UNIVERSITAT DE BARCELONA

B:KC Barcelona Knowledge Campus

Institut de Ciències del Cosmos

Light nuclei and hypernuclei from Lattice QCD (A=2,3,4)

Assumpta Parreño (U Barcelona)

for the NPLQCD Collaboration

Nucleus Nucleus Collisions 2015

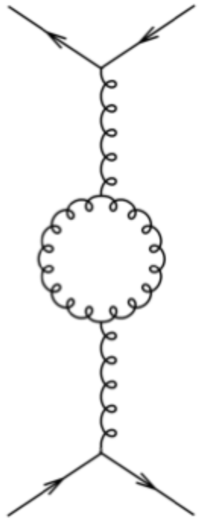
June 22, 2015 @ Università di Catania, Italy

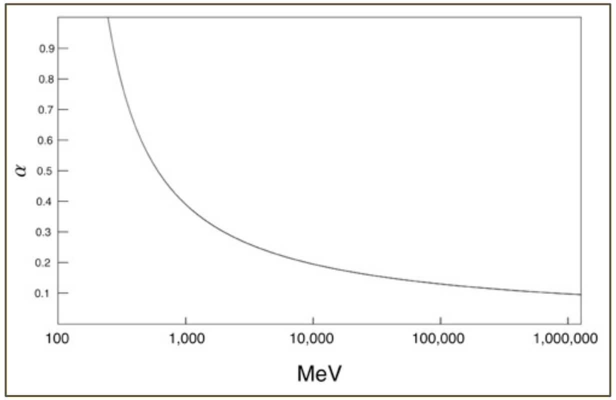
PRACE

Study of Strongly Interacting Matter HadronPhysics3

GOBIERNO DE ESPAÑA MINISTERIO DE CIENCIA E INNOVACIÓN

Generalitat de Catalunya Departament d'Economia i Coneixement





0.9
0.8
0.7
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α
100
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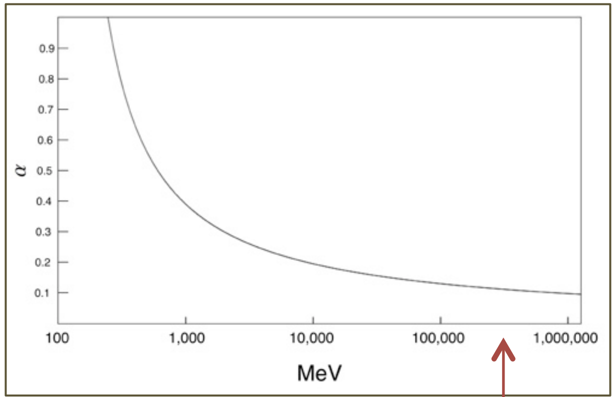
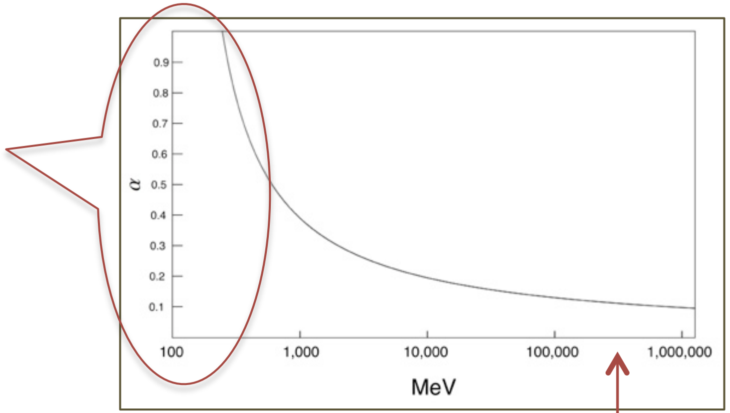
MeV



Perturbation theory applicable

Nuclear physics, the low-energy regime

- ✓ quarks and gluons confined into coulourless bound states: hadrons (p , n , π…)
- ✓ quarks strongly coupled
- ✓ perturbation theory breaks down: need of other formulations, computational techniques



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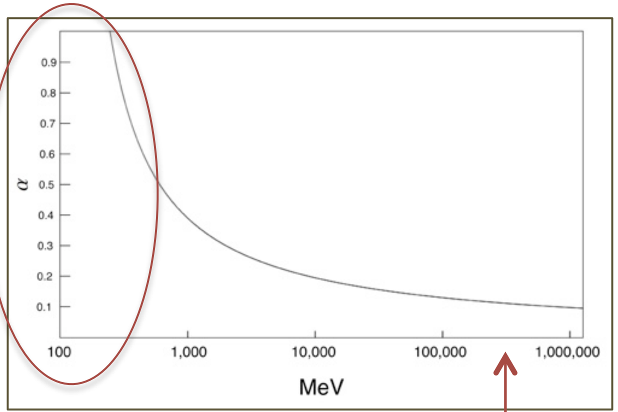
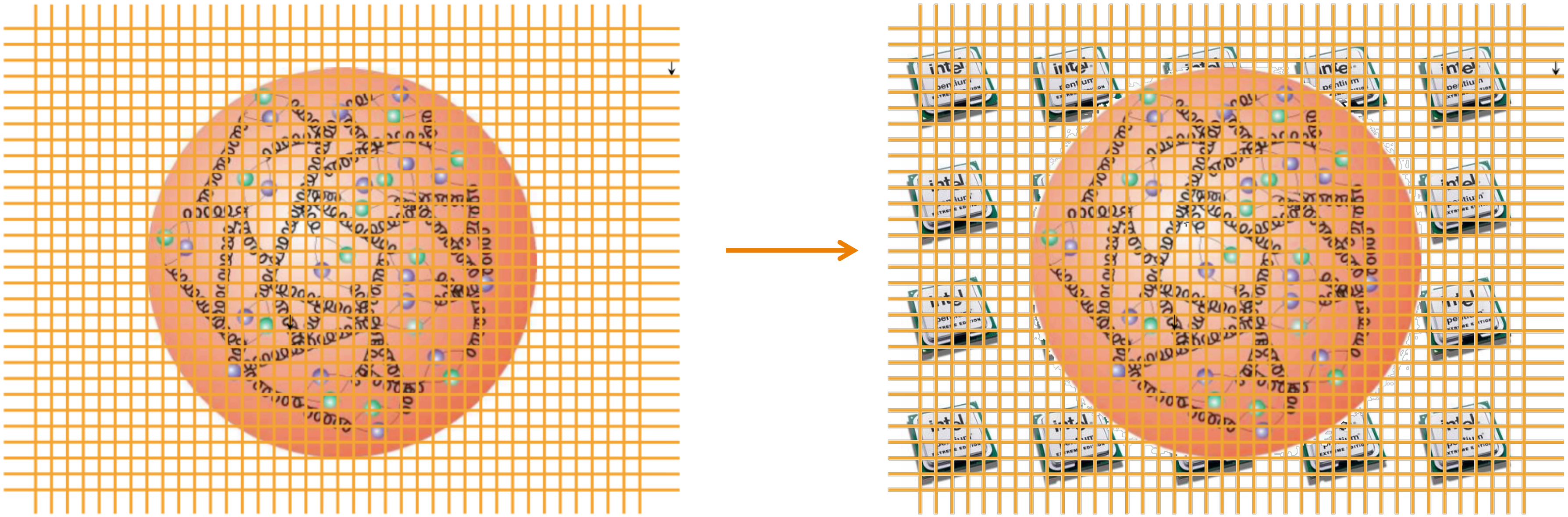


Perturbation theory applicable

- ✓ quarks and gluons confined into coulourless bound states: hadrons (p , n , π…)
- ✓ quarks strongly coupled
- ✓ perturbation theory breaks down: need of other formulations, computational techniques
- ✓ keep the basic degrees of freedom (quarks, gluons) and develop non-perturbative methods (Lattice QCD)



(nn-online.org)

Total elastic cross-section $\Lambda p \to \Lambda p$

momentum range (MeV/c)	number of events	SGT (mb)	reference
120 – 150	34	212 +/- 36	AL68
120 – 150	14	209 +/- 58	SE68
150 – 180	48	141 +/- 20	AL68
150 – 180	28	177 +/- 38	SE68
150 – 400	11	34 +/- 10	PI64
150 – 600	6	22 +/- 10	GR63
180 – 210	80	141 +/- 16	AL68
180 – 210	49	153 +/- 27	SE68
200 – 400		24.0 +/- 5.0	KA71
210 – 240	88	95 +/- 10	AL68
210 – 240	54	111 +/- 18	SE68
240 – 270	92	81 +/- 8	AL68
240 – 270	59	87 +/- 13	SE68
270 – 320	36	56 +/- 9	AL68
270 – 330	20	46 +/- 11	SE68
300 – 400		17.2 +/- 8.6	HA77
400 – 500		26.9 +/- 7.8	HA77
400 – 600		9.0 +/- 2.0	KA71
400 – 638	7	24.7 +/- 9.3	AL61
400 – 700		14.0 +/- 3.5	CL67
400 – 1500		42 +/- 16	AR62
500 – 600		7.0 +/- 4.0	HA77
500 – 1000	4	40 +/- 20	CR59
500 – 1000	20	22.2 +/- 5.0	CH70
500 – 1200	86	25 +/- 4	BE64
600 – 700		16.5 +/- 3.5	KA71
600 – 700		9.0 +/- 4.0	HA77
600 – 1500	20	19 +/- 5	GR63
639 – 1000	7	20.4 +/- 7.7	AL61
700 – 800		10.8 +/- 2.7	KA71
700 – 800		13.6 +/- 4.5	HA77
700 – 1000		15.5 +/- 3.5	CL67
800 – 900		10.2 +/- 2.7	KA71
800 – 900		11.3 +/- 3.6	HA77
900 – 1000		9.0 +/- 2.7	KA71
900 – 1000		11.3 +/- 3.8	HA77

Total elastic cross-section $\Sigma^- p \to \Sigma^- p$

momentum range (MeV/c)	number of events	SGT (mb)	reference
100 – 1900	6	10 + 6 – 4	ST61
125 – 165	25	166 +/- 33	RU67
130 – 140	14	184 +/- 52	EI71
135 – 145		207 +/- 85	DO66
140 – 145	19	152 +/- 38	EI71
145 – 150	30	146 +/- 30	EI71
145 – 155		198 +/- 48	DO66
150 – 155	49	142 +/- 25	EI71
155 – 160	81	164 +/- 32	EI71
155 – 165		189 +/- 32	DO66
160 – 165	107	138 +/- 19	EI71
165 – 170	106	113 +/- 16	EI71
500 – 1500	6	13.2 +/- 4.7	CH70
1500 – 2500	11	13.9 +/- 4.1	CH70
2500 – 4000	4	7.5 +/- 3.8	CH70

(nn-online.org)

