

Holographic Goldstino



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based on **1412.6499**

w/ Argurio, Musso, Porri & Redigolo

[plus work in progress w/ Musso, Papadimitriou & Raj]

see also: 1205.4709, 1208.3615, 1304.1481, 1310.6897

w/ Argurio, Di Pietro, Porri & Redigolo

Motivations

- Spontaneously broken global symmetries leave universal fingerprints in the IR behavior of QFTs. In particular, Goldstone theorem ensures that massless modes should appear in IR spectrum.
- In supersymmetric QFTs with spontaneously broken supersymmetry, a massless **Goldstino** is expected.
- Goldstino appears as massless pole in **supercurrent** 2-point correlator. Pole residue is related to vacuum energy by specific supersymmetry **Ward identities**.

Motivations

- If QFT dynamics **strongly coupled** (e.g. DSB) things get hard → *AdS/CFT* provides a powerful tool, allowing a dual weakly coupled supergravity description.
- Need a complete control of the holographic map in the supercurrent sector, i.e. the **gravitino**. Not much so far for non-AdS bg (necessary condition for broken SUSY).
- *Primary goal*: fill above gap + provide a holographic derivation of SUSY **Ward identities** and show, in turn, the appearance of **Goldstino** in theories which are inherently strongly coupled.

Supersymmetric RG-flows

- Let us consider 4d SQFTs described by RG-flows triggered by some **relevant** operator perturbing a strongly coupled UV fixed point. Most generally one can write

[GREEN ET AL. '10 + OTHERS]

$$\mathcal{L} = \mathcal{L}_{\text{SCFT}} + \lambda \int d^2\theta \mathcal{O} = \mathcal{L}_{\text{SCFT}} + \lambda \mathcal{O}_{\text{F}}$$

where $\mathcal{O} = \mathcal{O}_{\text{S}} + \sqrt{2}\theta \mathcal{O}_{\psi} + \theta^2 \mathcal{O}_{\text{F}}$.

- The theory can flow to a SUSY or **SUSY breaking** vacuum, depending whether the operator \mathcal{O}_{F} acquires a VEV.
→ 1 & 2-point functions of supercurrent multiplet can tell!
- Note:** we do not consider QFTs without an interacting UV-fixed point, e.g. cascading gauge theories as KS/KT or quiver embeddings of ISS-like models. [WORK IN PROGRESS...]

Supercurrent multiplet

- The **supercurrent** multiplet, which contains the EM tensor and the SUSY current, can be described as (**FZ** multiplet)

$$(\mathcal{J}_\mu, \mathbf{X}) \quad \begin{array}{l} \swarrow \text{Chiral superfield} \\ \searrow \text{Real superfield} \end{array} \quad \sigma_{\alpha\dot{\alpha}}^\mu \bar{\mathbf{D}}^{\dot{\alpha}} \mathcal{J}_\mu = -\frac{1}{2} \mathbf{D}_\alpha \mathbf{X}$$

$$\mathcal{J}_\mu = \mathbf{j}_\mu + \theta \mathbf{S}_\mu + \bar{\theta} \bar{\mathbf{S}}_\mu + \theta \sigma^\nu \bar{\theta} \mathbf{2T}_{\mu\nu} + \dots$$

$$\mathbf{X} = \mathbf{x} + \frac{2}{3} \theta \mathbf{S} + \theta^2 \left(\frac{2}{3} \mathbf{T} + \mathbf{i} \partial^\mu \mathbf{j}_\mu \right) + \dots$$

- For a SCFT $\mathbf{X} = \mathbf{0}$. In our case, choosing without loss of generality $\Delta_{\mathcal{O}} = \mathbf{2}$, we have $\mathbf{X} = \mathbf{4/3} \lambda \mathcal{O}$.

