What we really know about the Neutrino Mixing Matrix !

Stephen Parke, Fermilab

with Mark Ross-Lonergan, Durham University

invisibles
neutrinos, dark matter & dark energy physics

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PMNS matrix

500 km/GeV

15,000 km/GeV

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Mass Eigenstates Labeled by Decreasing ν_e content

flavor states

Mass Eigenstates

- $|\delta m^2_{31}| \approx 30\ \delta m^2_{21} > 0$ SNO

- Normal Ordering: $m_1^2 < m_2^2 < m_3^2$
and Inverted Ordering: $m_3^2 < m_1^2 < m_2^2$

NOνA, LBNF, HyperK
PINGU, ORCA $\cdots$

Usual representation:

23	13	12	$0\nu\beta\beta$ Decay

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \mathrm{diag}(1,\ e^{i\frac{\alpha_{21}}{2}},\ e^{i\frac{\alpha_{31}}{2}})$$

Atmospheric	Reactor/Interference	Solar
$\boxed{\mu \to \tau}$ 500 Km/GeV	$\boxed{\mu \leftrightarrow e}$ 500 Km/GeV	$\boxed{\mu \to e}$ 15,000 Km/GeV

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

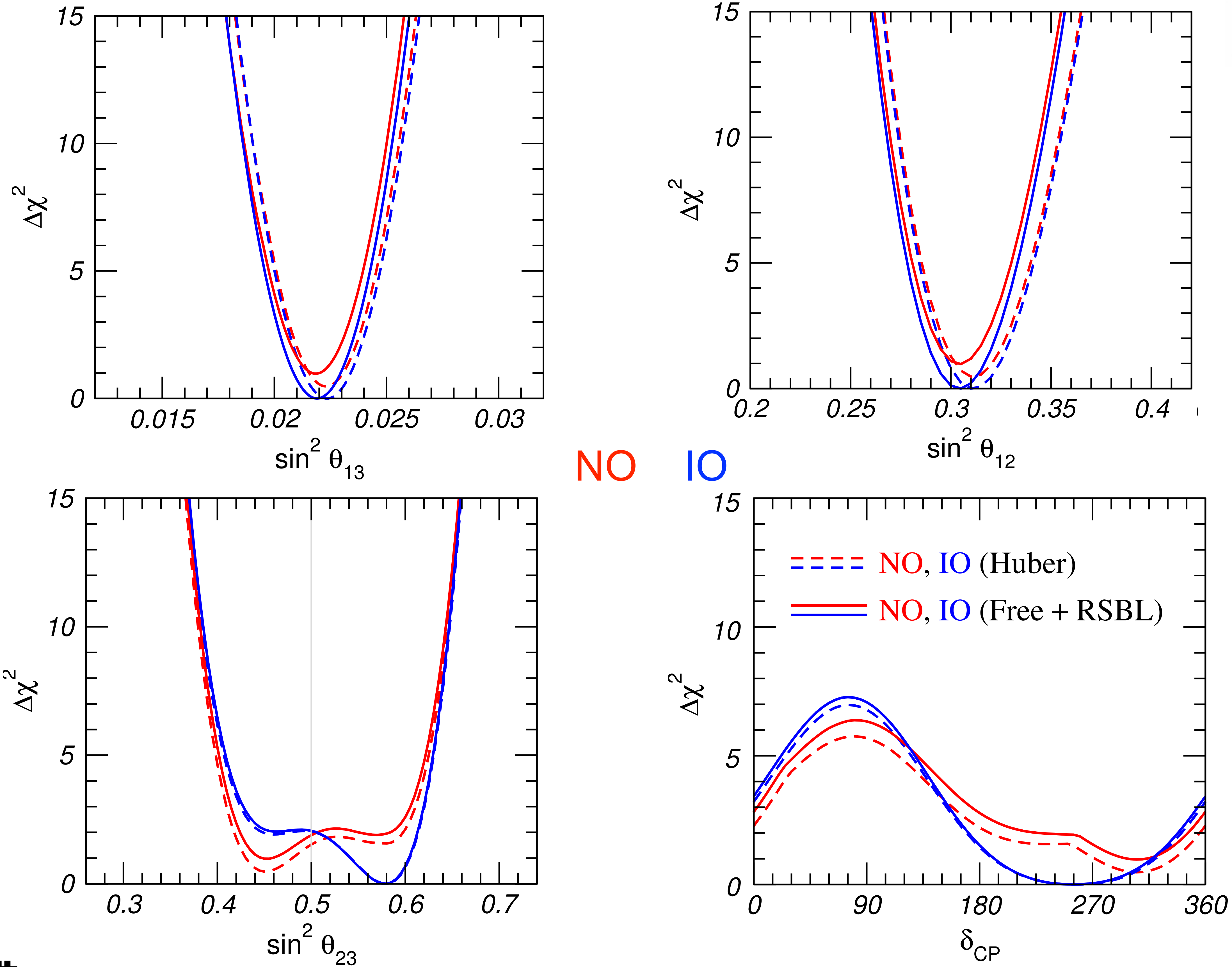
$$\times\ \mathrm{diag}(1,\ e^{i\frac{\alpha_{21}}{2}},\ e^{i\frac{\alpha_{31}}{2}})\ .$$

ignore !!!

UNITARITY IS BUILT IN: $U^\dagger U = 1$

Global Fits:

www.nu-fit.org

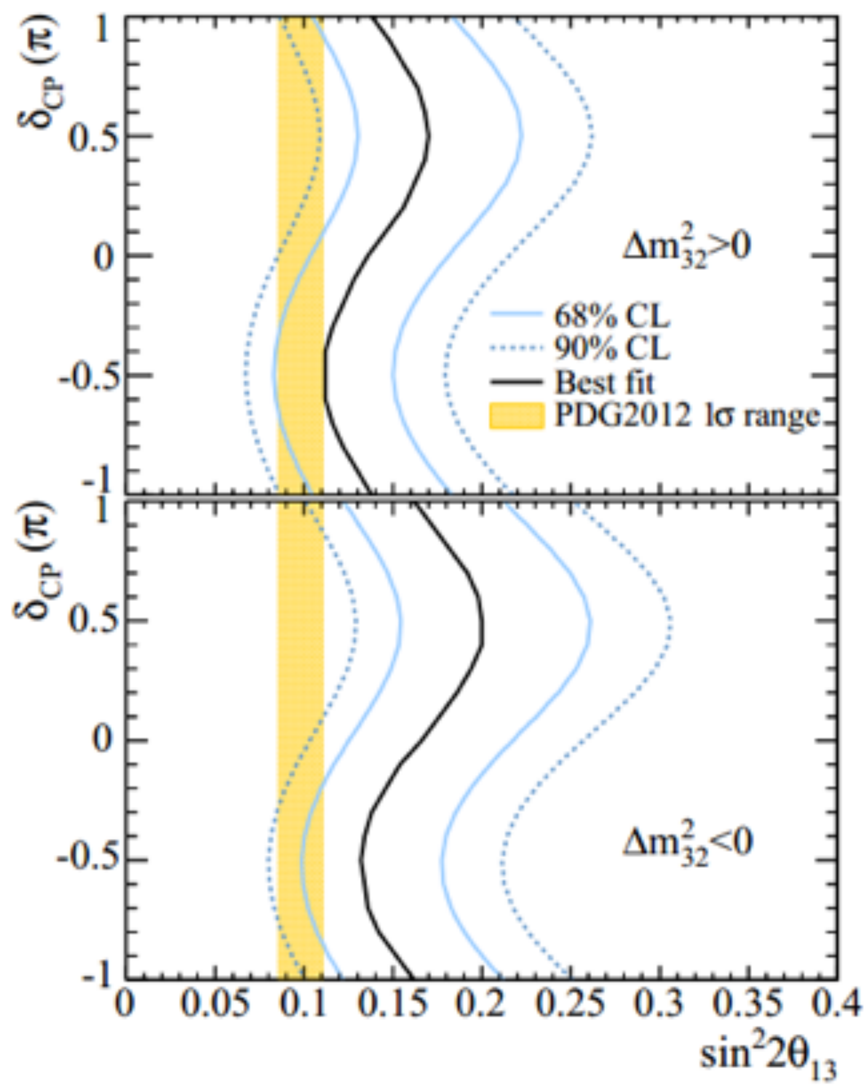


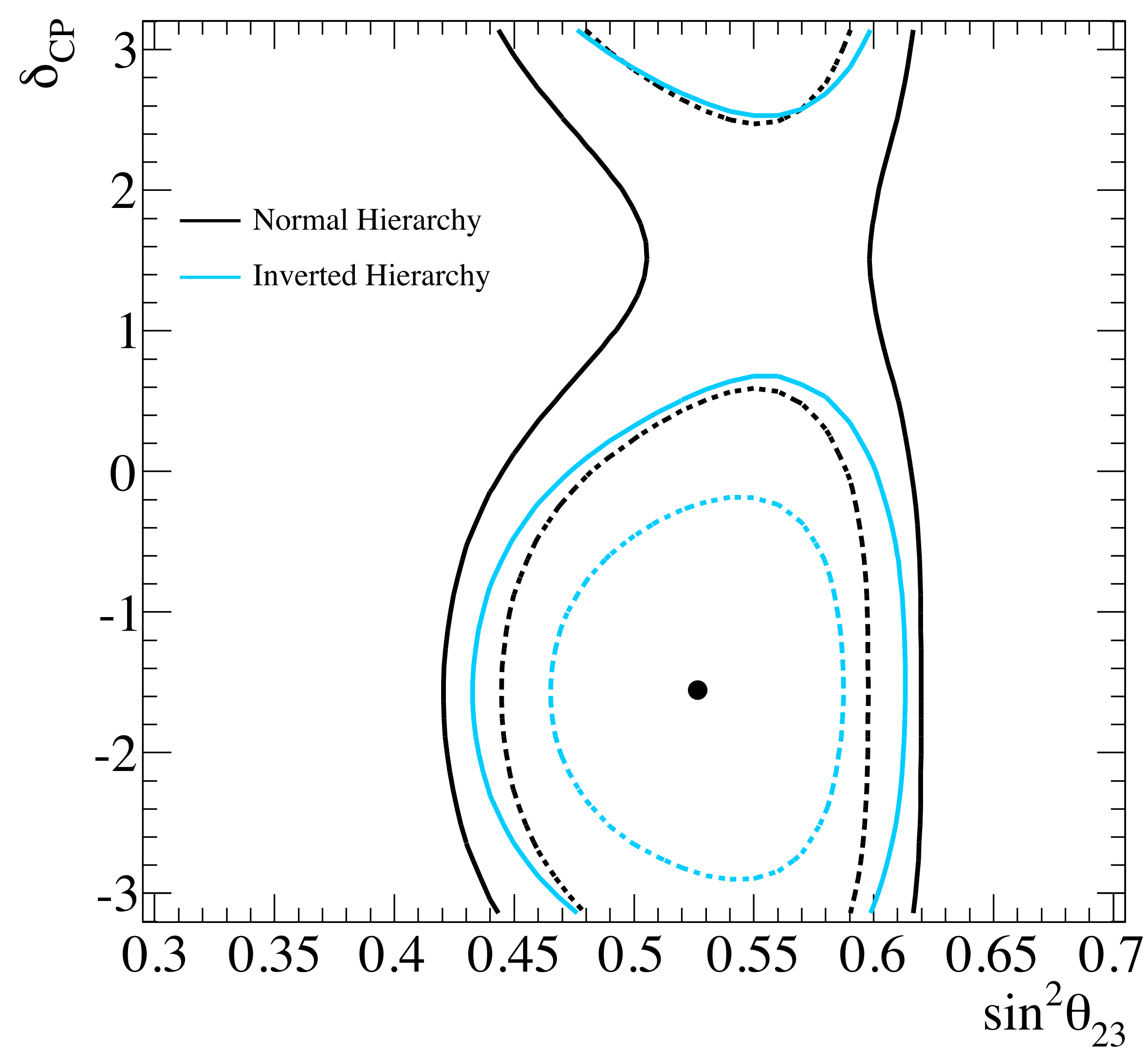
Reactors and $\nu_\mu \rightarrow \nu_e$ Appearance

$$1 - P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 4 \sin^2\theta_{13} \ \sin^2\Delta_{ee}$$

$$P(\nu_\mu \rightarrow \nu_e) = 4 \sin^2\theta_{23} \sin^2\theta_{13} \ \sin^2\Delta_{ee} + \cdots$$

