

Theoretical Models for Neutrino Masses

Luca Merlo



**Les Rencontres de Physique
de la Vallée d'Aoste**

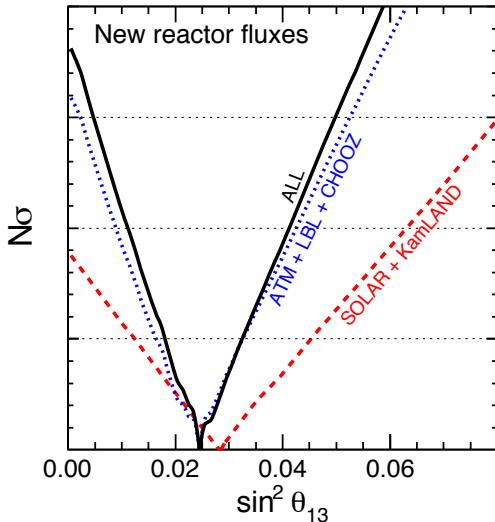
La Thuile, Aosta Valley, Italy
February 26 - March 3, 2012

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Assuming **3 light active oscillating neutrinos:**

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with both T2K and MINOS data and new reactor fluxes



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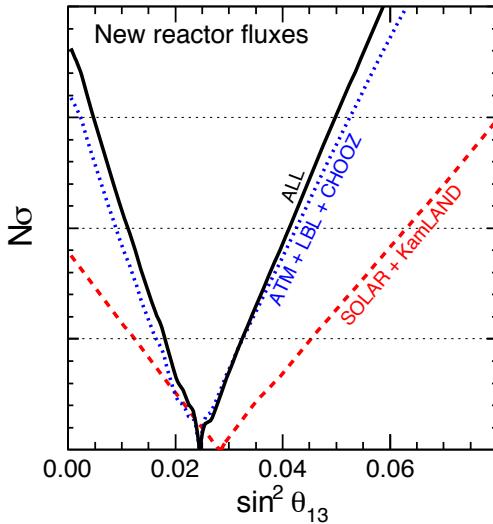
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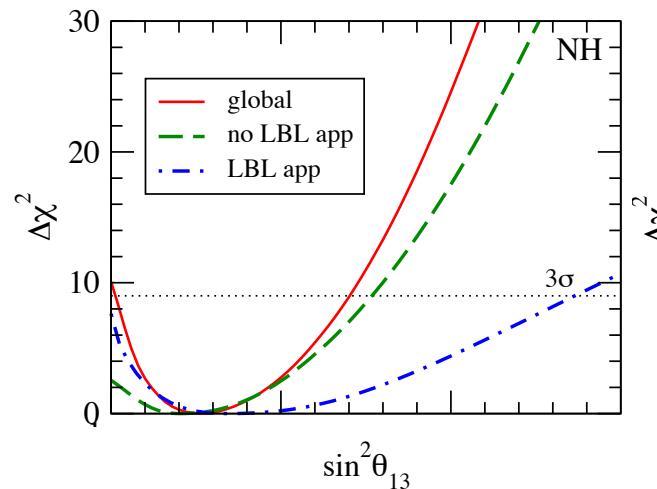
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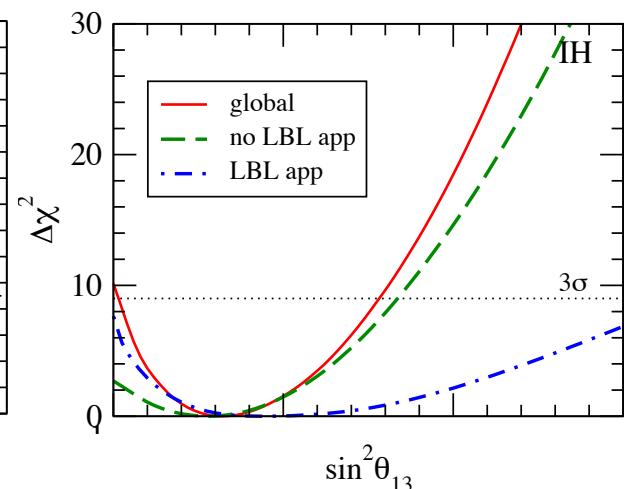
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→ Different definition of the atmospheric mass square difference:

$$\Delta m_{\text{atm}}^2 \equiv \left| m_3^2 - \frac{m_1^2 + m_2^2}{2} \right| \quad \longleftrightarrow \quad \Delta m_{\text{atm}}^2 \equiv |m_3^2 - m_1^2|$$

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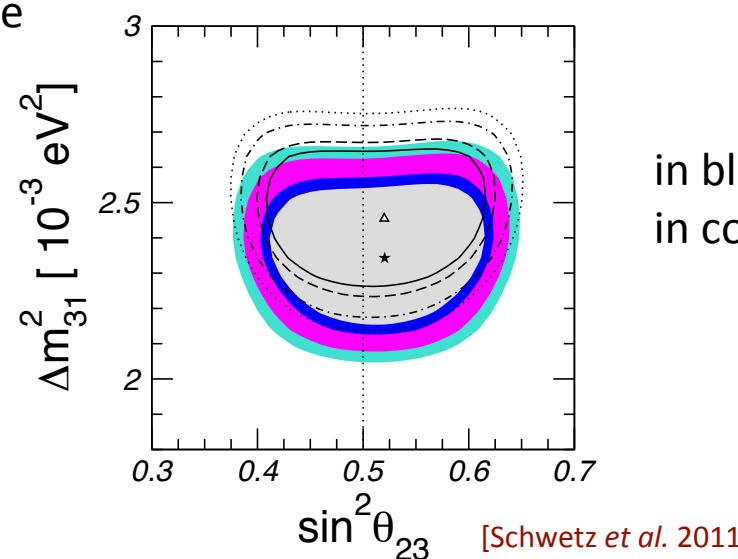
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The maximal angle solution is still allowed and the two minima are almost degenerate



in black the NH
in colours the IH

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different numerical results for $\sin^2 \theta_{13}$

- After the re-evaluation of the reactor neutrino fluxes, **tension** with SBL experiments (<100 m): maybe sterile neutrino with $\Delta m^2 \approx \mathcal{O}(1) \text{ eV}^2$
[Giunti & Laveder 2011]
- Fogli *et al.* do not consider these SBL data set, while Schwetz *et al.* do
- Schwetz *et al.* give also the results when SBL data are not considered:
much more similar to Fogli *et al.*

$$\sin^2 \theta_{13} = 0.022 \pm 0.008 [0.026 \pm 0.009]$$

Neutrino Mass Patterns

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In the past:

- large atmospheric angle
- only upper bound on the reactor angle



$$\sin^2 \theta_{23} = \frac{1}{2}$$
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**mu-tau
symmetry**

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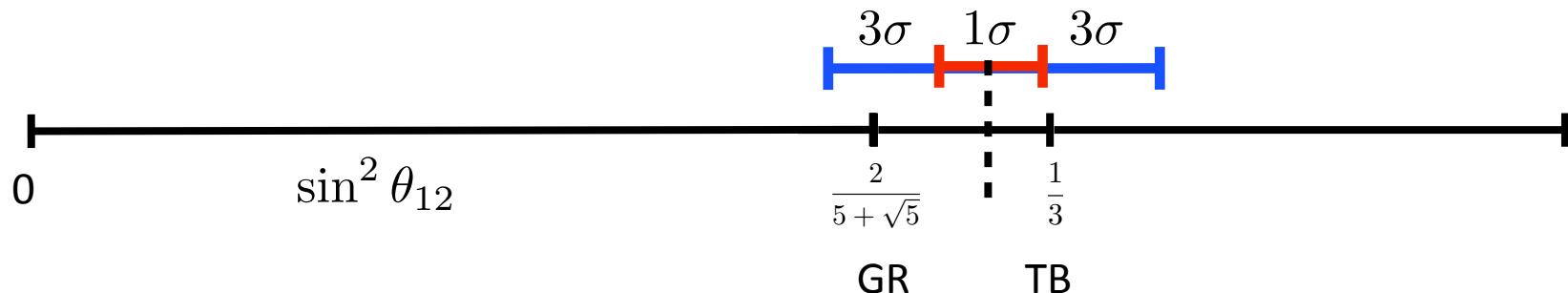
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TRI-BIMAXIMAL (TB)

[Harrison, Perkins & Scott 2002; Zhi-Zhong Xing 2002]

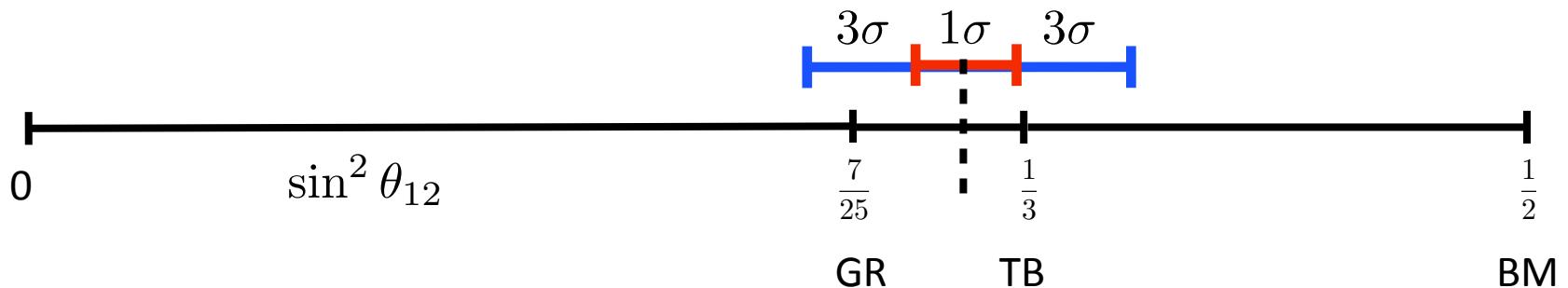
$$\sin^2 \theta_{23} = \frac{1}{2} \quad \sin^2 \theta_{13} = 0 \quad \sin^2 \theta_{12} = \frac{1}{3} \quad \rightarrow \quad \theta_{12} = 35.26^\circ$$

GOLDEN RATIO (GR)

[Kajiyama, Raidal & Strumia 2007]

$$\sin^2 \theta_{23} = \frac{1}{2} \quad \sin^2 \theta_{13} = 0 \quad \tan \theta_{12} = \frac{1}{\phi} \quad \rightarrow \quad \theta_{12} = 31.72^\circ$$
$$\phi \equiv \frac{1 + \sqrt{5}}{2}$$

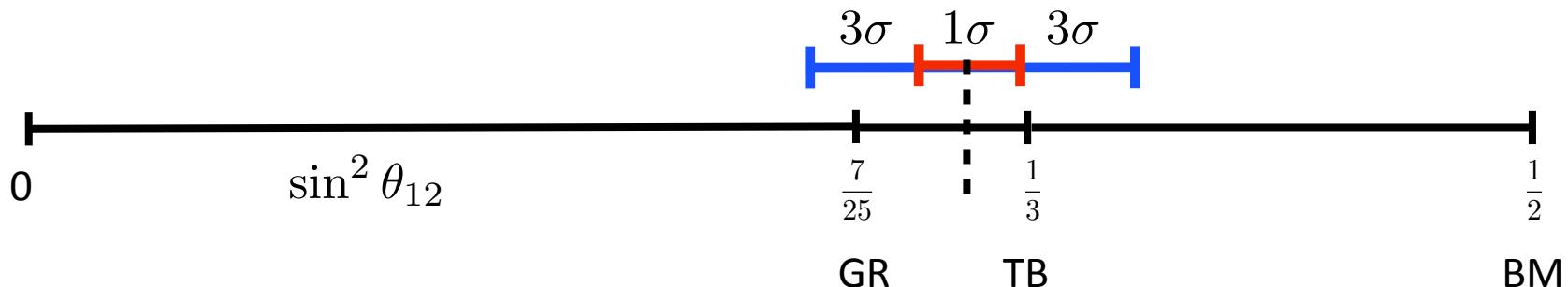
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BIMAXIMAL (BM) [Vissani 1997; Barger et al. 1998]

$$\sin^2 \theta_{23} = \frac{1}{2} \quad \sin^2 \theta_{13} = 0 \quad \sin^2 \theta_{12} = \frac{1}{2} \quad \longrightarrow \quad \theta_{12} = 45^\circ$$

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Maybe related to the
Quark-Lepton Complementarity:
[Smirnov; Raidal; Minakata & Smirnov 2004]

$$\pi/4 \approx \theta_{12} + \lambda$$

$$\rightarrow \quad \theta_{12}^{Exp} \approx \theta_{12}^{BM} - \lambda$$

[Altarelli, Feruglio and LM 2009,
Adelhart, Bazzocchi and LM 2010,
Meloni 2011]

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- at NLO, some corrections arise and they are proportional to the VEV of the flavons: larger is the VEV and larger are the corrections; loss of predictivity.