Supersymmetry Ward identities

- Regardless the vacuum, the SUSY algebra implies that the following **Ward identities** should hold

$$\langle \partial^\mu \mathbf{S}_{\mu\alpha}(\mathbf{x}) \bar{\mathbf{S}}_{\nu\dot{\beta}}(\mathbf{0}) \rangle = -2 \delta^4(\mathbf{x}) \langle \delta_\alpha \bar{\mathbf{S}}_{\nu\dot{\beta}} \rangle = -2 \sigma_{\alpha\dot{\beta}}^\mu \langle \mathbf{T}_{\mu\nu} \rangle \delta^4(\mathbf{x})$$

$$\langle \partial^\mu \mathbf{S}_{\mu\alpha}(\mathbf{x}) \mathcal{O}_{\psi\beta}(\mathbf{0}) \rangle = -2 \delta^4(\mathbf{x}) \langle \delta_\alpha \mathcal{O}_{\psi\beta} \rangle = \sqrt{2} \langle \mathcal{O}_{\mathbf{F}} \rangle \delta^4(\mathbf{x})$$

- The Ward identities imply

$$\begin{aligned} \langle \mathbf{S}_{\mu\alpha}(\mathbf{x}) \bar{\mathbf{S}}_{\nu\dot{\beta}}(\mathbf{0}) \rangle &= \dots - \frac{\mathbf{i}}{4\pi^2} \langle \mathbf{T} \rangle (\sigma_\mu \bar{\sigma}^\rho \sigma_\nu)_{\alpha\dot{\beta}} \frac{\mathbf{x}_\rho}{\mathbf{x}^4} \\ \langle \mathbf{S}_{\mu\alpha}(\mathbf{x}) \mathcal{O}_{\psi\beta}(\mathbf{0}) \rangle &= \dots - \frac{\mathbf{i}}{2\pi^2} \sqrt{2} \langle \mathcal{O}_{\mathbf{F}} \rangle \epsilon_{\alpha\beta} \frac{\mathbf{x}_\mu}{\mathbf{x}^4} \end{aligned}$$

[ARGURIO ET AL. '13]

Upon Fourier transform it displays the massless pole associated to **Goldstino**, which is the lowest energy excitation in \mathbf{S}_μ : in IR $\mathbf{S}_\mu = \sigma_\mu \bar{\mathbf{G}}$.

Field/operator map

- The SQFT

$$\mathcal{L} = \mathcal{L}_{\text{SCFT}} + \lambda \int d^2\theta \mathcal{O}$$

can be described holographically by 5d **N=2 SUGRA** coupled to **one hypermultiplet** (made of one hyperino ζ + two complex scalars ρ and ϕ)

- Since $\Delta(\mathcal{O}) = 2$ dual bg is AAdS and $\mathbf{m}_{(\rho,\phi)}^2 = -4, -3$.

$$ds^2 = \frac{1}{z^2} (F(z)dx^2 + dz^2) \quad , \quad F(0) = 1$$

$$\rho \sim z^2(c \log z + d) + \mathcal{O}(z^4) \quad , \quad \phi \sim z(a + bz^2) + \mathcal{O}(z^5)$$

$$\rho \longleftrightarrow \mathcal{O}_S$$

$$\phi \longleftrightarrow \mathcal{O}_F$$

$$c \longleftrightarrow \text{source } \mathcal{O}_S \quad (\text{SUSY})$$

$$a \longleftrightarrow \text{source } \mathcal{O}_F \quad (\text{SUSY})$$

$$d \longleftrightarrow \text{VEV } \mathcal{O}_S \quad (\text{SUSY})$$

$$b \longleftrightarrow \text{VEV } \mathcal{O}_F \quad (\text{SUSY})$$

Supergravity model

- The model we consider is **N=2 SUGRA** coupled to one **hypermultiplet** with σ -model metric $\frac{\text{SU}(2,1)}{\text{U}(1) \times \text{SU}(2)}$ of which graviphoton gauges a **U(1)** isometry. [CERESOLE ET AL. '01]

- One can choose a **gauging** for which the two complex hyperscalars (ρ, ϕ) have $\mathbf{m}^2 = -4, -3$. We further truncate the action to $\rho = 0$ and take ϕ real. Bosonic action becomes

$$\mathbf{S}_{5D}^B = \int d^5x \sqrt{-\mathbf{G}} \left\{ \frac{1}{2} \mathbf{R} - \partial_M \phi \partial^M \phi - \mathbf{U}(\phi) \right\}$$

$$\mathbf{U}(\phi) = \frac{1}{12} (10 - \cosh(2\phi))^2 - \frac{51}{4}$$

- We look for AAdS solutions preserving 4d Poincare' invariance, hence we require $\phi = \phi(\mathbf{z})$.

