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QCD thermodynamics. Hard-thermal-loop perturbation theory versus lattice data

XQCD 2017 Pisa

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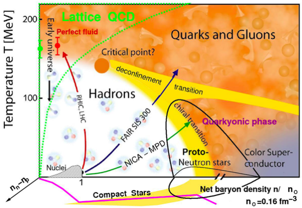
2. References: JHEP 1405 (2014) 027 and JHEP 1312 (2013) 055

Introduction

1. QCD equation of state - early universe, heavy-ion collisions, and compact stars



QCD phase diagram

2. Quark-gluon plasma strongly coupled for T close to to T_c. Weakly interacting for higher temperatures? Quasiparticle picture? Resummation? Dimensional reduction?

3. Lattice. (Karsch's talk). Sign problem at finite μ_B. Finite μ_I ok.

Weak-coupling expansion

1. Weak-coupling expansion of the thermodynamic functions has a long history. $\mathcal{F}$ is known to order $g^6 \log g$.[1]

2. Expansion poorly convergent (generic) and seems to be associated with the soft scale gT.

3. Goal: gauge-invariant framework with better convergence properties+able to describe dynamical properties+easy to generalize to finite μ_q.

[1] Arnold and Zhai, Braaten and Nieto, Kastening '94/'95, Kajantie, Laine, Rummukainen, and Schröder '02, Vuorinen '03.

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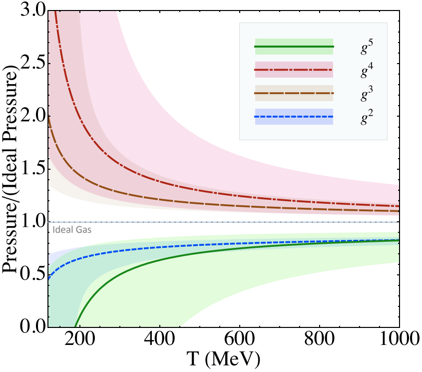
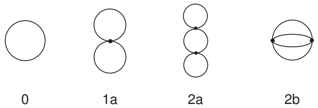


Weak-coupling expansion of the normalized pressure in three-flavor massless QCD as a function of the temperature T for zero μ_B.

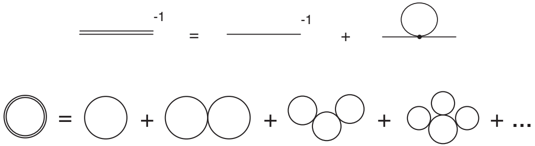
Screened perturbation theory

1. Massless scalar field theory

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}(\partial_\mu \phi)^2 + \frac{g^2}{24}\phi^4 \,, \\
\mathcal{L}^{\text{free}} &= \frac{1}{2}(\partial_\mu \phi)^2 \,, \\
\mathcal{L}^{\text{int}} &= \frac{g^2}{24}\phi^4 \,.
\end{aligned}
$$



2. Infrared divergences at higher orders



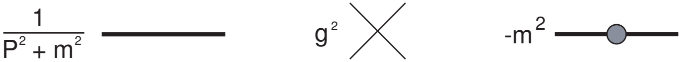
3. Reorganization of the perturbative series [2]

$$\begin{aligned}\mathcal{L}_0 &= \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}m^2\phi^2, \\ \mathcal{L}_{\text{int}} &= -\frac{1}{2}m^2\phi^2 + \frac{g^2}{24}\phi^4 \,.\end{aligned}$$

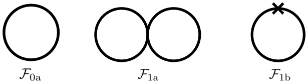
4. Expansion about an ideal gas of massive quasiparticles

[2] F. Karsch, A. Patkos, and P. Petreczky '97

5. Feynman rules:



6. Vacuum diagrams

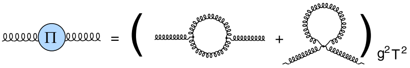


7. Calculate vacuum diagrams to desired order and finally give a prescription for the mass parameter m.

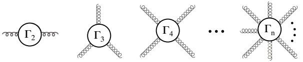
Hard-thermal-loop perturbation theory

$$\mathcal{L}_{\rm QCD} \;=\; -\frac{1}{2}{\rm Tr}[G_{\mu\nu}G^{\mu\nu}] + i\bar{\psi}\gamma^\mu D_\mu\psi + \mathcal{L}_{\rm gh} + \mathcal{L}_{\rm gf} + \Delta\mathcal{L}_{\rm QCD}\ .$$