Typical Tri-Bimaximal

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In the basis of diagonal charged leptons:

$$M_\nu^{TB} = \begin{pmatrix} x & y & y \\ y & z & x + y - z \\ y & x + y - z & z \end{pmatrix}$$

mu-tau sym
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The diagram shows a green arrow pointing from the TB mixing parameters to the CKM matrix U^{TB} . Above the matrix, there are two rows of colored boxes representing the neutrino mass basis and the lepton flavor basis.

Neutrino mass basis (left): ν_e (red), ν_μ (yellow), ν_τ (blue)

Lepton flavor basis (right): ν_1 , ν_2 , ν_3

CKM matrix U^{TB} :

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Discrete Symmetries:

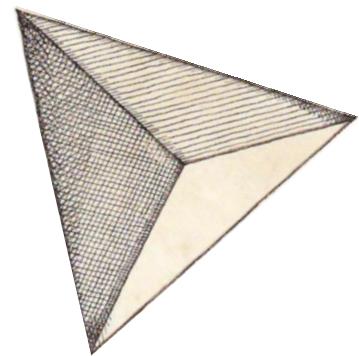
A_4 : Adhikary; Altarelli; Aristizabal Sierra; Babu; Bazzocchi; Bertuzzo; Di Bari; Branco; Brahmachari; Chen; Choubey; Ciafaloni; Csaki; Delaunay; Felipe; Feruglio; Frampton; Frigerio; Ghosal; Grimus; Grojean; Grossmann; Hagedorn; He; Hirsch; Honda; Joshipura; Kaneko; Keum; King; Koide; Kuhbock; Lavoura; Lin; Ma; Machado; Malinsky; Matsuzaki; de Medeiros Varzielas; Meloni; LM; Mitra; Molinaro; Morisi; Nardi; Parida; Paris; Petcov; Pleitez; Picariello; Rajasekaran; Riazuddin, Romao; Serodio; Skadhauge; Tanimoto; Torrente-Lujan; Urbano; Valle; Villanova del Moral; Volkas; Yin; Zee; ...;

$S_4, T', \Delta(3n^2)$: de Adelhart Toorop; Altarelli, Bazzocchi; Chen; Ding; Hagedorn; Feruglio; Frampton; Kephart; King; Lam; Lin; Luhn; Ma; Mahanthappa; Matsuzaki; de Medeiros Varzielas; LM; Morisi; Nasri; Ramond; Ross;
...]

The Altarelli-Feruglio Model

[Altarelli & Feruglio 2005]

A_4 is the group of even permutations of 4 objects isomorphic to the group of the rotations which leave a regular tetrahedron invariant (Subgroup of $SO(3)$).
It has 12 elements and 4 representations: $3, 1, 1', 1''$

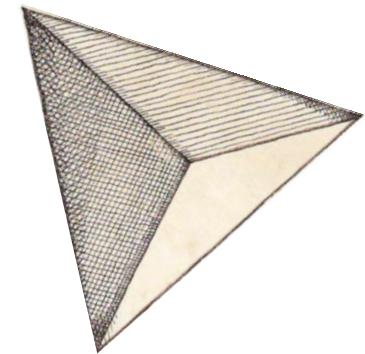


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$$G_f = A_4 \times G_{aux}$$

- keeps separated the two sectors
- explains the hierarchy

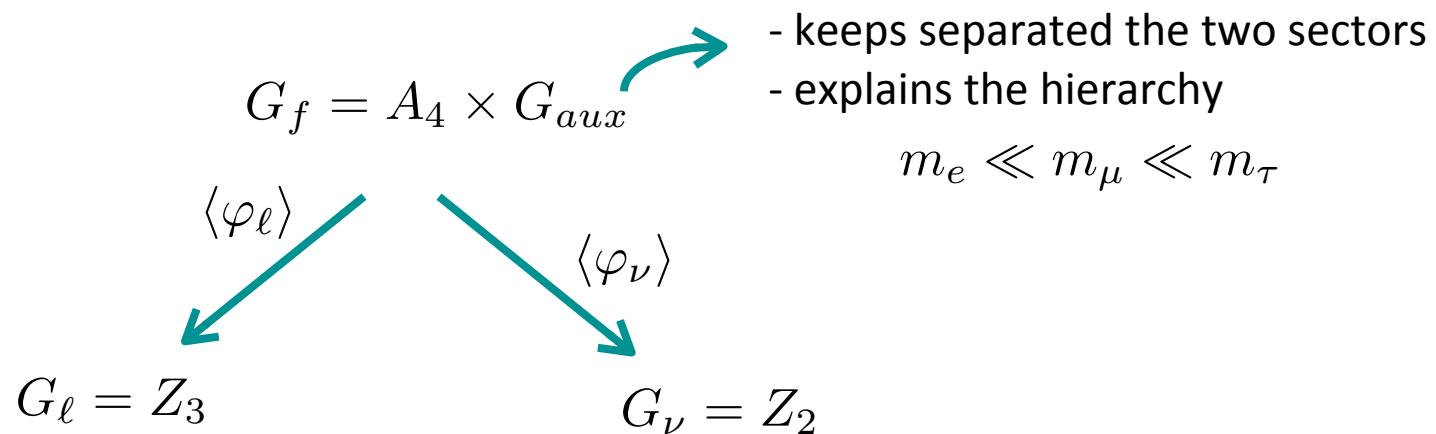
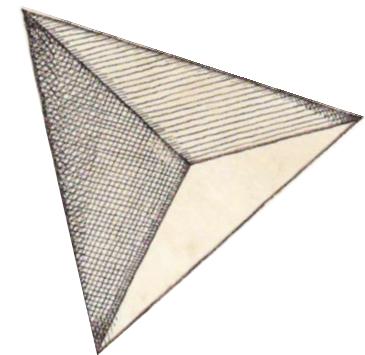
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A_4 is the group of even permutations of 4 objects isomorphic to the group of the rotations which leave a regular tetrahedron invariant (Subgroup of $SO(3)$).

It has 12 elements and 4 representations: $3, 1, 1', 1''$

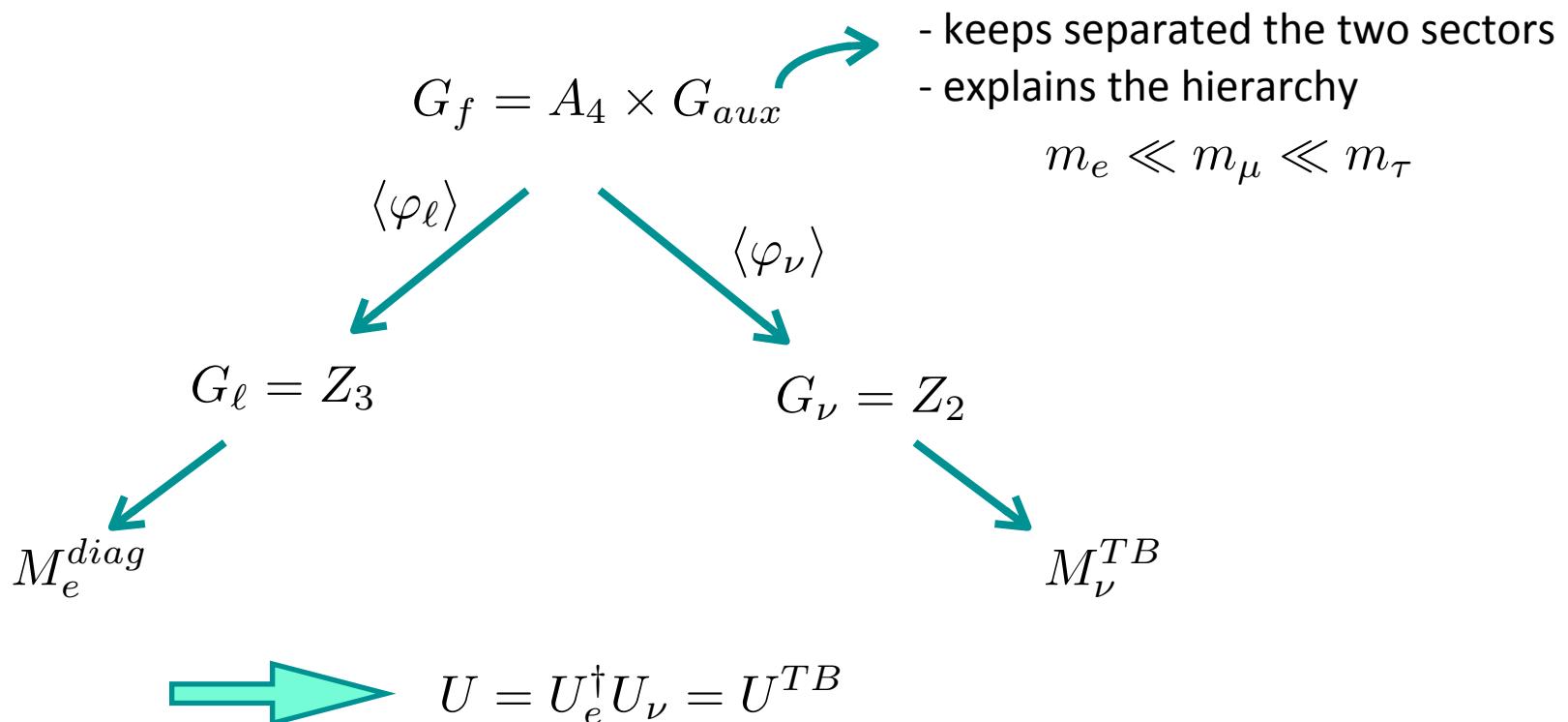
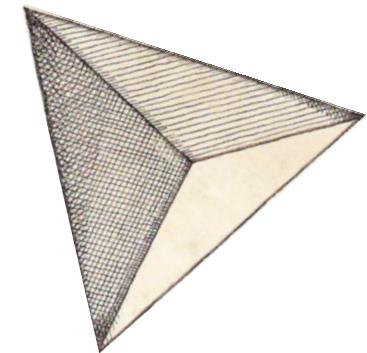


