GRAVITATIONAL WAVES FROM COALESCING BINARIES: THE THEORY

Alessandro Nagar
Centro Fermi, INFN Torino & IHES

The TEOBResumS code: eob.ihes.fr

EGO - Virgo

CENTRO FERMI
MUSEO STORICO DELLA FISICA E CENTRO STUDI E RICERCHE ENRICO FERMI

INFN
Istituto Nazionale di Fisica Nucleare

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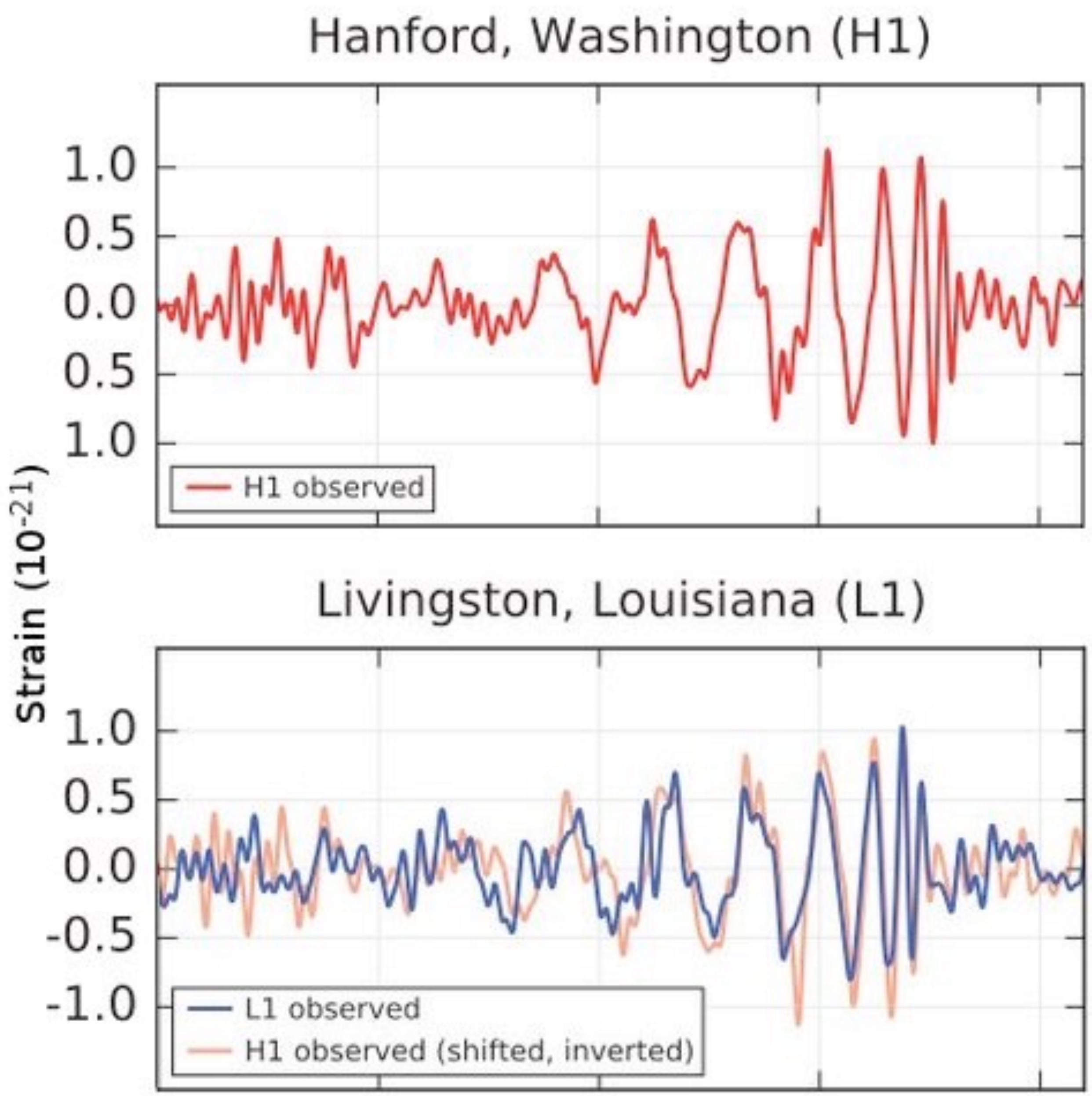
GW150914



$$\text{strain} = \frac{\delta L}{L}$$

Masses & Spins

GW150914 parameters:

$$\boxed{\begin{aligned} m_1 &= 35.7 M_\odot \\ m_2 &= 29.1 M_\odot \\ M_f &= 61.8 M_\odot \end{aligned}}$$

$$a_1 \equiv S_1/(m_1^2) = 0.31^{+0.48}_{-0.28}$$

$$a_2 \equiv S_2/(m_2^2) = 0.46^{+0.48}_{-0.42}$$

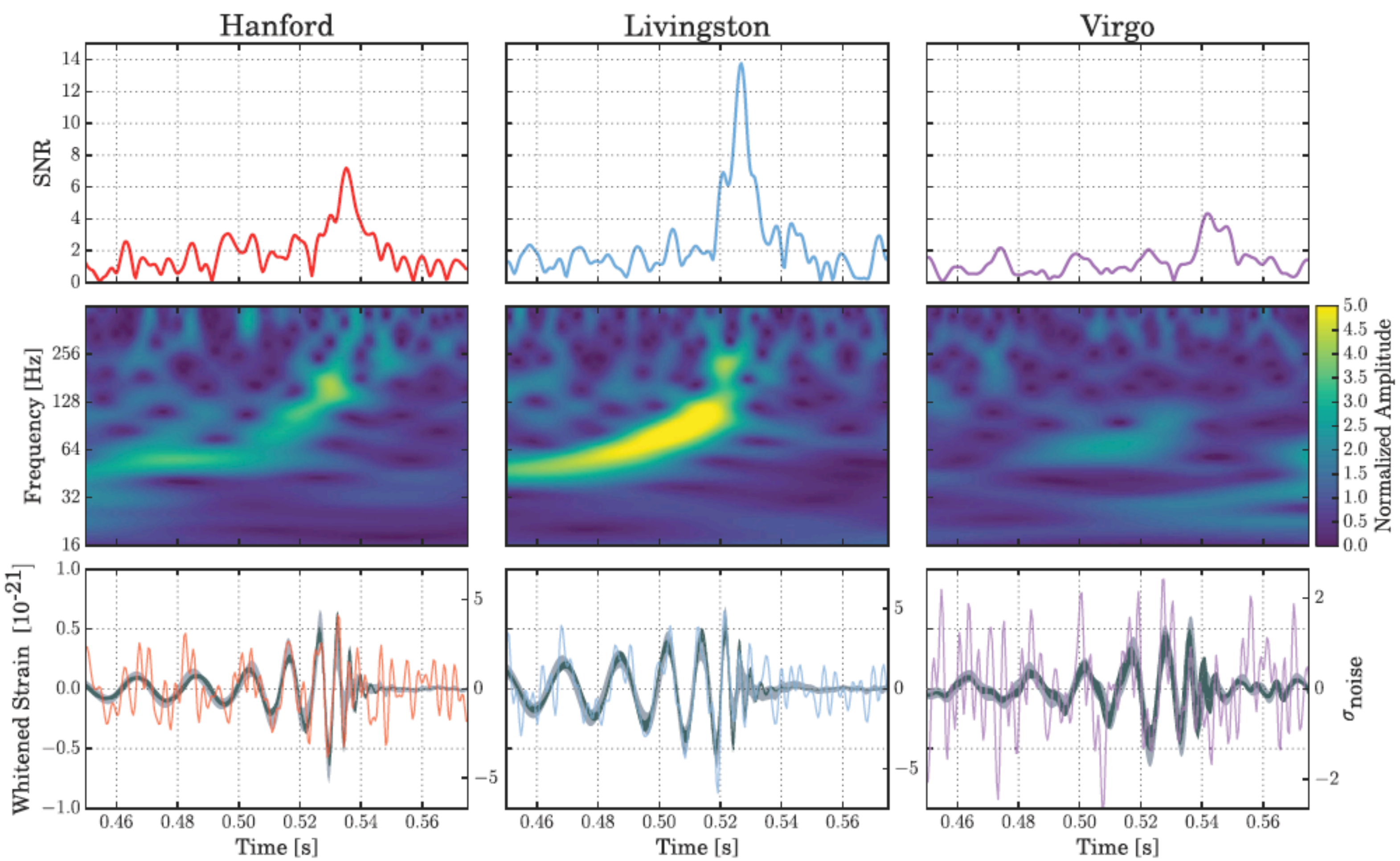
$$a_f \equiv \frac{J_f}{M_f^2} = 0.67$$

$$q \equiv \frac{m_1}{m_2} = 1.27$$

Symmetric mass ratio

$$\boxed{\nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2} = 0.2466}$$

JOINT LIGO-VIRGO DETECTION



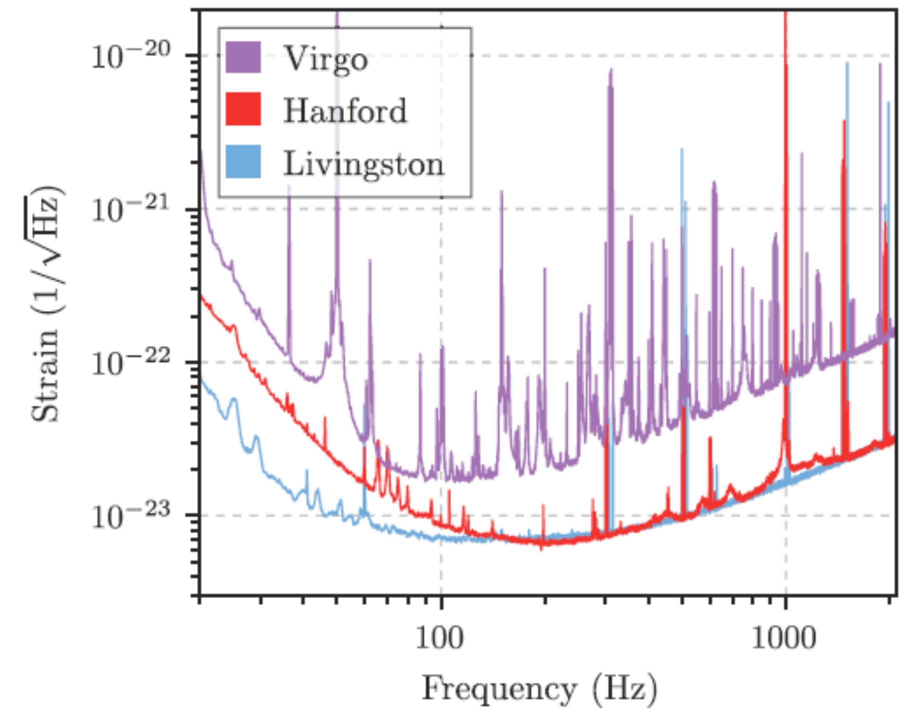
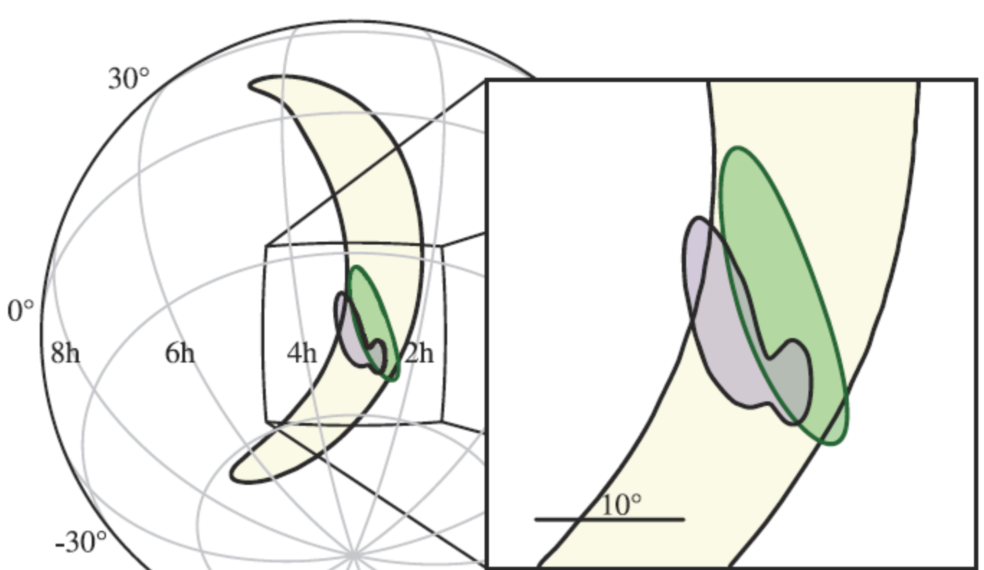


