ECT*

EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

LFC17: Old and New Strong Interaction from LHC to Future colliders

Top-quark mass determination

Peter Uwer

HUMBOLDT-UNIVERSITÄT ZU BERLIN

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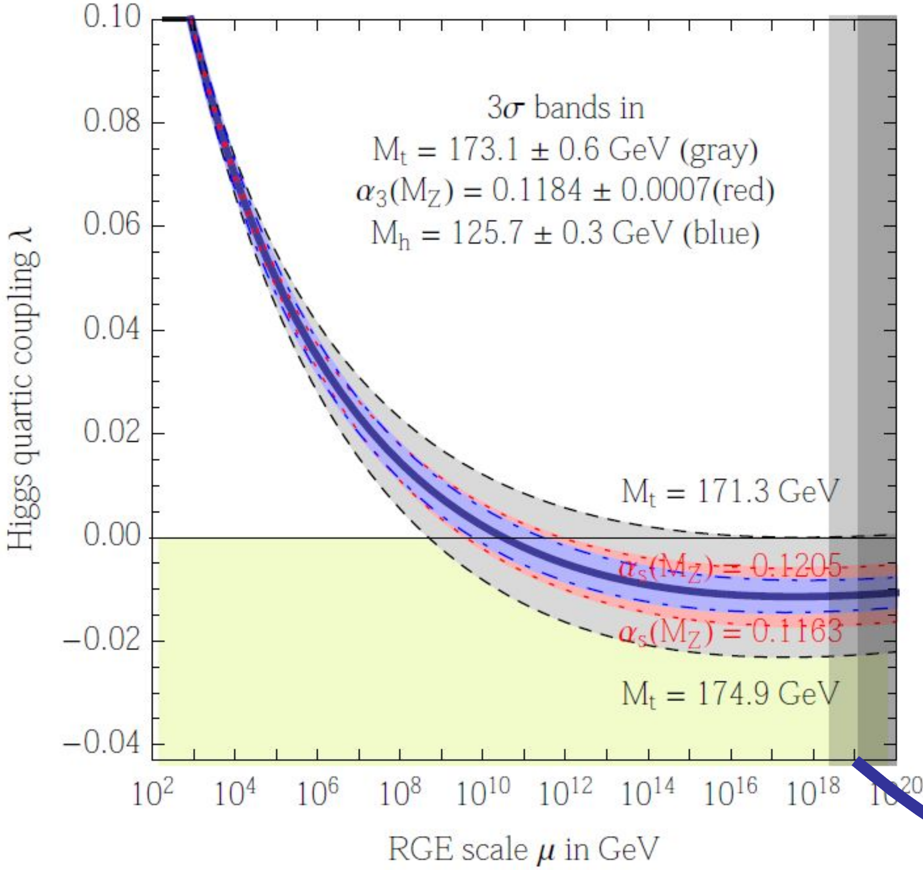
Outline

1. Motivation
2. Some preliminaries
3. Overview of existing methods and recent results
4. Summary

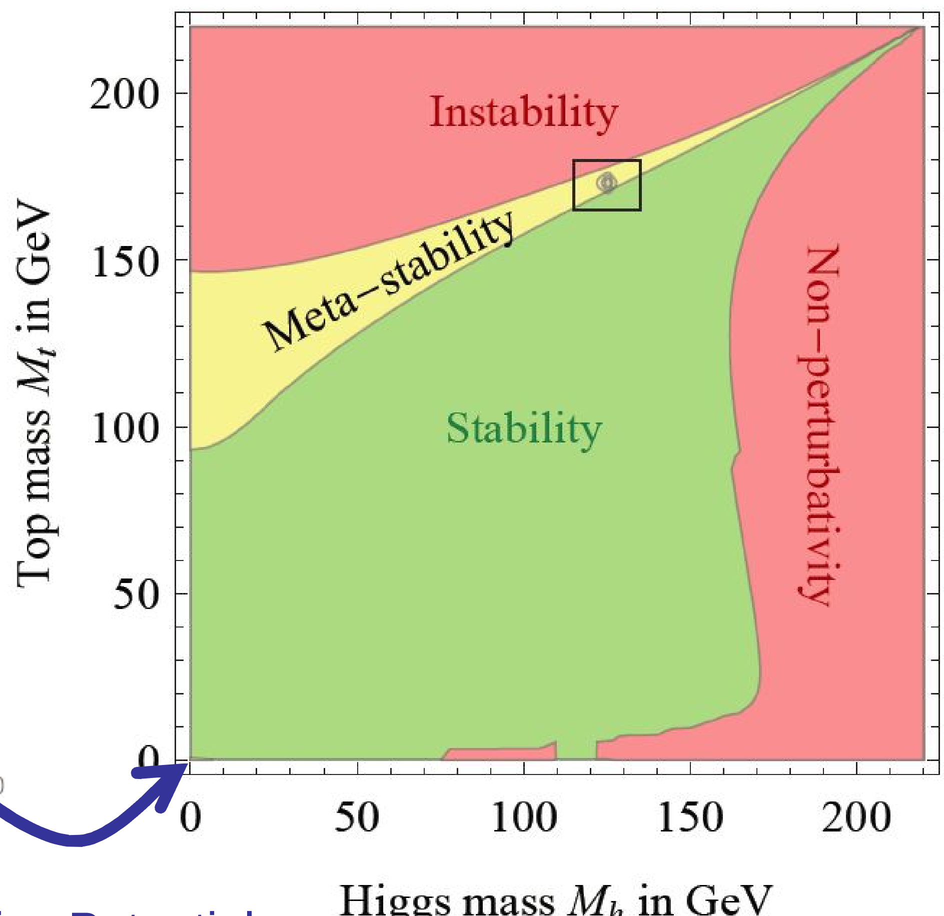
Why do we care about the top-quark mass ?

[Degrassi, Di Vita, Elias-Miro,Spinosa,Giudici ’12,
Alekhin, Djouadi, Moch ’12]

RGE running
quartic Higgs couling



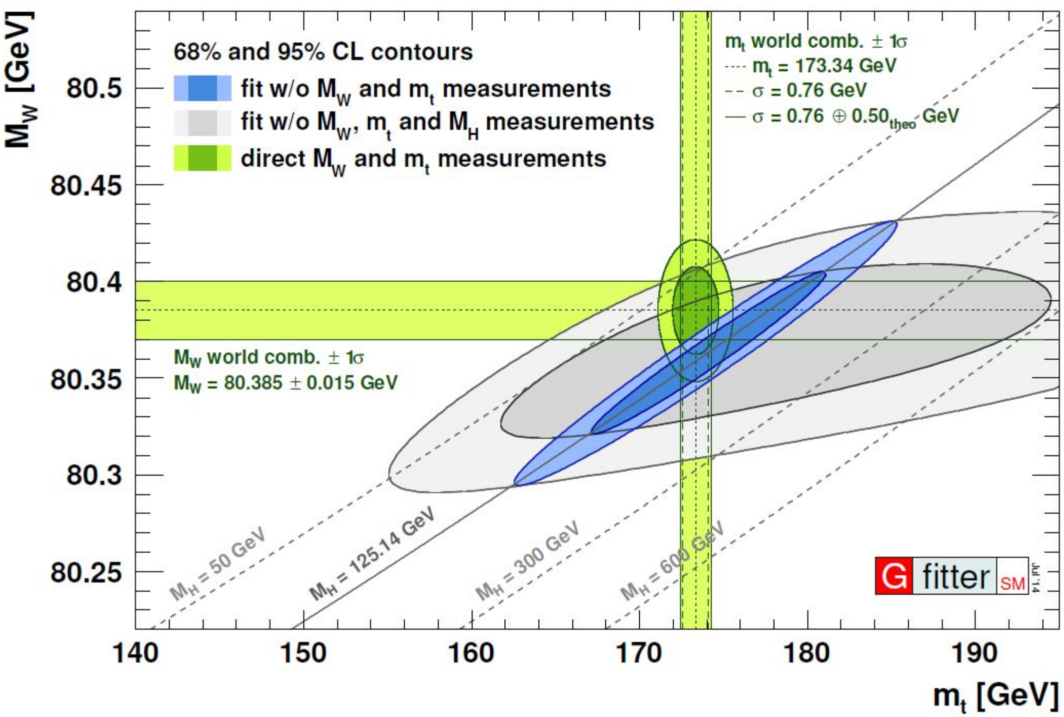
Vacuum stability

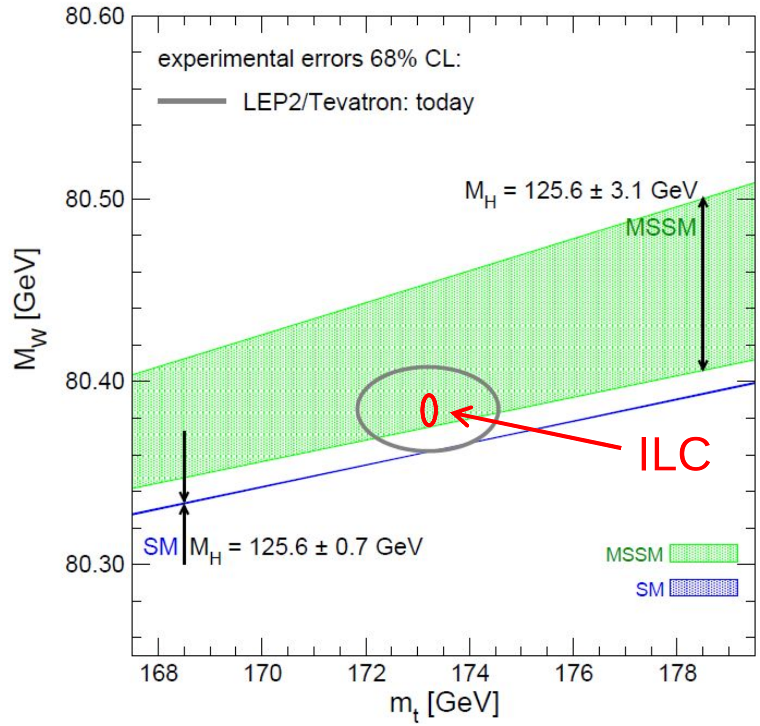


Effective Potential

Why do we care about the top-quark mass ?

