Simulation of the Effect of Errors in Multiple Pulse Laser Wakefield Acceleration

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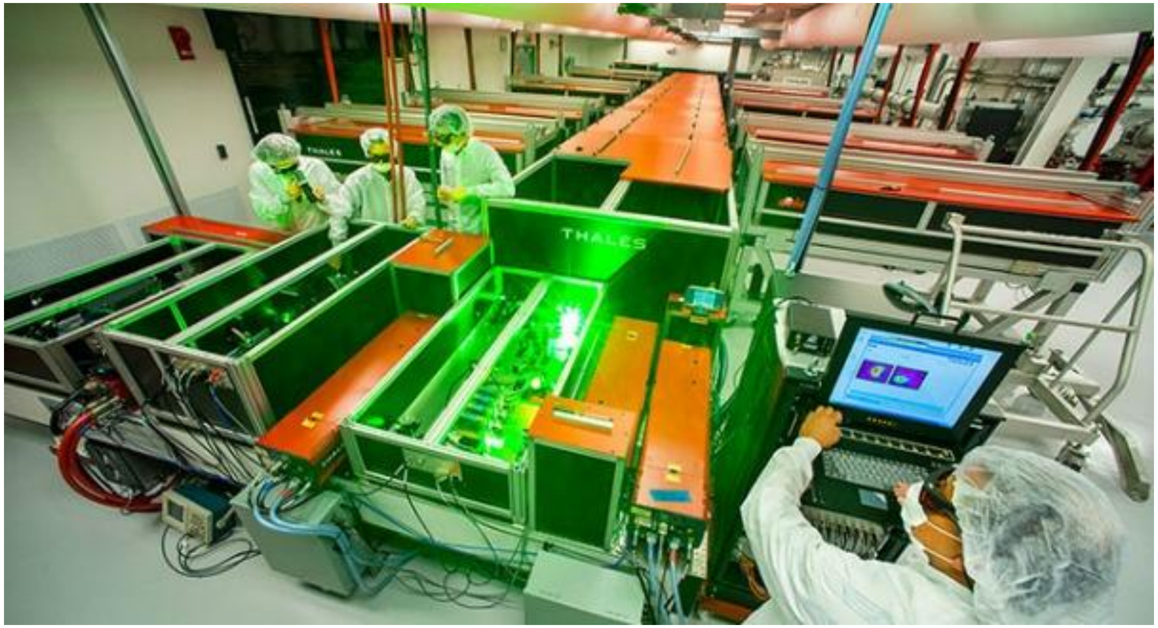
Overview

- Why is the Multiple Pulse technique needed?
- What are the effects of errors in pulse train spacing?
  - Systematic - tuning
  - Random - jitter
- How well do we need to control these errors?
- Develop simple analytic theory and compare to PIC simulations

Motivation

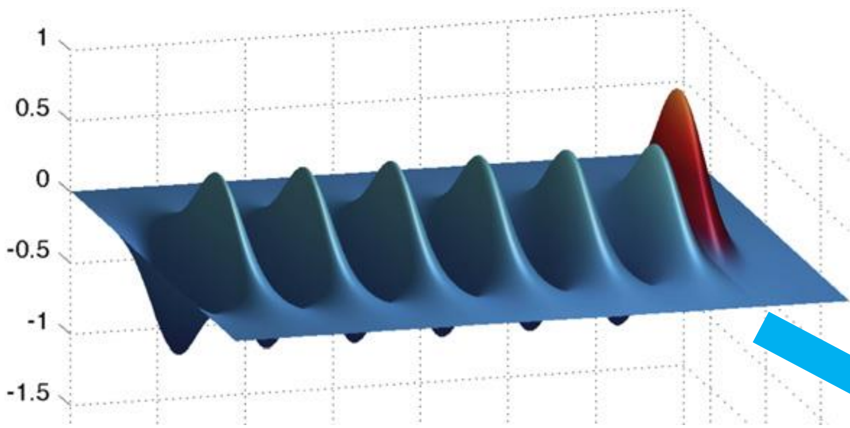
- Current laser systems use **single** laser pulses
  - Energy few J, Pulse duration tens of fs
- But repetition rates and efficiencies very low
  - eg. Ti:sapphire: Rep rate < 10 Hz, Efficiency < 0.1% wall-plug (Hooker et al 2014)
- Many applications require kHz repetition rates