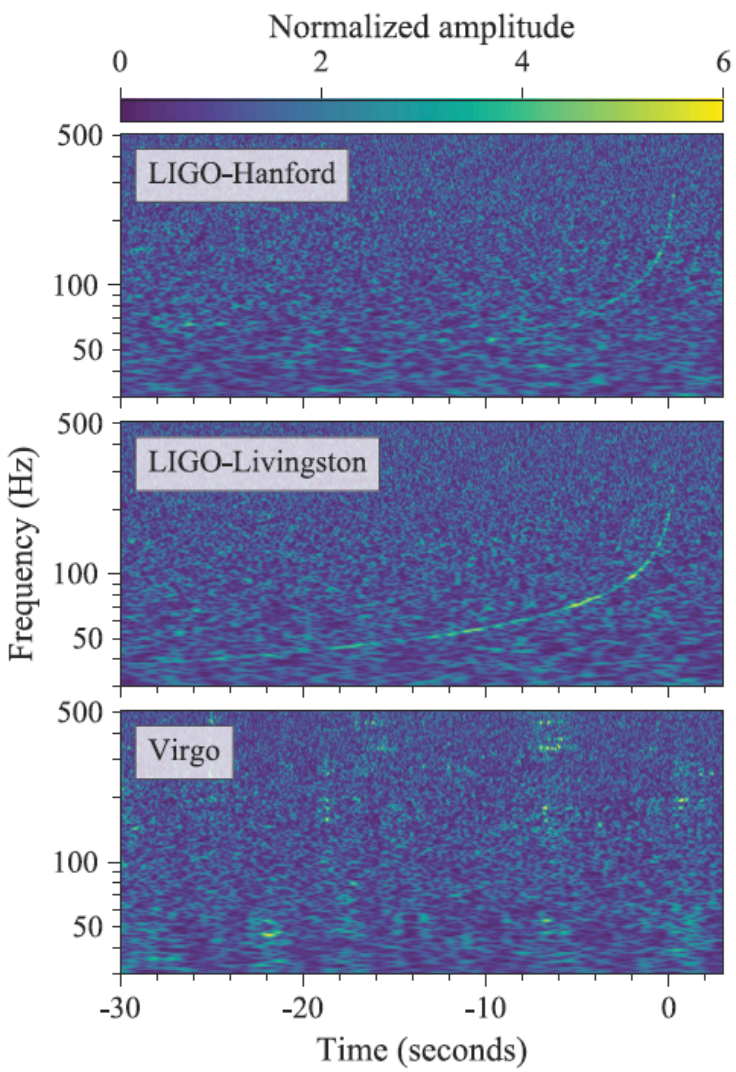
TABLE I. Source parameters for GW170814: median values with 90% credible intervals. We quote source-frame masses; to convert to the detector frame, multiply by $(1+z)$ [126,127]. The redshift assumes a flat cosmology with Hubble parameter $H_0 = 67.9\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ and matter density parameter $\Omega_m = 0.3065$ [128].

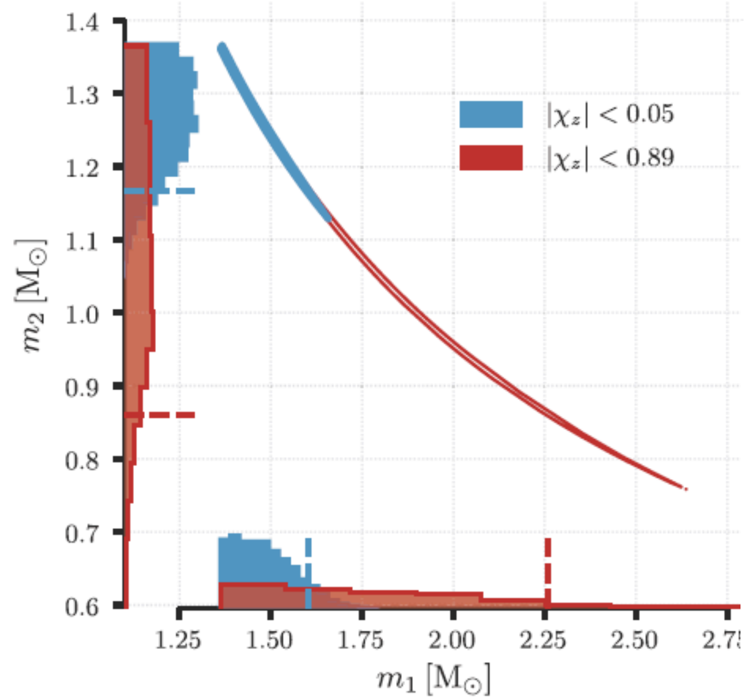
Primary black hole mass m_1	$30.5^{+5.7}_{-3.0} M_\odot$
Secondary black hole mass m_2	$25.3^{+2.8}_{-4.2} M_\odot$
Chirp mass $\mathcal{M}$	$24.1^{+1.4}_{-1.1} M_\odot$
Total mass M	$55.9^{+3.4}_{-2.7} M_\odot$
Final black hole mass M_f	$53.2^{+3.2}_{-2.5} M_\odot$
Radiated energy E_{rad}	$2.7^{+0.4}_{-0.3} M_\odot\ c^2$
Peak luminosity ℓ_{peak}	$3.7^{+0.5}_{-0.5} \times 10^{56}\ \mathrm{erg\,s^{-1}}$
Effective inspiral spin parameter χ_{eff}	$0.06^{+0.12}_{-0.12}$
Final black hole spin a_f	$0.70^{+0.07}_{-0.05}$
Luminosity distance D_L	540^{+130}_{-210} Mpc
Source redshift z	$0.11^{+0.03}_{-0.04}$

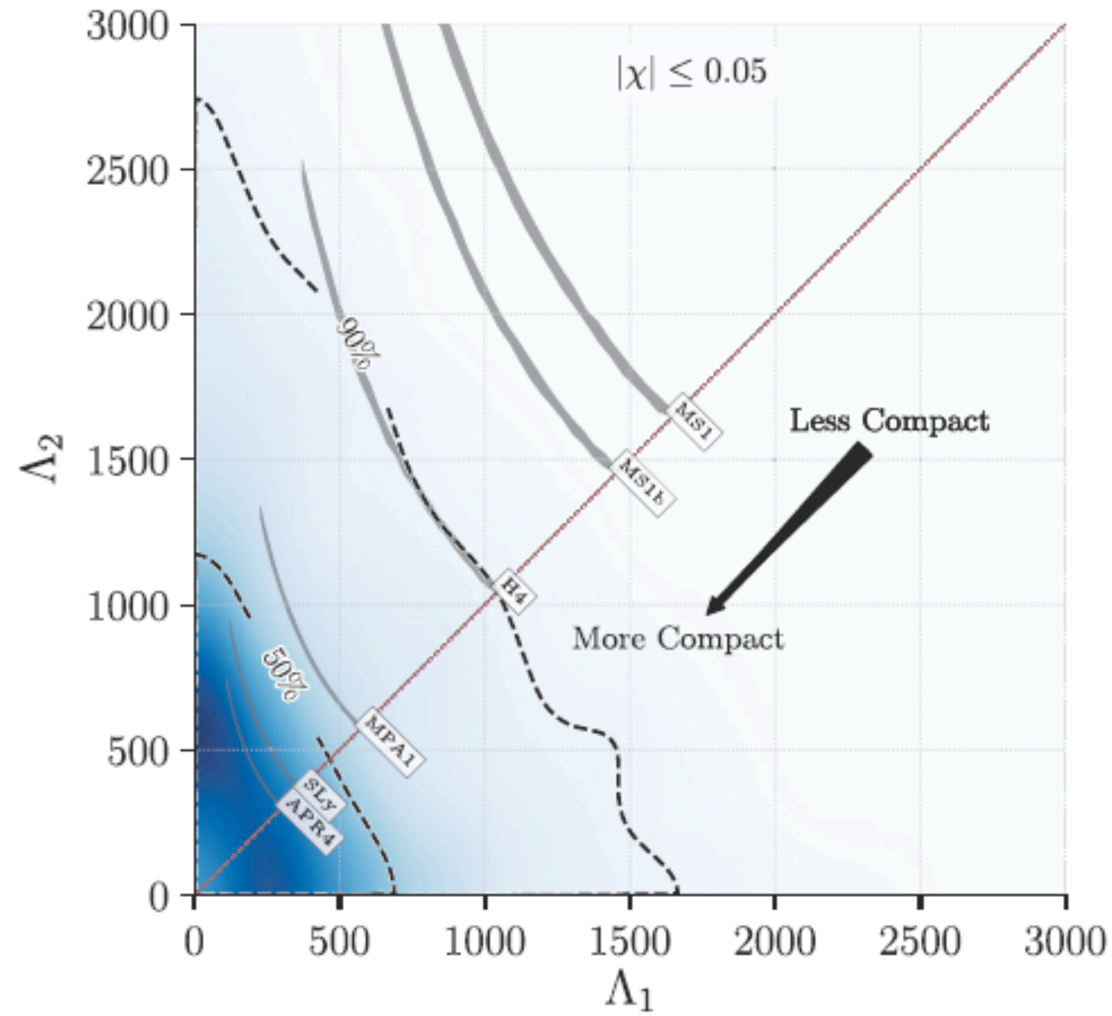
Localization



Binary neutron star: GW170817







Equation of state

$$\Lambda = \frac{2}{3} k_2 \left(\frac{c^2}{G} \frac{R_*}{M_*} \right)^5$$

THE THEORY?

Is needed to compute waveform templates for characterizing the source (GWs were detected...but WHAT was detected?)

Theory is needed to study the 2-body problem in General Relativity (dynamics & gravitational wave emission)

Theory: SYNERGY between

Analytical and Numerical General Relativity

(AR/NR)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

HOW TO MEASURE: MATCHED FILTERING!

To extract/do parameter estimation of the GW signal from detector's output (lost in broadband noise $S_n(f)$)

$$\langle output | h_{\rm template} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\rm template}(f)$$

Detector's output

Template of expected GW signal

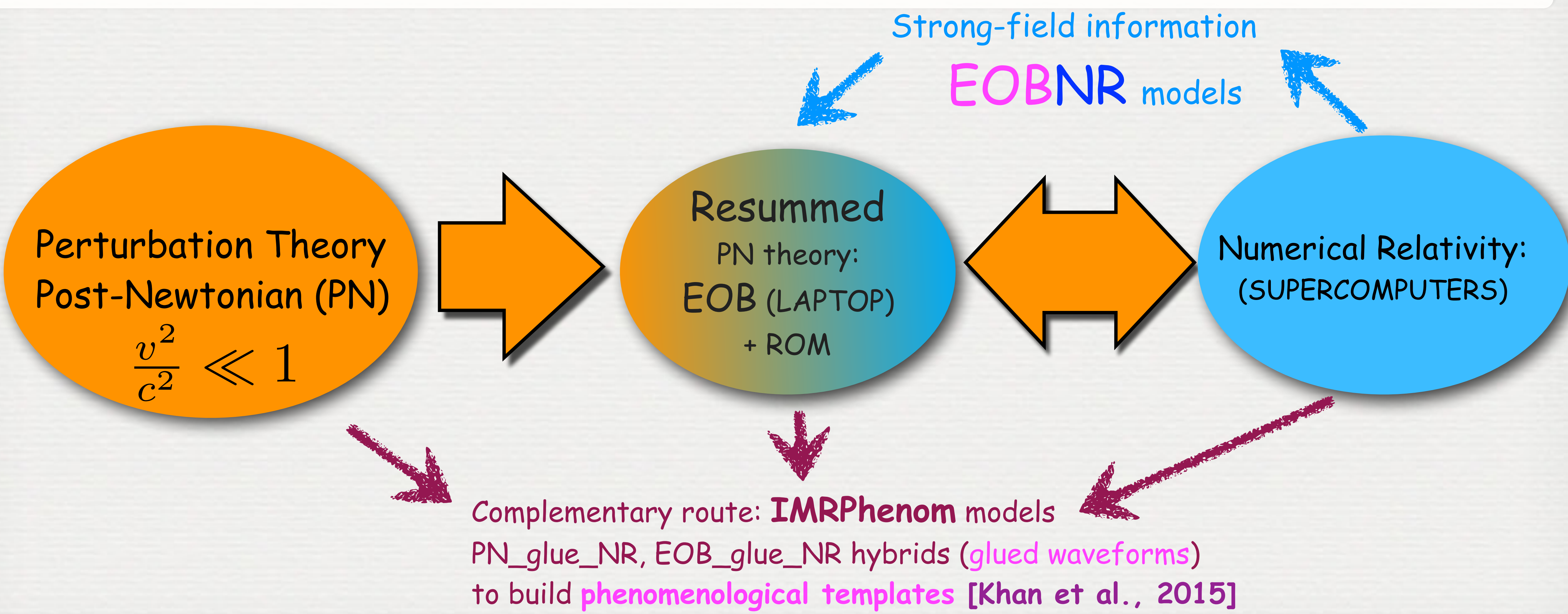
Need waveform templates!

IMPORTANCE OF AN ANALYTICAL FORMALISM

- **Theoretical**: physical understanding of the coalescence process, especially in complicated situations (e.g., precessing spins).

