TOMSK POLYTECHNIC UNIVERSITY

Effects of spatial coherence in coherent polarization radiation (transition radiation, Cherenkov radiation , parametric X-ray radiation)

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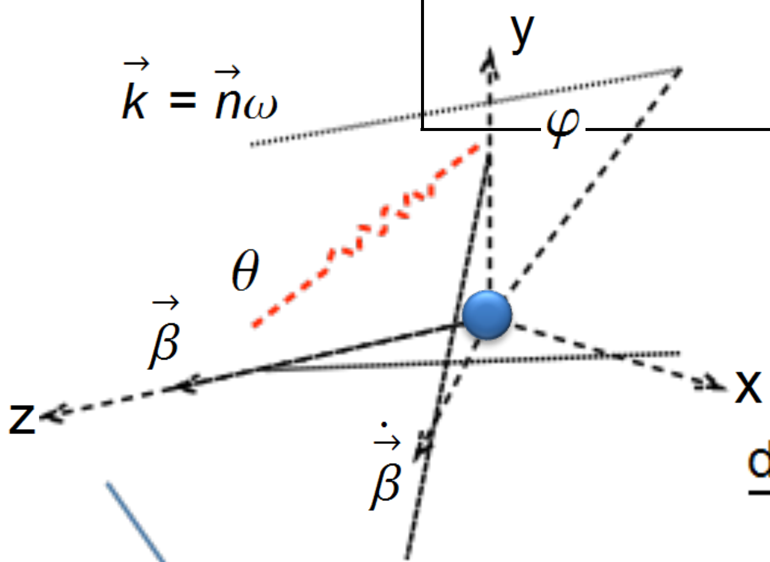
Outline of the talk

- Radiation from accelerated charge and polarization radiation
- Coherent polarization radiation, phase relation, formfactors
- Coherent transition radiation from “pancake-like” bunches
- Coherent BTR from a short bunch as Cherenkov radiation from the superluminal source
- Cherenkov radiation from a charge moving in vacuum near the dielectric target
- Coherent Cherenkov radiation from “pancake-like” bunches
- Coherent parametric X-ray radiation
- Conclusion

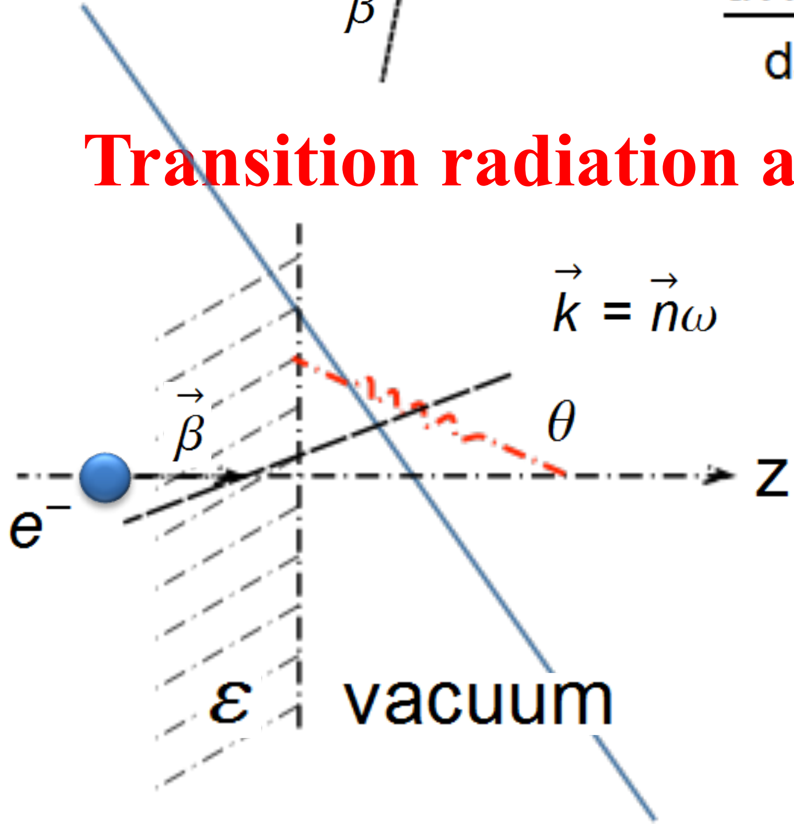
Radiation from accelerated charge (SR, UR)



$$\vec{E} = \frac{e}{cR}\frac{\left[\vec{n}\left[\left(\vec{n}-\vec{\beta}\right)\dot{\vec{\beta}}\right]\right]}{\left(1-\vec{n}\vec{\beta}\right)^3}$$ for SR $\dot{\vec{\beta}} \perp \vec{\beta}$

$$\frac{dW_{SR}}{d\Omega} = W_0\left\{\frac{\sin^2\varphi}{(1-\beta\cos\theta)^3} + \frac{(\beta-\cos\theta)^2\cos^2\varphi}{(1-\beta\cos\theta)^5}\right\}$$

Transition radiation as example of polarization radiation



$$\vec{E} = \frac{e}{cR}\left\{\frac{\left|\left[\vec{n}\,\vec{\beta}\right]\right|}{1-\vec{n}\,\vec{\beta}} - \frac{\left|\left[\vec{n}\,\vec{\beta}\right]\right|}{1-\sqrt{\varepsilon}\,\vec{n}\,\vec{\beta}}\right\} =$$

$$= \frac{e}{cR}\left\{\frac{\beta\sin\theta}{1-\beta\cos\theta} - \frac{\beta\sin\theta}{1-\sqrt{\varepsilon}\,\beta\cos\theta}\right\}$$

For ideal conductor $|\varepsilon|\to\infty$

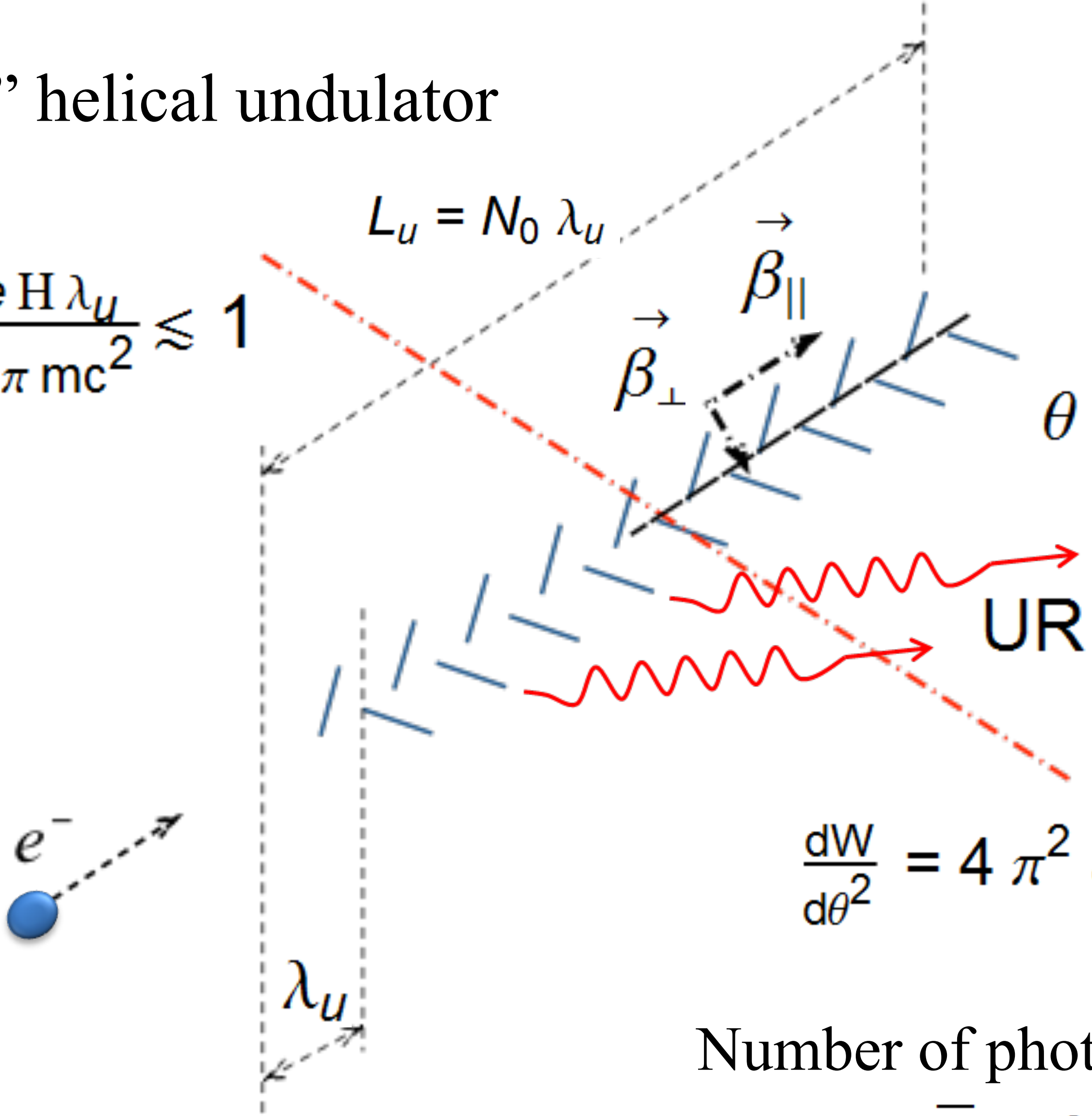
$$\frac{dW_{TR}}{d\Omega} = \frac{\alpha}{4\pi^2}\frac{\beta^2\sin^2\theta}{(1-\beta\cos\theta)^2}$$

Number of photons per one electron: $\vec{k}_{TR} \sim \alpha$

Undulator radiation

"Weak" helical undulator

$$k = \frac{e\,H\,\lambda_u}{2\,\pi\,mc^2} \lesssim 1$$



$$k = \sqrt{2}\,\gamma\ <\beta_{\perp}>$$

$$\gamma_{\parallel} = \frac{\gamma}{\sqrt{1 + k^2/2}}$$

$$\beta_{\parallel} \approx 1 - \frac{\gamma^{-2}}{2}\left(1 + \frac{k^2}{2}\right)$$

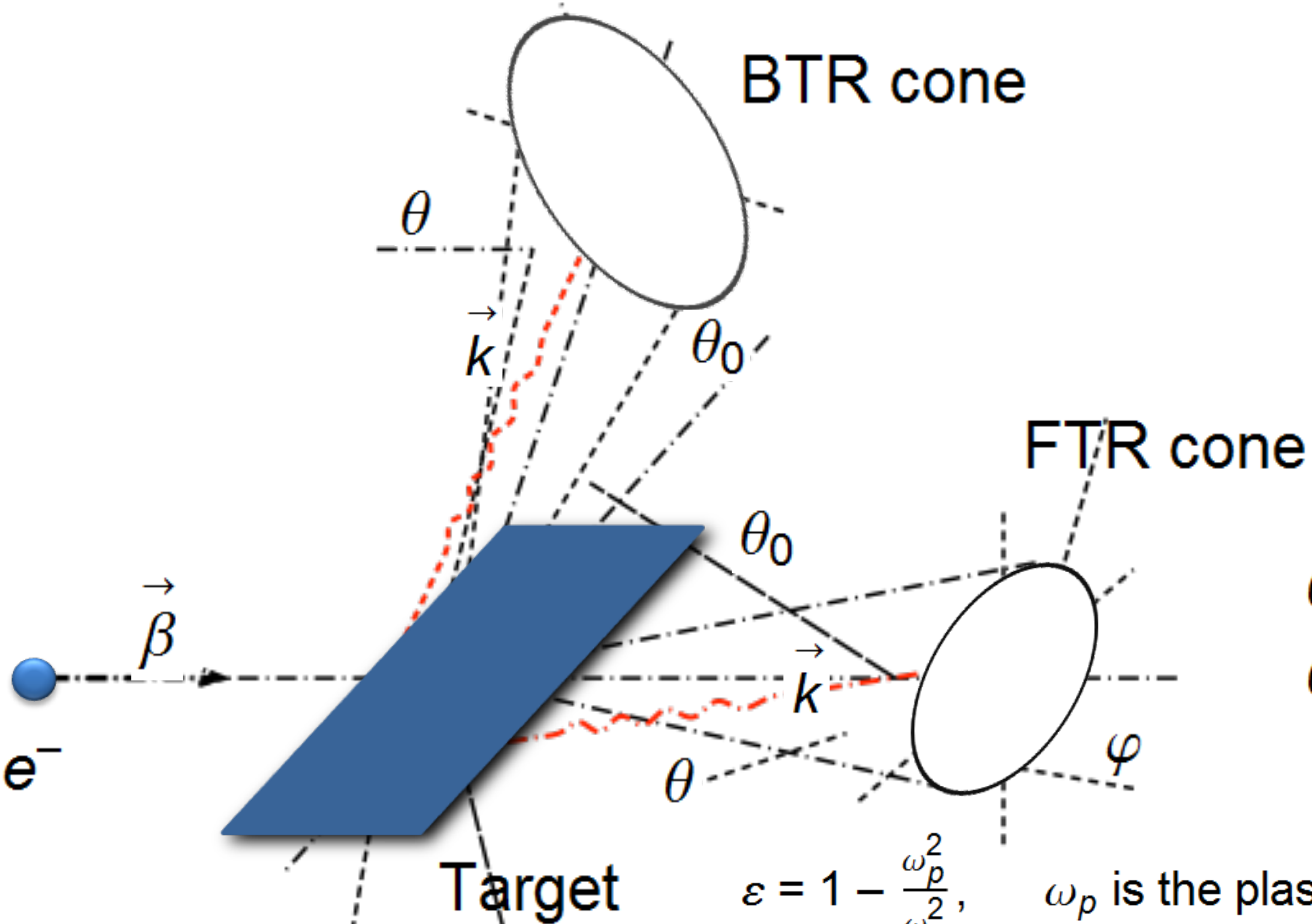
$$\frac{dW}{d\theta^2} = 4\,\pi^2\,\alpha\hbar c\,N_0\,k^2\,\frac{\beta_{\parallel}\,\gamma_{\parallel}^4}{\lambda_u}\,\frac{1 + \gamma_{\parallel}^2\,\theta^2}{\left(1 + \gamma_{\parallel}^2\,\theta^2\right)^5}$$

Number of photons per electron in undulator

$$\overline{k_{UR}} = \frac{2}{3}\,\pi\alpha\,k^2\,N_0 \sim \alpha\,N_0$$

For $N_0 \sim 10^2$ $\quad \overline{k_{UR}} \sim 1$

Transition radiation from ultrarelativistic charge



$$\theta_x = \theta \sin\varphi$$

$$\theta_y = \theta \cos\varphi$$

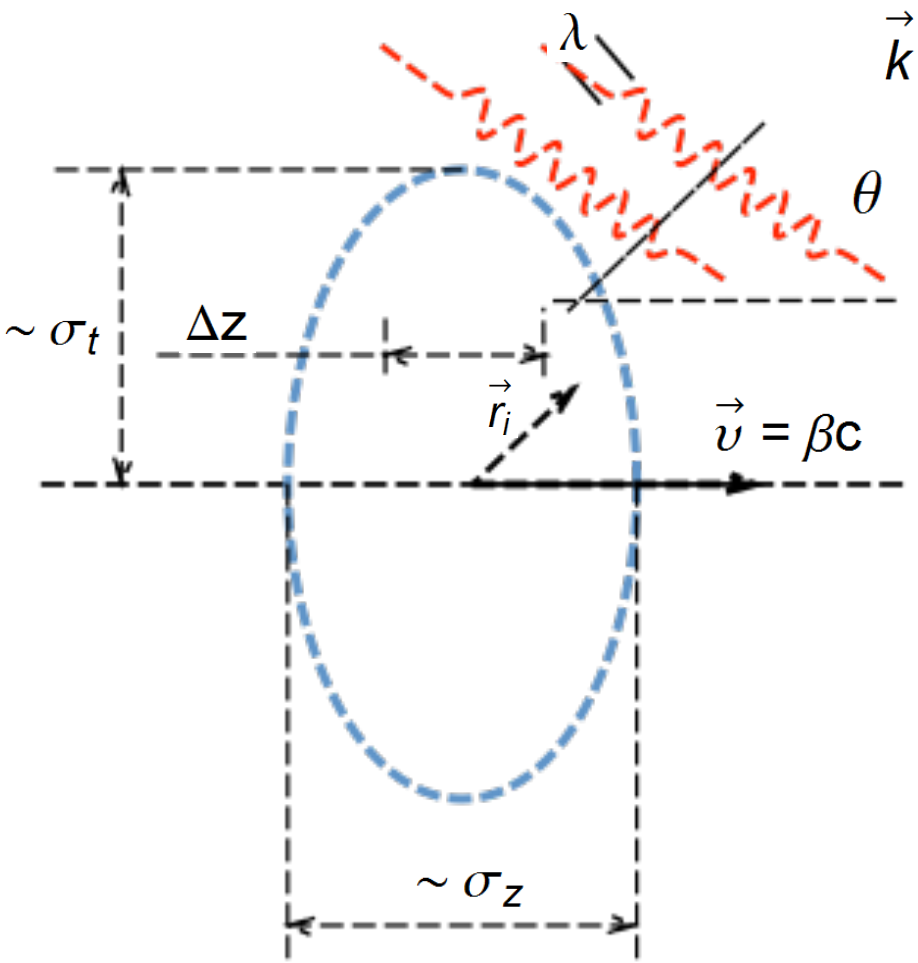
$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$, ω_p is the plasmon frequency

$\hbar\omega_p \sim 30$ eV for Al

$\frac{dW_{FTR}}{d\omega} \approx \text{const}$ for $\omega << \gamma\omega_p$

$\frac{dW_{BTR}}{\omega} \approx \text{const}$ for $\omega << \omega_p$

Coherent Synchrotron radiation (radiation from accelerated charged particles)



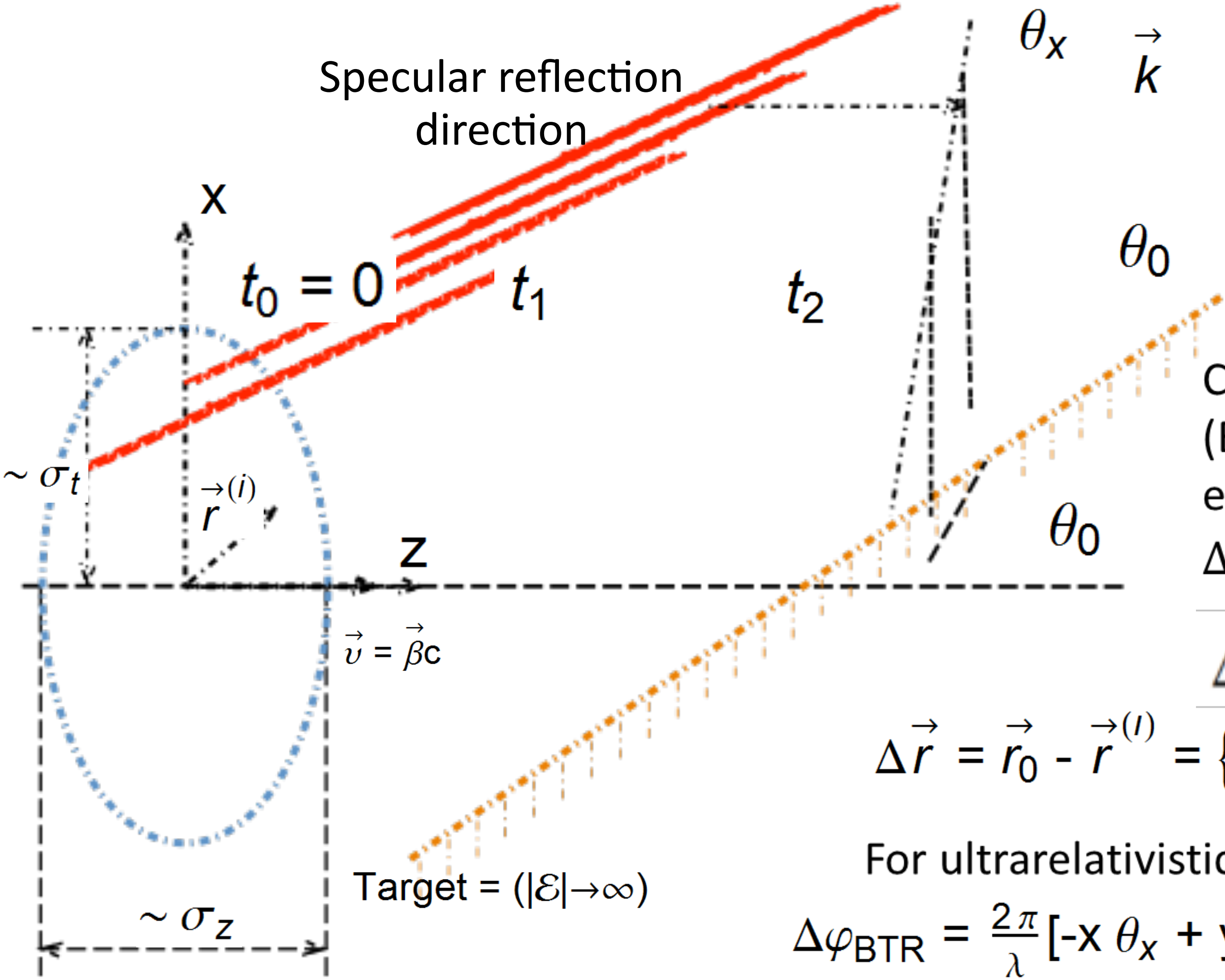
Condition of coherence (simultaneous emission from an arbitrary point along trajectory)

$$\Delta z \cos\theta \ll \lambda$$

In general case:

$$\Delta\varphi = \vec{k}\,\vec{r} = \frac{2\pi}{\lambda}\,\vec{n}\,\vec{r} \ll 2\pi$$