Supergravity model

- **SUSY** solutions

$$\phi(\mathbf{z}) = \frac{1}{2} \log \left(\frac{1 + \mathbf{a}\mathbf{z}}{1 - \mathbf{a}\mathbf{z}} \right), \quad \mathbf{F}(\mathbf{z}) = (1 - \mathbf{a}^2 \mathbf{z}^2)^{1/3}$$

- **SUSY breaking** solutions

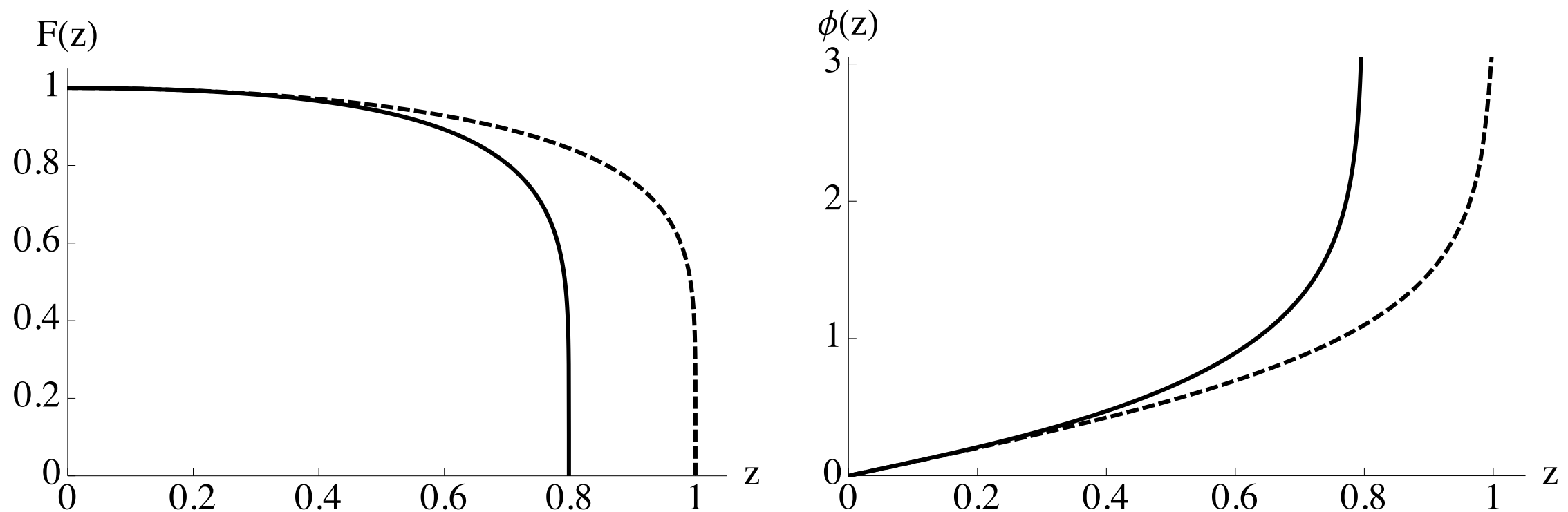
$$\phi(\mathbf{z}) = \mathbf{a}\mathbf{z} + \mathbf{b}\mathbf{z}^3 + \mathbf{O}(\mathbf{z}^5)$$

$$\mathbf{F}(\mathbf{z}) = 1 - \frac{\mathbf{a}^2}{3} \mathbf{z}^2 + \frac{\mathbf{a}^4 - 9\mathbf{a}\mathbf{b}}{18} \mathbf{z}^4 + \mathbf{O}(\mathbf{z}^6)$$

- **Note:** $\mathbf{b} = \mathbf{a}^3/3$ is the SUSY solution $\rightarrow \beta = \mathbf{a}^3/3 - \mathbf{b}$ is SUSY breaking parameter (related to VEV of $\mathcal{O}_{\mathbf{F}}$).
- **Comment:** all solutions are singular. But remarkably, this won't affect final results!

Supergravity model

- The BPS and ~~SUSY~~ **solutions** for the warp factor **F** and the scalar ϕ look



where **dashed** line is the BPS solutions and **solid** line the ~~SUSY~~ ones (parameters are $\mathbf{a} = \mathbf{1}$ and $\beta = \mathbf{0}, -\mathbf{2/3}$).