T2K: 1502.01550

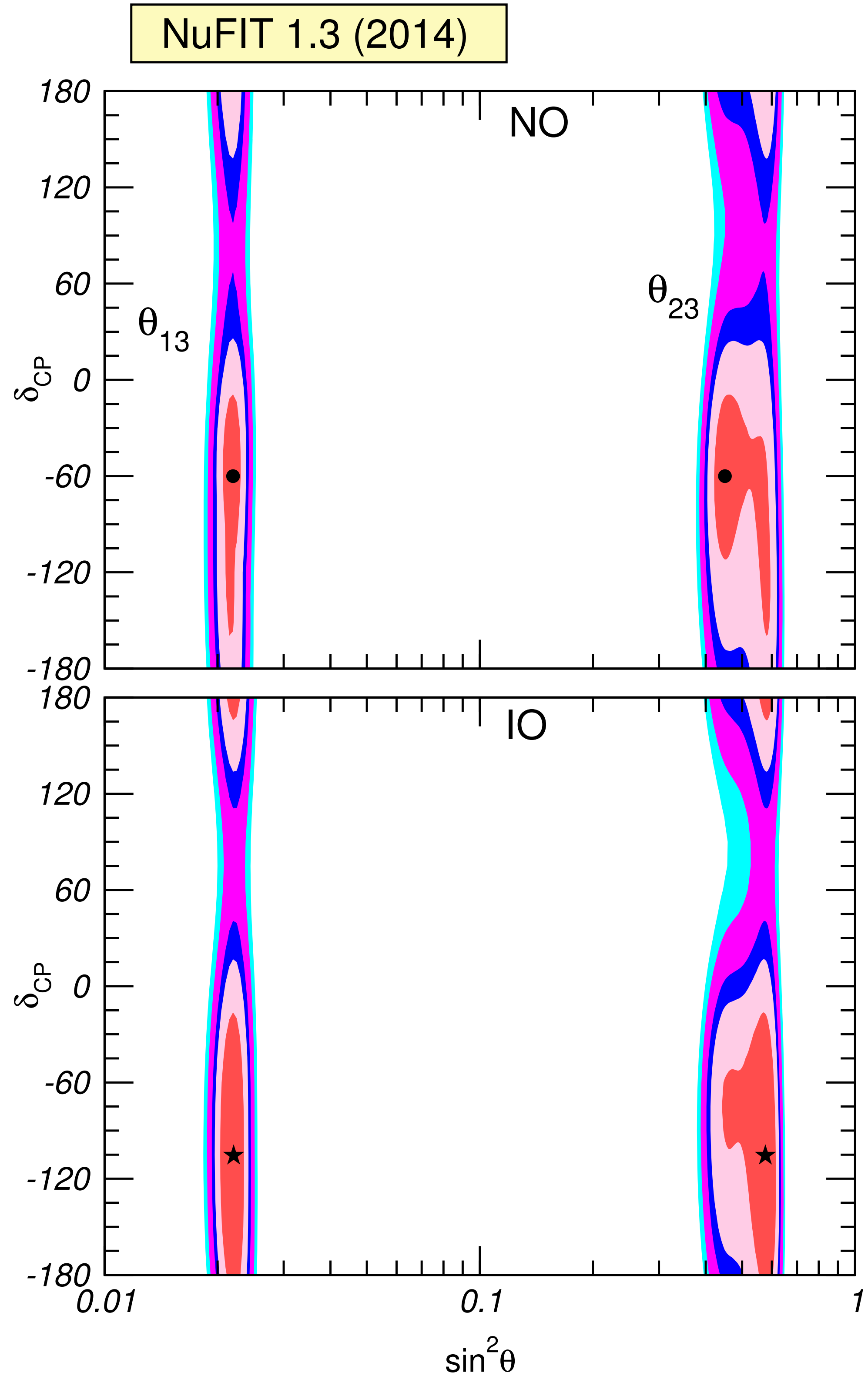




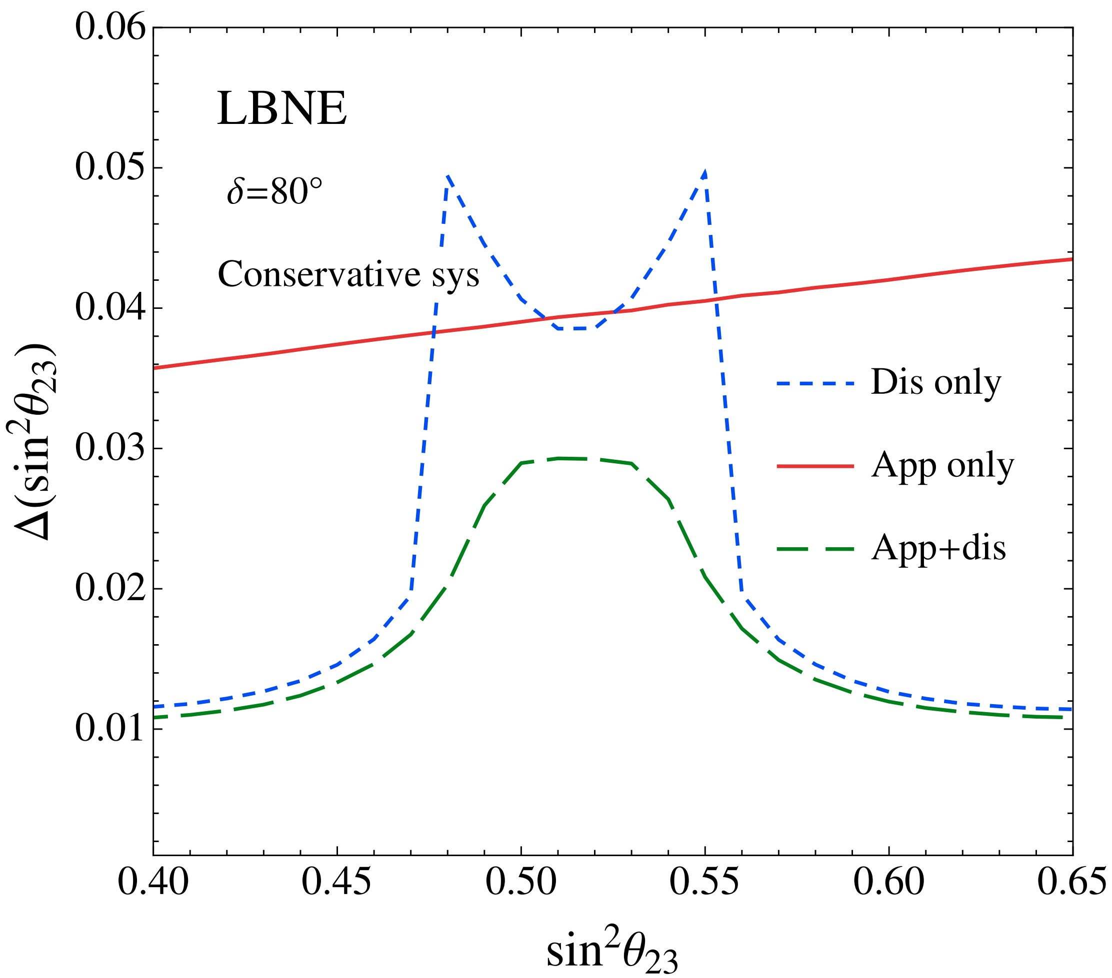
Marginalized over θ_{23}

Marginalized over θ_{13}

Together:

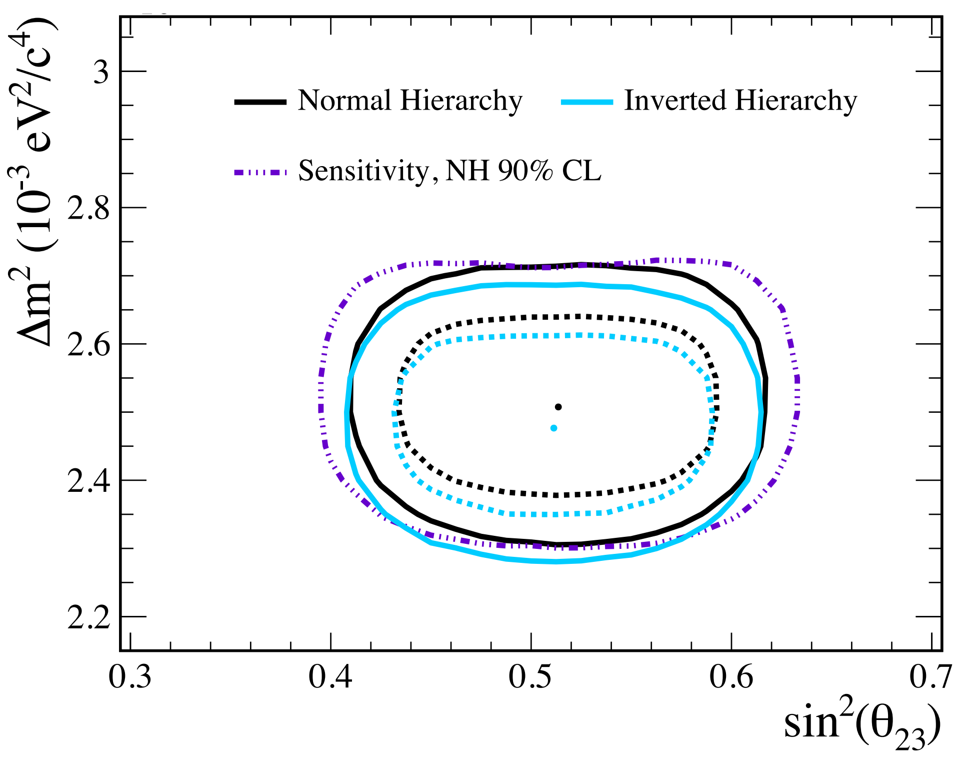

NuFIT 1.3 (2014)
NO
IO
θ_{13}
θ_{23}
δ_{CP}
180
120
60
0
-60
-120
-180
0.01
0.1
1
$\sin^2\theta$

θ_{23} from Appearance:



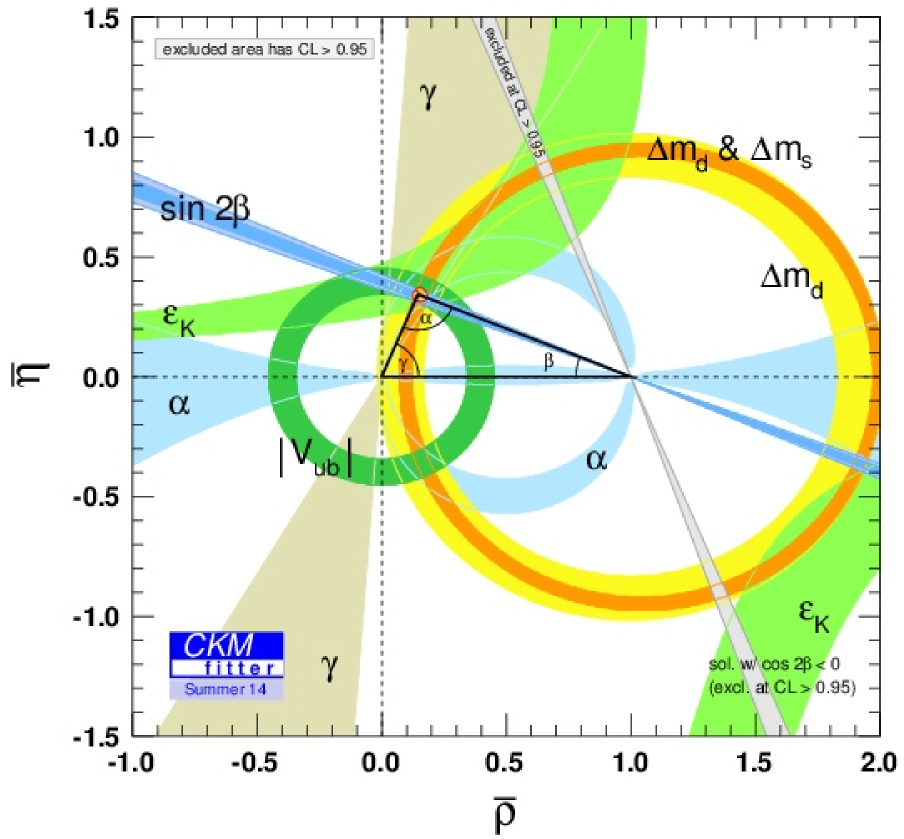
Coloma, Minakata and SP 1406.2551

T2K: 1502.01550



Unitarity Triangles:

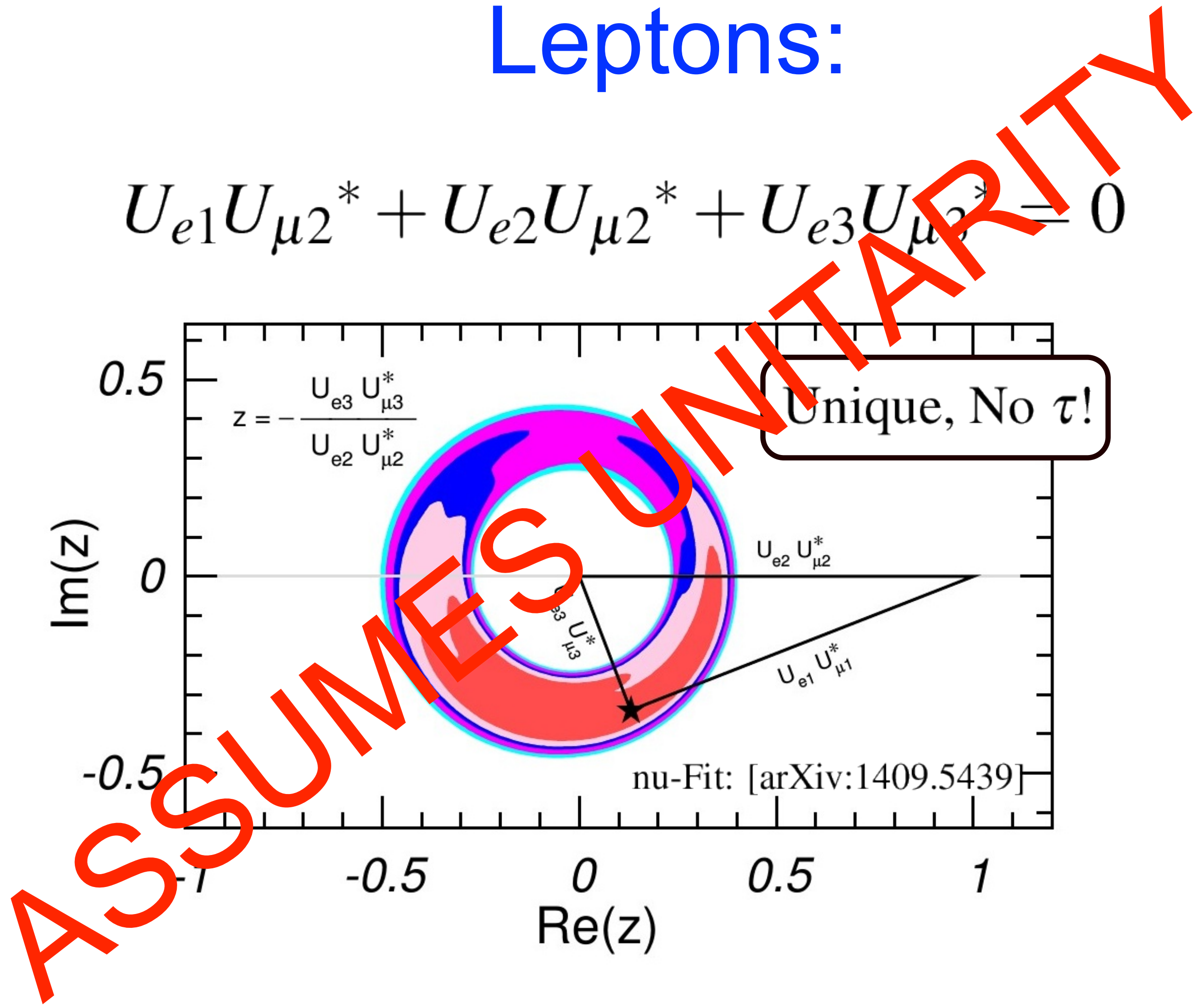
Quarks:



Unitarity *Not* assumed

Leptons:

$$U_{e1}U_{\mu 2}{}^* + U_{e2}U_{\mu 2}{}^* + U_{e3}U_{\mu 3}{}^* = 0$$

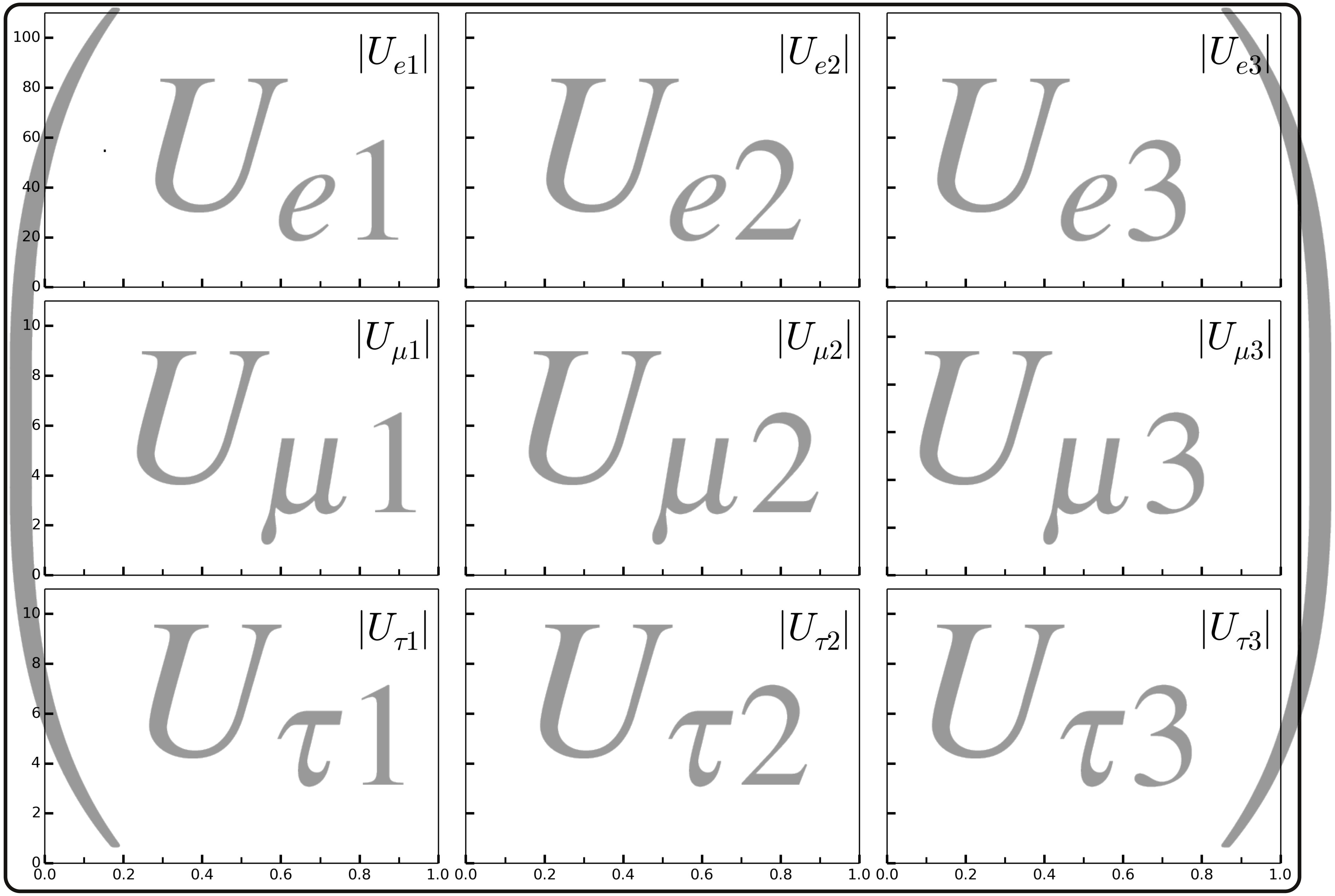


Unitarity *Is* assumed.

$$|J| = 2 \times \text{Area}$$
$$= |s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin\delta_{CP}|$$

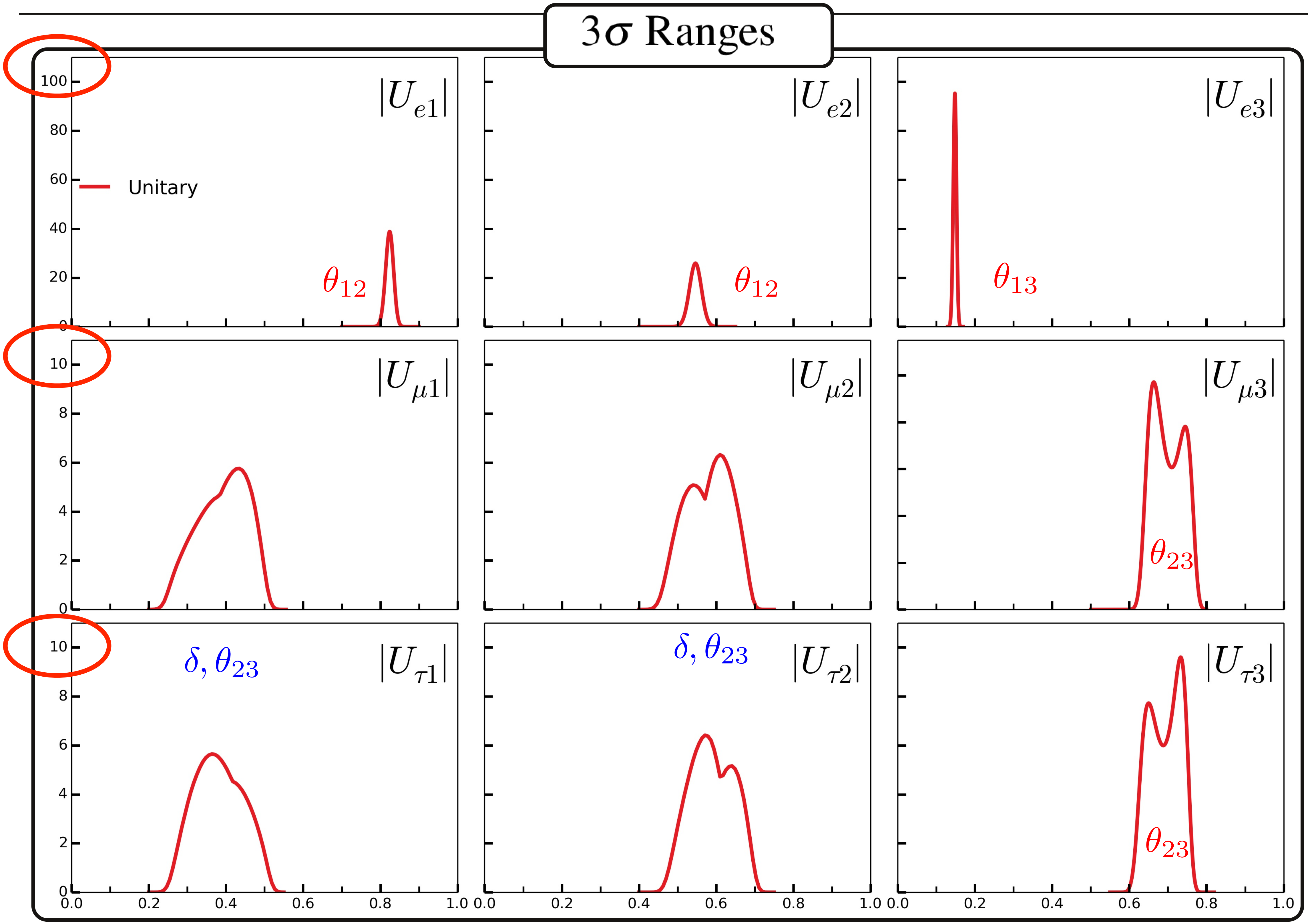
$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Visualisation of precision



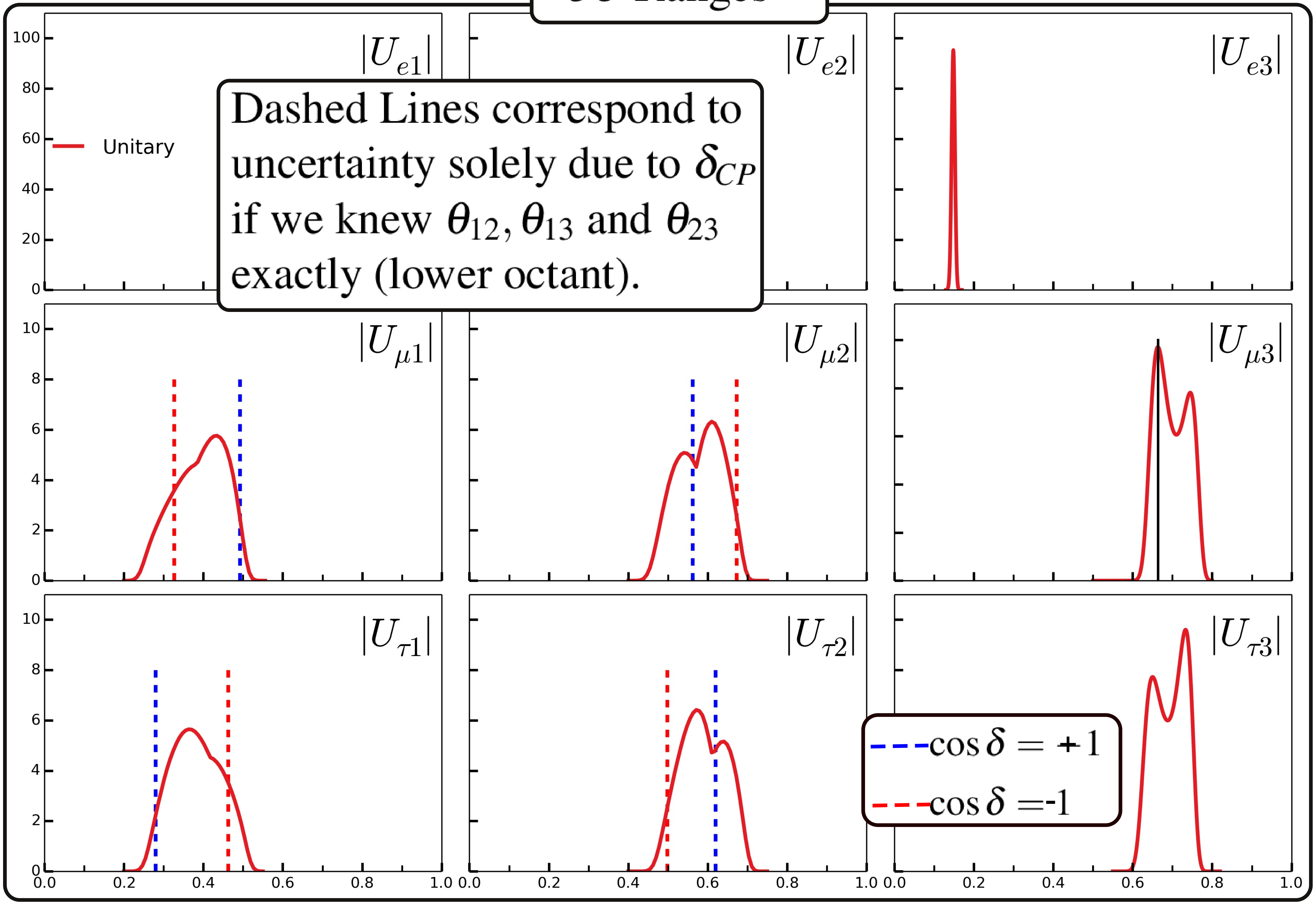
Probability Distribution for |U|

note scales



*Agrees with contemporary global fits to within $\mathcal{O}(1\%)$ precision at 3 σ.

3σ Ranges



Unitary
$|U_{e1}|$
$|U_{e2}|$
$|U_{e3}|$
$|U_{\mu1}|$
$|U_{\mu2}|$
$|U_{\mu3}|$
$|U_{\tau1}|$
$|U_{\tau2}|$
$|U_{\tau3}|$
Dashed Lines correspond to uncertainty solely due to δ_{CP} if we knew θ_{12}, θ_{13} and θ_{23} exactly (lower octant).
$\cos\delta = +1$
$\cos\delta =$-1

Non-Unitary 3x3

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$U^{3\times 3}_{PMNS} = \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}|e^{i\delta_{\mu 1}} & |U_{\mu 2}|e^{i\delta_{\mu 2}} & |U_{\mu 3}| \\ |U_{\tau 1}|e^{i\delta_{\tau 1}} & |U_{\tau 2}|e^{i\delta_{\tau 2}} & |U_{\tau 3}| \end{pmatrix}$$

- 13 real parameters after rephrasing the leptonic fields !
- compared to 4 real parameters for unitary case.

