π^0: the lightest hadron
– in the precision era –

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Outline:

- Introduction
- Leading logs
- processes:
  - $\pi^0 \to \gamma\gamma$
  - $\pi^0 \to e^+e^-$
  - Dalitz decay
- summary

Overview of low energy QCD

- history: current algebra
- modern form: ChPT [Weinberg'79], [Gasser,Leutwyler'84-'85]
- clash between the order of perturbation theory (number of LECs) and precision
- present status: ChPT for 2 and 3-flavours up to NNLO for the even sector, and up to NLO for the odd sector (chiral anomaly)
- number of parameters for 3-flavour ChPT: 2, 10, 90 for LO, NLO, NNLO, resp.
- ChPT described dynamics of Goldstone bosons (pions, kaons, eta)
- can be extended systematically by other particles (photon, resonances, etc.)
- first two-loop calculation: $\gamma\gamma \to \pi^0\pi^0$ [Bellucci, Gasser, Sainio '94]
- see talks by H.Leutwyler, G.Ecker, S.Scherer, M.Kolesar, ...

π^0's CV

- conception: Yukawa '35, Kemmer '38
- long birth: Lewis, Oppenheimer, Wouthuysen '48, Carlson, Hooper, King '50, Bjorklund, Crandall, Moyer, York '50 "The existence of a neutral meson is clearly not required at the present stage of the experiments, but is the only one of the above five hypotheses which seems to fit the experimental data."
  Steinberger, Panofsky, Steller '50: "It is clear from these properties that the gamma-rays are the decay products of neutral mesons."
  Ekspong'97: "It was generally felt that the neutral pion marked the end for particle searches."
- two siblings: π^+, π^-, born: Lattes, Muirhead, Occhialini, Powell, '47

Leading logarithms

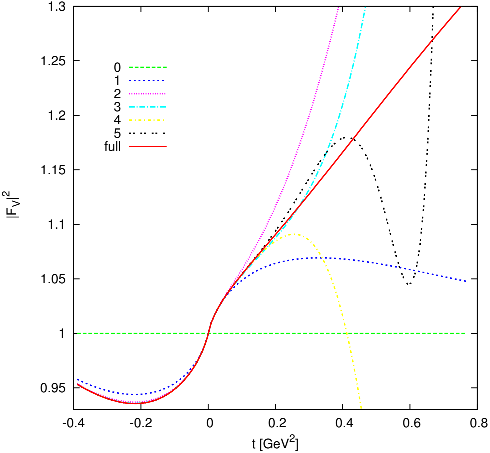
summary

- see J.Bijnens' *leading talk* on leading logs
- renormalizable vs. effective theory
- running coupling constant vs. algorithms how to calculate all orders
- known resumation only in few cases

see: [Weinberg'79], [Büchler, Colangelo'03], [M.Polyakov et al.'08,'09,'10], [Bijnens,Carloni '10,'11], [Bijnens,KK,Lanz '12,'13]

Leading logarithms, one example: vector formfactor

For the $O(N)$ sigma model: the expansions of the leading logarithms order by order for the vector form-factor in the chiral limit and next-to-large N limit ($F = 0.090$ GeV, $\mu = 0.77$ GeV and $N = 3$)

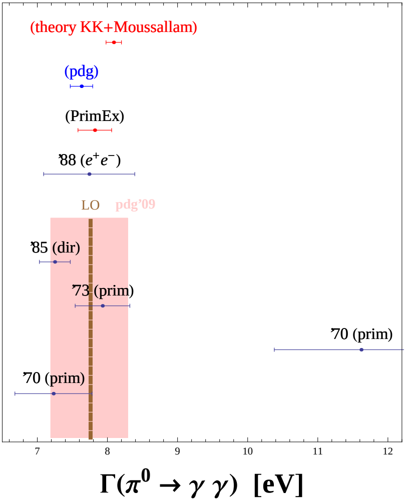


see [Kivel,Polyakov,Vladimirov '09], [Bijnens,Carloni'11], [Bijnens,KK,Lanz'12]