Coherent transition radiation
(radiation due to dynamical polarization of target atomic shells)



$t_2 > t_1 > t_0$

Condition of coherence (Emission from rest target electrons during lag time $\Delta t = t_2 - t_1 \sim \frac{\sigma_z}{\beta c}$):

$$\Delta\varphi = \vec{k}\,\Delta\vec{r} - \omega\,\Delta t$$

$$\Delta\vec{r} = \vec{r}_0 - \vec{r}^{(i)} = \{x^{(i)}, y^{(i)}, x^{(i)}/\tan\theta_0\}$$

For ultrarelativistic particles ($\beta \to 1$):

$$\Delta\varphi_{BTR} = \frac{2\pi}{\lambda}[-x\,\theta_x + y\,\theta_y + z\,(1 - 1/2\gamma^2)]$$

Coherent SR:

temporal coherence $\rightarrow \lambda >> \sigma_z \cos\theta \approx \sigma_z\,(1 - \theta^2/2)$

spatial coherence $\rightarrow \lambda >> \sigma_t \sin\theta \approx \sigma_t\,\theta$

Coherent BTR:

temporal coherence $\rightarrow \lambda >> \sigma_z/\beta \approx \sigma_z\,(1 - 1/2\gamma^2)$

spatial coherence $\rightarrow \lambda >> \sigma_x\,\theta_x; \sigma_y\,\theta_y,\;\; \sigma_t = \sqrt{\sigma_x^2 + \sigma_y^2}$

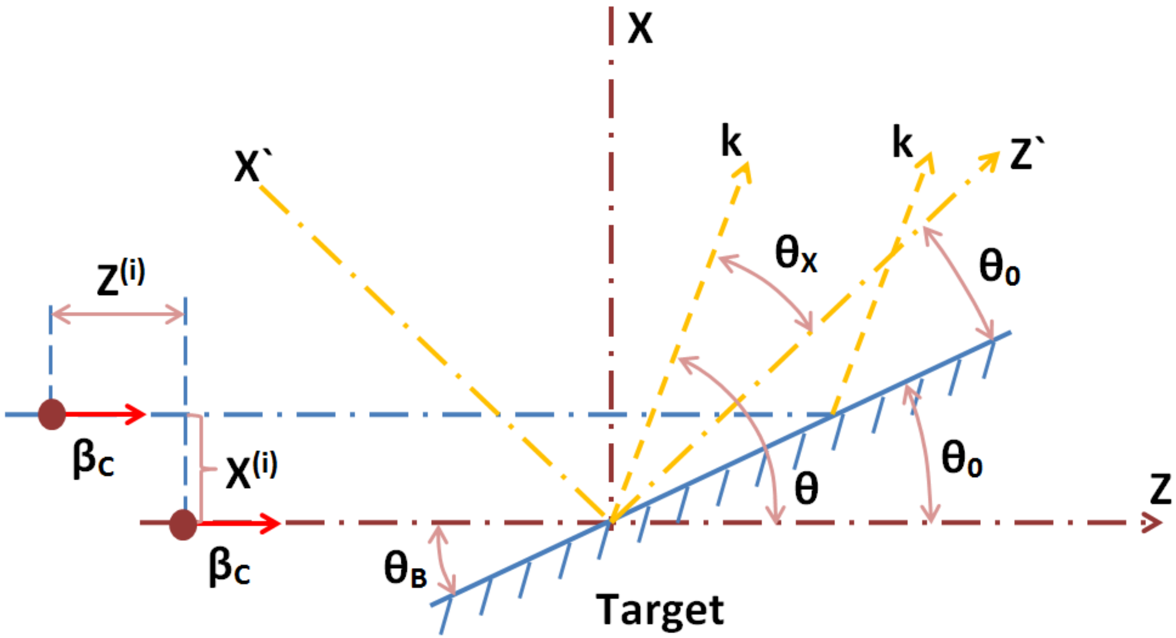
Angles θ_x, θ_y are defined relative to the specular reflection direction

Intensity of Coherent Transition Radiation

$$\frac{d^2 W_{CTR}}{d\omega d\Omega} = [N + N(N-1)\,F(k)]\,\frac{d^2 W_{TR}}{d\omega d\Omega}$$

Formfactor: $F(k) = \left|\int \rho(r)\exp[-i\,\Delta\varphi]\,dr\right|^2, \quad \Delta\varphi = k\Delta r - \omega\Delta t$

Kinematics of BTR



$$\Delta r = r_0 - r^{(i)} = \{x^{(i)},\, y^{(i)},\, x^{(i)}/\tan\theta_0\},$$

$$\Delta t^{(i)} = \frac{1}{\beta c}\,(x^{(i)}/\tan\theta_0 - z)$$

In the frame x`, z`

$$k = \frac{2\pi}{\lambda}\left\{\theta_x, \theta_y, 1 - \frac{\theta_x + \theta_y}{2}\right\}, \quad \theta_x, \theta_y \sim \gamma^{-1}$$

$$x' = x\cos 2\theta_0 - z\sin 2\theta_0,$$

$$y' = y,$$

$$z' = x\sin 2\theta_0 + z\cos 2\theta_0$$

Subsequently,

$$\Delta\varphi = k\Delta r - \omega\Delta t = \frac{2\pi}{\lambda}\left\{-x\theta_x + y\theta_y + \frac{z}{\beta} - x\frac{1 - \frac{\theta_x^2 + \theta_y^2}{2}}{2\gamma^2 \tan\theta_0}\right\}$$

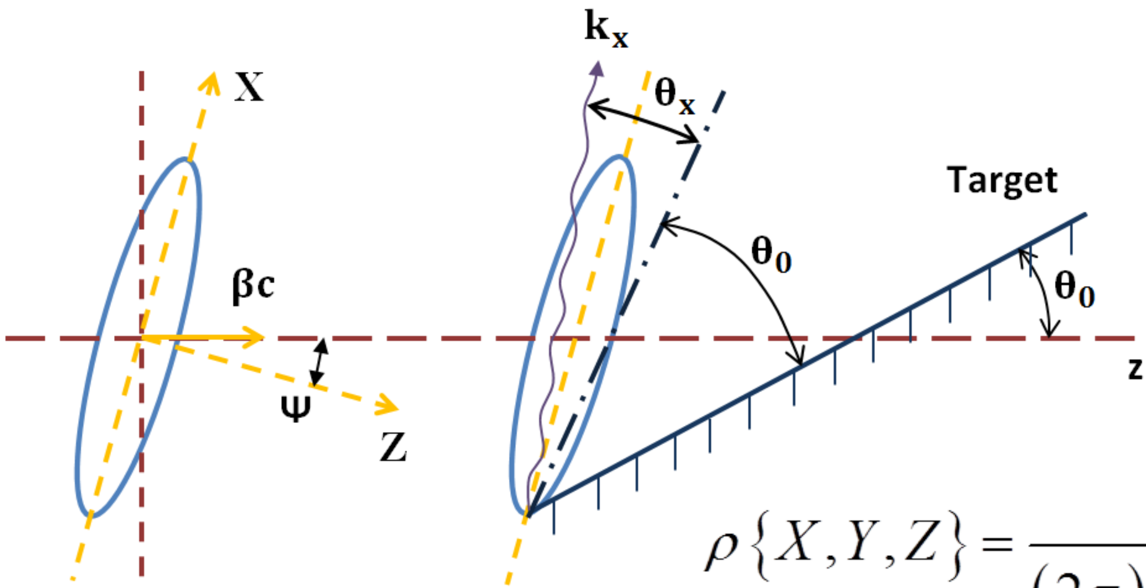
neglecting by γ^{-2} terms

$$\Delta\varphi_{BTR} \approx \frac{2\pi}{\lambda}\left\{-x\theta_x + y\theta_y + z\right\}$$

Phase shift for forward transition radiation:

$$\Delta\varphi_{FTR} \approx \frac{2\pi}{\lambda}\left\{x\theta_x + y\theta_y + z\right\}$$

Generation of coherent BTR by a tilted "pancake-like" bunch



In the frame $\{X, Y, Z\}$

$$\rho\{X,Y,Z\} = \frac{1}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z}\exp\left\{-\frac{1}{2}\left(\frac{X^2}{\sigma_x^2}+\frac{Y^2}{\sigma_y^2}+\frac{Z^2}{\sigma_z^2}\right)\right\}$$

And in the initial frame $\{x, y, z\}$

$$\rho\{\mathrm{x},\mathrm{y},\mathrm{z}\} = \frac{1}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z}\times \mathrm{Exp}\left\{-\frac{1}{2}\left[\frac{(x\cos\psi - z\sin\psi)^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}+\frac{(x\sin\psi + z\cos\psi)^2}{\sigma_z^2}\right]\right\}.$$

Formfactor for such configuration is calculated analytically:

$$F(\boldsymbol{k}) = F\left(\omega, \theta_x, \theta_y\right) =$$
$$\mathrm{Exp}\left\{-\frac{2\pi^2}{\lambda^2}\left[\sigma_x^2 \sin^2\psi + \sigma_z^2 \cos^2\psi + \theta_x\left(\sigma_x^2 - \sigma_z^2\right)\times \sin 2\psi\right.\right.$$
$$\left.\left. + \theta_x^2\left(\sigma_x^2 \cos^2\psi + \sigma_z^2 \sin^2\psi\right) + \theta_y^2\sigma_y^2\right]\right\}.$$

For the conventional configuration $\sigma_z >> \sigma_z, \sigma_y; \Psi = 0,$ the effects

of spatial coherency are negligible:

$$F_0(\omega) \approx \mathrm{Exp}\left\{-2\pi^2\left(\frac{\sigma_z^2}{\lambda^2} + \frac{\sigma_x^2\theta_x^2}{\lambda^2} + \frac{\sigma_y^2\theta_y^2}{\lambda^2}\right)\right\} \approx$$
$$\mathrm{Exp}\left\{-2\pi^2\frac{\sigma_z^2}{\lambda^2}\right\}.$$

But for a "pancake-like" bunch the "effective" longitudinal size will be defined by the value:

$$\sigma_\ell^2 = \sigma_x^2 \sin^2\psi + \sigma_z^2 \cos^2\psi_0 > \sigma_z^2,$$

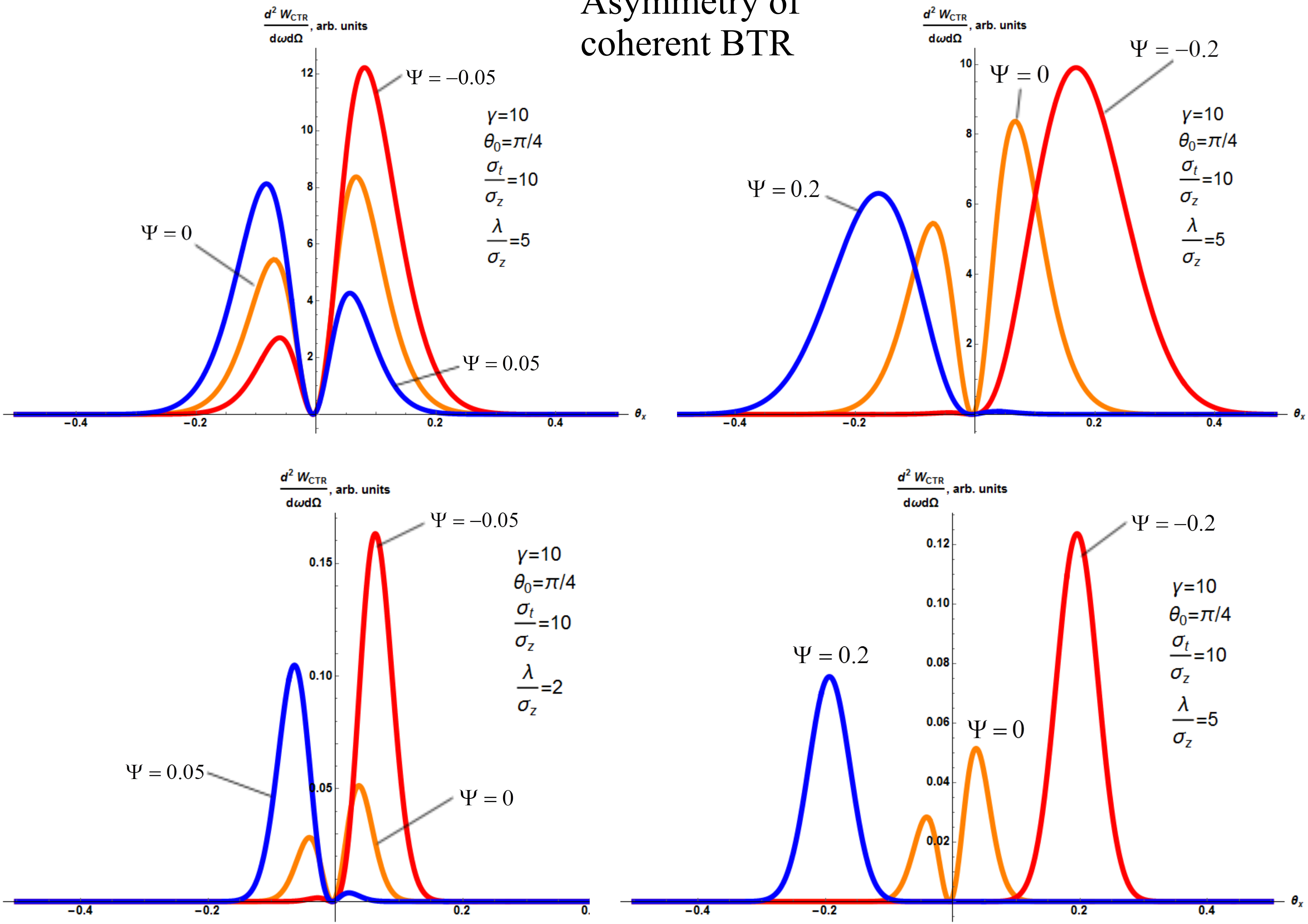
Spectral-angular distribution for coherent BTR can be calculated using well-known expression:

$$\frac{d^2W_{CTR}}{d\omega d\Omega} \approx N^2 F\left(\omega, \theta_x, \theta_y\right) \frac{e^2}{\pi^2 c} \frac{\theta_x^2 + \theta_y^2 + \theta_x\left(\gamma^{-2} + \theta_x^2 + \theta_y^2\right) / \tan\theta_0}{(\gamma^{-2} + \theta_x^2 + \theta_y^2)^2}.$$

For small tilt angle ($\psi \ll 1$) formfactor can be written as:

$$F\left\{\omega, \theta_x, \theta_y\right\} = \exp\left\{-\frac{2\pi^2}{\lambda^2}\left[\sigma_x^2\left(\psi + \theta_x\right)^2 + \sigma_y^2\theta_y^2 + \sigma_z^2\right]\right\}.$$

Asymmetry of coherent BTR



Spectral – angular distribution of coherent BTR, generated by a “pancake-like” bunch is determined by the ratio σ_t / σ_z, tilt angle and wavelength:

- if there is asymmetry which depends on the sign of ;
- if the distribution has a single maximum;
- the angle of maximum even if