[Gfitter ´14] [Heinemeyer, Hollik, Weiglein, Zeune ,13]





- Fundamental parameter of the SM
- Important consistency tests of the SM
- Important to constrain BSM physics

How do we measure a quark mass ?

…at least in theory*)

- We don't see free quarks, there is no pole in the *S*-matrix

→ top-quark mass is not an observable,
mass is just a parameter of the underlying theory

Precise value depends on the definition / renormalisation scheme (i.e. pole mass, $\overline{\text{MS}}$ mass)

- Determine / fit parameter from comparison of theoretical predictions and measurements

To fix the renormalisation scheme at least a NLO calculation is required

*) In theory there is no big difference between theory and practice – in practice there is [Yogi Bera]

Renormalization and scheme definition

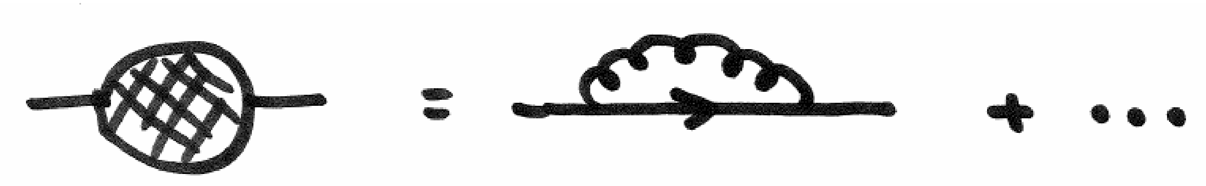
bare quantities

$$m_0 = Z_m^R m_R \qquad \psi_0 = Z_\psi^R \psi_R$$

renormalized quantities

Renormalization constants fixed through self energy correction:

$$\Sigma =$$



Dyson summation:

$$\frac{1}{\not{p} - m_0} \rightarrow \frac{1}{\not{p} - m_R - \Sigma(p)}$$

full propagator

Pole mass scheme:

Fix ren. constants such that propagator has pole at $m_R = m_{\text{pole}} \; (\to \Sigma(m_R) = 0)$

$\overline{\text{MS}}$ mass / running mass

Chose Z's such that only divergences are absorbed in renormalization constants

Renormalization and scheme definition

- Other schemes are possible:

1S mass, Potential Subtracted (PS) mass,…

(useful for e+e- not so relevant for pp)

- Different schemes can be related within pert. theory:

$$m_t = \overline{m}(\mu)\left(1+\frac{\alpha_s(\mu)}{\pi}\left[\frac{4}{3}+\ln\left(\frac{\mu^2}{\overline{m}(\mu)^2}\right)\right]+O(\alpha_s^2)\right)$$

Pole mass

$\overline{\text{MS}}$ mass

Relation known up to four loops:

[Marquard, Smirnov, Smirnov, Steinhauser, Wellmann '16]

$$\begin{aligned} M_t &= m_t(m_t)\left(1+0.4244\,\alpha_s+0.8345\,\alpha_s^2+2.375\,\alpha_s^3+(8.615\pm0.017)\,\alpha_s^4\right) \\ &= 163.508+7.529+1.606+0.496+(0.195\pm0.0004)\ \text{GeV} \end{aligned}$$ =173.34GeV

$$\overline{m}(\overline{m}) = 163.508,\ \alpha_s^{(6)}(\overline{m}) = 0.1085$$

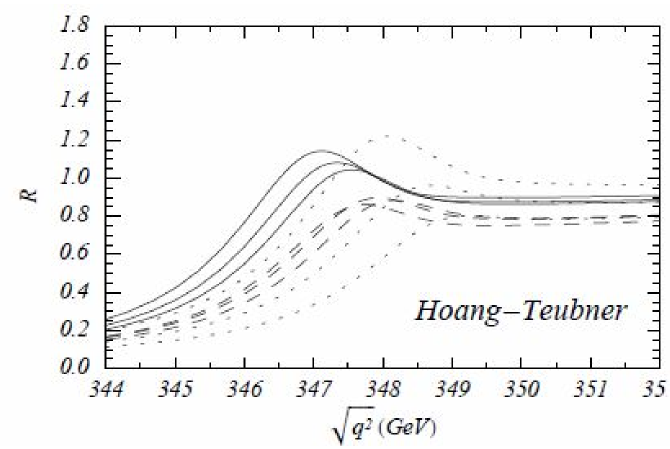
(much better convergence when relating short distance masses)

Potential issues:

1. Schemes may behave differently within perturbation theory, e.g. differences with respect to convergence possible
2. Schemes may have intrinsic limitation on reachable precision

Examples:

1) [Hoang et al '00]

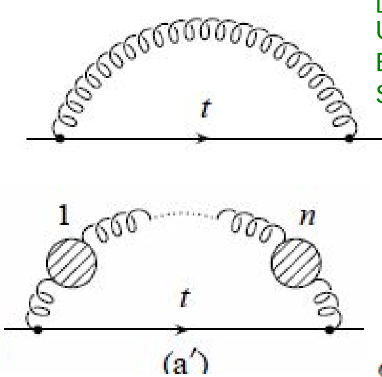


R-ratio at threshold (LO/NLO/NNLO) in e+e- annihilation using the pole mass
Significant changes in LO/NLO/NNLO

2) Renormalon ambiguity in pole mass

[Bigi, Shifman, Uraltsev, Vainshtein 94
Beneke, Braun,94
Smith, Willenbrock 97]

$\Sigma^{(1)} =$



$\sum_{n=0}^{\infty}$

$$\frac{16 m_R}{3\beta_0} \sum_{n=0}^{\infty} c_n a^{n+1}$$

$$c_n \overset{n\to\infty}{\longrightarrow} e^{-C/2} 2^n n!$$

Intrinsic uncertainty of pole mass concept due to ill behaved pert. theory

Which scheme should we use ?

Both problems are most likely not relevant for LHC

- Renormalon ambiguity in pole mass:

  Recent estimates of uncertainties yield ~70 MeV instead of $O(\Lambda_{QCD})$ estimated previously [Beneke,Marquard,Nason, Steinhauser '16]

- Pole mass and running mass at equal footing concerning convergence as long as $\overline{m}(\overline{m})$ is used:

$$m_t = \overline{m}(\overline{m})\left(1 + \frac{\alpha_s(\mu)}{\pi}\frac{4}{3} + O(\alpha_s^2)\right)$$

→ no large logs in conversion

(kinematical effects may lead to slight improvement, this is however most likely an artifact)

Picture may change if m(μ) is used to describe events at large momentum transfer

From theory to practice

Crude categorization of measurements: [CMS?]

Standard methods	Pole mass methods	Alternative methods

Features:

Standard methods

- Methods used since the beginning (with many refinements)
- Few observables related to top-quark decay
- All decay channels (all hadronic, semi-leptonic, di-leptonic)
- Most precise results apart from scheme issue
- Included in averages

Pole mass methods

- Closest to idealized measurement outlined before
- well defined renormalization scheme
- Not as precise as standard methods
- Only few observables/measurement so far

Alternative methods

- large variety of different observables and decay channels
- Some rather precise measurements
- Others still limited by statistics
- Highly correlated with other measurements (→often not yet included in averages)
- Will play more important role in the future

Pole mass measurements

Comparison of observable calculated (including higher order corrections) within the pole mass scheme with measurements

Prominent examples:

$\sigma_{t\bar{t}}$

- NNLO/NNLL QCD predictions
- limited sensitivity: $\frac{\Delta\sigma}{\sigma} \sim 5\frac{\Delta m}{m}$
- limited by achievable exp./th. precision