$\pi^0 \to \gamma\gamma$

- one of the most important processes for theory of particle physics
- π^0 lightest hadron $\Rightarrow$ dominant decay mode $\pi^0 \to \gamma\gamma$ (br=98.82%)
- non-existence of logarithmic correction to the current algebra result at NLO
- connection with the non-renormalization theorem ?
- new experimental activities: JLab, KLOE
- see also talks by L. Gan, S. Giovannella, A.Bernstein, A. Gasparian
- theory – NNLO calculation: [KK,Moussallam'09] $\rightarrow$ see next

π^0 life time



π^0 mean life, PDG history:

1985 $(8.4 \pm 0.6) \times 10^{-17}$ s
...
2009 $(8.4 \pm 0.6) \times 10^{-17}$ s
2010 $(8.4 \pm 0.5) \times 10^{-17}$ s
2011 $(8.4 \pm 0.4) \times 10^{-17}$ s
2012 $(8.52 \pm 0.18) \times 10^{-17}$ s $\leftarrow$ PrimEx col.
...
today $(8.52 \pm 0.18) \times 10^{-17}$ s

theory: [KK,Moussallam] $(8.04 \pm 0.11) \times 10^{-17}$ s

$\pi^0 \to \gamma\gamma$: chiral expansion

- in chiral limit exact due to QCD axial anomaly:

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{m_{\pi^0}^3}{64\pi}\left(\frac{\alpha N_C}{3\pi F_\pi}\right)^2 = 7.73\, eV$$

Correction to the current algebra prediction:

- using [Pagels and Zepeda '72] sum rules in [Kitazawa '85]
- NLO corrections are hidden in $F \to F_{\pi^0}$ and $O(p^6)$ LECs [Donoghue, Holstein, Lin '85] [Bijnens, Bramon, Cornet '88]
- in 3-flavour case we can study π^0, η, η' mixing, resulting to [Goity, Bernstein, Holstein '02]:

$$\Gamma^{\mathsf{NLO}} = 8.1 \pm 0.08\,\mathrm{eV}$$

  in 2-flavour case EM corrections [Ananth., Moussallam '02]:

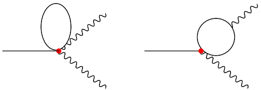
$$\Gamma^{\mathsf{NLO}} = 8.06 \pm 0.02 \pm 0.06\,\mathrm{eV}$$

- Quite recently another study based on dispersion relations, QCD sum rules, using only the value $\Gamma(\eta \to \gamma\gamma)$ gives [Ioffe, Oganesian '07]:

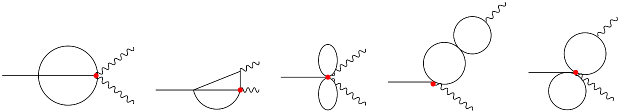
$$\Gamma^{\mathsf{NLO}} = 7.93 \pm 0.11\,\mathrm{eV}$$

$\pi^0 \to \gamma\gamma$ at NNLO in 2 flavour ChPT: technical part

- NLO: a) One-loop diagrams with one vertex from $\mathcal{L}^{WZ}$, b) tree diagrams with one vertex from $\mathcal{L}^{WZ}$ and one vertex from $O(p^4)$ Lagrangian, c) tree diagrams with one vertex from $O(p^6)$ anomalous-parity sector
- $O(p^6)$ anomalous-parity sector from [Bijnens, Girlanda, Talavera '02]
- representation of chiral field: $U = \sigma + i\frac{\tau.\pi}{F}$, $\sigma = \sqrt{1-\vec{\pi}^2/F^2}$ (no $\gamma 4\pi$ vertex at LO)
- one-loop



- two-loop



- verification of Z-factor, F_π/F [Bürgi '96], [Bijnens, Colangelo, Ecker, Gasser, Sainio '02]
- double log checked by Weinberg consistency rel. [Colangelo '95]