$$\Psi < \frac{\sigma_z}{\sigma_t}$$

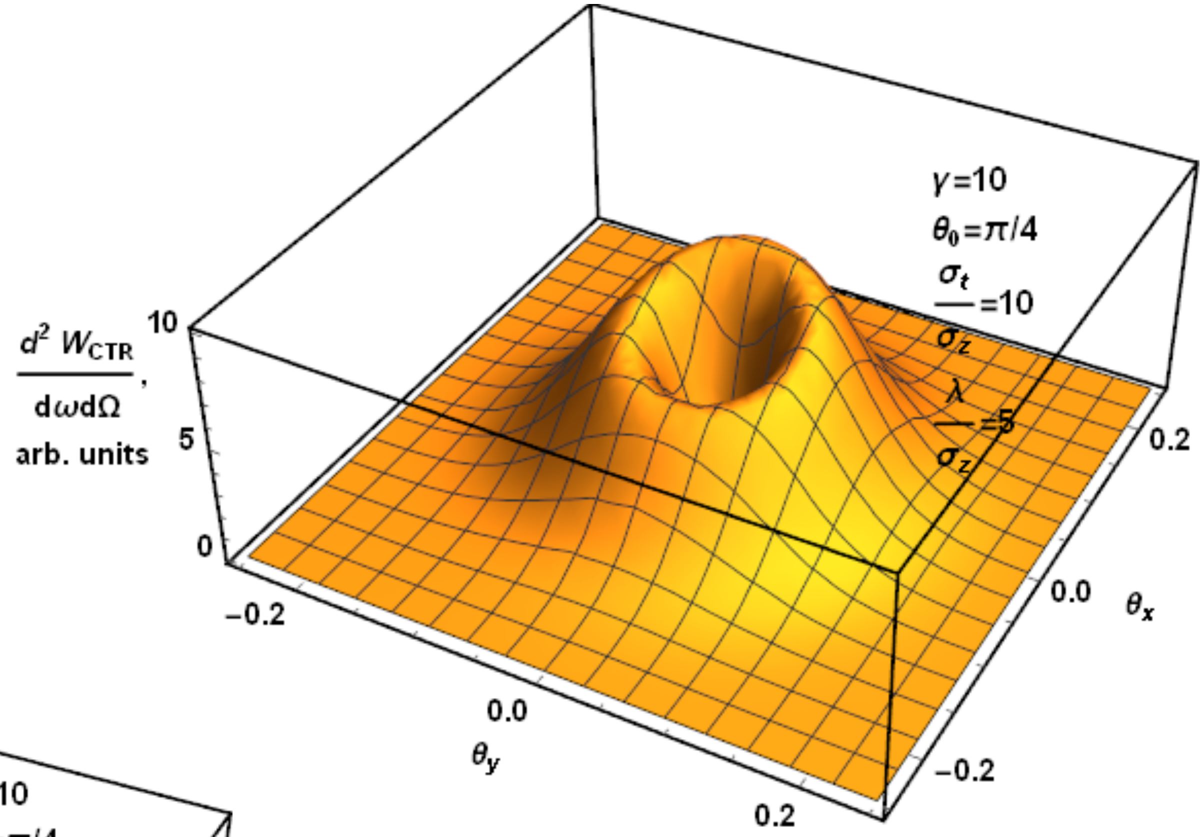
$$\Psi$$

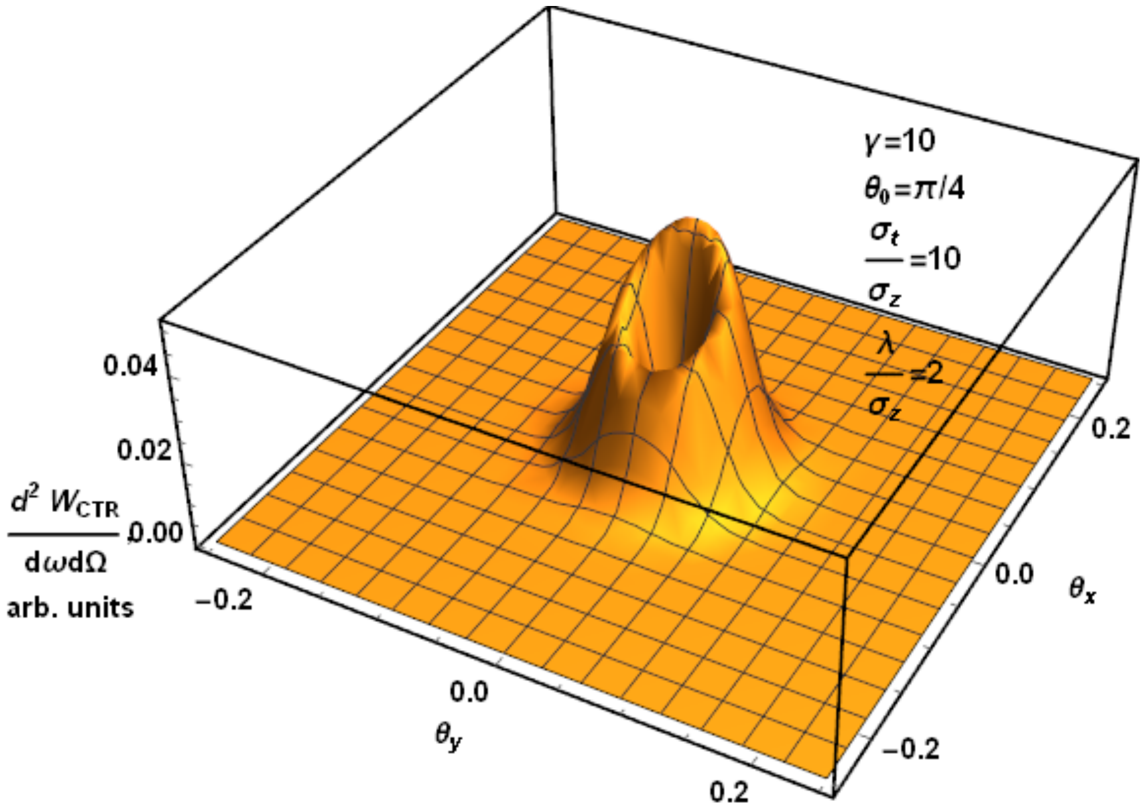
$$\Psi > \frac{\sigma_z}{\sigma_t}$$

$$\theta_x \approx -\Psi \qquad \Psi \gg \gamma^{-1}$$

2D distributions of coherent BTR

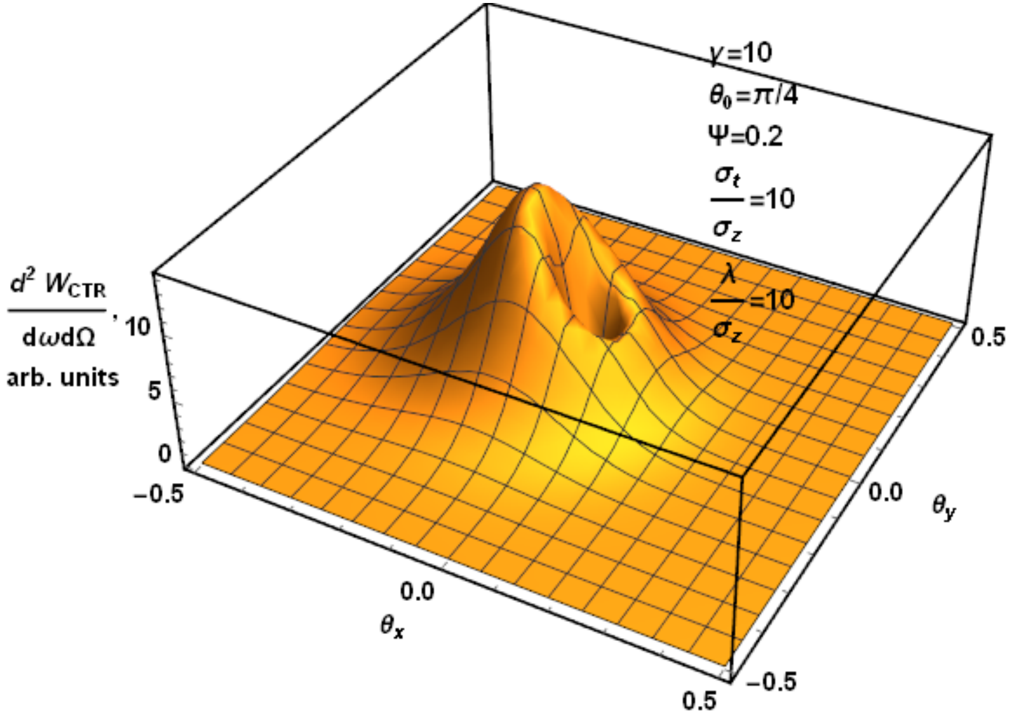
$$\Psi = 0$$

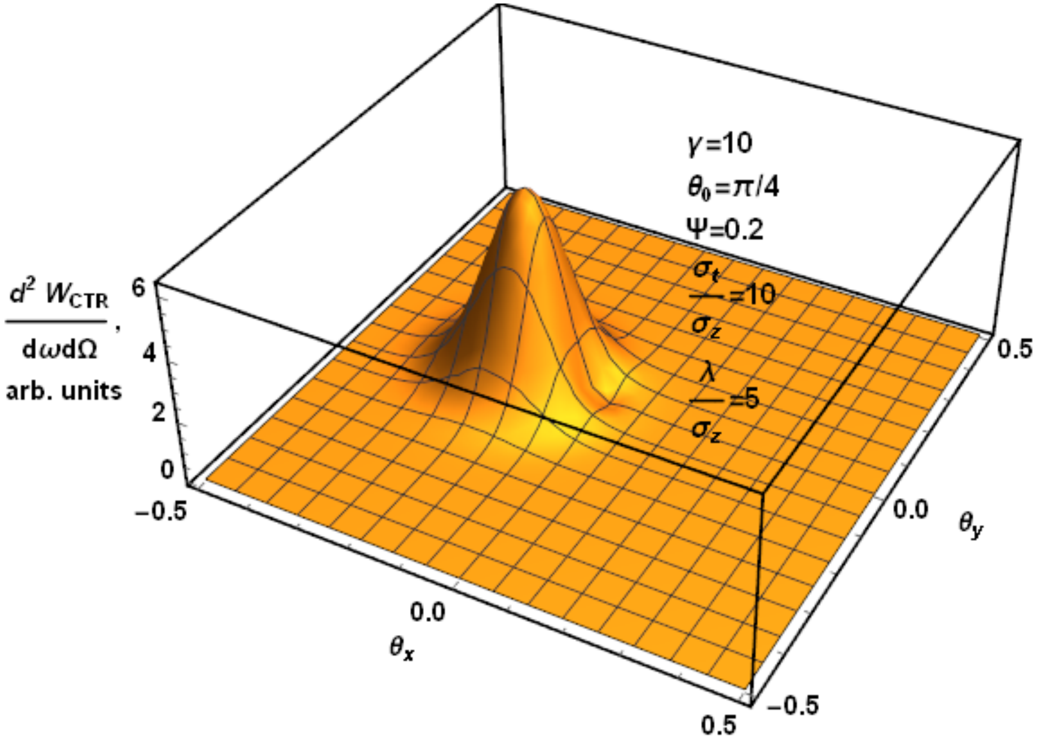


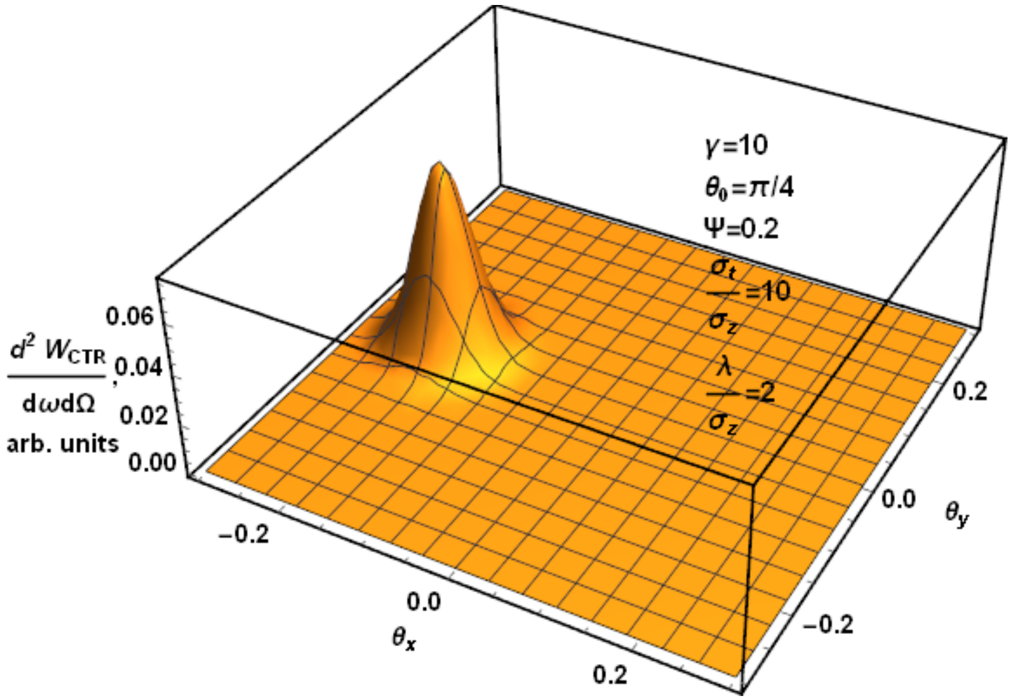


2D distributions of coherent BTR

$$\Psi = 0.2$$







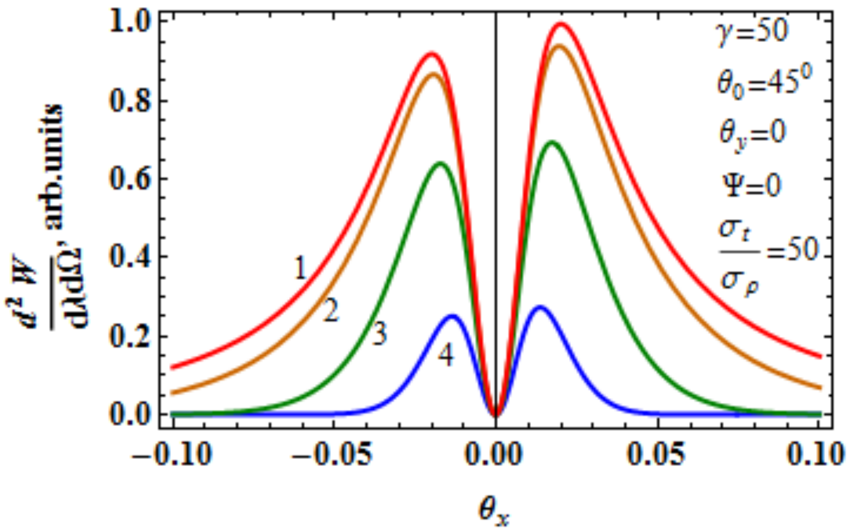
For the perpendicular pancake $(\psi = 0)$ and for ultrarelativistic case $(\gamma \gg 1)$

$$\frac{d^2W_{CTR}}{d\omega d\Omega} \approx N^2 \mathrm{Exp}\left\{-\frac{2\pi^2}{\lambda^2}\sigma_z^2\right\} \mathrm{Exp}\left\{-\frac{2\pi^2}{\lambda^2}\left(\sigma_x^2\theta_x^2 + \sigma_y^2\theta_y^2\right)\right\} \times$$

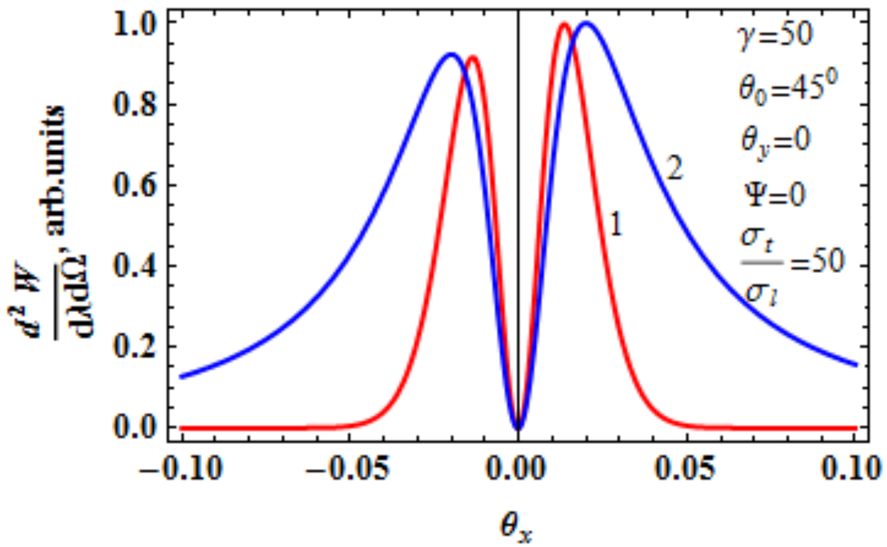
$$\frac{e^2}{\pi^2 c} \frac{\theta_x^2 + \theta_y^2}{\left(\gamma^{-2} + \theta_x^2 + \theta_y^2\right)^2}.$$

Transversal formfactor is closed to unity for the case: $\sigma_x, \sigma_y \ll \lambda / \sqrt{2}\,\pi.$

In the opposite case the spatial coherency effect suppressed BTR yield



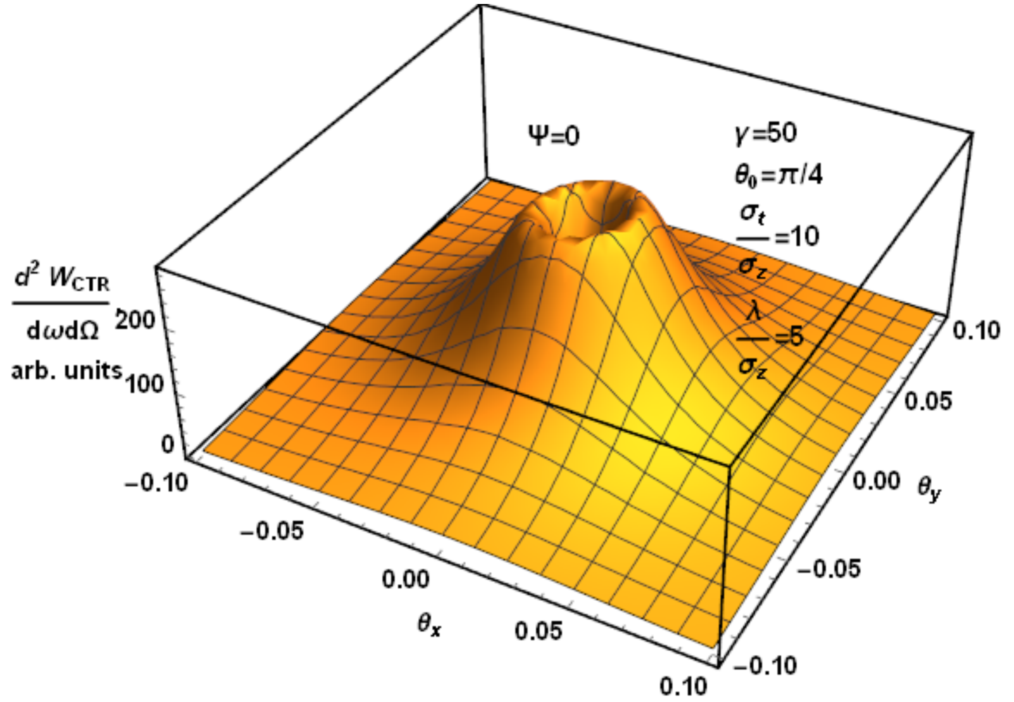
$1)\,\lambda = 100\sigma_z;\ 2)\lambda = 25\sigma_z;\ 3)\lambda = 10\sigma_z;\ 4)\lambda = 5\sigma_z$

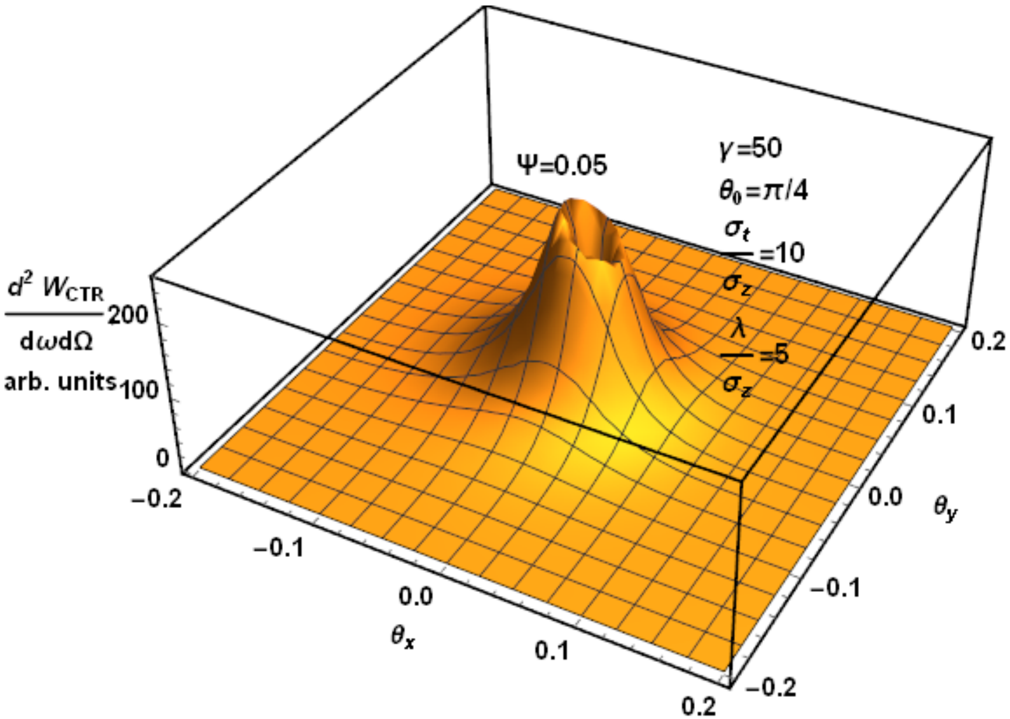


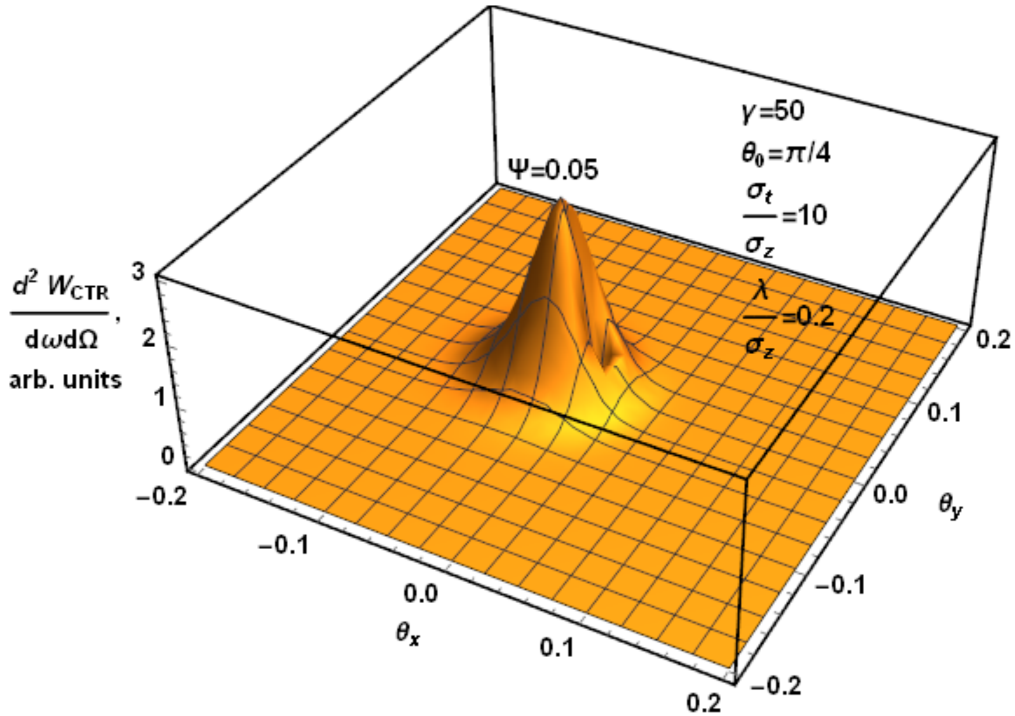
$1)\,\lambda = 5\sigma_z;\ 2)\,incoherent\ TR$

