LD QCD sum rules for strong IB in decay constants of heavy mesons

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We propose a new method for calculating the dependences of the decay constants of heavy-light mesons on the light-quark mass m_q based on QCD sum rules at infinitely large Borel mass parameter (local-duality limit). For a specific choice of the correlation functions, all condensate contributions vanish and the m_q-dependence of the decay constants is shown to be mainly determined by the known analytic m_q-dependence of the diagrams of perturbative QCD.

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QCD sum rule for 2 – point function

A typical Borel QCD sum rule for the decay constant f_H of a heavy (pseudoscalar or vector) $\bar{Q}q$ meson H of mass M_H, (heavy quark Q with mass m_Q and light quark q with mass m_q):

$$f_H^2 (M_H^2)^N \exp(-M_H^2 \tau) = \int\limits_{(m_Q+m_q)^2}^{s_{\rm eff}^{(N)}(\tau, m_Q, m_q, \alpha_s)} ds \exp(-s\tau) s^N \rho_{\rm pert}(s, m_Q, m_q, \alpha_s) + \Pi_{\rm power}^{(N)}(\tau, m_Q, m_q, \alpha_s, \langle\bar{q}q\rangle, ...).$$

Nonperturbative effects appear at two places:

(i) power corrections

(ii) effective threshold $s_{\rm eff}^{(N)}(\tau, m_Q, m_q, \alpha_s)$.

Depending on N, nonperturbative effects are distributed in a different way between power corrections and the effective threshold.

The usual procedure: work in a "windows" of (nonzero) τ, fix $s_{\rm eff}$ and obtain the decay constant.

<u>QCD input parameters:</u>

α_s, quark masses, perturbative spectral densities, condensates (+ in some cases hadron masses).

Another perspective: one is interested in particular dependence of the hadron observable on m_q and can make use of some "external" inputs. A different approach is promising.

Borel QCD sum rule in LD limit $\tau \to 0$

LD limit in Borel sum rules was introduced by Radyushkin in the context of relations between 2-point and 3-point sum rules and obtaining predictions for hadron form factor in a broad range of momentum transfers.

We propose a different application of this limit.

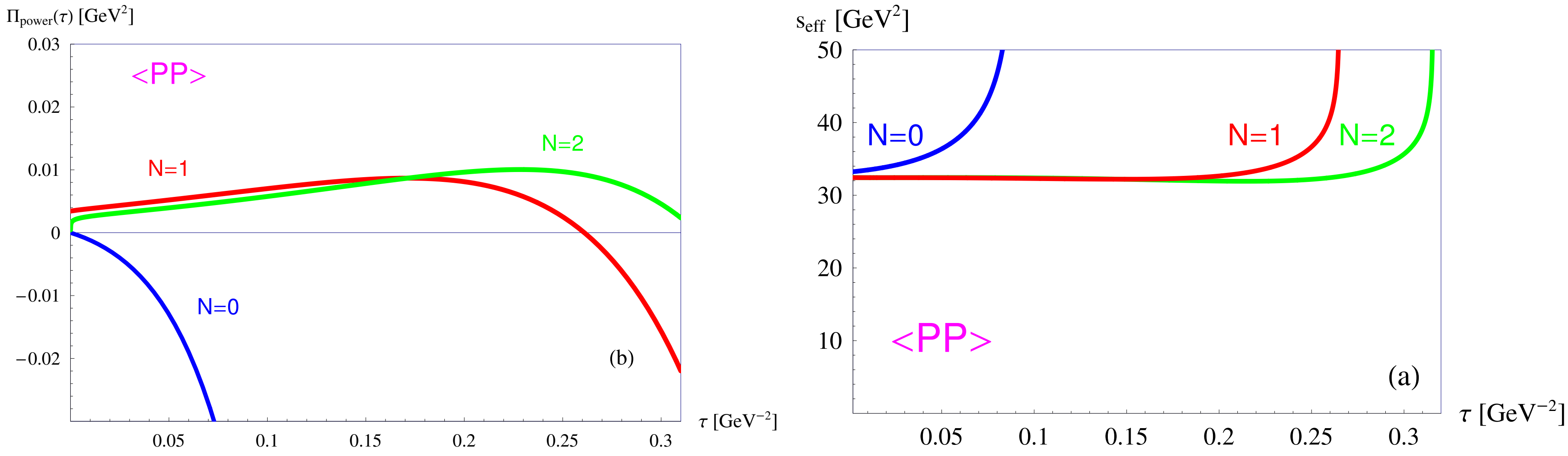
(i) Consider Borelized correlation function of dimension-2 in LD limit. *LD limit is well-defined.*