$\pi^0 \to \gamma\gamma$ at NNLO, result

$$
\begin{aligned}
A_{NNLO} = \frac{e^2}{F_\pi}\Big\{ & \frac{1}{4\pi^2} \\
& + \frac{16}{3} m_\pi^2 (-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr}) + \frac{64}{9} B(m_d - m_u)(5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr}) \\
& - \frac{M^4}{24\pi^2 F^4}\left(\frac{1}{16\pi^2} L_\pi\right)^2 + \frac{M^4}{16\pi^2 F^4} L_\pi \left[\frac{3}{256\pi^4} + \frac{32F^2}{3}\left(2c_2^{Wr} + 4c_3^{Wr} + 2c_6^{Wr} + 4c_7^{Wr} - c_{11}^{Wr}\right)\right] \\
& + \frac{32M^2 B(m_d - m_u)}{48\pi^2 F^4} L_\pi \left[-6c_2^{Wr} - 11c_3^{Wr} + 6c_4^{Wr} - 12c_5^{Wr} - c_7^{Wr} - 2c_8^{Wr}\right] \\
& + \frac{M^4}{F^4}\lambda_+ + \frac{M^2 B(m_d - m_u)}{F^4}\lambda_- + \frac{B^2(m_d - m_u)^2}{F^4}\lambda_{--}\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\lambda_+ &= \frac{1}{\pi^2}\left[-\frac{2}{3} d_+^{Wr}(\mu) - 8c_6^r - \frac{1}{4}(l_4^r)^2 + \frac{1}{512\pi^4}\left(-\frac{983}{288} - \frac{4}{3}\zeta(3) + 3\sqrt{3}\,\mathrm{Cl}_2(\pi/3)\right)\right] \\
&\quad + \frac{16}{3} F^2 \left[8 l_3^r (c_3^{Wr} + c_7^{Wr}) + l_4^r(-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr})\right] \\
\lambda_- &= \frac{64}{9}\left[d_-^{Wr}(\mu) + F^2 l_4^r (5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr})\right] \\
\lambda_{--} &= d_{--}^{Wr}(\mu) - 128 F^2 l_7 (c_3^{Wr} + c_7^{Wr}) \;.
\end{aligned}
$$

- 4 LECs in 2 combinations of NLO
- additional 4 LECs in 3 combinations of NNLO

Is it at all possible to make some reliable prediction?

$\pi^0 \to \gamma\gamma$: modified counting

- Use of $SU(3)$ phenomenology via $c_i^{Wr} \leftrightarrow C_i^{Wr}$ connection (analogously to [Gasser,Leutwyler'85] and [Gasser; Haefeli, Ivanov, Schmid '07,'08])

$$c_i^{Wr} = \frac{\alpha_i}{m_s} + \left(\beta_i + \gamma_{ij} C_j^{Wr} + \delta_i \ln \frac{B_0 m_s}{\mu^2}\right) + O(m_s)$$

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- implementation of modified counting

$$m_u, m_d \sim O(p^2) \quad \text{and} \quad m_s \sim O(p)$$

Result:

$$A_{NNLO}^{mod} = \frac{e^2}{F_\pi}\left\{\frac{1}{4\pi^2} - \frac{64}{3} m_\pi^2 C_7^{Wr} + \frac{1}{16\pi^2}\frac{m_d - m_u}{m_s}\left[1 - \frac{3}{2}\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right]\right.$$
$$+ 32B(m_d - m_u)\left[\frac{4}{3}C_7^{Wr} + 4C_8^{Wr}\left(1 - 3\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)\right.$$
$$\left.\left. - \frac{1}{16\pi^2 F_\pi^2}\left(3L_7^r + L_8^r - \frac{1}{512\pi^2}(L_K + \frac{2}{3}L_\eta)\right)\right] - \frac{1}{24\pi^2}\left(\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)^2\right\}$$

$\pi^0 \to \gamma\gamma$: Phenomenology

- $F_\pi = 92.22 \pm 0.07$ MeV (using [Marciano, Sirlin'93] with updated value of V_{ud} [Towner, Hardy'08]).