Ultrarelativistic case

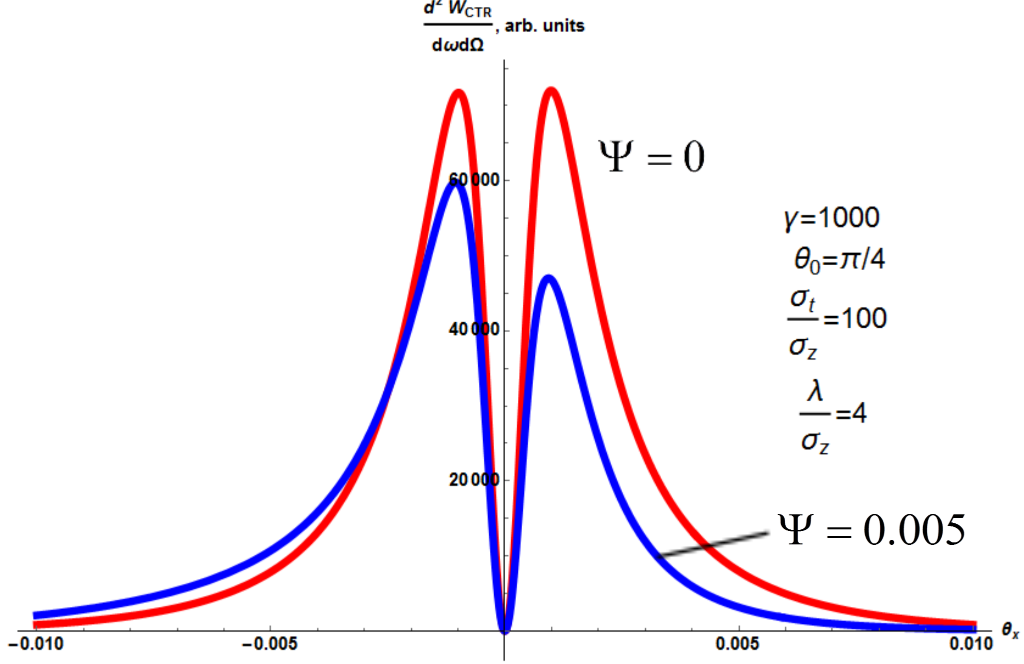
$(\gamma = 50)$

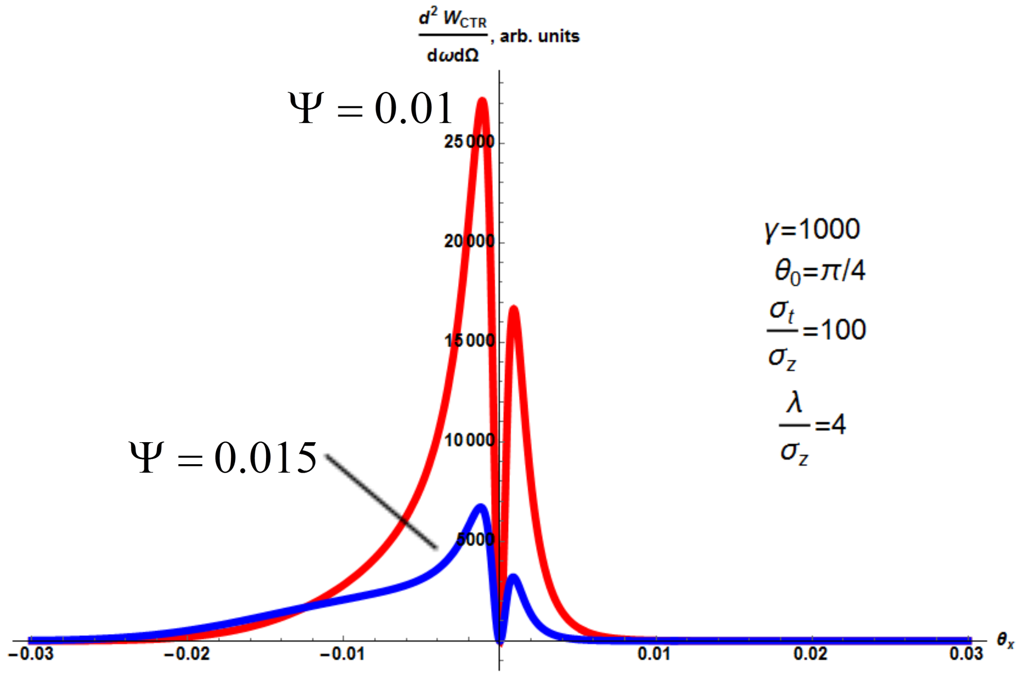


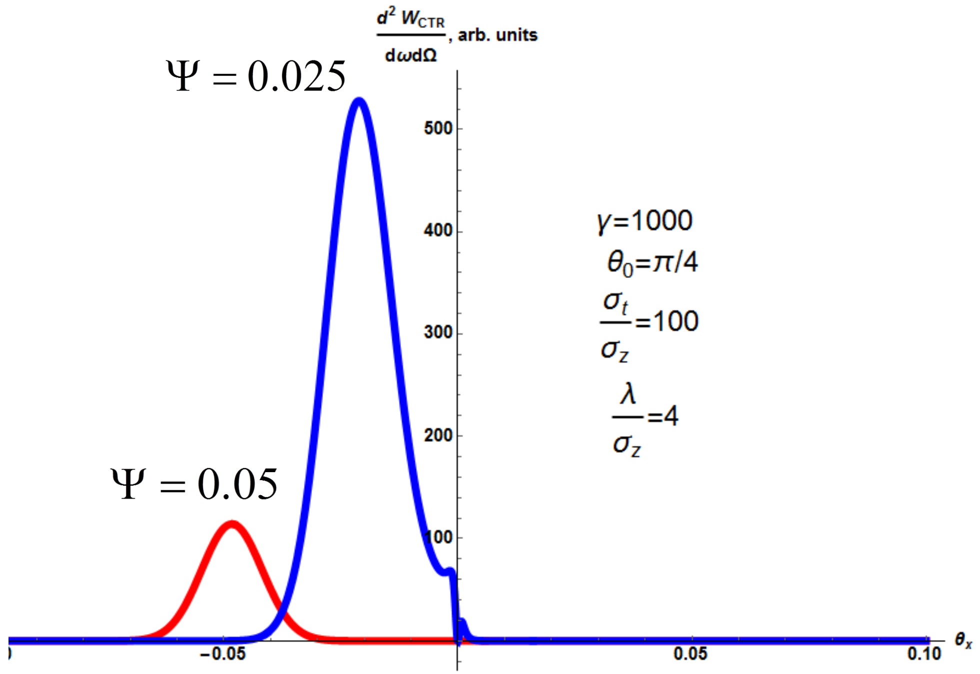




Ultrarelativistic case $(\gamma = 1000)$ distortion of the symmetrical angular BTR distribution

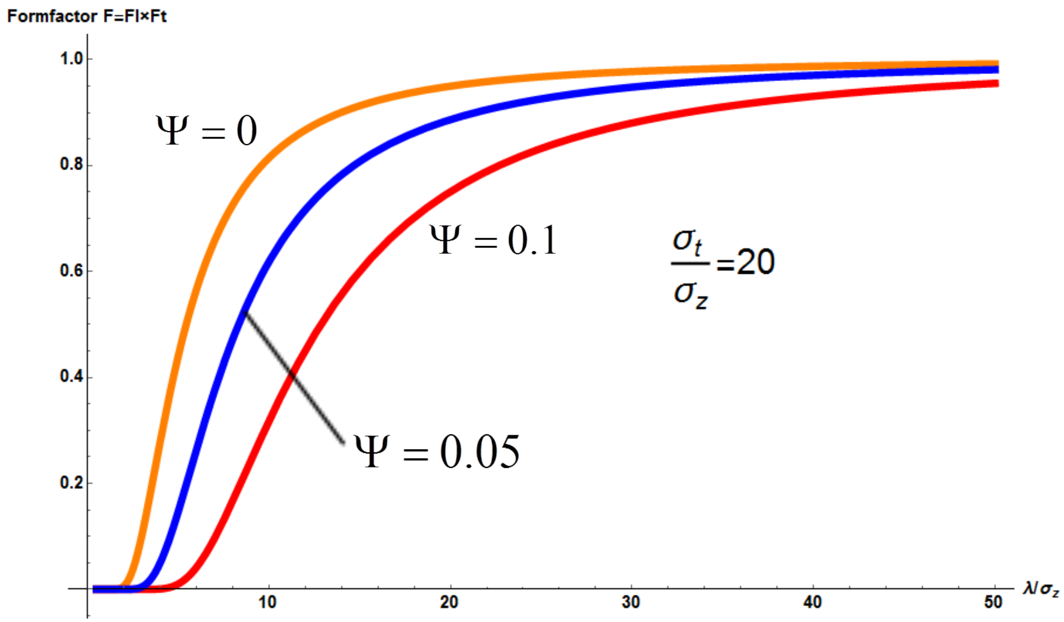


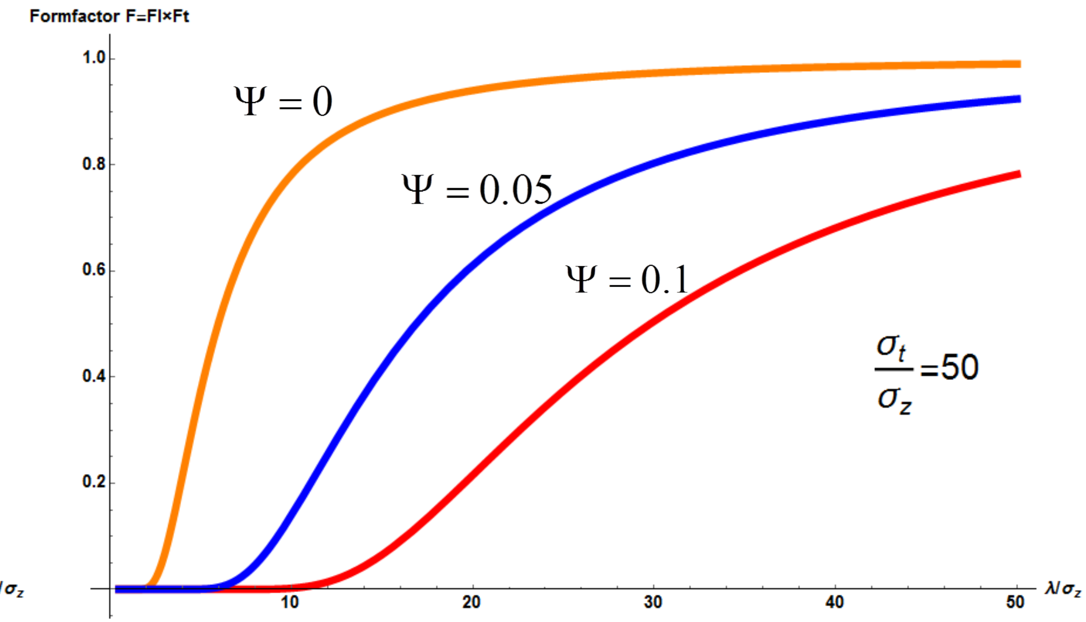




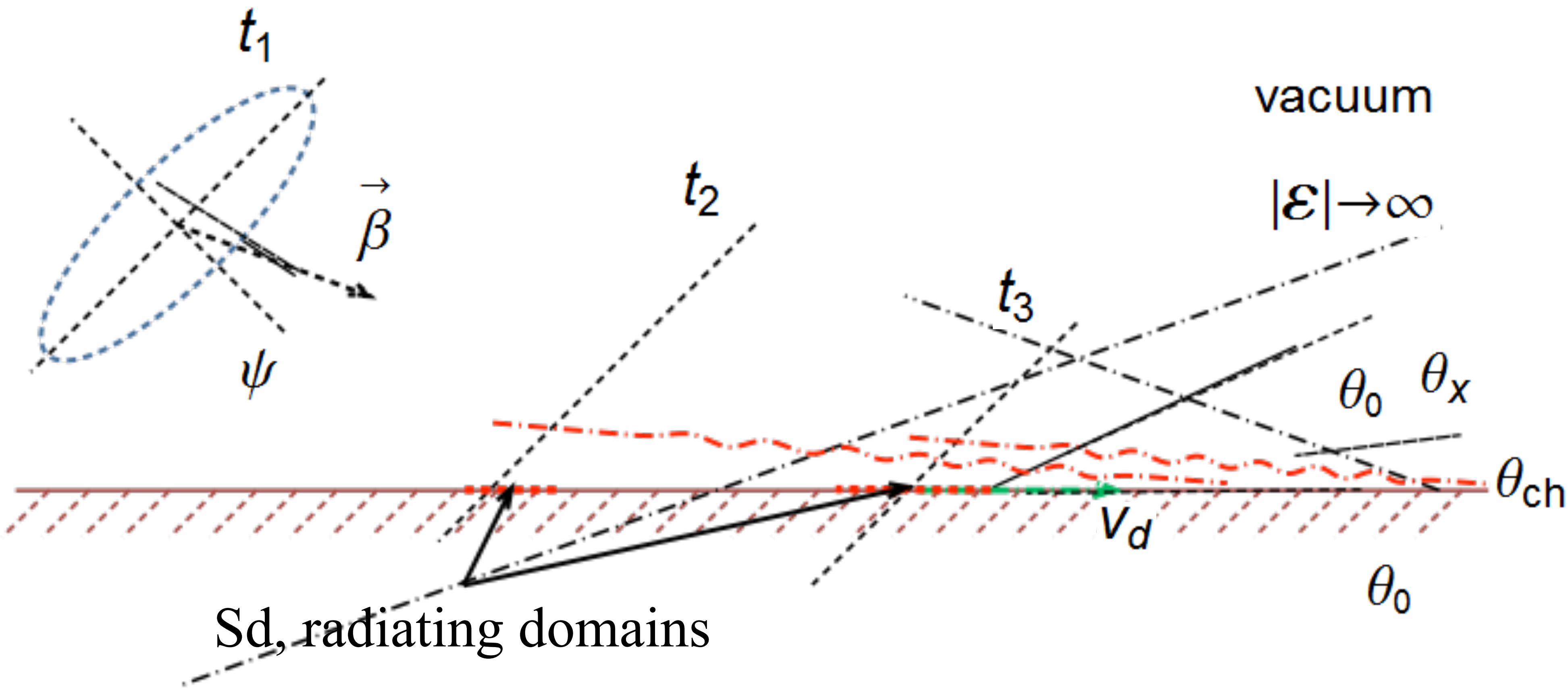
In the ultrarelativistic case the conventional “lobe-shape” distribution of BTR transforms into a single-mode one only for tilt angles $\Psi \gg \gamma^{-1}$ (for instance, when $\Psi \sim 10\gamma^{-1}$)

A dependence of “full” formfactor on wavelength is determined by tilt angle and ratio σ_t / σ_z





Coherent BTR as radiation produced by superluminal motion of the radiating domain



A velocity of the radiating domain:

$$v_d = \beta c \, \frac{\cos\psi}{\cos(\theta_0 - \psi)}$$

Cherenkov condition:

$$v_d > c \qquad \textbf{or} \qquad \cos\theta_{ch} = \frac{c}{v_d} = \frac{\cos(\theta_0 - \psi)}{\beta\cos\psi}$$

For $\psi << 1$ and $\beta \to 1$

$$\cos\theta_{ch} \approx \cos(\theta_0 - \psi), \qquad \theta_{ch} \approx \theta_0 - \psi$$

Passing to the variable θ_x $\longrightarrow$ $\theta_x = \theta_{ch} - \theta_0 = -\psi$

Just at this angle there is occurred the single intensity maximum of BTR

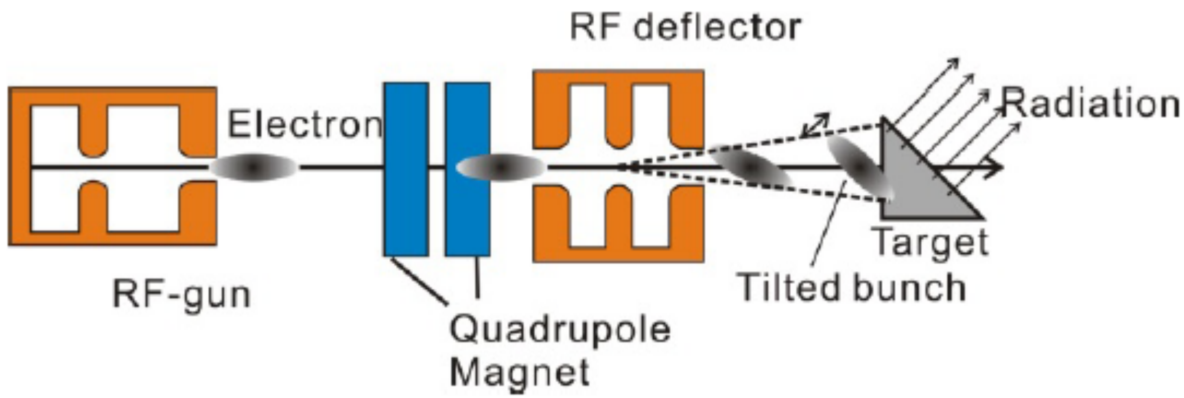
For perpendicular bunch $(\psi = 0)$

$$v_d = \frac{\beta c}{\cos\theta_0} > c \qquad \Rightarrow \qquad \beta > \cos\theta_0 \qquad \Rightarrow \qquad \frac{1}{\gamma} > \theta_0$$

Such an effect can be observed only for moderately relativistic beams.