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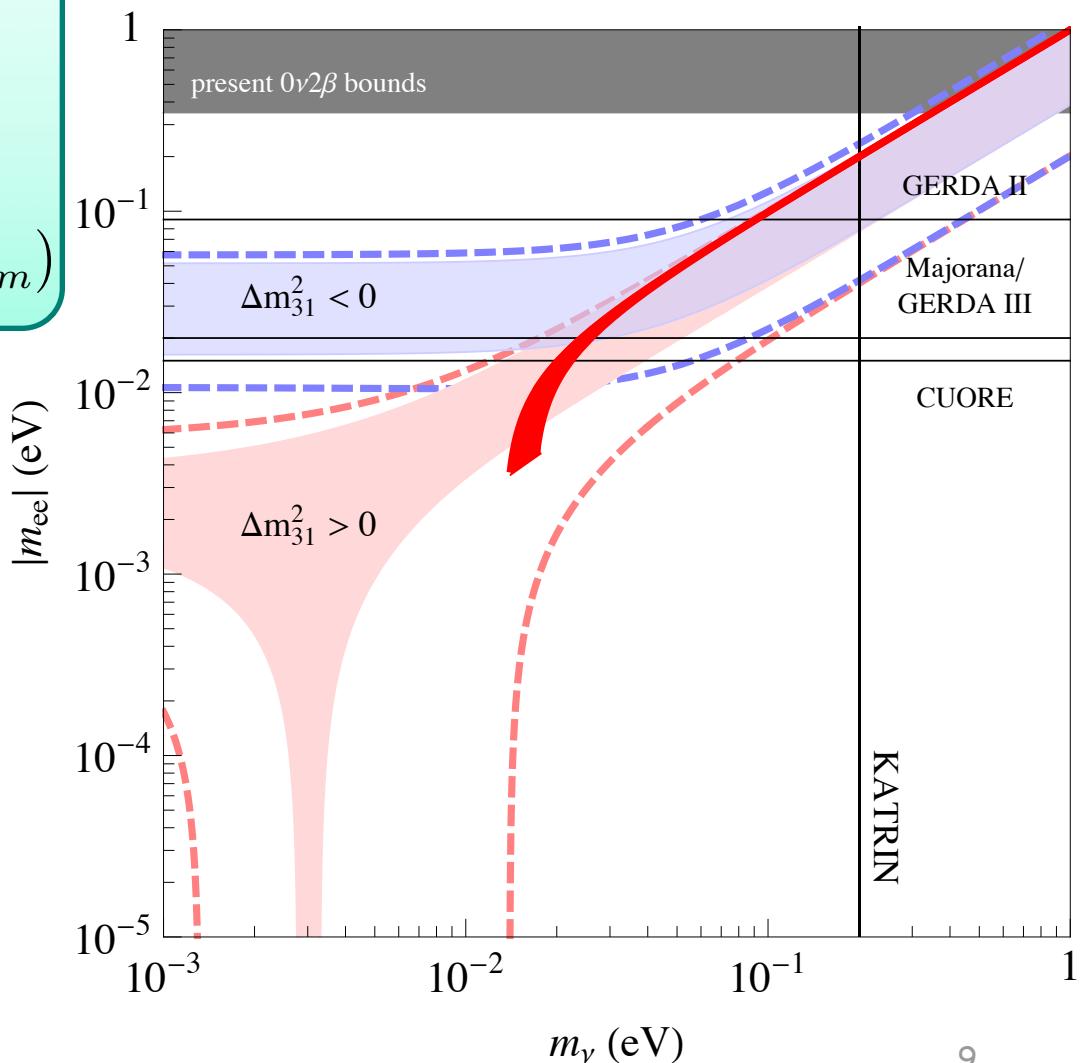
LO neutrino spectrum:

Only Normal Hierarchy

$$m_1 \geq 0.014 \text{ eV}$$

$$\sum m_i \geq 0.09 \text{ eV}$$

$$|m_{ee}|^2 = \frac{1}{9} (9m_1^2 + 5\Delta m_{sol}^2 - \Delta m_{atm}^2)$$



@ LO:

$$\sin^2 \theta_{12}^{TB} = \frac{1}{3}$$

$$\sin^2 \theta_{23}^{TB} = \frac{1}{2}$$

$$\sin \theta_{13}^{TB} = 0$$

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@ NLO:



$$M_\nu = M_\nu^{TB} + \delta M_\nu$$

$$M_\ell = M_\ell^{\text{diag}} + \delta M_\ell$$

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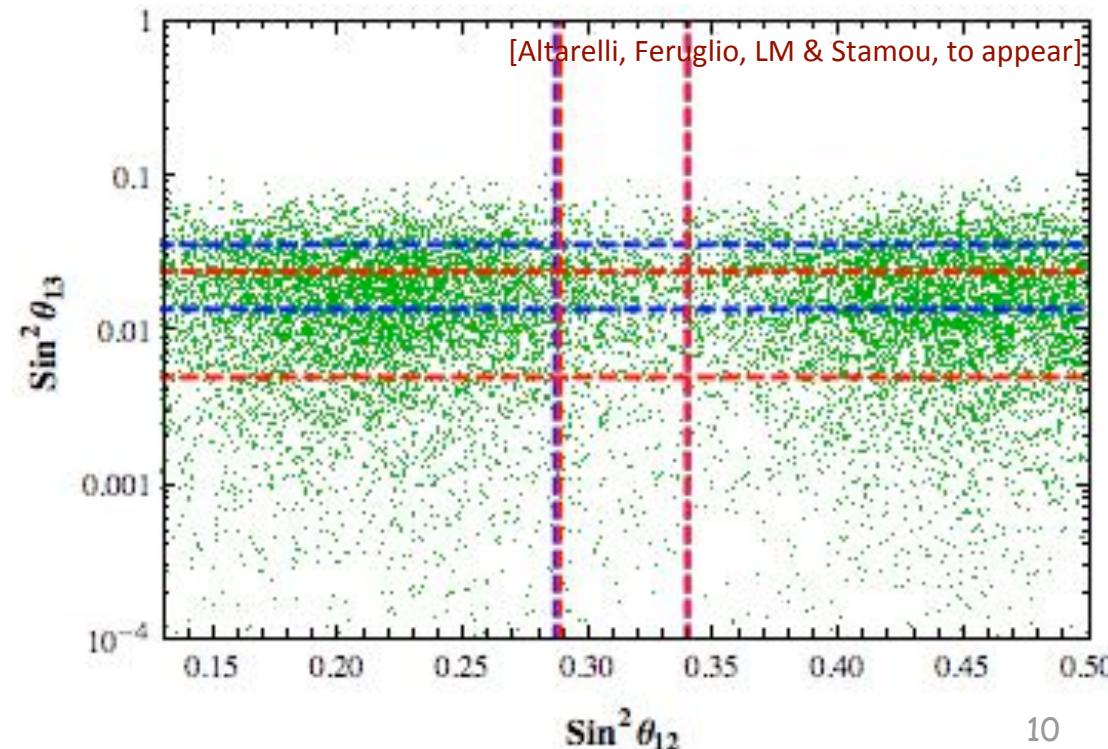
$$\sin \theta_{13} = \delta_{13}$$

In a typical model, the corrections are democratic in all the angles:

$$\delta_{12} \approx \delta_{23} \approx \delta_{13}$$

To maximize the success rate for a **reactor angle** inside the 90% C.L.

$$\delta_{ij} \approx 0.12$$



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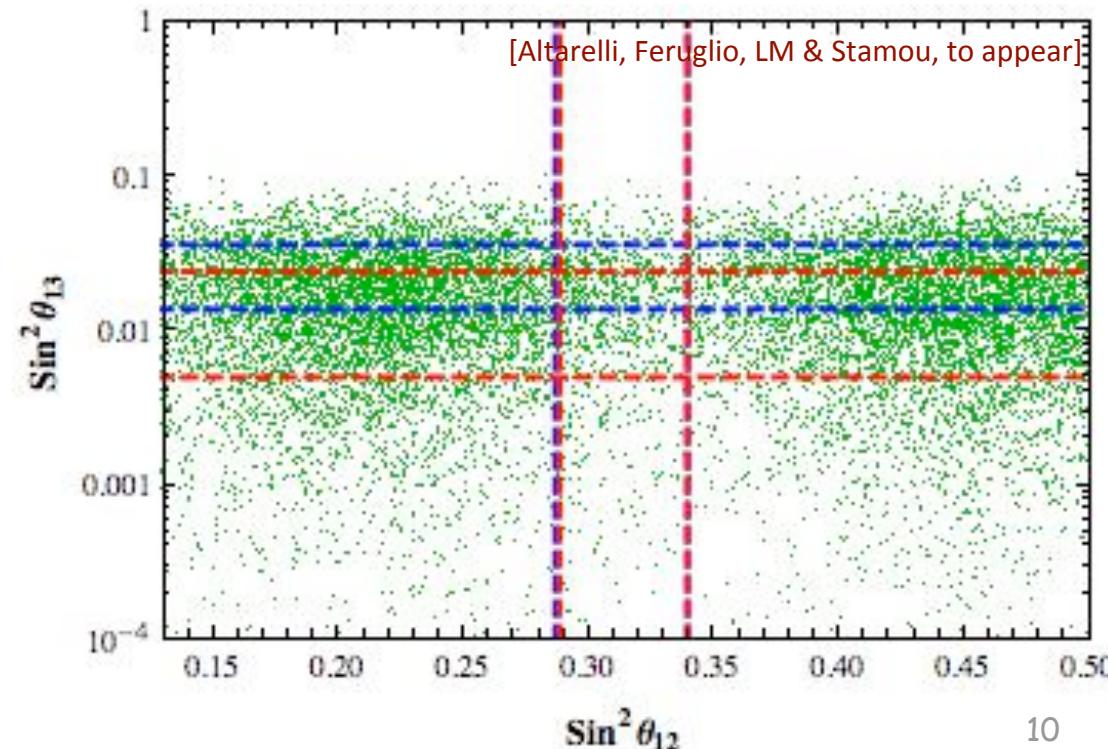
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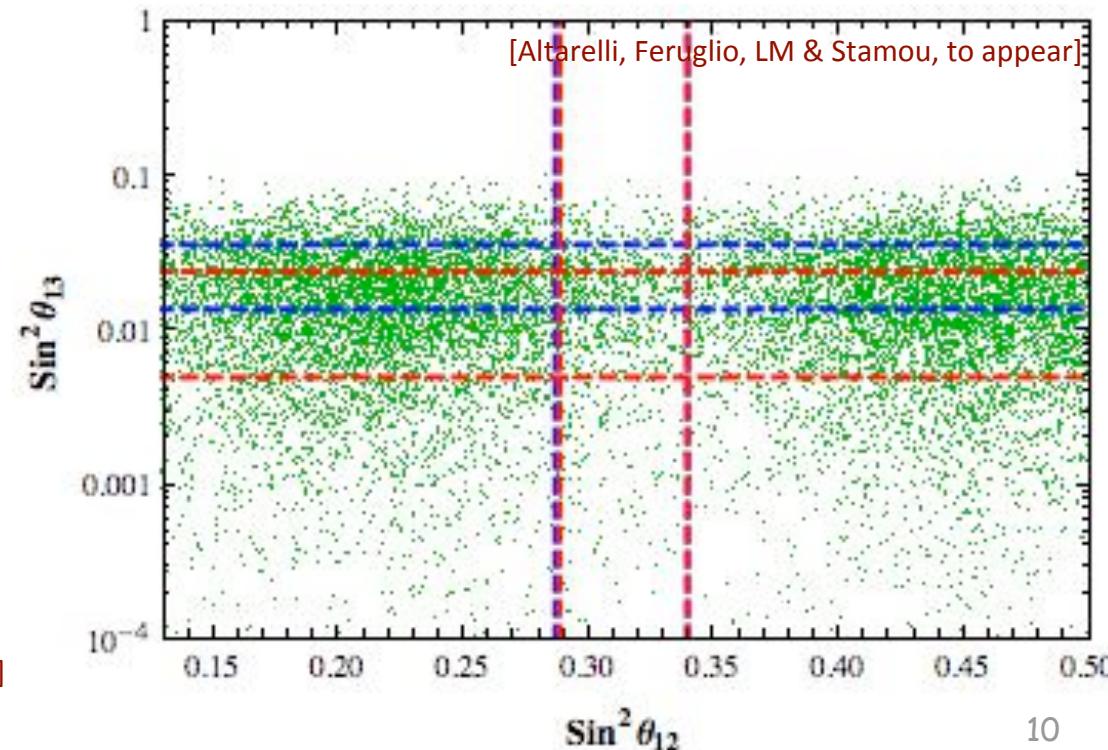
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However, not so good for the **solar angle!!!**

Similar conclusions are valid for the Golden Ratio pattern.