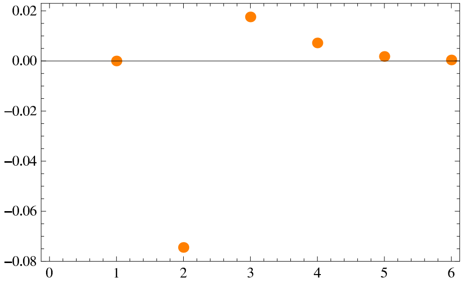
  rem.: if SM violated: $F_\pi \to \hat{F}_\pi$ [Bernard, Oertel, Passemar, Stern '08]
- $\frac{m_d - m_u}{m_s} = (2.29 \pm 0.23)\, 10^{-2}$
- $B(m_d - m_u) = (0.32 \pm 0.03)\, M_{\pi^0}^2$
- $3L_7 + L_8^r(\mu) = (0.10 \pm 0.06)\, 10^{-3} \quad (\mu = M_\eta)$ (from pseudo-scalar meson masses formula [Gasser, Leutwyler '85])
- $C_7^W = 0$ (more precisely $C_7^W \ll C_8^W$, motivated by simple resonance saturation)
- $C_8^W = (0.58 \pm 0.2) 10^{-3}$GeV^{-2} (from $\eta \to 2\gamma$)
- incl.of EM corrections [Ananthanarayan, Moussallam '02]

result $\boxed{\Gamma_{\pi^0 \to 2\gamma} = (8.09 \pm 0.11)\text{eV}}$

$\pi^0 \to \gamma\gamma$: leading logs

[Bijnens,KK,Lanz'12]

Leading logarithm contribution of individual orders in percent of the leading order:



Adler-Lee-Treiman-Zee-Terentev theorem on triangle and box anomaly

$$F^{3\pi}(0,0,0) = \frac{1}{eF_\pi^2} F_{\pi\gamma\gamma}(0,0)$$

is valid up to 2-loop order for LL beyond the soft-photon limit

$\pi^0 \to e^+e^-$

- first studied by [S. Drell '59]
- radiative corrections: [L.Bergström '83]
- most recent experiment: KTeV E799-II [Abouzaid'07]
- radiative corrections play important role
- see talk by P. Sanchez-Puertas

$\pi^0 \to e^+e^-$

KTeV's measurement:

$$\frac{\Gamma(\pi^0 \to e^+e^-,\, x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma,\, x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}\,.$$

by extrapolating the Dalitz branching ratio to the full range of x

$$B(\pi^0 \to e^+e^-(\gamma),\, x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}\,.$$

Extrapolating the radiative tail using Bergström:

$$B^{\text{no-rad}}_{\text{KTeV}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}\,.$$

Theoretical prediction [Dorokhov, Ivanov '07, '10]

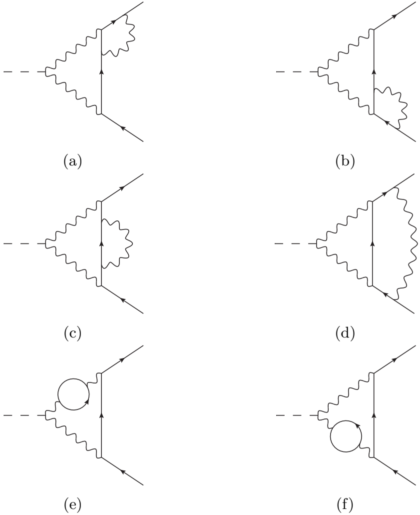
$$B^{\text{no-rad}}_{\text{SM}}(\pi^0 \to e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}\,. \qquad (1)$$

3.3 $\sigma \Rightarrow$ New physics?

In any case, radiative corrections play an important role in the analysis

$\pi^0 \to e^+e^-$

Radiative corrections $\to$ two-loop graphs



$\pi^0 \to e^+e^-$: results

- all technical details in [Vasko,Novotny '11], [Husek,KK,Novotny'14]
- electromagnetic corrections at NLO without approximation
- stability in the strong sector (leading logarithm suppressed)
- original discrepancy down to 2σ level
- possibility to use this precise measurement for the VVP correlator
  - counter-term chiral Lagrangian for $\pi^0 l\bar{l}$ [Savage et al'92]
  - modelled using the resonances [Knecht et al '99]

$$\chi^{(r)}_{\mathsf{LMD}}(M_\rho) = 2.2 \pm 0.9$$

  - KTeV implies [Husek,KK,Novotny'14]