$$\frac{d\sigma_{t\bar{t}+1\,jet}}{d\rho_s}$$

- NLO QCD
- gluon emission leads to higher sensitivity

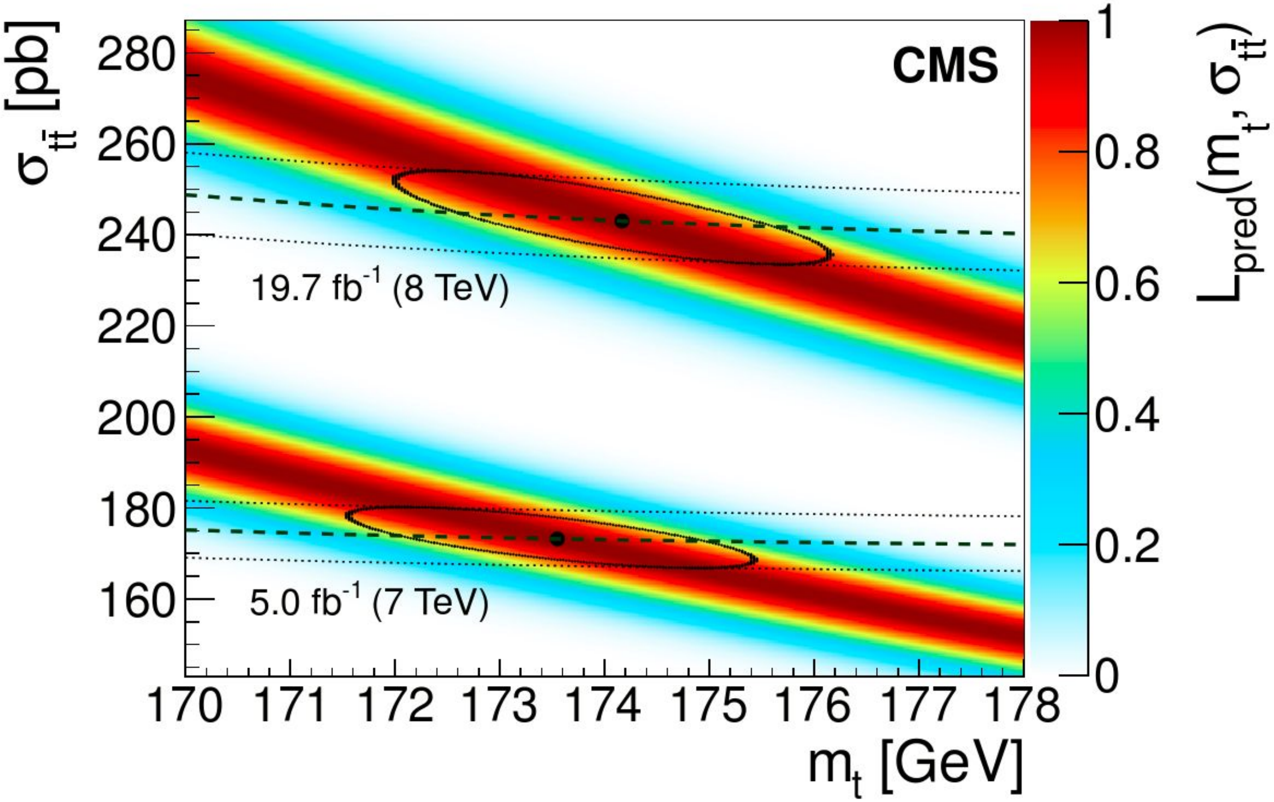
$$\frac{d\sigma_{t\bar{t}}}{dX}$$

- NNLO QCD
- slightly higher sensitivity than incl. cross section
- still in its infancy

Note: all methods rely on using MC's

Top-quark pole mass from σ_{tt} (NNLO/NNLL)

[CMS-TOP-13-004, JHEP 08 (2016) 029]



PDF dependence combined measurement:

	m_t [GeV]
NNPDF3.0	$173.8^{+1.7}_{-1.8}$
MMHT2014	$174.1^{+1.8}_{-2.0}$
CT14	$174.3^{+2.1}_{-2.2}$

Dashed and dotted lines show result of cross section measurement (~3.5% uncertainty), depends on mt because of efficiencies and phase space extrapolation!

Theory predictions for m=172.5 GeV:

$$\sigma_{t\bar{t}} = 177.3^{+4.7}_{-6.0}\,(\text{scale}) \pm 9.0\,(\text{PDF}+\alpha_s)\ \text{pb}, \quad \text{at } \sqrt{s} = 7\ \text{TeV and}$$
$$\sigma_{t\bar{t}} = 252.9^{+6.4}_{-8.6}\,(\text{scale}) \pm 11.7\,(\text{PDF}+\alpha_s)\ \text{pb}, \quad \text{at } \sqrt{s} = 8\ \text{TeV}.$$

NNLO/NNLL

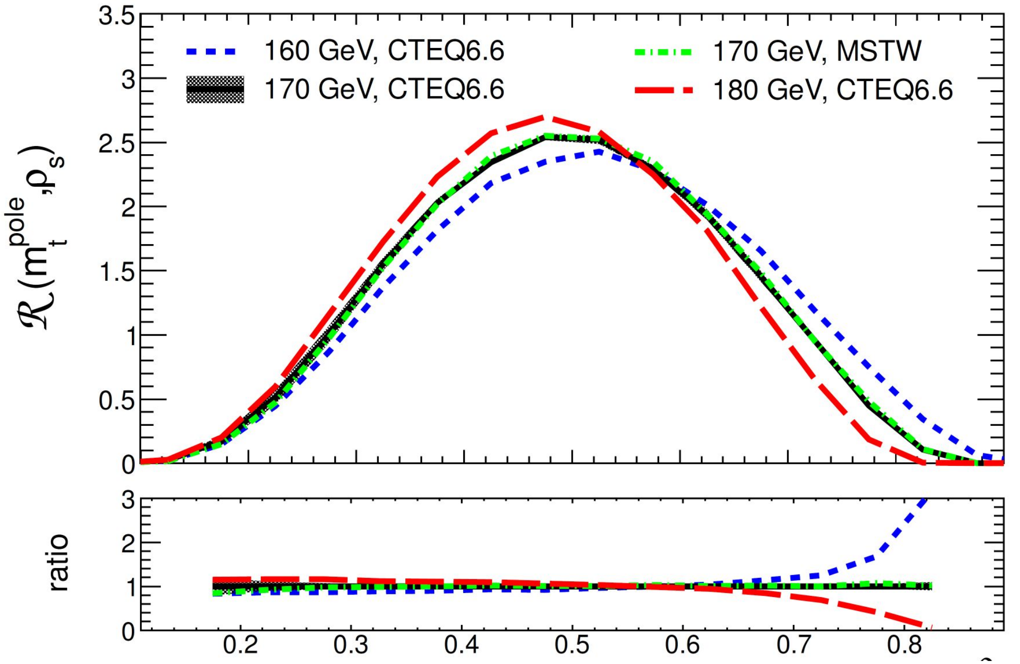
Pole mass from tt+1jet

[S. Alioli, P.Fernandez, J.Fuster, A. Irles, S. Moch, PU, M. Vos]

Consider tt+1-jet events:

$$\mathcal{R}(m_{\text{Pole}}, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\text{Jet}}} \frac{d\sigma_{t\bar{t}+1\text{Jet}}}{d\rho_s}(m_{\text{Pole}})$$

$$\rho_s = \frac{2m_0}{\sqrt{s_{t\bar{t}+1\text{Jet}}}},$$ m_0 scale of order mt, for example m_0 =170GeV



high energy ρ_s threshold

Compare theory to unfolded data

ATLAS: (7TeV) $m_t = 173.7 \pm 1.5(stat) \pm 1.4(sys.)^{+1.0}_{-0.5}(th)$GeV [JHEP10 (2015) 121]

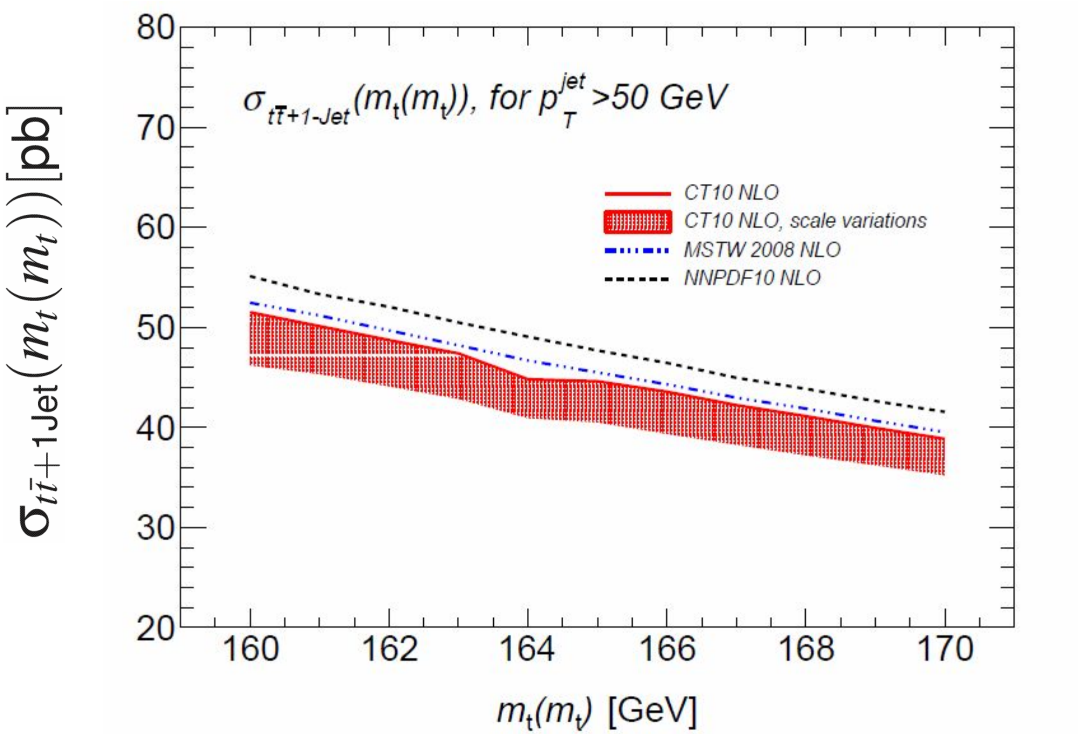
CMS 8TeV $m_t = 169.9 \pm 1.1(stat)^{+2.5}_{-3.1}(sys.)^{+3.6}_{-1.6}(th)$GeV [CMS PAS TOP-13-006]

Running mass from tt+1jet

[J.Fuster, A. Irles, D. Melini, PU, M. Vos '17]