1. For soft momenta, one needs effective propagators [3]



2. Dressed n-point vertices



[3] Braaten and Pisarski '90

3. Expansion point is gas of massive quasiparticles by adding the HTL Lagrangian [4]

$$\mathcal{L}_{\rm HTL} \quad = \quad -\frac{1}{2}(1-\delta)m_D^2 \mathrm{Tr}\left(G_{\mu\alpha}\left\langle \frac{y^\alpha y_\beta}{(y\cdot D)^2}\right\rangle_{\hat{\mathbf{y}}} G^{\mu\beta}\right) + (1-\delta) i m_q^2 \bar{\psi}\gamma^\mu \left\langle \frac{y_\mu}{y\cdot D}\right\rangle_{\hat{\mathbf{y}}} \psi \,,$$

4. δ is formal HTLpt expansion parameter. $\delta = 1$ at the end.
5. Expansion generates effective propagators and vertices.
6. Prescription for m_D and m_q.

[4] Braaten and Pisarski 1990

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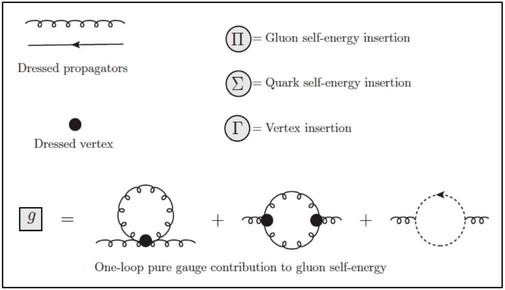
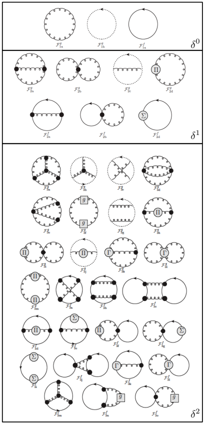




Final result

$$
\begin{aligned}
\frac{\Omega_{\rm NNLO}}{\Omega_0} \quad = \quad & \frac{7}{4}\frac{d_F}{d_A}\frac{1}{N_f}\sum_f\left(1+\frac{120}{7}\hat\mu_f^2+\frac{240}{7}\hat\mu_f^4\right)-\frac{s_F\alpha_s}{\pi}\frac{1}{N_f}\sum_f\left[\frac{5}{8}\left(1+12\hat\mu_f^2\right)\left(5+12\hat\mu_f^2\right)\right.\\
& \left.-\frac{15}{2}\left(1+12\hat\mu_f^2\right)\hat m_D-\frac{15}{2}\left(2\ln\frac{\hat\Lambda}{2}-1-\aleph(z_f)\right)\hat m_D^3+90\hat m_q^2\hat m_D\right]\\
& +\frac{s_{2F}}{N_f}\left(\frac{\alpha_s}{\pi}\right)^2\sum_f\left[\frac{15}{64}\left\{35-32\left(1-12\hat\mu_f^2\right)\frac{\zeta'(-1)}{\zeta(-1)}+472\hat\mu_f^2+1328\hat\mu_f^4\right.\right.\\
& \left.+64\left(-36i\hat\mu_f\aleph(2,z_f)+6(1+8\hat\mu_f^2)\aleph(1,z_f)+3i\hat\mu_f(1+4\hat\mu_f^2)\aleph(0,z_f)\right)\right\}\\
& \left.-\frac{45}{2}\hat m_D\left(1+12\hat\mu_f^2\right)\right]
\end{aligned}
$$

$$+\left(\frac{s_F\alpha_s}{\pi}\right)^2\frac{1}{N_f}\sum_f\frac{5}{16}\Bigg[96\left(1+12\hat{\mu}_f^2\right)\frac{\hat{m}_q^2}{\hat{m}_D}+\frac{4}{3}\left(1+12\hat{\mu}_f^2\right)\left(5+12\hat{\mu}_f^2\right)\ln\frac{\hat{\Lambda}}{2}$$

$$+\frac{1}{3}+4\gamma_E+8(7+12\gamma_E)\hat{\mu}_f^2+112\mu_f^4-\frac{64}{15}\frac{\zeta'(-3)}{\zeta(-3)}-\frac{32}{3}(1+12\hat{\mu}_f^2)\frac{\zeta'(-1)}{\zeta(-1)}$$

