

Heavy hadrons decays

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On behalf of the BaBar Collaboration

Outline

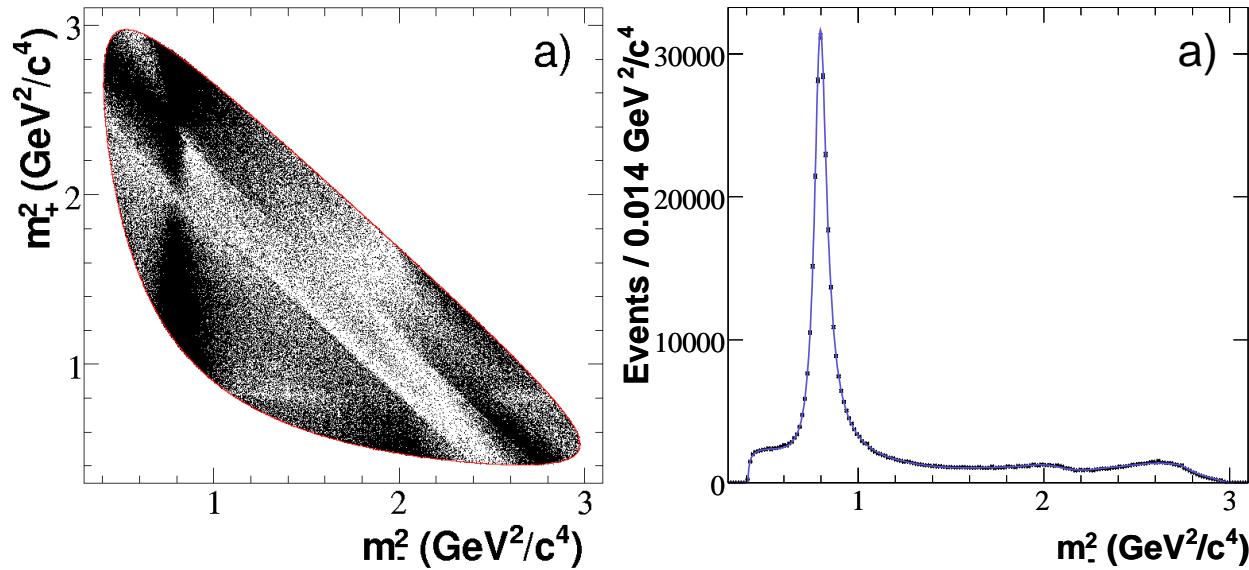
- Measurement of the $I=1/2$ $K\pi$ S -wave amplitude from a Dalitz plot analysis of η_c decays in two-photon interactions.
- Dalitz plot analysis of three J/ψ three-body hadronic decays.

IWHSS17, Cortona, Italy

() Work supported in part by Jefferson Lab, VA, USA*

The problem of the $K\pi$ S-wave. (I)

- An accurate description of the $K\pi$ S-wave is a fundamental issue for many important physics topics.
- Examples are: Measurement of γ , study and search for CP violations in heavy flavors decays through Dalitz plot analyses of 3-body or 4-body decays.
- $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ Dalitz plot and $K_S^0 \pi^-$ mass² projection.



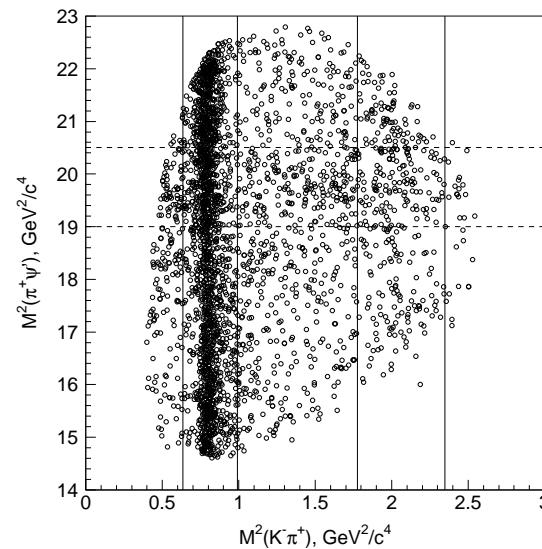
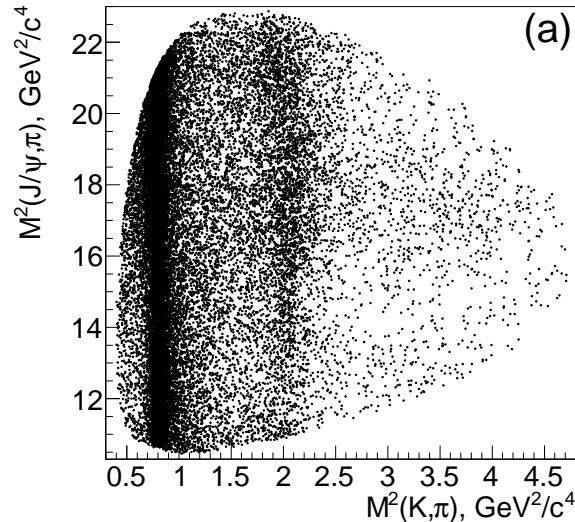
- Very high statistics are currently available.
- Strong resonance production along the $K\pi$ axis which need to be correctly described. (from arXiv:0804.2089, BaBar: Phys.Rev.D78:034023,2008)

The problem of the $K\pi$ S-wave. (II)

- Recent evidences/observations of exotic states in the decays of heavy flavors.
- The observation of the Z particles in $B \rightarrow \psi/\psi' K\pi$ is strongly correlated with an accurate description of the Dalitz plot.
- In this description we also need a full understanding of the K^* resonances and in particular of the $K\pi$ S-wave.
- Data from Belle.

(arXiv:0905.2869, Phys. Rev. D 80:031104, 2009), (arXiv:1408.6457, Phys. Rev. D 90, 112009 (2014))

$$\bar{B}^0 \rightarrow J/\psi K^- \pi^+ \quad B \rightarrow \psi' \pi^+ K$$

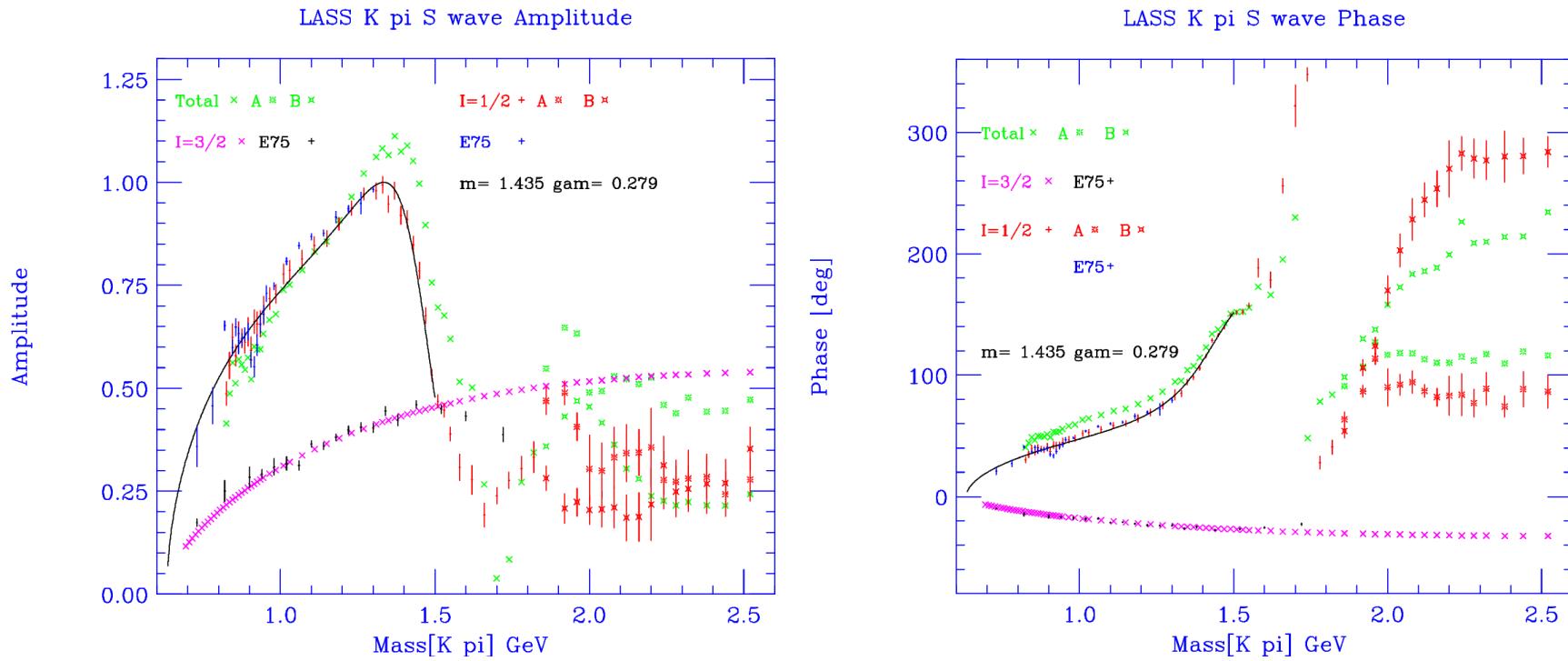


- Strong resonance production along the $K\pi$ axis.

The $K\pi$ S-wave from LASS

- The best measurement of the $K\pi$ S-wave comes from the LASS (Nucl. Phys. B296, 493 (1988))

$$K^- p \rightarrow K^- \pi^+ p$$
- The $K\pi$ S-wave is described by a coherent sum of a scattering length and a relativistic Breit-Wigner.
- However the LASS PWA is affected by a two-fold ambiguity for $m(K\pi) > 1.9$ GeV.