- **Practical**: need hundreds of thousands of accurate GWs templates for detection and data analysis. Need analytical templates: $h\left(m_1, m_2, \vec{S}_1, \vec{S}_2\right)$

- **Solution**: synergy between analytical & numerical relativity



GWS FROM COMPACT BINARIES: BASICS

Newtonian binary systems in circular orbits: Kepler's law

$$GM = \Omega^2 R^3$$

$$\frac{v^2}{c^2} = \frac{GM}{c^2 R} = \left(\frac{GM\Omega}{c^3}\right)^{2/3}$$

$$M = m_1 + m_2$$

Einstein (1918) quadrupole formula: GW luminosity (energy flux)

$$P_{\rm gw} = \frac{dE_{\rm gw}}{dt} = \frac{32}{5}\frac{c^5}{G}\nu^2 x^5$$

$$x = \left(\frac{v}{c}\right)^2$$

$$\nu = \frac{\mu}{M} = \frac{m_1 m_2}{M^2}$$

GWS FROM COMPACT BINARIES: BASICS

$$E^{\rm orbital} = E^{\rm kinetic} + E^{\rm potential} = -\frac{1}{2}\frac{m_1 m_2}{R} = -\frac{1}{2}\mu x$$

Balance argument

$$\frac{dE^{\rm orbital}}{dt} = P_{GW} = \frac{dE_{\rm GW}}{dt}$$

$$\omega_{22}^{\rm GW} = 2\pi f_{22}^{\rm GW} = 2\Omega^{\rm orbital}$$

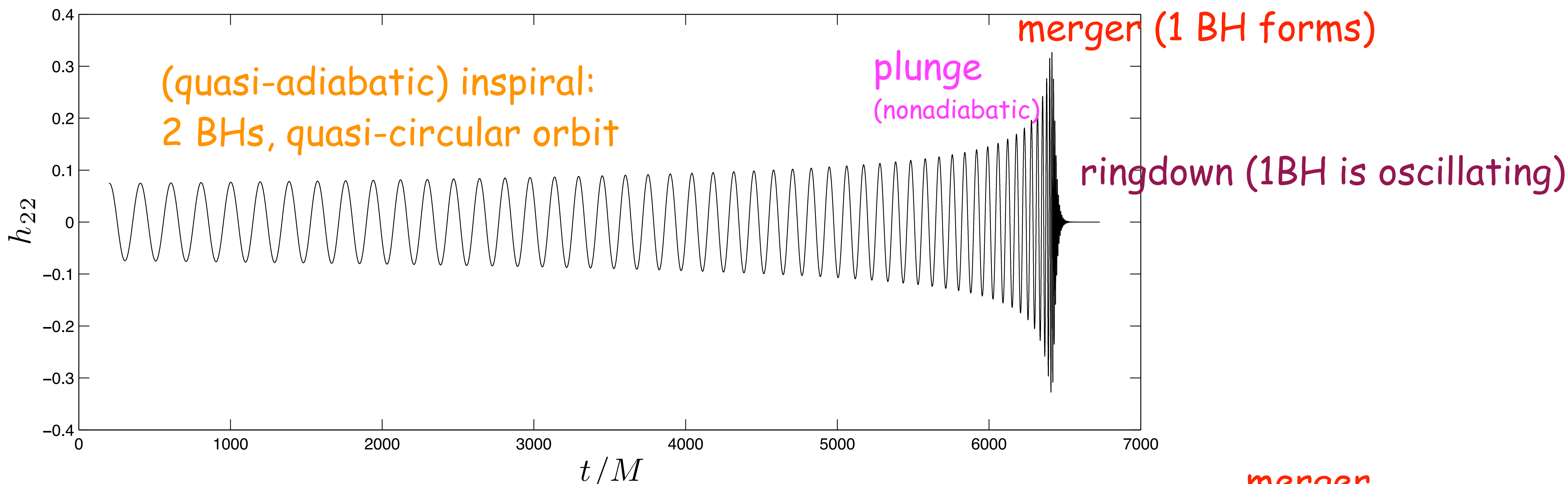
$$\boxed{f_{GW}^{22} = \frac{1}{\pi}\left(\frac{5}{256\nu}\right)^{3/8}\left(\frac{1}{t - t_{\rm coalescence}}\right)^{3/8}}$$

MONOTONICALLY GROWING FREQUENCY: CHIRP!

BBHS: WAVEFORM OVERWIEV

$$h_+ - i h_\times = \frac{1}{r}\sum_{\ell m} h_{\ell m}\; {}_{-2}Y_{\ell m}(\theta,\phi)$$

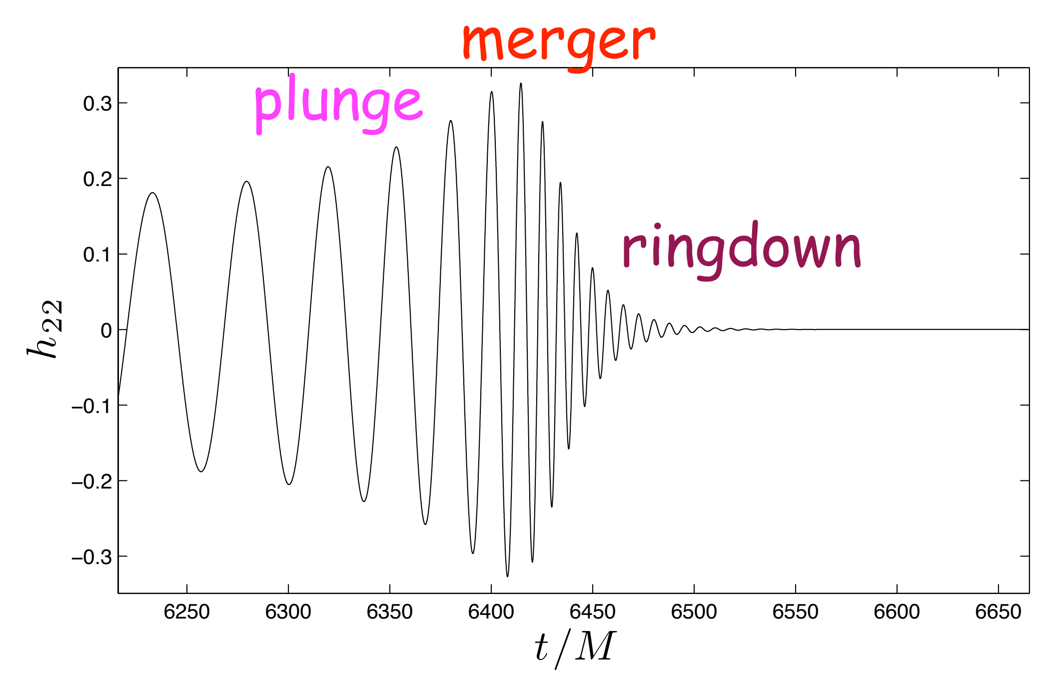
$$h\left(m_1, m_2, \vec{S}_1, \vec{S}_2\right)$$



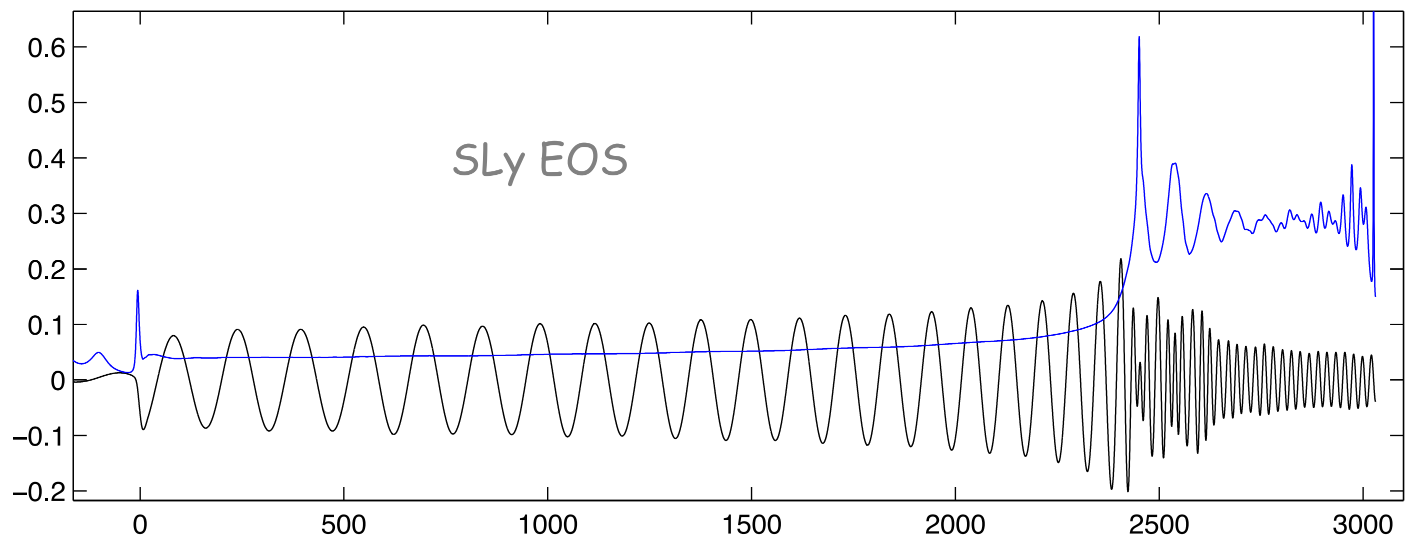
e.g: equal-mass BBH, aligned-spins

$$\chi_1 = \chi_2 = +0.98$$

- SXS (Simulating eXtreme Spacetimes) collaboration
- www.blackholes.org
- Free catalog of waveforms (downloadable)



BINARY NEUTRON STARS (BNS)?



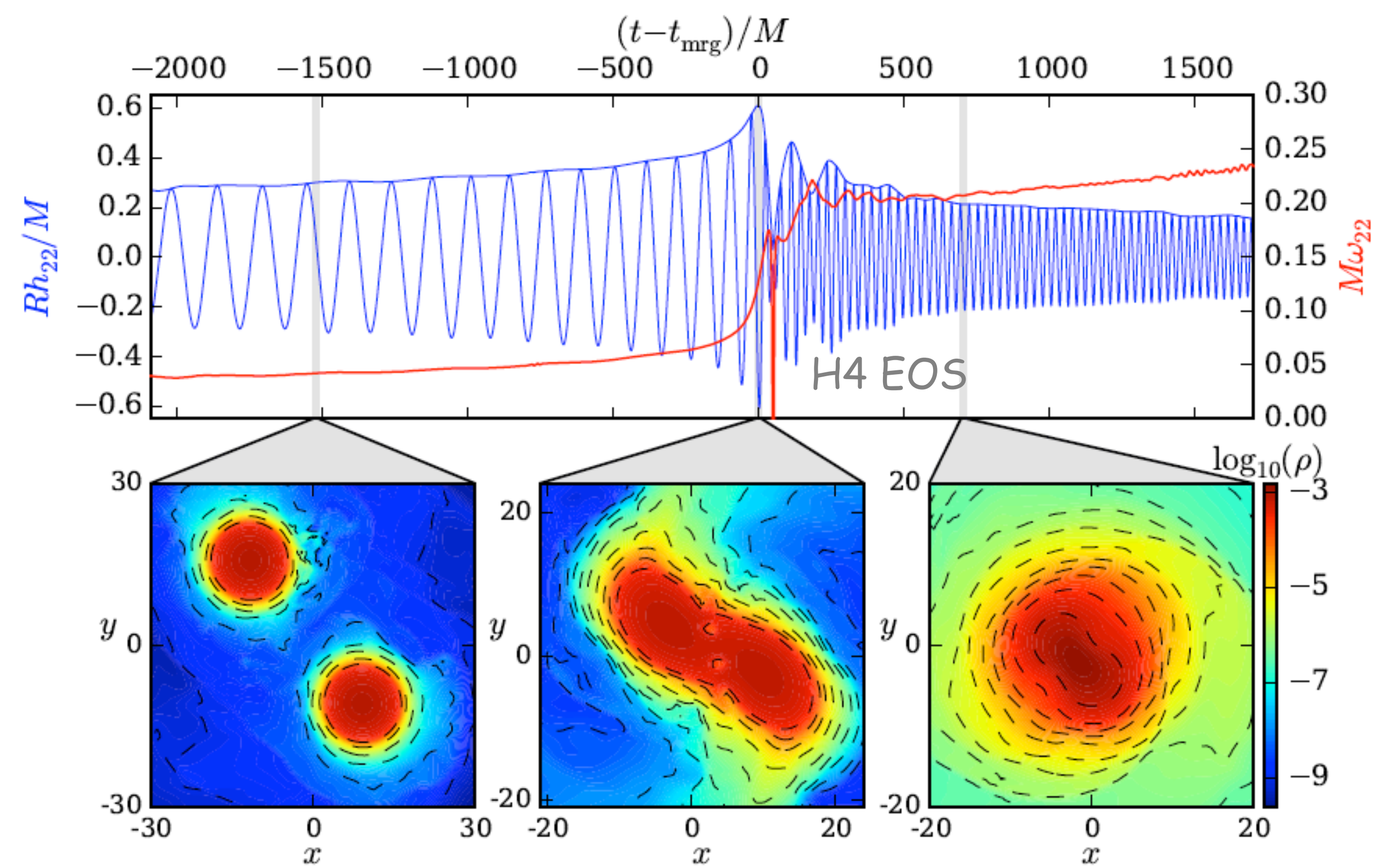
All BNS need is Love!