(ii) In dimension-2 correlator ALL power corrections vanish in LD

$$f_q^2 = \int\limits_{(m_Q+m_q)^2}^{s_{\rm eff}^{(q)}} ds \rho_{\rm pert}(s, m_Q, m_q, \alpha_s | m_{\rm sea})$$

LD limit $\tau \to 0$ in a sum rule : is it well defined?

Power corrections vs τ and the "exact" τ-dependent threshold:



- $N = 0$ **and** $N = 1$ **power corrections are regular at** $\tau = 0$**;**
- $N = 2$ **power corrections contain** $\log(\tau)$

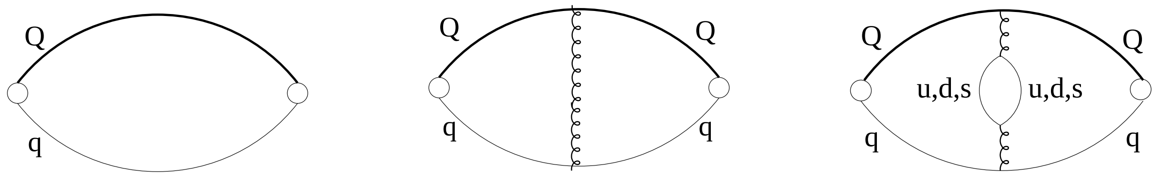
$$f_q^2 = \int\limits_{(m_Q+m_q)^2}^{s_{\rm eff}^{(q)}} ds \rho_{\rm pert}(s, m_Q, m_q, \alpha_s | m_{\rm sea}) \equiv \Pi_{\rm dual}(s_{\rm eff}^q, m_q | m_{\rm sea})$$

Here $m_q = m_u$, m_d, or m_s, and $m_{\rm sea}$ denotes the set of values m_u, m_d, and m_s.
The IB is related to

$$\delta\Pi_{\rm dual} = \Pi_{\rm dual}(s_{\rm eff}^d, m_d | m_{\rm sea}) - \Pi_{\rm dual}(s_{\rm eff}^u, m_u | m_{\rm sea}).$$

• **What is known in QCD? $\rho_{\rm pert}$ is calculated as expansion in powers of $a \equiv \alpha_s/\pi$:**

$$\rho_{\rm pert}(s, m_Q, m_q, \alpha_s) = \rho^{(0)}(s, m_Q, m_q) + a\rho^{(1)}(s, m_Q, m_q) + a^2\rho^{(2)}(s, m_Q, m_q | m_{\rm sea}) + \dots$$



Order a^2 is the first order where the "sea-quark" masses appear. The second-order spectral density is known approximately, i.e. for massless quarks only, $\rho^{(2)}(s, m_Q, m_q = 0 | m_{\rm sea} = 0)$.

Does this approximate OPE allow us to obtain IB and with what accuracy?