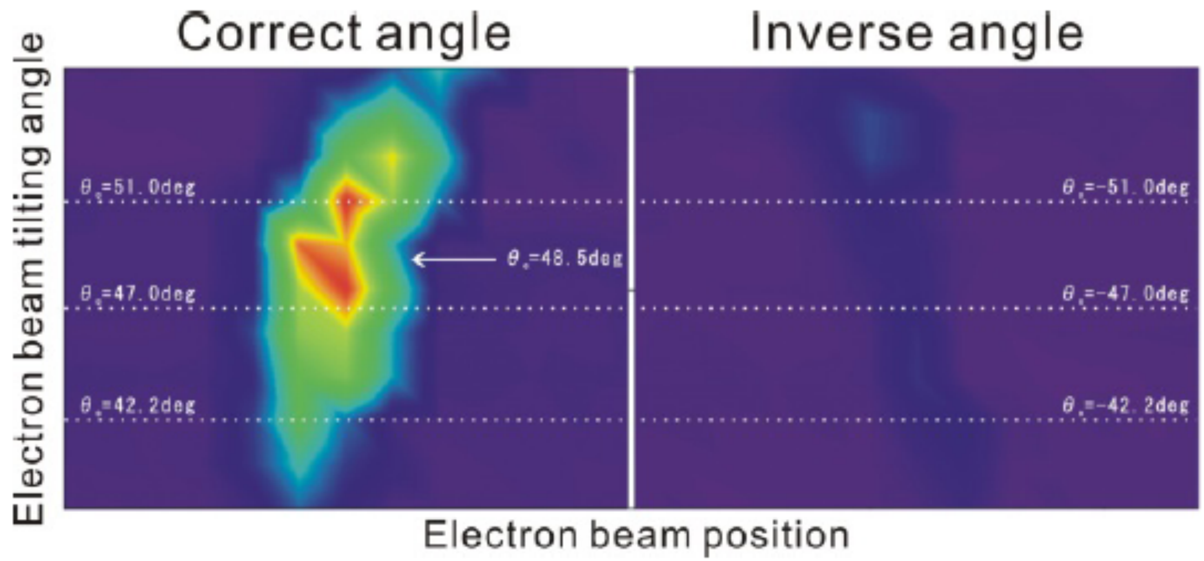
See, for instance, B.M. Bolotovsii et al. Physics Uspekhi 48 (9) 903-915 (2005)

5 Mev ($\gamma \approx 10$)	electron energy
quasi-optical	detector
0.1÷2 THz	viewed a range of frequency
1.52	TOPAS refractive index
48.5 deg	Cherenkov angle
width 1 mm	Prizm target size
thickness 1 mm	



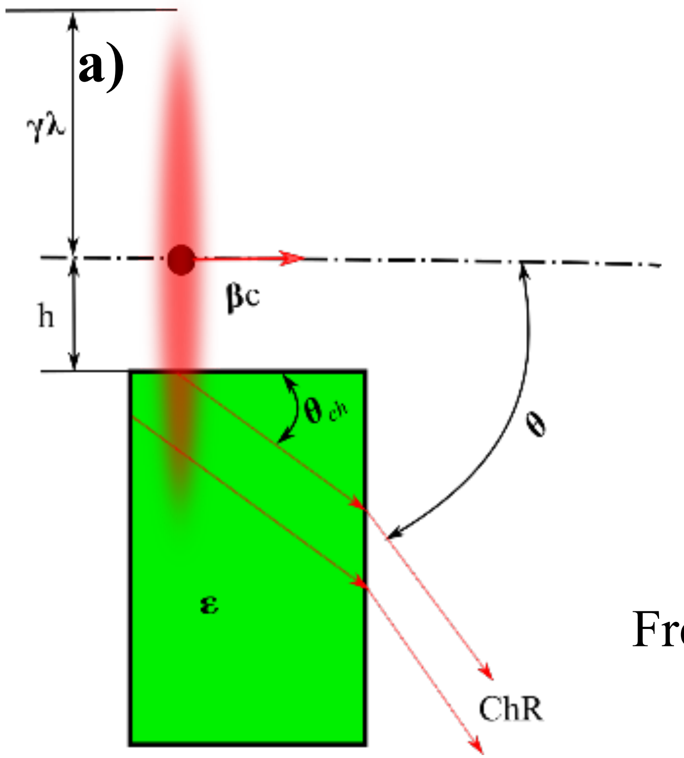
Experimental setup for coherent Cherenkov radiation by using tilted electron bunch.

THz intensity with 1 THz as a function of the electron bunch position and electron bunch tilting angle →



The Cherenkov mechanism may be realized for charge passing in vacuum near a dielectric target.

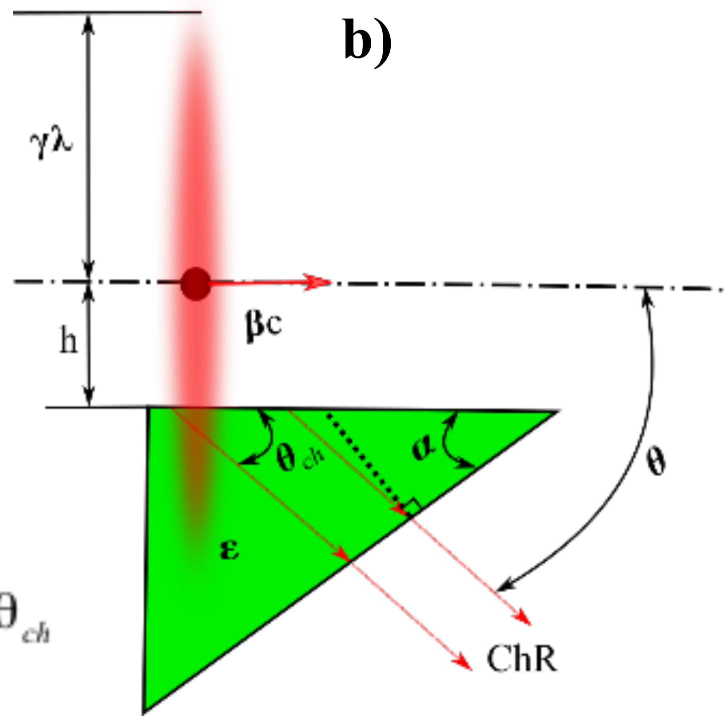
For the high Lorentz-factor γ and if the condition $h \leq \gamma\lambda$ is fulfilled (h is the impact parameter, λ is the ChR wavelength) ChR can be produced.



$$\cos\theta_{ch} = \frac{1}{\beta\sqrt{\varepsilon}}$$

$$\theta_{ch} = \arccos\left(\frac{1}{\beta\sqrt{\varepsilon}}\right)$$

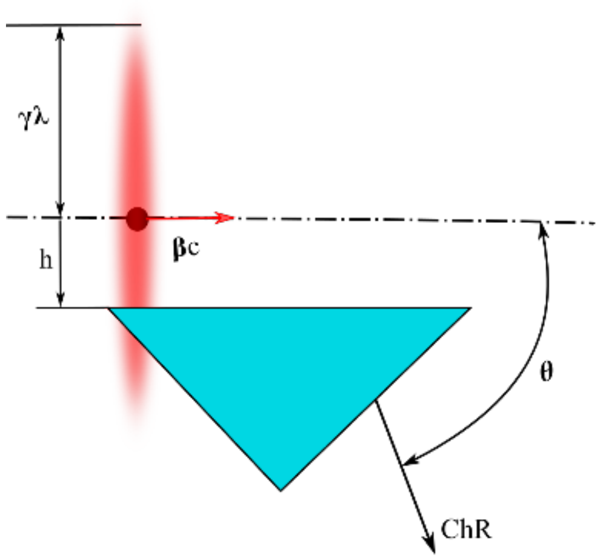
Fresnel's law $\sin\theta = \sqrt{\varepsilon}\sin\theta_{ch}$

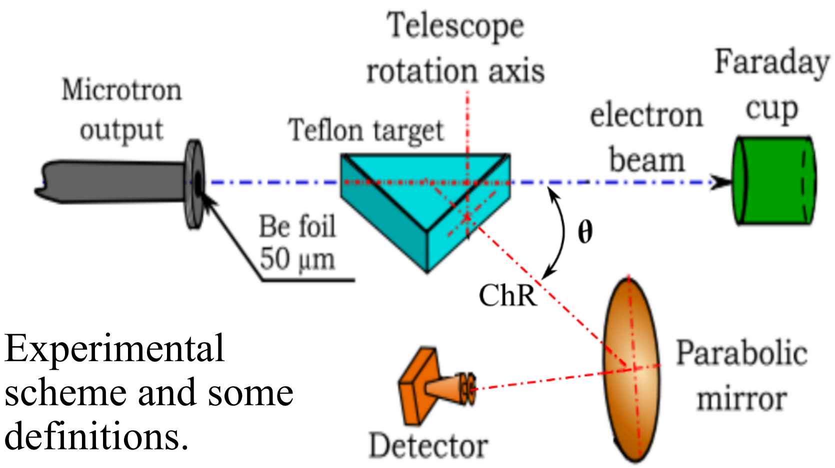


$$\theta = \arcsin\left(\sqrt{\varepsilon}\sin(\theta_{ch})\right)$$

$$\theta = 90^{\circ} - \alpha - \arcsin\left(\sqrt{\varepsilon}\cos(\alpha + \theta_{ch})\right)$$

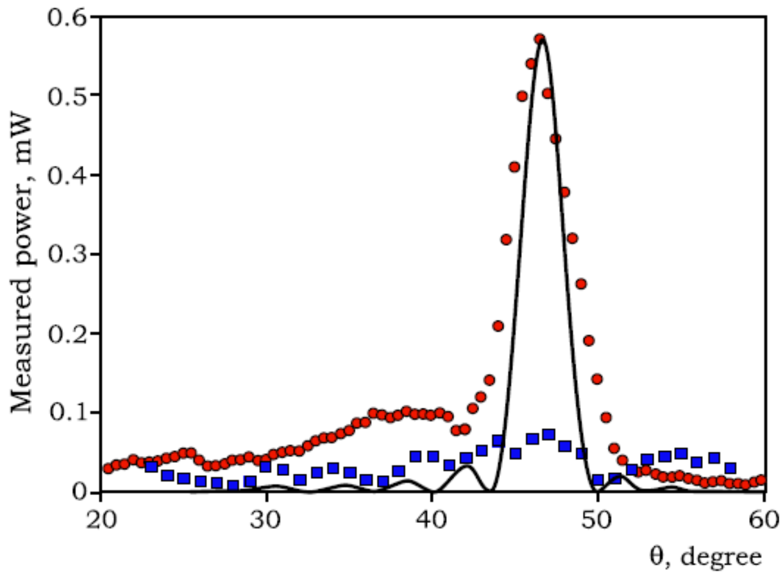
If $\varepsilon < 2 - \gamma^{-2}$ then the geometry a) can be realized





Experimental scheme and some definitions.

6.1 Mev (γ≈12)	**electron energy**
30 mA	average beam current
108	maximal bunch population
10526	Bunches in a train
1.1 mm	longitudinal size (rms) of electrons
4×4 mm2	transverse sizes of electron beam
4 μs	train duration
25 mm	impact-parameter
170 mm	Diameter of paraboloidal mirror
151 mm	focal distance
DP-21M	detector
1÷17 mm	viewed a range of wavelengths
1.45	Teflon refractive index



The angular dependence of CChR:
the red circles – horizontal polarization component,
the blue squares – vertical polarization component,
the solid line – theoretical simulation.

Polarization current model (D.V. Karlovets, A.P. Potylitsyn) PLA

$$\frac{d^2W_{ChR}}{d\omega\, d\Omega}=\frac{e^2\beta^2\cos^2(\theta)}{4\pi^2 c\left(1+\gamma^2\sin^2(\theta)\sin^2(\phi)\beta^2\right)}\left|\frac{(\varepsilon-1)\left(e^{\frac{i2\pi L\left(1-\beta\sqrt{\varepsilon-\sin^2(\theta)}\right)}{\lambda\beta}}-1\right)}{\varepsilon\left(1-\beta\sqrt{\varepsilon-\sin^2(\theta)}\right)}\right|^2$$

$$\Bigg[\left|\frac{\varepsilon}{\varepsilon\cos(\theta)+\sqrt{\varepsilon-\sin^2(\theta)}}\right|^2\left|\left(\Phi_2 i\gamma\cos(\phi)\sqrt{\varepsilon-\sin^2(\theta)}\sqrt{1+\gamma^2\sin^2(\theta)\sin^2(\phi)\beta^2}+\Phi_1\left(\sin(\theta)\cos^2(\phi)+\sin(\theta)\sin^2(\phi)\gamma^2\left(1-\beta^2-\beta\sqrt{\varepsilon-\sin^2(\theta)}\right)\right)\right)^2\right|+\left|\frac{\sqrt{\varepsilon}}{\varepsilon\cos(\theta)+\sqrt{\varepsilon-\sin^2(\theta)}}\right|^2$$

$$\Bigg|\sin^2(\theta)\sin^2(\phi)\gamma^2\left(\Phi_1\gamma\sin(\theta)\cos(\phi)\beta+i\Phi_2\sqrt{1+\gamma^2\sin^2(\theta)\sin^2(\phi)\beta^2}\right)^2+\Bigg(\sin(\phi)\left(\Phi_1\sin(\theta)\cos(\phi)+i\Phi_2\gamma\sqrt{\varepsilon-\sin^2(\theta)}\sqrt{1+\gamma^2\sin^2(\theta)\sin^2(\phi)\beta^2}\right)-$$

$$-\Phi_1\cos(\phi)\sin(\theta)\sin(\phi)\gamma^2\left(1-\beta^2-\beta\sqrt{\varepsilon-\sin^2(\theta)}\right)\Bigg)^2\Bigg|\Bigg] \qquad (11)$$

$$\Phi_{1,2}=\frac{\exp\left(-\frac{a\pi\left(-i\gamma\sin(\theta)\cos(\phi)\beta+\sqrt{1+\gamma^2\sin^2(\theta)\sin^2(\phi)\beta^2}\right)}{\gamma\lambda\beta}\right)\left(1-\exp\left(-\frac{2\pi H\left(-i\gamma\sin(\theta)\cos(\phi)\beta+\sqrt{1+\gamma^2\sin^2(\theta)\sin^2(\phi)\beta^2}\right)}{\gamma\lambda\beta}\right)\right)}{\left(-i\gamma\sqrt{\grave{o}}ny\beta+\sqrt{1+\gamma^2\sin^2(\theta)\sin^2(\phi)\beta^2}\right)}\pm$$

$$\pm\frac{\exp\left(-\frac{a\pi\left(i\gamma\sin(\theta)\cos(\phi)\beta+\sqrt{1+\gamma^2\sin^2(\theta)\sin^2(\phi)\beta^2}\right)}{\gamma\lambda\beta}\right)\left(1-\exp\left(-\frac{2\pi H\left(i\gamma\sin(\theta)\cos(\phi)\beta+\sqrt{1+\gamma^2\sin^2(\theta)\sin^2(\phi)\beta^2}\right)}{\gamma\lambda\beta}\right)\right)}{\left(i\gamma\sin(\theta)\cos(\phi)\beta+\sqrt{1+\gamma^2\sin^2(\theta)\sin^2(\phi)\beta^2}\right)} \qquad (12)$$

V.E. Pafomov, JETP, 33, 1074 (1957) (Russian)

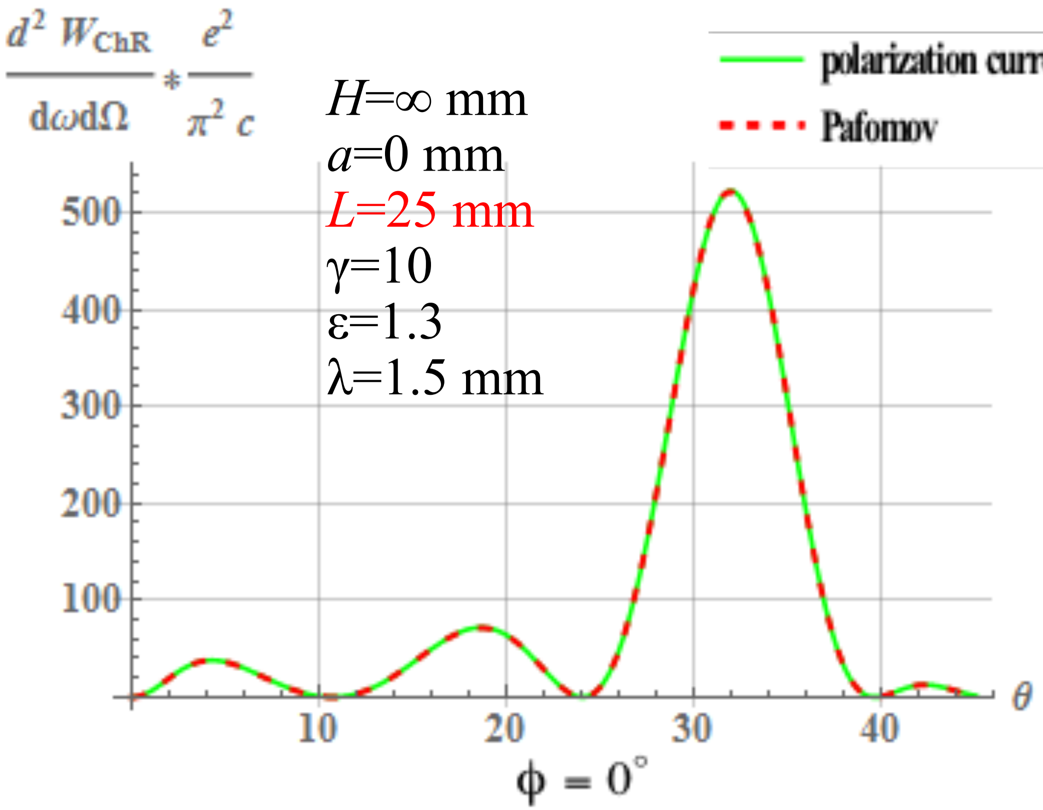
Spectral and angular distribution of the radiation, calculated using method of images

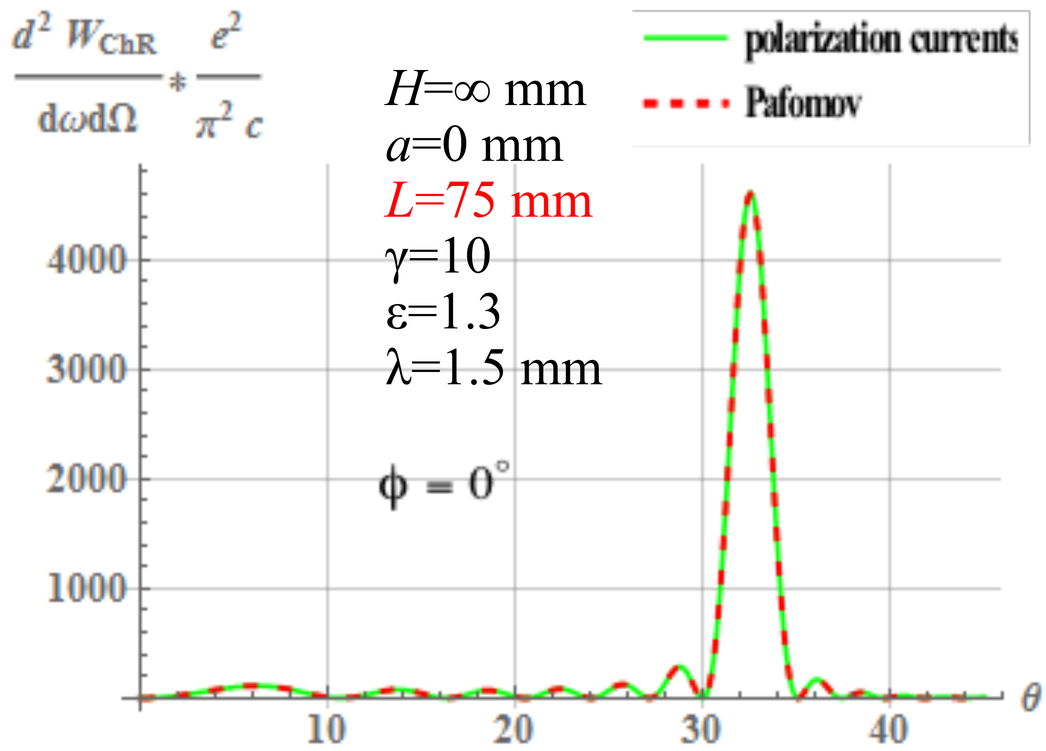
$$\frac{d^2W_{ChR}}{d\omega\, d\Omega}=\frac{e^2\beta^2\sin^2(\theta)\cos^2(\theta)\left|(\varepsilon-1)\right|^2}{\pi^2 c\left|\left(1-\beta^2\cos^2(\theta)\right)\left(1-\beta^2\left(\varepsilon-\sin^2(\theta)\right)\right)\right|^2}$$

$$\left|\frac{e^{-\frac{i2\pi L\sqrt{\varepsilon-\sin^2(\theta)}}{\lambda}}\left(\sqrt{\varepsilon-\sin^2(\theta)}-\varepsilon\cos(\theta)\right)\left(1-\beta\sqrt{\varepsilon-\sin^2(\theta)}\right)\left(1-\beta^2+\beta\sqrt{\varepsilon-\sin^2(\theta)}\right)}{\left(e^{\frac{i2\pi L\sqrt{\varepsilon-\sin^2(\theta)}}{\lambda}}\left(\sqrt{\varepsilon-\sin^2(\theta)}+\varepsilon\cos(\theta)\right)-e^{-\frac{i2\pi L\sqrt{\varepsilon-\sin^2(\theta)}}{\lambda}}\left(\sqrt{\varepsilon-\sin^2(\theta)}-\varepsilon\cos(\theta)\right)\right)}+\frac{e^{\frac{i2\pi L\sqrt{\varepsilon-\sin^2(\theta)}}{\lambda}}\left(\sqrt{\varepsilon-\sin^2(\theta)}+\varepsilon\cos(\theta)\right)\left(1+\beta\sqrt{\varepsilon-\sin^2(\theta)}\right)\left(1-\beta^2-\beta\sqrt{\varepsilon-\sin^2(\theta)}\right)}{\left(e^{\frac{i2\pi L\sqrt{\varepsilon-\sin^2(\theta)}}{\lambda}}\left(\sqrt{\varepsilon-\sin^2(\theta)}+\varepsilon\cos(\theta)\right)-e^{-\frac{i2\pi L\sqrt{\varepsilon-\sin^2(\theta)}}{\lambda}}\left(\sqrt{\varepsilon-\sin^2(\theta)}-\varepsilon\cos(\theta)\right)\right)}-\right.$$

$$\left.-\frac{2e^{\frac{i2\pi L}{\lambda\beta}}\sqrt{\varepsilon-\sin^2(\theta)}\left(1+\beta\cos(\theta)\right)\left(1-\beta\cos(\theta)-\beta^2\varepsilon\right)}{\left(e^{\frac{i2\pi L\sqrt{\varepsilon-\sin^2(\theta)}}{\lambda}}\left(\sqrt{\varepsilon-\sin^2(\theta)}+\varepsilon\cos(\theta)\right)-e^{-\frac{i2\pi L\sqrt{\varepsilon-\sin^2(\theta)}}{\lambda}}\left(\sqrt{\varepsilon-\sin^2(\theta)}-\varepsilon\cos(\theta)\right)\right)}\right|^2 \qquad (13)$$