[Feruglio and Paris 2011]



Special Tri-Bimaximal

@ LO:

$$M_{\nu}^{TB} = \begin{pmatrix} x & y & y \\ y & z & x + y - z \\ y & x + y - z & z \end{pmatrix} \quad \rightarrow \quad \begin{aligned} \sin^2 \theta_{12}^{TB} &= \frac{1}{3} \\ \sin^2 \theta_{23}^{TB} &= \frac{1}{2} \\ \sin^2 \theta_{13}^{TB} &= 0 \end{aligned}$$
$$M_{\ell} = M_{\ell}^{\text{diag}}$$

[Lin 2009]

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@ NLO:

$$M_\nu = \begin{pmatrix} x & y-w & y+w \\ y-w & z+w & x+y-z \\ y+w & x+y-z & z-w \end{pmatrix} \quad \rightarrow \quad \begin{aligned} \sin^2 \theta_{12} &= \frac{1}{3} \\ \sin^2 \theta_{23} &= \frac{1}{2} + \frac{1}{\sqrt{2}} \sin \theta_{13} \cos \delta \\ \sin \theta_{13} &= \sqrt{\frac{2}{3}} \delta_{13} \end{aligned}$$

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@ NNLO:

Generic corrections to all the angles

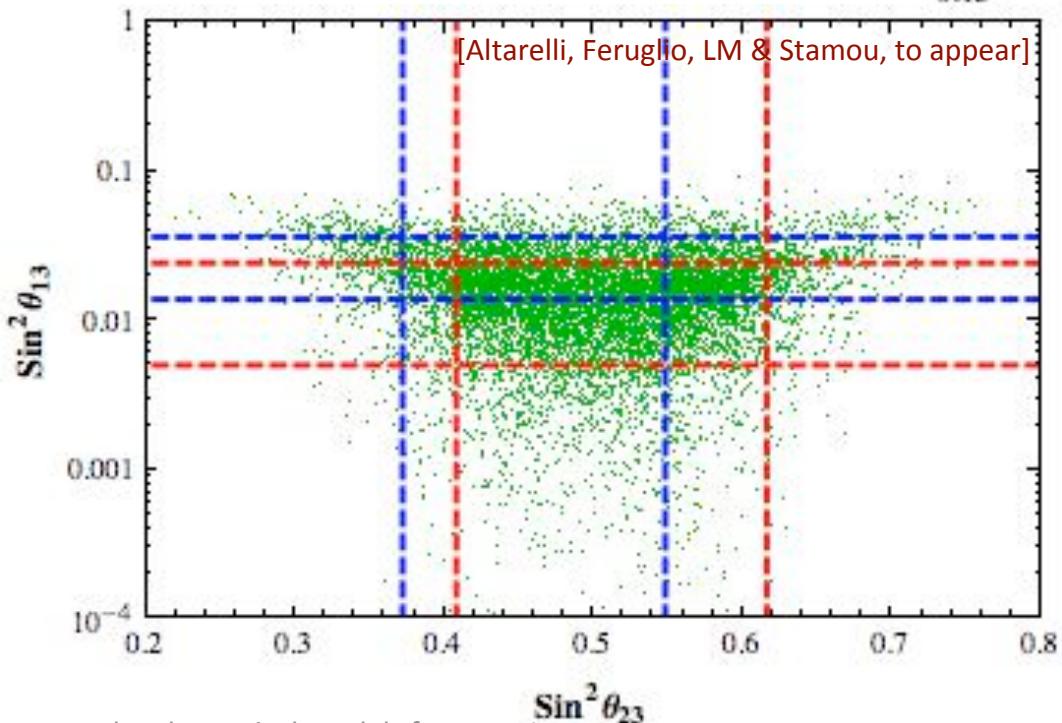
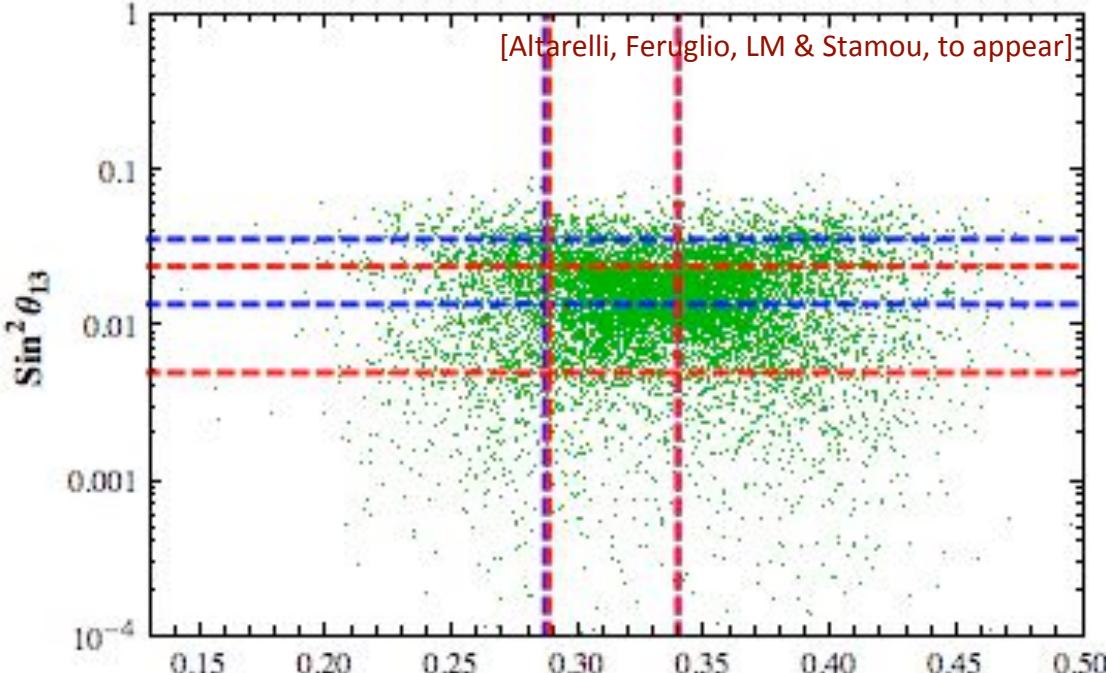


$$\begin{aligned} \theta_{13} &\approx \mathcal{O}(\delta) \\ \sin^2 \theta_{12} - \sin^2 \theta_{12}^{TB} &\approx \mathcal{O}(\delta^2) \end{aligned}$$

To maximize the success rate for a **reactor angle** inside the 90% C.L., again

$$\delta \approx 0.15$$

and now the **solar angle** is also inside its 90% C.L.



[Altarelli, Feruglio, LM & Stamou, to appear]

and even the **atmospheric angle** is also inside its 90% C.L.

Bimaximal

@ LO:

[Altarelli, Feruglio and LM 2009]

$$\begin{aligned}\sin^2 \theta_{12}^{BM} &= 1/2 \\ \sin^2 \theta_{23}^{BM} &= 1/2 \\ \sin \theta_{13}^{BM} &= 0\end{aligned}$$



$$U^{BM} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/2 & 1/2 & +1/\sqrt{2} \end{pmatrix}$$

$$M_\ell = M_\ell^{\text{diag}}$$

$$M_\nu^{BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$



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@ NLO:

$$M_\nu = M_\nu^{BM}$$

$$M_\ell = \begin{pmatrix} m_e & \epsilon m_e & \epsilon m_e \\ \epsilon m_\mu & m_\mu & 0 \\ \epsilon m_\tau & 0 & m_\tau \end{pmatrix} \longrightarrow$$

$$\sin^2 \theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}(\delta_{12} + \delta_{13})$$

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[Altarelli, Feruglio and LM 2009]

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$$\sin^2 \theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}(\delta_{12} + \delta_{13})$$

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@ NNLO:

Generic corrections to all the angles

$$\theta_{13} \approx \mathcal{O}(\delta)$$

$$\sin^2 \theta_{12} = 1/2 - \mathcal{O}(\delta)$$

The model is not in agreement with the data and large corrections are necessary.