Total elastic cross-section $\Lambda\, p \rightarrow \Lambda\, p$

momentum range (MeV/c)	number of events	SGT (mb)	reference
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180 – 210	80	141 +/- 16	AL68
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400 –			
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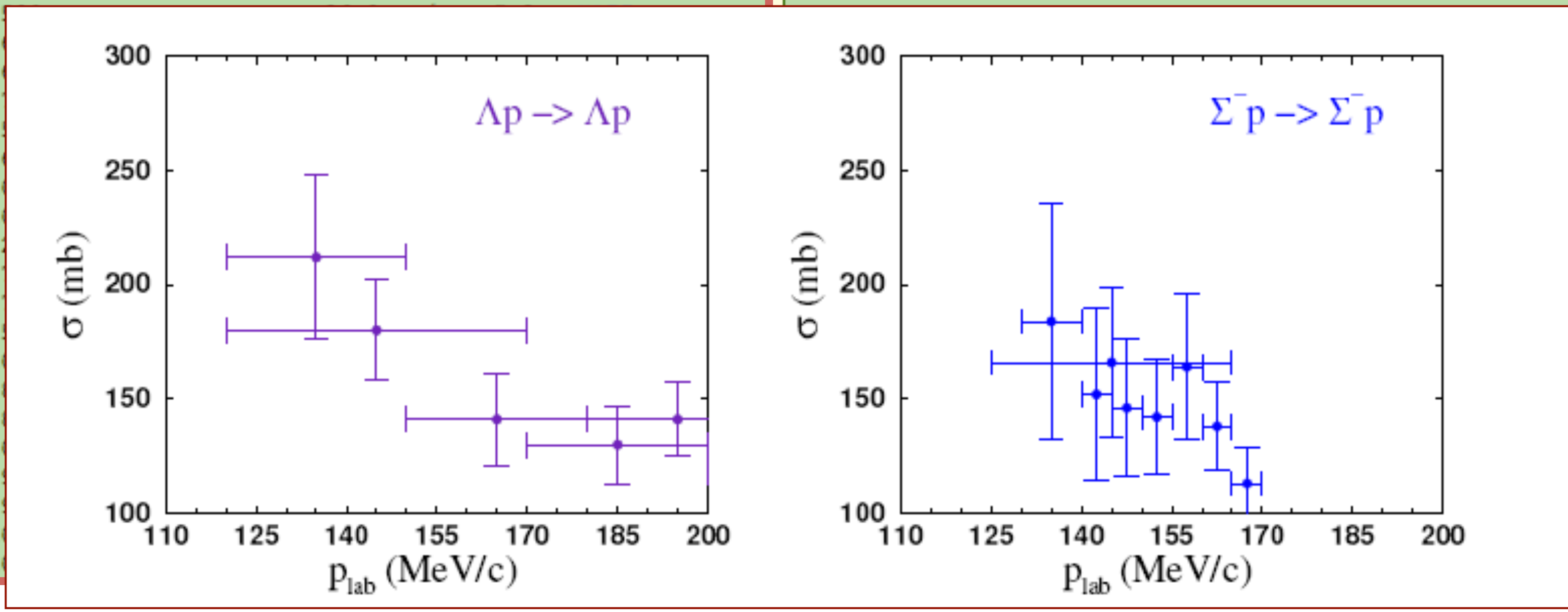
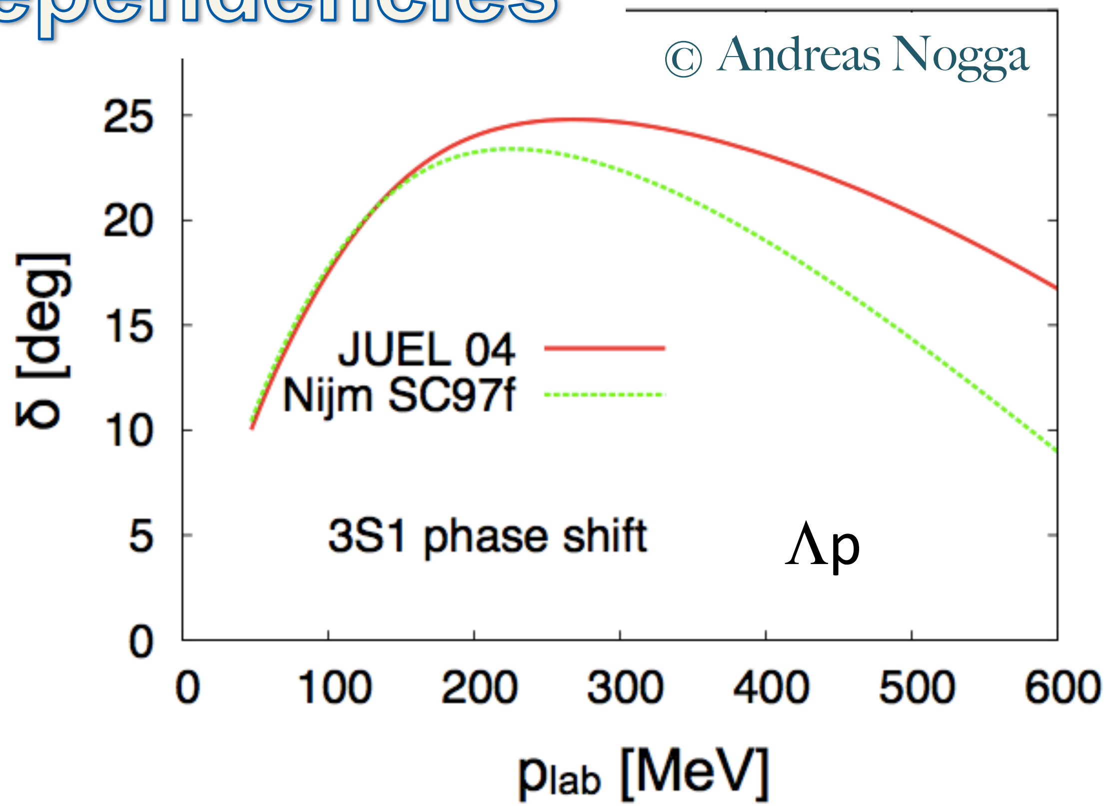
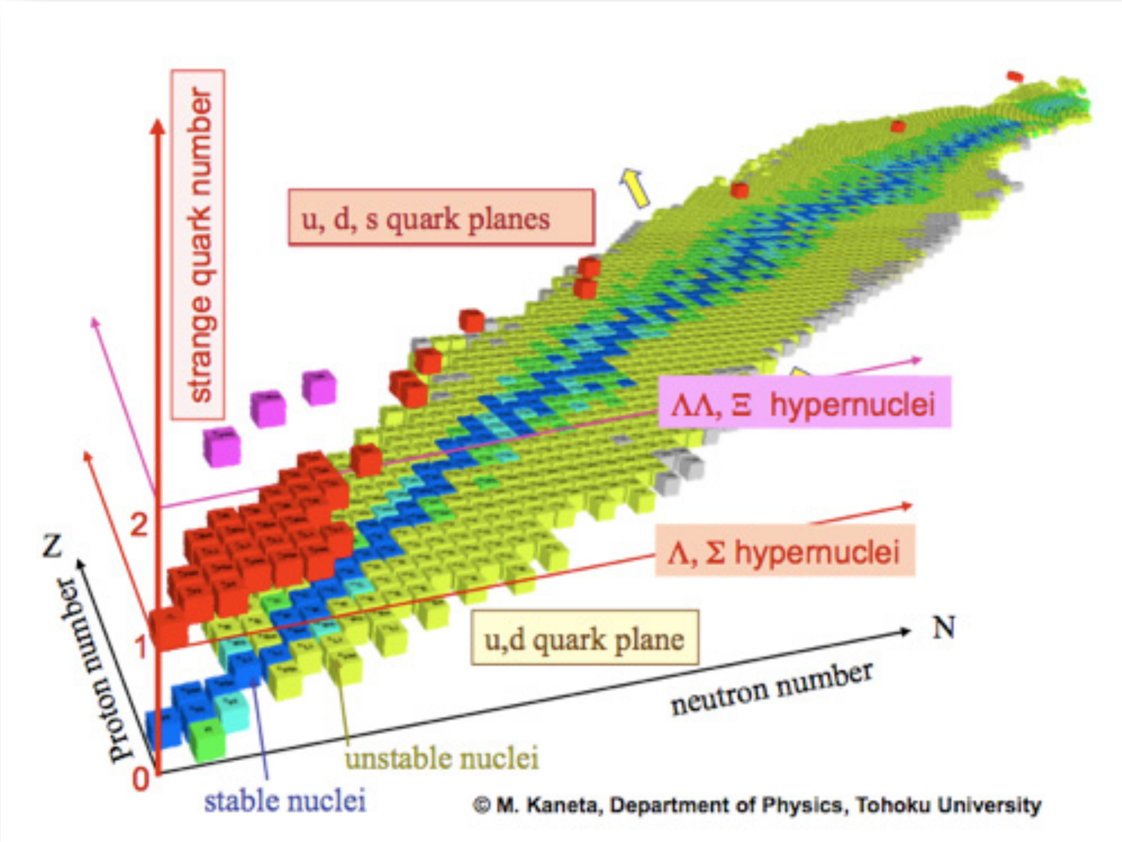