$$q = 1 \qquad M = 2.7 M_\odot$$

- Tidal effects
- Love numbers (tidal "polarization" constants)
- EOS dependence & "universality"
- EOB/NR for BNS

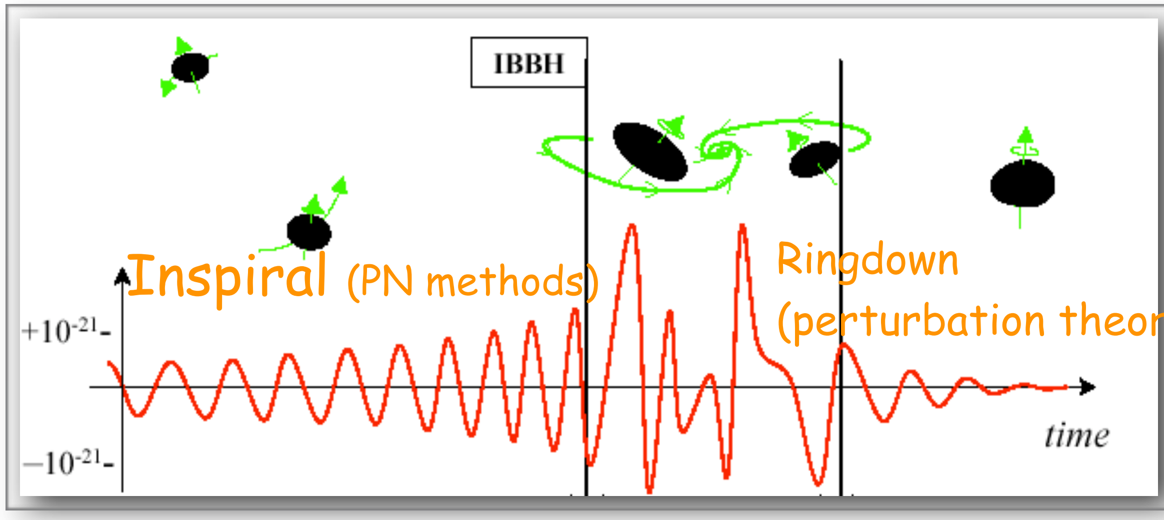
See:

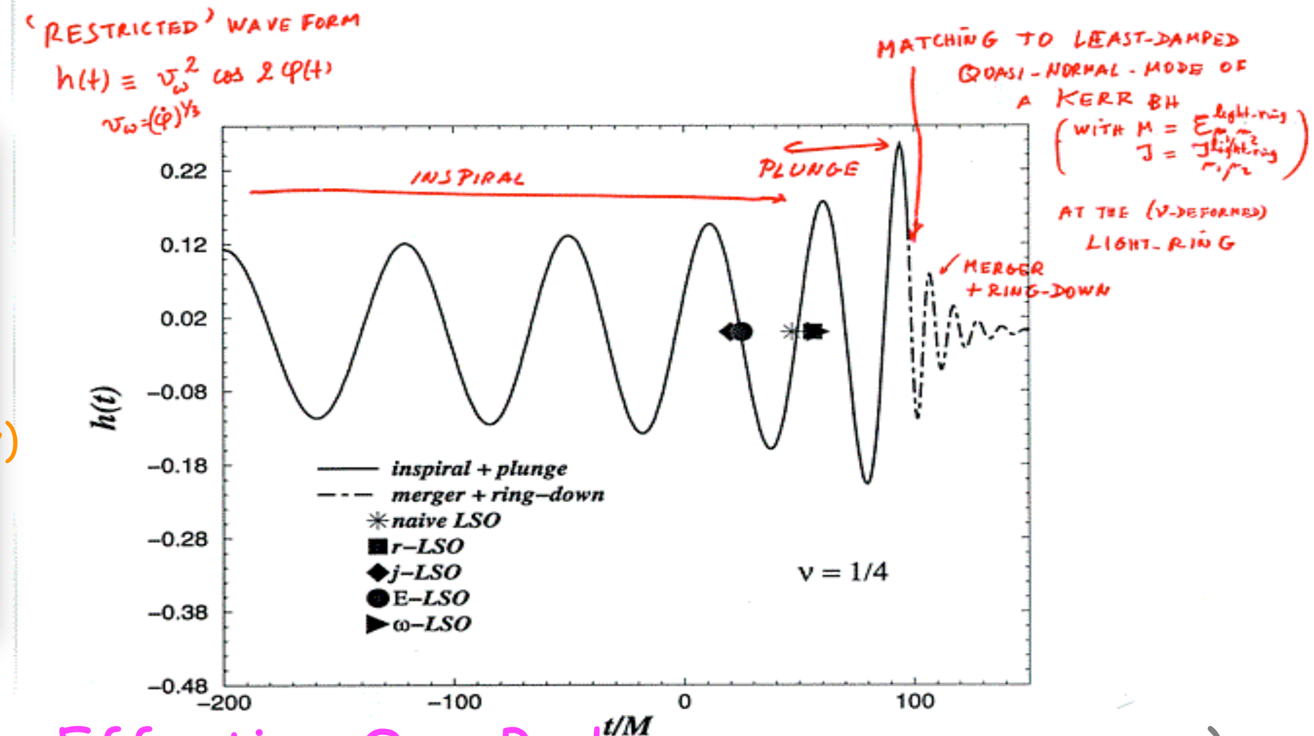
Damour&Nagar, PRD 2009
Damour&Nagar, PRD 2010
Damour,Nagar et al., PRL 2011
Bini,Damour&Faye, PRD2012
Bini&Damour, PRD 2014
Bernuzzi, Nagar, et al, PRL 2014
Bernuzzi, Nagar, Dietrich, PRL 2015
Bernuzzi, Nagar, Dietrich & Damour,PRL, 2015



TEMPLATES FOR GWS FROM BBH COALESCENCE

Brady, Craighton & Thorne, 1998





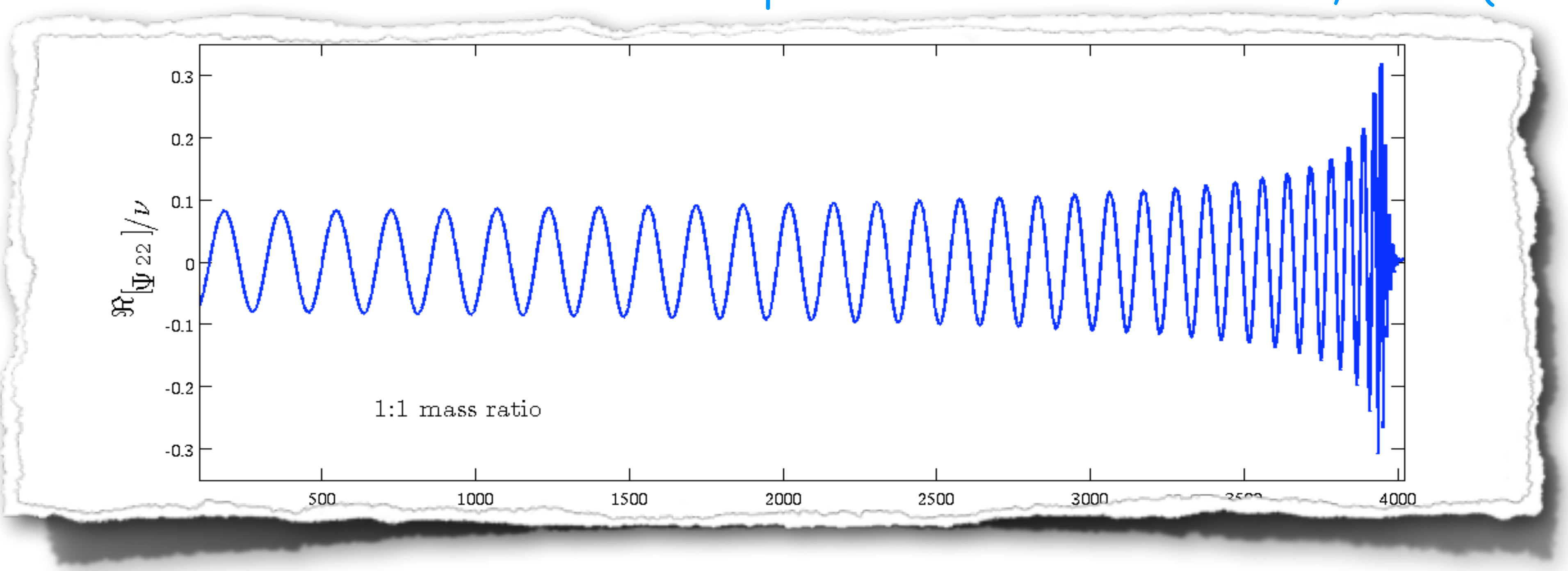
Merger:
Numerical Relativity

Effective-One-Body (Buonanno & Damour (2000))

PN-resummation (Damour,Iyer, Sathyaprakash (1998))

Numerical Relativity: >= 2005 (F. Pretorius, Campanelli et al., Baker et al.)

Most accurate data: Caltech-Cornell spectral code: M. Scheel et al., 2008 (SXS collaboration)



Spectral code

Extrapolation (radius & resolution)

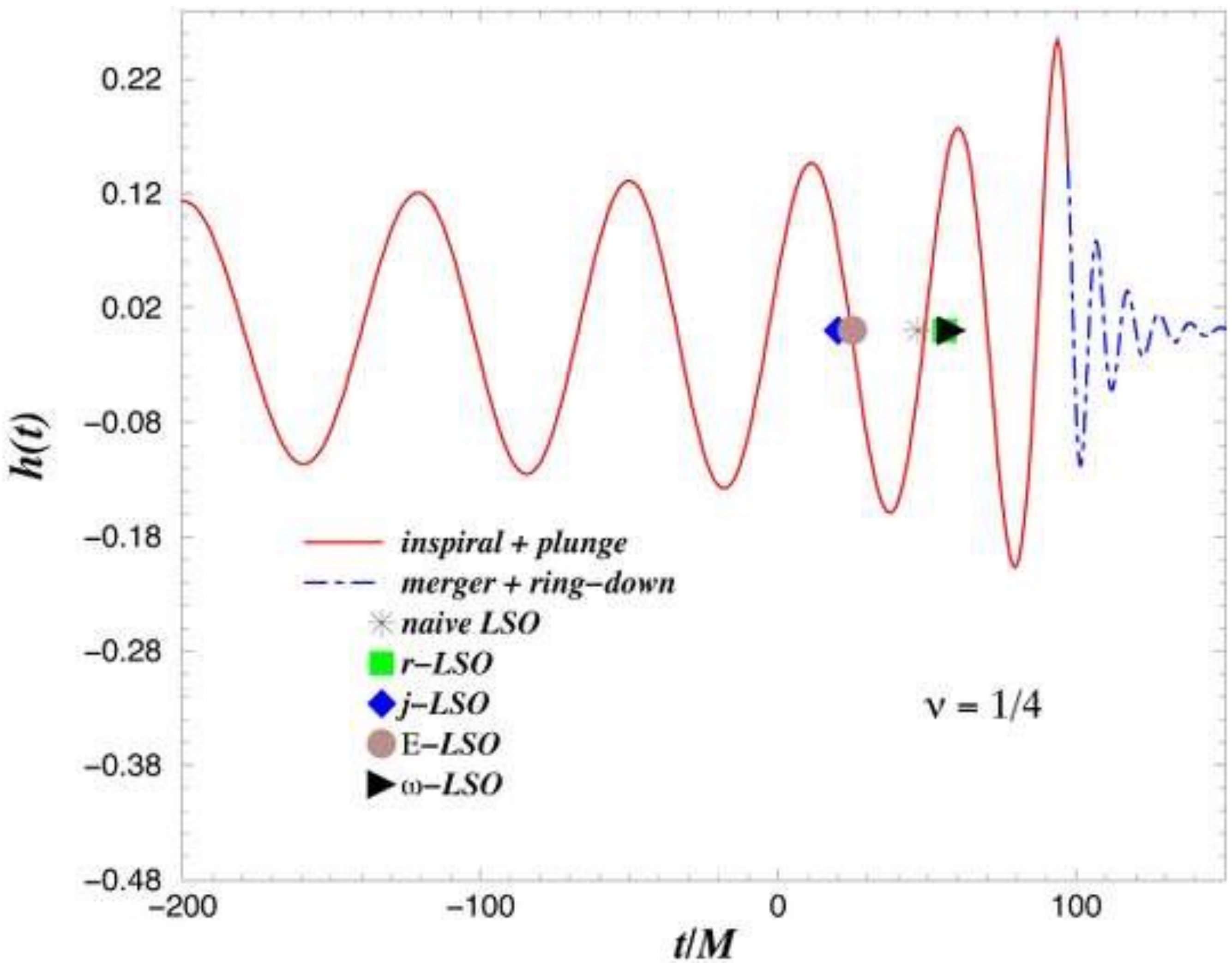
Phase error:

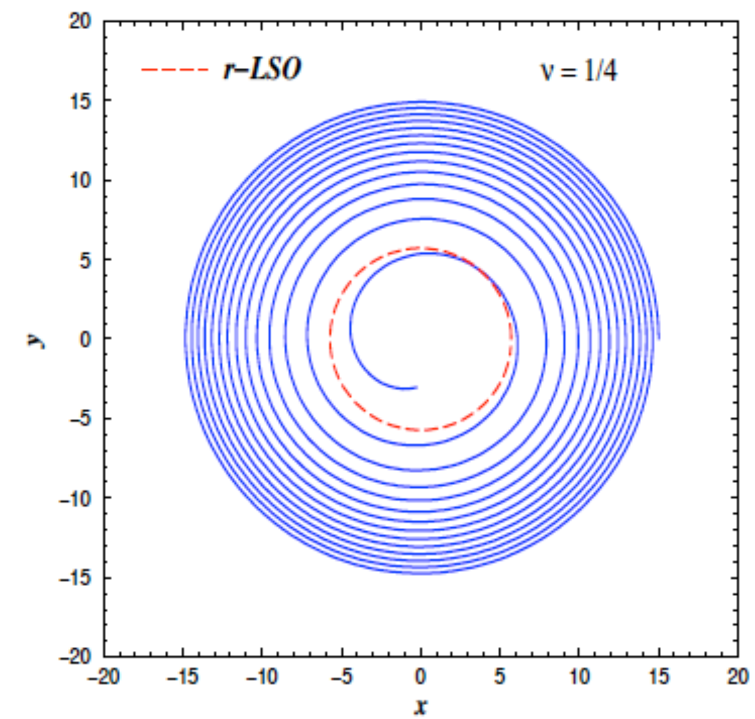
< 0.02 rad (inspiral)

<0.1 rad (ringdown)

EFFECTIVE ONE BODY (EOB): 2000

Numerical Relativity was not working (yet...)
EOB formalism was predictive, qualitatively and semi-quantitatively correct (10%)





- Blurred transition from inspiral to plunge
- Final black-hole mass
- Final black hole spin
- Complete waveform

A. Buonanno & T. Damour, PRD 59 (1999) 084006
A. Buonanno & T. Damour, PRD 62 (2000) 064015

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2} = \frac{\mu}{M}$$

> 2005: Developing EOB & interfacing with NR
2 groups did (and are doing) it
- A.Buonanno+ (AEI)
- T.Damour & AN + (>2005)

$$\boxed{h_+ - i h_\times = \frac{1}{r}\sum_{\ell m} h_{\ell m}\, {}_{-2}Y_{\ell m}(\theta,\phi)}$$

ANALYTICALLY: MOTION AND GW IN GR

Hamiltonian: conservative part of the dynamics

Radiation reaction: mechanical energy/angular momentum goes away in GWs and backreacts on the system.

The (closed) orbit CIRCULARIZES and SHRiNks with time

Waveform

General Relativity is NONLINEAR!

Post-Newtonian (PN) approximation: expansion in $\frac{v^2}{c^2}$

POST-NEWTONIAN HAMILTONIAN (C.O.M)

$$\widehat{H}^{\rm NR}_{\rm real}(\mathbf{q},\mathbf{p}) = \widehat{H}_{\rm N}(\mathbf{q},\mathbf{p}) + \widehat{H}_{\rm 1PN}(\mathbf{q},\mathbf{p}) + \widehat{H}_{\rm 2PN}(\mathbf{q},\mathbf{p}) + \widehat{H}_{\rm 3PN}(\mathbf{q},\mathbf{p}), \tag{4.27}$$

where

$$\widehat{H}_{\rm N}(\mathbf{q},\mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{q}, \quad \text{Newton} \quad \text{(0PN)} \tag{4.28a}$$

$$\widehat{H}_{\rm 1PN}(\mathbf{q},\mathbf{p}) = \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\left[(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\right]\frac{1}{q} + \frac{1}{2q^2}, \quad \text{(1PN, 1938)} \tag{4.28b}$$

- [Einstein-Infeld-Hoffman]

$$\widehat{H}_{\rm 2PN}(\mathbf{q},\mathbf{p}) = \frac{1}{16}\left(1-5\nu+5\nu^2\right)(\mathbf{p}^2)^3 + \frac{1}{8}\left[\left(5-20\nu-3\nu^2\right)(\mathbf{p}^2)^2 - 2\nu^2(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - 3\nu^2(\mathbf{n}\cdot\mathbf{p})^4\right]\frac{1}{q}$$
$$+ \frac{1}{2}\left[(5+8\nu)\mathbf{p}^2 + 3\nu(\mathbf{n}\cdot\mathbf{p})^2\right]\frac{1}{q^2} - \frac{1}{4}(1+3\nu)\frac{1}{q^3}, \quad \text{(2PN, 1982/83)} \tag{4.28c}$$

- [Damour-Deruelle]

$$\widehat{H}_{\rm 3PN}(\mathbf{q},\mathbf{p}) = \frac{1}{128}\left(-5+35\nu-70\nu^2+35\nu^3\right)(\mathbf{p}^2)^4$$
$$+\frac{1}{16}\left[\left(-7+42\nu-53\nu^2-5\nu^3\right)(\mathbf{p}^2)^3 + (2-3\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + 3(1-\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - 5\nu^3(\mathbf{n}\cdot\mathbf{p})^6\right]\frac{1}{q}$$
$$+\left[\frac{1}{16}\left(-27+136\nu+109\nu^2\right)(\mathbf{p}^2)^2 + \frac{1}{16}(17+30\nu)\nu(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \frac{1}{12}(5+43\nu)\nu(\mathbf{n}\cdot\mathbf{p})^4\right]\frac{1}{q^2} \quad \text{(3PN, 2000)}$$
$$+\left\{\left[-\frac{25}{8} + \left(\frac{1}{64}\pi^2 - \frac{335}{48}\right)\nu - \frac{23}{8}\nu^2\right]\mathbf{p}^2 + \left(-\frac{85}{16} - \frac{3}{64}\pi^2 - \frac{7}{4}\nu\right)\nu(\mathbf{n}\cdot\mathbf{p})^2\right\}\frac{1}{q^3}$$
$$+\left[\frac{1}{8} + \left(\frac{109}{12} - \frac{21}{32}\pi^2 + \omega_{\rm static}\right)\nu\right]\frac{1}{q^4}. \tag{4.28d}$$

- [Damour, Jaranowski,Schaefer]

...and 4PN too, [Damour, Jaranowski&Schaefer 2014/2015] - 4 loop calculation

$$\mathbf{q} = \mathbf{q}_1 - \mathbf{q}_2$$

$$\mathbf{p} = \mathbf{p}_1 = -\mathbf{p}_2$$

FLUX & WAVEFORM (3.5PN)

$$\frac{dE}{dt} = -\mathcal{L}$$

balance equation

Mechanical loss GW luminosity

$$\mathcal{L} = \frac{32c^5}{5G}\nu^2 x^5 \Big\{ 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2$$

Newtonian quadrupole

$$+ \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2}$$
$$+ \Big[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16\,x)$$
$$+ \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\Big] x^3$$
$$+ \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Big\}.$$

$$h^{22} = -8\sqrt{\frac{\pi}{5}}\frac{G\nu m}{c^2 R}e^{-2i\phi}x\Big\{ 1 - x\left(\frac{107}{42} - \frac{55}{42}\nu\right) + x^{3/2}\left[2\pi + 6i\ln\left(\frac{x}{x_0}\right)\right] - x^2\left(\frac{2173}{1512} + \frac{1069}{216}\nu - \frac{2047}{1512}\nu^2\right)$$
$$- x^{5/2}\left[\left(\frac{107}{21} - \frac{34}{21}\nu\right)\pi + 24i\nu + \left(\frac{107i}{7} - \frac{34i}{7}\nu\right)\ln\left(\frac{x}{x_0}\right)\right]$$
$$+ x^3\Big[\frac{27\,027\,409}{646\,800} - \frac{856}{105}\gamma_E + \frac{2}{3}\pi^2 - \frac{1712}{105}\ln 2 - \frac{428}{105}\ln x$$
$$- 18\left[\ln\left(\frac{x}{x_0}\right)\right]^2 - \left(\frac{278\,185}{33\,264} - \frac{41}{96}\pi^2\right)\nu - \frac{20\,261}{2772}\nu^2 + \frac{114\,635}{99\,792}\nu^3 + \frac{428i}{105}\pi + 12i\pi\ln\left(\frac{x}{x_0}\right)\Big] + O(\epsilon^{7/2})\Big\},$$

$$C = \gamma_E = 0.5772156649...$$

$$x = (M\Omega)^{2/3} \sim v^2/c^2$$

EFFECTIVE-ONE-BODY (EOB)

approach to the general relativistic two-body problem

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

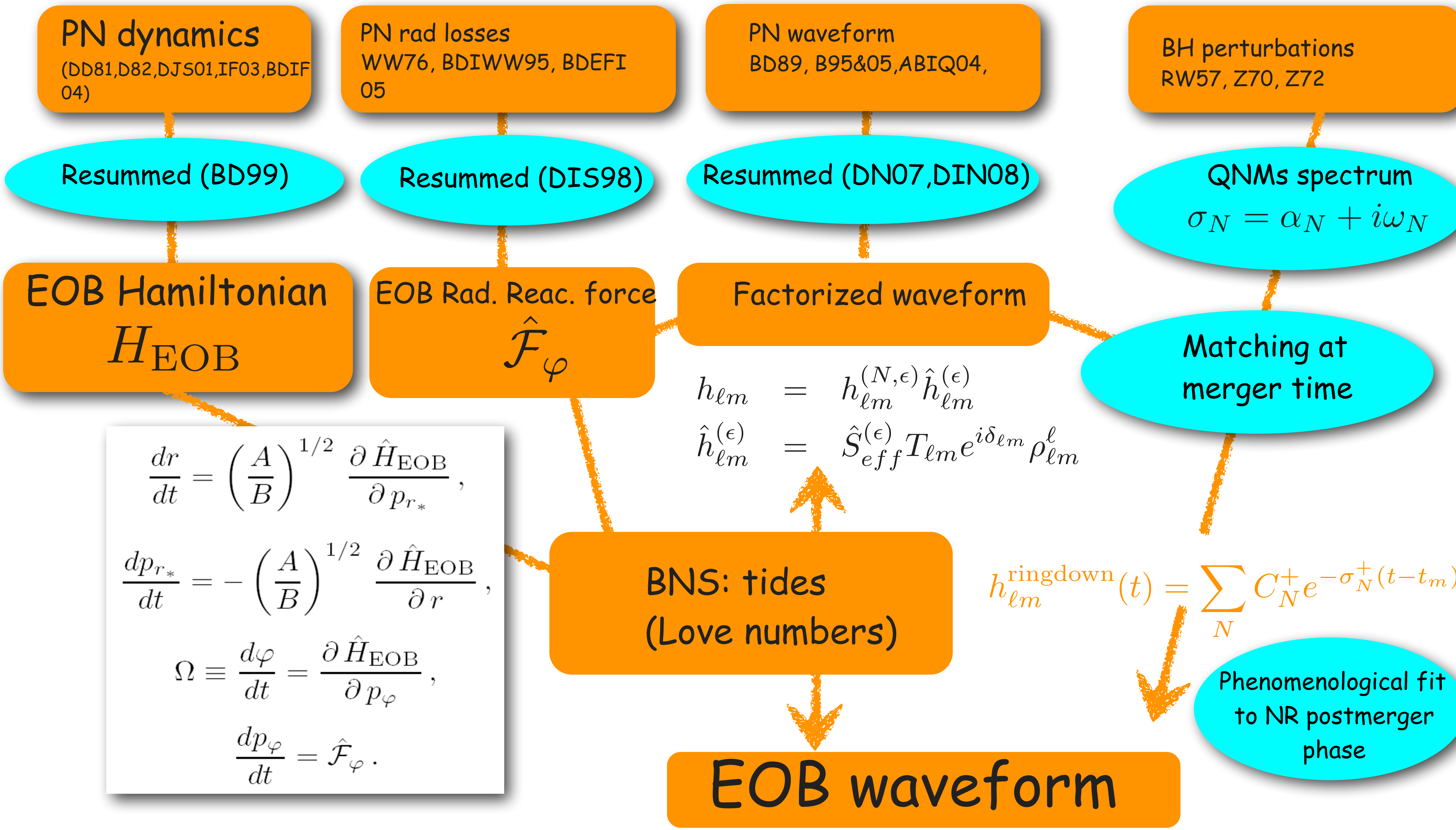
(1) Replace two-body dynamics (m_1, m_2) by dynamics of a particle $(\mu \equiv m_1 m_2/(m_1 + m_2))$ in an effective metric $g_{\mu\nu}^{\text{eff}}(u)$, with

$$u \equiv GM/c^2 R\,, \quad M \equiv m_1 + m_2$$

(2) Systematically use RESUMMATION of PN expressions (both $g_{\mu\nu}^{\text{eff}}$ and $\mathcal{F}_{RR}$) based on various physical requirements