$$-96\left\{8\aleph(3,z_f)+12i\hat{\mu}_f\aleph(2,z_f)-2(1+2\hat{\mu}_f^2)\aleph(1,z_f)-i\hat{\mu}_f\aleph(0,z_f)\right\}\Bigg]$$

$$+\left(\frac{s_F\alpha_s}{\pi}\right)^2\frac{1}{N_f^2}\sum_{f,g}\Bigg[\frac{5}{4\hat{m}_D}\left(1+12\hat{\mu}_f^2\right)\left(1+12\hat{\mu}_g^2\right)+90\Bigg\{2\left(1+\gamma_E\right)\hat{\mu}_f^2\hat{\mu}_g^2$$

$$-\Big\{\aleph(3,z_f+z_g)+\aleph(3,z_f+z_g^*)+4i\hat{\mu}_f\left[\aleph(2,z_f+z_g)+\aleph(2,z_f+z_g^*)\right]-4\hat{\mu}_g^2\aleph(1,z_f)$$

$$-(\hat{\mu}_f+\hat{\mu}_g)^2\aleph(1,z_f+z_g)-(\hat{\mu}_f-\hat{\mu}_g)^2\aleph(1,z_f+z_g^*)-4i\hat{\mu}_f\hat{\mu}_g^2\aleph(0,z_f)\Big\}\Bigg\}$$

$$
\begin{aligned}
&-\frac{15}{2}\left(1+12\hat\mu_f^2\right)\left(2Ł-1-\aleph(z_g)\right)\hat m_D\Bigg]\\
&+\left(\frac{c_A\alpha_s}{3\pi}\right)\left(\frac{s_F\alpha_s}{\pi N_f}\right)\sum_f\Bigg[\frac{15}{2\hat m_D}\left(1+12\hat\mu_f^2\right)-\frac{235}{16}\Bigg\{\left(1+\frac{792}{47}\hat\mu_f^2+\frac{1584}{47}\hat\mu_f^4\right)\ln\frac{\hat\Lambda}{2}\\
&-\frac{144}{47}\left(1+12\hat\mu_f^2\right)\ln\hat m_D+\frac{319}{940}\left(1+\frac{2040}{319}\hat\mu_f^2+\frac{38640}{319}\hat\mu_f^4\right)-\frac{24\gamma_E}{47}\left(1+12\hat\mu_f^2\right)\\
&-\frac{44}{47}\left(1+\frac{156}{11}\hat\mu_f^2\right)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{268}{235}\frac{\zeta'(-3)}{\zeta(-3)}-\frac{72}{47}\Bigg[4i\hat\mu_f\aleph(0,z_f)+\left(5-92\hat\mu_f^2\right)\aleph(1,z_f)\\
&+144i\hat\mu_f\aleph(2,z_f)+52\aleph(3,z_f)\Bigg]\Bigg\}+90\frac{\hat m_q^2}{\hat m_D}+\frac{315}{4}\Bigg\{\left(1+\frac{132}{7}\hat\mu_f^2\right)Ł\\
&+\frac{11}{7}\left(1+12\hat\mu_f^2\right)\gamma_E+\frac{9}{14}\left(1+\frac{132}{9}\hat\mu_f^2\right)+\frac{2}{7}\aleph(z_f)\Bigg\}\hat m_D\Bigg]+\frac{\Omega_{\rm NNLO}^{\rm YM}}{\Omega_0}\,, \qquad (1)
\end{aligned}
$$

Results

1. Use one-loop running with $\alpha_s(1.5\mathrm{GeV}) = 0.326$ [5]

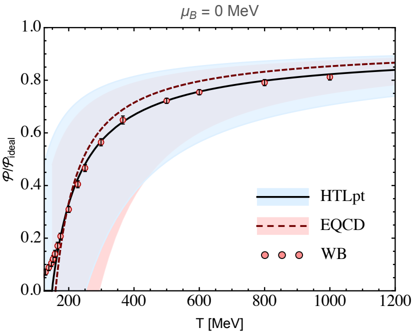
2. Mass prescription: use $m_f = 0$ and m_D from EQCD [6]