Comparison of both models

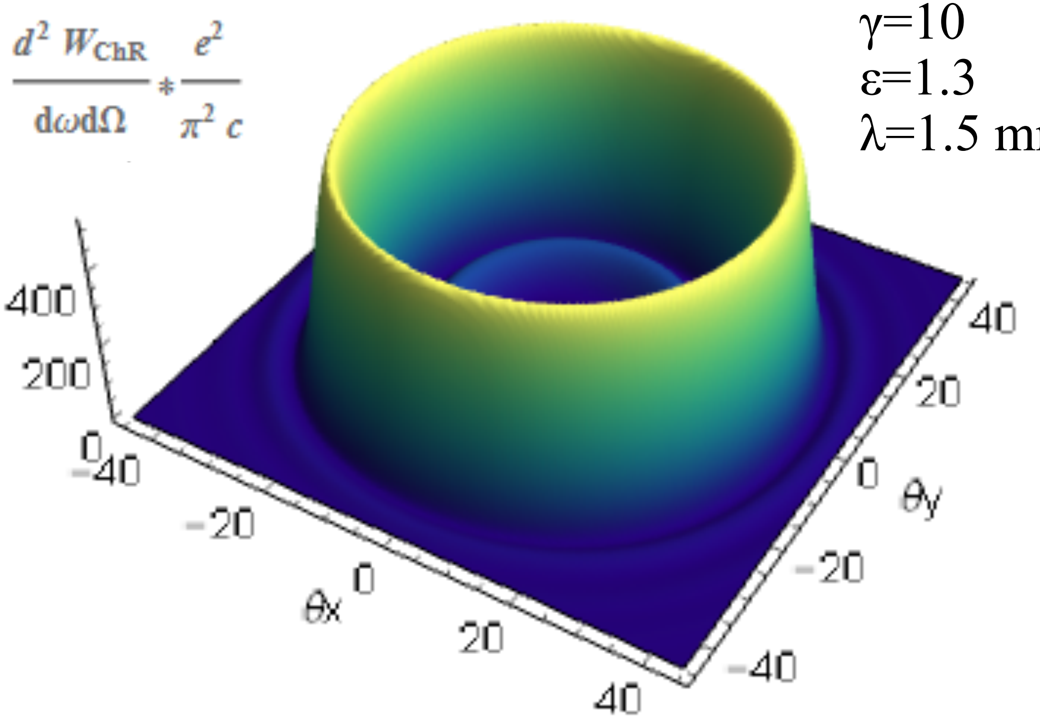


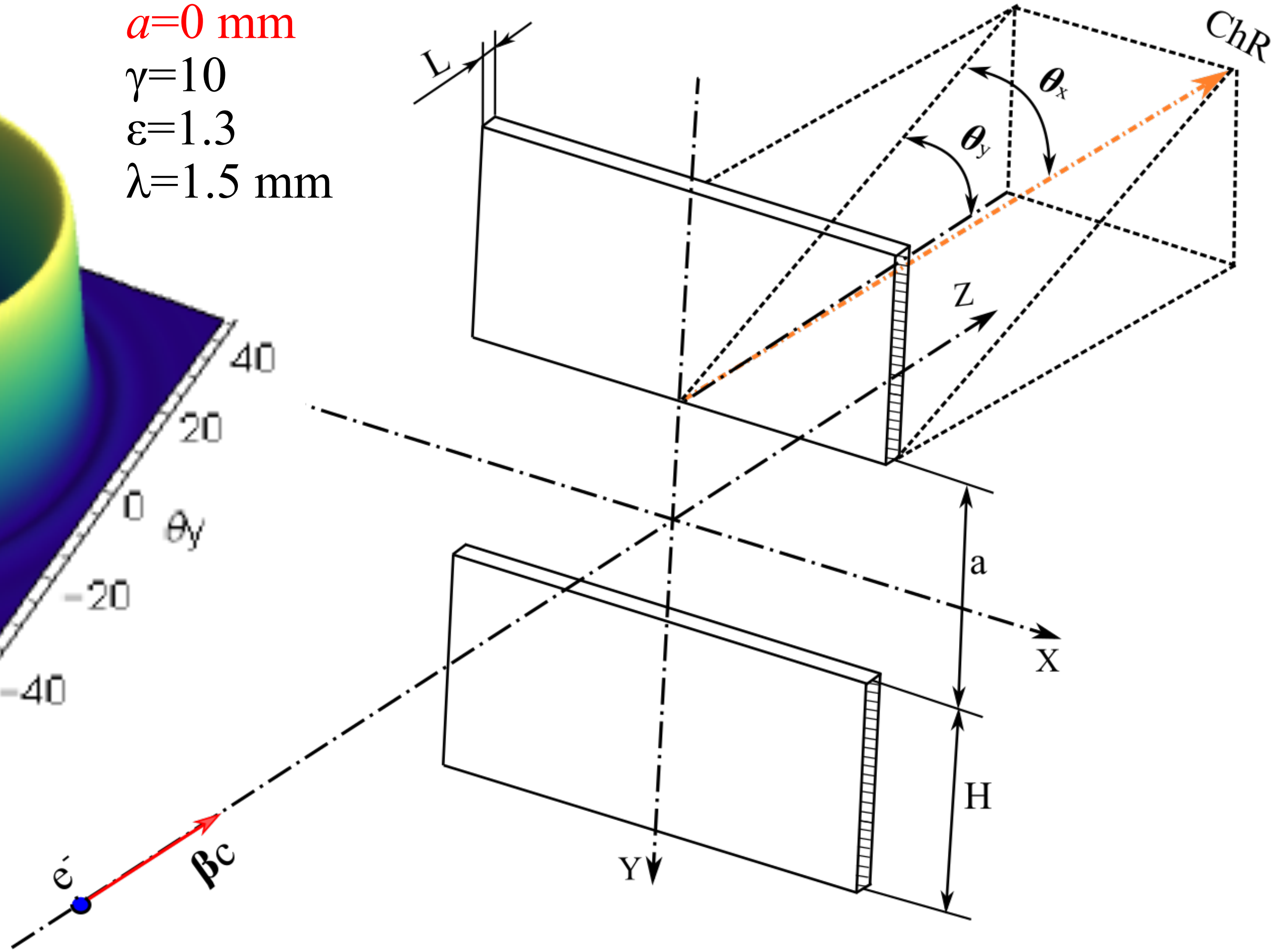


Angular distribution of incoherent ChR

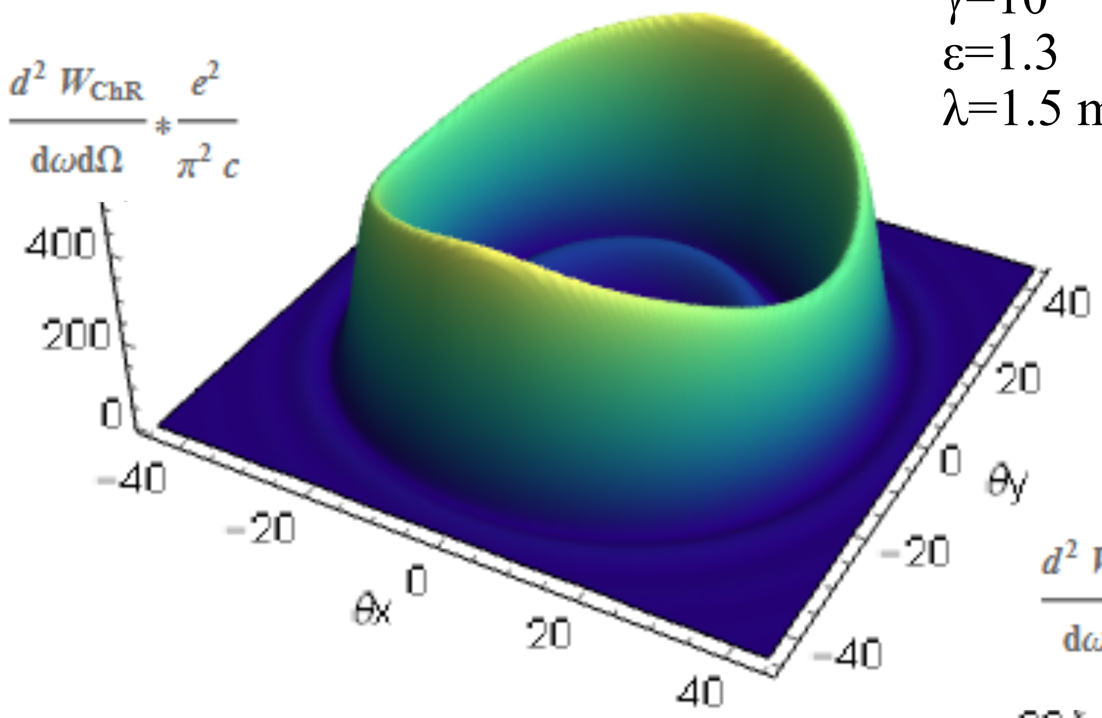
$$\vec{k} = \frac{2\pi}{\lambda}\left\{\sin(\theta_x), \cos(\theta_x)\sin(\theta_y), \sqrt{\varepsilon - \left(1 - \cos^2(\theta_x)\cos^2(\theta_y)\right)}\right\}$$

H=50 mm
L=25 mm
a=0 mm
γ=10
ε=1.3
λ=1.5 mm

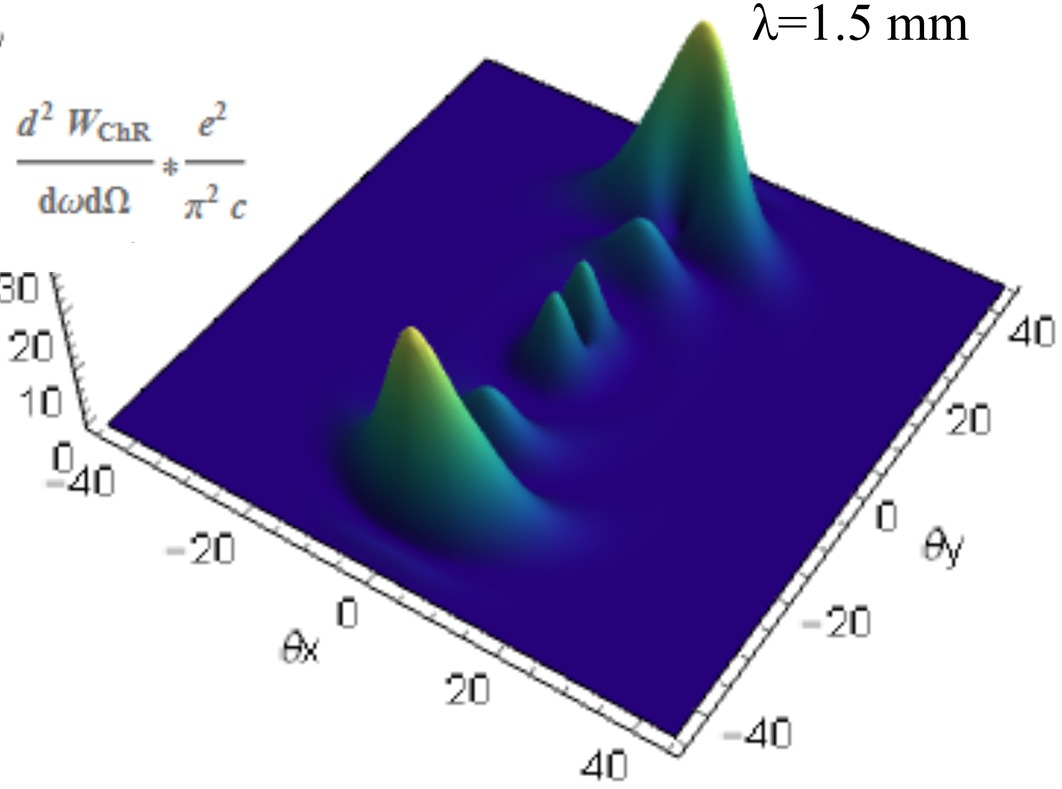




H=50 mm
L=25 mm
a=0.1 mm
γ=10
ε=1.3
λ=1.5 mm



H=50 mm
L=25 mm
a=5 mm
γ=10
ε=1.3
λ=1.5 mm

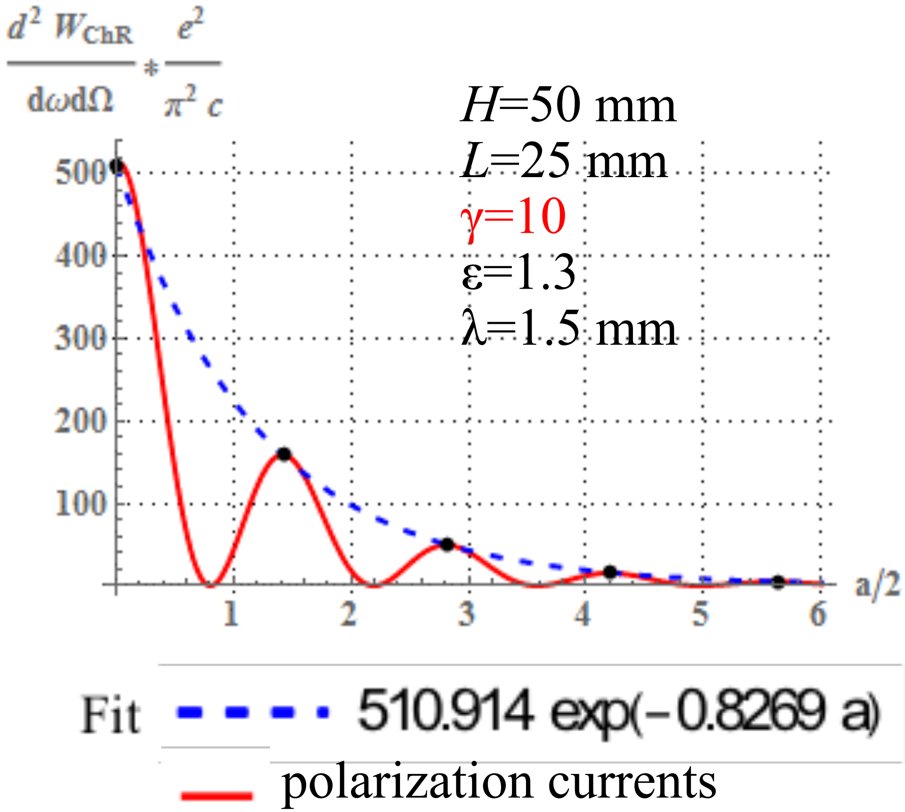


Dependence on impact parameter

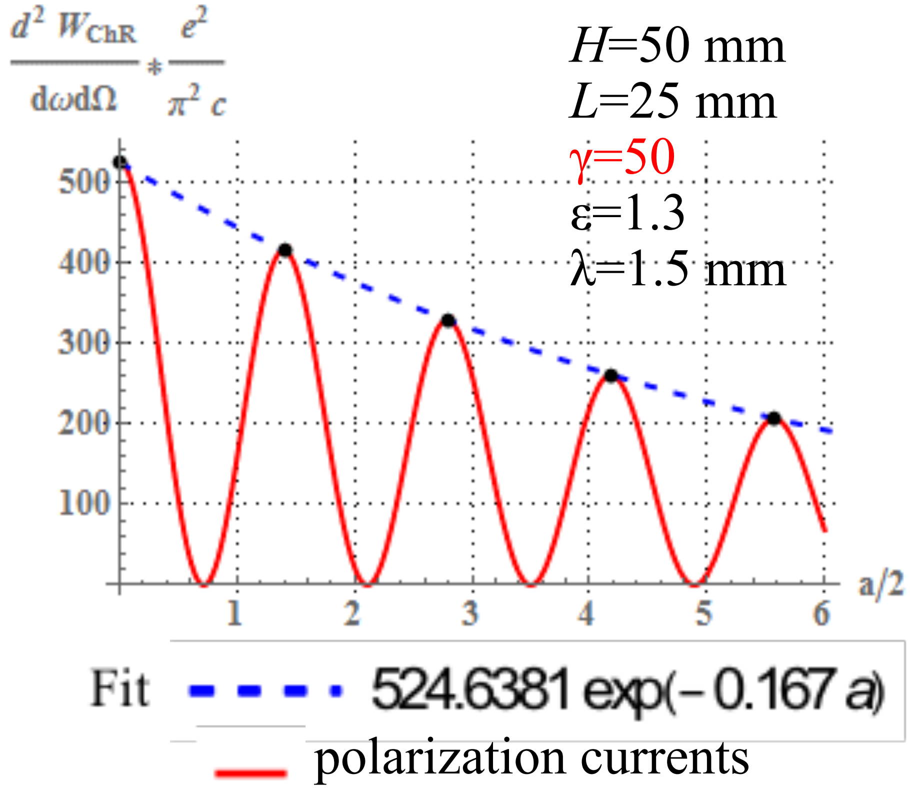
see, Norihiro Sei, Takeshi Sakai, et. al., Physics Letters A, 379 (2015) 2399–2404