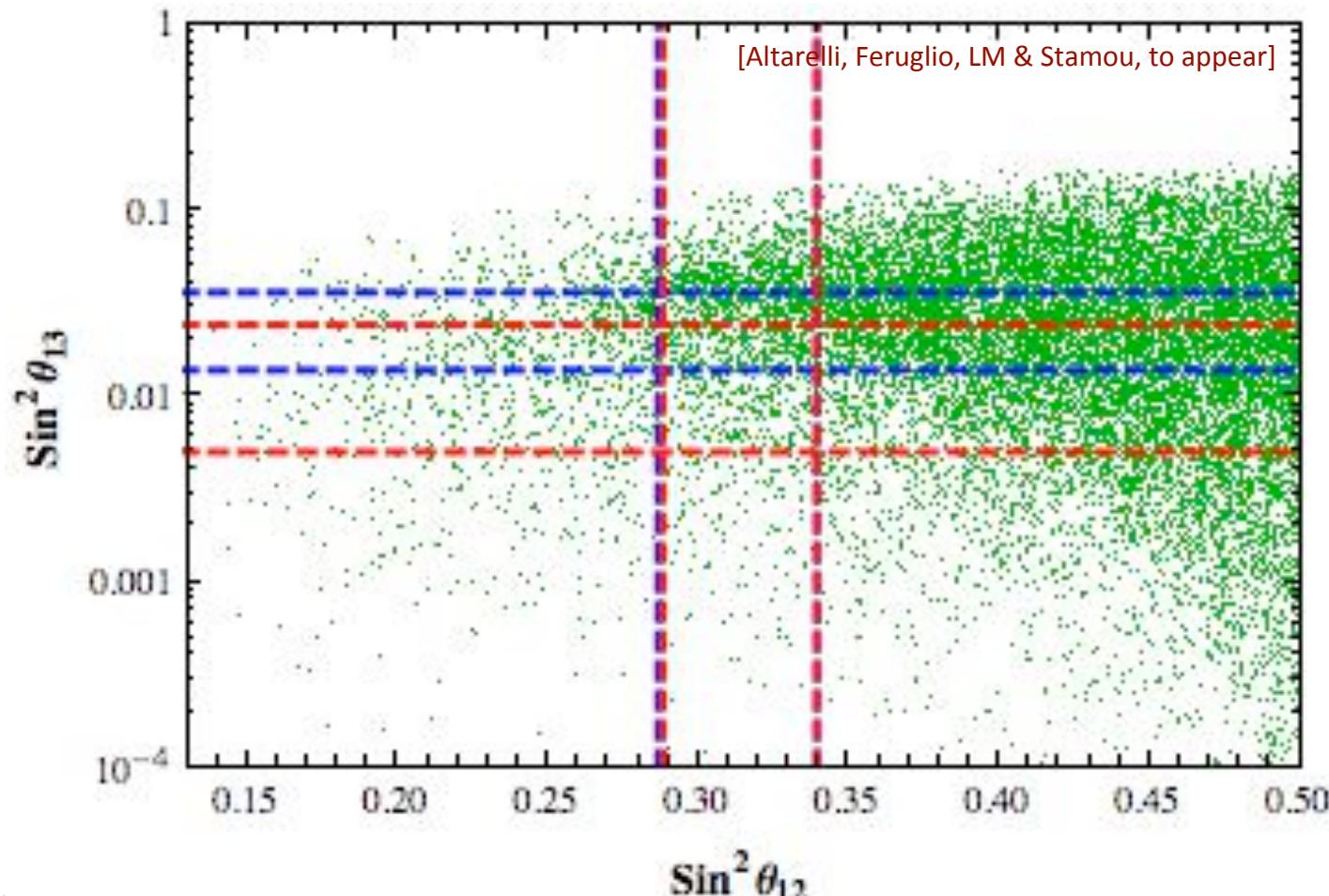
The question is whether the model can arrange both the **solar** and the **reactor angles** inside their own 90% C.L. at the same time: best result with

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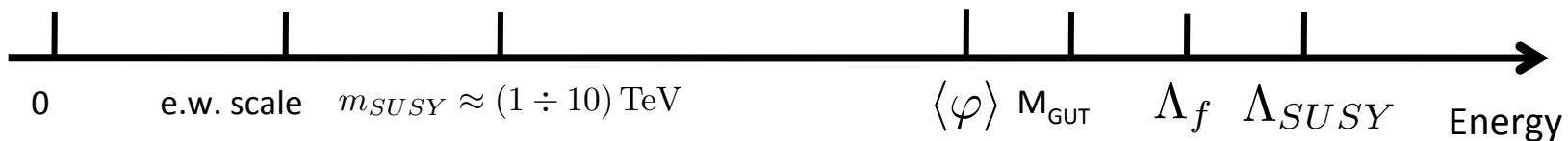


Impact on LFV

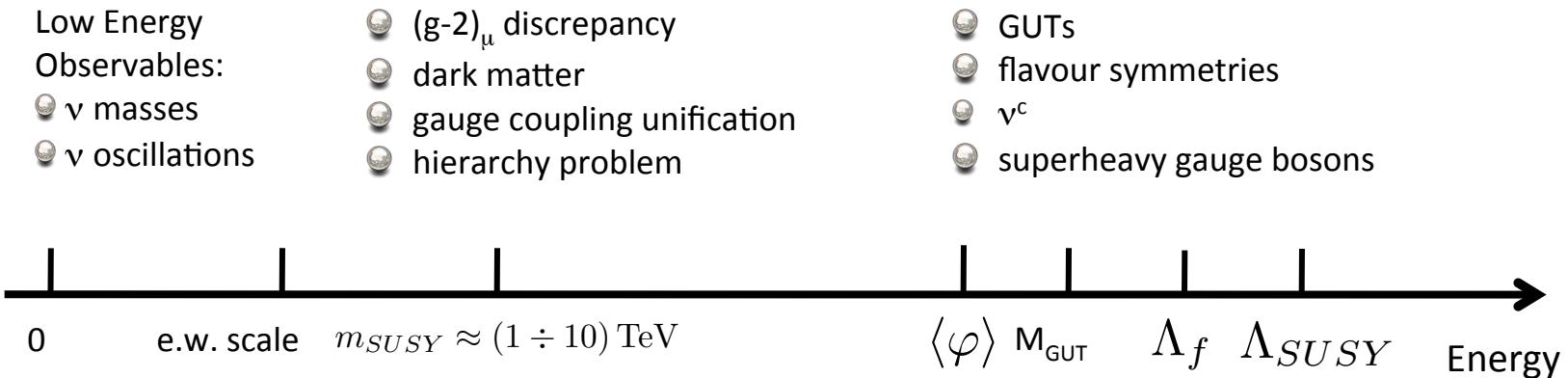
Low Energy
Observables:
• neutrino masses
• neutrino oscillations

- (g-2)_μ discrepancy
- dark matter
- gauge coupling unification
- hierarchy problem

- GUTs
- flavour symmetries
- ν^c
- superheavy gauge bosons



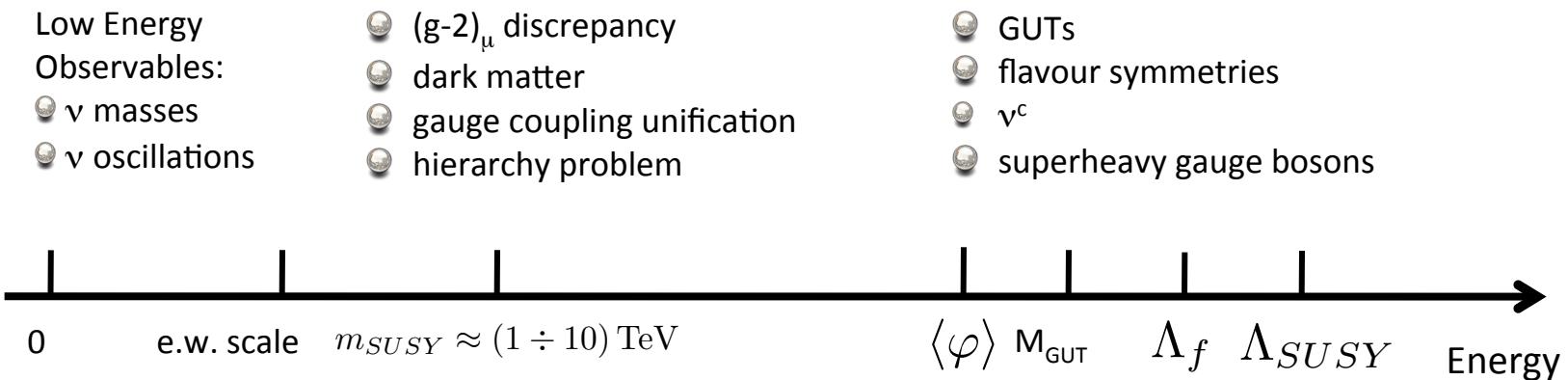
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The Flavour symmetry at the High-scale affects the low-energy observables **indirectly**:

- the flavons φ do not lead to direct contributions (tree-level or loops are suppressed by the heavy mass)
- the soft-SUSY breaking parameters are governed by the flavour symmetry and its breaking mechanism

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- non-universal boundary conditions for the soft terms
- different results wrt m-SUGRA scenario

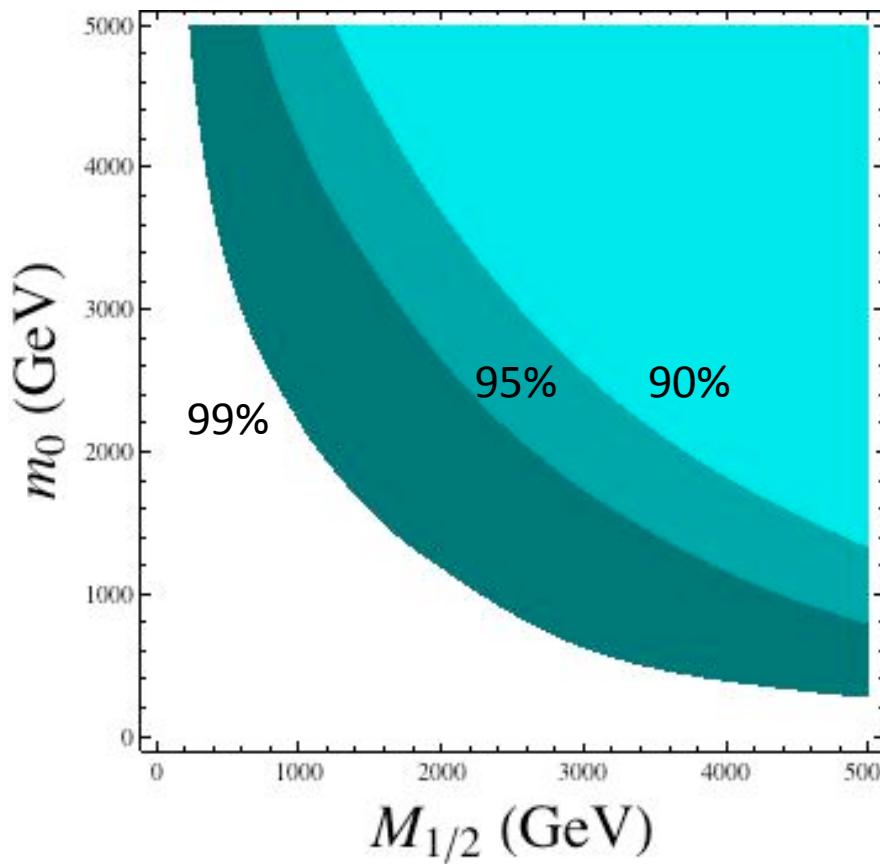
Typical Tri-Bimaximal

[Altarelli, Feruglio, LM & Stamou, to appear]

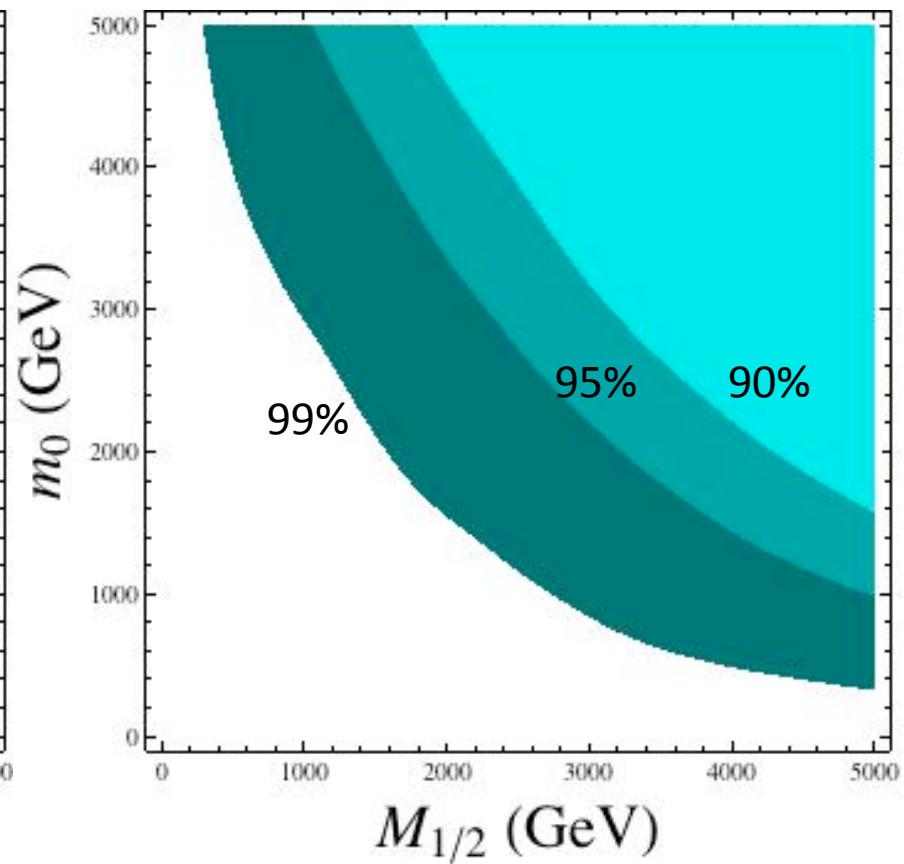
$$BR(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12}$$