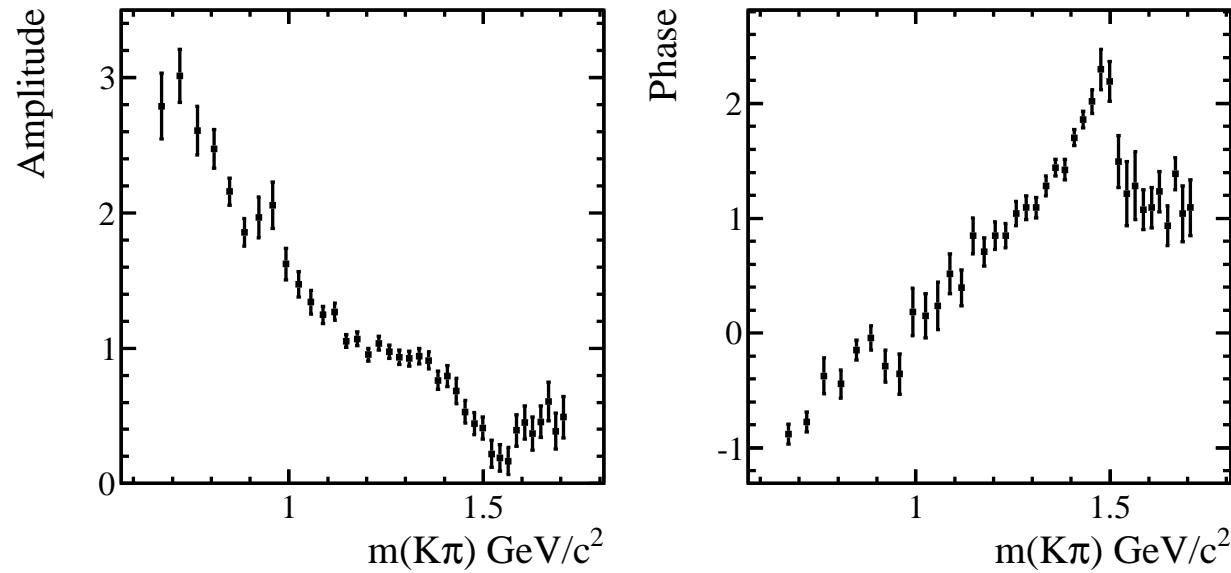


The $K\pi$ S -wave from D^+ decays

- Other measurements of the $K\pi$ S -wave come from the Dalitz plot analysis of:

$$D^+ \rightarrow K^- \pi^+ \pi^+$$

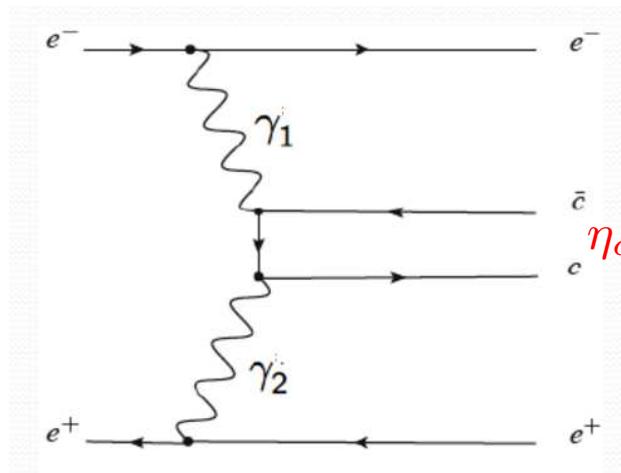
- The Model Independent Partial Wave method was introduced for the first time (E791, arXiv:hep-ex/0506040v2, Phys. Rev. D 73, 032004)).
- The $K\pi$ S -wave amplitude and phase was measured up to a mass of 1.5 GeV.



- Also in this case the final state is affected by the presence of a not well known contribution from I=3/2.

The $K\pi$ S-wave from η_c decays

- Charmonium decays can be used to obtain new information on light meson spectroscopy.
- In e^+e^- interactions, samples of charmonium decays can be obtained using different processes.
- In two-photon interactions we select events in which the e^+ and e^- beam particles are scattered at small angles and remain undetected.
- Only resonances with $J^{PC} = 0^{\pm+}, 2^{\pm+}, 3^{++}, 4^{\pm+} \dots$ can be produced.



Use of η_c decays produced in two-photon interactions

- In these BaBar analyses we make use of the following final states.

$$\gamma\gamma \rightarrow K_S^0 K^+ \pi^- \text{ (*)},$$

$$\gamma\gamma \rightarrow K^+ K^- \pi^0,$$

$$\begin{aligned}\gamma\gamma &\rightarrow K^+ K^- \eta \\ &\quad \rightarrow \gamma\gamma \\ &\quad \rightarrow \pi^+ \pi^- \pi^0\end{aligned}$$

- We find that the η_c three-body hadronic decays proceed almost entirely through:

$$\eta_c \rightarrow pseudoscalar + scalar$$

- Therefore three body decays of the η_c are a unique window to study the properties of the scalar mesons.

(*) Charge conjugation is implied through all this work.

Phys.Rev. D89 (2014) no.11, 112004, Phys.Rev. D93 (2016) 012005

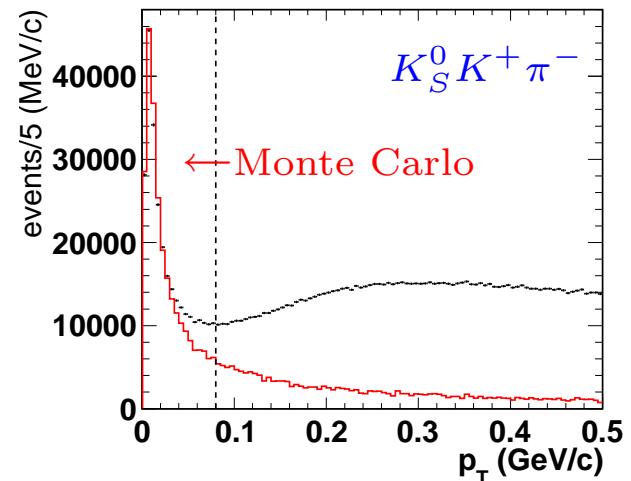
Example of selection: $\gamma\gamma \rightarrow K_S^0 K^+ \pi^-$

- Select events having only four tracks.
- p_T : transverse momentum of the $K_S^0 K^+ \pi^-$ system with respect to the beam axis.

- The signal at low p_T evidences the presence of two-photon events. We require $p_T < 0.08 \text{ GeV}/c$.

- We define M_{rec}^2 as:

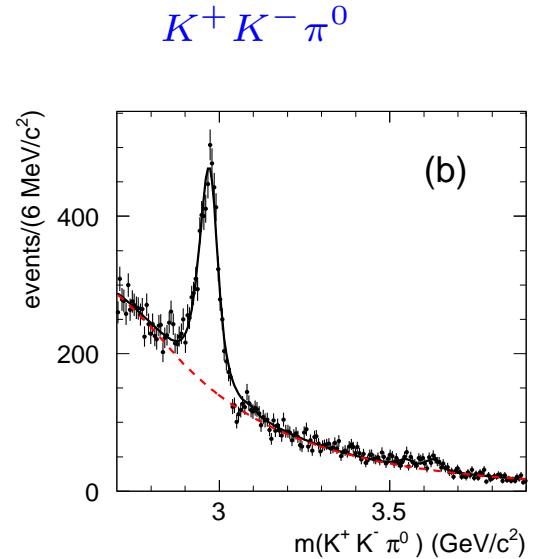
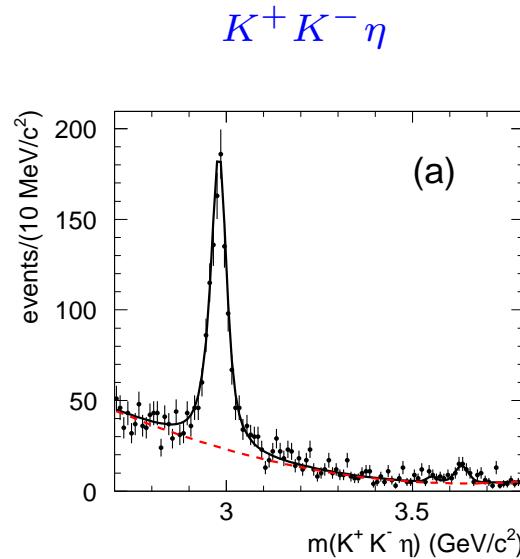
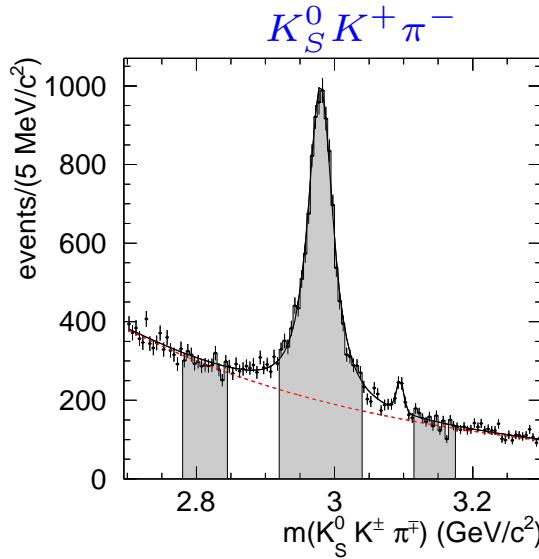
$$M_{\text{rec}}^2 \equiv (p_{e^+ e^-} - p_{\text{rec}})^2$$



- $p_{e^+ e^-}$ is the four-momentum of the initial state and p_{rec} is the four-momentum of the $K_S^0 K^+ \pi^-$ system.
- We remove ISR events requiring $M_{\text{rec}}^2 > 10 \text{ GeV}^2/c^4$.

The $K\bar{K}\pi$ mass spectra in the η_c region

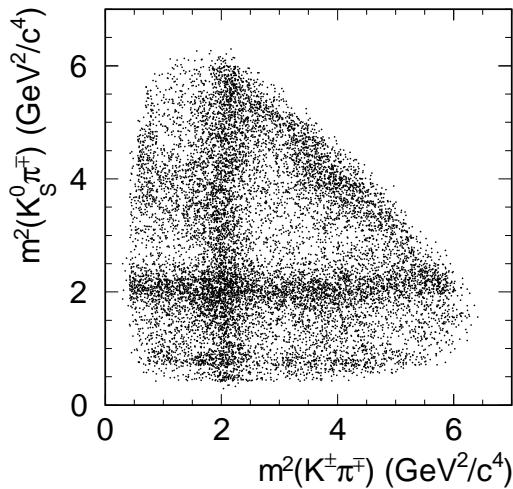
- $\eta_c \rightarrow K_S^0 K^\pm \pi^\mp$, 12849 evts with $(64.3 \pm 0.4)\%$ purity.
- $\eta_c \rightarrow K^+ K^- \pi^0$, 6494 evts with $(55.2 \pm 0.6)\%$ purity.
- $\eta_c \rightarrow K^+ K^- \eta$, 1161 evts with $(76.1 \pm 1.3)\%$ purity.



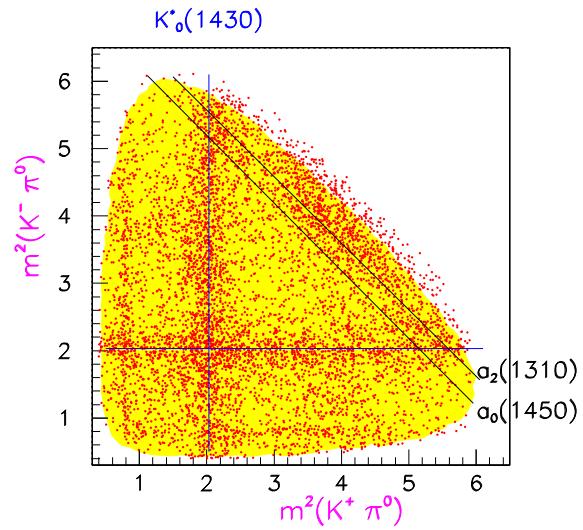
- $Purity = Signal / (Signal + Background)$

Dalitz plots.