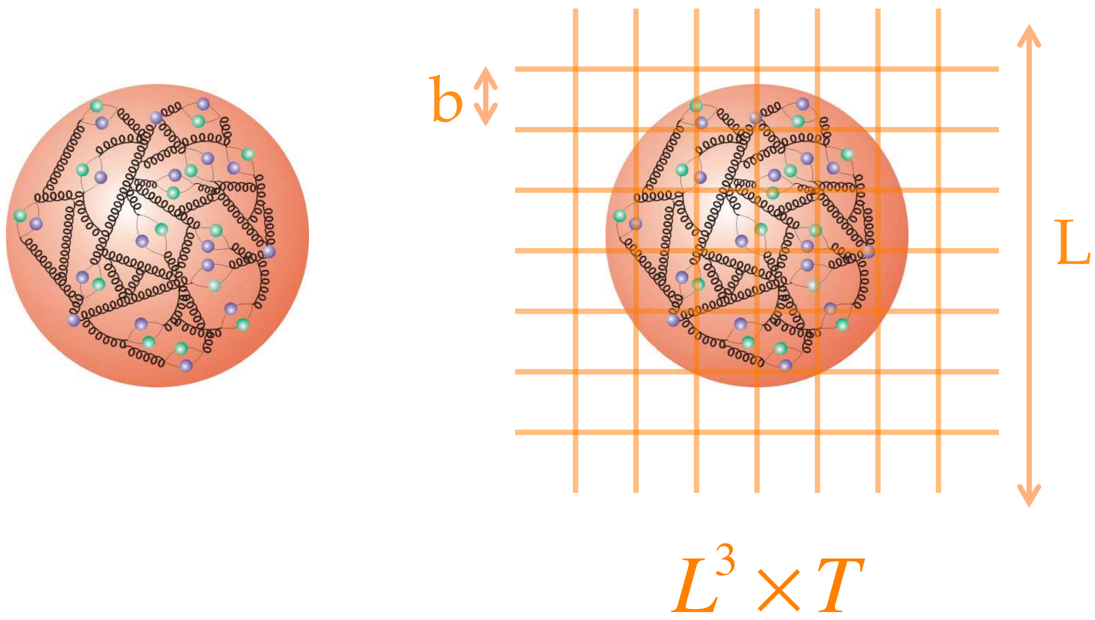
model dependencies



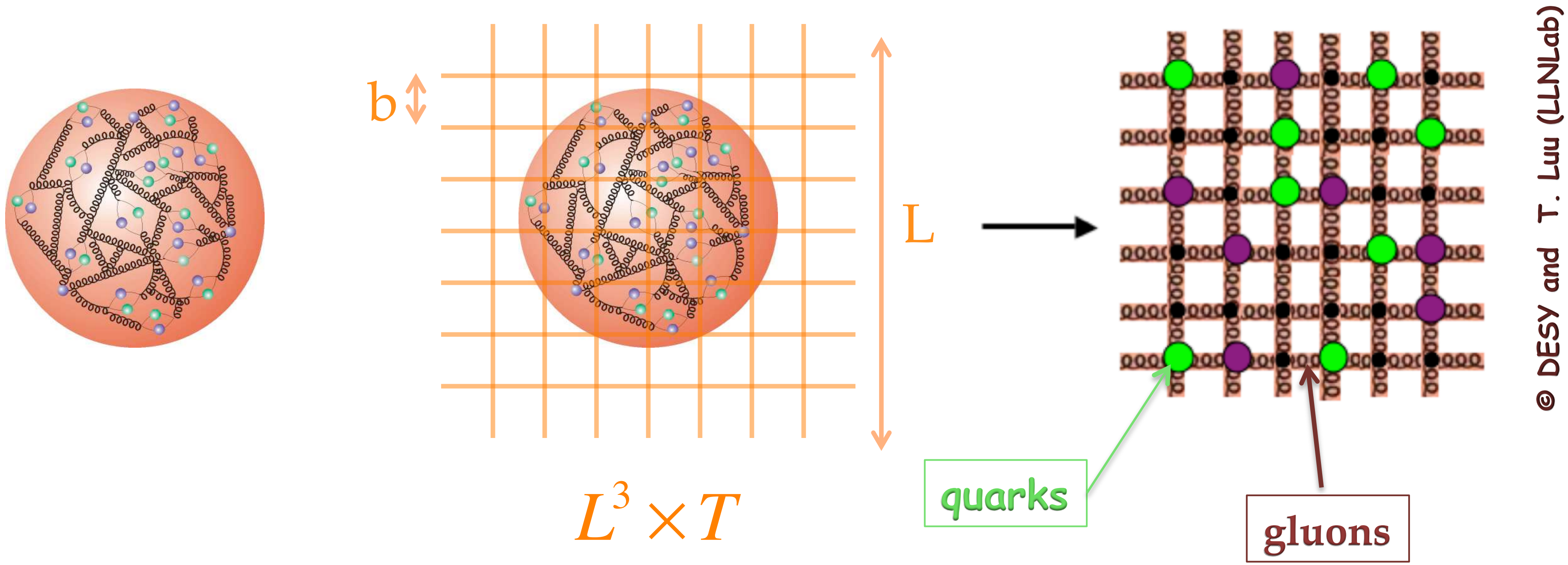


Hypernuclei:
Bound nuclear states containing strange baryons (hyperons)

JPARC, DAPHNE, FAIR, GSI, JLAB, …

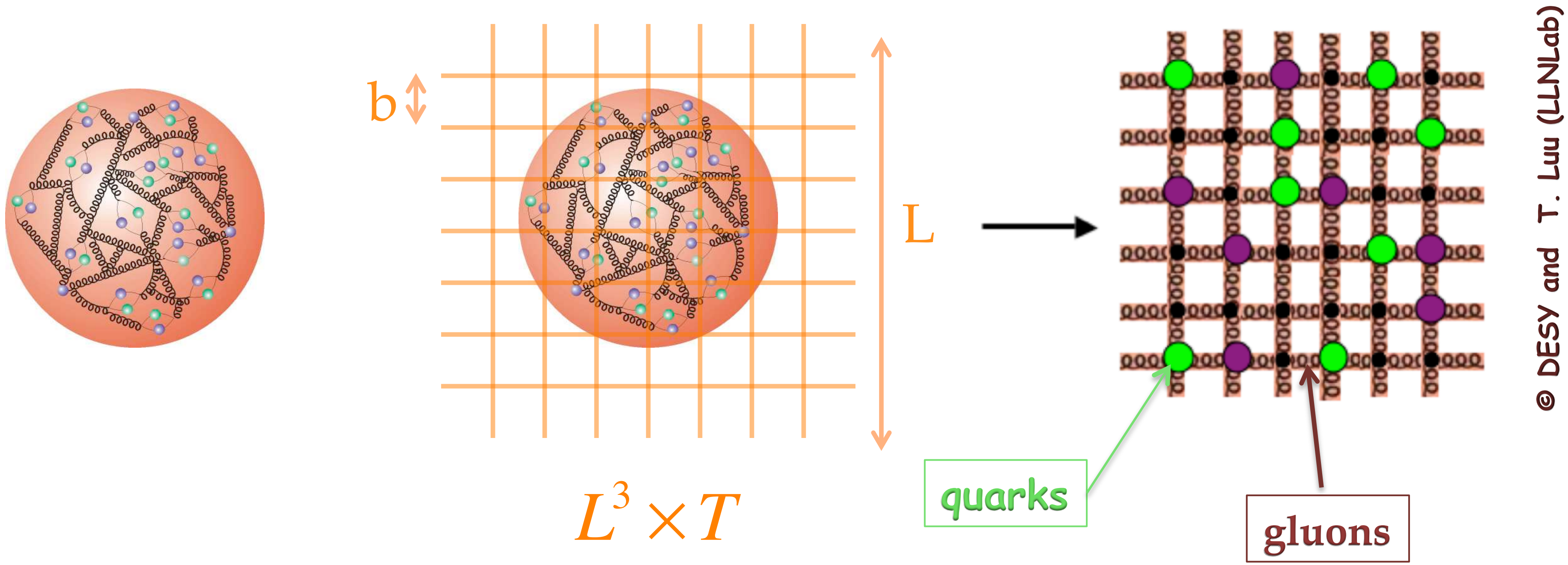


b
L
$L^3 \times T$



For numerical calculations in QCD, the theory is formulated on a (Euclidean) space-time lattice

((anti) periodic (time) spatial boundary conditions) $N_s \times N_s \times N_s \times N_t$



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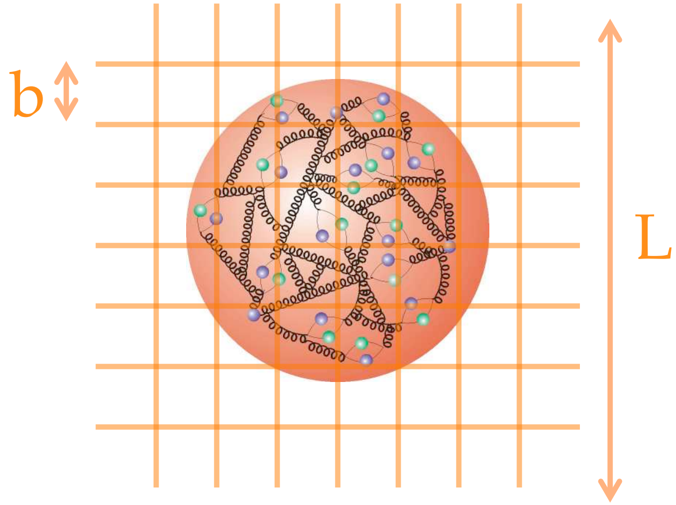
$$L >> \text{ relevant scales } >> b \quad \left(\frac{1}{L} << m_\pi << \Lambda_\chi << \frac{1}{b} \right)$$

→ finite number of d.o.f.
(finite volume)

$b\,L >> 1$ fm (fundamental scale of QCD)
with $b << 1$ fm

nucleon-nucleon scattering
$\Rightarrow L > 2\pi \, m_\pi^{-1}$



$N_s \times N_s \times N_s \times N_t$

$$\boxed{\text{Cost} \approx \left[\frac{1}{m_q}\right] [L]^a \left[\frac{1}{\text{b}}\right]^\gamma}$$

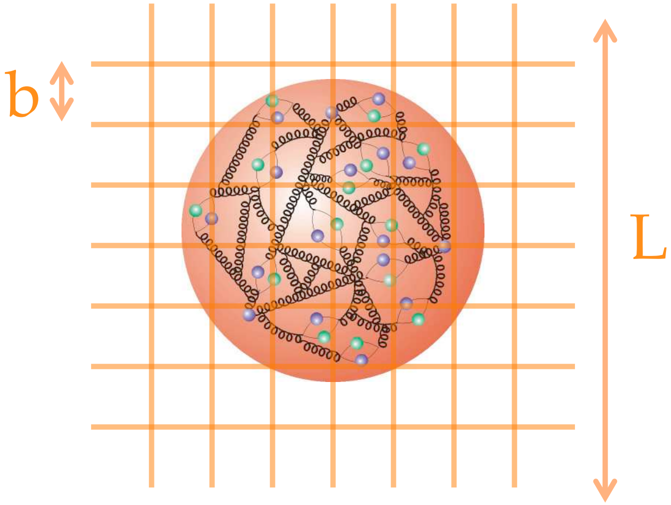
Significant computational resources required for calculations @ $\mathrm{m}_{\mathrm{u,d}}^{\mathrm{phys}}$ (petascale)

USE UNPHYSICAL VALUES OF THESE PARAMETERS

Published works have L $\leq$ 7 fm, b < ~ 0.1 fm and $\mathrm{m}_\pi \geq 200$ MeV

sources of systematic errors in the numerical simulation

finite volume L, discretization (finite spacing) b, value of the light quark masses



$N_s \times N_s \times N_s \times N_t$

$$\text{Cost} \approx \left[\frac{1}{m_q}\right] [L]^{a} \left[\frac{1}{\mathrm{b}}\right]^{\gamma}$$

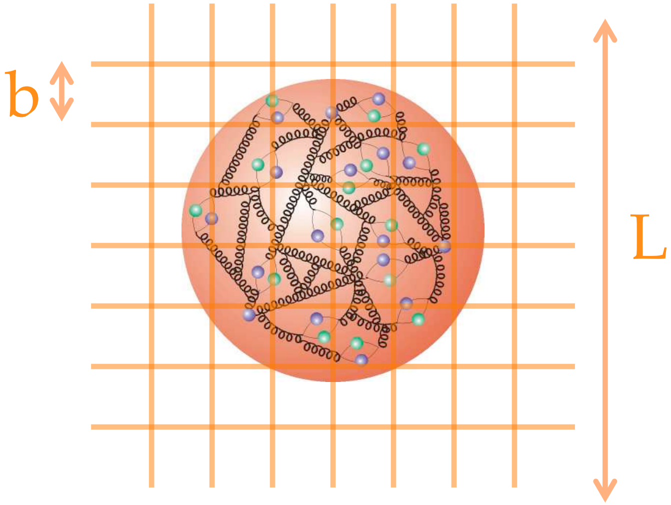
USE UNPHYSICAL VALUES OF THESE PARAMETERS