@ 90% C.L. [MEG coll. 2011]

$$\tan \beta = 2$$



$$\tan \beta = 15$$



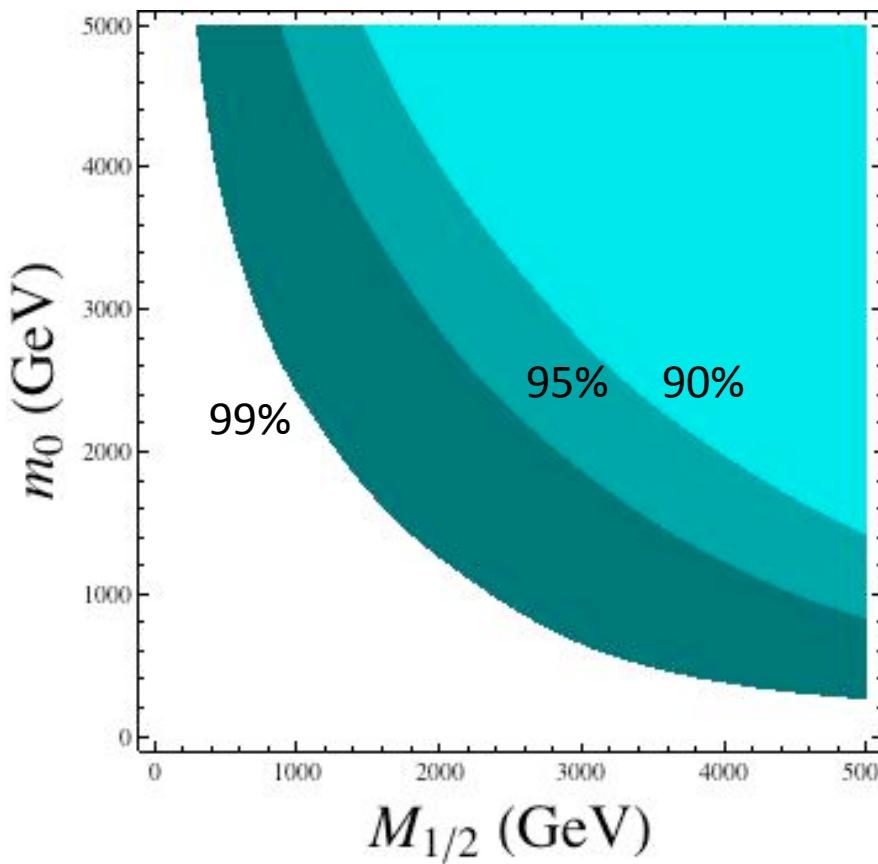
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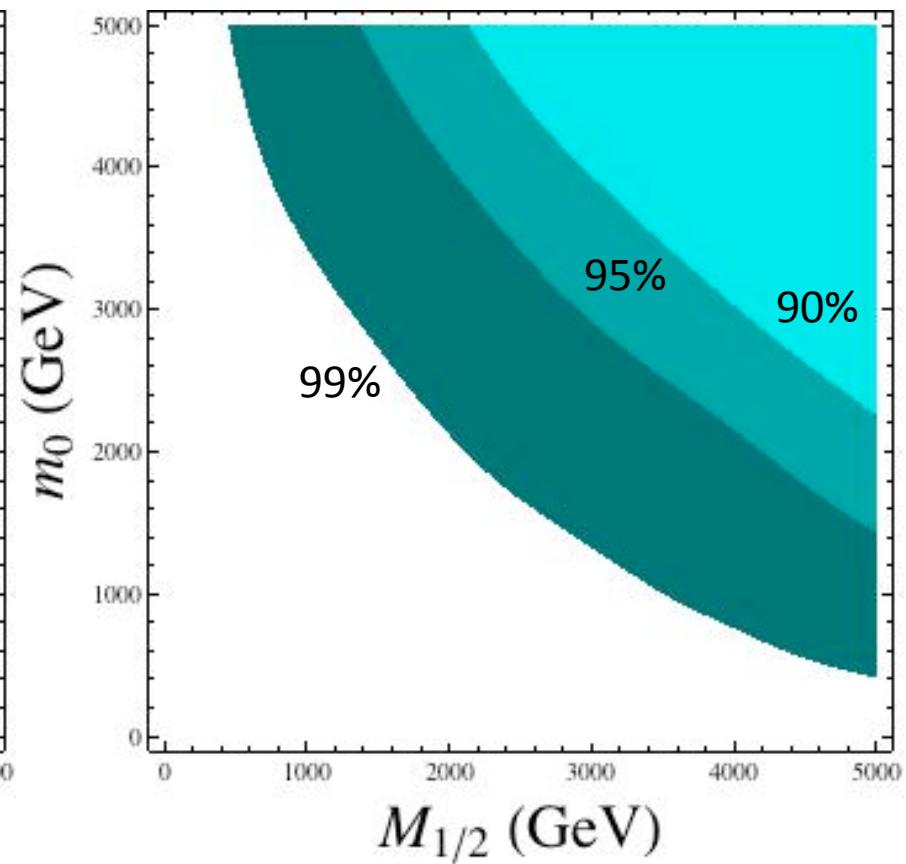
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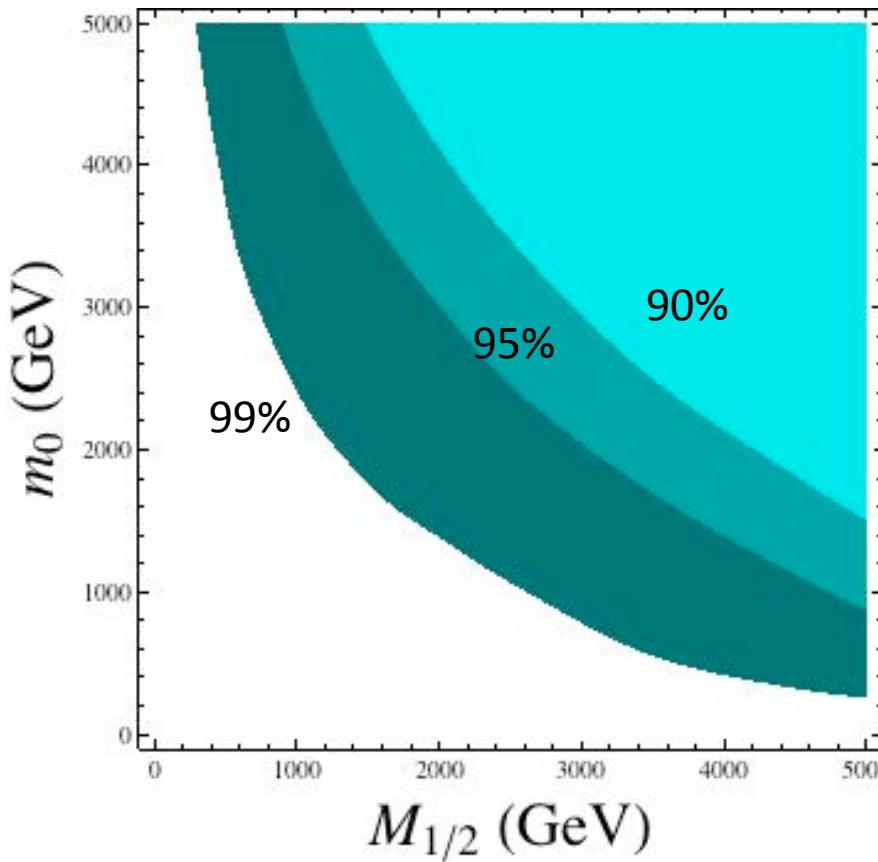
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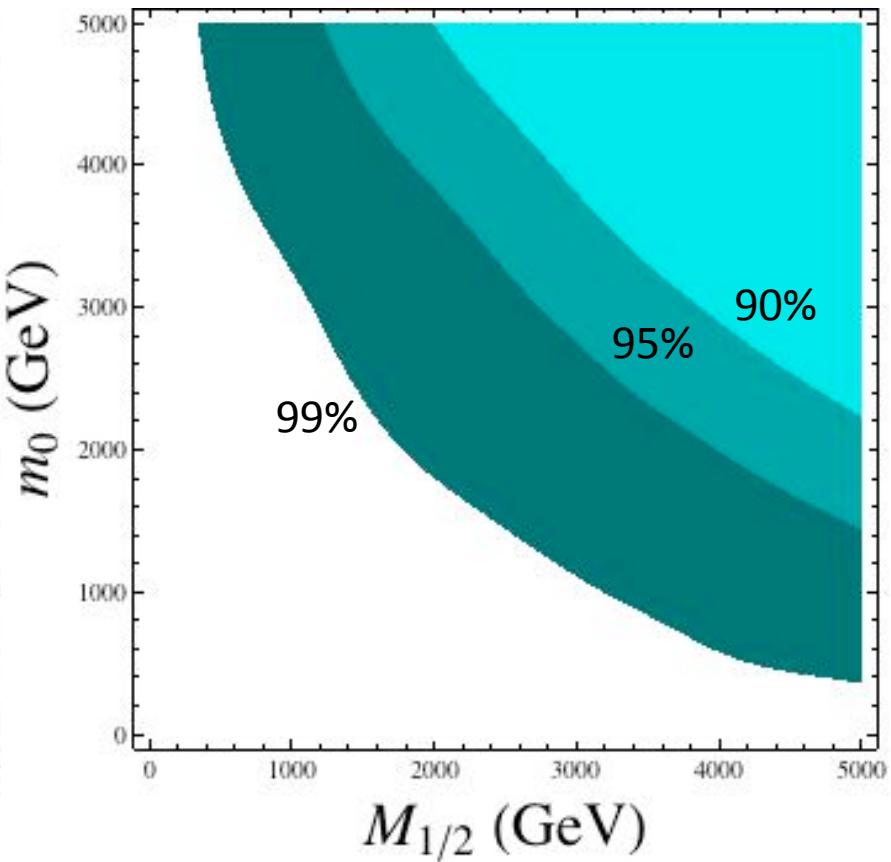
$$BR(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12}$$

@ 90% C.L. [MEG coll. 2011]

$$\tan \beta = 3$$



$$\tan \beta = 23$$



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Still a lot of work to find the final neutrino flavour model

Thanks for your attention

Numerical Accidents?