$$\begin{aligned}\hat{m}_D^2 \quad = \quad & \frac{\alpha_s}{3\pi}\left\{c_A + \frac{c_A^2\alpha_s}{12\pi}\left(5 + 22\gamma_E + 22\ln\frac{\hat{\Lambda}_g}{2}\right)\right. \\ & + \frac{1}{N_f}\sum_f\left[s_F\left(1+12\hat{\mu}_f^2\right) + \frac{c_A s_F \alpha_s}{12\pi}\left(\left(9+132\hat{\mu}_f^2\right)\right.\right. \\ & \left. +22\left(1+12\hat{\mu}_f^2\right)\gamma_E + 2\left(7+132\hat{\mu}_f^2\right)\text{Ł} + 4\aleph(z_f)\right) \\ & \left.\left. + \frac{s_F^2\alpha_s}{3\pi}\left(1+12\hat{\mu}_f^2\right)\left(1 - 2\text{Ł} + \aleph(z_f)\right) - \frac{3}{2}\frac{s_{2F}\alpha_s}{\pi}\left(1+12\hat{\mu}_f^2\right)\right]\right\}.\end{aligned}$$

[5] Bazavov '12

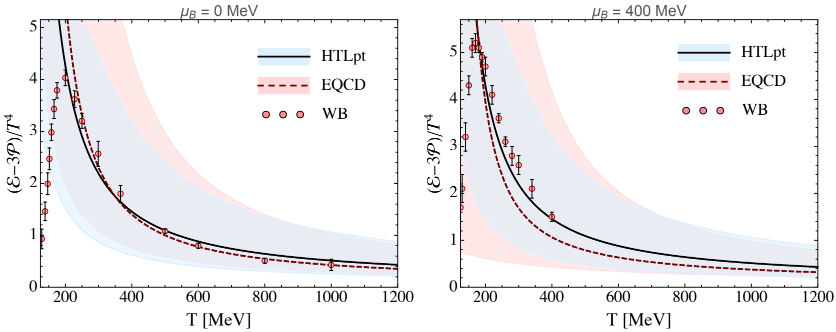
[6] Vuorinen '03

3. Pressure [7]



[7] Borsanyi et al '10 and '12

4. Trace anomaly



5. Expansion of pressure

$$\frac{\mathcal{P}}{T^4} = \frac{\mathcal{P}_0}{T^4} + \sum_{ijk} \frac{1}{i!j!k!} \chi_{ijk} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

Quark susceptibilities

$$\chi_{ijk\ldots} = \left.\frac{\partial^{i+j+k+\cdots}\mathcal{P}(T,\mu)}{\partial\mu_u^i\partial\mu_d^j\partial\mu_s^k}\right|_{\mu_q=0} .$$

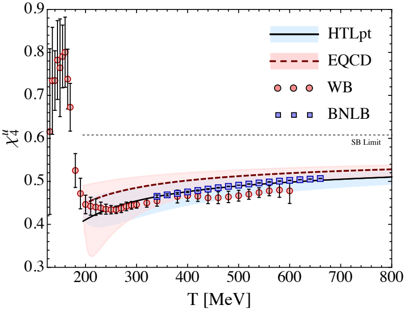
Baryon susceptibilities

$$\chi_n^B = \left.\frac{\partial^n\mathcal{P}}{\partial\mu_B^n}\right|_{\mu_B=0} .$$

Relations

$$\chi_2^B = \frac{1}{9}\left[\chi_2^{uu} + \chi_2^{dd} + \chi_2^{ss} + 2\chi_2^{ud} + 2\chi_2^{us} + 2\chi_2^{ds}\right] .$$

6. Susceptibilities [8]



[8] WB: R. Bellwied, S. Borsanyi, Z. Fodor, S. D. Katz, A. Pasztor, C. Ratti, K. K. Szabo,:1507.04627. BLNB = H.-T. Ding, S. Mukherjee, H. Ohno, P. Petreczky, H.-P. Schadler, 1507.06637

Summary and outlook

1. Hard-thermal-loop perturbation theory represents a gauge-invariant reorganization of the perturbative series
2. Analytic result for the three-loop QCD thermodynamic potential at finite T and μ. We also have results for $\mu_B = 0$ and $\mu_I \neq 0$ (lattice simulations possible!)
3. Agreement with lattice data for a number of variables is good, in particular considering that there are no fit parameters.
4. HTLpt is formulated in Minkowski space and can therefore be applied to real-time quantities as well.
5. Resummed DR also in good agreement with lattice
6. Decrease sensitivity to the renormalization scale [9] (Kneur's talk in 5min)

[9] Kneur and Pinto PRL '15