considered above.

There are still two photon jets pointing along initial and final electron lines. They are linked by a “belt", which represents the volume contribution.





It may as well suffice to use at all ω the double time integral representation (7). The result of integrations has a structure similar to that for radiation at double scattering

$$\frac{dI}{d\omega}=\frac{dI_{\text{syn}}}{d\omega}+\frac{2e^2}{\pi}\left[2J_{\text{edge}}(\Omega)+2A_1\left(\Omega X^3\right)F_j(\Omega X^2)+A_2\left(\Omega X^3\right)\right],$$

with $\Omega=\frac{\omega R}{2\gamma^3}$, $X=\frac{\gamma T}{R}=\gamma\chi$,

$$\frac{dI_{\text{syn}}}{d\omega}=2e^2X\left\{-(2\Omega)^{1/3}\mathrm{Ai}'\left[(2\Omega)^{2/3}\right]-\Omega\int_{(2\Omega)^{2/3}}^{\infty}d\alpha\mathrm{Ai}(\alpha)\right\},$$

the same F_j given by Eq. (17), and different antenna formfactors

$$A_1=-\frac{2}{\Omega X^3}\int_0^\infty\frac{du}{(1+u)^2}\left[\sin\frac{\Omega X^3}{2}\left(\frac{2}{3}+u\right)-\sin\frac{\Omega X^3}{3}(1+u)\right],$$

$$A_2=\int_0^\infty\frac{du}{1+u}\left[\cos\frac{\Omega X^3}{12}(1+3u)-\cos\frac{\Omega X^3}{12}(1+u)+\frac{2}{1+u}\cos\frac{\Omega X^3}{12}(1+u)^3\right],$$

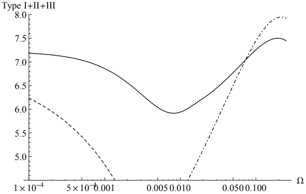
and with a different semi-bare electron contribution

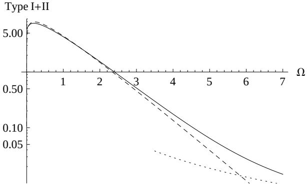
$$2J_{\text{edge}}(\Omega)=(2\Omega)^{\frac{2}{3}}\pi\mathrm{Gi}\left[(2\Omega)^{\frac{2}{3}}\right]-1$$

$$+\int_1^\infty\frac{dw}{w-\frac{3}{4}}\left\{1+\Omega^{\frac{2}{3}}\left[2\left(1-\frac{3}{4w}\right)^{\frac{2}{3}}-w\left(1-\frac{3}{4w}\right)^{-\frac{1}{3}}\right]\pi\mathrm{Gi}\left(\Omega^{\frac{2}{3}}w\left(1-\frac{3}{4w}\right)^{-\frac{1}{3}}\right)\right\}-2.$$

The latter represents a kind of transition radiation without atomic matter.

Note that here J_{edge} is non-oscillatory (although it changes its sign two times), because due to the trajectory bending, there is no collinear-collinear interference. Also note that J_{edge} is not positive definite, so it cannot be regarded as an independent radiation intensity, but rather as an edge effect.





At high ω, the semi-bare electron (edge) contribution falls off according to a power law

$$\frac{dI}{d\omega} \underset{\Omega\to\infty}{\simeq} \frac{2e^2}{\pi} 2J_{\mathrm{edge}}(\Omega) \simeq \frac{7e^2}{15\pi\Omega^2}. \tag{18}$$

The scaling law $\frac{dI}{d\omega} \underset{\omega\to\infty}{\sim} \omega^{-2}$ is the key prediction made in [8]; here we have also determined its coefficient.