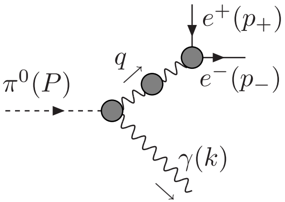
$$\chi^{(r)}_{\mathsf{KTeV}}(M_\rho) = 4.5 \pm 1.0$$

Dalitz decay

[K.K., Knecht, Novotný '06]
[Husek,K.K., Novotný '15]
see also poster by S. Gonzalez-Solis

History

- First calculated by [Dalitz '51].
- Radiative corrections studied by [Joseph '60], [Lautrup, Smith'71], [Mikaelian, Smith'72]
- and during the 1980s by Tupper, Grose, Samuel, Lambin, Pestieau, Roberts...



$$x = m_{ee}^2/M_\pi^2, \quad y = \frac{E_+ - E_-}{E_\gamma}\Big|_{\pi^0\to 0}$$

NLO studied via $\delta(x,y)$ and $\delta(x)$:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x\mathrm{d}y} = \delta(x,y)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x\mathrm{d}y}, \quad \frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \delta(x)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x}.$$

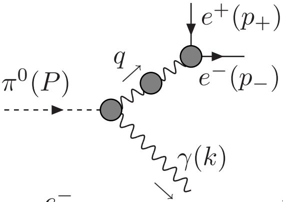
with (point-like pion)

$$\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x\mathrm{d}y} = \frac{\alpha^3}{(4\pi)^4}\frac{M_{\pi^0}}{F_\pi^2}\frac{(1-x)^3}{x^2}\left[M_{\pi^0}^2 x(1+y^2) + 4m^2\right],$$

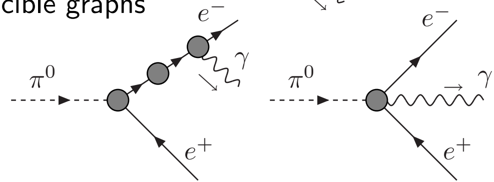
$$\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} = \frac{\alpha^3}{(4\pi)^4}\frac{8}{3}\frac{M_{\pi^0}}{F_\pi^2}\frac{(1-x)^3}{x^2}(xM_{\pi^0}^2 + 2m^2).$$

Dalitz decay: Anatomy of the amplitude

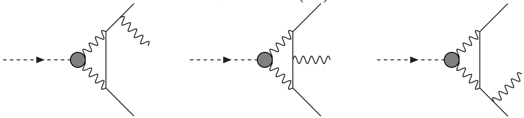
- one-photon reducible graphs: electron-positron pair is produced by a single photon (Dalitz pair)



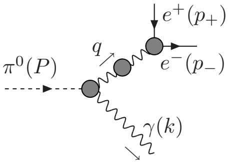
- one-photon irreducible graphs



LO, order $O(e^5)$



Dalitz decay: slope parameter



$$\Gamma_\mu^{1\gamma R}(p_+,p_-,k) = \mathrm{i}e^2\varepsilon_\mu{}^{\nu\alpha\beta} q_\alpha k_\beta\, \mathcal{F}_{\pi^0\gamma\gamma^*}(q^2)\, \mathrm{i}D^T_{\nu\rho}(q)(-\mathrm{i}e)\Lambda^\rho$$

$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2)$ is related to the doubly off-shell form factor $\mathcal{A}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)$

$$\int d^4x\, e^{il\cdot x}\langle 0|T(j^\mu(x)j^\nu(0)|\pi^0(P)\rangle = -\mathrm{i}\varepsilon^{\mu\nu\alpha\beta} l_\alpha P_\beta\, \mathcal{A}_{\pi^0\gamma^*\gamma^*}(l^2,(P-l)^2)$$

One can define a slope parameter a_π

$$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2) \;=\; \mathcal{F}_{\pi^0\gamma\gamma^*}(0)\left[1 \,+\, a_\pi\,\frac{q^2}{M^2_{\pi^0}} \,+\cdots\right],$$

$$\frac{\mathrm{d}\Gamma^{exp}}{\mathrm{d}x} - \delta_{QED}(x)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} \;=\; \frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x}\,[1+2x\,a_\pi].$$

Dalitz decay: Low's theorem

Motivation: new interest in low-energy behaviour of gauge theories (see eg. [Bern et al '14], [Plefka et al '14,'15]).