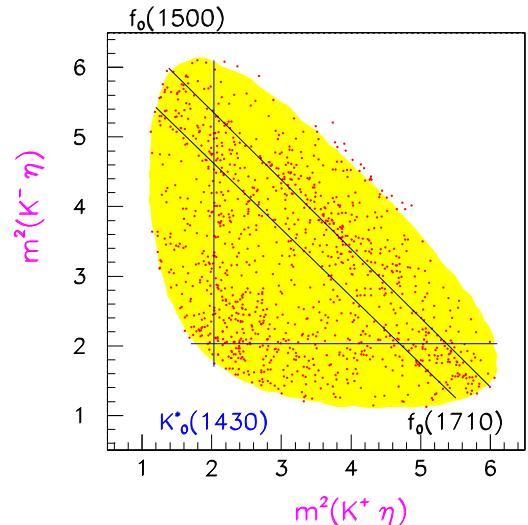
$\eta_c \rightarrow K_S^0 K^+ \pi^-$



$\eta_c \rightarrow K^+ K^- \pi^0$



$\eta_c \rightarrow K^+ K^- \eta$



- Dominated by the presence of scalar mesons.
- In particular, strong contribution from $K_0^*(1430)$ in the three Dalitz plots.

Dalitz plot analysis

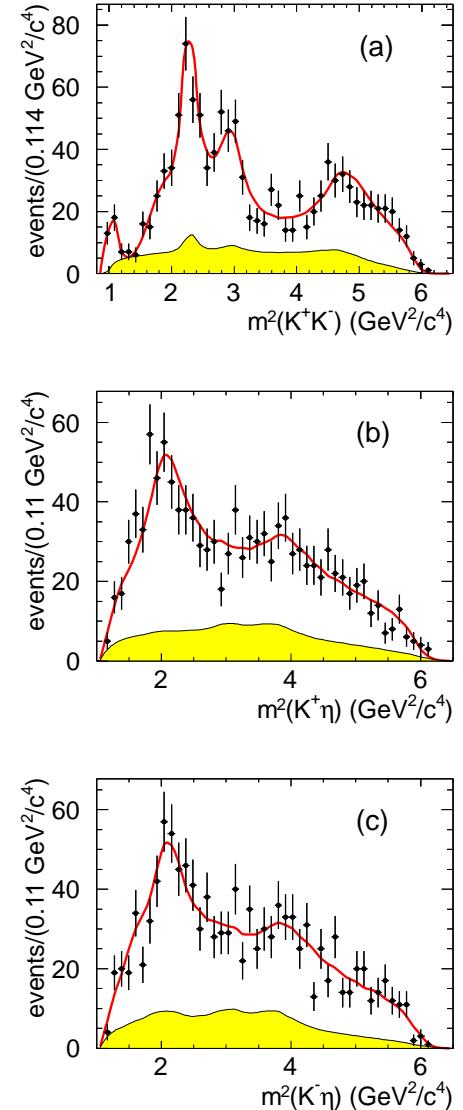
- Unbinned maximum likelihood fits.
- Fits performed using:
 - Isobar model: resonances described by Breit-Wigner functions.
(D. Asner, "Review of Particle Physics", Phys. Lett. B 592, 1 (2004)).
 - Model Independent Partial Wave Analysis (MIPWA) (Phys. Rev. D 73, 032004 (2006)).
 - The $K\pi$ S -wave (A_1) is taken as the reference amplitude.
$$A = A_1 + c_2 A_2 e^{i\phi_2} + c_3 A_3 e^{i\phi_3} + \dots$$
 - The $K\pi$ mass spectrum is divided into 30 equally spaced mass intervals 60 MeV wide and for each bin we add to the fit two new free parameters, the amplitude and the phase of the $K\pi$ S -wave (constant inside the bin).
 - We also fix the A_1 amplitude to 1.0 and its phase to $\pi/2$ in an arbitrary interval of the mass spectrum (bin 14 which corresponds to a mass of 1.45 GeV/ c^2).
 - The number of additional free parameters is therefore 58.

$\eta_c \rightarrow \eta K^+ K^-$ Dalitz plot analysis.

- Results from the Dalitz analysis and fit projections.
- Charge conjugated amplitudes symmetrized.

Final state	Fraction %	Phase (radians)
$f_0(1500)\eta$	$23.7 \pm 7.0 \pm 1.8$	0.
$f_0(1710)\eta$	$8.9 \pm 3.2 \pm 0.4$	$2.2 \pm 0.3 \pm 0.1$
$f_0(2200)\eta$	$11.2 \pm 2.8 \pm 0.5$	$2.1 \pm 0.3 \pm 0.1$
$f_0(1350)\eta$	$5.0 \pm 3.7 \pm 0.5$	$0.9 \pm 0.2 \pm 0.1$
$f_0(980)\eta$	$10.4 \pm 3.0 \pm 0.5$	$-0.3 \pm 0.3 \pm 0.1$
$f'_2(1525)\eta$	$7.3 \pm 3.8 \pm 0.4$	$1.0 \pm 0.1 \pm 0.1$
$K_0^*(1430)^+ K^-$	$16.4 \pm 4.2 \pm 1.0$	$2.3 \pm 0.2 \pm 0.1$
$K_0^*(1950)^+ K^-$	$2.1 \pm 1.3 \pm 0.2$	$-0.2 \pm 0.4 \pm 0.1$
NR	$15.5 \pm 6.9 \pm 1.0$	$-1.2 \pm 0.4 \pm 0.1$
Sum	$100.0 \pm 11.2 \pm 2.5$	
χ^2/ν	87/65	

- Largest amplitudes are $f_0(1500)\eta$ and $K_0^*(1430)K$.
- First observation of $K_0^*(1430) \rightarrow \eta K$.

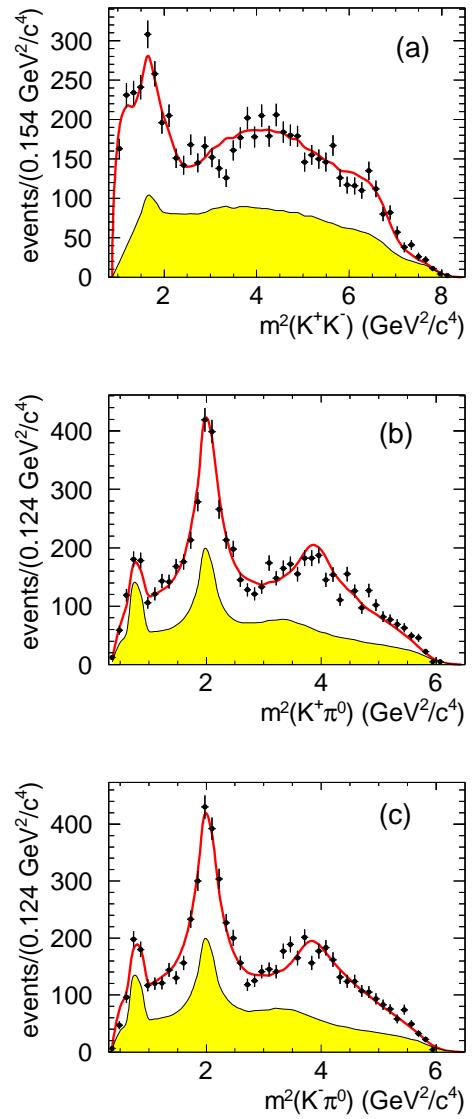


$\eta_c \rightarrow \pi^0 K^+ K^-$ Dalitz analysis.

- Results from the Dalitz analysis and fit projections.

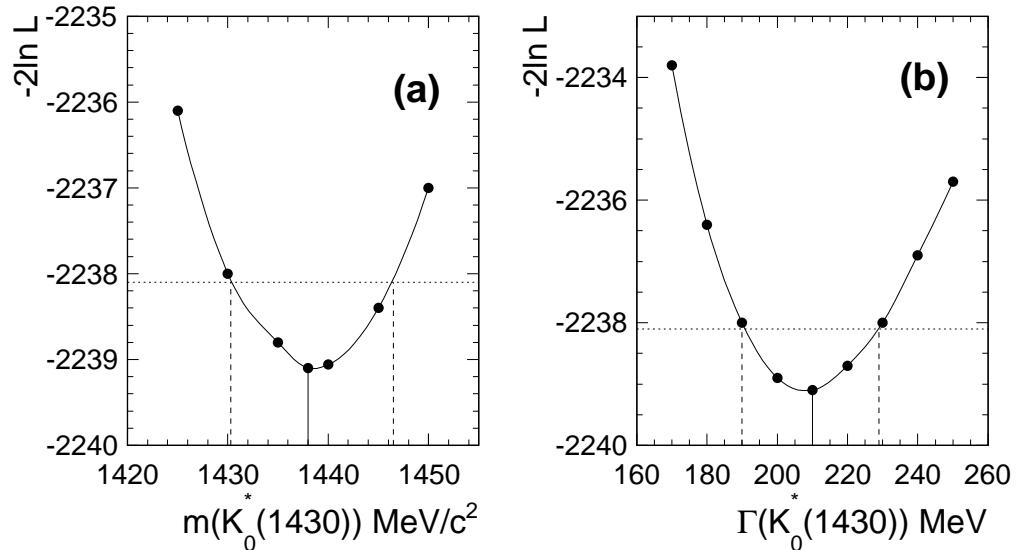
Final state	Fraction %			Phase (radians)
$K_0^*(1430)^+ K^-$	33.8 ±	1.9 ±	0.4	0.
$K_0^*(1950)^+ K^-$	6.7 ±	1.0 ±	0.3	-0.67 ± 0.07 ± 0.03
$K_2^*(1430)^+ K^-$	6.8 ±	1.4 ±	0.3	-1.67 ± 0.07 ± 0.03
$a_0(980)\pi^0$	1.9 ±	0.1 ±	0.2	0.38 ± 0.24 ± 0.02
$a_0(1450)\pi^0$	10.0 ±	2.4 ±	0.8	-2.4 ± 0.05 ± 0.03
$a_2(1320)\pi^0$	2.1 ±	0.1 ±	0.2	0.77 ± 0.20 ± 0.04
NR	24.4 ±	2.5 ±	0.6	1.49 ± 0.07 ± 0.03
Sum	85.8 ±	3.6 ±	1.2	
χ^2/ν	212/130			

- Largest amplitudes are $K_0^*(1430)K$ and $a_0(1450)\pi^0$.
- $K_1^*(890)K$ amplitude consistent with zero.
- Spin-one resonances consistent to originate entirely from background.
- The isobar model does not fit very well the data.



The $K_0^*(1430)$ parameters.

- In the $\eta_c \rightarrow \pi^0 K^+ K^-$ Dalitz plot analysis we scan the likelihood as a function of the $K_0^*(1430)$ mass and width.



- We obtain:

$$m(K_0^*(1430)) = 1438 \pm 8 \pm 4 \text{ MeV}/c^2$$

$$\Gamma(K_0^*(1430)) = 210 \pm 20 \pm 12 \text{ MeV}$$

$K_0^*(1430)$ branching fraction.