Express $\mathcal{R}$ in terms of the running mass:

$$\mathcal{R}(m_{\text{Pole}}, \rho_s) \to \mathcal{R}(m(m), \rho_s)$$



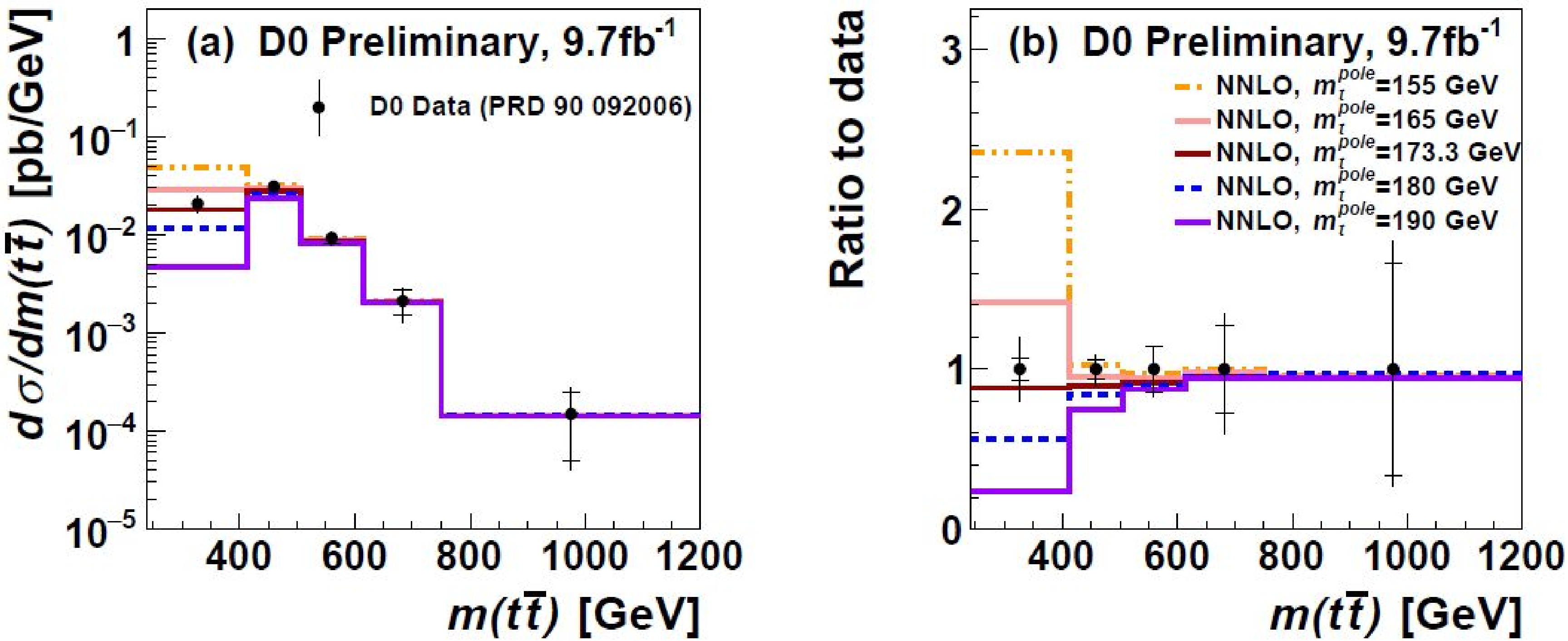
Using ATLAS 7TeV results:

$$m(m) = 165.9 \pm 1.4(stat) \pm 1.3(sys.)^{+1.5}_{-0.5}(th)\text{GeV}$$

- Consistent with pole mass determinations
- No improvement of perturbation theory as expected

Pole mass from $\frac{d\sigma_{t\bar{t}}}{dX}$

[FERMILAB-CONF-16-383-PPD]



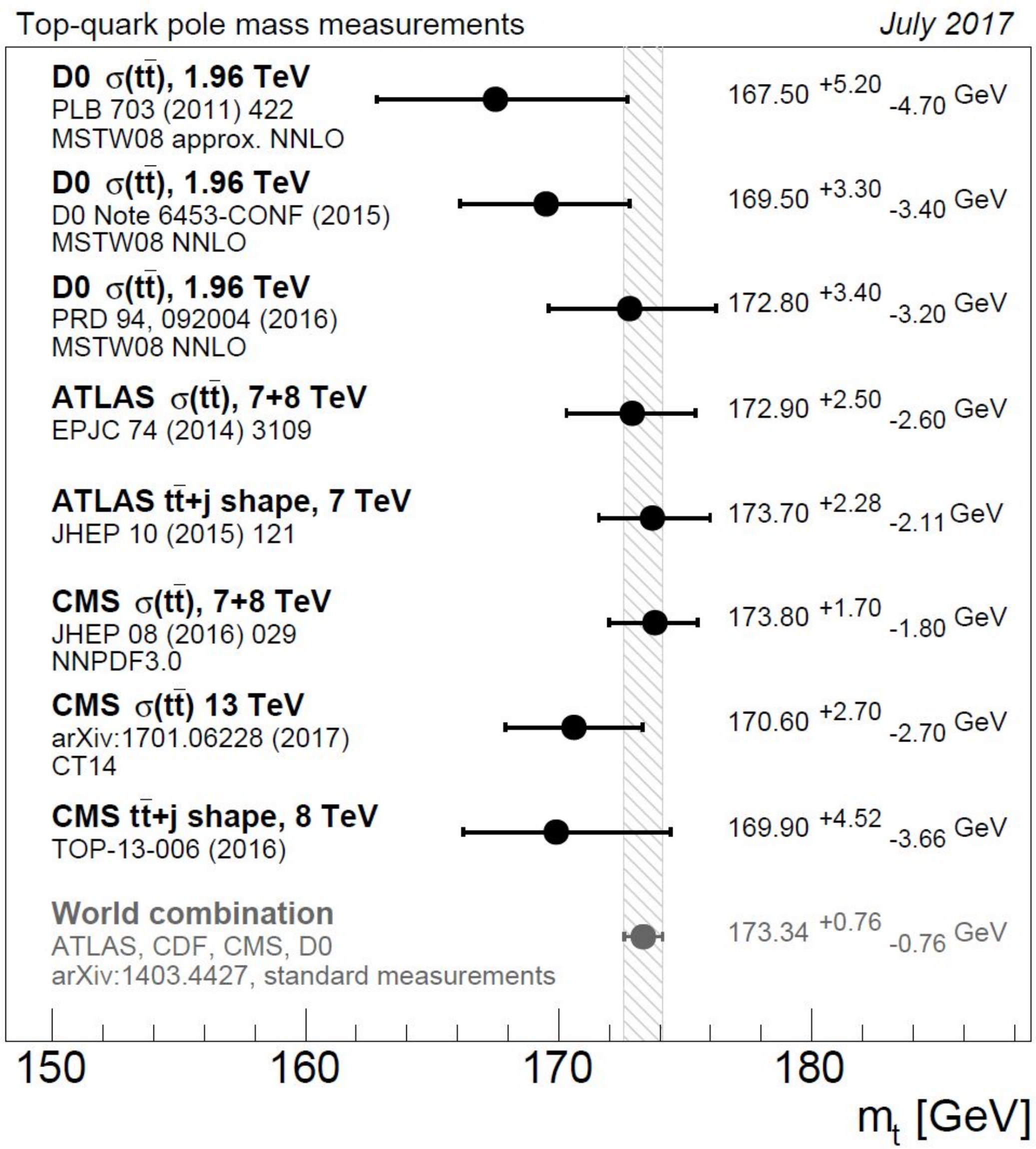
- Based on unfolded data
- Results for $\frac{d\sigma}{dm_{t\bar{t}}}, \frac{d\sigma}{dp_\perp^{t,\bar{t}}}$ combined

Preliminary result:

$$m_t = 167.3 \pm 2.1(exp) \pm 1.5(scale) \pm 0.2(PDF)\text{GeV} = 167.3 \pm 2.6\text{GeV}$$ NLO

$$m_t = 169.1 \pm 2.2(exp) \pm 0.8(scale) \pm 1.2(PDF)\text{GeV} = 169.1 \pm 2.5\text{GeV}$$ NNLO

Pole mass measurements



- Theory predictions as function of mt are compared to measured observables
- Measurements consistent among each other and with standard measurements
- Exp. Determination of observables still relies on theory and MC's (efficiencies,unfolding,…)

Weighted average ignoring any correlation yields:

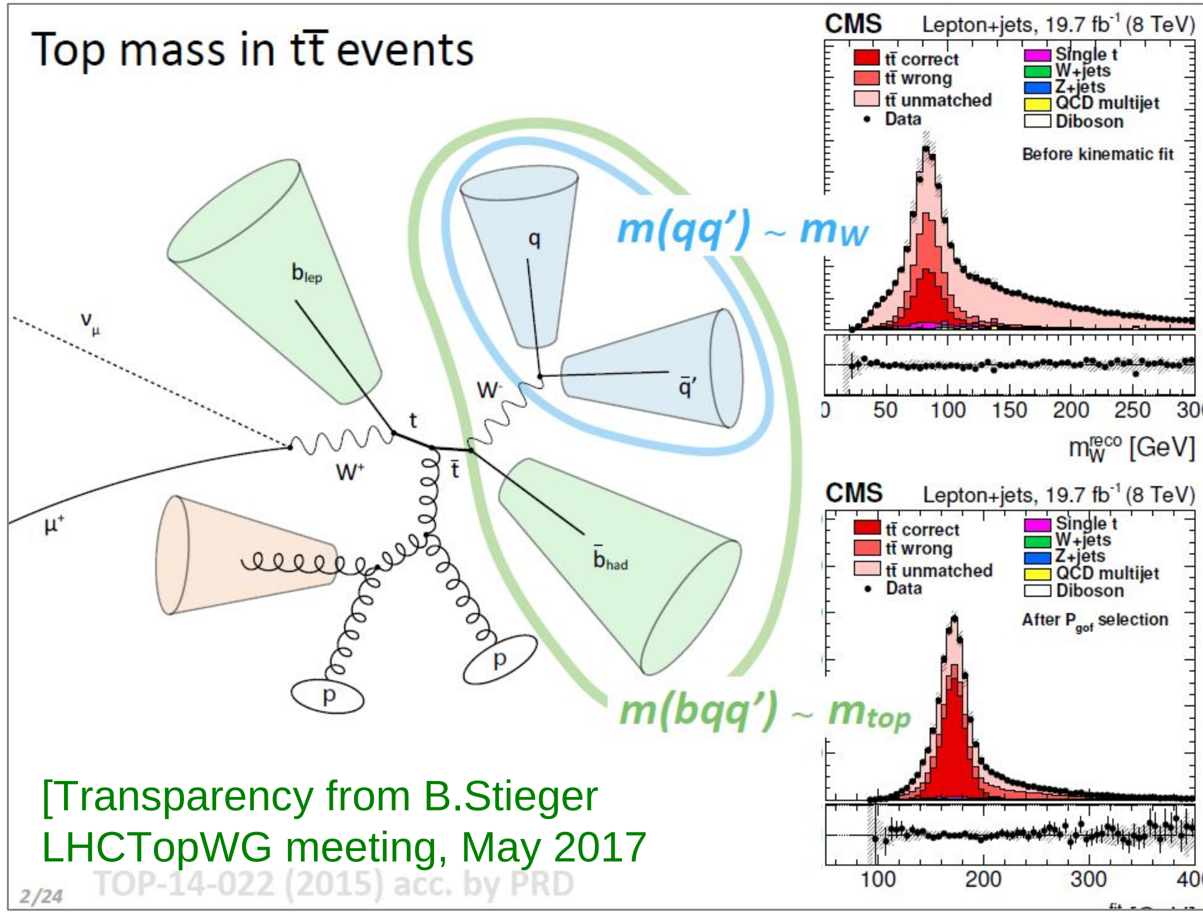
$$172.39 \pm 0.98 \text{ GeV}$$

Standard measurements

Study kinematical distributions related to the top-quark decay products

Extract mass through template fits

Higher statistics allows multi-dimensional fits to constrain dominant uncertainties like JES



→ Requires reliable differential predictions depending on a variety of cuts and jet dynamics required incorporating shower and hadronization effects, taking pert. as well as non-pert. effects into account

Standard measurements

- Reliable predictions are based on LO/NLO matrix elements and include parton shower and hadronization
- Apart from matrix elements top-quark mass appears also as parameter in other parts of the MC (e.g. shower)
- While the mass used in NLO matrix elements is in a well defined scheme, not obvious for other parts
- The mass parameter determined from a comparison of data and MC is often called MC-mass
- No compact scheme definition for MC mass like for pole mass / running mass
- However, \`relation´ is encoded in the MC (interplay part.←→hadr.)

What is the precise relation

$$m_t \leftrightarrow m_{MC} \quad ?$$

- universal?
- observable dependent?
- MC dependent?
- tune dependent?
- just another calibration?

Pole mass versus MC mass

[Butenschoen,Dehnadi,Hoang,Mateu,Preisser,Stewart, PRL 117 (2016) 232001]

Idea: Compare hadronic observable calculable from \`first princples´ using well defined renormalisation scheme to MC prediction

$$O_{th}^{R}(m_t^R) = O_{\text{MC}}(m_t^{\text{MC}})$$

$$R \in \{\text{pole}, \overline{\text{MS}}, \text{MSR}, \ldots\}$$

→ Relation between m_t^R and m_t^{MC}

Challenge:

Only very few observables calculable from first principles, requires consitent factorisation and non-perturbative input

→ So far only results for e+e- annihilation available

Pole mass versus MC mass

[Butenschoen,Dehnadi,Hoang,Mateu,Preisser,Stewart, PRL 117 (2016) 232001]

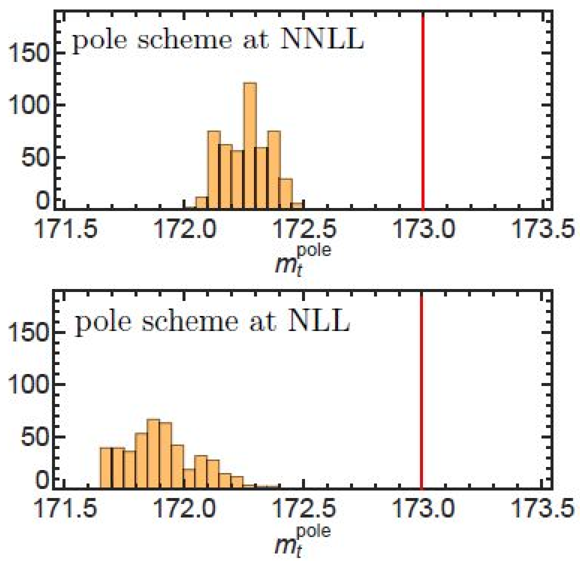
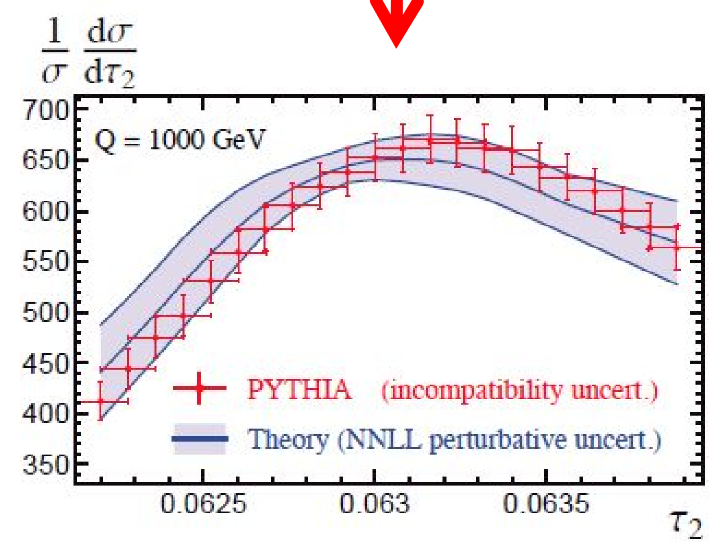


FIG. 1. Distribution of best-fit mass values from the scan over parameters describing perturbative uncertainties. Results are shown for cross sections employing the MSR mass $m_t^{\text{MSR}}(1\,\text{GeV})$ (top two panels) and the pole mass m_t^{pole} (bottom two panels), both at N^{2}LL and NLL. The PYTHIA datasets use $m_t^{\text{MC}} = 173\,\text{GeV}$ as an input (vertical red lines).

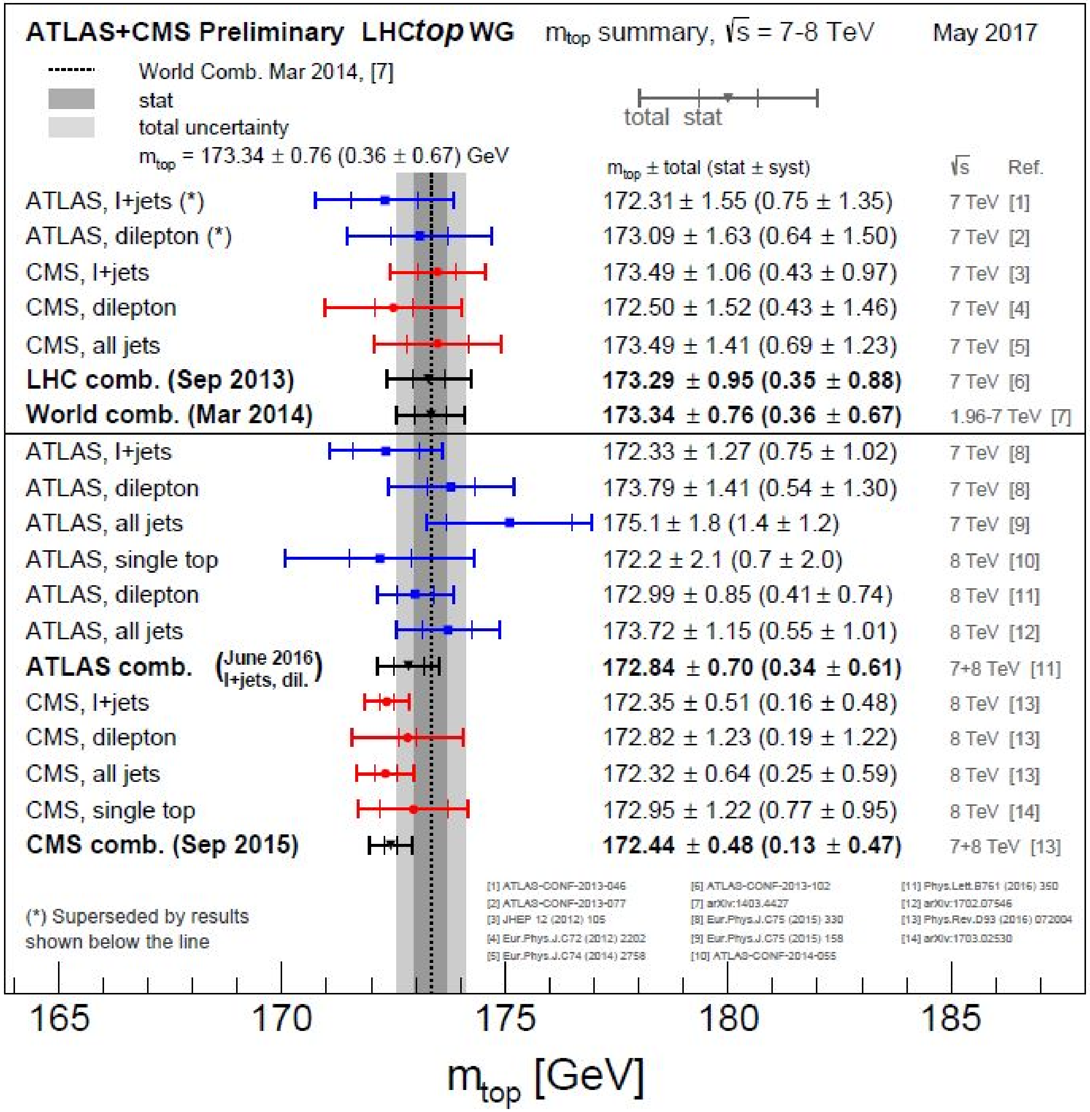
$m_t^{\text{MC}} = 173\,\text{GeV}\ \ (\tau_2^{e^+e^-})$