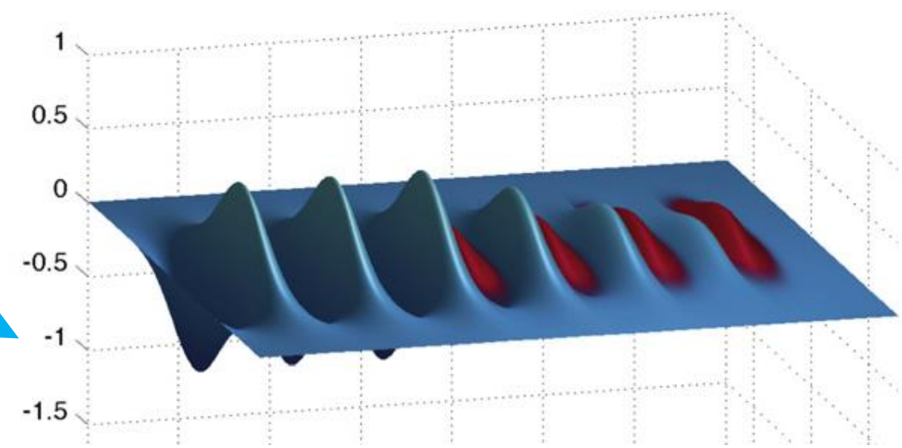


BELLA laser, LBNL
Credit: Mark Gable

Multiple Pulse-LWFA

- **Use smaller, more efficient laser systems in resonance**
- Several small pulses as effective as one large pulse
- Higher repetition rates become possible
- Vital for pulses to remain in resonance with wakefield
    - Throughout both i) pulse train and ii) length of accelerator





S. M. Hooker, R. Bartolini et al,
J. Phys. B: At. Mol. Opt. Phys. 47
pp. 234003, (2014).

Effect of Pulse Train Spacing

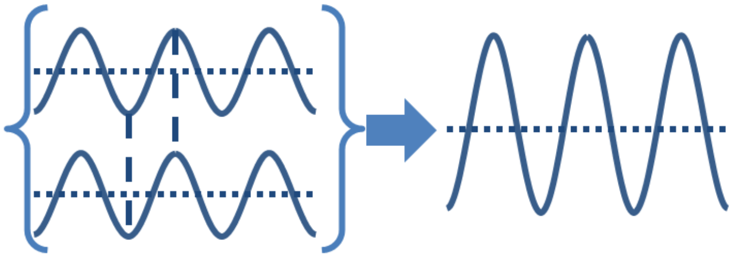
- In linear regime each pulse produces a sinusoidal plasma wave
- What happens if these are mistimed with error $\epsilon_n = \Delta\tau_n/\tau_\mathrm{p}$?

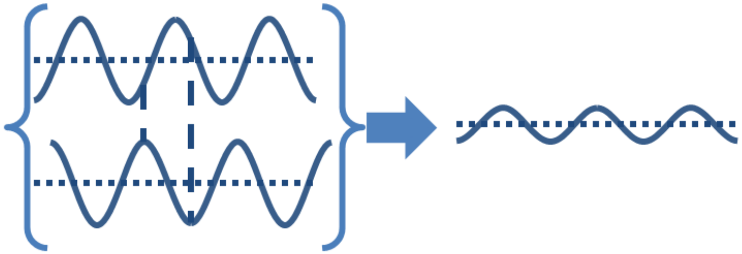
$$g_n(t) = a\cos\left[\omega_\mathrm{p}\left(t - \{n + \epsilon_n\}\tau_\mathrm{p}\right)\right]$$

$\sum_n^N g_n = A\, a\cos\left(\omega_\mathrm{p} t + \varphi\right)$ for:

$$A^2 = \left[\sum\nolimits_n \cos(2\pi\epsilon_n)\right]^2 + \left[\sum\nolimits_n \sin(2\pi\epsilon_n)\right]^2$$
$$= \boxed{N} + \boxed{\sum_{n=1}^N \sum_{m\neq n}^N \cos[2\pi(\epsilon_n - \epsilon_m)]}$$