- The observation of $K_0^*(1430)$ in both $K\eta$ and $K\pi^0$ decay modes allows a measurement of the relative branching fraction.
- The Dalitz plot analysis of $\eta_c \rightarrow K^+ K^- \eta$ decay gives a total $K_0^*(1430)^+ K^-$ contribution of

$$f_{\eta K} = 0.164 \pm 0.042 \pm 0.010$$

- The Dalitz plot analysis of the $\eta_c \rightarrow K^+ K^- \pi^0$ decay mode gives a total $K_0^*(1430)^+ K^-$ contribution of

$$f_{\pi^0 K} = 0.338 \pm 0.019 \pm 0.004$$

- Using the measurement of $\mathcal{R}(\eta_c)$, we obtain the $K_0^*(1430)$ branching ratio

$$\frac{\mathcal{B}(K_0^*(1430) \rightarrow \eta K)}{\mathcal{B}(K_0^*(1430) \rightarrow \pi K)} = \mathcal{R}(\eta_c) \frac{f_{\eta K}}{f_{\pi K}} = 0.092 \pm 0.025^{+0.010}_{-0.025}$$

where $f_{\pi K}$ denotes $f_{\pi^0 K}$ after correcting for the $K^0 \pi$ decay mode.

Model Independent Partial Wave Analysis

- Interference between the two $K\pi$ modes is determined by Isospin conservation.
- For $\eta_c \rightarrow K_S^0 K^+ \pi^-$:

$$A_{S-wave} = \frac{1}{\sqrt{2}}(a_j^{K^+\pi^-} e^{i\phi_j^{K^+\pi^-}} + a_j^{\bar{K}^0\pi^-} e^{i\phi_j^{\bar{K}^0\pi^-}})$$

where $a^{K^+\pi^-}(m) = a^{\bar{K}^0\pi^-}(m)$ and $\phi^{K^+\pi^-}(m) = \phi^{\bar{K}^0\pi^-}(m)$

- For $\eta_c \rightarrow K^+ K^- \pi^0$:

$$A_{S-wave} = \frac{1}{\sqrt{2}}(a_j^{K^+\pi^0} e^{i\phi_j^{K^+\pi^0}} + a_j^{K^-\pi^0} e^{i\phi_j^{K^-\pi^0}})$$

where $a^{K^+\pi^0}(m) = a^{K^-\pi^0}(m)$ and $\phi^{K^+\pi^0}(m) = \phi^{K^-\pi^0}(m)$

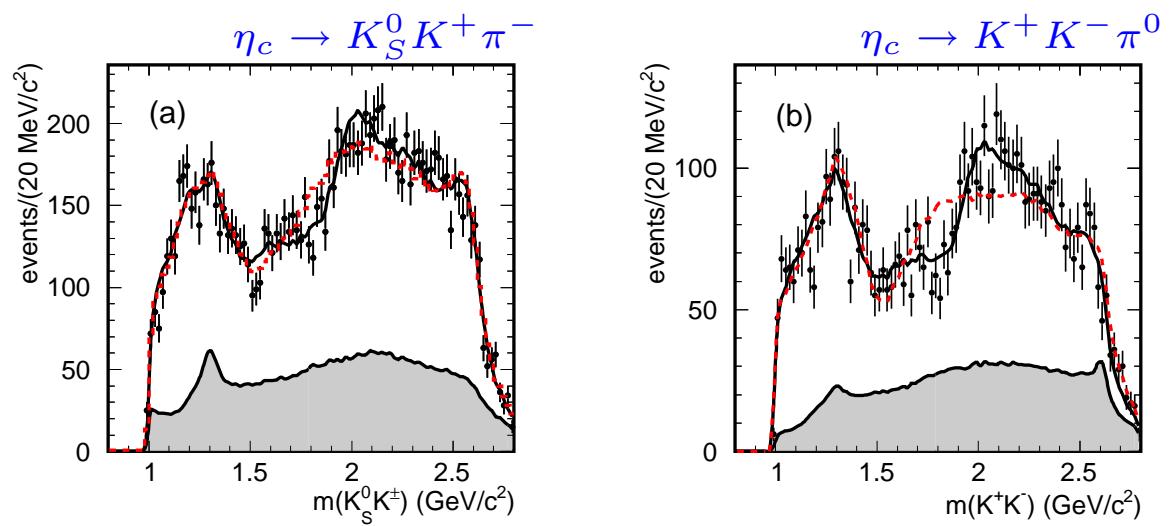
- The $K_2^*(1420)$, $a_0(980)$, $a_0(1400)$, $a_2(1310)$, ... contributions are modeled as relativistic Breit-Wigner functions multiplied by the corresponding angular functions.
- Backgrounds are fitted separately and interpolated into the η_c signal regions.

An additional $a_0(1950)$ resonance

- The fits improves when an additional high mass $a_0(1950) \rightarrow K\bar{K}$ I=1 resonance is included with free parameters in both η_c decay modes.
- The fits return the following parameters:

Final state	Mass (MeV/ c^2)	Width (MeV)
$\eta_c \rightarrow K_S^0 K^\pm \pi^\mp$	$1949 \pm 32 \pm 76$	$265 \pm 36 \pm 110$
$\eta_c \rightarrow K^+ K^- \pi^0$	$1927 \pm 15 \pm 23$	$274 \pm 28 \pm 30$
Weighted mean	$1931 \pm 14 \pm 22$	$271 \pm 22 \pm 29$

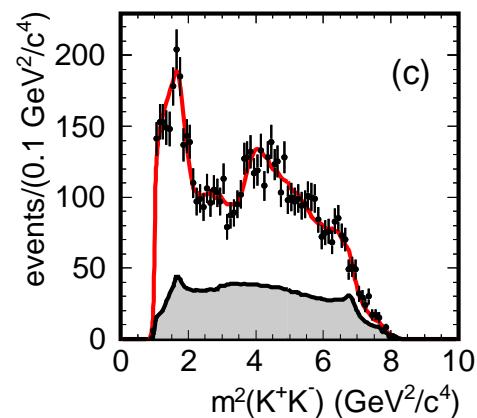
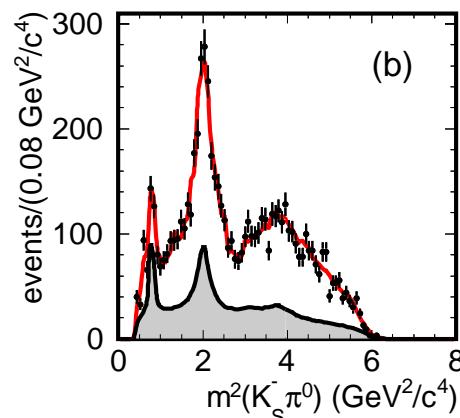
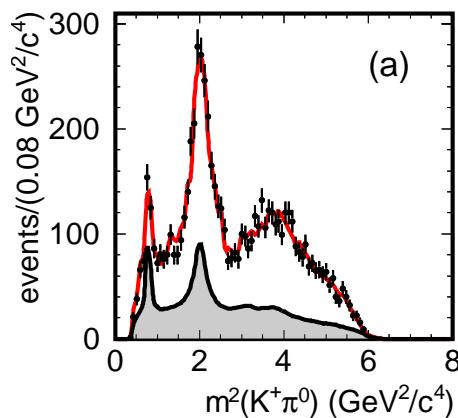
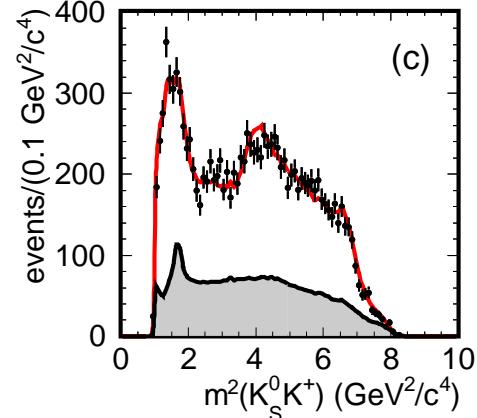
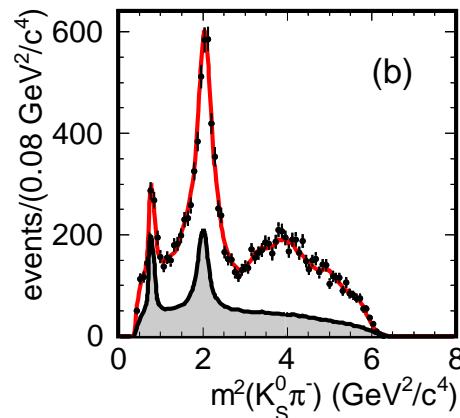
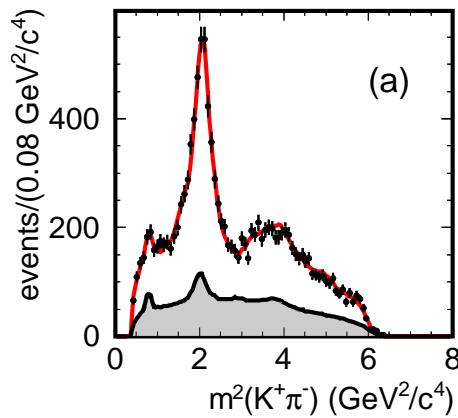
red line: no $a_0(1950)$



- Statistical significances for the $a_0(1950)$ effect (including systematics) are 2.5σ for $\eta_c \rightarrow K_S^0 K^+ \pi^-$ and 4.2σ for $\eta_c \rightarrow K^+ K^- \pi^0$.

Dalitz plots mass projections

- Dalitz plot projections with fit results for $\eta_c \rightarrow K_S^0 K^+ \pi^-$ (top) and $\eta_c \rightarrow K^+ K^- \pi^0$ (bottom)



- Shaded is contribution from the interpolated background.
- $K^*(890)$ contributions entirely from background.