mass	order	central	perturb.	incompatibility	total
$m_{t,1\,\text{GeV}}^{\text{MSR}}$	NLL	172.80	0.26	0.14	0.29
$m_{t,1\,\text{GeV}}^{\text{MSR}}$	N^{2}LL	172.82	0.19	0.11	0.22
m_t^{pole}	NLL	172.10	0.34	0.16	0.38
m_t^{pole}	N^{2}LL	172.43	0.18	0.22	0.28



- Sizeable effects, ~400 MeV
- pp adds new features (ISR, color reconnection, add. Hadronization), results applicable to pp?

Standard measurements — recent results



So far in agreement with pole mass measurements,
May become relevant in the future

Alternative measurements

- Many additional observables under investigation

  Lepton+b-jet inv. mass, lepton+J/Ψ inv. mass, dilepton kinematics, b-jet energy peak, lepton+secondary vertex, kinematic endpoints, MT2,single top-quarks,…

- Not all measurements already competitive, large stat. required
- Additional measurements provide valuable cross checks

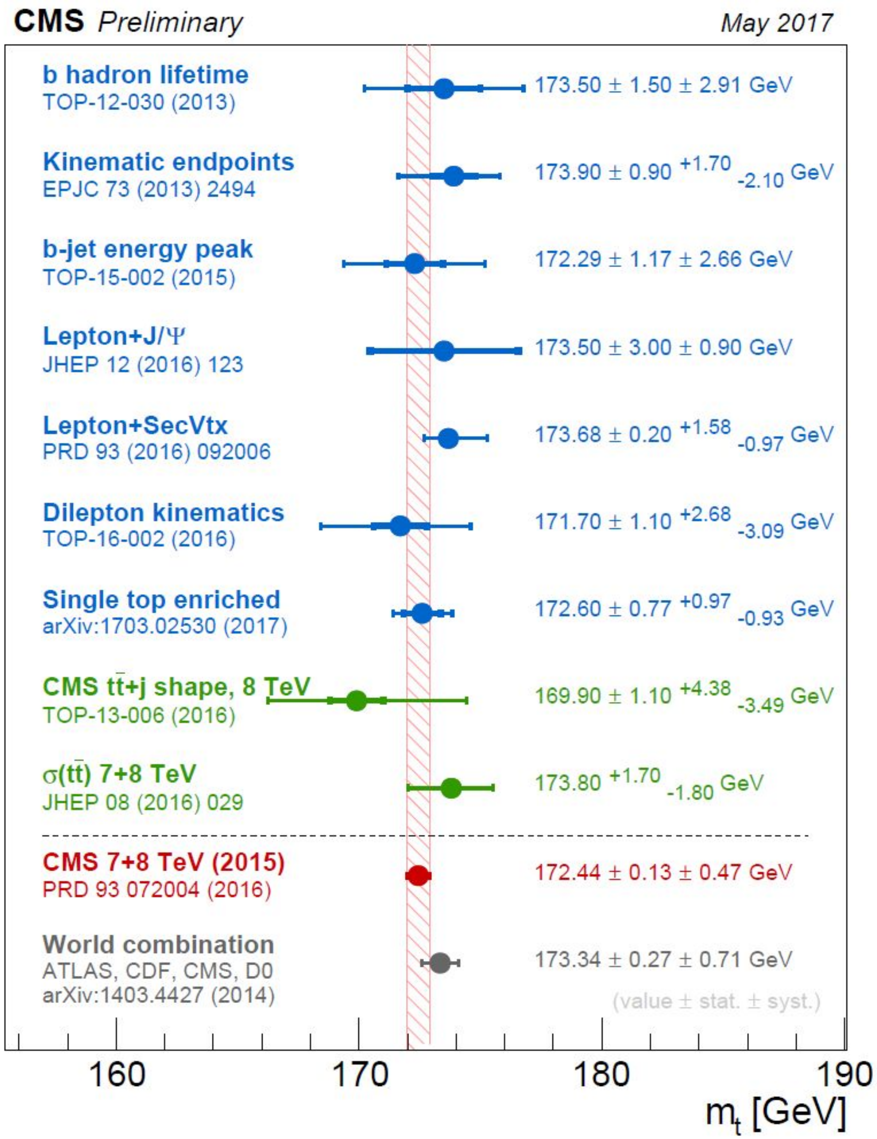
  Example:

  Effects of color reconnection in single-top are expected to be different from top-quark pair production

- In many cases alternative measurements suffer also from the pole mass ←→ MC mass issue

May provide useful information about known and unknown unknowns

Alternative measurements



Top-quark mass using the matrix element method

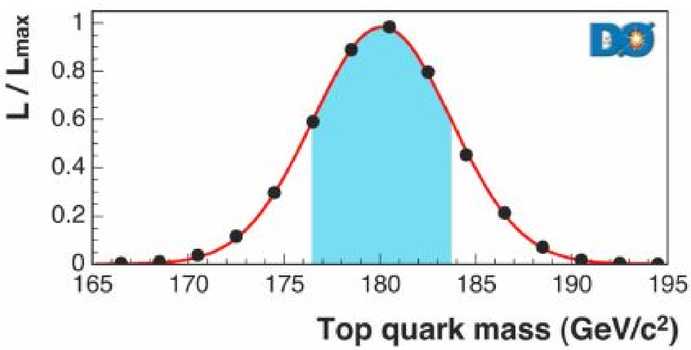
Idea: [Kondo'88,'91]

Construct likelihood using the diff. cross section/matrix elements for event sample $\{\vec{x}_i\}$

$$\mathcal{L}(m_t) = \prod_{\text{events } i} \frac{1}{\sigma(m_t)} \int d\vec{y} \frac{d\sigma(m_t)}{d\vec{y}} W(\vec{x}_i, y)$$

Maximizing likelihood wrt to m_t yields estimator

Most efficient estimator since all information from event sample is used



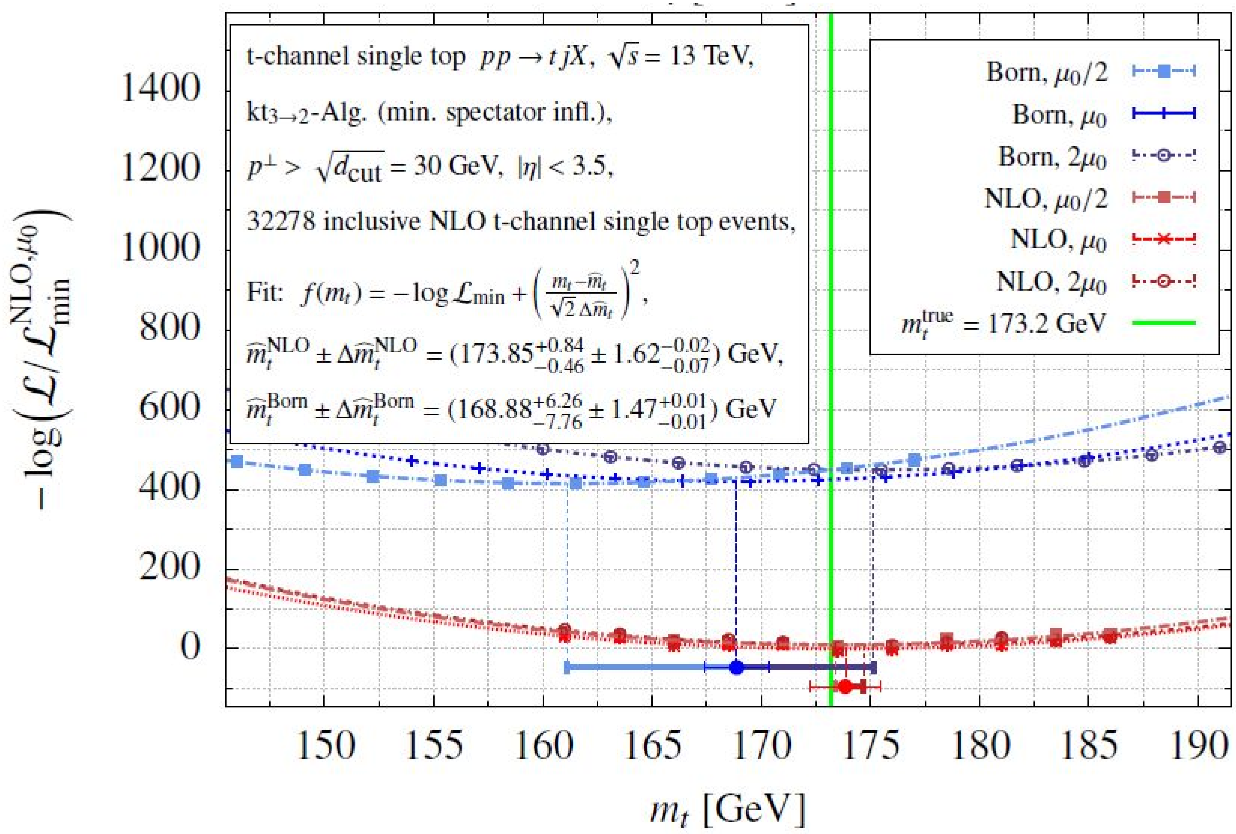
top mass measurement at Tevatron based on O(70) events!