Incoherent Coherent





- Now compare to simulations with Particle In Cell code EPOCH…

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Simulation Parameters

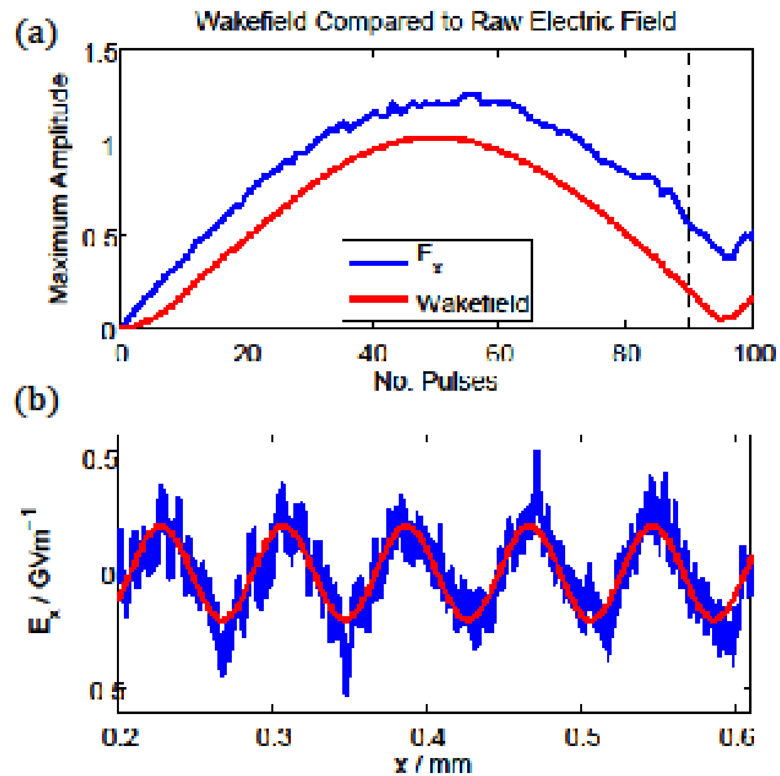
- Parameters in line with concept paper (Hooker et al 2014)
  - Linear regime, $a_0 = 0.052$
  - Broad Gaussian pulses, $t_{FWHM} = 100$ fs
  - Low density, $n_e = 1.75\text{x}10^{17}\ \text{cm}^{-3}$
  - Plasma wavelength, $\lambda_p = 80\ \mu\text{m}$

- EPOCH PIC code
  - Resolution 25 cells per feature size ($\lambda_L = 1\ \mu\text{m}$)
  - 20 pseudo-particles per cell – avoids numerical heating
  - Window 80 µm to capture exactly one plasma wavelength
  - Output taken once per plasma period, 266 fs
  - 100 repeats over random jitter

Calculating a Wakefield Amplitude

- Use Fourier amplitude to calculate wakefield amplitude
  - Require a consistent plasma wave, moving with electrons
  - Smoothing is better than a low pass filter
  - Faster than a fitting technique

⇒ Simulation window of length $L = n\,\lambda_p$
  - Best captures Fourier peak
  - Introduces artefact at beginning
  - Only works close to linear regime