Extrapolations to connect with real life

$L \to \infty$

$b \to 0$

$m_q \to m_q^{physical}$



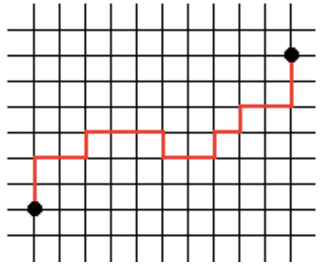
N_s x N_s x N_s x N_t

$$\boxed{\text{Cost} \approx \left[\frac{1}{m_q}\right][L]^a\left[\frac{1}{\mathrm{b}}\right]^\gamma}$$

$$\underbrace{64^3 \times 256}_{L^3 \times T} \times 3_{\text{color}} \times 4_{\text{spin}} \approx 10^{11} \text{ degrees of freedom}$$

only practical way for this type of computation → **Monte Carlo integration**

LQCD is a non-perturbative implementation of Field Theory, which uses the Feynman path-integral approach to evaluate transition matrix elements



Our continuous Path-Integral (QCD partition function):

$$Z = \int D\varphi(x)\, e^{-iS_{QCD}[\varphi(x)]} = \int DU\ D\bar{\psi}\ D\psi\ e^{-iS_{QCD}[U,\bar{\psi},\psi]}$$

oscillating phase

$(U_{x\mu} \sim e^{igbA_{x\mu}})$

each path contributes a phase

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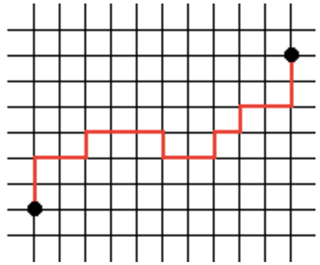

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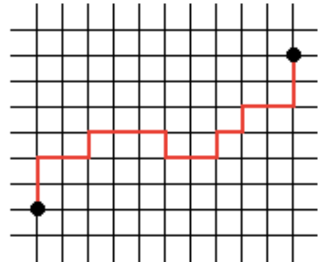
By rotating to Euclidean time:

$$-i\,S_{QCD}[\varphi(x)] = -i\int d^3x\,dt\,\mathscr{L} \xrightarrow[t\to -i\tilde{t}]{} -\int d^4x\,\tilde{\mathscr{L}} = -\tilde{S}_E$$

decaying exponential

Now, the weight of each path is a real positive quantity

BASIS OF NUMERICAL SIMULATIONS



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oscillating phase

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decaying exponential

Now, the weight of each path is a real positive quantity

BASIS OF NUMERICAL SIMULATIONS

$$Z = \int DU \left\{ \int D\psi\ D\bar{\psi}\ e^{-\bar{\psi}Q(U)\psi} \right\} e^{-S_g[U]} = \int DU\ \det Q(U)\, e^{-S_g[U]} \quad \sim \mathrm{P(U)}$$

$$\det\left[Q_f(A)\right] \equiv \det\left(\not{D}[A] + m\right)$$

(quark matrix)

1. Generate an ensemble of N gauge-field configurations $\{U_i\}$ according to the probability distribution P(U)

$$Z = \int DU \left\{ \int D\psi \, D\bar{\psi} \; e^{-\bar{\psi} Q(U) \psi} \right\} e^{-S_g[U]} = \int DU \; \boxed{\det Q(U) \; e^{-S_g[U]}} \quad \sim \mathbf{P(U)}$$

$$\det\left[Q_f(A)\right] \equiv \det\left(\not{D}[A] + m\right)$$

(quark matrix)

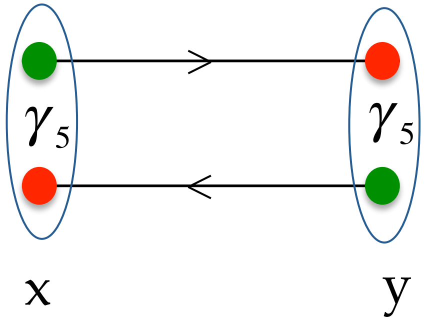
expectation values

$$\langle O \rangle = \frac{1}{Z} \int DU \;\; D\bar{\psi} \;\; D\psi \, O[\psi, \bar{\psi}, U] \, e^{-S_E[U, \bar{\psi}, \psi]}$$

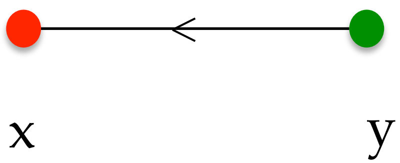
When computing expectation values of any given operator *O*, the quark fields in *O* are re-expressed in terms of quark propagators using Wick's Theorem: **write all possible contractions for the fields** (removing the dependence of quarks as dynamical fields)

$$\pi^+ = \bar{d}\gamma_5 u$$ only possible (Wick) contractions:

$$\left\langle \pi^\dagger(x)\, \pi(y) \right\rangle = \left\langle \bar{u}(x)\gamma_5 d(x)\, \bar{d}(y)\gamma_5 u(y) \right\rangle$$

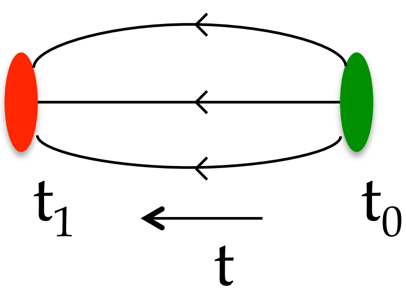


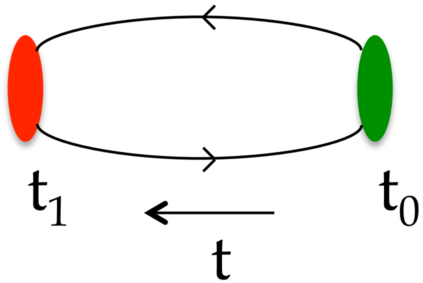
2. Use the N gauge-field configurations previously generated to calculate the quark propagators on each configuration Q^{-1} [U_i]



$$Q_u^{-1}(x,y) = u(y)\bar{u}(x)$$

3. Contract propagators onto correlation functions C_i (t) (t_0=0)





$$C(\Gamma^\nu,\vec{p},t) = \sum_{\vec{x}_1} e^{-i\vec{p}\vec{x}_1}\,\Gamma^\nu \left\langle J(\vec{x}_1,t)\bar{J}(\vec{x}_0,0)\right\rangle$$

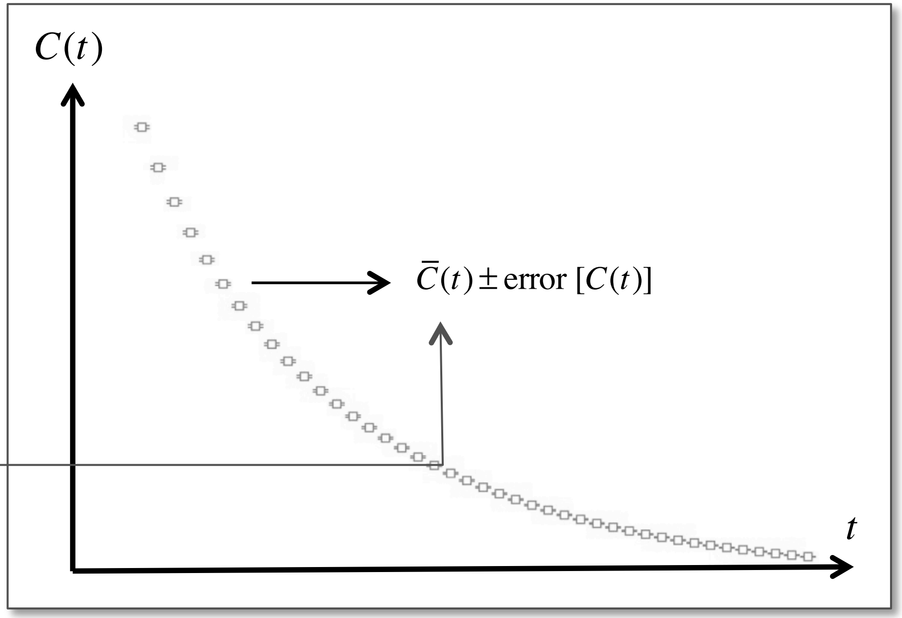
How do we get masses?

$$C(t)=\langle 0|\phi(t)\phi^{\dagger}(0)|0\rangle \longrightarrow \langle\phi|e^{-Ht}|\phi\rangle = \sum_n \langle\phi|e^{-Ht}|n\rangle\langle n|\phi\rangle = \sum \left|\langle\phi|n\rangle\right|^2 e^{-E_n t}$$

$$\phi(t)=e^{Ht}\phi\, e^{-Ht}$$

$$\xrightarrow{t\to\infty} Z_0\, e^{-E_0 t}$$

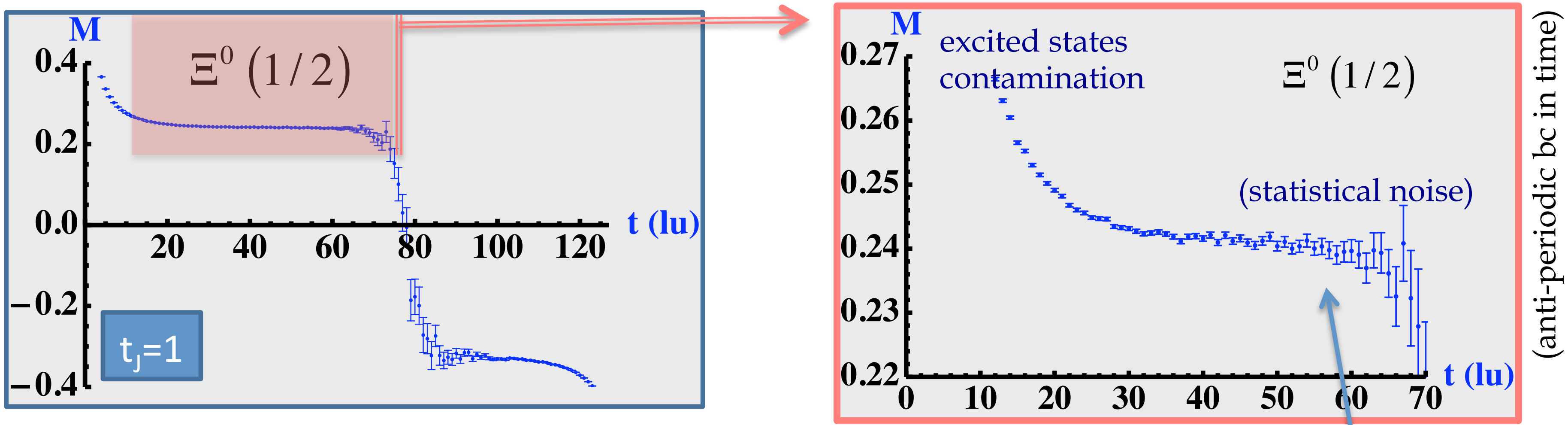
mass



average over N gauge-field configurations

Baryons, an example: Ξ^0 mass (uss)

$$\Xi^0_\alpha(\vec{x},t) = \varepsilon^{ijk} s^i_\alpha(\vec{x},t)\left(u^{jT}_\alpha(\vec{x},t)\, C\gamma_5\, s^k(\vec{x},t)\right)$$



$$\frac{1}{t_J}\log\frac{C_A(t)}{C_A(t+t_J)} = E_{0A} \quad (m_A)$$ Generalized effective plots

ground states dominate in this region

$$C_A(t) \sim Z_{0A}\, e^{-E_0 t}$$
$$C_A(t+t_J) \sim Z_{0A}\, e^{-E_0 (t+t_J)}$$