Are these patterns only numerical accidents? If Yes what?

→ **Anarchy (?)**

[Review: Altarelli, Feruglio & Masina 2002]

[Recently: Buchmuller, Domcke & Schmitz 2011]

Consist in using a simple U(1) as flavour symmetry:

- many parameters already at the LO
- low predictive power
- no correlations among observables

→ but the mixing angles and the r parameter can be accommodated

$$SU(5) \times U(1)$$

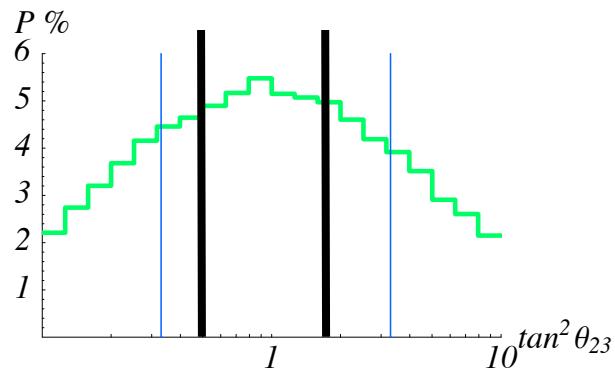
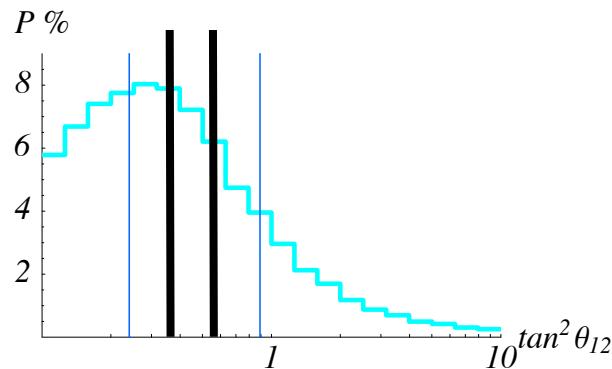
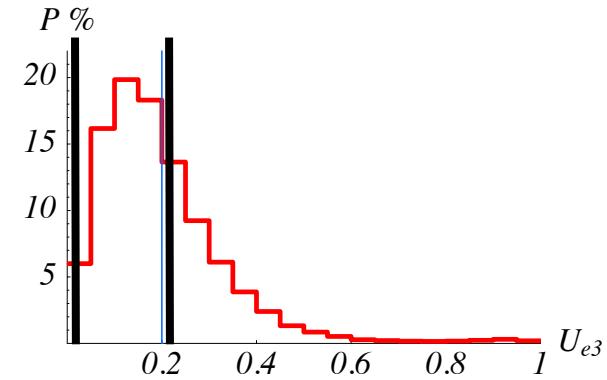
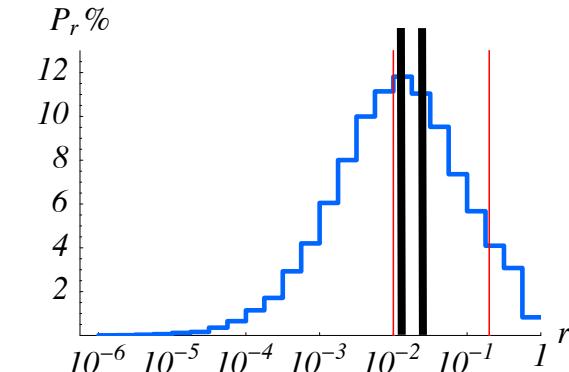
$$\Psi_{10} = (5, 3, 0)$$

$$\Psi_{\bar{5}} = (2, 0, 0)$$

$$\Psi_1 = (1, -1, 0)$$

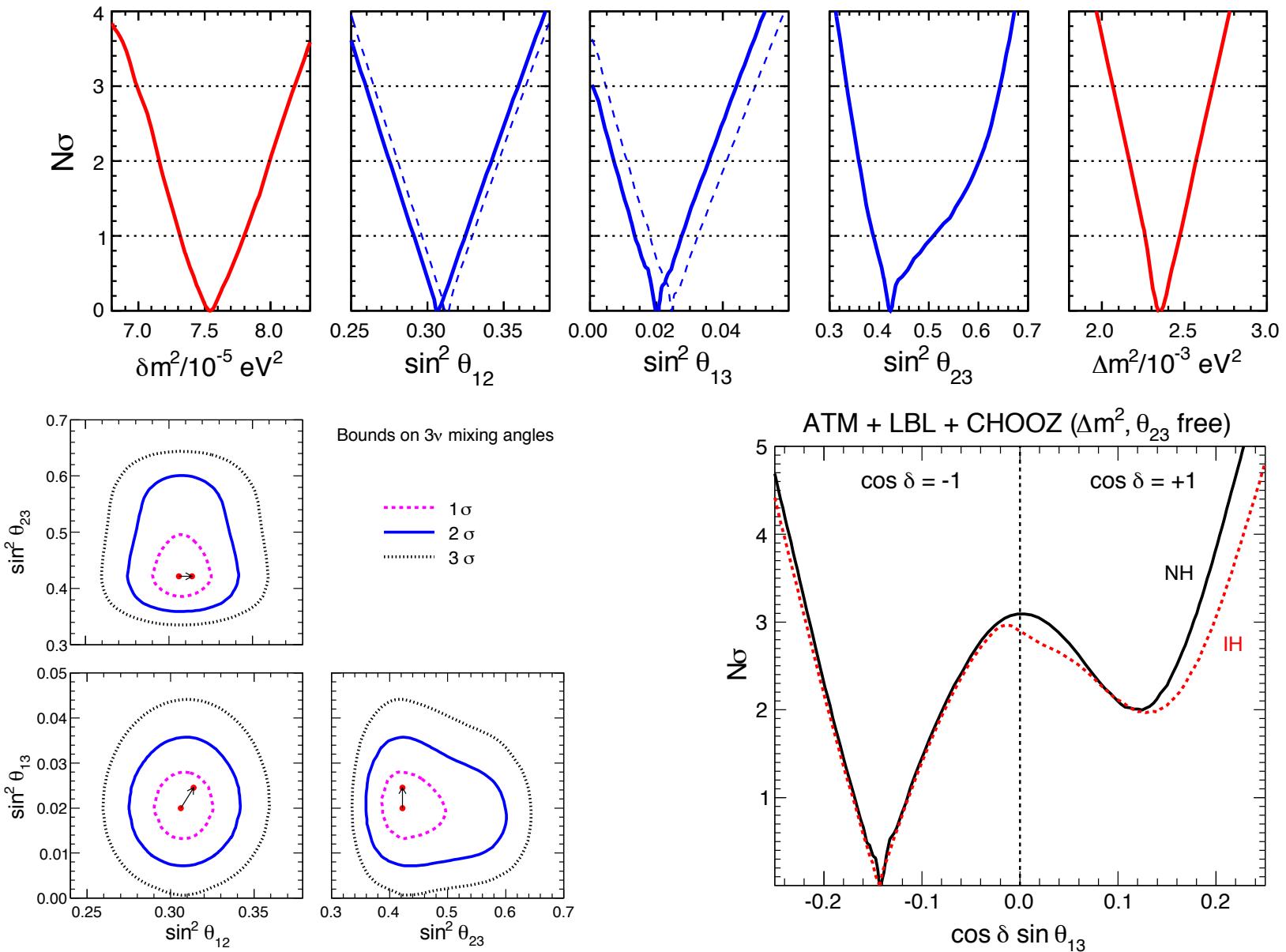
$$m_\nu = \begin{pmatrix} \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 0 & 0 \\ \epsilon^2 & 0 & 0 \end{pmatrix}$$

$$\epsilon \approx 0.5$$



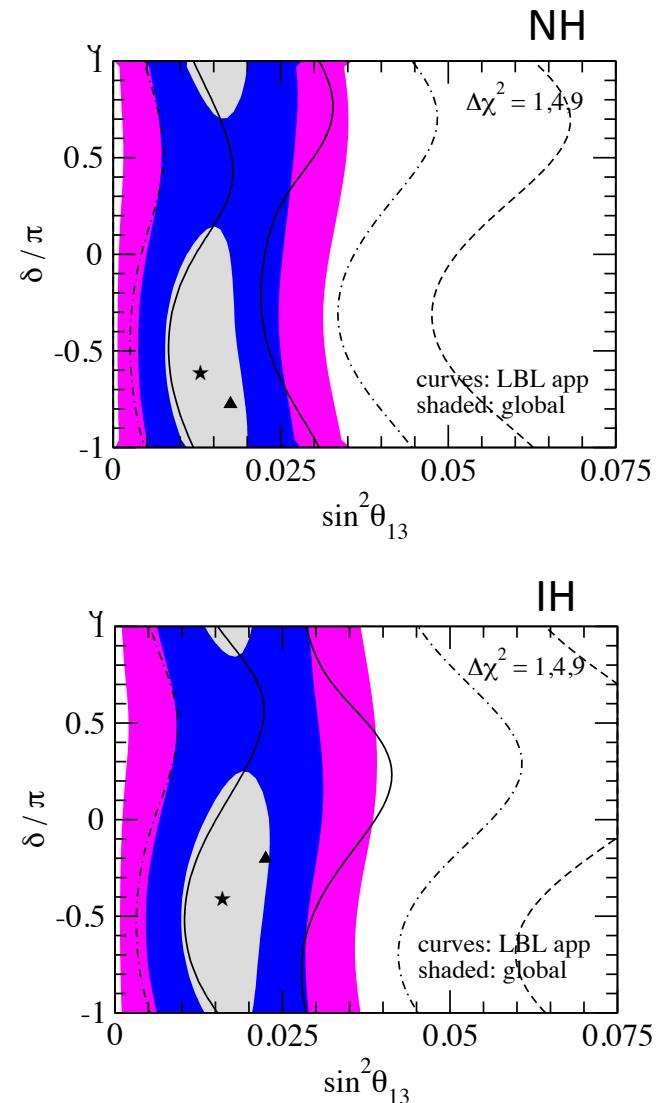
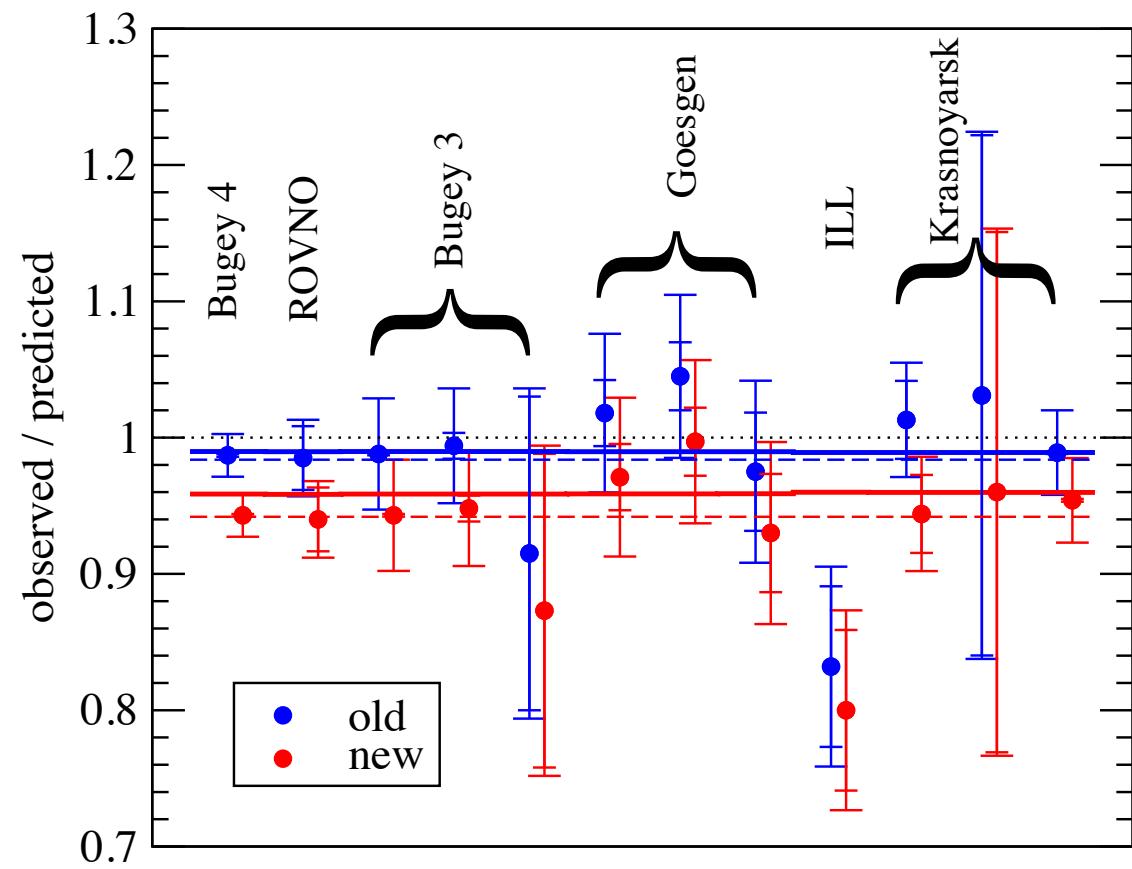
Neutrino Oscillation Fits