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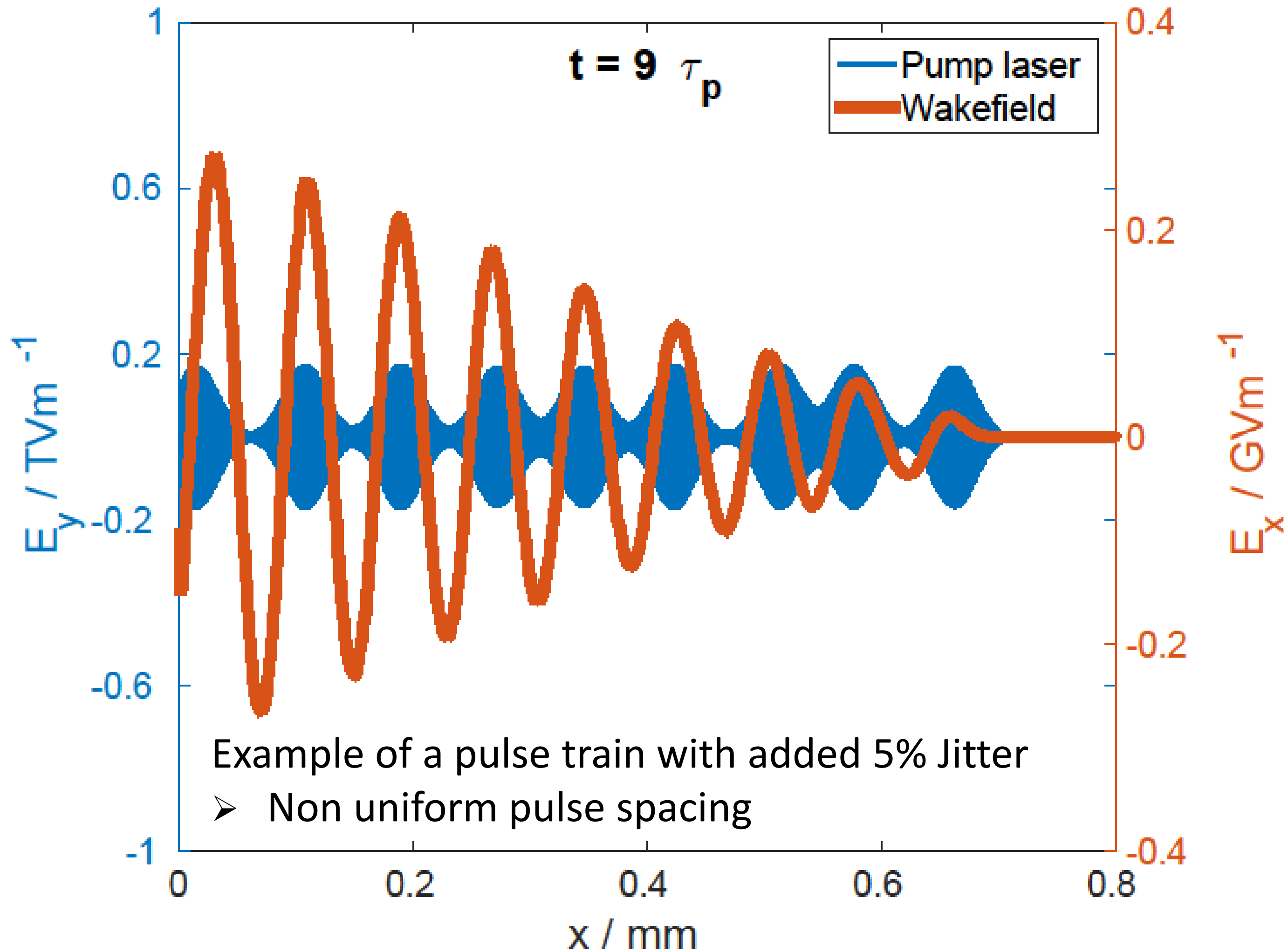
Example Simulation Results