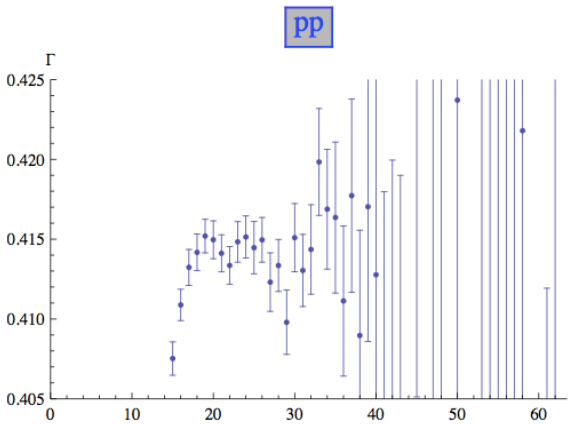
g.s. energy (mass) from plateau

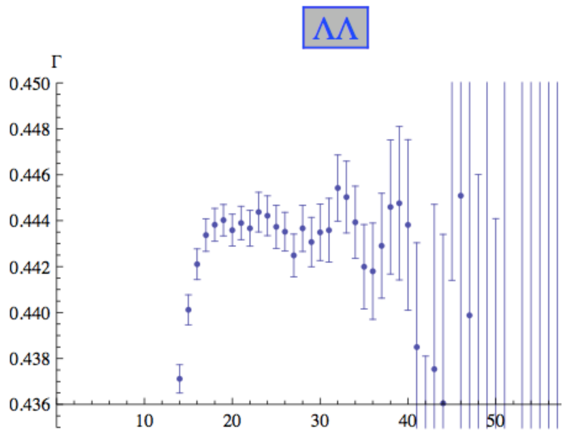
nucleons:

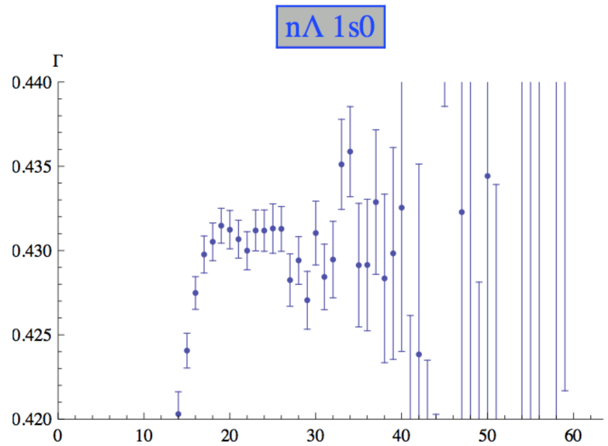
$$\frac{\sigma}{\langle C \rangle} \sim \frac{\exp\left(M_N - \frac{3m_\pi}{2}\right)t}{\sqrt{N}}$$

for baryons, the noise grows exponentially with time
poor signal-to-noise ratio

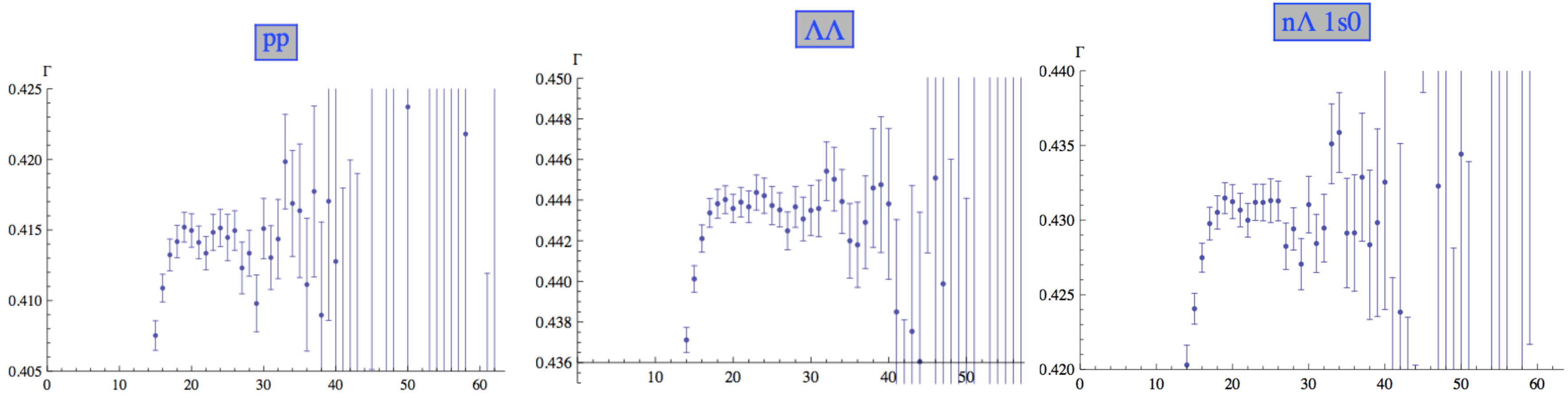
m_π ~390 MeV, L_s ~ 2.5 fm, b=0.123 fm NPLQCD, Phys.Rev. D81 (2010) 054505







$m_\pi \sim 390$ MeV, $L_s \sim 2.5$ fm, b=0.123 fm NPLQCD, Phys.Rev. D81 (2010) 054505



LQCD calculations involving A > 2 particles

Greater complexity of multinucleon systems as compared to single meson and baryon calcs

Wick contractions at que quark level, to form the correlation function is naively $N_u!\, N_d!\, N_s!$

$$\big((A+Z)!\,(2A-Z)!\big)$$

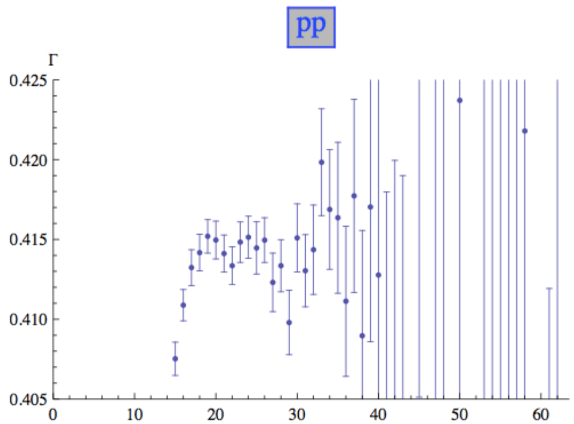
$$\boxed{{}^3\mathrm{H} \rightarrow 2880 \qquad {}^4\mathrm{He} \rightarrow 518\,400}$$

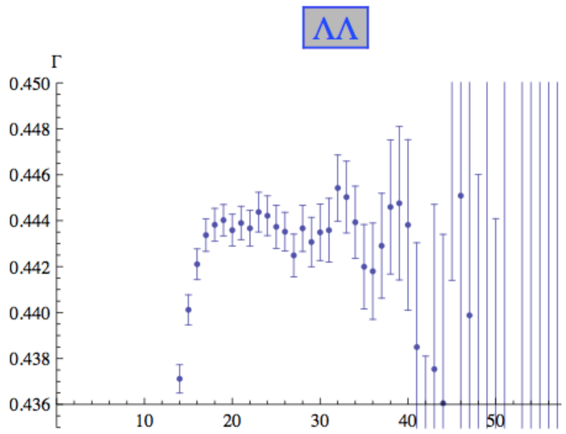
Expectation is that for A nucleons:

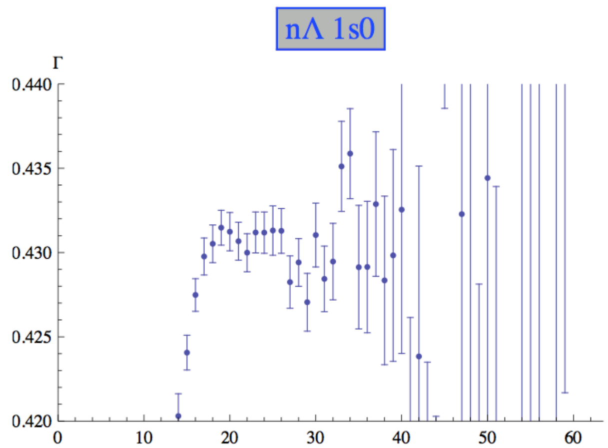
$$\frac{\sigma}{\langle C \rangle} \sim \frac{\exp\left[A\left(M_N - \frac{3m_\pi}{2} \right) t \right]}{\sqrt{N}}$$