General case. Separation of volume and edge contributions

Whenever there is a finite uniform target,

$$\frac{dI}{d\omega} = \frac{dI_{\mathrm{vol}}}{d\omega} + \frac{2e^2}{\pi}\left[2J_{\mathrm{edge}}\left(\frac{\omega}{\tilde{\omega}}\right) + J_{\mathrm{interf}}\right].$$

with $\tilde{\omega}$ – a “typical" frequency ($\tilde{\omega} = \max\{\frac{2\gamma^2}{T}, \frac{2\gamma^3}{R}\}$), and

$$J_{\mathrm{interf}} = 2A_1\left(\frac{\omega T\chi^2}{2}\right)F_j\left(\frac{\omega T\chi}{\gamma}\right) + A_2\left(\frac{\omega T\chi^2}{2}\right).$$

Generally, “antenna" may be defined as the *sufficiently bent* electron's trajectory without the electron itself, along which the electric current (the electron motion) flows exactly at the speed of light ($\gamma \to \infty$). In fact, antenna resums all γ-independent contributions through all orders [beyond $\mathcal{O}(\omega)$, cf. Eq. (8)].
In case if the target edge has a non-zero width ΔT,

$$J_{\mathrm{edge}} \to J_{\mathrm{edge}}F_{\mathrm{edge}}(\omega\Delta T/\gamma^2).$$

N.B.: Single-edge contribution is always logarithmically divergent at $\omega \to 0$ (cf. [7]), but this divergence is cancelled by the antenna (edge interference) contribution.

Quadrupole form factor for an amorphous finite target

A similar decomposition must exist also for radiation at electron passage through finite slab of amorphous target.
But instead of repeating the corresponding procedure, though, let us emphasize that in principle, non-dipole interference effects can be discernible even at relatively weak scattering strengths.

$$\frac{dI}{d\omega} = \frac{2e^2}{3\pi}\gamma^2\overline{\chi^2}\left[1 - \frac{3\gamma^2\overline{\chi^2}}{10}F_q\left(\frac{\omega T}{2\gamma^2}\right) + \mathcal{O}\left(\gamma^4\overline{\chi^4}\right)\right],$$

with

$$F_q(\Omega) = \frac{80}{\Omega^2}\int_0^\infty \frac{duu}{(1+u)^8}\sin^2\frac{\Omega}{2}(1+u)$$

the quadrupole formfactor normalized by $F_q(0) = 1$. Here $\int_0^\infty d\Omega F(\Omega) \neq 0$ (in fact, $F_q > 0$), because averaging in a continuous medium permits photon emission points to coalesce: $t_1 \to t_2$.

Summary

- Jet effects can be essential even in angle-integral radiation spectra.
- Transition radiation exists for all kinds of boundaries, not necessarily between vacuum and atomic matter.
- There is ‘antenna’ radiation from an ultra-relativistic, significantly deflecting electron.
- For separation of volume, edge and antenna contributions there exists a rigorous non-dipole decomposition.
- Weakly non-dipole radiation spectrum can be described by a quadrupole form factor.

Some references

R. Blankenbecler and S.D. Drell,
Phys. Rev. D **53**, 6265 (1996).

K.K. Andersen *et al.*,
Phys. Lett. B **732**, 309 (2014).

M.V. Bondarenco and N.F. Shul'ga,
Phys. Rev. D **90**, 116007 (2014).

V.N. Baier, V.M. Katkov, V.M. Strakhovenko,
Electromagnetic Processes at High Energies in Oriented Crystals, Singapore, 1998.

V.N. Baier and V.M. Katkov,
Phys. Rev. D **60**, 076001 (1999).

X. Artru,
in: *Advanced Radiation Sources and Applications, NATO Sci. Ser. II*, **199**, 387 (2006).

I.I. Gol'dman,
Sov. Phys. JETP **11**, 1341 (1960).

V.G. Bagrov, N.I. Fedosov, and I.M. Ternov,
Sov. Phys. JETP **55**, 835 (1982)]; *Phys. Rev. D* **28**, 2464 (1983).

Y. Takabayashi, V.G. Bagrov, O.V. Bogdanov, Yu.L. Pivovarov, and T.A. Tukhfatullin,
NIM B **355**, 188 (2015).