[Fogli *et al.* 2011]



Neutrino Oscillation Fits

[Schwetz *et al.* 2011]



The Flavour Puzzle

Charged Fermions

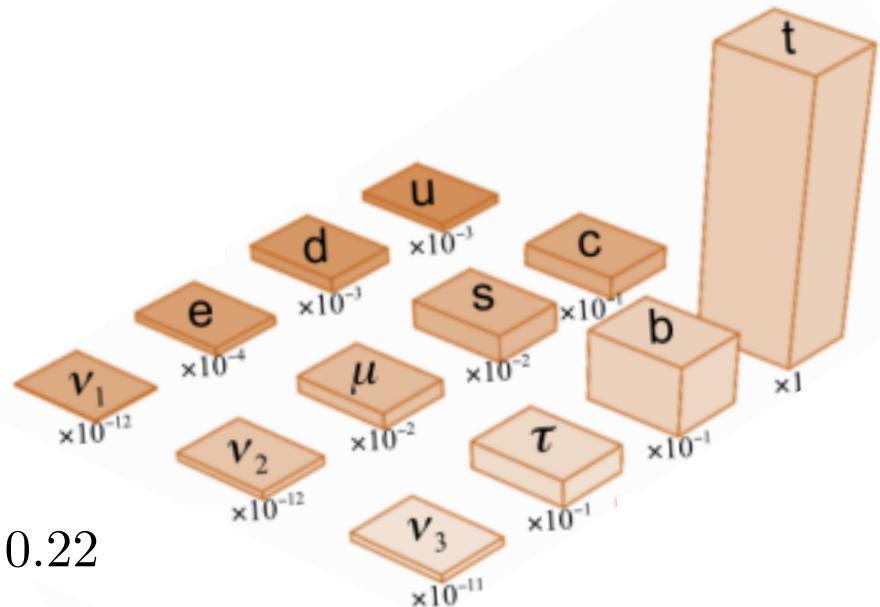
$$m_u : m_c : m_t \approx \lambda^{7/8} : \lambda^3 : 1$$

$$m_d : m_s : m_b \approx \lambda^{4/5} : \lambda^{2.5} : 1$$

$$m_e : m_\mu : m_\tau \approx \lambda^5 : \lambda^2 : 1$$

$$m_\tau : m_b : m_t \approx \lambda^3 : \lambda^{2.5} : 1$$

$$\lambda \approx \theta_C \approx 0.22$$



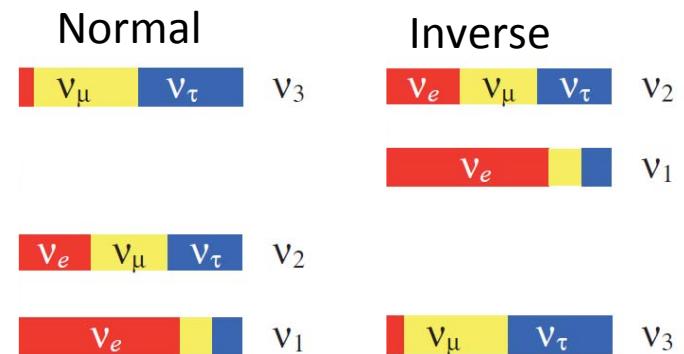
Neutrinos

Considering the conservative case with 3 active neutrinos:

$$m_\nu \lesssim \mathcal{O}(\text{eV})$$

$$\Delta m_{sol}^2 \equiv m_2^2 - m_1^2 \sim 2 \times 10^{-5} \text{ eV}^2$$

$$|\Delta m_{atm}^2| \equiv |m_3^2 - m_1^2| \sim 8 \times 10^{-3} \text{ eV}^2$$



The Mixing Matrices

CKM Matrix

$$V = R_{23}(\theta_{23}) \cdot R_{13}(\theta_{13}, \delta) \cdot R_{12}(\theta_{12})$$

$\sin \theta_{12}$	\approx	0.22
$\sin \theta_{23}$	\approx	0.04
$\sin \theta_{13}$	\approx	0.004
δ	\approx	77°

PMNS Matrix (end of 2008)

Atmospheric Reactor Dirac Solar Majorana

$$U = R_{23}(\theta_{23}) \cdot R_{13}(\theta_{13}, \delta) \cdot R_{12}(\theta_{12}) \cdot P$$

$\sin^2 \theta_{12}$	$= 0.312^{+0.019}_{-0.018}$
$\sin^2 \theta_{23}$	$= 0.466^{+0.073}_{-0.058}$
$\sin^2 \theta_{13}$	$= 0.020^{+0.016}_{-0.016}$

[Fogli et al. 0809.2936]

$(0.26 - 0.37)$	$\rightarrow \theta_{12} \approx 34^\circ$
$(0.331 - 0.644)$	$\rightarrow \theta_{23} \approx 43^\circ$
(≤ 0.046)	$\rightarrow \theta_{13} \approx 8^\circ$

2011 News on the Reactor Angle

Jun.	T2K claimed observation of $\nu_\mu \rightarrow \nu_e$ appearance: for NH (IH) @1.6 σ
[1106.2822]	$\sin^2 \theta_{13} = 0.028(0.036)$ $0.0076(0.01) < \sin^2 \theta_{13} < 0.076(0.094)$
Aug.	MINOS, in the $\nu_\mu \rightarrow \nu_e$ appearance, found for NH (IH) @1.6 σ
[1108.0015]	$\sin^2 \theta_{13} = 0.01(0.02)$ $\sin^2 \theta_{13} < 0.031(0.053)$
Dec.	Double Chooz, in the $\bar{\nu}_e \rightarrow \bar{\nu}_e$ disappearance, found for NH @1.6 σ
[1112.6353]	$\sin^2 \theta_{13} = 0.022$ $\sin^2 \theta_{13} < 0.03$

Sensitivity at 3 years:

Double Chooz	$\sin^2 \theta_{13} \sim 0.005 - 0.008$
RENO	$\sin^2 \theta_{13} \sim 0.005 - 0.008$
Daya Bay	$\sin^2 \theta_{13} \sim 0.0025$

More in the future >2015-2020: β -beams and Neutrino Factories

The Altarelli-Feruglio Model

[Altarelli & Feruglio 2005]

	Matter fields				Higgs		Flavons		
	ℓ	e^c	μ^c	τ^c	$h_{u,d}$	θ	φ_T	φ_S	ξ
A_4	3	1	1''	1'	1	1	3	3	1

$$w_e = y_e \frac{\theta^2}{\Lambda^3} e^c (\varphi_T \ell) h_d + y_\mu \frac{\theta}{\Lambda^2} \mu^c (\varphi_T \ell)' h_d + y_\tau \frac{1}{\Lambda} \tau^c (\varphi_T \ell)'' h_d$$

$$w_\nu = x_a \frac{\xi}{\Lambda} \frac{h_u \ell h_u \ell}{\Lambda_L} + x_b \left(\frac{\varphi_S}{\Lambda} \frac{h_u \ell h_u \ell}{\Lambda_L} \right)$$

Expansion in ϕ/Λ

vacuum alignment:

$$\frac{\langle \varphi_T \rangle}{\Lambda} = (u, 0, 0)$$

$$\frac{\langle \varphi_S \rangle}{\Lambda} = c_b(u, u, u)$$

$$\frac{\langle \xi \rangle}{\Lambda} = c_a u$$

$$\frac{\langle \theta \rangle}{\Lambda} = t$$

$$M_e = \text{diag}(y_e t^2, y_\mu t, y_\tau) v_d u$$

$$\frac{m_e}{m_\mu} = \frac{m_\mu}{m_\tau} = t \approx 0.05$$

$$M_\nu = \begin{pmatrix} a + \frac{2}{3}b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3}b & a - \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & \frac{2}{3}b \end{pmatrix} v_u^2$$

$$M_\nu^{diag} = v_u^2 \text{diag}(a+b, a, -a+b)$$

$$r \equiv \frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} \approx \frac{1}{35}$$

Radiative Lepton Decays

$$-\mathcal{L}_m \supset (\tilde{e} \quad \tilde{e}^c) \mathcal{M}_e^2 \begin{pmatrix} \tilde{e} \\ \tilde{e}^c \end{pmatrix} + \bar{\nu} m_{\nu LL}^2 \tilde{\nu}$$

$$\mathcal{M}_e^2 = \begin{pmatrix} m_{eLL}^2 & m_{eLR}^2 \\ m_{eRL}^2 & m_{eRR}^2 \end{pmatrix}$$

$$R_{ij} = \frac{48\pi^3\alpha}{G_F^2 m_{SUSY}^4} \left(|A_L^{ij}|^2 + |A_R^{ij}|^2 \right)$$

$$A_L^{ij} = \underline{a_{LL}(\delta_{ij})_{LL}} + a_{RL} \frac{m_{SUSY}}{m_i} (\delta_{ij})_{RL}$$

$$A_R^{ij} = a_{RR}(\delta_{ij})_{RR} + a_{LR} \frac{m_{SUSY}}{m_i} (\delta_{ij})_{LR}$$

$$\tan \beta = \{2, 25\} \quad \left\{ \begin{array}{l} a_{LL} = \{2, 27\} \\ a_{RR} = \{-1.9, -0.6\} \\ a_{RL} = a_{LR} = 0.3 \end{array} \right. \quad (\delta_{ij})_{CC'} = \frac{(\hat{m}_{eCC'}^2)_{ij}}{m_{SUSY}^2}$$