m_π~390 MeV, L_s ~ 2.5 fm, b=0.123 fm

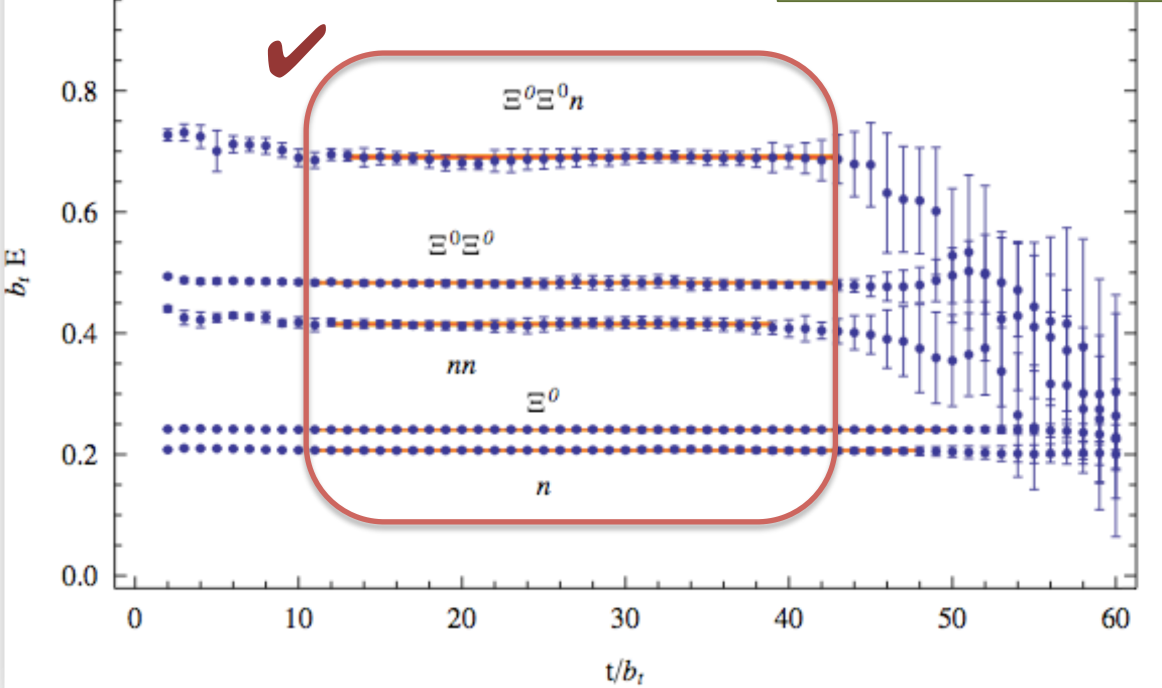
NPLQCD, Phys.Rev. D81 (2010) 054505







NPLQCD, Phys. Rev. D80 (2009) 074501

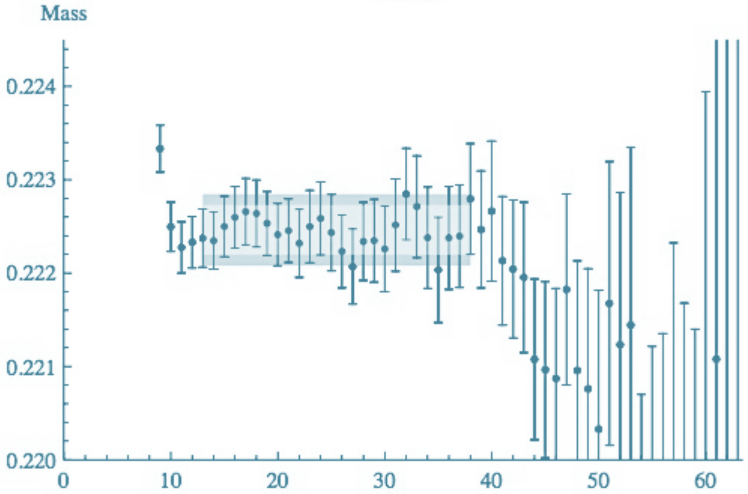


cheapest 3-baryon system:
$\Xi^0\Xi^0$n, with 3! 2! 4! = 288
Wick contractions

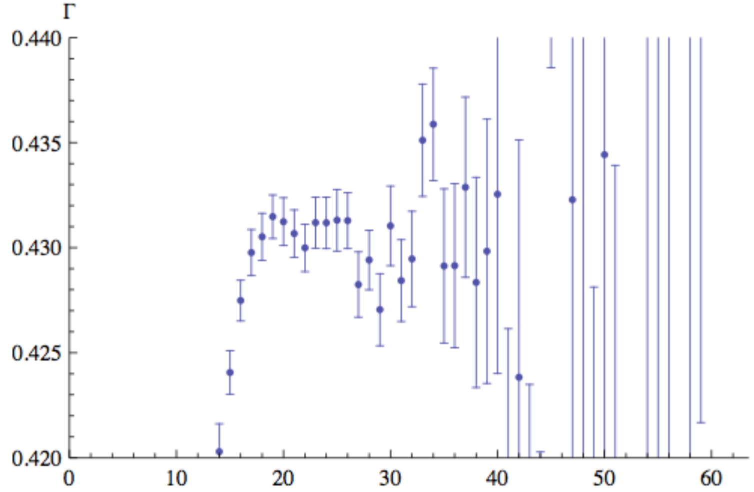
m_π~390 MeV
L_s ~ 2.5 fm

We can also extract the energy of the interacting system for a given {m$_\pi$, L, b} set

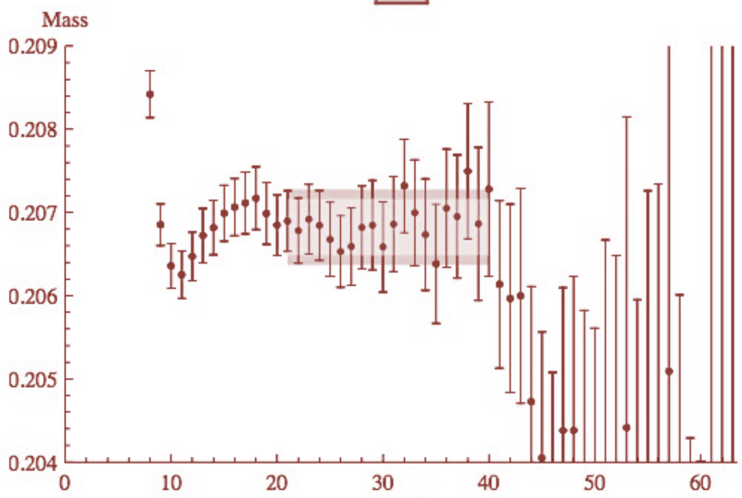
Λ



nΛ 1s0



N



$$G_{\Lambda N({}^1S_0)}(t) = \frac{C_{\Lambda N({}^1S_0)}(t)}{C_\Lambda(t)\, C_N(t)} \to A_0\, e^{-\Delta E_{\Lambda N({}^1S_0)} t}$$

Effective Mass method

$$\frac{1}{t_J} \log \frac{G(t)}{G(t+t_J)} \to \text{extract} \quad \Delta E$$

Scattering states:

Beane, Bedaque, Parreño, Savage, Phys. Lett. B585 (2004) 106-114

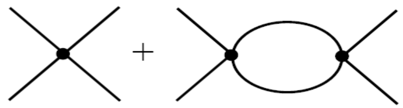
$$\Delta E_0 = \frac{p^2}{M} = \frac{4\pi a}{ML^3}\left[1 - c_1 \frac{a}{L} + c_2\left(\frac{a}{L}\right)^2 + \ldots\right]$$

Ground state energy shift

Recovering M. Lüscher, Commun. Math. Phys. 105, 153 (1986) (L>>a)

Bound states?

$\mathcal{A} \sim$



$+ \cdots = \dfrac{4\pi}{M}\dfrac{1}{p\cot\delta(p) - ip}$

infinite volume

b.s. $\quad p^2 = -\gamma^2$

$$\cot\delta(i\gamma) = i$$

finite volume:

$$\cot\delta(i\gamma)\Big|_{k=i\gamma} = i - i\sum_{\vec{m}\neq\vec{0}} \frac{e^{-|\vec{m}|\gamma L}}{|\vec{m}|\gamma L}$$

$k^2 < 0, \quad k = i\kappa$

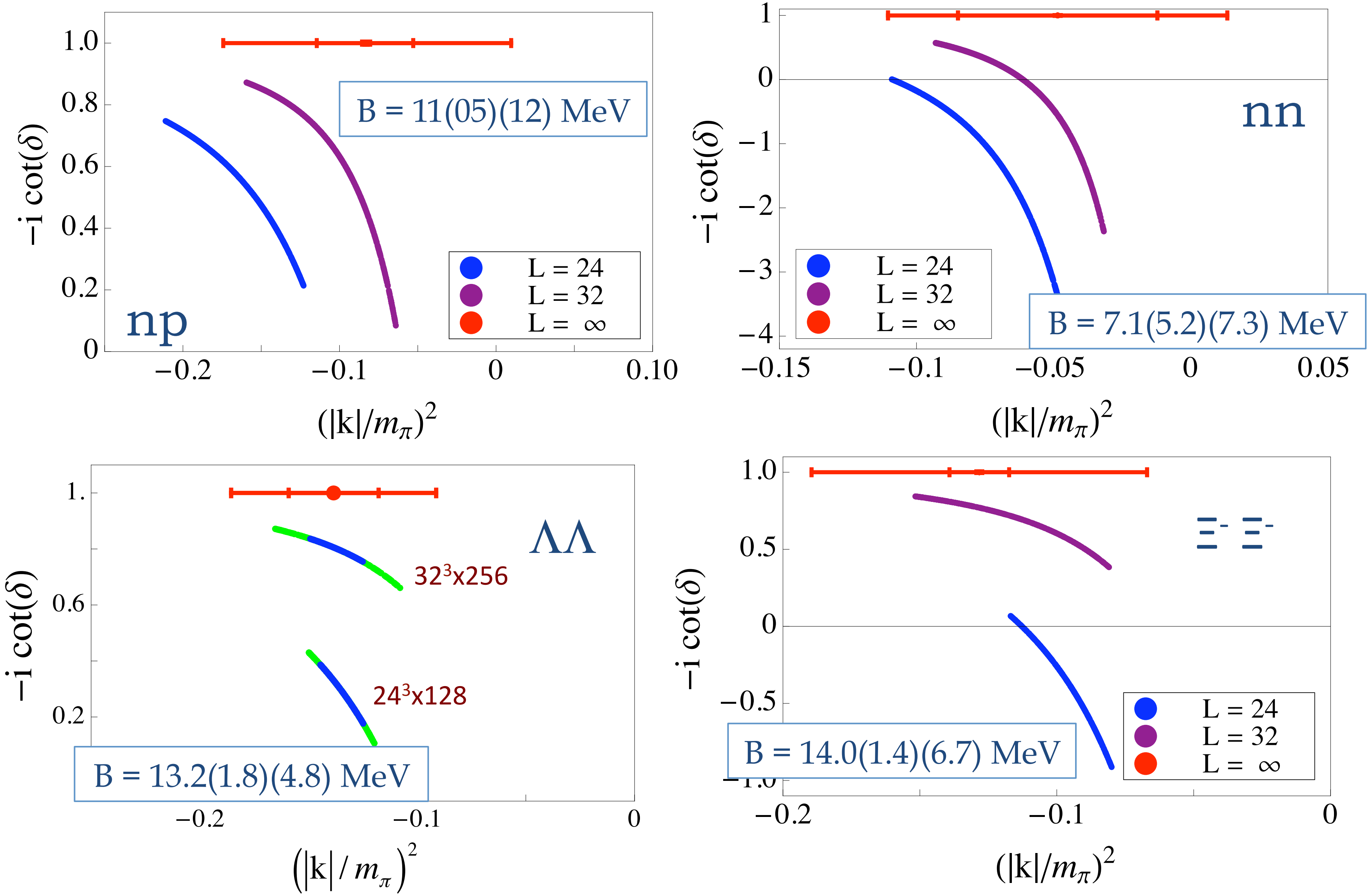
$$\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L} + \ldots\right)$$

$\kappa \to \gamma \quad$ for large L

$$B_\infty = \frac{\gamma^2}{M}$$

m_π ~390 MeV, L_s ~ 2, 2.5, 3, 4 fm, b=0.123 fm

NPLQCD, Phys.Rev. D85 (2012) 054511



Going beyond A=2

infinite volume extrapolations with enough statistics for (hyper) nuclear systems

energy splittings in nuclear physics are small → Need of high statistics calculations