Example of a pulse train with added 5% Jitter

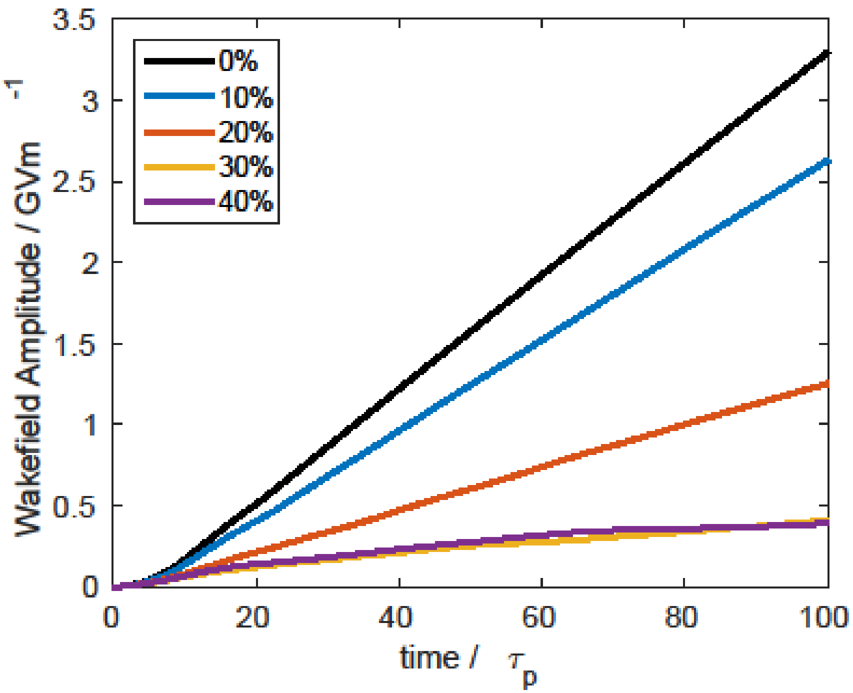
- Non uniform pulse spacing

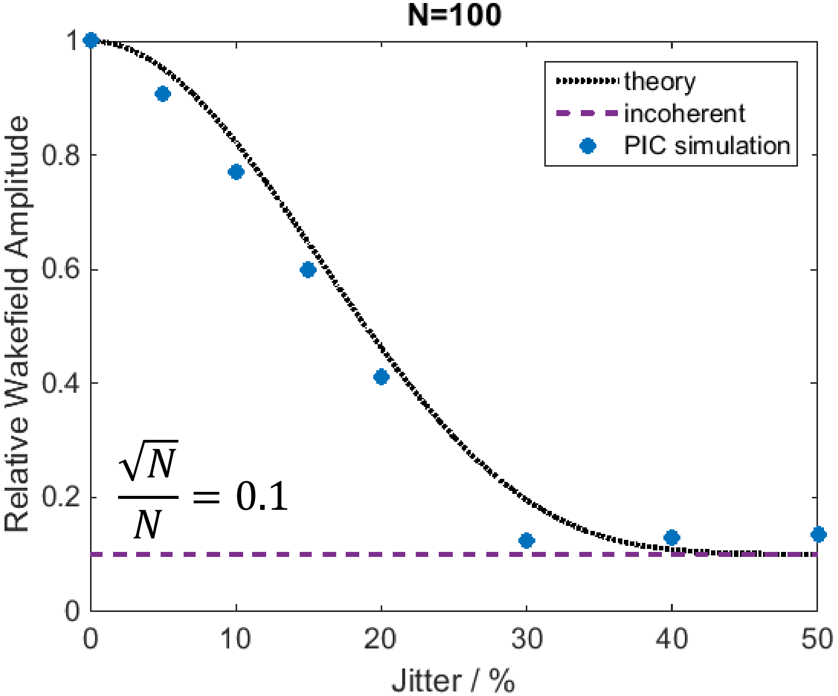
Random Spacing Error-Jitter

- Eg. vibrations on optic bench, thermal fluctuations

$$\epsilon_n \sim \mathrm{Gaussian}(0,\sigma) \;\Rightarrow\; A \approx \sqrt{\boxed{N} + \boxed{N^2 \exp[-(2\pi\sigma)^2]}}$$