can be approximated as $\exp\left(\frac{-4\pi a}{\gamma\lambda\beta}\right)$

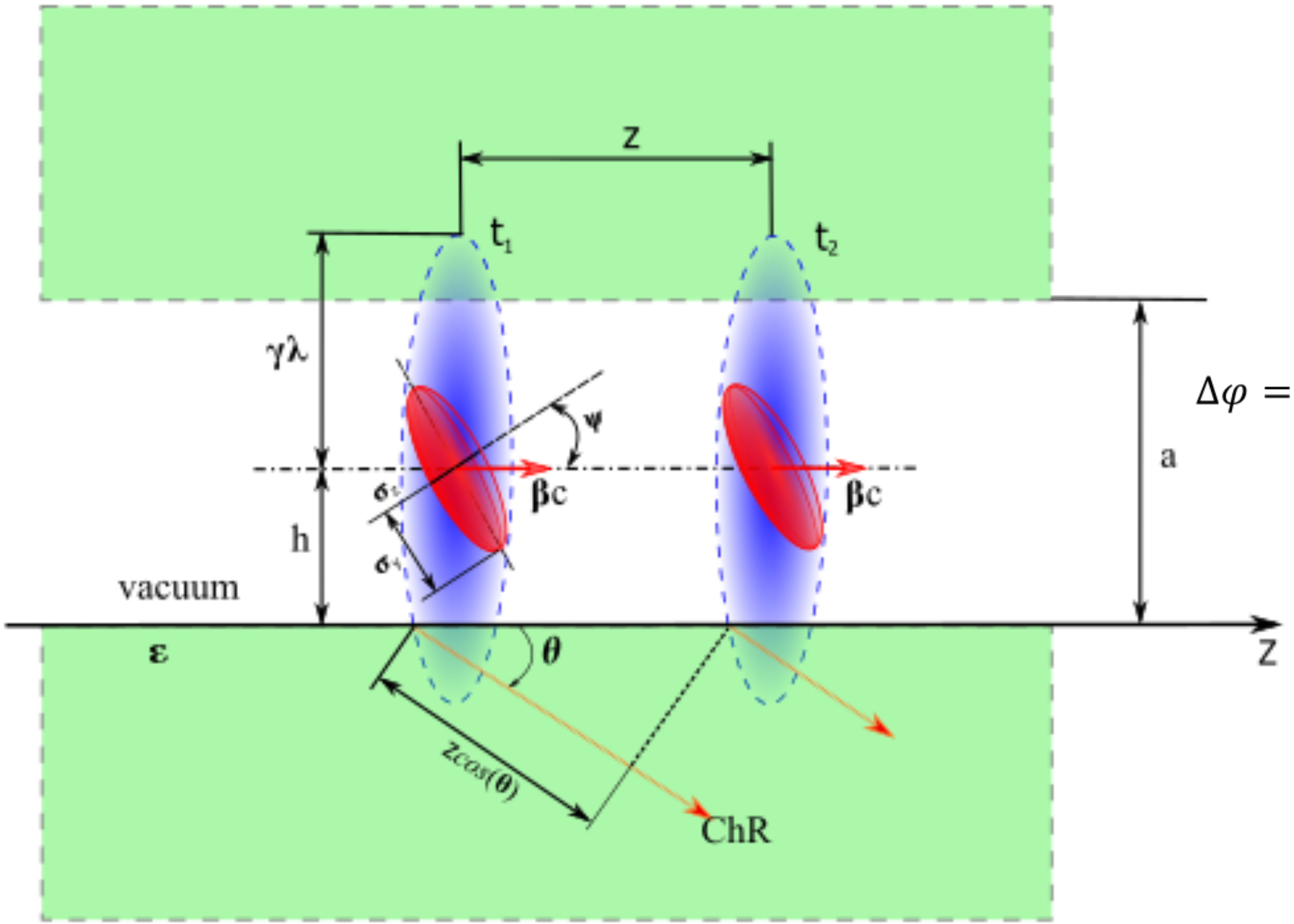
$$\exp\left(\frac{-4\pi a}{\gamma\lambda\beta}\right)=\exp\left(-\frac{4\pi a}{10\cdot 1.5\cdot 0.9949}\right)=\boxed{\exp(-0.8419\,a)}$$



$$\exp\left(\frac{-4\pi a}{\gamma\lambda\beta}\right)=\exp\left(-\frac{4\pi a}{50\cdot 1.5\cdot 0.9998}\right)=\boxed{\exp(-0.1675\,a)}$$



Form factor a tilted electron bunch with distribution of electrons in the bunch is a Gaussian:



$$\Delta\varphi = \frac{2\pi}{\lambda}\left(x\sin(\theta)\sin(\phi) + \right.$$
$$\left. + y\sin(\theta)\cos(\phi) + \frac{z}{\beta}\right)$$

Coherent Cherenkov radiation

The spectral-angular density of Coherent Cherenkov radiation (CChR) from a bunch with population N:

$$\frac{d^2W_{CChR}}{d\omega\, d\Omega} = N\left(1+(N-1)F(\vec{k})\right)\frac{d^2W_{ChR}}{d\omega\, d\Omega},$$

$$\text{Form factor} \quad F(\vec{k}) = \left|\int_{-\infty}^{\infty} \rho(\vec{r})\exp\left(-i\left[\frac{\omega}{c}sin(\theta)\sin(\phi)x + \frac{\omega}{c}sin(\theta)\cos(\phi)y + \frac{\omega}{c\beta}z\right]\right)d\vec{r}\right|^2,$$

$\rho(\vec{r})$ is a charge distribution of electrons in the bunch is a Gaussian,

$$\rho(\vec{r}) = \rho(x,y,z) = \frac{1}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z}\exp\left(-\frac{1}{2}\left[\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z}{\sigma_z}\right)^2\right]\right). \qquad (*)$$

$\frac{d^2W_{ChR}}{d\omega d\Omega}$ is thespectral-angular density for a single charge. $$\frac{d^2W_{ChR}}{d\omega\, d\Omega} = cr'^2\,|\vec{E}_{vac}(\vec{r},\omega)|^2 \quad (1)$$

$$\overline{E}_{vac}(\vec{r},\omega) = \frac{1}{|\varepsilon|^2}\left(\left|\sqrt{\varepsilon}F_E\right|^2\left|H_{\parallel}^{R}(\vec{r},\omega)\right|^2 + \left|F_H\right|^2\left|H_{\perp}^{R}(\vec{r},\omega)\right|^2\right) \quad (2)$$

$$H_{\parallel}^{R}(\vec{r},\omega) = \sqrt{H_z^R(\vec{r},\omega)^2 + \left(H_x^R(\vec{r},\omega)\sin(\phi) + H_y^R(\vec{r},\omega)\cos(\phi)\right)^2} \quad (3)$$

$$H_{\perp}^{R}(\vec{r},\omega) = H_x^R(\vec{r},\omega)\cos(\phi) - H_y^R(\vec{r},\omega)\sin(\phi) \quad (4)$$

$$F_H = \frac{2\varepsilon\cos\theta}{\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta}} \quad (5)$$

$$F_E = \frac{2\cos\theta}{\cos\theta + \sqrt{\varepsilon - \sin^2\theta}} \quad (6)$$

$$\vec{H}^{R}(k_x, y, z, \omega) = \frac{(\varepsilon - 1)\omega}{2c} \frac{e^{i\frac{\omega}{c} r' \sqrt{\varepsilon}}}{r'} \vec{k} \times \int_0^L \left(\int_{a/2}^{a/2+H} \vec{E}^{e}(k_x, y, z, \omega) e^{-i(k_y y + k_z z)} dy + \int_{-a/2-H}^{-a/2} \vec{E}^{*e}(k_x, y, z, \omega) e^{-i(k_y y + k_z z)} dy \right) dz \quad (7)$$

$$\boxed{\vec{E}^{*e}(k_x, y, z, \omega) = -\frac{ie}{2\pi\beta c\sqrt{1+\varepsilon(\beta\gamma n_x)^2}} \left\{ \sqrt{\varepsilon}\,\beta\gamma n_x, -i\sqrt{1+\varepsilon(\beta\gamma n_x)^2}, \gamma^{-1} \right\} e^{i\frac{\omega}{\beta c} z} e^{y\frac{\omega}{\beta c\gamma}\sqrt{1+\varepsilon(\beta\gamma n_x)^2}} \quad (8)}$$

$$\boxed{\vec{E}^{e}(k_x, y, z, \omega) = -\frac{ie}{2\pi\beta c\sqrt{1+\varepsilon(\beta\gamma n_x)^2}} \left\{ \sqrt{\varepsilon}\,\beta\gamma n_x, i\sqrt{1+\varepsilon(\beta\gamma n_x)^2}, \gamma^{-1} \right\} e^{i\frac{\omega}{\beta c} z} e^{-y\frac{\omega}{\beta c\gamma}\sqrt{1+\varepsilon(\beta\gamma n_x)^2}} \quad (9)}$$

$$\vec{k} = \vec{n}\sqrt{\varepsilon}\frac{\omega}{c}, \quad \vec{n} = \frac{1}{\sqrt{\varepsilon}} \left\{ sin(\theta)\sin(\phi), sin(\theta)\cos(\phi), \sqrt{\varepsilon - sin^2(\theta)} \right\} \quad (10)$$

Form factor for tilted electron bunch

$$F(\vec{k}) = \left| \int_{-\infty}^{\infty} \rho(\vec{r}) \exp\left(-i \left[\frac{\omega}{c} sin(\theta)\sin(\phi) x + \frac{\omega}{c} sin(\theta)\cos(\phi) y + \frac{\omega}{c\beta} z \right] \right) d\vec{r} \right|^2 ,$$

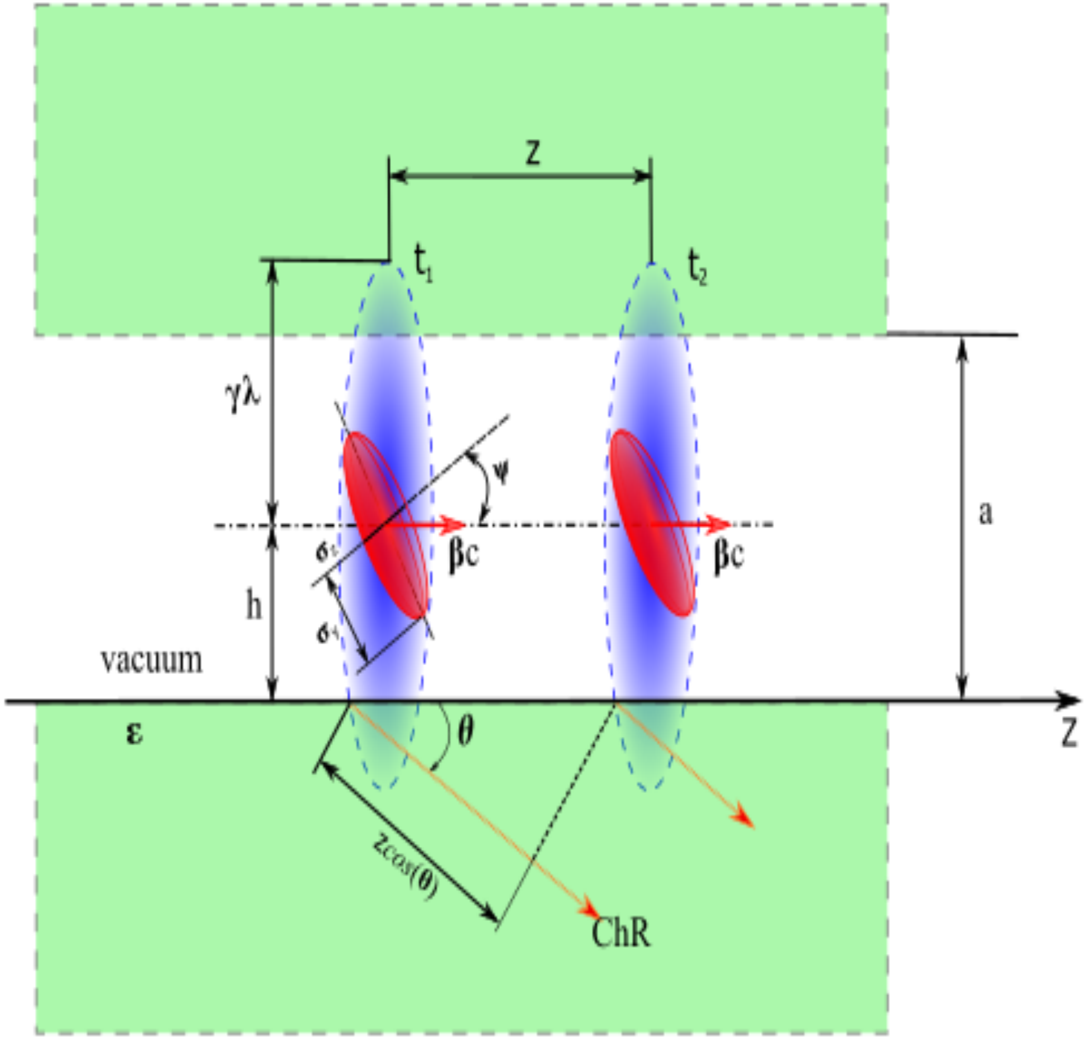
$$\rho(\vec{r}) = \frac{1}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} \exp\left(-\frac{1}{2} \left[\left(\frac{x}{\sigma_x} \right)^2 + \left(\frac{y\cos(\psi) - z\sin(\psi)}{\sigma_y} \right)^2 + \left(\frac{y\sin(\psi) + z\cos(\psi)}{\sigma_z} \right)^2 \right] \right). \quad (**)$$

$$F(\vec{k}) = \exp\left(-k_x^2 \sigma_x^2 + \frac{1}{2} \left[-\left(k_y^2 + k_z^2\right)\left(\sigma_y^2 + \sigma_z^2\right) - \left(k_y^2 - k_z^2\right)\left(\sigma_y^2 - \sigma_z^2\right)\cos(2\psi) + 2k_y k_z \left(\sigma_y^2 - \sigma_z^2\right)\sin(2\psi) \right] \right), \quad (14)$$

$$\left\{k_x, k_y, k_z\right\} = \frac{2\pi}{\lambda} \left\{ sin(\theta)\sin(\phi), sin(\theta)\cos(\phi), \frac{1}{\beta} \right\} \quad (15)$$

$$\left\{k_x, k_y, k_z\right\} = \frac{2\pi}{\lambda} \left\{ \sin(\theta_x), \cos(\theta_x)\sin(\theta_y), \frac{1}{\beta} \right\} \quad (16)$$

Form factor a tilted electron bunch with distribution of electrons in the bunch is a Gaussian:



$$\Delta\varphi = \frac{2\pi}{\lambda}\left(x\sin(\theta)\sin(\phi) + y\sin(\theta)\cos(\phi) + \frac{z}{\beta} \right)$$

Form factor for tilted electron bunch

$$F(\vec{k}) = \left| \int_{-\infty}^{\infty} \rho(\vec{r}) \exp\left(-i \left[\frac{\omega}{c} sin(\theta) \sin(\phi) x + \frac{\omega}{c} sin(\theta) \cos(\phi) y + \frac{\omega}{c\beta} z \right] \right) d\vec{r} \right|^2 ,$$

$$\rho(\vec{r}) = \frac{1}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} \exp\left(-\frac{1}{2} \left[\left(\frac{x}{\sigma_x} \right)^2 + \left(\frac{y\cos(\psi) - z\sin(\psi)}{\sigma_y} \right)^2 + \left(\frac{y\sin(\psi) + z\cos(\psi)}{\sigma_z} \right)^2 \right] \right). \quad (**)$$

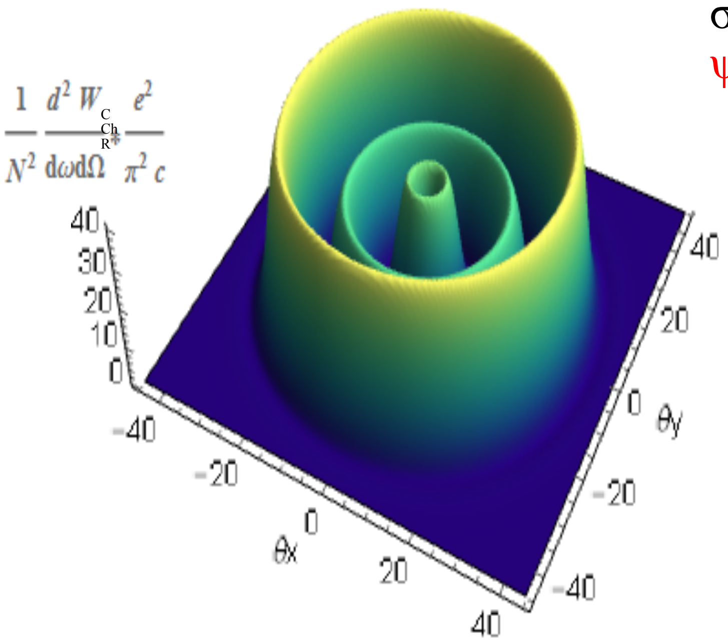
$$F(\vec{k}) = \exp\left(-k_x^2 \sigma_x^2 + \frac{1}{2} \left[-\left(k_y^2 + k_z^2\right)\left(\sigma_y^2 + \sigma_z^2\right) - \left(k_y^2 - k_z^2\right)\left(\sigma_y^2 - \sigma_z^2\right)\cos(2\psi) + 2k_y k_z \left(\sigma_y^2 - \sigma_z^2\right)\sin(2\psi) \right] \right), \quad (14)$$

$$\left\{k_x, k_y, k_z\right\} = \frac{2\pi}{\lambda} \left\{ sin(\theta)\sin(\phi), sin(\theta)\cos(\phi), \frac{1}{\beta} \right\} \quad (15)$$

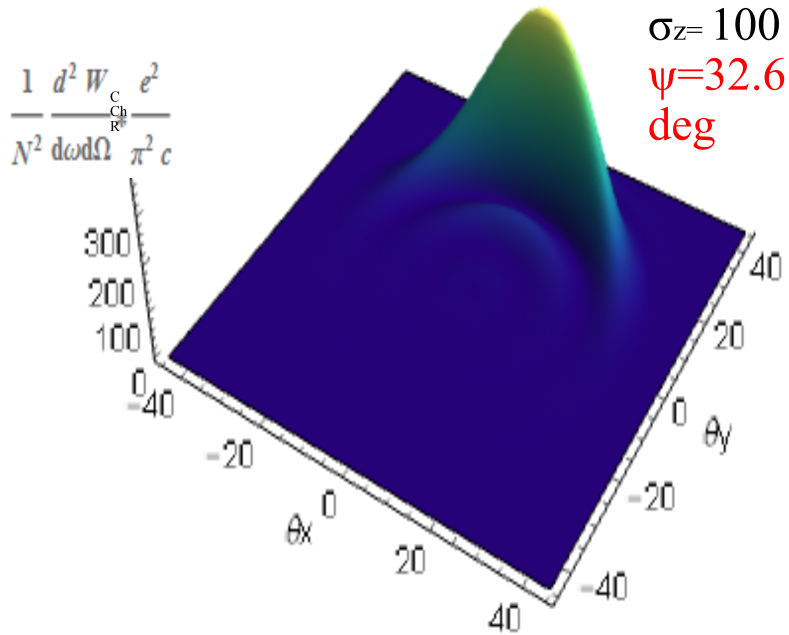
$$\left\{k_x, k_y, k_z\right\} = \frac{2\pi}{\lambda} \left\{ \sin(\theta_x), \cos(\theta_x)\sin(\theta_y), \frac{1}{\beta} \right\} \quad (16)$$

Angular distribution of coherent Cherenkov radiation from a tilted bunch passing through a target

H=50 mm
L=25 mm
a=0 mm
γ=10
ε=1.3
λ=1.5 mm
$\sigma_x = \sigma_y$=707 μm
σ_z= 100 μm
ψ=0 deg

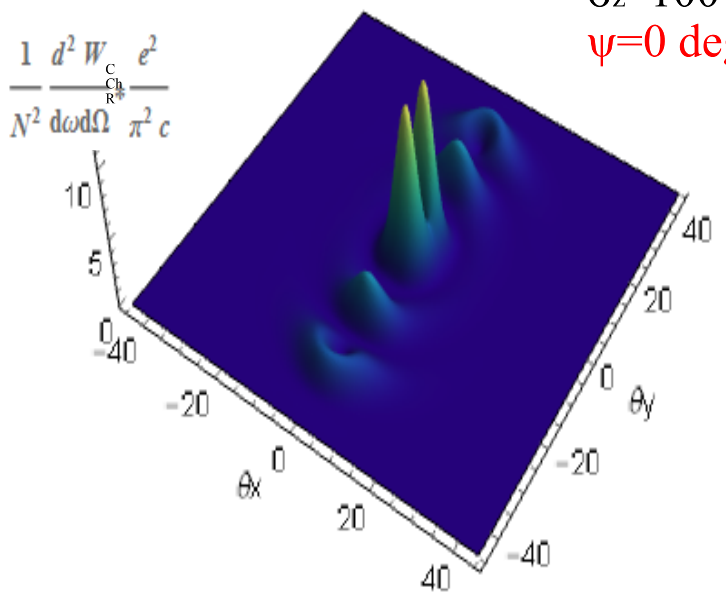


H=50 mm
L=25 mm
a=0 mm
γ=10
ε=1.3
λ=1.5 mm
$\sigma_x = \sigma_y$=707 μm
σ_z= 100 μm
ψ=32.6 deg