AdS/CFT & Hol. Renormalization

- QFT correlators are computed in AdS/CFT as

$$\langle \mathcal{O}(\mathbf{x}_1) \dots \mathcal{O}(\mathbf{x}_n) \rangle = \frac{\delta^n \mathbf{S}_{\text{on-shell}}}{\delta \phi_0(\mathbf{x}_1) \dots \delta \phi_0(\mathbf{x}_n)} \Big|_{\phi_0=0}$$

- $\mathbf{S}_{\text{on-shell}}$ diverges at $\mathbf{z} = 0$ (because of the infinite volume of (A)AdS space) and should then be **holographically renormalized**.
[BIANCHI-FREEDMAN-SKENDERIS '01A]

-  Regularize the action at $\mathbf{z} = \epsilon$, add a 4d covariant counter-term action $\mathbf{S}_{\text{c.t.}}$ and define a renormalized action

$$\mathbf{S}_{\text{ren}} = \lim_{\epsilon \rightarrow 0} (\mathbf{S}_{\text{on-shell}}[\epsilon] + \mathbf{S}_{\text{c.t.}}[\epsilon])$$

- **Note:** in general $\mathbf{S}_{\text{c.t.}}$ provides extra finite terms, which corresponds to arbitrariness in renormalization scheme.

Hol Ren: Bosonic sector

- Have to evaluate 1-pt functions only → focus on on-shell action at **linear order** in the fluctuations of metric and scalar

$$\phi = \phi(\mathbf{z}) + \varphi(\mathbf{z}, \mathbf{x})$$

$$ds^2 = \frac{1}{z^2} [\mathbf{dz}^2 + \mathbf{F}(\mathbf{z}) (\eta_{\mu\nu} + \mathbf{h}_{\mu\nu}(\mathbf{z}, \mathbf{x})) d\mathbf{x}^\mu d\mathbf{x}^\nu]$$

where we fixed the gauge $\mathbf{h}_{zz} = \mathbf{h}_{\mu z} = 0$ and we decompose the metric as

$$\mathbf{h}_{\mu\nu} = \mathbf{h}_{\mu\nu}^{\text{tt}} + \eta_{\mu\nu} \mathbf{h} + \partial_{(\mu} \mathbf{H}_{\nu)}$$

- Evaluating the on-shell SUGRA action

$$\mathbf{S} = \mathbf{S}_{5\text{D}}^{\text{B}} + \mathbf{S}_{\text{GH}} \quad , \quad \mathbf{S}_{\text{GH}} = \int d^4\mathbf{x} \sqrt{-\mathbf{g}} \mathcal{K}$$

one gets a **boundary term**, only.

Hol Ren: Bosonic sector

- The boundary action (to linear order) reads

$$S_{\text{bdy}}^{\text{B}} = \int d^4x \sqrt{-g} \left[\left(3 - \frac{3zF'}{2F} \right) (1 + 2h + \dots) + 2z\phi'\varphi \right]$$

$\textcircled{F^2/z^4}$ \nearrow

- $S_{\text{bdy}}^{\text{B}}$ diverges at $z = 0$. The (SUSY preserving scheme) c.t. action is

[BIANCHI-FREEDMAN-SKENDERIS '01 B + OTHERS]

$$S_{\text{c.t.}}^{\text{B}} = - \int_{z=\epsilon} d^4x \sqrt{-g} 3 \mathbf{W}(\phi + \varphi)$$

with \mathbf{W} the SUGRA superpotential evaluated at the bdy.

- The end result is

$$S_{\text{ren}}^{\text{B}} = - \int d^4x [\mathbf{a}\beta (1 + 2h_0 + \dots) + 4\beta\varphi_0]$$

Consistently, the renormalized action vanishes for $\beta = 0$.