- Good agreement with theory
- 10% scale jitter acceptable



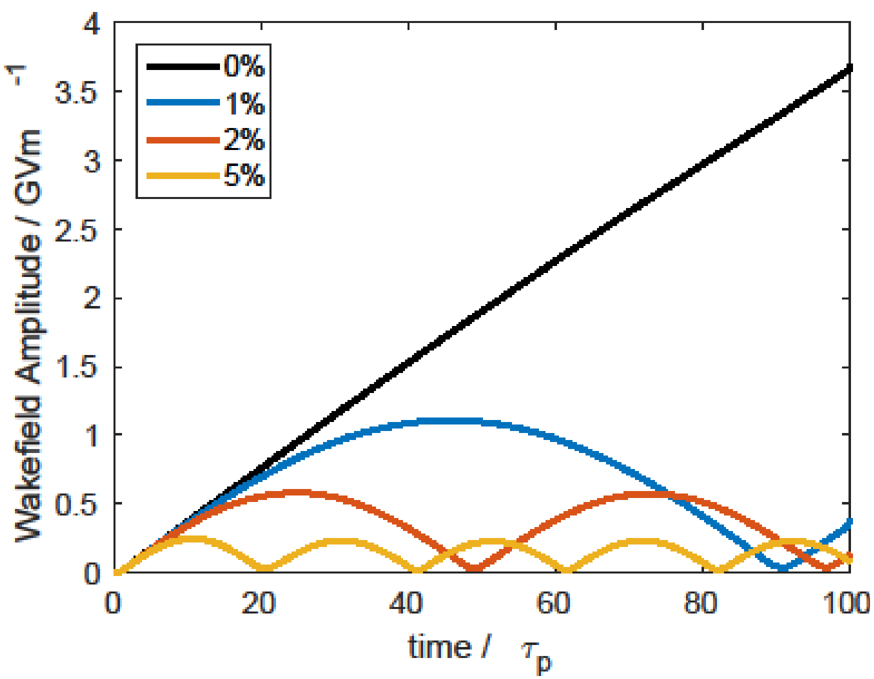


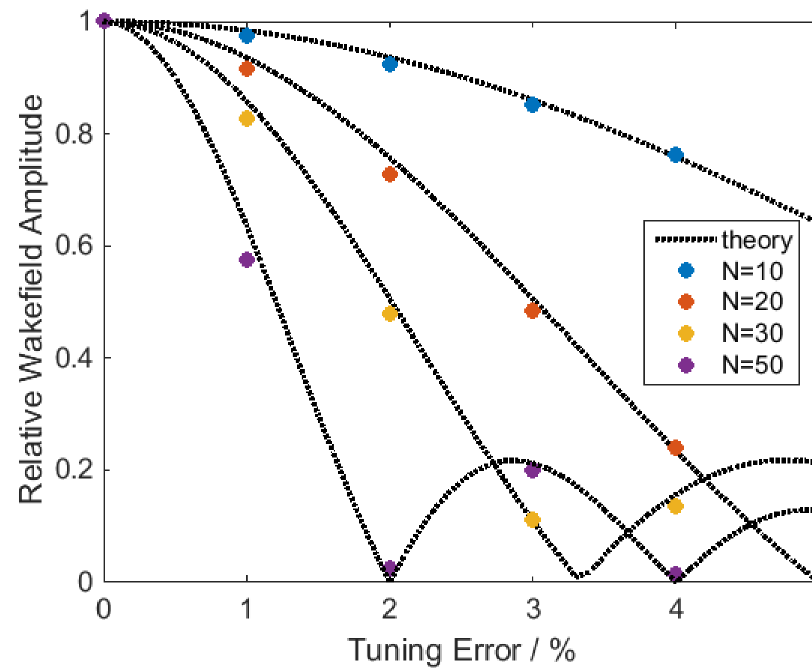
Systematic Spacing Error-Tuning

- Eg. target density error, misaligned optics error

$$\epsilon_n = an \;\Rightarrow A = |\sin(\pi a N)/\sin(\pi a)|$$

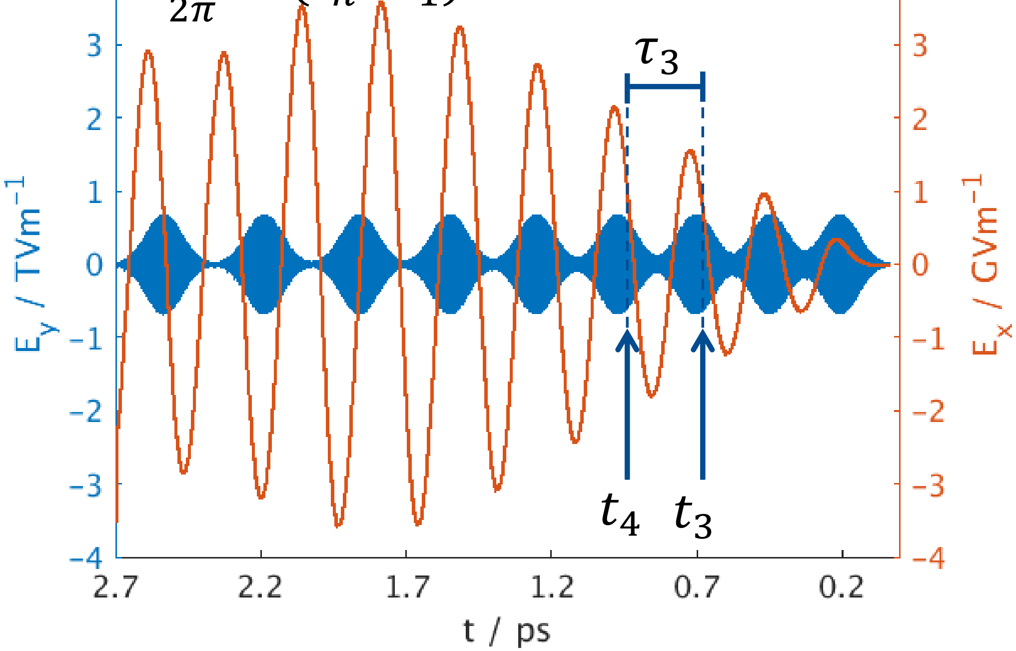
- Required accuracy: a < N^{-1}
- Limits useful pulse train length: N < $(2a)^{-1}$