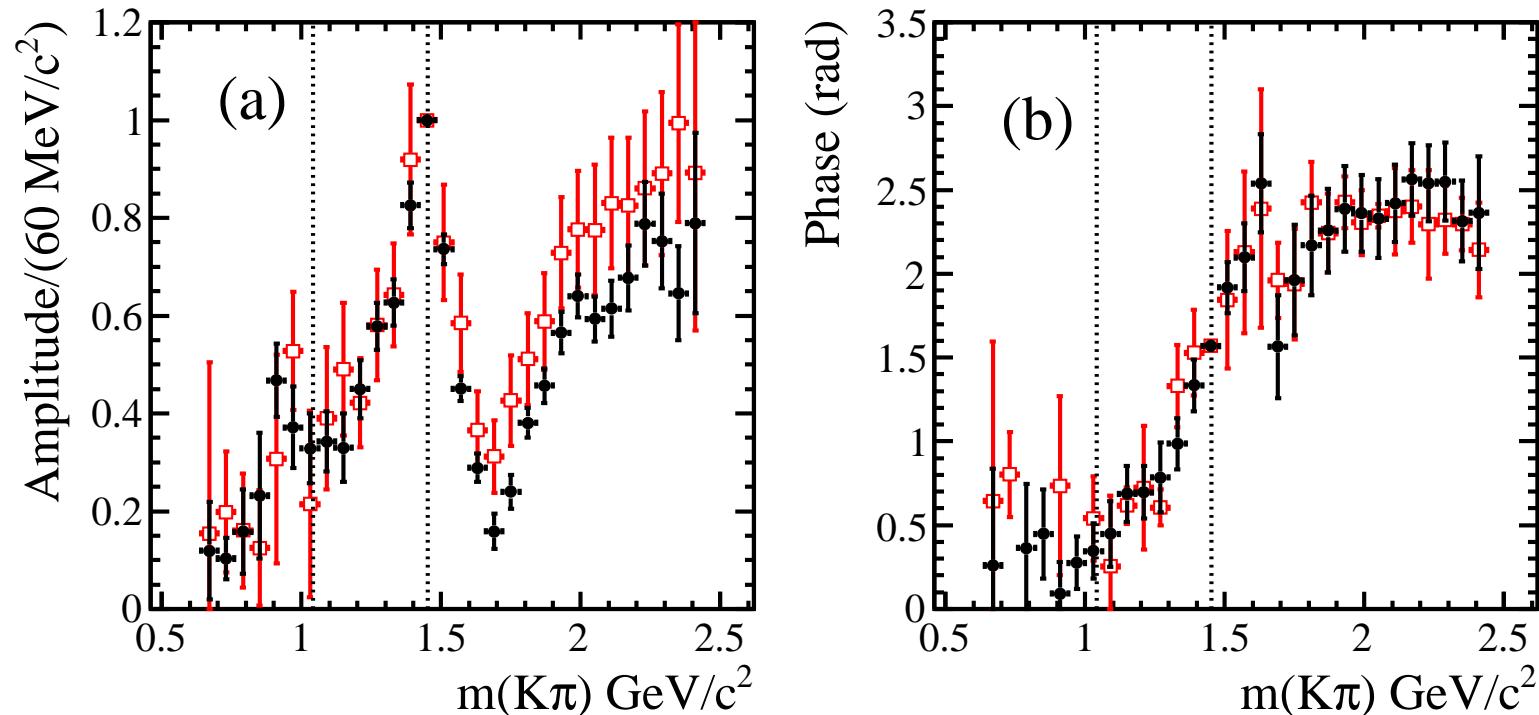
Fit fractions from the MIPWA. Comparison with the Isobar Model

	$\eta_c \rightarrow K_S^0 K^+ \pi^-$	$\eta_c \rightarrow K^+ K^- \pi^0$
Amplitude	Fraction (%)	Fraction (%)
$(K\pi \text{ } S\text{-wave}) K$	$107.3 \pm 2.6 \pm 17.9$	$125.5 \pm 2.4 \pm 4.2$
$a_0(1950)\pi$	$3.1 \pm 0.4 \pm 1.2$	$4.4 \pm 0.8 \pm 0.7$
$K_2^*(1430)^0 K$	$4.7 \pm 0.9 \pm 1.4$	$3.0 \pm 0.8 \pm 4.4$
χ^2/N_{cells}	$301/254=1.17$	$283.2/233=1.22$
Isobar Model		
$(K_0^*(1430)K) +$	73.6 ± 3.7	63.6 ± 5.6
$(K_0^*(1950)K) +$		
<i>Nonresonant</i>		
χ^2/N_{cells}	$457/254=1.82$	$383/233=1.63$

- For MIPWA, good agreement between the two η_c decay modes.
- $(K\pi \text{ } S\text{-wave}) K$ amplitude dominant with small contributions from $K_2^*(1430)^0 K$ and $a_0(1950)\pi$ amplitudes.
- Spin-1 resonances consistent to come entirely from background.
- Good description of the data with MIPWA.
- Worse description of the data with the Isobar Model.

The I=1/2 $K\pi$ S-wave

- Fitted amplitude and phase. Average systematic uncertainty is 16%.
- Red: $\eta_c \rightarrow K^+ K^- \pi^0$. Black: $\eta_c \rightarrow K_S^0 K^+ \pi^-$.
- Clear $K_0^*(1430)$ resonance and corresponding phase motion.
- At high mass broad $K_0^*(1950)$ contribution.



- Dashed lines are $K\eta$ and $K\eta'$ thresholds.
- Good agreement between the two η_c decay modes.

Comparison with the LASS and E791 experiments

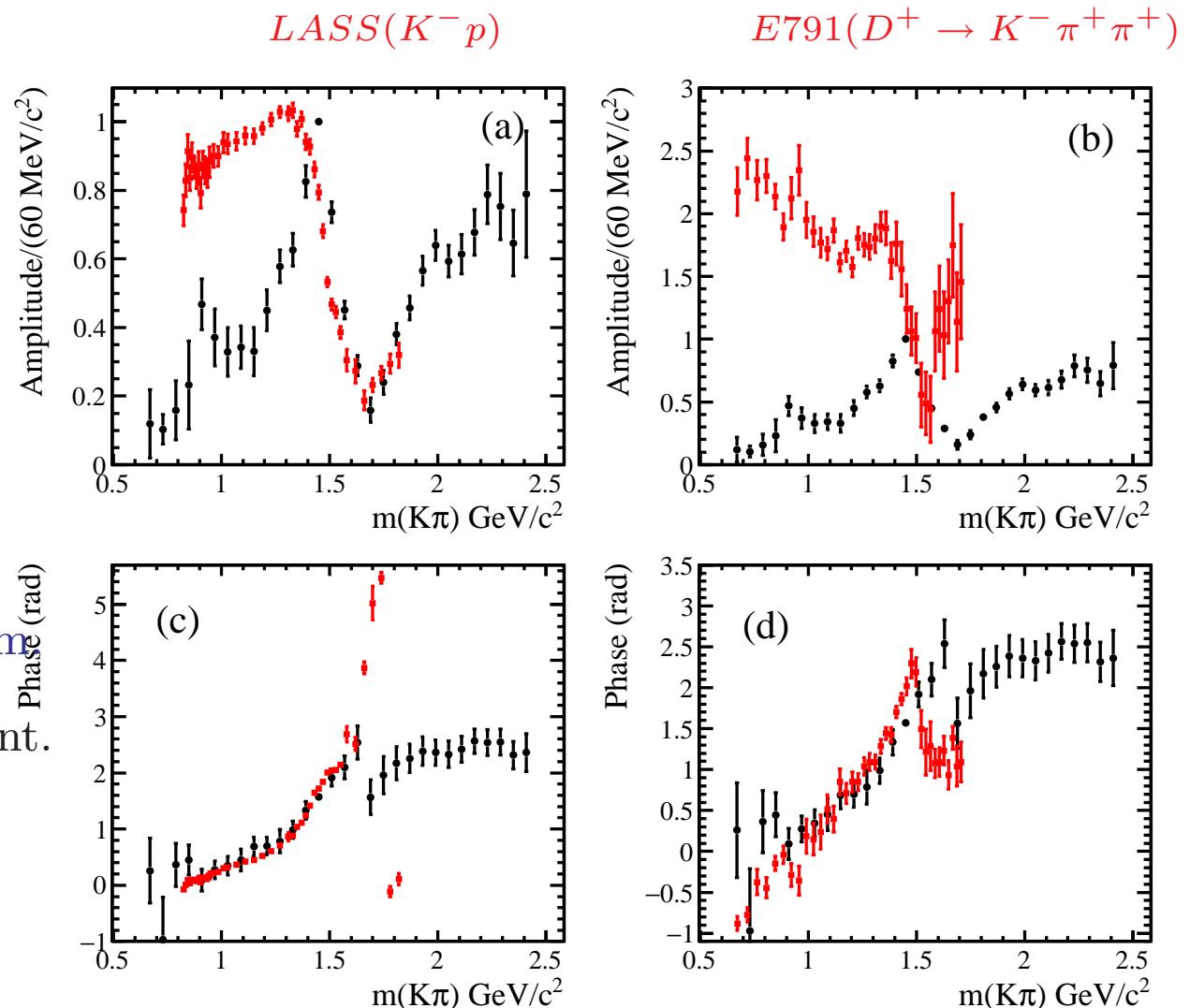
- Black is $\eta_c \rightarrow K_S^0 K^+ \pi^-$.

- Normalization is arbitrary.

- LASS analysis has two solutions above 1.9 GeV.

- Phases before the $K\eta'$ threshold are similar, as expected from Watson theorem

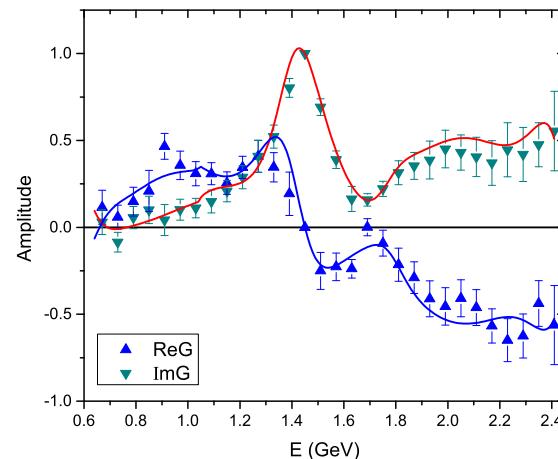
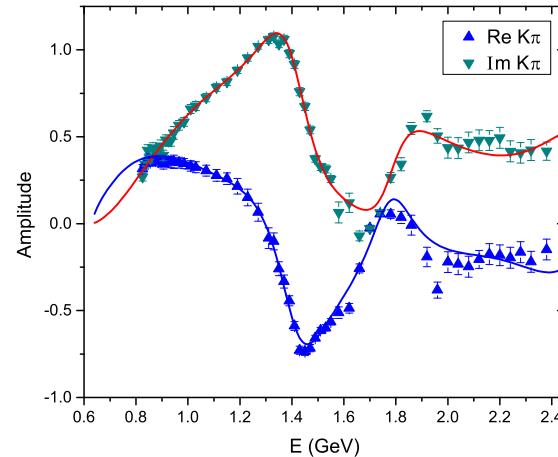
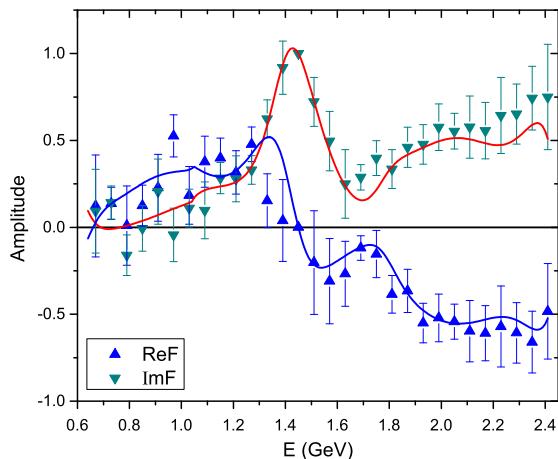
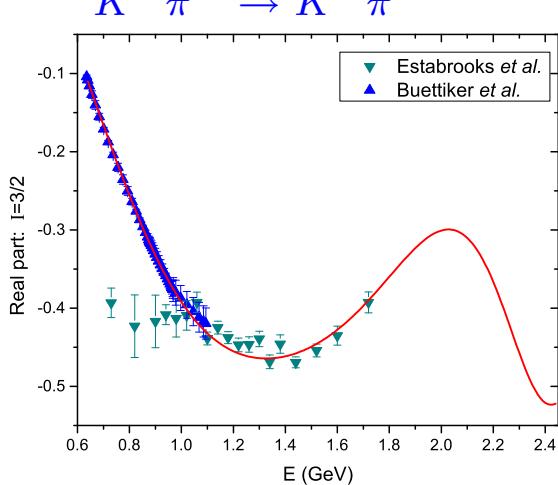
- Amplitudes are very different.