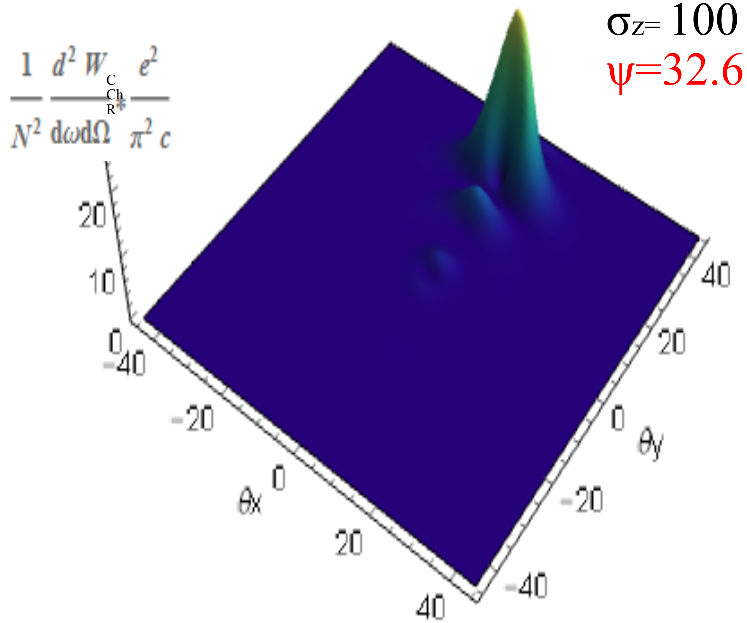


Angular distribution of coherent Cherenkov radiation from a tilted bunch passing through a slit in target

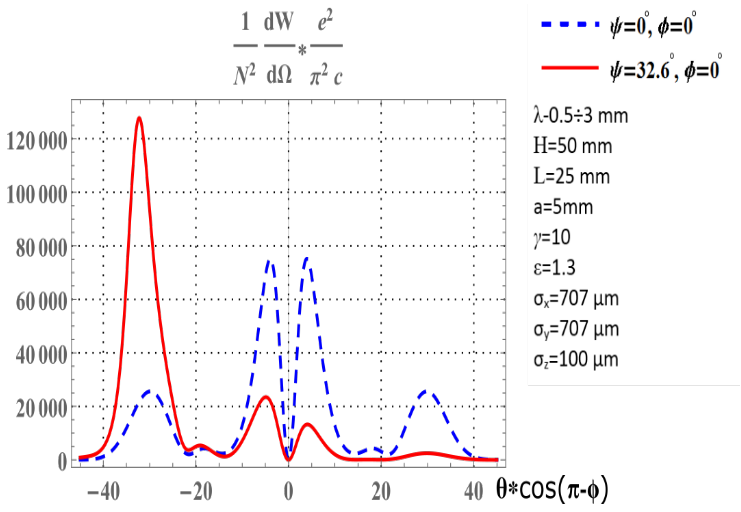
H=50 mm
L=25 mm
a=5 mm
γ=10
ε=1.3
λ=1.5 mm
$\sigma_x = \sigma_y$=707 µm
σ_z= 100 µm
ψ=0 deg

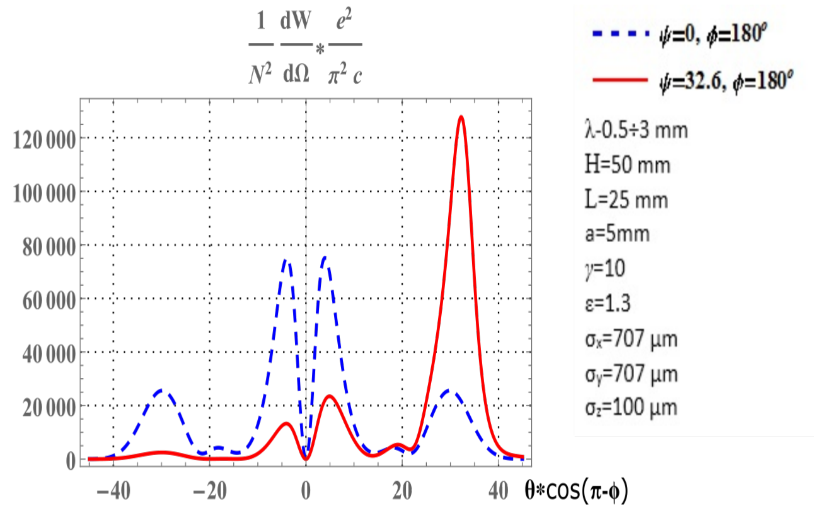


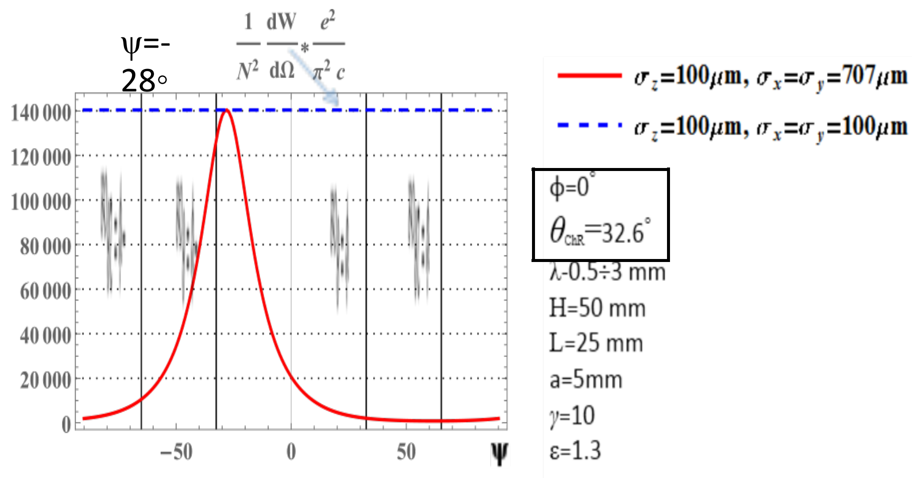
H=50 mm
L=25 mm
a=5 mm
γ=10
ε=1.3
λ=1.5 mm
$\sigma_x = \sigma_y$=707 µm
σ_z= 100 µm
ψ=32.6 deg

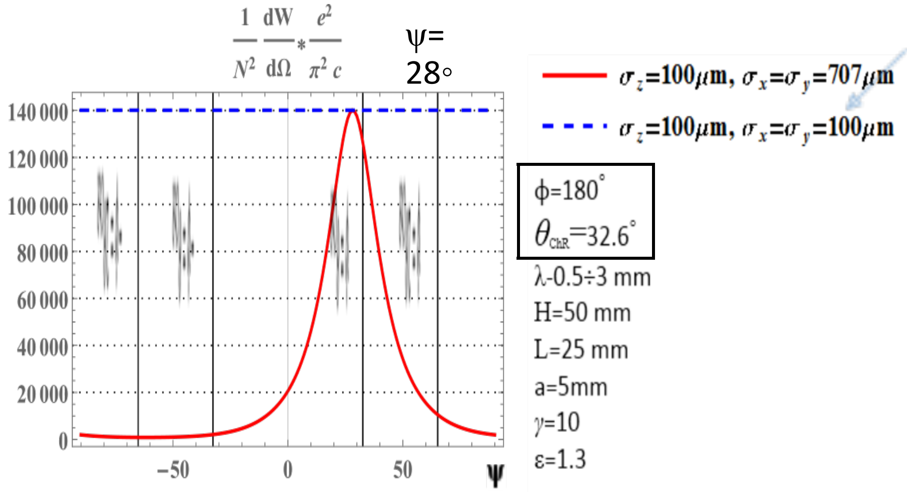


Coherent Cherenkov radiation from a tilted bunch

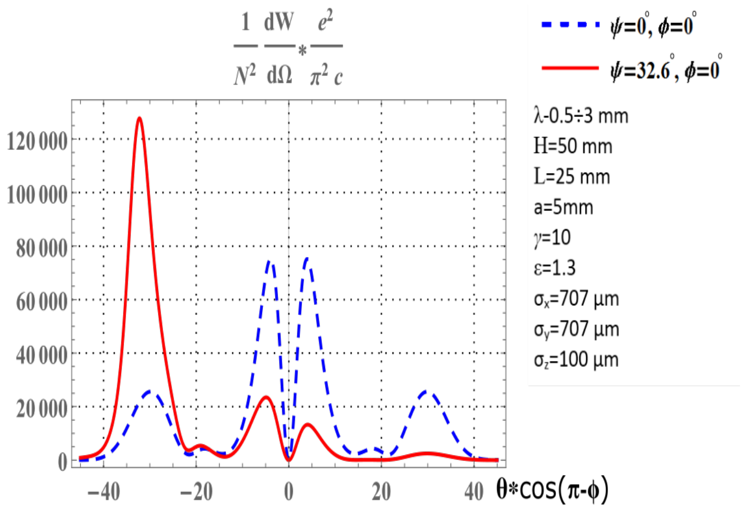


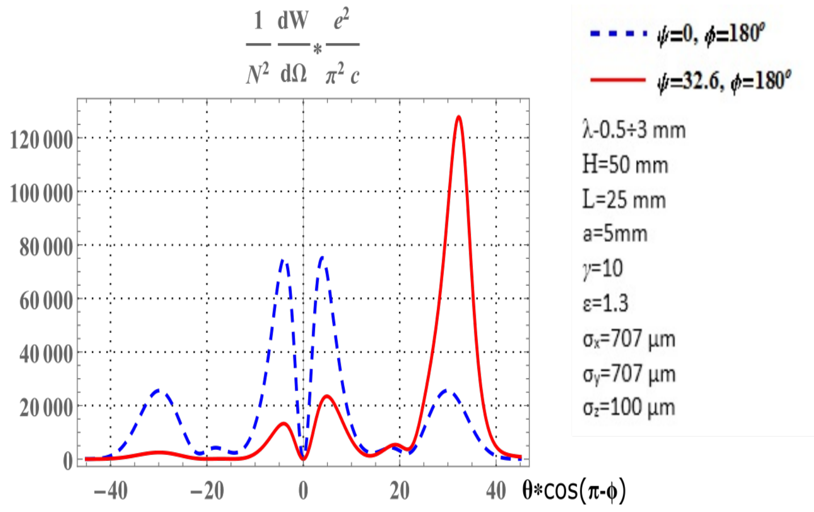


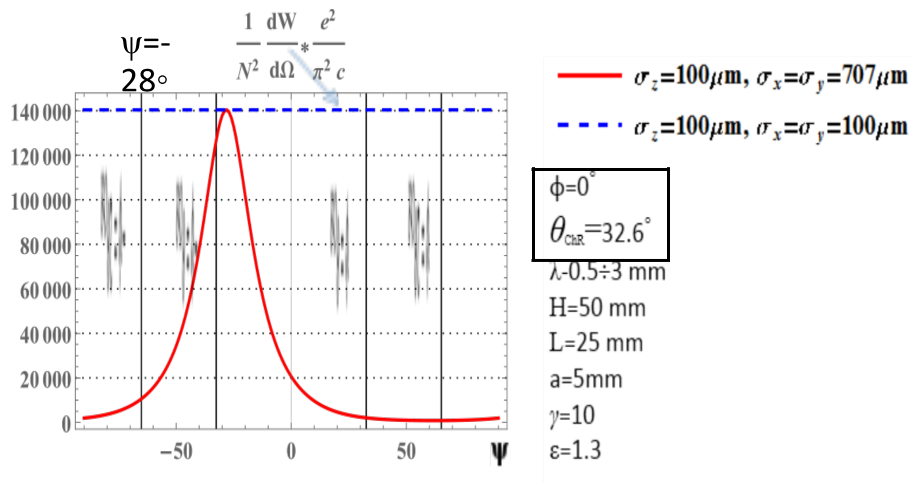


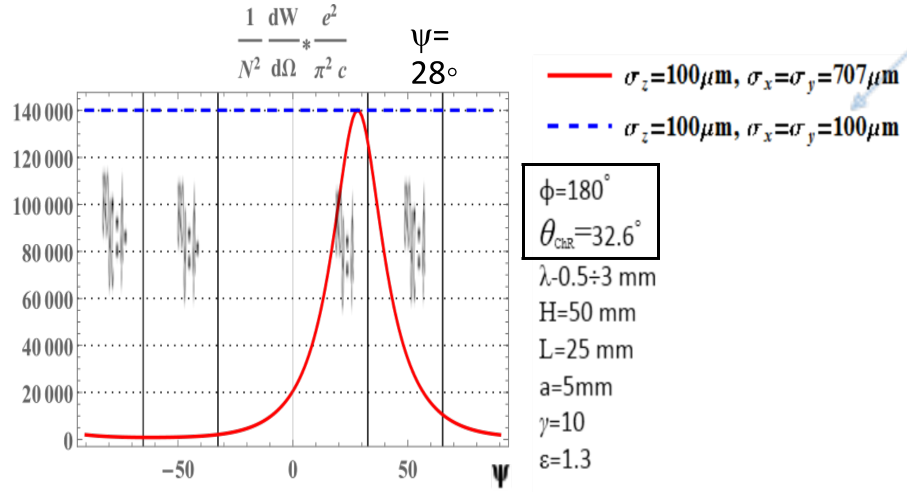


Coherent Cherenkov radiation from a tilted bunch

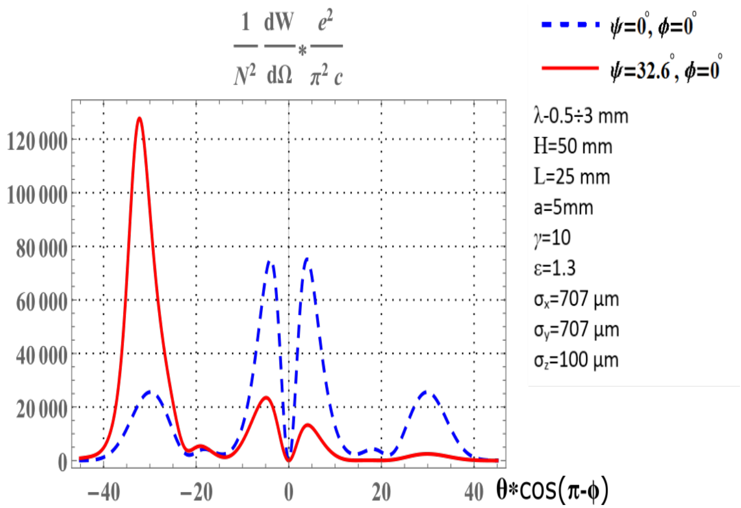


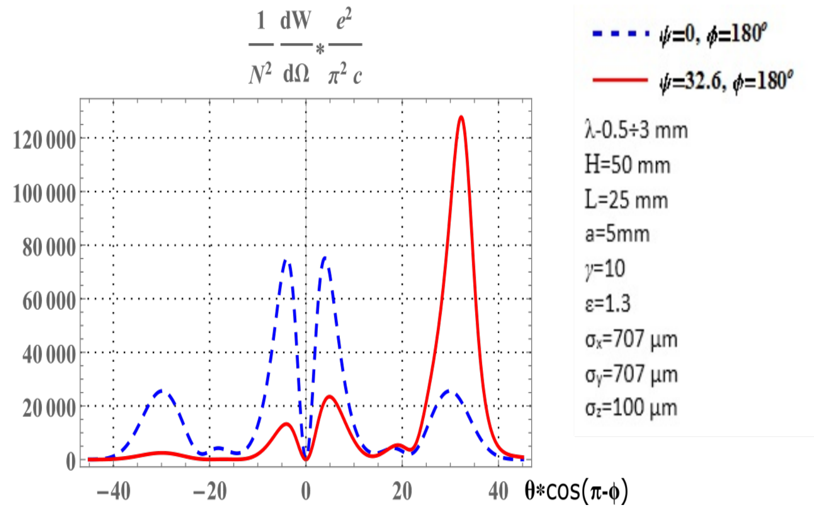


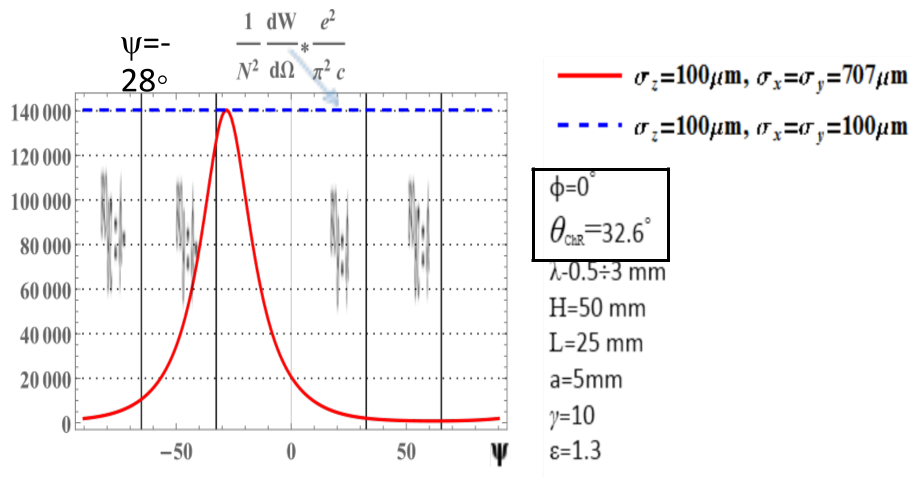


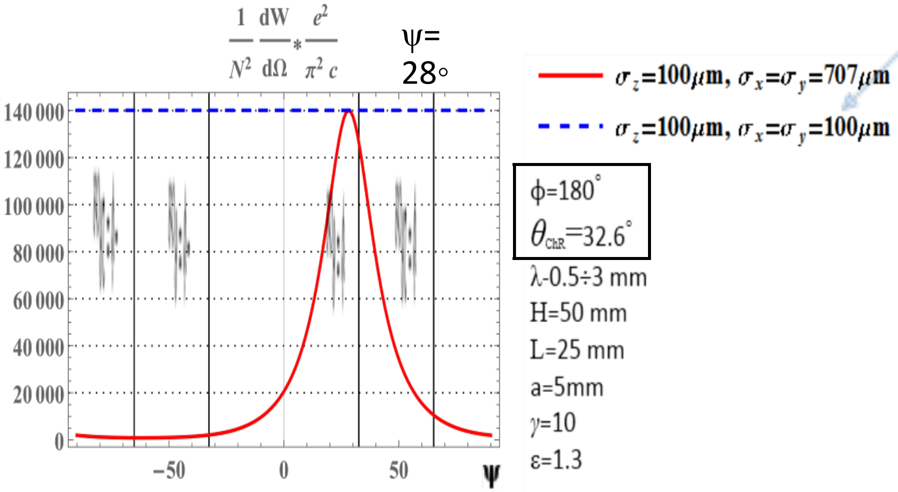


Coherent Cherenkov radiation from a tilted bunch









Conclusion

- Due to the spatial coherence spectral-angular distributions of coherent BTR from "pancake-like" bunches depend on 3 independent parameters: $\boldsymbol{\sigma_t, \sigma_z, \Psi}$;
- Angular distributions of coherent BTR if $\boldsymbol{\sigma_t > \sigma_z}$ is "narrower" in comparison with incoherent BTR and becomes asymmetric if $\boldsymbol{\Psi \neq 0}$;
- The measurement of longitudinal bunch size $\boldsymbol{\sigma_z}$ can be carried out using spectral measurements if two other parameters $\boldsymbol{\sigma_t}$ and $\boldsymbol{\Psi}$ are known;
- The tilt angle $\boldsymbol{\Psi}$ can be determined from measurements of angular distribution $\boldsymbol{\frac{d^2W_{CTR}}{d\omega d\Omega}}$
- The developed model allows to simulate the spectral-angular distribution of ChR for target geometry where charge passes through the slit in dielectric.
- Coherent ChR produced by tilted electron bunch possesses the strong azimuthal asymmetry.
- Simulation results show that the maximal ChR yield is directed in the plane coinciding with the bunch axes and confirm the experimental data.