Move to heavier quark masses:
resources required to generate configurations and q-propagators are smaller
degradation in the signal-to-noise ratio in multinucleon correlation functions is reduced

calculations at the SU(3)-flavor symmetric point

L/b	T/b	β	$b\,m_q$	b (fm)	L (fm)	T (fm)	m_π (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
24	48	6.1	−0.2450	0.145	3.4	6.7	806.5(0.3)(0)(8.9)	14.3	28.5	3822	96
32	48	6.1	−0.2450	0.145	4.5	6.7	806.9(0.3)(0.5)(8.9)	19.0	28.5	3050	72
48	64	6.1	−0.2450	0.145	6.7	9.0	806.7(0.3)(0)(8.9)	28.5	38.0	1905	54

NPLQCD, PRD 87, 034506 (2013); PRC 88, 024003 (2013)

physical strange quark mass

no physical light-quark masses yet
only one lattice spacing

Anisotropic lattices: $N_t >> N_s$

(N_f=2+1 clover-improved Wilson fermion actions)

higher resolution in the time direction:

better study of noisy states
better extraction of excited states
reduce the systematic due to fitting
(confident plateaus)

Going beyond A=2 $SU(3)_f$

NPLQCD Phys.Rev. D87 (2013), 034506

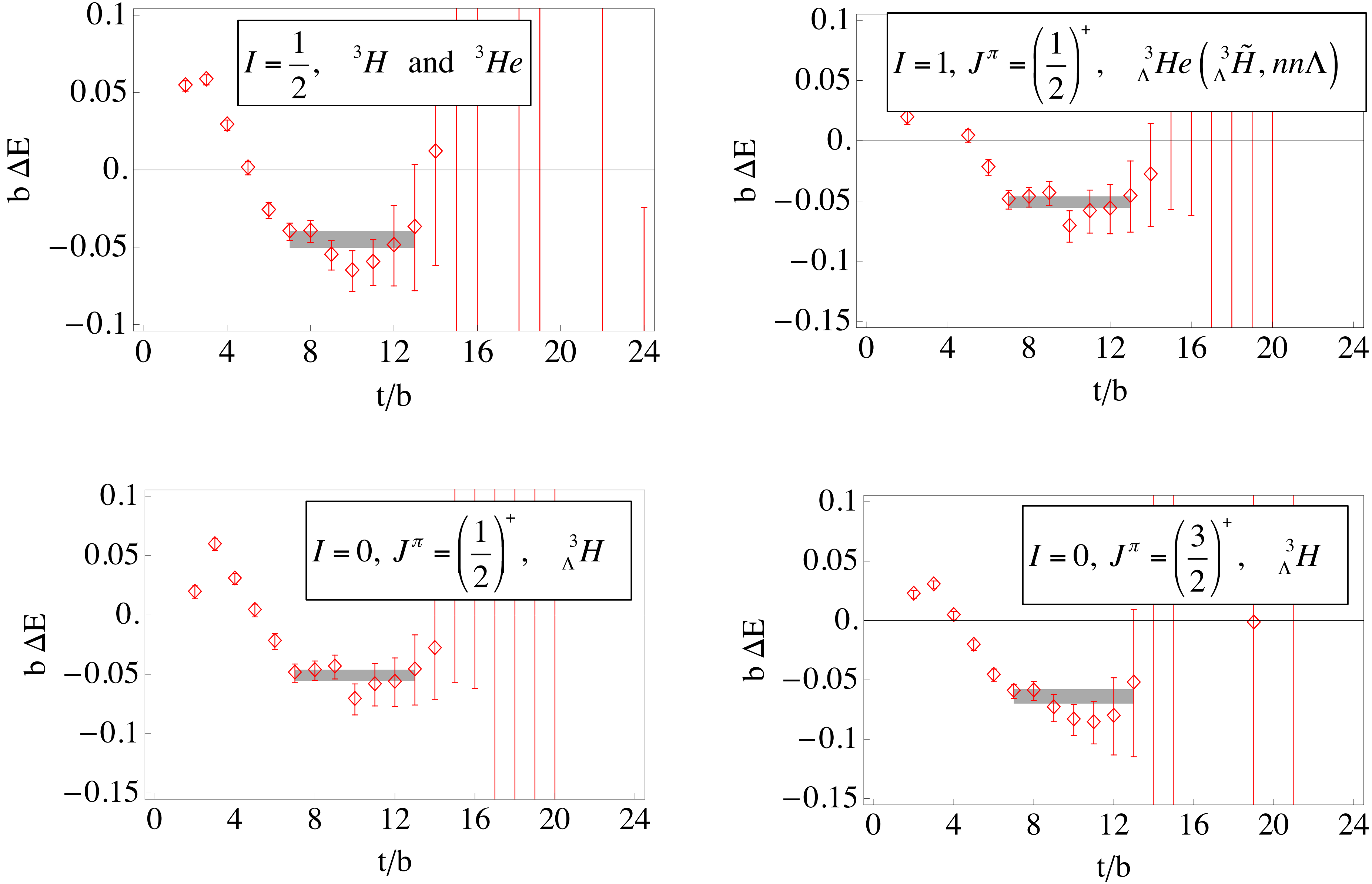
$(\pi, J^2, J_z, s, A, I^2, I_z)$

(no e.m. interactions)

Label	A	s	I	J^π	Local SU(3) irreps	This work
N	1	0	1/2	$1/2^+$	$\mathbf{8}$	$\mathbf{8}$
Λ	1	-1	0	$1/2^+$	$\mathbf{8}$	$\mathbf{8}$
Σ	1	-1	1	$1/2^+$	$\mathbf{8}$	$\mathbf{8}$
Ξ	1	-2	1/2	$1/2^+$	$\mathbf{8}$	$\mathbf{8}$
d	2	0	0	1^+	$\overline{\mathbf{10}}$	$\overline{\mathbf{10}}$
nn	2	0	1	0^+	$\mathbf{27}$	$\mathbf{27}$
$n\Lambda$	2	-1	1/2	0^+	$\mathbf{27}$	$\mathbf{27}$
$n\Lambda$	2	-1	1/2	1^+	$\mathbf{8}_A$, $\overline{\mathbf{10}}$	—
$n\Sigma$	2	-1	3/2	0^+	$\mathbf{27}$	$\mathbf{27}$
$n\Sigma$	2	-1	3/2	1^+	$\mathbf{10}$	$\mathbf{10}$
$n\Xi$	2	-2	0	1^+	$\mathbf{8}_A$	$\mathbf{8}_A$
$n\Xi$	2	-2	1	1^+	$\mathbf{8}_A$, $\mathbf{10}$, $\overline{\mathbf{10}}$	—
H	2	-2	0	0^+	$\mathbf{1}$, $\mathbf{27}$	$\mathbf{1}$, $\mathbf{27}$
^{3}H, ^{3}He	3	0	1/2	$1/2^+$	$\overline{\mathbf{35}}$	$\overline{\mathbf{35}}$
${}^3_\Lambda$H$(1/2^+)$	3	-1	0	$1/2^+$	$\overline{\mathbf{35}}$	—
${}^3_\Lambda$H$(3/2^+)$	3	-1	0	$3/2^+$	$\overline{\mathbf{10}}$	$\overline{\mathbf{10}}$
${}^3_\Lambda$He, ${}^3_\Lambda\tilde{\mathrm{H}}$, $nn\Lambda$	3	-1	1	$1/2^+$	$\mathbf{27}$, $\overline{\mathbf{35}}$	$\mathbf{27}$, $\overline{\mathbf{35}}$
${}^3_\Sigma$He	3	-1	1	$3/2^+$	$\mathbf{27}$	$\mathbf{27}$
^{4}He	4	0	0	0^+	$\overline{\mathbf{28}}$	$\overline{\mathbf{28}}$
${}^4_\Lambda$He, ${}^4_\Lambda$H	4	-1	1/2	0^+	$\overline{\mathbf{28}}$	—
${}^{\ 4}_{\Lambda\Lambda}$He	4	-2	0	0^+	$\mathbf{27}$, $\overline{\mathbf{28}}$	$\mathbf{27}$, $\overline{\mathbf{28}}$