ν_μ disappearance: L/E ~ 500 km/GeV

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ \boxed{U_{\mu 1} \quad U_{\mu 2} \quad U_{\mu 3}} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

SK, K2K,
MINOS, T2K,
NOvA, ...

$$|U_{\mu 3}|^2(1-|U_{\mu 3}|^2) \quad \Rightarrow \quad \frac{|U_{\mu 3}|^2(\ |U_{\mu 1}|^2+|U_{\mu 2}|^2\ )}{(\ |U_{\mu 1}|^2+|U_{\mu 2}|^2+|U_{\mu 3}|^2\ )}$$

Solar:

SNO (CC/NC ratio), ...

$$\begin{pmatrix} U_{e1} & \boxed{U_{e2}} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$|U_{e2}|^2 \quad \Rightarrow \quad \frac{|U_{e2}|^2}{(\ |U_{e2}|^2+|U_{\mu 2}|^2+|U_{\tau 2}|^2\ )}$$

- also SNO's NC fluxes constrains $|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2$

ν_e disappearance: L/E ~ 500 m/MeV

$$\begin{pmatrix} \boxed{U_{e1} \quad U_{e2} \quad U_{e3}} \\ U_{\mu 1} \quad U_{\mu 2} \quad U_{\mu 3} \\ U_{\tau 1} \quad U_{\tau 2} \quad U_{\tau 3} \end{pmatrix}$$

Daya Bay,
RENO,
Double Chooz

$$|U_{e3}|^2(1-|U_{e3}|^2) \quad \Rightarrow \quad \frac{|U_{e3}|^2(\ |U_{e1}|^2+|U_{e2}|^2\ )}{(\ |U_{e1}|^2+|U_{e2}|^2+|U_{e3}|^2\ )}$$

ν_e disappearance: L/E ~ 15 km/MeV

KamLAND wiggles

$$\begin{pmatrix} \boxed{U_{e1} \quad U_{e2} \quad U_{e3}} \\ U_{\mu 1} \quad U_{\mu 2} \quad U_{\mu 3} \\ U_{\tau 1} \quad U_{\tau 2} \quad U_{\tau 3} \end{pmatrix}$$

$$|U_{e1}|^2 |U_{e2}|^2 \quad \Rightarrow \quad \frac{|U_{e1}|^2 |U_{e2}|^2}{(\ |U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2\ )}$$

ν_τ appearance: L/E ~ 500 km/GeV

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Opera and SK

$$|U_{\tau 3}|^2 |U_{\mu 3}|^2$$

$$\Rightarrow \mathcal{R}\{-U^*_{\tau 3} U_{\mu 3} \, (U_{\tau 1} U^*_{\mu 1} + U_{\tau 2} U^*_{\mu 2})\}$$

ν_e appearance: L/E ~ 500 km/GeV

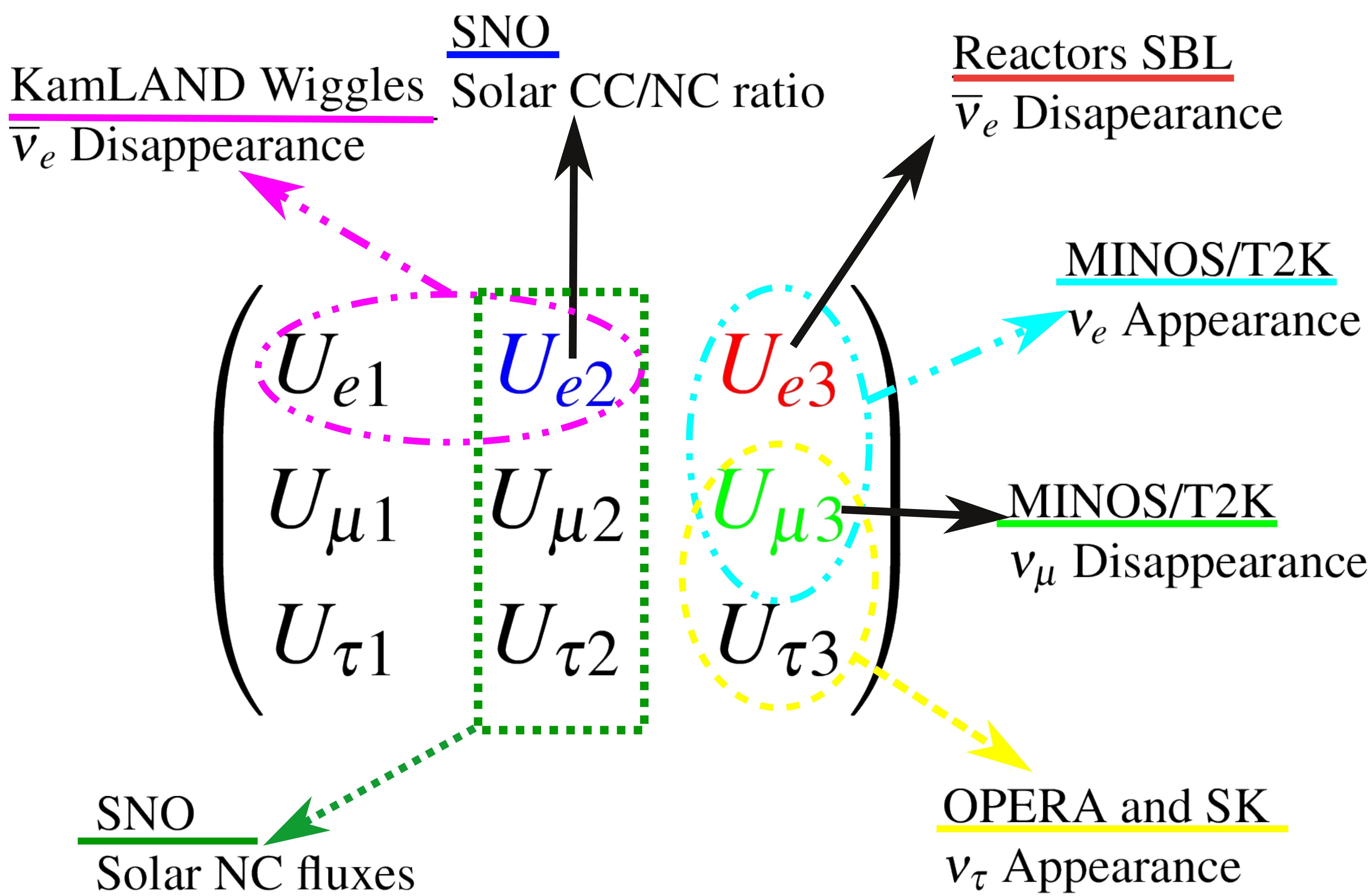
T2K, MINOS
NOvA,
LBNF, HyperK,
SuperPINGU, ...

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$|U_{e3}|^2 |U_{\mu 3}|^2 + \cdots$$

$$\Rightarrow \mathcal{R}\{-U^*_{e3} U_{\mu 3} \, (U_{e1} U^*_{\mu 1} + U_{e2} U^*_{\mu 2})\} + \cdots$$

Summary (unitary case):

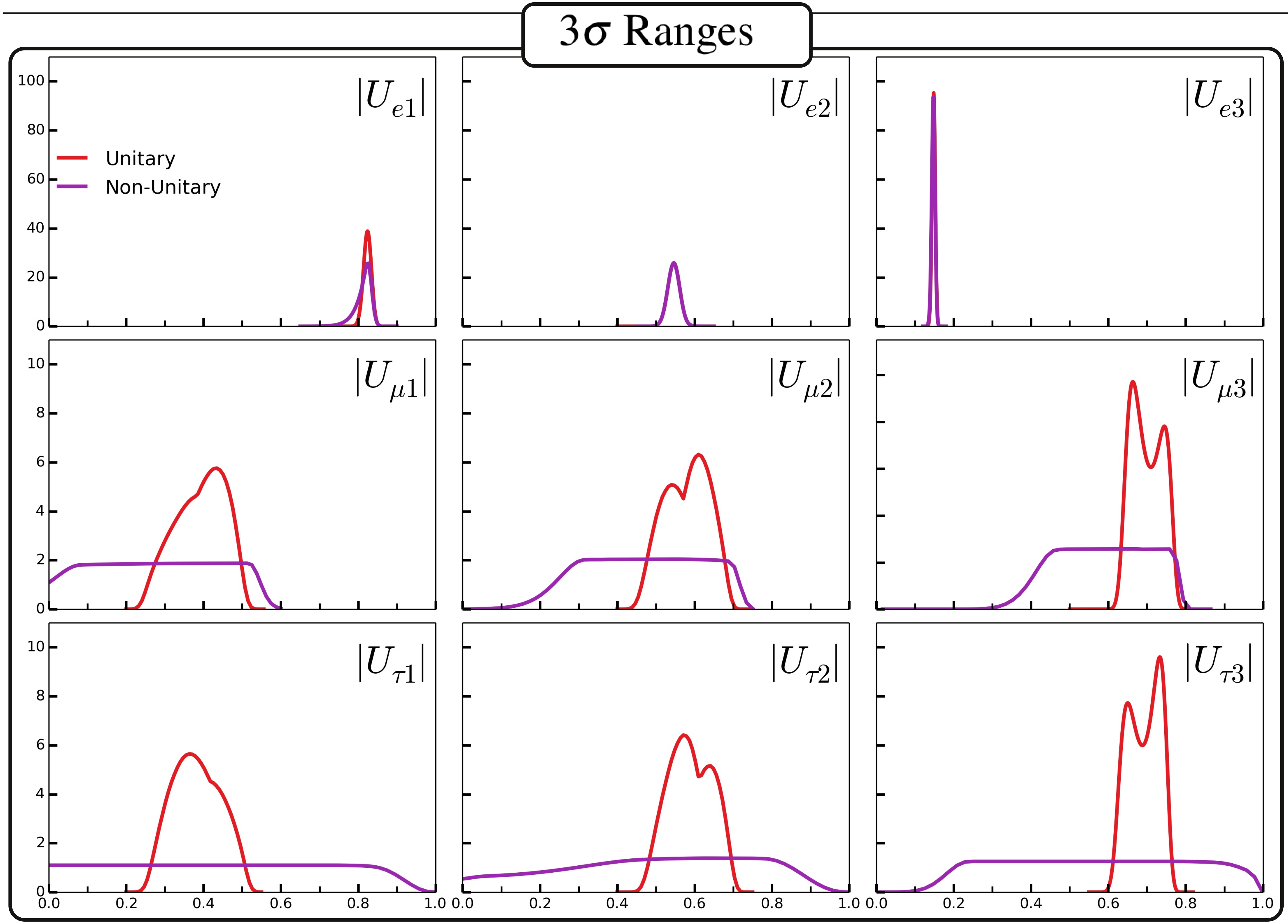


where is our information ?
non-unitary case:

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- Only places the degeneracy is broken between $|U_{\alpha 1}|$ and $|U_{\alpha 2}|$:
- KamLAND wiggles and SNO's NC flux plus feed through ! ! !

Non-Unitary ! ! !



3σ Ranges
|U_e1|
|U_e2|
|U_e3|
|U_μ1|
|U_μ2|
|U_μ3|
|U_τ1|
|U_τ2|
|U_τ3|
Unitary
Non-Unitary
100
80
60
40
20
0
10
8
6
4
2
0.0
0.2
0.4
0.6
0.8
1.0

What about Theory ? ? ?

The Minimal Unitary Violation (MUV) Scheme

- Assume extra fermionic singlets introduced via some new high energy physics. New high scale physics is still $SU(2)_L \times U(1)_Y$ symmetric.

$\propto (\overline{L}\phi)(\phi^\dagger L)$: Usual neutrino mass upon electroweak breaking

- $\mathscr{L}_{\text{MUV}} = \mathscr{L}_{\text{SM}} + \delta\mathscr{L}^{d=5} + \delta\mathscr{L}^{d=6}$

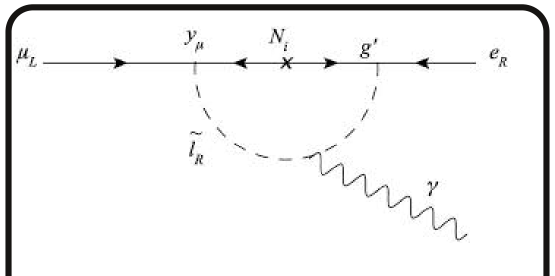
$\propto (\overline{L}\phi)\, i\partial\!\!\!/\, (\phi^\dagger L)$

Extra neutrino kinetic terms which upon canonical normalization, lead to non-unitary mixing

- Experimentally bounded by a plethora of experiments;
- Oscillation experiments, Lepton Universality, Rare Lepton Decays, Electroweak precision measurements, CKM precision measurements, Gauge Boson Decays ... etc ..

S. Antusch, C. Biggio, E. Fernandez-Martinez, M. Gavela, and J. Lopez-Pavon, JHEP 0610, 084 (2006), arXiv:hep-ph/0607020.

Rare Lepton Decays : $\mu \to e\gamma$
MEG Experiment



$BR < 5.7 \times 10^{-13}$

$$|U_{e1}U_{\mu 2}{}^* + U_{e2}U_{\mu 2}{}^* + U_{e3}U_{\mu 3}{}^*| < 1.5 \times 10^{-5}$$

Post Neutrino 2014 results, at the 90 % C.L, the bounds on the unitarity violation of U_{PMNS} is given by

Experimentally unitary at $\mathcal{O}(0.1\%)$ level!

$$|U^\dagger U| = \begin{pmatrix} 0.9978 - 0.9998 & < 10^{-5} & < 0.0021 \\ < 10^{-5} & 0.9996 - 1.0 & < 0.0008 \\ < 0.0021 & < 0.0008 & 0.9947 - 1.0 \end{pmatrix}$$

S. Antusch and O. Fischer, (2014), arXiv:1407.6607 [hep-ph]

Lite Sterile Neutrinos

- Eg. $\mathscr{O}(eV)$ sterile neutrino and $\mu \to e\gamma$.

	SM	SM $+ \nu$ Mass	MUV	$\mathscr{O}(eV)$ Sterile
$\mu \to e\gamma$	No	Yes	Yes	Yes
GIM	Yes	Supressed $\frac{m_\nu^4}{m_W^4}$	No	Supressed $\frac{m_s^4}{m_W^4}$
BR	0	$\approx 10^{-40}$	$\approx 10^{-13}$	$\approx 10^{-30} \to 10^{-40}$

- In MUV, the GIM mechanism cannot take place at all, meaning branching ratio's of 10^{-13} can be obtained for % level unitarity violation. This is highly constraining based on MEG's most recent results
- If, however, the non-unitarity is due to low-energy physics then the branching ratio merely increases mildly, still well below what's experimentally possible to measure.