Autoresonance Set Up

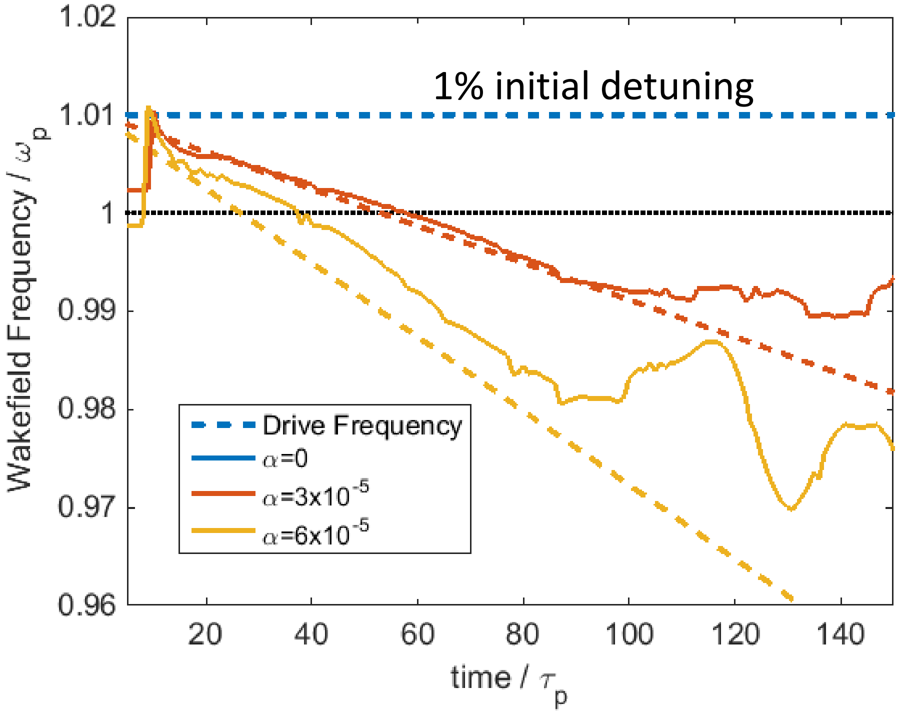
- Exploit autoresonance to overcome errors
  - R.R. Lindberg, A.E. Charman et al, Phys.Rev.Lett., 93 5, pp. 055001, (2004).
- Change pulse spacing to pass downwards through resonance
  - Pulse rate chirped: $\omega_{\mathrm{pulses}} = \omega_{\mathrm{initial}} - \alpha {\omega_{\mathrm{p}}}^2 t$ for $\omega_{\mathrm{initial}} > \omega_{\mathrm{p}}$
  - After the n^{th} pulse at t_n, the next separation is $\tau_n \approx \tau_1 + \frac{\alpha {\omega_{\mathrm{p}}}^2 {\tau_1}^2}{2\pi}(t_n - t_1)$