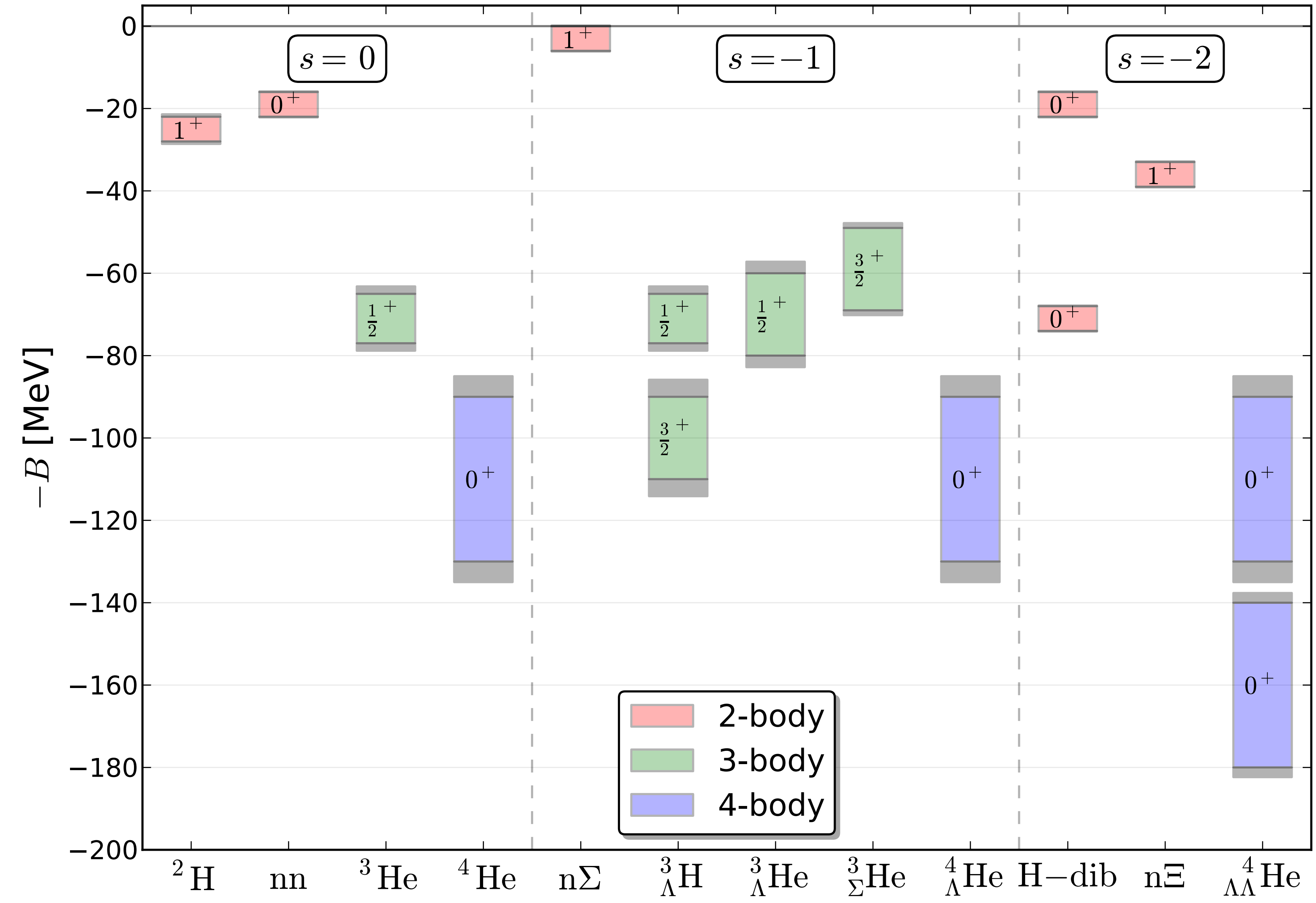
NPLQCD Phys.Rev. D87 (2013), 034506

For example, for the A=3 system: $(48^3 \times 64)$



NPLQCD Phys. Rev. D87 (2013) 3, 034506

$SU(3)_f$



LQCD calculations of magnetic moments of light nuclei $SU(3)_f$

Use background magnetic fields

$m_\pi \sim 800$ MeV

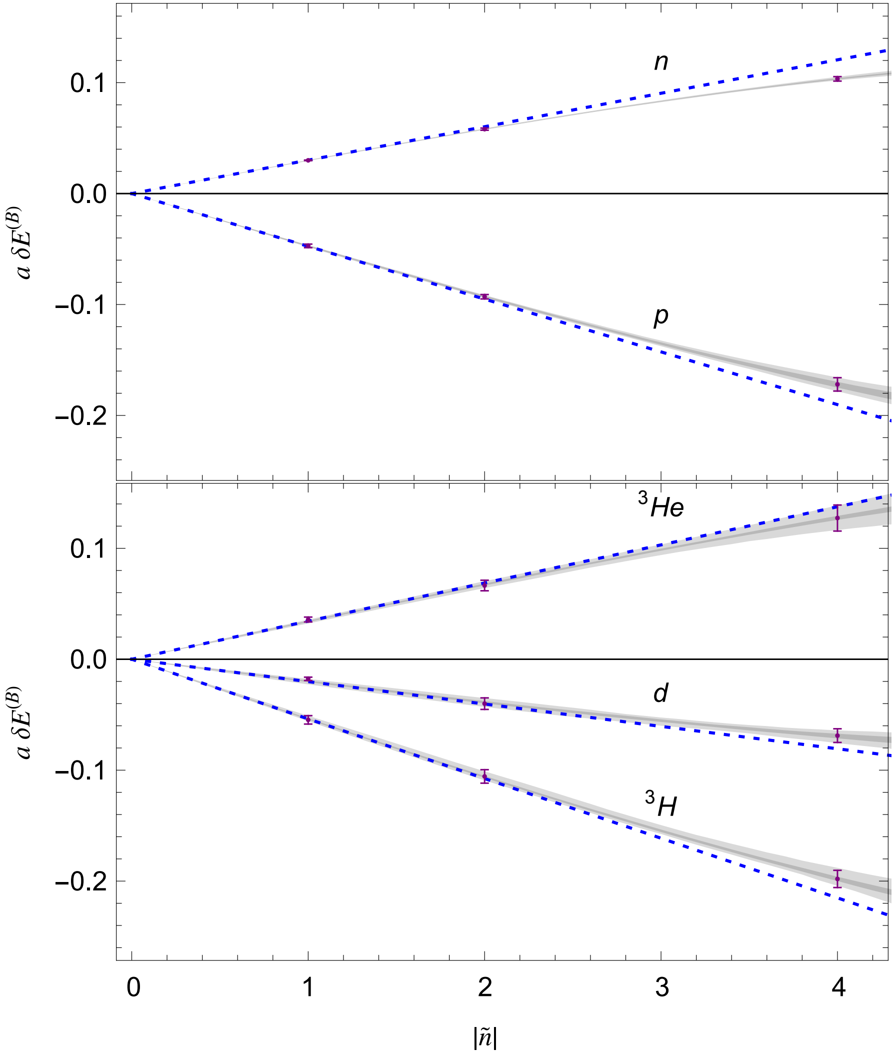
$$e|B| = \frac{6\pi}{L^2}\tilde{n}, \quad \vec{B} = \hat{z}\cdot B \quad (e|B| \sim 0.046\,\tilde{n}\ \mathrm{GeV}^2)$$

$$E(B) = M + \frac{|QeB|}{2M} - \mu\cdot B - 2\pi\beta|B|^2 + \ldots$$

$$\delta E^B = E^B_{+j} - E^B_{-j}$$

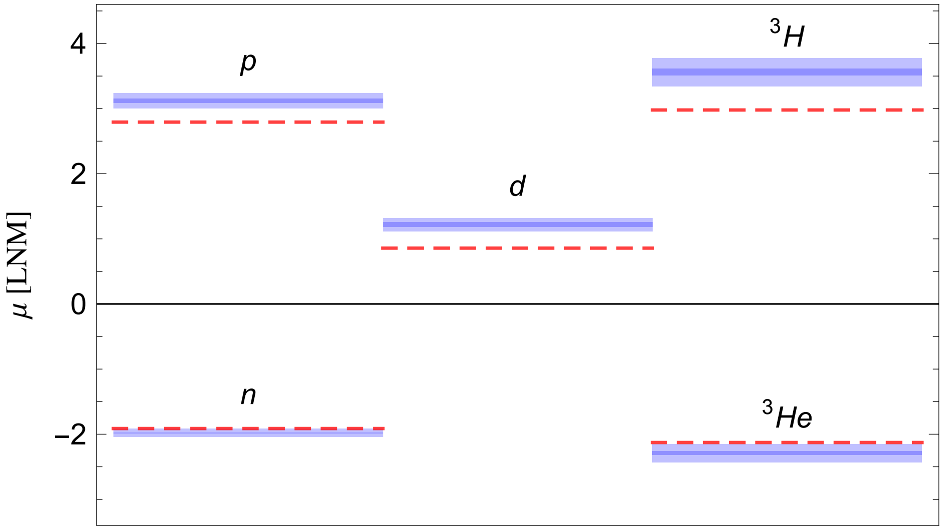
$$R(B) = \frac{C^B_{j_z}(t)\,C^0_{-j_z}(t)}{C^B_{-j_z}(t)\,C^0_{j_z}(t)} \xrightarrow{t\to\infty} Ze^{-\delta E^B t}$$

$$\delta E^B = -2\mu|B| + \gamma_3|B|^3$$



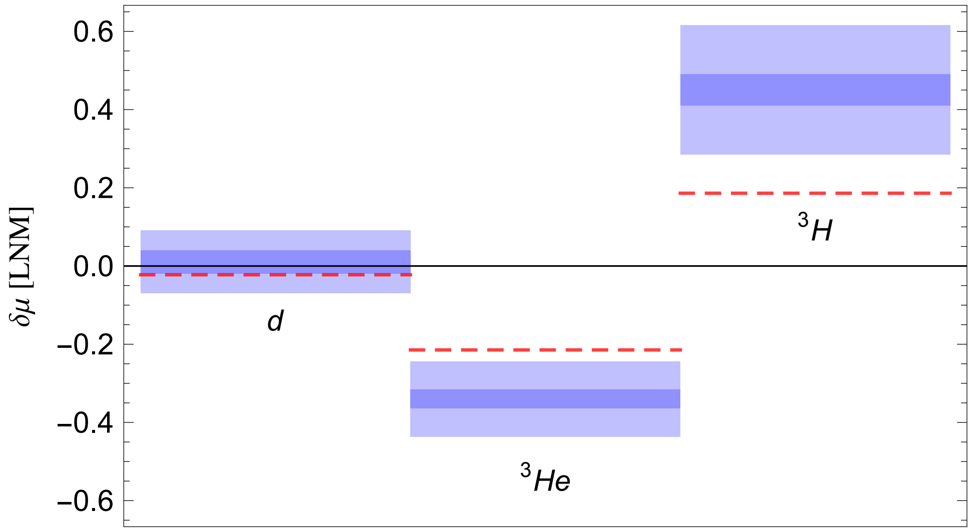
NPLQCD, Phys. Rev .Lett. 113 (2014) 25, 252001

LQCD calculations of magnetic moments of light nuclei $SU(3)_f$



$m_\pi \sim 800$ MeV

NPLQCD, Phys. Rev .Lett. 113 (2014) 25, 252001



$$\mathrm{LNM} = \frac{e}{2M_N^{\mathrm{latt}}}$$

Light nuclei and hypernuclei from Lattice QCD (A $\leq$ 4)

Can we understand the properties of (small) nuclei directly from QCD?

Calculation of important quantities in nuclear physics with LQCD is only now becoming practical, with first calculations of simple multibaryon interactions being performed, although at unphysical quark masses.

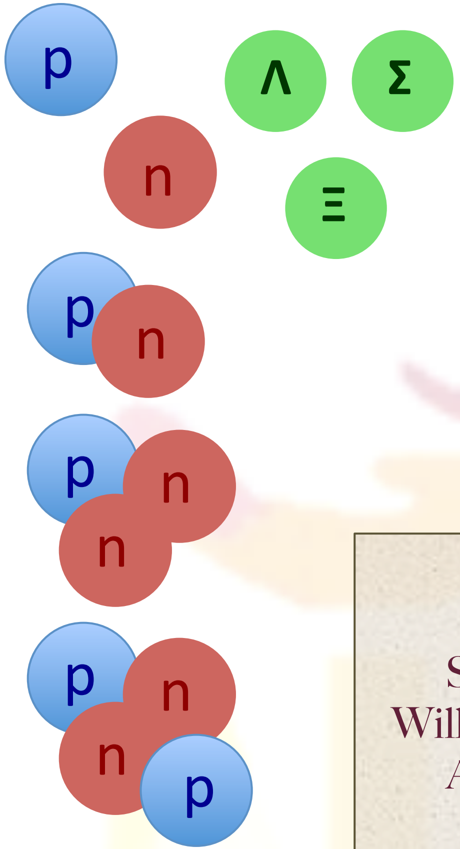
Present day computing power begins to be sufficient to calculate nuclear properties in lattice QCD with near physical quark masses (completed calculations @ 430 MeV & on-going calcs. @ 300 MeV)

A chiral extrapolation to the physical pion mass may be possible if sufficiently many results at various pion masses are available.

Also, calculations at different lattice spacings are now at reach (b ~ 0.1, 0.08 fm)

LQCD calculations would be specially of interest for systems that are not accessible experimentally → Complementary information to experimental programs not too far in the future.

"This is a long term project that can be done given sufficient resources. Once completed will definitely be transformative although each individual step may not be."
(encouraging referee's comment)



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