Theoretical Geometric Bounds:

Non-Unitarity soley from extended PMNS matrix

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & \cdots & U_{eN} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & \cdots & U_{\mu N} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & \cdots & U_{\tau N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ U_{s_n 1} & U_{s_n 2} & U_{s_n 3} & \cdots & U_{s_n N} \end{pmatrix}$$

- Form Cauchy–Schwarz inequalities using new sterile elements

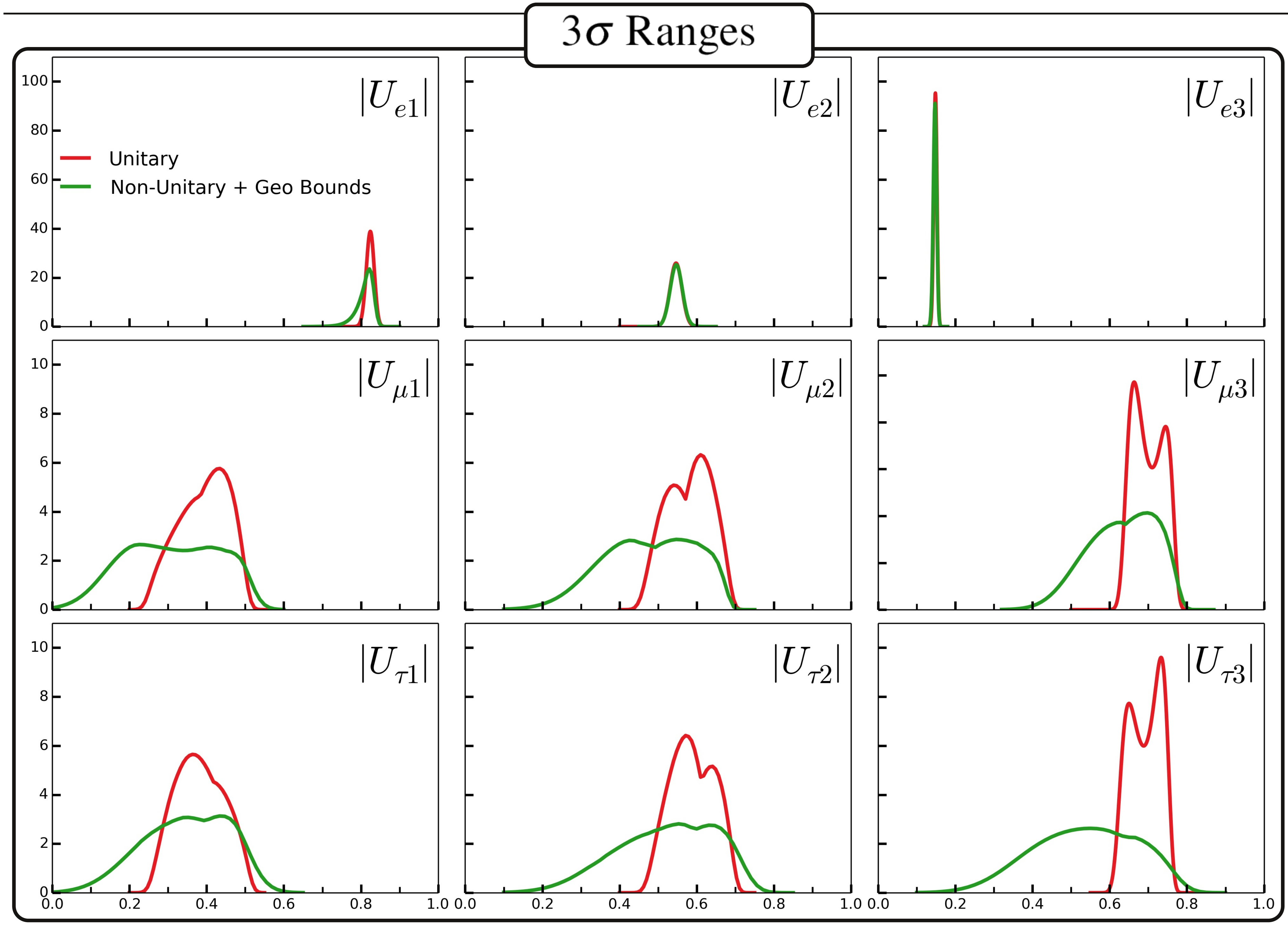
$$|U_{e4}U^*_{\mu 4} + \cdots U_{eN}U^*_{\mu N}|^2 \leq (|U_{e4}|^2 + \cdots |U_{eN}|^2)(|U_{\mu 4}|^2 + \cdots |U_{\mu N}|^2)$$

and as total $N \times N$ mixing matrix is unitary,

$$|U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e3}U^*_{\mu 3}|^2 \leq (1 - |U_{e1}|^2 - |U_{e2}|^2 - |U_{e3}|^2)(1 - |U_{\mu 1}|^2 - |U_{\mu 2}|^2 - |U_{\mu 3}|^2)$$

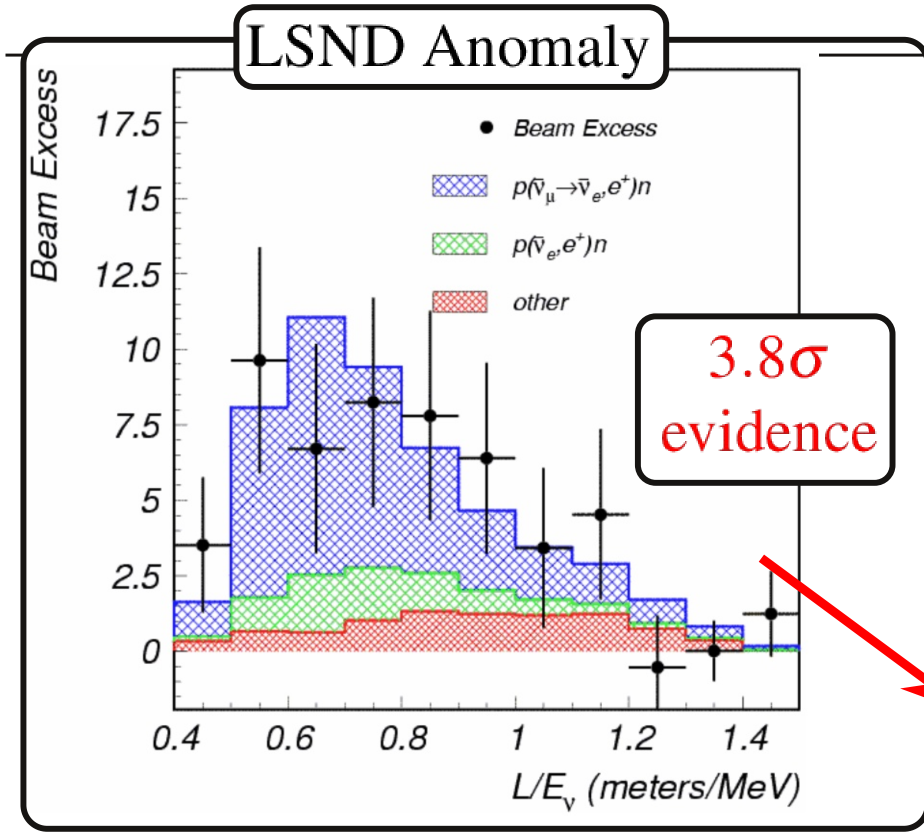
$$\mathcal{O}(\varepsilon^2)$$

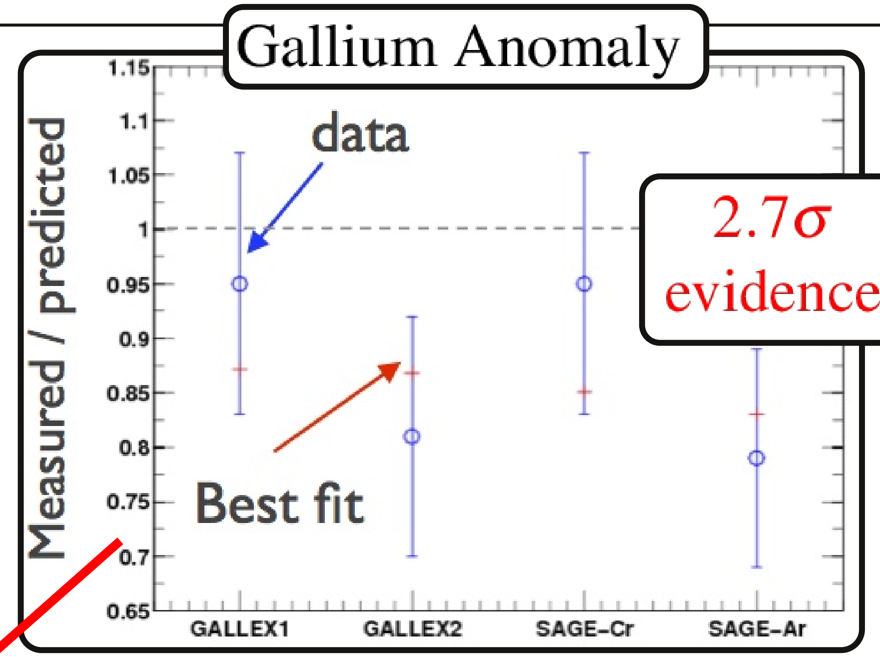
Theoretical Geometric Bounds:



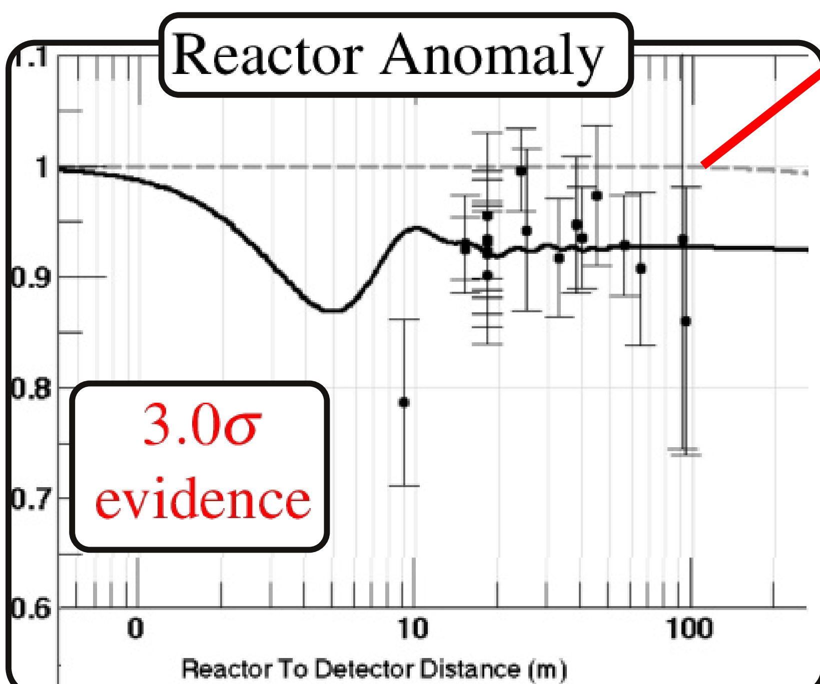
Most Assumption Independent that is theoretically motivated !

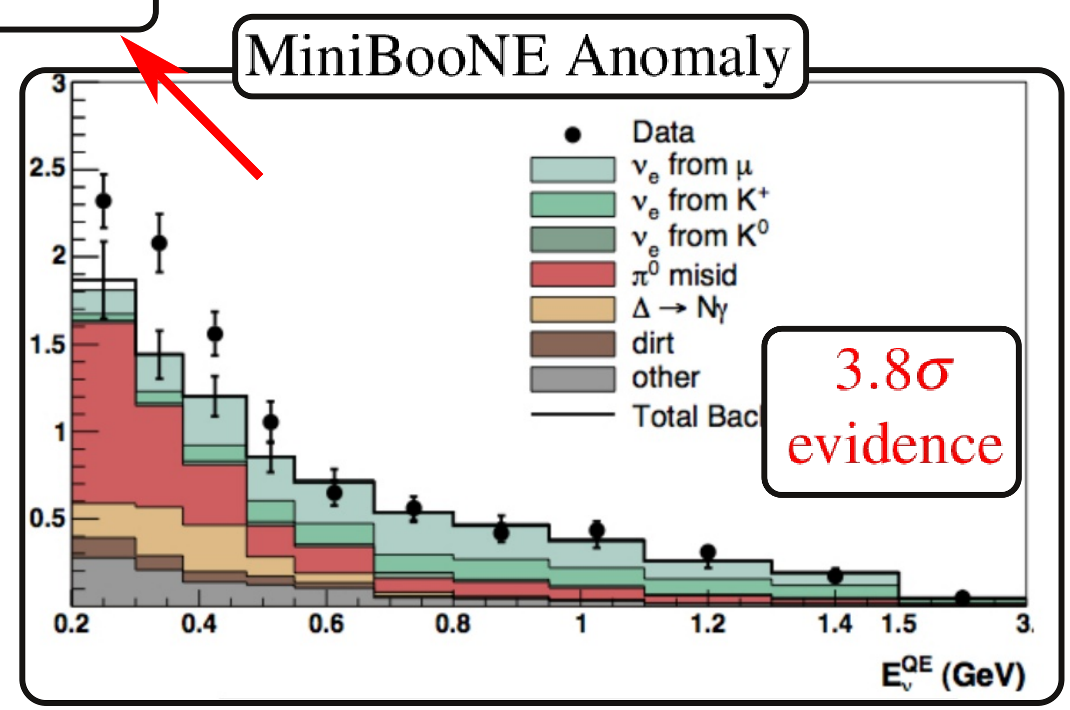
Current Anomalies !



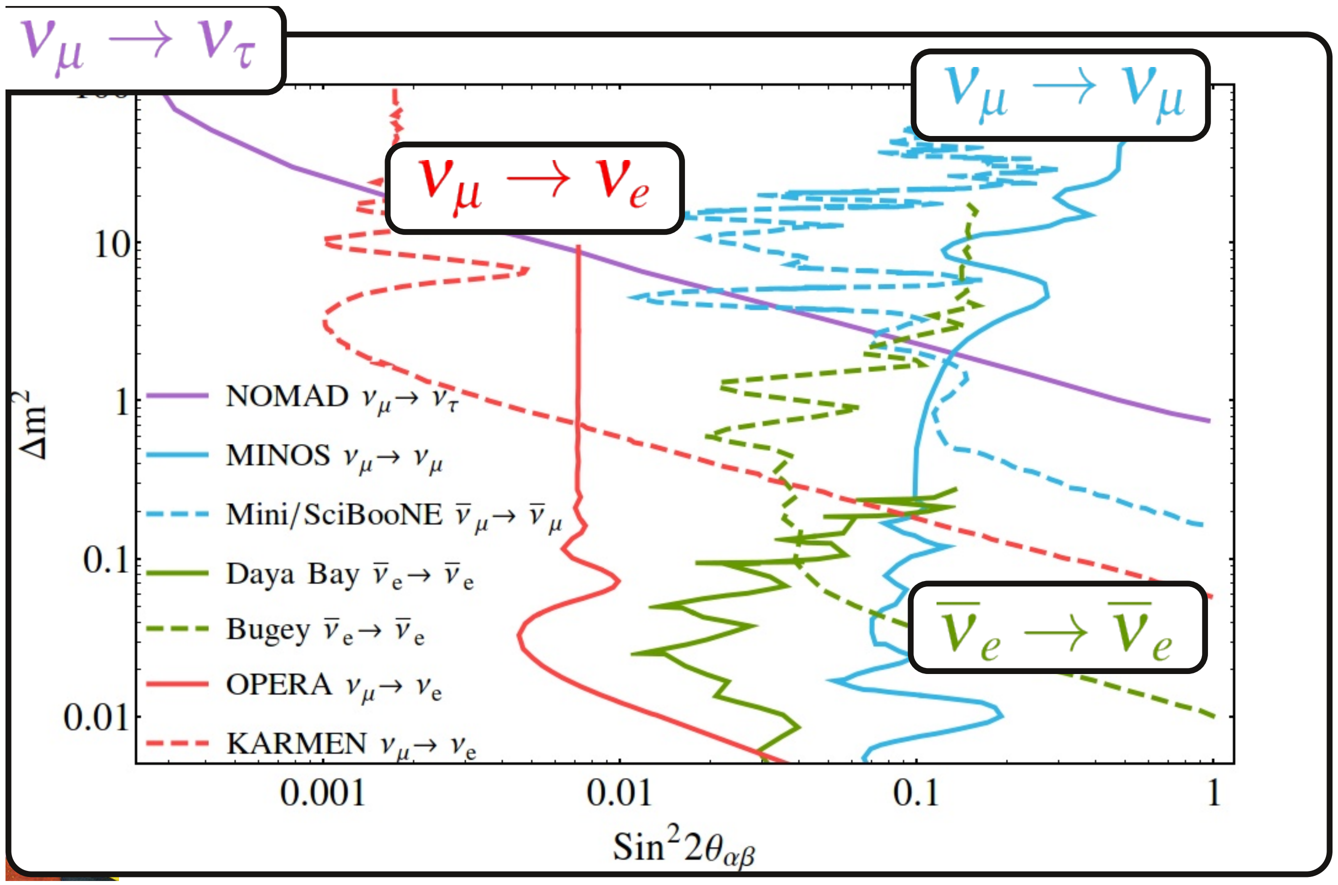


$\approx \mathcal{O}(0.1 - 1\mathrm{eV}^2)$
New Physics?