Soft limit due to the Low's theorem, naively:

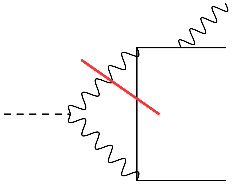
$$\mathcal{M}_{\pi^0\to e^+e^-\gamma} = \mathcal{M}^{\text{Low}} + O(k)$$

$$\mathcal{M}^{\text{Low}} = (s^{(0)} + s^{(1)}) P_{\pi^0 e^- e^+}$$

or equivalently (the LO is of the order $O(k)$)

$$\delta^{1\gamma IR}(x,y) = \delta^{\text{Low}}(x,y) + O(1)$$

However, one should be careful. Due to the non-analyticity (the branch-cut of the intermediate $e\gamma$ state starts at m^2)



one should expect the logarithms i.e. →

Dalitz decay: Low's theorem

Motivation: new interest in low-energy behaviour of gauge theories (see eg. [Bern et al '14], [Plefka et al '14,'15]).

Soft limit due to the Low's theorem, properly:

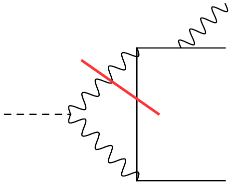
$$\mathcal{M}_{\pi^0\to e^+e^-\gamma} = \mathcal{M}^{\mathsf{Low}} + O(k\ln k) + O(k)$$

$$\mathcal{M}^{\mathsf{Low}} = (s^{(0)} + s^{(1)})P_{\pi^0 e^- e^+}$$

or equivalently (the LO is of the order $O(k)$)

$$\delta^{1\gamma IR}(x,y) = \delta^{\mathsf{Low}}(x,y) + O(\ln(1-x)) + O(1)$$

However, one should be careful. Due to the non-analyticity (the branch-cut of the intermediate $e\gamma$ state starts at m^2)



one should expect the logarithms

for details see [KK,Knecht,Novotny '06]

Dalitz decay: summary of [KK,Knecht,Novotny'06] & [Husek,KK,Novotny'15]

- Our works provide a detailed analysis of NLO radiative corrections to the Dalitz decay amplitude.
- The off-shell pion-photon transition form factor was included:
  this requires a treatment of non perturbative strong interaction effects
- The one-photon irreducible contributions, which had been usually neglected, were included.
  We have shown that, although these contributions are negligible as far as the corrections to the total decay rate are concerned, they are however sizeable in regions of the Dalitz plot which are relevant for the determination of the slope parameter a_π of the pion-photon transition form factor.
- Our prediction for the slope parameter $a_\pi = 0.029 \pm 0.005$ is in good agreement with the determinations obtained from the (model dependent) extrapolation of the CELLO and CLEO data.
  Unfortunately, the experimental error bars on the latest values of a_π extracted from the Dalitz decay are still too large
- used both soft-photon approximation and hard-photon corrections with no approximations
- used in NA48 analysis for the search of dark photon [1504.00607]

Summary

Short overview of the π^0 decays presented. Based on the following projects

- leading logs
  - with J.Bijnens,S.Lanz: Leading logarithms in the anomalous sector of two-flavour QCD [1201.2608]
- $\pi^0 \to \gamma\gamma$
  - with B.Moussallam: Chiral expansions of the pi0 lifetime [0901.4688]
- $\pi^0 \to e^+e^-$
  - with T.Husek and J.Novotny: Rare decay $\pi^0 \to e^+e$: on corrections beyond the leading order [1405.6927]
- $\pi^0 \to e^+e^-\gamma$
  - with M.Knecht and J.Novotny: The Dalitz decay $\pi^0 \to e^+e^-\gamma$ revisited [hep-ph/0510021]
  - with T.Husek and J.Novotny: Radiative corrections to the Dalitz decay $\pi^0 \to e^+e^-\gamma$ revisited [1504.06178]

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Thank you.