• **For decay constants of heavy mesons, the OPE error is** $O(a^2 m_s) \sim O($**a few MeV**$)$**,** $a \le 0.1$

• **However, for IB effects, the strange sea-quark contributions cancel in the difference and the OPE accuracy increases strongly:**

$$\begin{aligned}\delta\Pi_{\rm dual} &= \Pi_{\rm dual}(\tau, s^d_{\rm eff}, m_d|m_u, m_d, m_s) - \Pi_{\rm dual}(\tau, s^u_{\rm eff}, m_u|m_u, m_d, m_s)\\ &= \Pi_{\rm dual}(\tau, s^d_{\rm eff}, m_d|m_{\rm sea} = 0) - \Pi_{\rm dual}(\tau, s^u_{\rm eff}, m_u|m_{\rm sea} = 0) + O(a^2\delta m).\end{aligned}$$

This relation suggests the following algorithm for the calculation of the IB effects:

(i) **Obtain** $f(m_q)$ **corresponding to the correlation function in which the light-quark mass in the LO and the NLO spectral densities is equal to** m_q **which we choose in the range** $m_{ud} < m_q < m_s$**, whereas in the NNLO spectral density the light** u**,** d **and** s **quarks are taken massless.**

(ii) **Calculates** $f(m_d) - f(m_u)$**; the OPE error in this quantity compared to the IB in "real" QCD (i.e. corresponding to the physical sea-quark masses) is of order** $O(a^2\delta m) \simeq \delta m/100$**. It is important to emphasize that the known OPE allows one to address properly the IB effects.**

What to do with the m_q*-dependence of* $s_{\rm eff}$*?*

Parametrize $s_{\rm eff} = s^{(0)}_{\rm eff} + m_q s^{(1)}_{\rm eff} + \cdots$ **and determine** $s^{(0)}_{\rm eff}$ **and** $s^{(1)}_{\rm eff}$ **using a few "external" lattice QCD results for strange mesons and isosymmetric mesons.**

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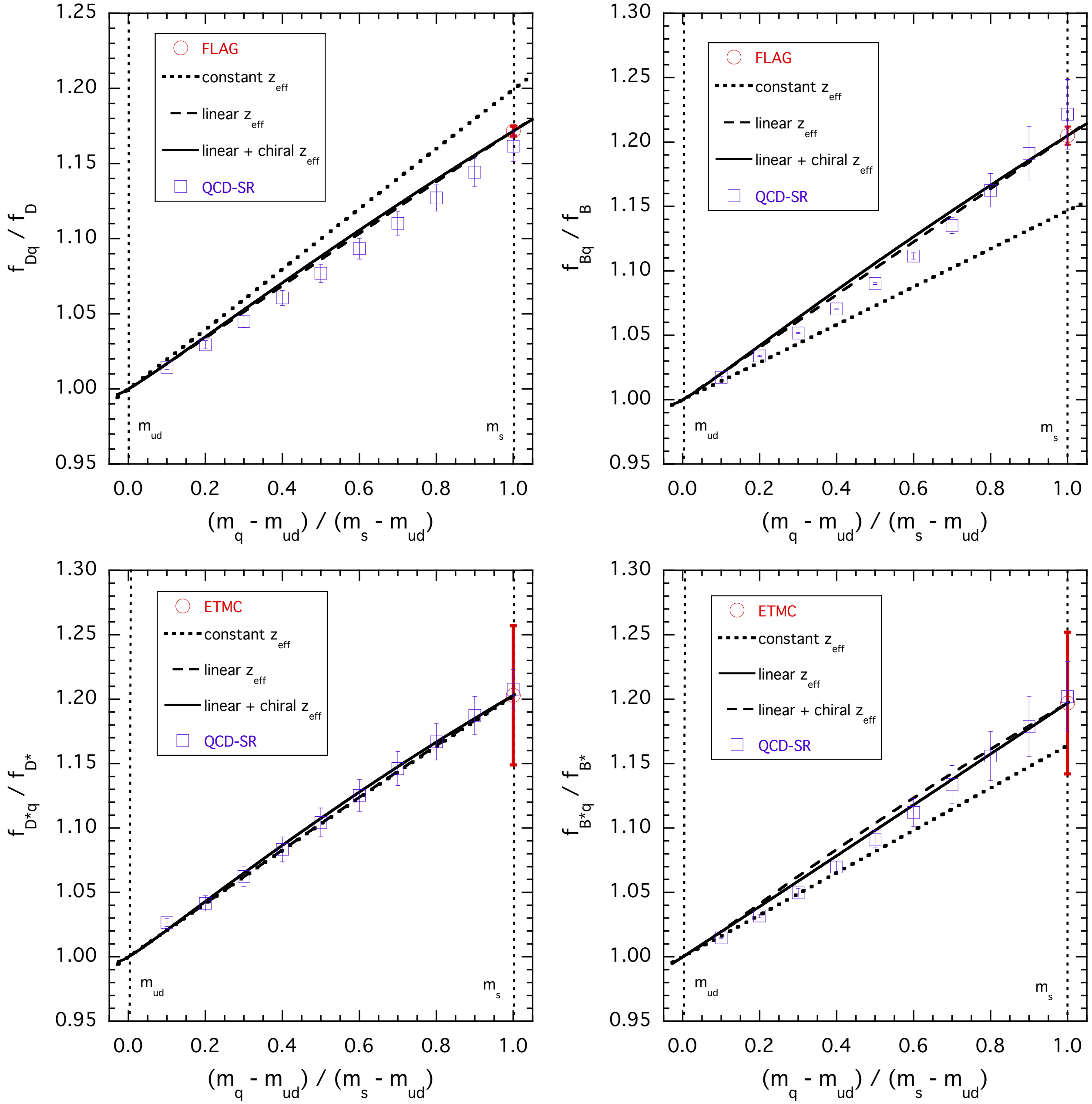
Interesting subtleties:

• HQ limit in pole mass, our LD sum rule reproduces correctly the full QCD result for f_V/f_P at LO, NLO, NNLO in α_s. A convincing evidence that LD sum rules are consistent.

• HQ limit in pole mass vs HQ limit in running mass. Subtleties about the corresponding effective thresholds

• Chiral logs, which may influence IB effects. Where do these chiral logs come from in our approach?

The answer: **the only source of nonperturbative effects now is $s_{\rm eff}$. ALL phenomena originating from lond-distances are now encoded in $s_{\rm eff}$. In particular, chiral logs, which are known in the HQ limit. We have taken these effects into account.**



$$R(x_q) = 1 + R_L x_q \log(x_q) + R_1 x_q + \ldots .$$

Summary and conclusions

• We present the first application of QCD sum rules in LD limit to IB in the decay constants of heavy mesons and show that it is possible to obtain accurate predictions for $\delta f/f$. This was not obvious since the typical accuracy of the SR predictions for f_{B,B^*,D,D^*} is about 10-15 MeV.

• The known OPE (full m_q-dependence at LO and NLO, massless light quarks in NNLO) allows one to access IB with $O(a^2 m_u, a^2 m_d)$ accuracy, whereas the accuracy of the individual f is $O(a^2 m_s)$.

• Knowing the explicit dependence of the OPE on m_q and obtaining the decay constants as a function $f(m_q)$ opens the possibility to access the IB effects.

• Making use of lattice QCD results for strange and isosymmetric heavy mesons, we report the following IB effects:

$$
\begin{aligned}
f_{D^+} - f_{D^0} &= (0.96 \pm 0.09)\ \text{MeV}\,, \qquad & f_{B^0} - f_{B^+} &= (1.01 \pm 0.10)\ \text{MeV} \\
f_{D^{*+}} - f_{D^{*0}} &= (1.18 \pm 0.35)\ \text{MeV}\,, \qquad & f_{B^{*0}} - f_{B^{*+}} &= (0.89 \pm 0.30)\ \text{MeV}
\end{aligned}
$$

(i) The main IB (70-80%) is due to the m_q dependence of the spectral densities
(ii) The accuracy is limited mainly by the uncertainties of the available lattice results for f_H.