(3) Require continuous deformation w.r.t. $\nu \equiv \mu/M \equiv m_1 m_2/(m_1 + m_2)^2$ in the interval $0 \leq \nu \leq \frac{1}{4}$

STRUCTURE OF THE EOB FORMALISM



$$h^{\rm EOB}_{\ell m} = \theta(t_m - t) h^{\rm insplunge}_{\ell m}(t) + \theta(t - t_m) h^{\rm ringdown}_{\ell m}(t)$$

EXPLICIT FORM OF THE EOB HAMILTONIAN

EOB Hamiltonian

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\hat{H}_{\rm eff} - 1\right)}$$

All functions are a ν-dependent deformation of the Schwarzschild ones

$$\underline{A(r) = 1 - 2u + 2\nu u^3 + a_4 \nu u^4}$$

$$a_4 = \frac{94}{3} - \frac{41}{32}\pi^2 \simeq 18.6879027$$

$$A(r)B(r) = 1 - 6\nu u^2 + 2(3\nu - 26)\nu u^3$$

$$u = GM/(c^2 R)$$

Simple effective Hamiltonian:

$$\hat{H}_{\rm eff} \equiv \sqrt{p_{r_*}^2 + A(r)\left(1 + \frac{p_\varphi^2}{r^2} + z_3\frac{p_{r_*}^4}{r^2}\right)} \qquad p_{r_*} = \left(\frac{A}{B}\right)^{1/2} p_r$$

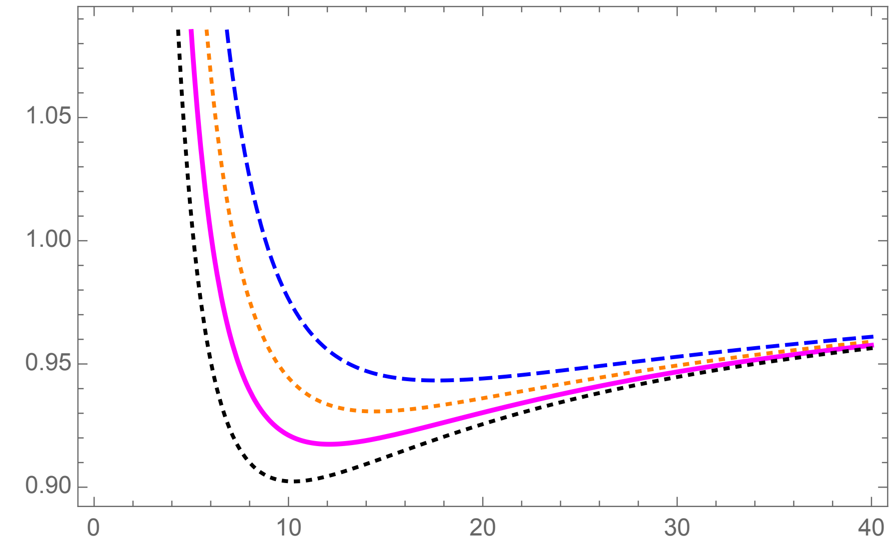
Crucial EOB radial potential

Contribution at 3PN

EFFECTIVE POTENTIALS

Newtonian gravity (any mass ratio):
circular orbits are always stable. No plunge.

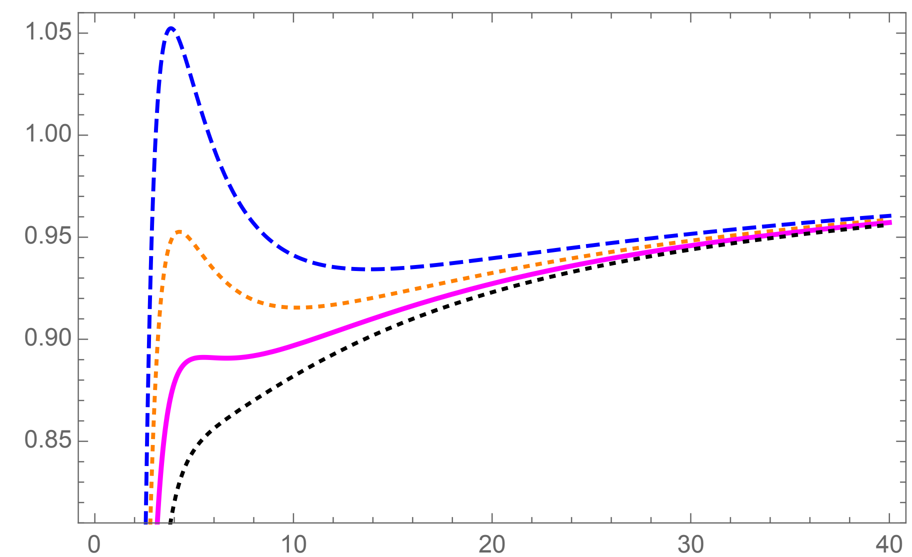
$$W^{\rm eff}_{\rm Newt} = 1 - \frac{2}{r} + \frac{p_\varphi^2}{r^2}$$



Test-body on Schwarzschild black hole:

last stable orbit (LSO) at **r=6M**; plunge

$$W^{\rm eff}_{\rm Schwarzschild} = \left(1 - \frac{2}{r}\right)\left(1 + \frac{p_\varphi^2}{r^2}\right)$$



EOB, Black-hole binary, any mass ratio:

last stable orbit (LSO) at **r<6M** plunge

$$W^{\rm eff}_{\rm EOB} = A(r;\,\nu)\left(1 + \frac{p_\varphi^2}{r^2}\right)$$

ν -deformation of the Schwarzschild case!

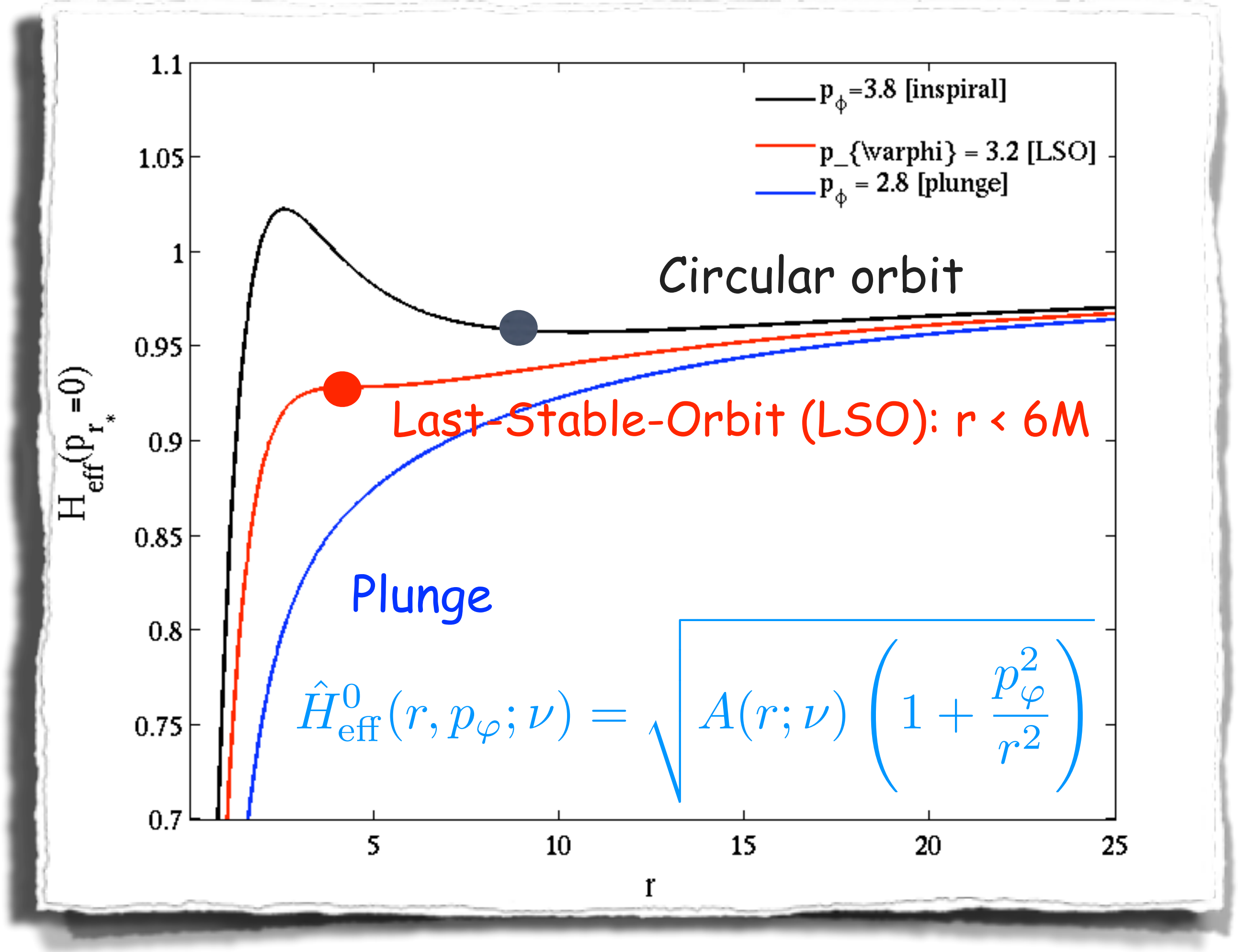
HAMILTON'S EQUATIONS & RADIATION REACTION

$$\dot{r} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}}$$

$$\dot{\varphi} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi} \equiv \Omega$$

$$\dot{p}_{r_*} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r} + \hat{\mathcal{F}}_{r_*}$$

$$\dot{p}_\varphi = \hat{\mathcal{F}}_\varphi$$



- The system must radiate angular momentum
- How?Use PN-based (Taylor-expanded) radiation reaction force (ang-mom flux)
- Need flux resummation

$$\hat{\mathcal{F}}_\varphi^{\rm Taylor} = -\frac{32}{5}\nu\Omega^5 r_\Omega^4 \hat{F}^{\rm Taylor}(v_\varphi)$$

Resummation multipole by multipole (Damour&Nagar 2007, Damour, Iyer & Nagar 2008, Damour & Nagar, 2009)

Plus horizon contribution [AN&Akcay2012]

4PN analytically complete + 5PN logarithmic term in the A(u) function:

[Damour 2009, Blanchet et al. 2010, Barack, Damour & Sago 2010, Le Tiec et al. 2011, Barausse et al. 2011,Akcay et al. 2012, Bini& Damour2013, DamourJaranowski&Schaefer 2014].

$$A_{5\mathrm{PN}}^{\mathrm{Taylor}} = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 + \nu[a_5^c(\nu) + a_5^{\ln}\ln u]u^5 + \nu[a_6^c(\nu) + a_6^{\ln}\ln u]u^6$$

1PN 2PN 3PN 4PN 5PN

$$a_5^{\log} = \frac{64}{5}$$
$$a_5^c = a_{5_0}^c + \nu a_{5_1}^c$$
$$a_{5_0}^c = -\frac{4237}{60} + \frac{2275}{512}\pi^2 + \frac{256}{5}\log(2) + \frac{128}{5}\gamma$$
$$a_{5_1}^c = -\frac{221}{6} + \frac{41}{32}\pi^2$$

4PN fully known ANALYTICALLY!