[D0: Nature 429, 638], [CDF: PRD 50, 2966]

Extension of the matrix-element method to NLO [Martini,PU '15]

Toy experiment: Generate unweigthed NLO jet events, use MEM to extract mass parameter

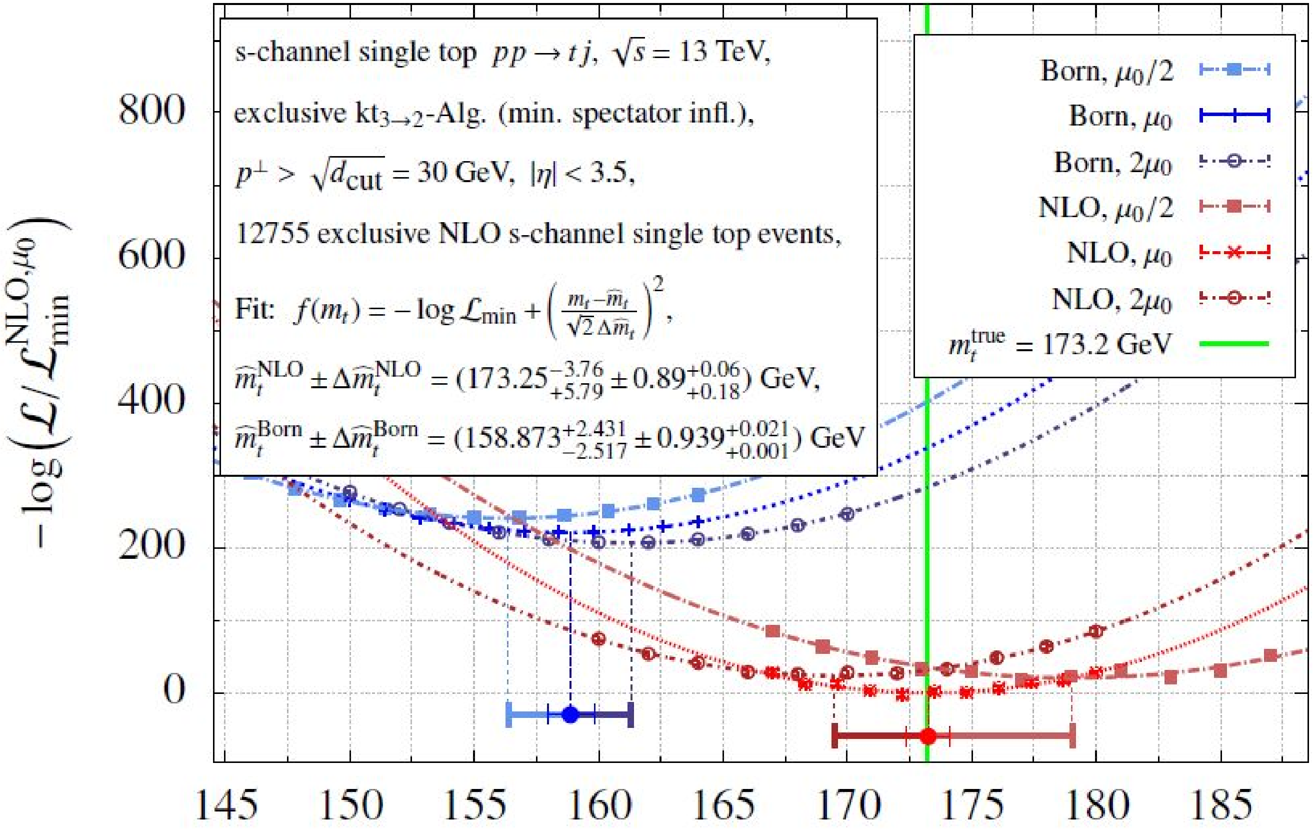
t-channel



[Martini,PU in preparation]

- scheme well defined
- NLO gives better description
- MEM in NLO recovers true value
- scale dependence reduced
- using MEM in LO requires calibration (→add. uncertainty)

s-channel

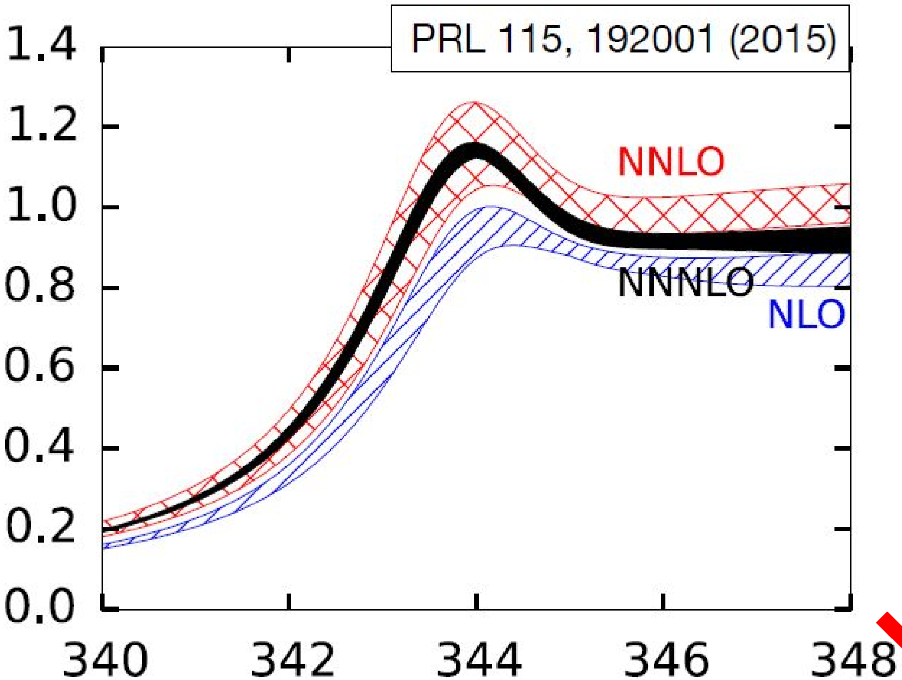


- LO scale variation does not provide reliable estimate of uncertainty (no surprise
- Scale variation gets worse (first reliable estimate of uncertainty?)
- In LO significant calibration required

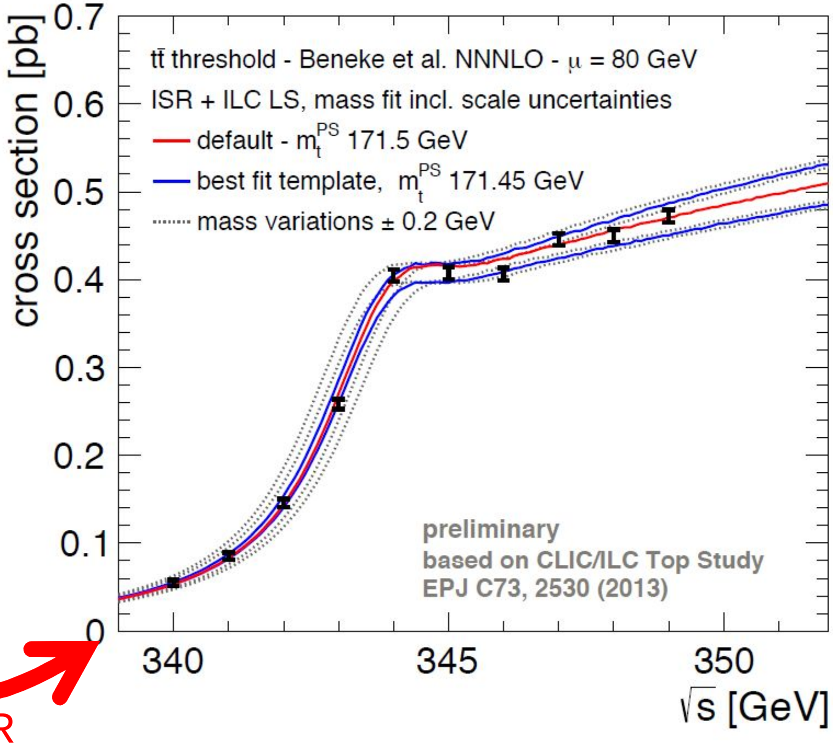
Mass measurement at future lineare collider

R-Ratio at threshold

[F. Simon presented at Top@LC 2016, see also this workshop]