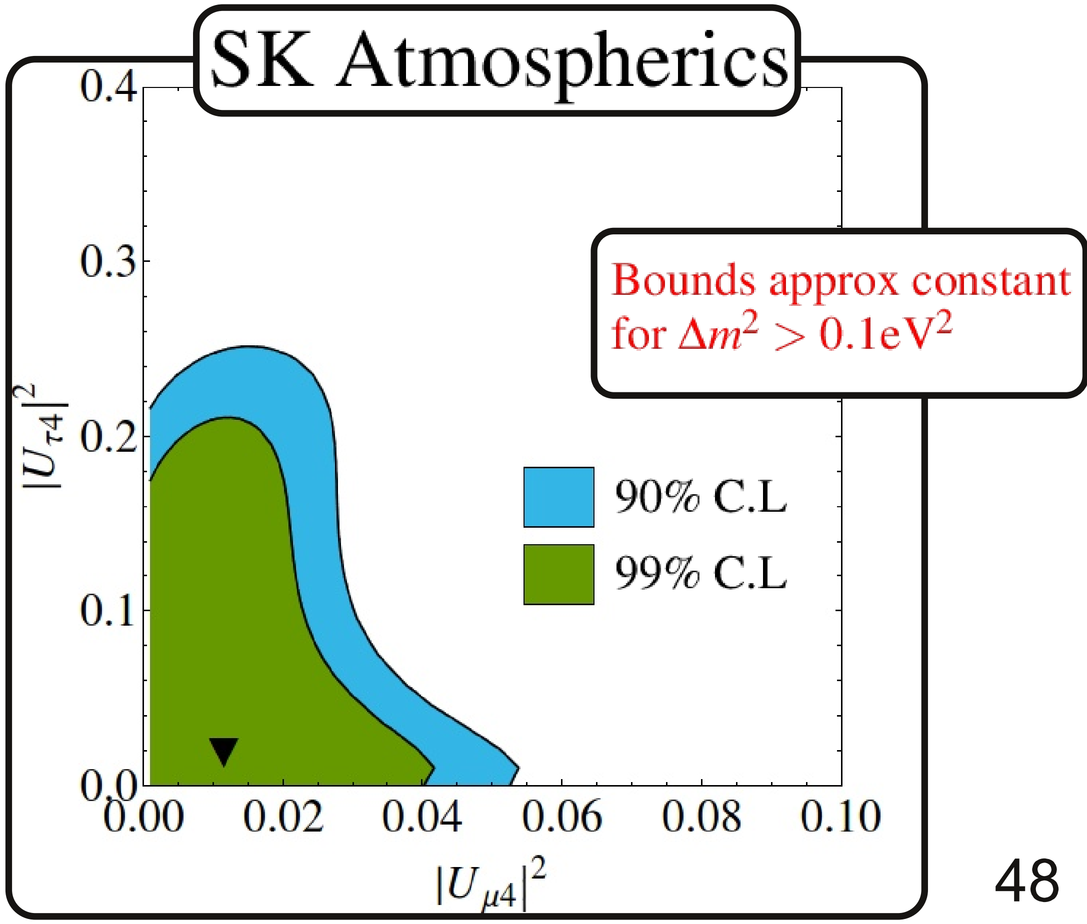




~ 1 eV^2



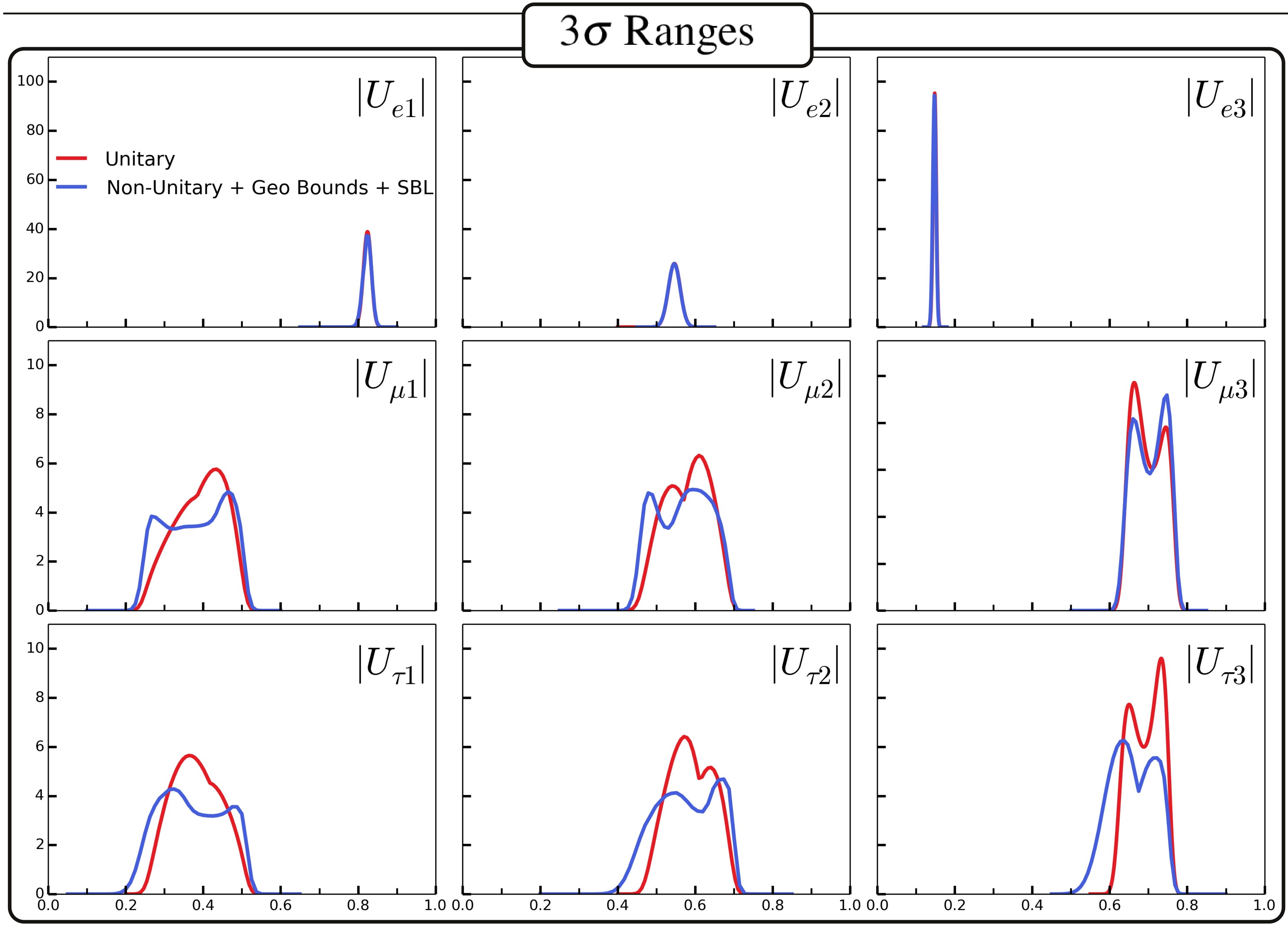
$\nu_\mu \to \nu_\tau$
$\nu_\mu \to \nu_\mu$
$\nu_\mu \to \nu_e$
$\bar{\nu}_e \to \bar{\nu}_e$
10
1
0.1
0.01
Δm^2
NOMAD $\nu_\mu \to \nu_\tau$
MINOS $\nu_\mu \to \nu_\mu$
Mini/SciBooNE $\bar{\nu}_\mu \to \bar{\nu}_\mu$
Daya Bay $\bar{\nu}_e \to \bar{\nu}_e$
Bugey $\bar{\nu}_e \to \bar{\nu}_e$
OPERA $\nu_\mu \to \nu_e$
KARMEN $\nu_\mu \to \nu_e$
0.001
0.01
0.1
1
$\mathrm{Sin}^2 2\theta_{\alpha\beta}$



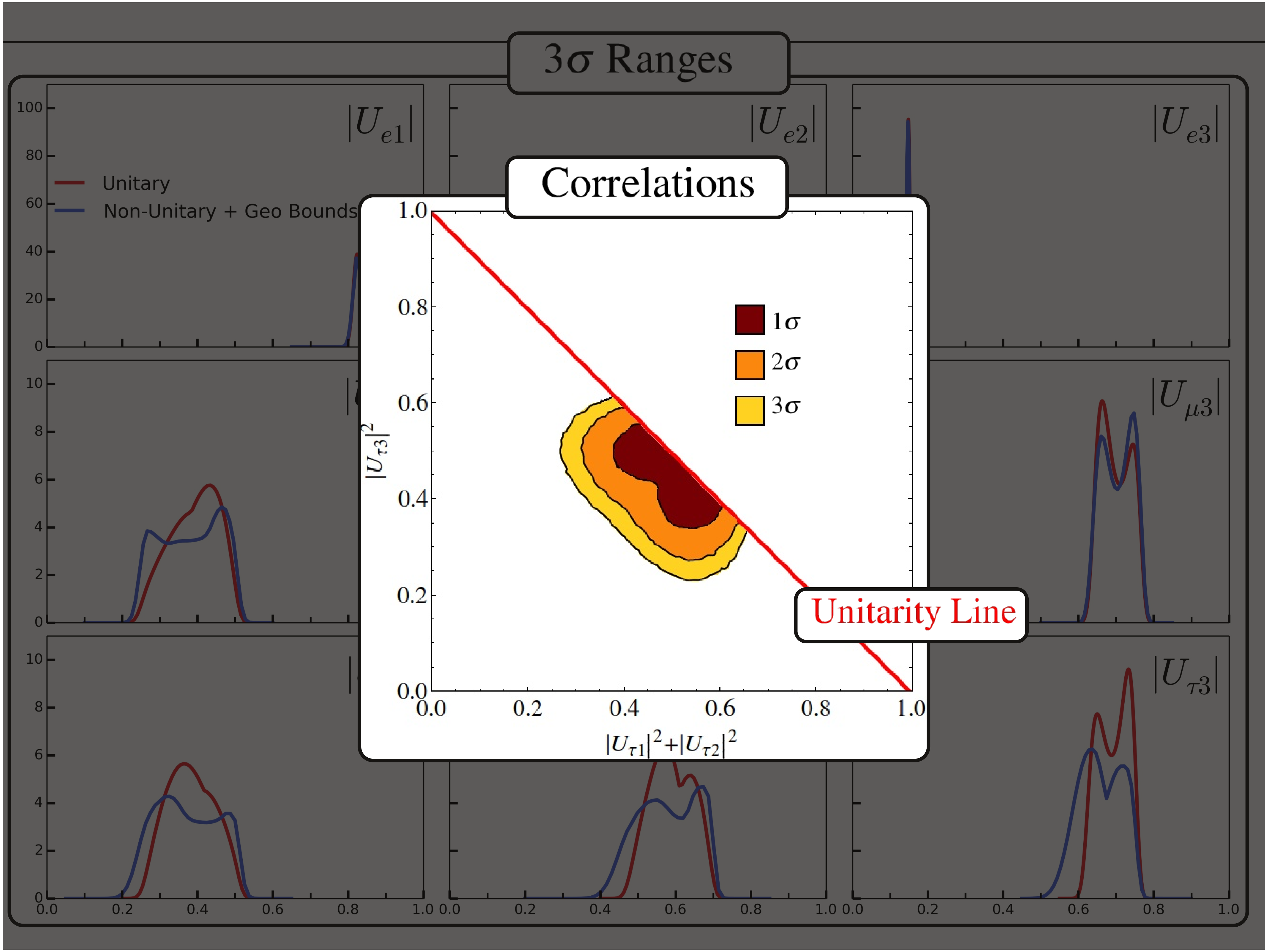
SK Atmospherics
Bounds approx constant for $\Delta m^2 > 0.1\mathrm{eV}^2$
90% C.L
99% C.L
0.4
0.3
0.2
0.1
0.0
$|U_{\tau 4}|^2$
0.00
0.02
0.04
0.06
0.08
0.10
$|U_{\mu 4}|^2$