(LASS: Nucl. Phys. B **296**, 493 (1988)), (E791: Phys. Rev. D **73**, 032004 (2006)), (K.M. Watson, Phys. Rev. 88, 1163 (1952))

Overall fit of LASS and η_c data.

- K-matrix fit. (M. Pennington, arXiv:1701.04881)



- Data fitted in terms of Real and Imaginary parts of the complex amplitudes.
- Solution A for the LASS data.

Overall K-matrix fit of LASS and η_c data.

- Measured pole positions.

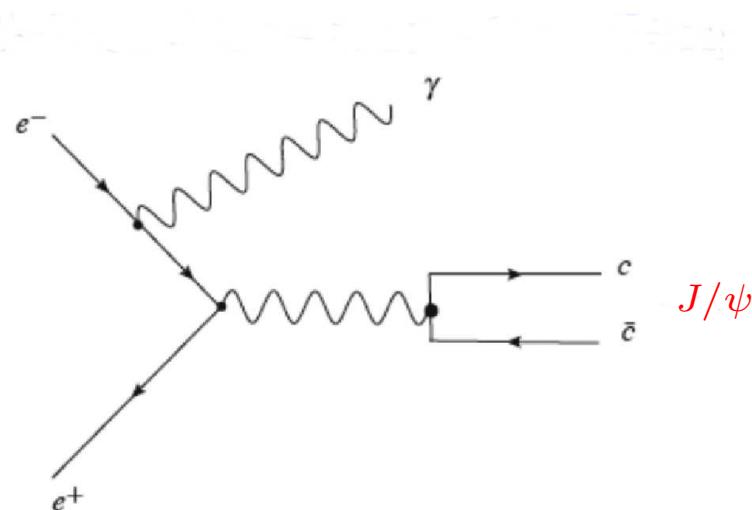
$$\text{Pole 1 } E_{P1} = 659 - i302 \text{ MeV on Sheet II,}$$
$$\text{Pole 2 } E_{P2} = 1409 - i128 \text{ MeV on Sheet III,} \quad (1)$$

$$\text{Pole 3 } E_{P3} = 1768 - i107 \text{ MeV on Sheet III.} \quad (2)$$

- Pole 1 is identified with the κ , the pole position of which was found to be at $[(658 \pm 7) - i(278 \pm 13)]$ MeV, in the dispersive analysis of (arXiv:0310283, Eur.Phys.J. C33, 409 (2004)).
- Pole 2 is identified with $K_0^*(1430)$, to be compared with $[(1438 \pm 8 \pm 4) - i(105 \pm 20 \pm 12)]$ MeV using the Breit-Wigner form.
- Pole 3 may be identified with the $K_0^*(1950)$ with a pole mass closer to that of the reanalysis of the LASS by Anisovich (Phys. Lett.B413, 137 (1997)) with a pole at $E = (1820 \pm 20) - i(125 \pm 50)$ MeV.
- For pole 2, the $K_0^*(1430)$, we have a ratio of $K\eta$ to $K\pi$ decay of 0.05 consistent with the branching ratio of $(0.092 \pm 0.025^{+0.010}_{-0.025})$ determined from the Dalitz plot analysis of $\eta_c \rightarrow K^+ K^- \eta/\pi^0$ decays.

Dalitz plot analysis of $J/\psi \rightarrow \text{three body decays. Introduction}$

- We make use of the Initial State Radiation (ISR) process to obtain clean J/ψ samples.
- In this process, we reconstruct events having a (mostly undetected) fast forward γ_{ISR} .



- Only $J^{PC} = 1^{--}$ states can be produced.

Dalitz plot analysis of $J/\psi \rightarrow \pi^+\pi^-\pi^0$, $J/\psi \rightarrow K^+K^-\pi^0$, and $J/\psi \rightarrow K_S^0K^\pm\pi^\mp$

- Only a preliminary result exists, to date, on a Dalitz-plot analysis of J/ψ decays to $\pi^+\pi^-\pi^0$ (SLAC-PUB-5674, (1991)).
- While large samples of J/ψ decays exist, some branching fractions remain poorly measured. In particular the $J/\psi \rightarrow K^+K^-\pi^0$ branching fraction has been measured by MarkII using only 25 events.
- $J/\psi \rightarrow K_S^0K^\pm\pi^\mp$ Dalitz plot analysis performed here for the first time.
- The BES III experiment has performed an angular analysis of $J/\psi \rightarrow K^+K^-\pi^0$. The analysis requires the presence of a broad $J^{PC} = 1^{--}$ state in the K^+K^- threshold region, which is interpreted as a multiquark state (Phys. Rev. Lett. **97**, 142002 (2006)).

Data selection

- We study the following reactions:

$$e^+ e^- \rightarrow \gamma_{\text{ISR}} \pi^+ \pi^- \pi^0,$$

$$e^+ e^- \rightarrow \gamma_{\text{ISR}} K^+ K^- \pi^0,$$

$$e^+ e^- \rightarrow \gamma_{\text{ISR}} K_S^0 K^\pm \pi^\mp,$$

where γ_{ISR} indicates the (undetected) ISR photon.

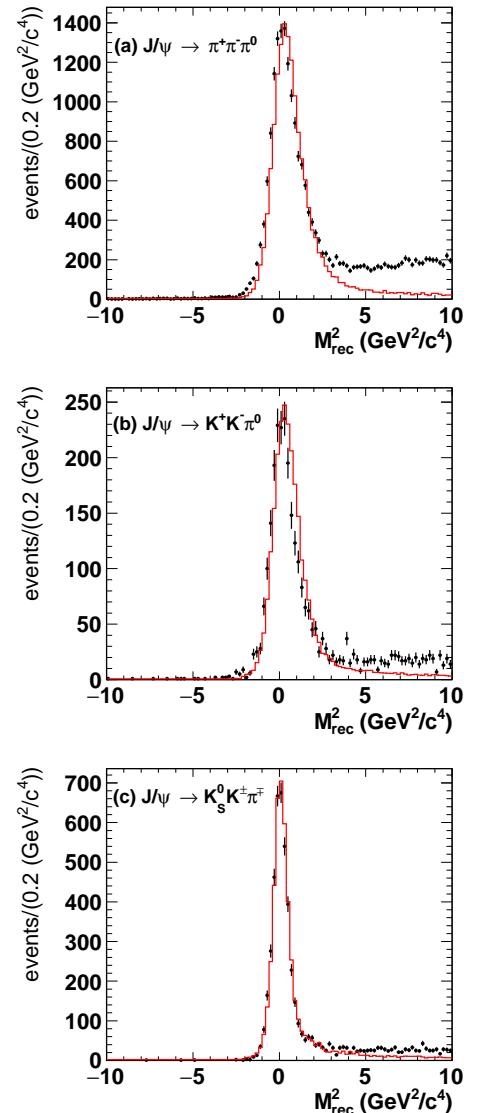
- We compute:

$$M_{\text{rec}}^2 \equiv (p_{e^-} + p_{e^+} - p_{h1} - p_{h2} - p_{h3})^2$$

where $h = \pi/K/K_S^0$.

- This quantity should peak near zero for ISR events.
- Plot of M_{rec}^2 in the J/ψ signal region.

In red are Monte Carlo simulations.

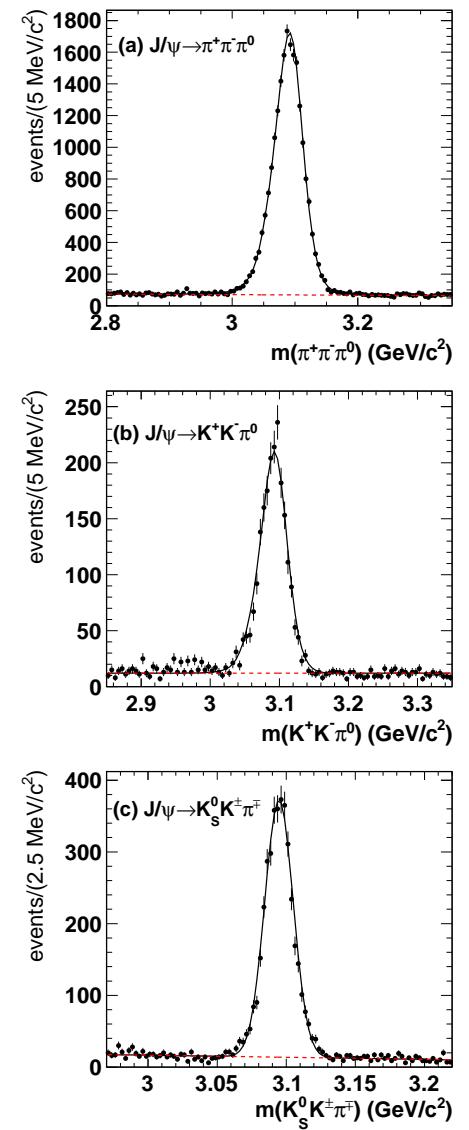


J/ ψ signals and yields

- We select events in the ISR region by requiring $|M_{\text{rec}}^2| < 2 \text{ GeV}^2/c^4$ ($|M_{\text{rec}}^2| < 1.5 \text{ GeV}^2/c^4$).

- We fit the mass spectra using the Monte Carlo resolution functions described by a Crystal Ball+Gaussian functions and obtain the yields:

J/ψ decay mode	Signal region (GeV/c^2)	Event yields	Purity %
$\pi^+\pi^-\pi^0$	3.028-3.149	20417	91.3 ± 0.2
$K^+K^-\pi^0$	3.043-3.138	2102	88.8 ± 0.7
$K_S^0 K^\pm \pi^\mp$	3.069-3.121	3907	93.1 ± 0.4



Branching fractions

- We measure the following branching fraction:

$$\mathcal{R}_1 = \frac{\mathcal{B}(J/\psi \rightarrow K^+ K^- \pi^0)}{\mathcal{B}(J/\psi \rightarrow \pi^+ \pi^- \pi^0)} = 0.120 \pm 0.003(\text{stat}) \pm 0.009(\text{sys}).$$

- PDG reports $\mathcal{B}(J/\psi \rightarrow \pi^+ \pi^- \pi^0) = (2.11 \pm 0.07) \times 10^{-2}$.
- Using $\mathcal{B}(J/\psi \rightarrow K^+ K^- \pi^0) = (2.8 \pm 0.8) \times 10^{-3}$ from Mark II (25 events), to be $(2.8 \pm 0.8) \times 10^{-3}$ we obtain $\mathcal{R}_1^{PDG} = 0.133 \pm 0.038$, in agreement with our measurement.
- We also measure the branching fraction:

$$\mathcal{R}_2 = \frac{\mathcal{B}(J/\psi \rightarrow K_S^0 K^\pm \pi^\mp)}{\mathcal{B}(J/\psi \rightarrow \pi^+ \pi^- \pi^0)} = 0.265 \pm 0.005(\text{stat}) \pm 0.021(\text{sys})$$

- Using $\mathcal{B}(J/\psi \rightarrow K_S^0 K^\pm \pi^\mp) = (26 \pm 7) \times 10^{-4}$ from Mark I (126 events), we obtain $\mathcal{R}_2^{PDG} = 0.123 \pm 0.033$, which deviates by 3.6σ from our measurement.