ISR
Luminosity spectrum



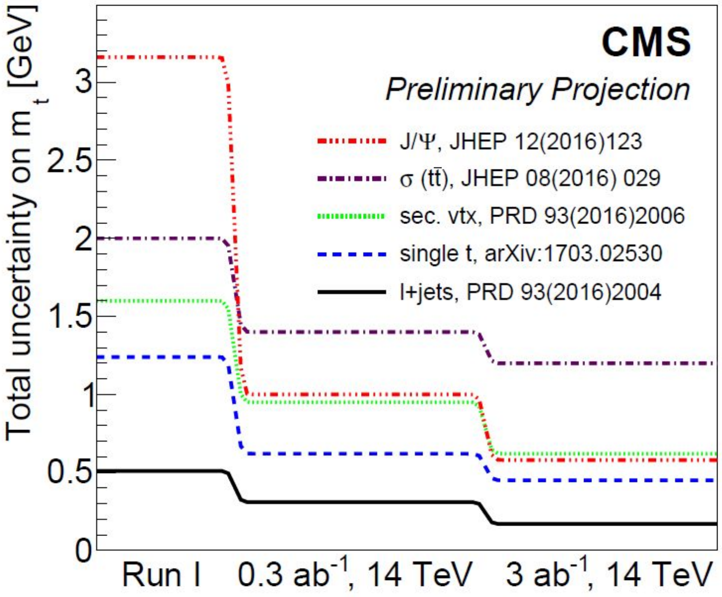
- Scale uncertainty ~40 MeV
- Parametric α_s uncertainty → 30 MeV
- Conversion of PS-mass to m(m)

~60 MeV uncertainty on m_t

Conclusion

- Renormalon ambiguity smaller than previously estimated → pole mass seems okay for most LHC applications
- Large variety of different measurements (standard measurements/pole mass measurements/alternative measurements)
- Pole mass ←→ MC mass, possible difference of a few 100 MeV
- Given current measurements no direct evidence
- Rather consistent picture so far, may change with decreasing uncertainty
- Alternative measurements may also suffer from pole mass – MC mass issue
- Key issue for the future: reliable calibration
- Time to prepare new world average

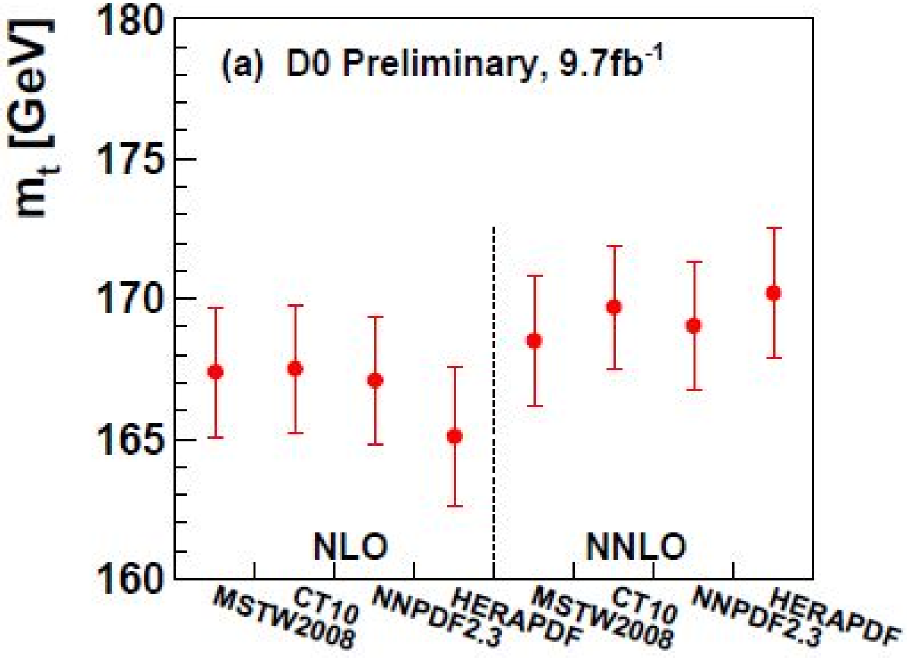
[CMS-FTR-16-006-PAS]

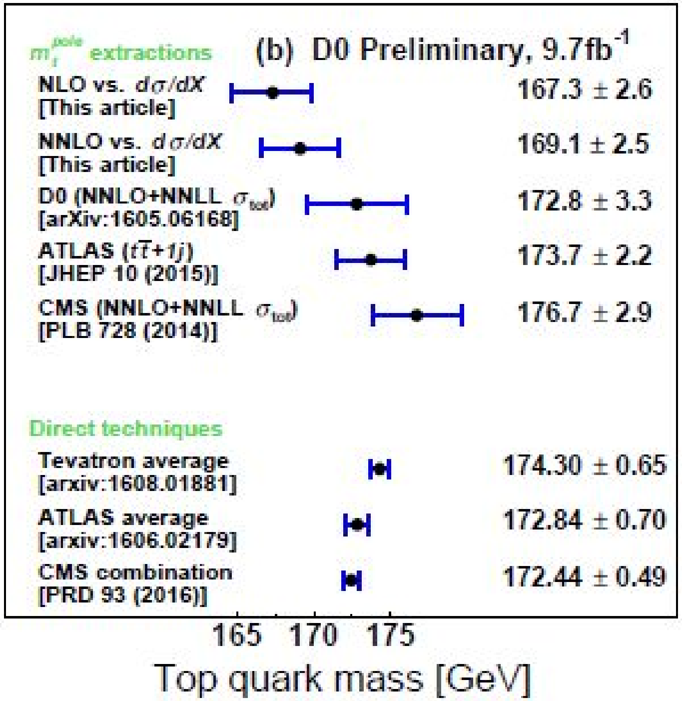


Need to take uncertainty of MC mass into account

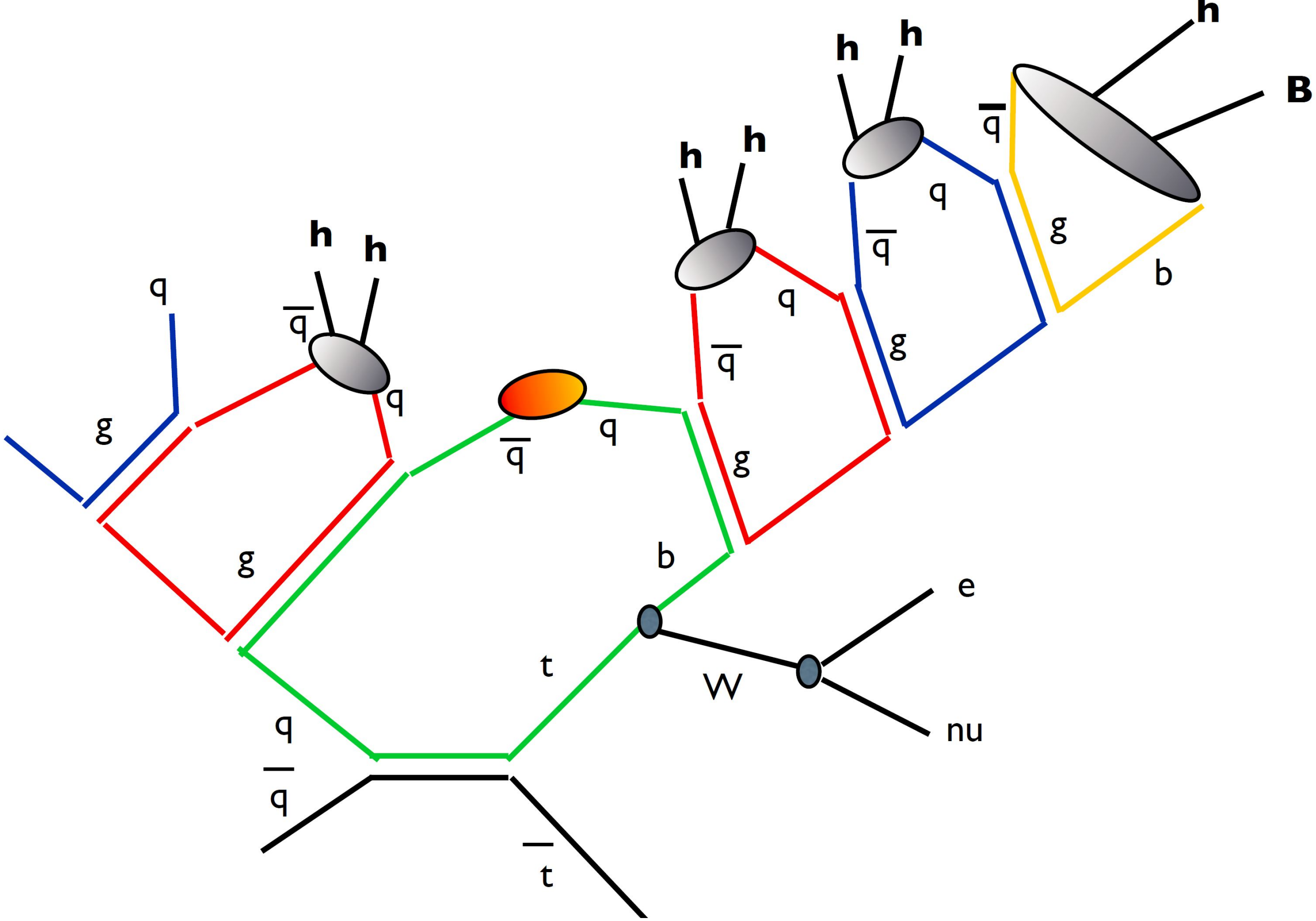
Thank you
for your attention

Top mass from differential cross section





[Mangano, Top workshop, July 2012, CERN]



$$p_t \neq p_W + \sum_i p^i_{\text{had}}$$