48

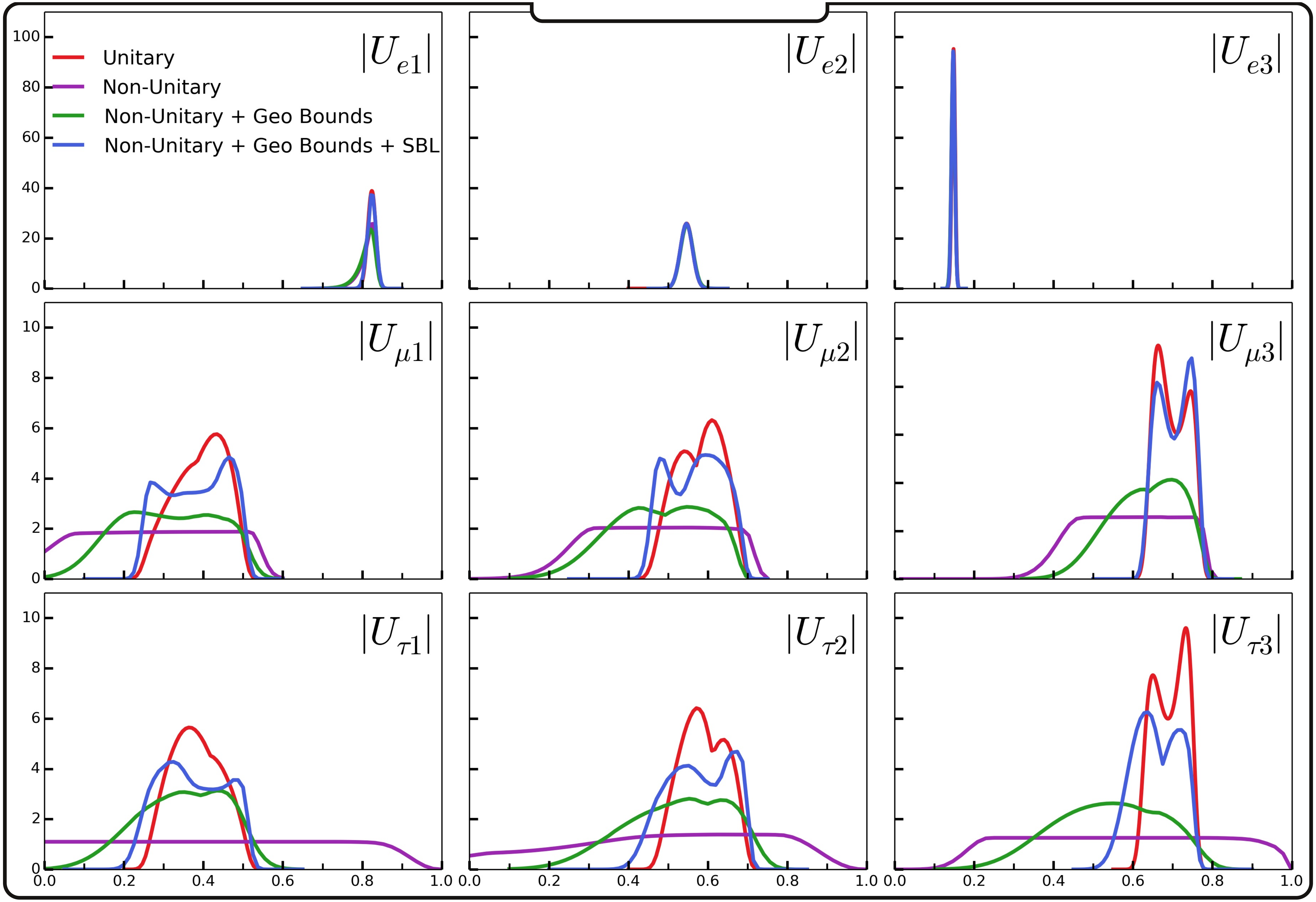
~ 1 eV^2


3σ Ranges
$|U_{e1}|$
$|U_{e2}|$
$|U_{e3}|$
$|U_{\mu1}|$
$|U_{\mu2}|$
$|U_{\mu3}|$
$|U_{\tau1}|$
$|U_{\tau2}|$
$|U_{\tau3}|$
Unitary
Non-Unitary + Geo Bounds + SBL
100
80
60
40
20
0
10
8
6
4
2
0.0
0.2
0.4
0.6
0.8
1.0

Correlations:



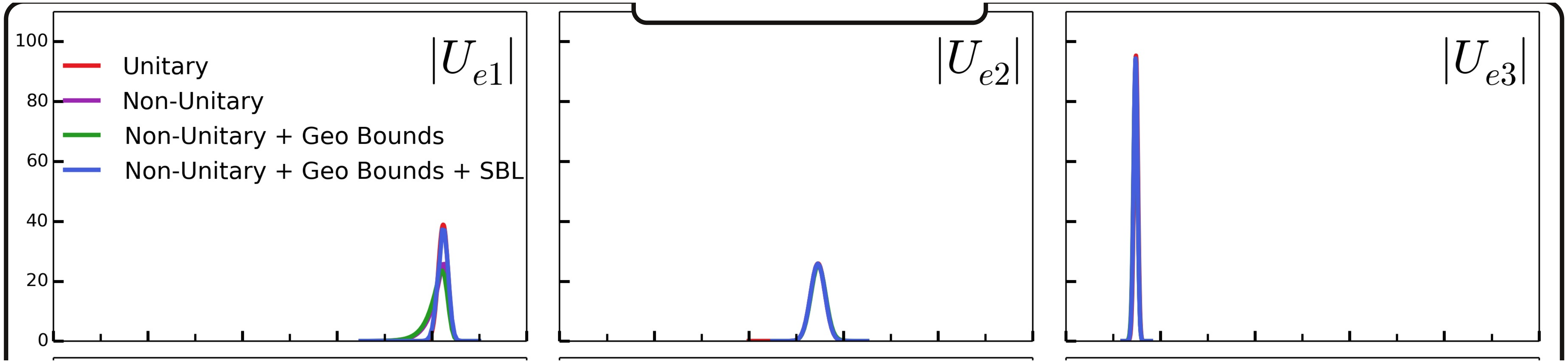
All



Unitary
Non-Unitary
Non-Unitary + Geo Bounds
Non-Unitary + Geo Bounds + SBL
$|U_{e1}|$
$|U_{e2}|$
$|U_{e3}|$
$|U_{\mu 1}|$
$|U_{\mu 2}|$
$|U_{\mu 3}|$
$|U_{\tau 1}|$
$|U_{\tau 2}|$
$|U_{\tau 3}|$
0 20 40 60 80 100
0 2 4 6 8 10
0.0 0.2 0.4 0.6 0.8 1.0

Future Prospects
and
Conclusions:

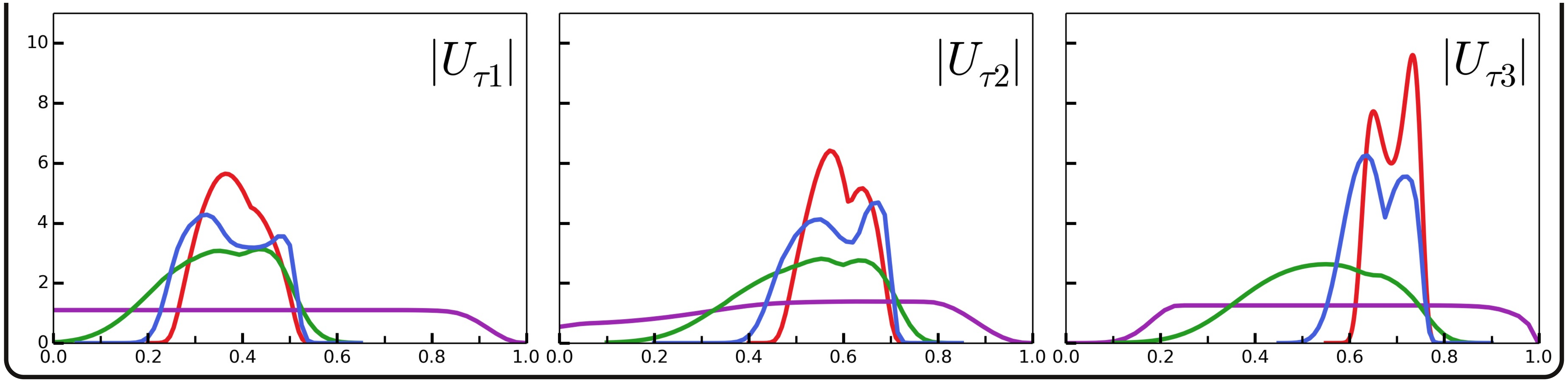
Future Prospects: ν_e-row



- Much better known than other rows:
- Will improve from
  - $|U_{e3}|$ from Daya Bay, RENO and Double Chooz
  - $|U_{e1}|$ and $|U_{e2}|$ JUNO and RENO-50: especially important !!!
  - only row we can easily separate 1st and 2nd columns L/E = 15 km/MeV
- Constraint to a few % level:

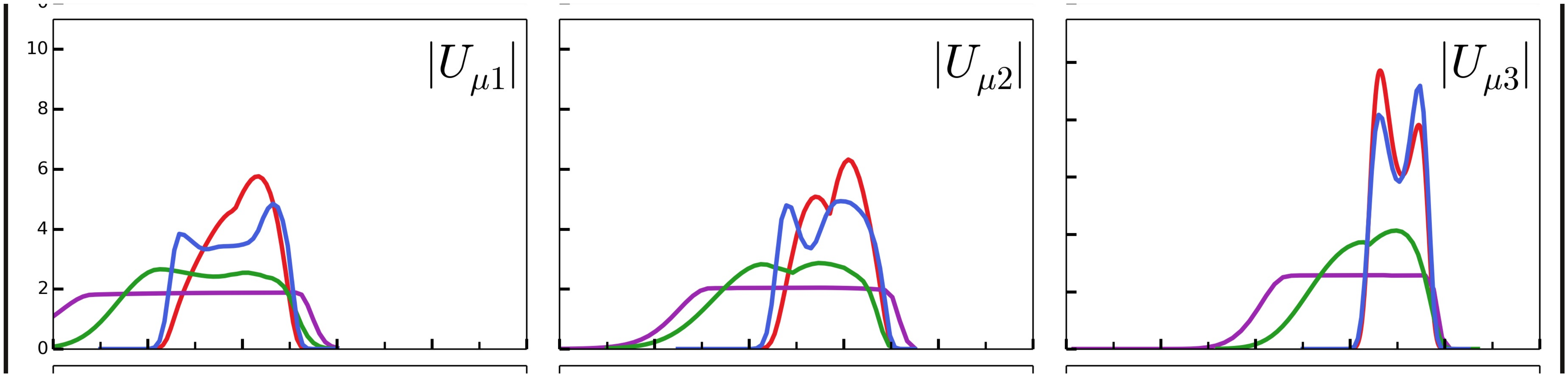
$$|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2$$

Future Prospects: ν_τ-row



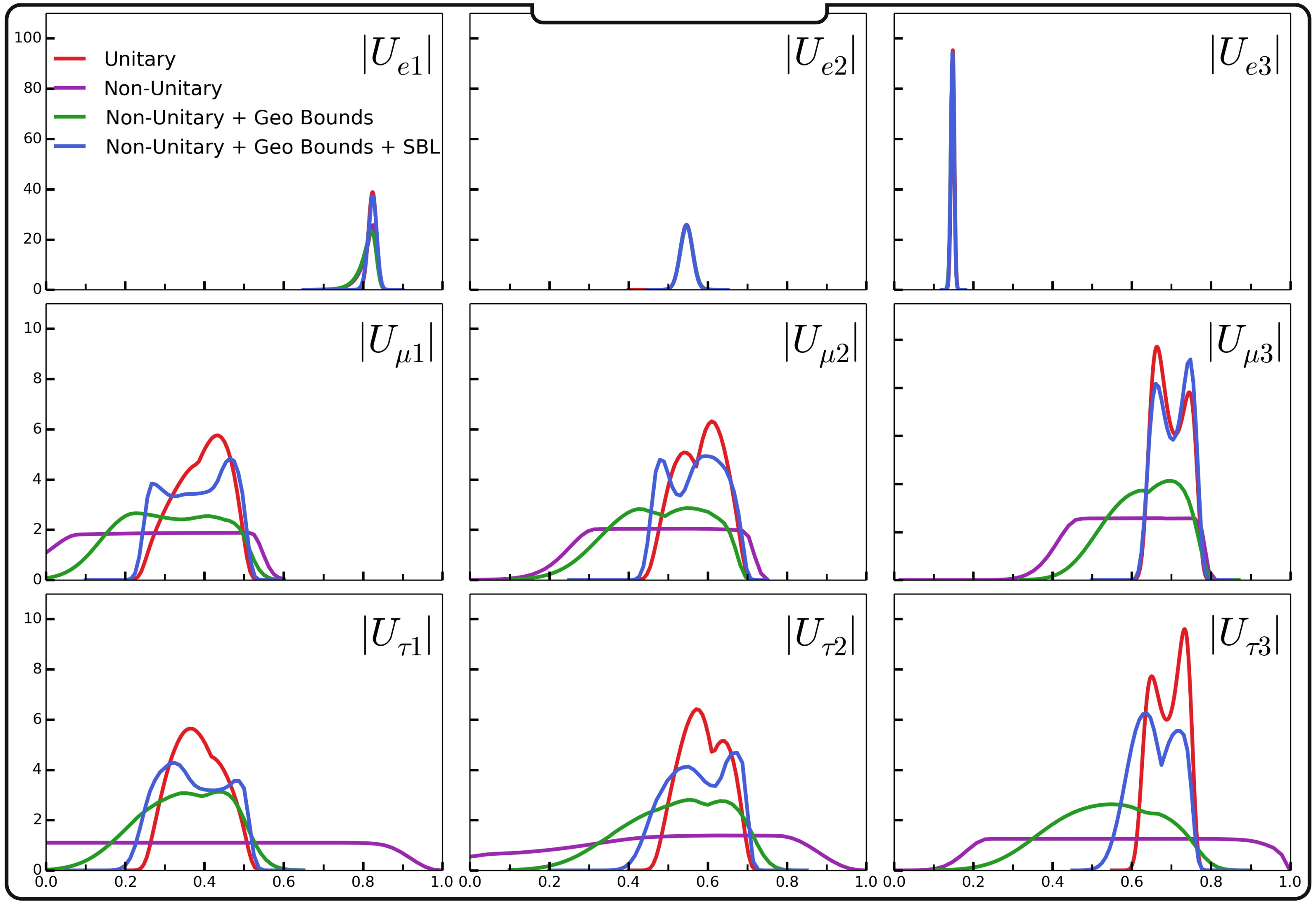
- Really challenging to make progress on this row:
  - $\nu_\mu \to \nu_\tau$ and $\nu_e \to \nu_\tau$ at Neutrino Factory (muon storage ring)
    - requires determination of tau charge !
  - any ideas on ν_τ disappearance ! ! !
- Separating $|U_{\tau1}|$ and $|U_{\tau2}|$ will require great innovation !
  - L/E = 15,000 km/GeV
- Geometric constraint from e-row will also improve our knowledge here.