Hol Ren: Fermionic sector

- As for the fermions, we need to work at quadratic order

$$S_{5D}^F = \int d^5x \sqrt{-G} \left\{ -\bar{\Psi}_M \Gamma^{MNP} D_N \Psi_P - 2 \bar{\zeta} \Gamma^M D_M \zeta \right. \\ \left. + i \partial_N \phi (\bar{\zeta} \Gamma^M \Gamma^N \Psi_M - \bar{\Psi}_M \Gamma^N \Gamma^M \zeta) - 2 \mathcal{M}(\phi) \bar{\zeta} \zeta \right. \\ \left. + 2 \mathcal{N}(\phi) (\bar{\Psi}_M \Gamma^M \zeta + \bar{\zeta} \Gamma^M \Psi_M) + m(\phi) \bar{\Psi}_M \Gamma^{MN} \Psi_N \right\}$$

- In AAdS space has to be supplemented by a bdy term, to make it stationary on the EOM. We get

$$S_{\text{bdy}}^F = \int d^4x \sqrt{-g} \left\{ -\frac{1}{2} \bar{\Psi}_m \Gamma^{mn} \Psi_n - \bar{\zeta} \zeta \right\}$$

- We work in axial gauge $\Psi_z = 0$ and split the gravitino as

$$\Psi_m = \psi_m^{\text{tt}} + \partial_m \vartheta + \Gamma_m \chi$$

→ EOM for ψ_m^{tt} **decouple**, those for ϑ and χ **coupled** to ζ .

Hol Ren: Fermionic sector

- The bulk action vanishes on-shell, only bdy term matters ... and diverges. The action has ϵ^{-2} and $\log \epsilon$ divergent terms and should be **holographically renormalized**. The **counter-term** action is

$$S_{\text{c.t.}}^{\text{F}} = \int_{\mathbf{z}=\epsilon} d^4 \mathbf{x} \sqrt{-g} \left\{ \frac{1}{2} \bar{\Psi}_m \Gamma^{mnr} \partial_r \Psi_n + \log \epsilon \left[-2 \bar{\zeta} \not{\partial} \zeta \right. \right. \\ - \frac{1}{4} \bar{\Psi}_m \Gamma^{mnr} \square \partial_r \Psi_n + \frac{1}{3} \phi^2 \bar{\Psi}_m \Gamma^{mnr} \partial_r \Psi_n \\ - \frac{1}{6} (\partial_n \bar{\Psi}_m \Gamma^{mn}) \not{\partial} (\Gamma^{rs} \partial_r \Psi_s) \\ \left. \left. - \frac{2}{3} i \phi [\bar{\zeta} (\Gamma^{rs} \partial_r \Psi_s) - (\partial_n \bar{\Psi}_m \Gamma^{mn}) \zeta] \right] \right\}$$

where m, n, \dots 4d curved indices.

Hol Ren: Fermionic sector

- The end result for the renormalized action is

$$\mathbf{S}_{\text{ren}}^{\text{F}} = \int d^4\mathbf{x} \left\{ \mathbf{i} \frac{\beta \mathbf{a}}{2} \bar{\vartheta}_0 \not{\partial} \vartheta_0 + 2\beta \vartheta_0 (\mathbf{i} \zeta_0 + \mathbf{a} \chi_0) \right. \\ \left. - 2\beta \bar{\vartheta}_0 (\mathbf{i} \bar{\zeta}_0 - \mathbf{a} \bar{\chi}_0) + \text{non-local} + \text{scheme-dep} \right\}$$

where ζ_0 , ϑ_0 and χ_0 are bdy leading modes of hyperino and gravitino, resp.

- Note:** **local** dependence of *all* subleading modes on leading ones can be uniquely fixed *just* by near boundary analysis + on-shell SUSY invariance of EOM! In particular we find

$$\tilde{\zeta}_1 = -\mathbf{i}\beta\vartheta_0 + \not{\partial}\mathbf{f}(\square)(\bar{\zeta}_0 + \mathbf{i}\mathbf{a}\bar{\chi}_0) + \bar{\mathbf{f}}(\square)\mathbf{a}(\zeta_0 - \mathbf{i}\mathbf{a}\chi_0)^{\text{T}}$$