$J/\psi \rightarrow \pi^+\pi^-\pi^0$ Dalitz plot and projections ■

□ Dominated by three $\rho(770)\pi$ contributions.

□ Dalitz plot analysis performed using:

- Isobar model using Zemach tensors;

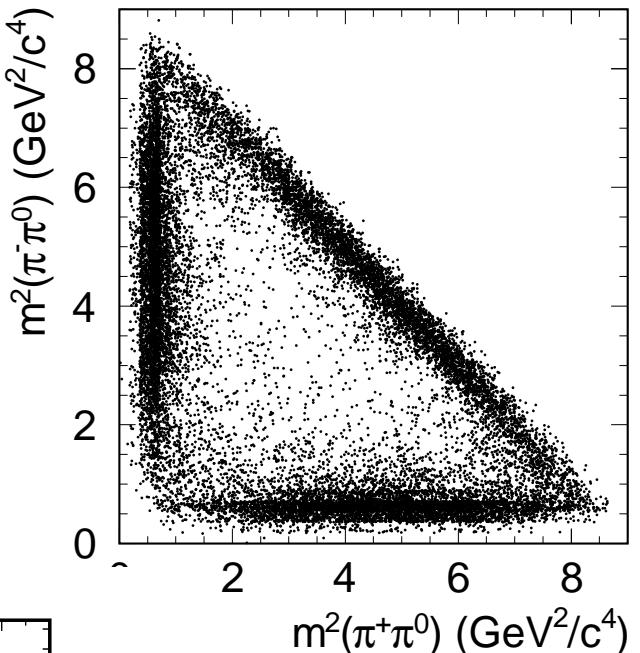
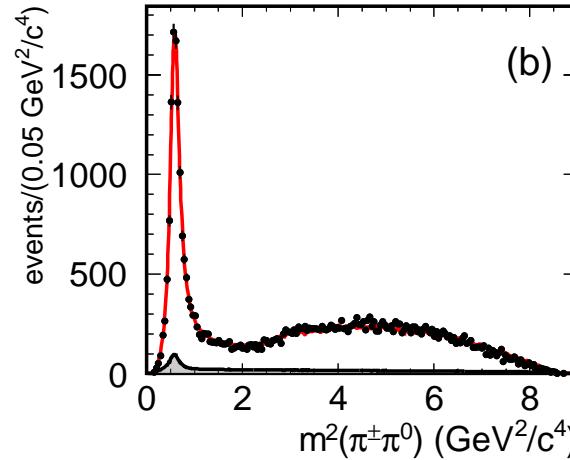
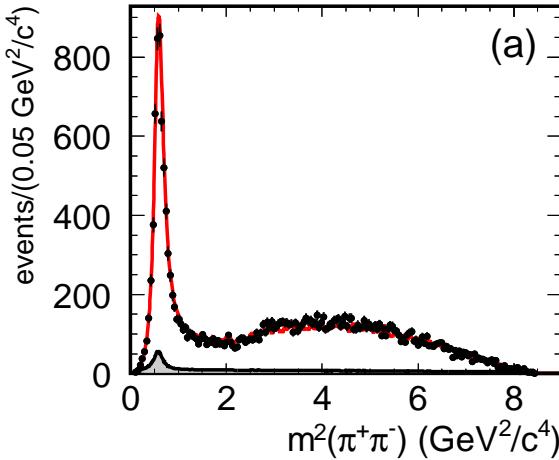
C. Zemach, Phys Rev. **133**, B1201 (1964),

C. Dionisi et. al., Nucl. Phys. **B169**, 1 (1980).

- Veneziano model.

(A. P. Szczepaniak, M.R. Pennington, Phys. Lett. **B737**, 283 (2014)).

□ Dalitz plot projections.

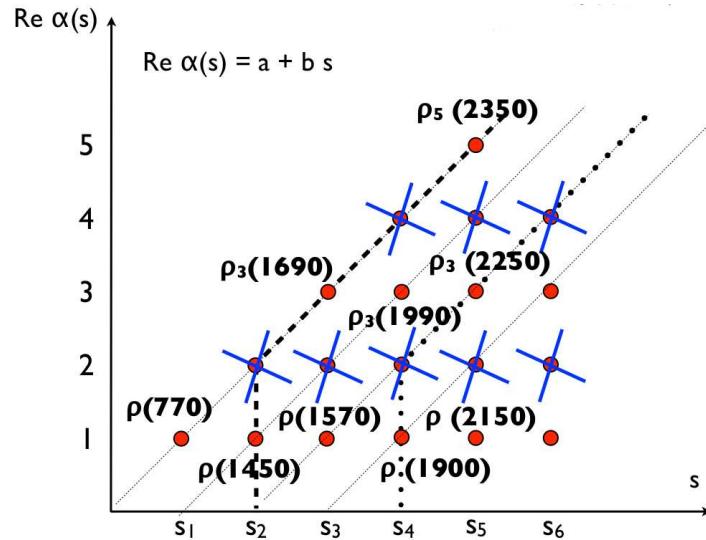


□ Shaded is the background interpolated by sidebands.

$J/\psi \rightarrow \pi^+ \pi^- \pi^0$ Dalitz plot analysis with Veneziano model

- The Veneziano model deals with trajectories rather than single resonances

(arXiv:1403.5782, Phys.Lett. N737, 283 (2014)).



- The amplitudes are written as:

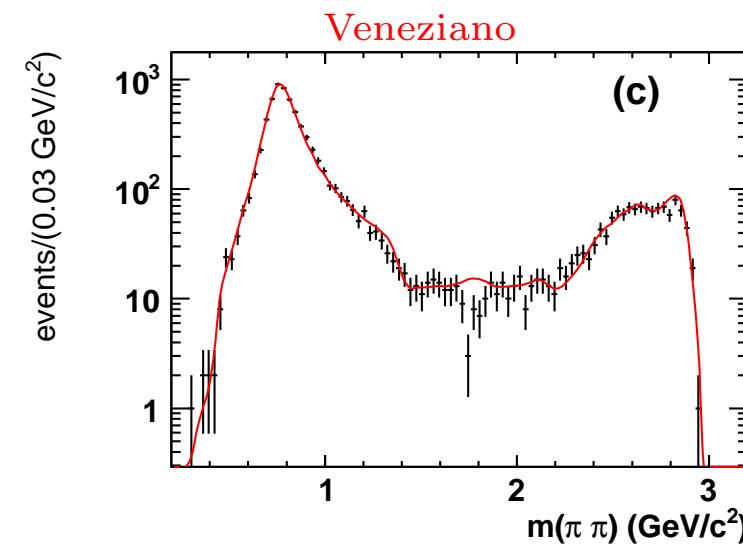
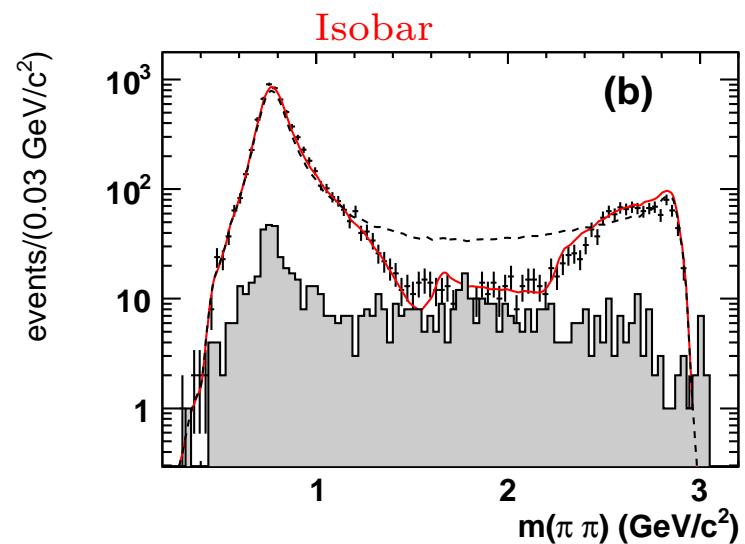
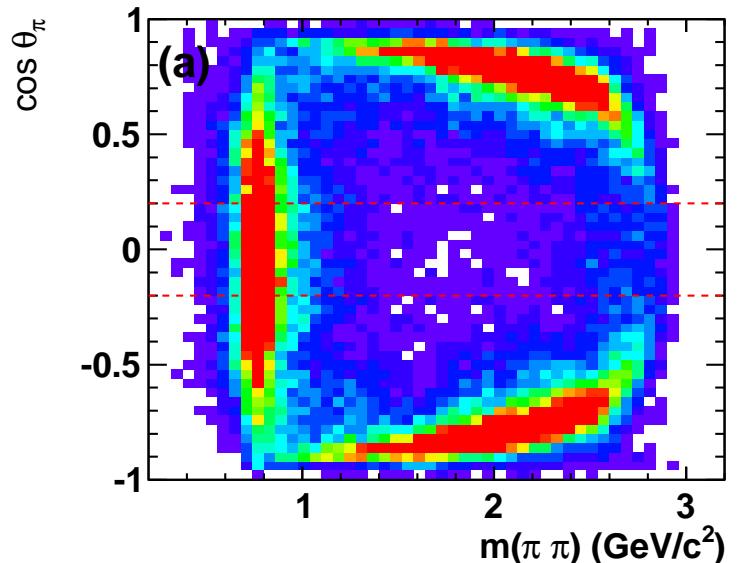
$$A_{X \rightarrow abc} = \sum_{n,m} c_X \rightarrow abc(n, m) A_{n,m}$$

with $1 \leq m \leq n$.

- The complexity of the model is related to n , the number of Regge trajectories included in the fit.
- The fit requires $n=7$, with 19 free parameters.

$J/\psi \rightarrow \pi^+ \pi^- \pi^0$ Dalitz plot analysis.

- Combinatorial π helicity angle vs. $m(\pi\pi)$.
- $m(\pi\pi)$ mass projections for $|\cos\theta_\pi| < 0.2$ in log scale.
- The cut removes the reflections from the other combinations.
- Dashed line: fit without ρ' contributions.



$J/\psi \rightarrow \pi^+ \pi^- \pi^0$ Dalitz plot analysis.