Future Prospects ! ν_μ-row



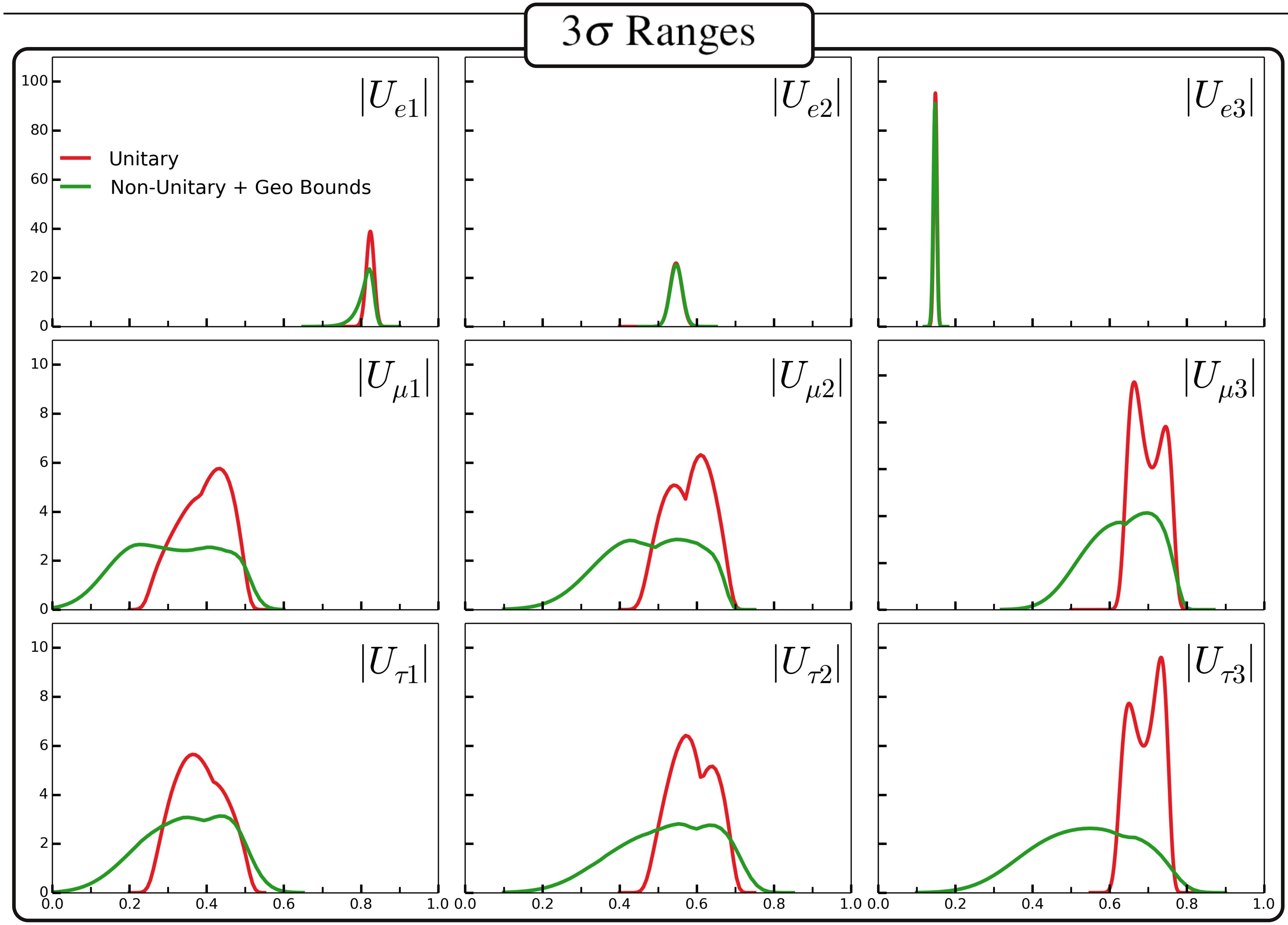
- T2K, NOvA, LBNF, HyperK, ESS, SuperPINGU, ……
  - ν_μ disappearance and $\nu_\mu \to \nu_e$ appearance will tighten this row considerable
    - $|U_{\mu3}|^2$ and some "J" (octant of θ_{23} and δ_{CP})
    - geometric constraint with e-row will also improve our knowledge here.
  - Wonderful Opportunity !
- Breaking the degeneracy between $|U_{\mu1}|$ and $|U_{\mu2}|$ will be challenging !!!
  - ν_μ disappearance at 15,000 km/GeV. (detector in geo-synchronous orbit ! ! !)

What we really know about the Neutrino Mixing Matrix !



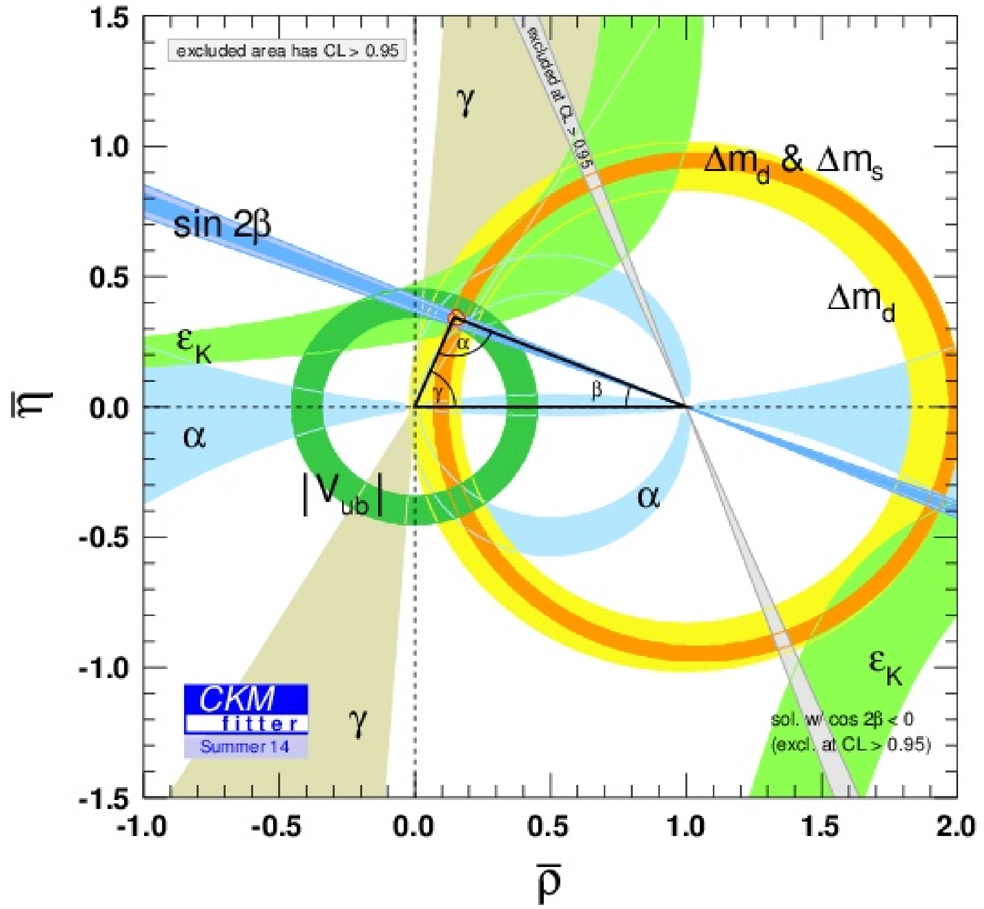
- Answer depends on what assumptions you make ! ! !
  - As Scientists we need to test these assumptions as best we can !

Theoretical Geometric Bounds:

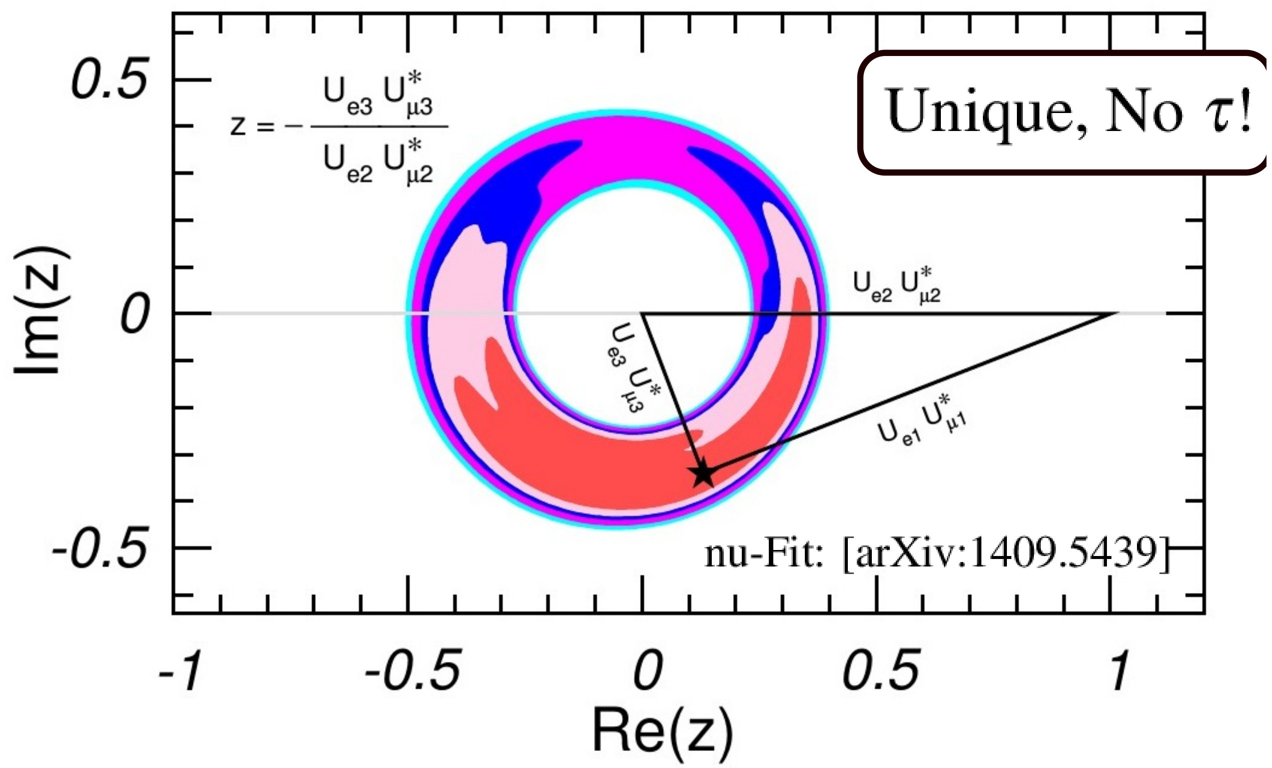


Most Assumption Independent that is theoretically motivated !

quarks v neutrinos !



Unitarity *Not* assumed



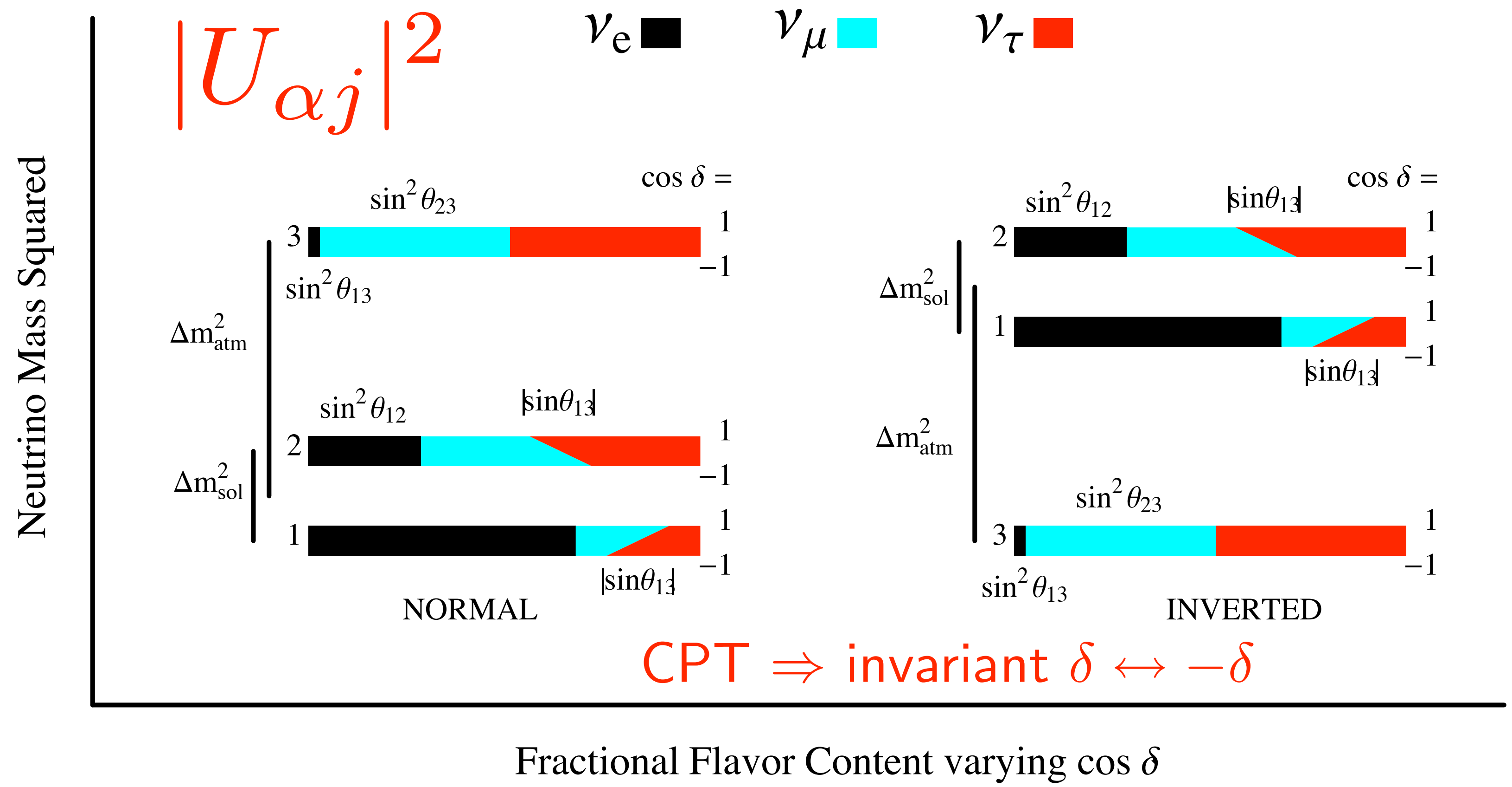
Unitarity *Is* assumed.

Thank You !

additional:

Flavor Content of Mass Eigenstates:

- Labeling massive neutrinos: $|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$



$$\sin^2\theta_{12} \sim \frac{1}{3}$$

$$\sin^2\theta_{23} \sim \frac{1}{2}$$

$$\sin^2\theta_{13} \sim 0.02$$

$$0 \leq \delta < 2\pi$$

- $0.06\ \text{eV} < \sum m_i < 0.5\ \text{eV} \approx m_e/10^6$