Holographic Ward identities

- The **field/operator map** can be read from couplings between FZ and \mathcal{O} multiplets with bulk fields at the boundary. This is

$$\int d^4x \left[\frac{1}{2} h_0^{\mu\nu} T_{\mu\nu} + \frac{1}{2} (i\Psi_0^\mu S_\mu + \text{c.c.}) + \right. \\ \left. + 2(\varphi_0 \mathcal{O}_F + \text{c.c.}) - \sqrt{2} (i\zeta_0 \mathcal{O}_\psi + \text{c.c.}) + \dots \right]$$

which gives

$$h_0^{\mu\nu} \longleftrightarrow \frac{1}{2} T_{\mu\nu} \quad , \quad \Psi_0^{\mu\alpha} \longleftrightarrow \frac{i}{2} S_{\mu\alpha} \\ \varphi_0 \longleftrightarrow 2 \mathcal{O}_F \quad , \quad \zeta_0^\alpha \longleftrightarrow -i\sqrt{2} \mathcal{O}_{\psi\alpha}$$

- As far as gravitational sector, the relevant maps hence is

$$h_0 \longleftrightarrow \frac{1}{2} T \quad , \quad \vartheta_0^\alpha \longleftrightarrow -\frac{i}{2} \partial^\mu S_{\mu\alpha} \quad , \quad \bar{\chi}_{0\dot{\alpha}} \longleftrightarrow \frac{1}{2} \bar{\sigma}^{\mu\dot{\alpha}\alpha} S_{\mu\alpha}$$

Holographic Ward identities

- Using AdS/CFT prescription, from $\mathbf{S}_{\text{ren}}^{\text{B}}$ we get

$$\langle \mathbf{T} \rangle = 2 \frac{\delta \mathbf{S}_{\text{ren}}^{\text{B}}}{\delta \mathbf{h}_0} = -4\beta \mathbf{a} , \quad \langle \mathcal{O}_{\text{F}} \rangle = \frac{1}{2} \frac{\delta \mathbf{S}_{\text{ren}}^{\text{B}}}{\delta \varphi_0} = -2\beta$$

and from $\mathbf{S}_{\text{ren}}^{\text{F}}$

$$\langle \partial^\mu \mathbf{S}_{\mu\alpha} (\sigma^\nu \bar{\mathbf{S}}_\nu)_\beta \rangle = -4 \frac{\delta^2 \mathbf{S}_{\text{ren}}^{\text{F}}}{\delta \vartheta_0^\alpha \delta \chi_0^\beta} = -8\beta \mathbf{a} \varepsilon_{\alpha\beta}$$

$$\langle \partial^\mu \mathbf{S}_{\mu\alpha} \mathcal{O}_{\psi\beta} \rangle = \sqrt{2}\mathbf{i} \frac{\delta^2 \mathbf{S}_{\text{ren}}^{\text{F}}}{\delta \vartheta_0^\alpha \delta \zeta_0^\beta} = -2\sqrt{2}\beta \varepsilon_{\alpha\beta}$$

which **exactly** reproduce the QFT Ward identities!

- Note:** dependence of $\mathbf{S}_{\text{ren}}^{\text{F}}$ on combination $\mathbf{i}\zeta_0 + \mathbf{a}\chi_0$ provides holographic derivation of QFT identity $\sigma^\mu \bar{\mathbf{S}}_\mu = 2\sqrt{2}\lambda \mathcal{O}_\psi$. Corresponding bosonic operator identity from $\mathbf{S}_{\text{ren}}^{\text{B}}$, too.

Summary

- We provided a holographic description of a general class of SQFT where SUSY is broken at strong coupling.
- Performing holographic renormalization in the **gravitino** sector, we have recovered a set of Ward identities, which encodes the presence of the **Goldstino** mode. Non-trivial check of AdS / CFT!
- Focus was on a prototype model, but results have wider applicability → **Notice**: relevant contact terms do not depend on deep interior, or on nature of IR singularity. This is consistent with QFT expectations!

Outlook

- **Short-term** goal:
 - Repeat the analysis for non-AAAdS backgrounds, where QFT ~~SUSY~~ vacua arise due to strong coupling dynamics (like in **cascading theories**, cfr. KS, quiver completions of ISS-like models, ...).
[WORK IN PROGRESS W/ MUSSO-PAPADIMITRIOU&RAJ]
- **Longer-term** goals:
 - Full control on fermionic sector, in particular gravitino's, could allow for holographic derivation of Goldstino **effective action**.
[Cfr. KOMARGODSKI-SEIBERG '09, HOYOS ET AL. '13]
 - Thorough study of **stability properties** of e.g. proposed SUSY breaking (string theory) vacua.
[Cfr. ARGURIO-MUSSO-REDIGOLO '14]