Final state	Amplitude	Isobar fraction (%)	Phase (radians)	Veneziano fraction (%)
$\rho(770)\pi$	1.	$114.2 \pm 1.1 \pm 2.6$	0.	133.1 ± 3.3
$\rho(1450)\pi$	0.513 ± 0.039	$10.9 \pm 1.7 \pm 2.7$	$-2.63 \pm 0.04 \pm 0.06$	0.80 ± 0.27
$\rho(1700)\pi$	0.067 ± 0.007	$0.8 \pm 0.2 \pm 0.5$	$-0.46 \pm 0.17 \pm 0.21$	2.20 ± 0.60
$\rho(2150)\pi$	0.042 ± 0.008	$0.04 \pm 0.01 \pm 0.20$	$1.70 \pm 0.21 \pm 0.12$	6.00 ± 2.50
$\omega(783)\pi^0$	0.013 ± 0.002	$0.08 \pm 0.03 \pm 0.02$	$2.78 \pm 0.20 \pm 0.31$	0.40 ± 0.08
$\rho_3(1690)\pi$				
Sum		$127.8 \pm 2.0 \pm 4.3$		142.5 ± 2.8
χ^2/ν		$687/519 = 1.32$		$596/508 = 1.17$

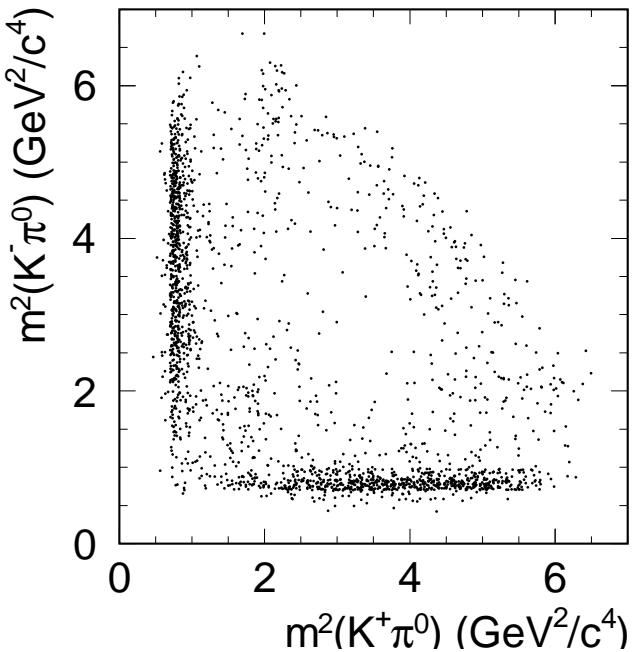
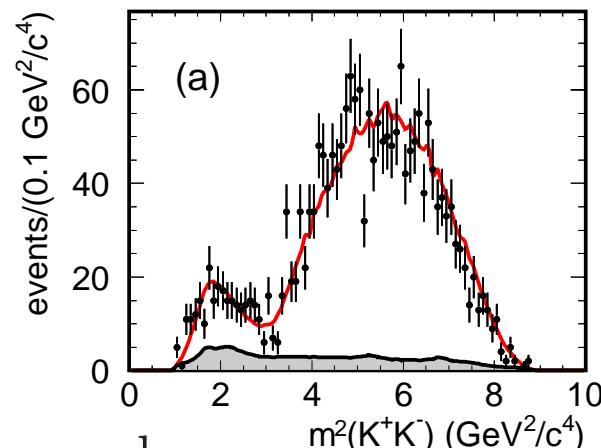
- The two models similar data representation, but different fractions. The Veneziano model fits better the data.
- This may indicate that other resonances are contributing to the decay.
- We note a small contribution in the amplitudes from the isospin violating decay $J/\psi \rightarrow \omega\pi^0$ with a statistical significance of 4.9σ .

$J/\psi \rightarrow K^+ K^- \pi^0$ Dalitz plot analysis ■

- Clear K^{*+} and K^{*-} bands.
- Broad structure in the low $K^+ K^-$ mass region.
- We make use of the Isobar model only.

Final state	fraction (%)	phase (radians)
$K^*(892)^\pm K^\mp$	$92.4 \pm 1.5 \pm 3.4$	0.
$\rho(1450)^0 \pi^0$	$9.3 \pm 2.0 \pm 0.6$	$3.78 \pm 0.28 \pm 0.08$
$K^*(1410)^\pm K^\mp$	$2.3 \pm 1.1 \pm 0.7$	$3.29 \pm 0.26 \pm 0.39$
$K_2^*(1430)^\pm K^\mp$	$3.5 \pm 1.3 \pm 0.9$	$-2.32 \pm 0.22 \pm 0.05$
Total	107.4 ± 2.8	
χ^2/ν	$132/137 = 0.96$	

- Dalitz plot projections:



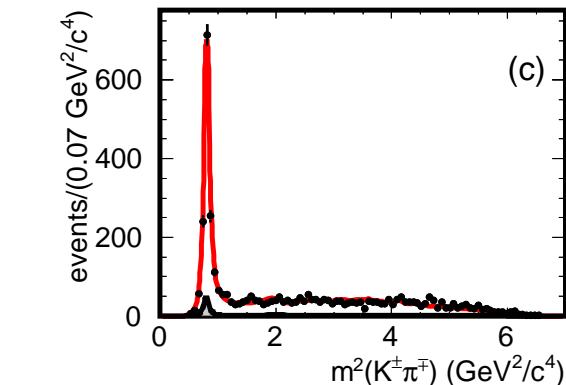
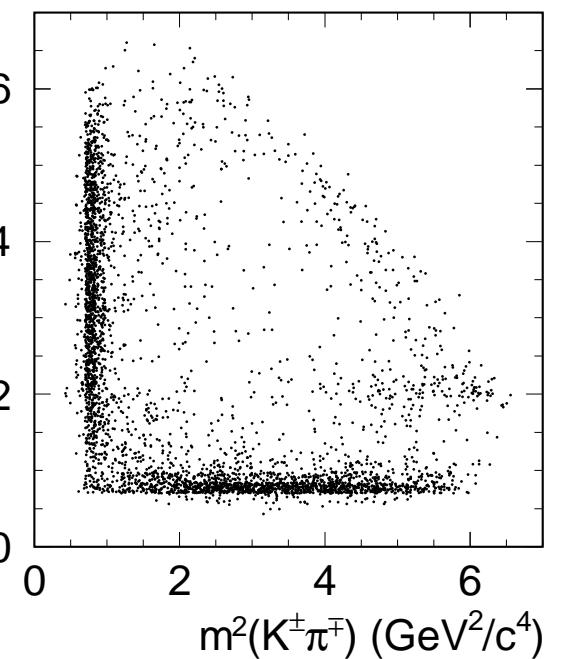
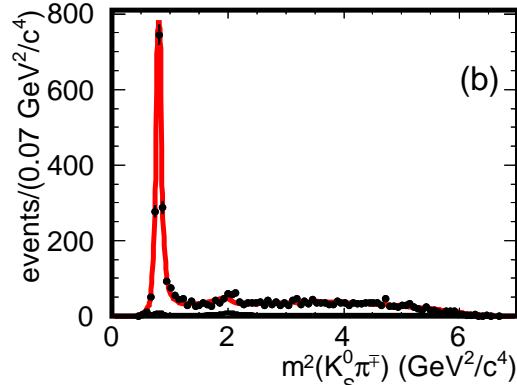
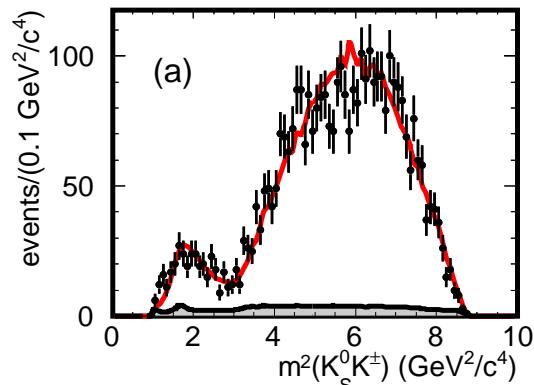
- Shaded is the background.

$J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$ Dalitz plot analysis

- Clear K^{*+} and K^{*-} bands.
- Broad structure in the low K^+K^- mass region.
- We make use of the Isobar model only.

Final state	fraction (%)	phase (radians)
$K^*(892)\bar{K}$	$90.5 \pm 0.9 \pm 3.8$	0.
$\rho(1450)^\pm \pi^\mp$	$6.3 \pm 0.8 \pm 0.6$	$-3.25 \pm 0.13 \pm 0.21$
$K_1^*(1410)\bar{K}$	$1.5 \pm 0.5 \pm 0.9$	$1.42 \pm 0.31 \pm 0.35$
$K_2^*(1430)\bar{K}$	$7.1 \pm 1.3 \pm 1.2$	$-2.54 \pm 0.12 \pm 0.12$
Total	105.3 ± 3.1	
χ^2/ν	$274/217 = 1.26$	

- Dalitz plot projections:



$\rho(1450)$ branching fraction

- We find the parameters of the low mass $K\bar{K}$ structure consistent for being associated to $\rho(1450)$.
- From the Dalitz-plot analysis of $J/\psi \rightarrow \pi^+ \pi^- \pi^0$ we obtain:

$$\begin{aligned}\mathcal{B}_1 &= \frac{\mathcal{B}(J/\psi \rightarrow \rho(1450)^0 \pi^0) \mathcal{B}(\rho(1450)^0 \rightarrow \pi^+ \pi^-)}{\mathcal{B}(J/\psi \rightarrow \pi^+ \pi^- \pi^0)} \\ &= (3.6 \pm 0.6(\text{stat}) \pm 0.9(\text{sys}))\%.\end{aligned}$$

- From the Dalitz-plot analysis of $J/\psi \rightarrow K^+ K^- \pi^0$ we obtain:

$$\begin{aligned}\mathcal{B}_2 &= \frac{\mathcal{B}(J/\psi \rightarrow \rho(1450)^0 \pi^0) \mathcal{B}(\rho(1450)^0 \rightarrow K^+ K^-)}{\mathcal{B}(J/\psi \rightarrow K^+ K^- \pi^0)} \\ &= (9.3 \pm 2.0(\text{stat}) \pm 0.6(\text{sys}))\%.\end{aligned}$$

- We therefore obtain:

$$\begin{aligned}\frac{\mathcal{B}(\rho(1450)^0 \rightarrow K^+ K^-)}{\mathcal{B}(\rho(1450)^0 \rightarrow \pi^+ \pi^-)} \\ = 0.307 \pm 0.084(\text{stat}) \pm 0.082(\text{sys}).\end{aligned}$$

Summary

- We show results on the Dalitz plot analyses of $\eta_c \rightarrow K_S^0 K^+ \pi^-$, $\eta_c \rightarrow K^+ K^- \pi^0$ and $\eta_c \rightarrow K^+ K^- \eta$ produced in two-photon interactions.
- We extract for the first time the $I=1/2$ $K\pi$ S -wave amplitude and phase using the MIPWA method up to a mass of 2.5 GeV/c^2 . We find a very different amplitude with respect to that measured by previous experiments in different processes.
- A K-matrix formalism is able to obtain a good description of the $I=1/2$ $K\pi$ S -wave.
- We show results on Dalitz plot analyses of $J/\psi \rightarrow \pi^+ \pi^- \pi^0$, $J/\psi \rightarrow K^+ K^- \pi^0$ and $J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$ produced in Initial State Radiation events using the isobar and Veneziano models.