$$a_6^{\log} = -\frac{7004}{105} - \frac{144}{5}\nu$$

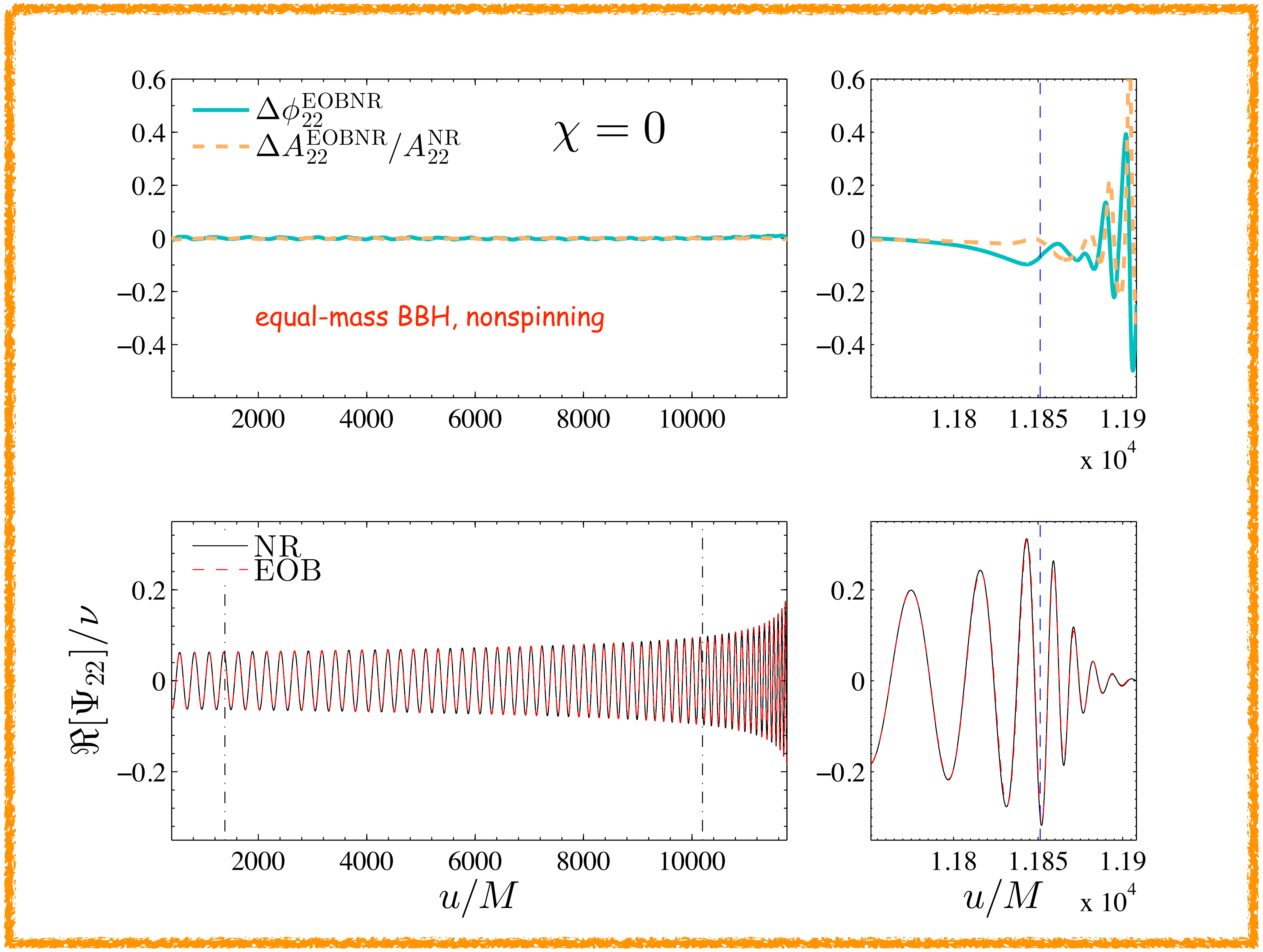
5PN logarithmic term (analytically known)

NEED ONE "effective" 5PN parameter from NR waveform data: $a_6^c(\nu)$

State-of-the-art EOB potential (5PN-resummed):

$$A(u;\nu,a_6^c) = P_5^1[A_{5\mathrm{PN}}^{\mathrm{Taylor}}(u;\nu,a_6^c)]$$

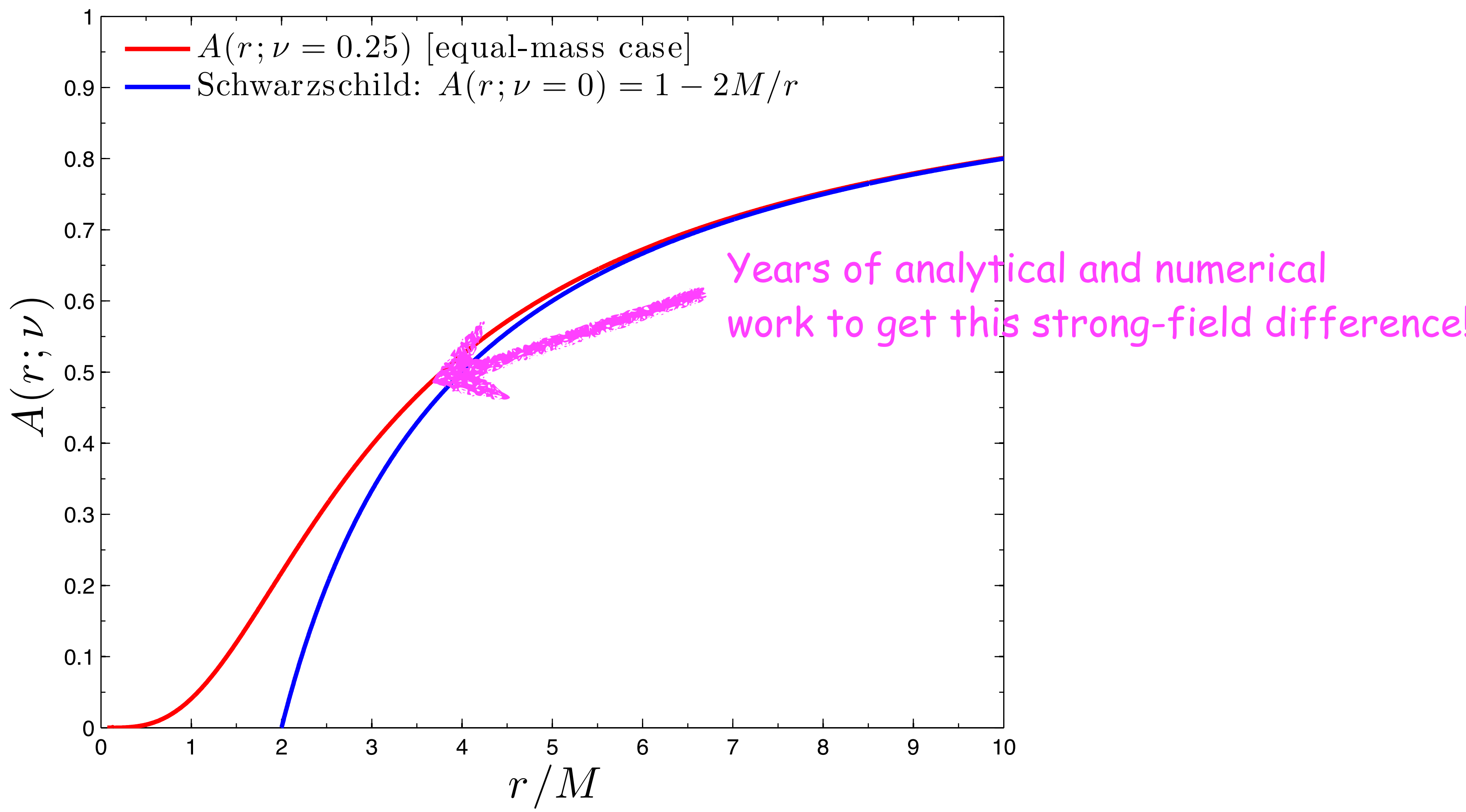
RESULTS: EOBNR/NR WAVEFORMS (NO SPIN)



Nagar, Damour, Reisswig & Pollney, PRD 93 (2016), 04404

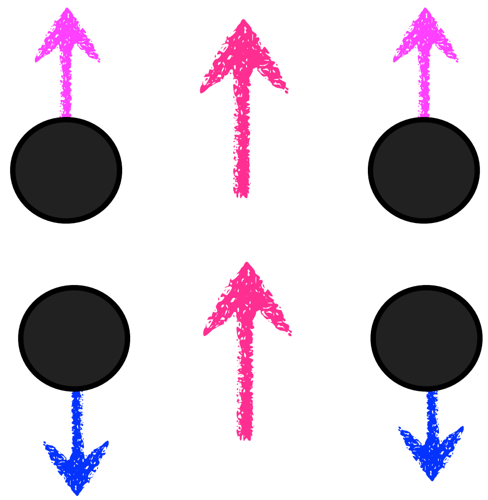
TEOBResumS: equal-mass, no spin

THE EOB[NR] POTENTIAL



From EOB/NR-fitting: $a_6^c(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.3804$

SPINNING BBHS



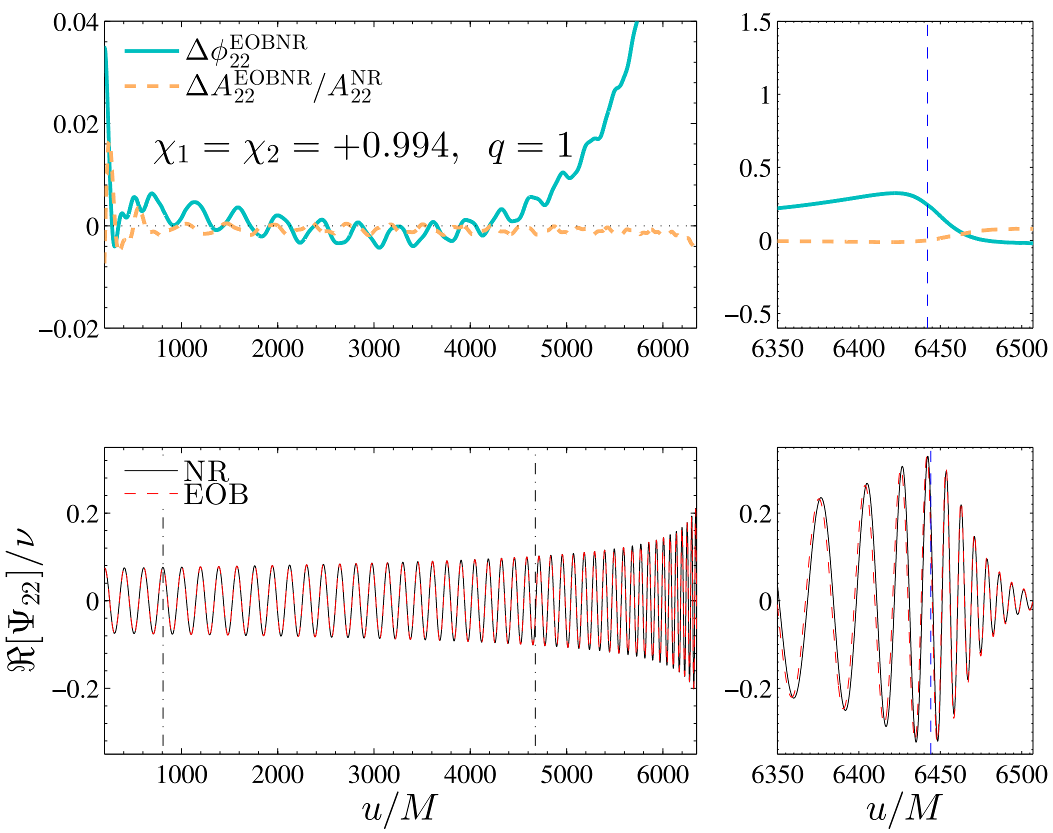
Spin-orbit & spin-spin couplings

(i) Spins aligned with L: repulsive (slower) L-o-n-g-e-r INSPIRAL

(ii) Spins anti-aligned with L: attractive (faster) Shorter INSPIRAL

(iii) Misaligned spins: precession of the orbital plane (waveform modulation)

$$\chi_{1,2} = \frac{c\,\mathbf{S}_{1,2}}{G m_{1,2}^2}$$



EOB/NR agreement: sophisticated (though rather simple) model for spin-aligned binaries

Damour&Nagar, PRD90 (2014), 024054 (Hamiltonian)
Damour&Nagar, PRD90 (2014), 044018 (Ringdown)
Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

AEI model, SEOBNRv4, Bohe et al., arXiv:1611.03703v1 (PRD in press)

PRECESSION

Different EOB Hamiltonian [Barausse & Buonanno11, Taracchini et al.12]