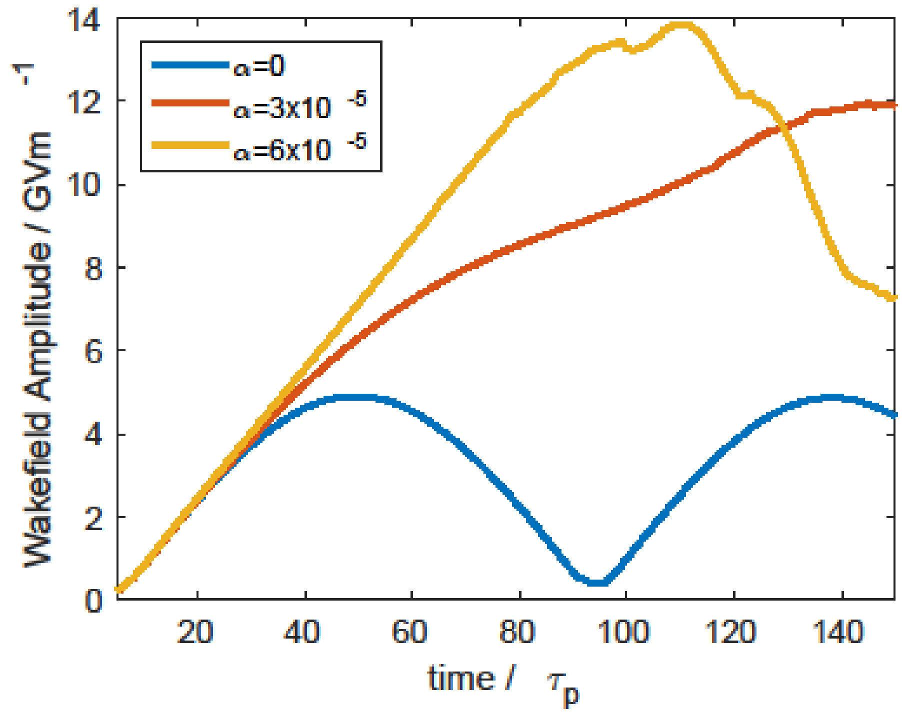


Example simulation
α=0.01, τ_0= 0.9 τ_p

Autoresonance Results

- Wakefield frequency becomes ‘locked’ to driver, not to plasma
- Pushes wakefield amplitude beyond the detuning limit
- Wakefield becomes resilient against plasma density fluctuations





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Conclusions

- High stability to random jitter (≈10%)
- But require accurate mean spacing (< 1% for 50 pulses)
  - Requires tuning plasma density within 2%
- PIC simulations in good agreement with simple linear theory

- Next steps
  - Experimental demonstration – J. Cowley
  - Ion motion further limits pulse train length – J. Holloway
  - Investigate autoresonance for resilience to pulse spacing and density error

Thank you

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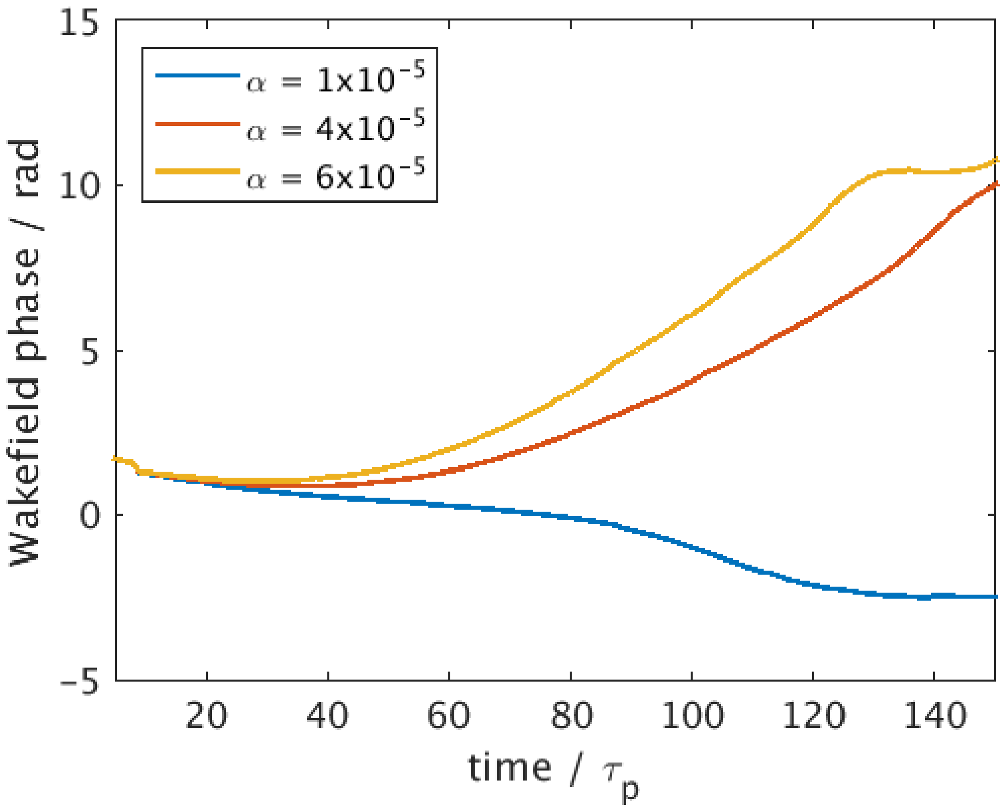
Further Autoresonance

- Wakefield phase changes smoothly with pulse spacing
- More work ahead on testing this in realistic density profiles
  - Will plasma waves match between different density regions?



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