SEOBNRv3: Taracchini, Buonanno et al., PRD 89, 061502 (R), 2014

Babak, Taracchini & Buonanno, 2016

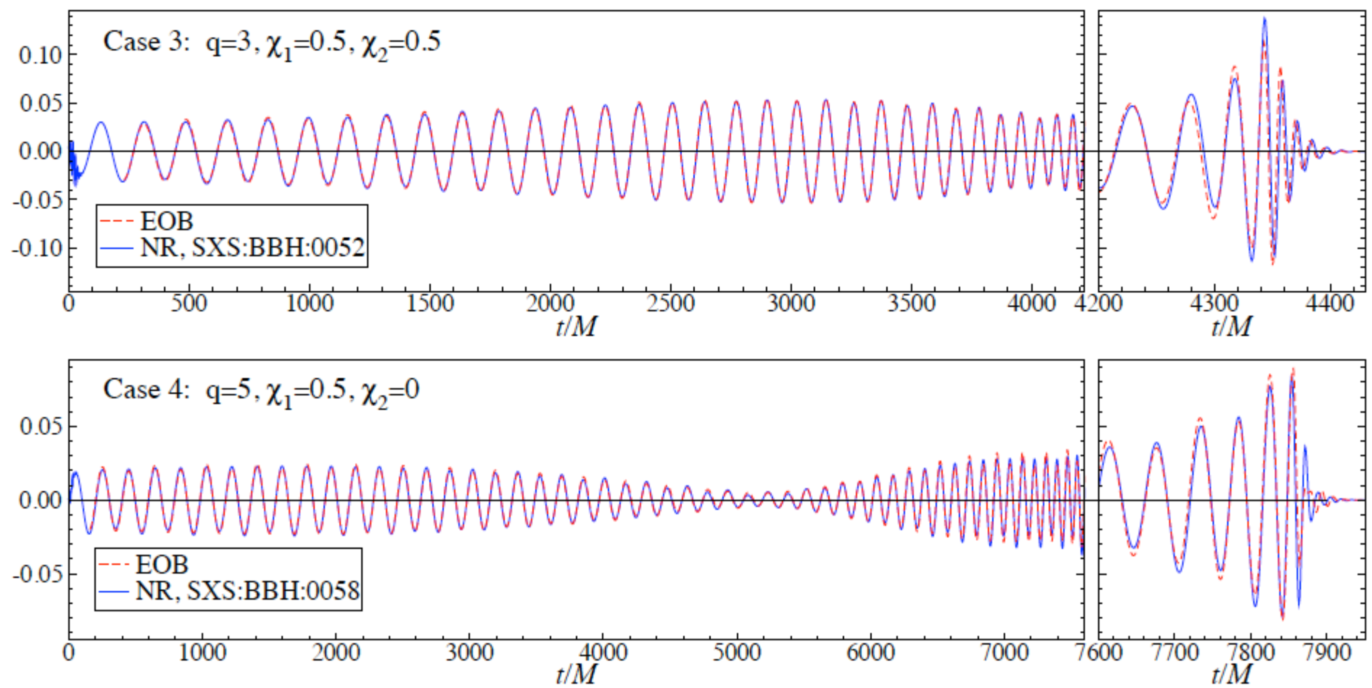


FIG. 9: We show for cases 3 and 4 of Table I the GW polarization h_+, containing contributions from $\ell = 2$ modes, that propagates along a direction $\hat{N}$ specified by spherical coordinates $\theta = \pi/3$ and $\phi = \pi/2$ associated with the inertial source frame $\{e_1^S, e_2^S, e_3^S\}$. The EOB waveforms start at the after-junk-radiation times of $t = 230M$ and $t = 160M$, respectively.

Good EOBNR/NR agreement.
The method works

Slow: analysis is time-consuming

Improvements in the implementation are needed

PhenomP: P. Schmidt et al. 2012/2014

Phenomenological Precessing model that takes into account precession effects at leading order by "twisting" nonprecessing waveforms.

Conclusion: no precession could be really seen.

BNS: ANALYTICAL NEEDS

- Study the response of each neutron star to the tidal field of the companion [theory of relativistic Love numbers (i.e. tidal polarizability coefficients) + tidal corrections to dynamics (beyond Newtonian accuracy)]

- Incorporate the corresponding tidal effects within a theoretical framework able to describe the gravitational wave signal emitted by inspiralling compact binaries (up to merger): EOB-resummed description of dynamics and waveforms

- Compare/check analytical models against NR simulations; possibly, inform, when needed, the EOB model with high-order tidal information extracted from NR

- Assess the measurability of tidal effects within the signal seen by interferometric detectors

- Construct surrogate models (e.g. ROMs) for fast template evaluation [Lackey et al. 2016]

A BIT OF HISTORY…

1983-Damour. Relativistic generalization of Love numbers

1989-1991-Damour-Soffel-Xu: magnetic&electric tidal polarization constants (tidal polarizabiility)

2007-Hinderer. First Love number computation in GR

2008-Hinderer& Flanagan. Polytropes. Inspiral only - not promising

2009-TD&AN - Binnington&Poisson - multipolar Love numbers

2009- Hinderer. Love numbers for realistic EOS. Measurability inspiral only - not promising

2009- TD&AN - Tidal Effective One Body model (TEOB). Up to merger (contact). 1PN dynamical effects

2010- Vines, Hinderer & Flanagan. 1PN effects in waveforms

2010 - EOB/NR comparisons [Baiotti, Damour, Nagar+]. **TaylorT4 ruled out.** Strong tides close to merger?

2011 - 2PN tidal terms [Bini, Damour & Faye]

2012 - Damour, Nagar & Villain. EOB. Strong indications for measurability of Love numbers up to merger.

2012 - Bernuzzi, Nagar et al.: high order reconstruction - E(j) correct. Disentangle tides & noise. Strong tides?

2013 - Del Pozzo+; Agathos+. TaylorF2 up to merger. Bayesian analysis. Measurability of Love numbers.

2014 - Bini& Damour: GSF-informed tidal potential

2015 - Bernuzzi, Nagar, Dietrich&Damour - EOB-GSF/NR compatibility up to merger. Several EOS.

2015 - Hotokezaka et al. Similar analysis confirming BNDD. **Negligible eccentricity.**

2015 - Hinderer +: inclusion of f-mode oscillations in EOB model. BHNS relevancy. H4 EOS.

2016 - Hotokezaka+; EOB-NR hybrids & new assessment of LN measurability; several EOS; PN finally ruled out

2016 - Lackey, Bernuzzi+: ROM of BNDD model

2017 - Dietrich-Hinderer: comparisons between TEOB: high level of consistency

2017 - Bernuzzi-Dietrich-Tichy: NR-based, closed form tidal approximants

2018 - Nagar+: incorporation of spin-orbit & spin-spin effect (wip)

TIDAL EFFECTS IN EOB FORMALISM

Tidal extension of EOB formalism: nonminimal worldline couplings

$$\Delta S_{\mathrm{nonminimal}} = \sum_A \frac{1}{4} \mu_2^A \int ds_A \, (u^\mu u^\nu R_{\mu\alpha\nu\beta})^2 + \dots$$

Damour&Esposito-Farèse96, Goldberger&Rothstein06, TD&AN09

Relativistic Love number

Modifications of the EOB effective metric...

$$\begin{aligned} A(r) &= A_r^0 + A^{\mathrm{tidal}}(r) \\ A^{\mathrm{tidal}}(r) &= -\kappa_2^T u^6 \left(1 + \bar{\alpha}_1 u + \bar{\alpha}_2 u^2 + \dots\right) + \dots \end{aligned}$$

And tidal modifications of GW waveform & radiation reaction

Need analytical theory for computing $\mu_2, \ \kappa_2^T, \ \bar{\alpha}_1 \dots$

(?)Need accurate NR simulations to check/inform the higher-order PN tidal contributions, that may be quite important during the late inspiral up to merger

LOVE NUMBERS IN GENERAL RELATIVITY

Relativistic star in an external gravito-electric & gravito-magnetic (multipolar) tidal field

The star acquires induced gravito-electric and gravito-magnetic multipole moments

Linear tidal polarization

Induced multipole moments

$$M_L^{(A)} = \mu_\ell^A G_L^{(A)}$$
$$S_L^{(A)} = \sigma_\ell^A H_L^{(A)}$$

External multipolar field

$$G\mu_\ell = [length]^{2\ell+1}$$
$$G\sigma_\ell = [length]^{2\ell+1}$$

$$2k_\ell \equiv (2\ell-1)!!\frac{G\mu_\ell}{R^{2\ell+1}}$$

$$j_\ell \equiv (2\ell-1)!!\frac{4(\ell+2)}{\ell-1}\frac{G\sigma_\ell}{R^{2\ell+1}}$$

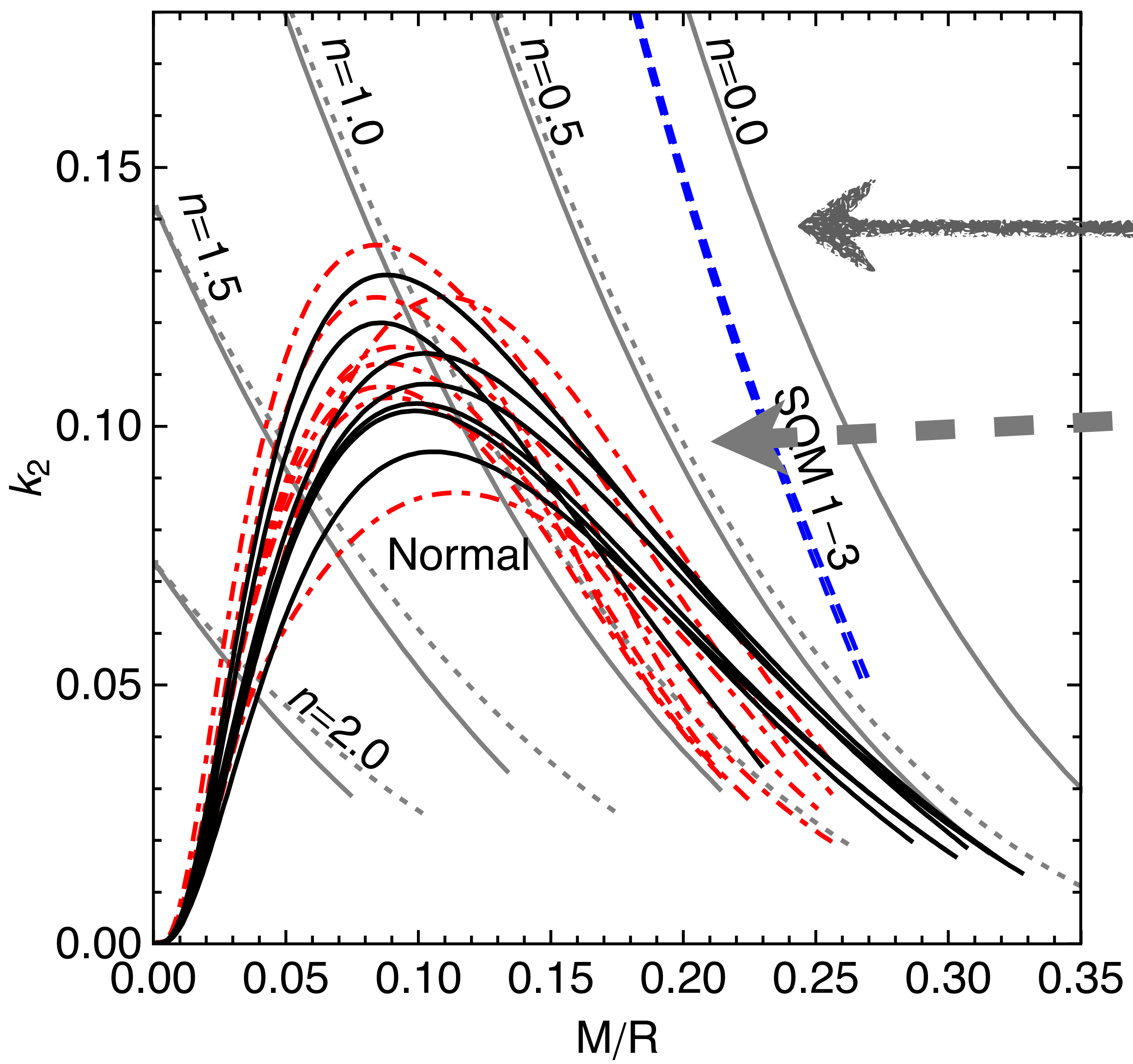
Dimensionless relativistic "second" Love numbers

Actual calculation based on star perturbation theory: Love numbers are obtained as boundary conditions (matching interior to exterior perturbations)

RELATIVISTIC LOVE NUMBERS: (REALISTIC EOS)

DN 2009 (SLy & FPS)

Hinderer et al. 2010 (several tabulated EOS)



Polytropic EOS

$$p = K\rho^{1+1/n}$$

$$p = K\epsilon^{1+1/n}$$

TIDAL INTERACTION POTENTIAL

Central tidal "coupling constant":

$$\kappa_\ell^T \equiv 2\left[\frac{1}{q}\left(\frac{X_A}{C_A}\right)^{2\ell+1} k_\ell^A + q\left(\frac{X_B}{C_B}\right)^{2\ell+1} k_\ell^B\right]$$

$$\begin{array}{c} M_A = M_B \\ \ell = 2 \\ k_2^A = k_2^B \end{array} \Rightarrow \kappa_2^T = \frac{1}{8}\frac{k_2}{\mathcal{C}^5}$$

$X_{A,B} \equiv M_{A,B}/M$

Function of: masses, compactnesses and relativistic Love numbers

$\kappa_2^T \sim 100$

In the dynamics:

$$A(u) = A^0(u) + A^{\text{tidal}}$$

$$A^{\text{tidal}} = \sum_{\ell \geq 2} -\kappa_\ell^T u^{2\ell+2} \hat{A}_\ell^{\text{tidal}}(u)$$

"Newtonian" (LO) part

+ PN corrections (NLO, NNLO, ...)

NLO & NNLO tidal PN corrections known analytically

[Bini, Damour& Faye 2011]

$$\hat{A}_2^{\text{tidal}} = 1 + \frac{5}{4}u + \frac{85}{14}u^2$$

RESUMMED TIDAL INTERACTION

Bini&Damour (2015) resummed expression for $\hat{A}_\ell^{\rm tidal}$

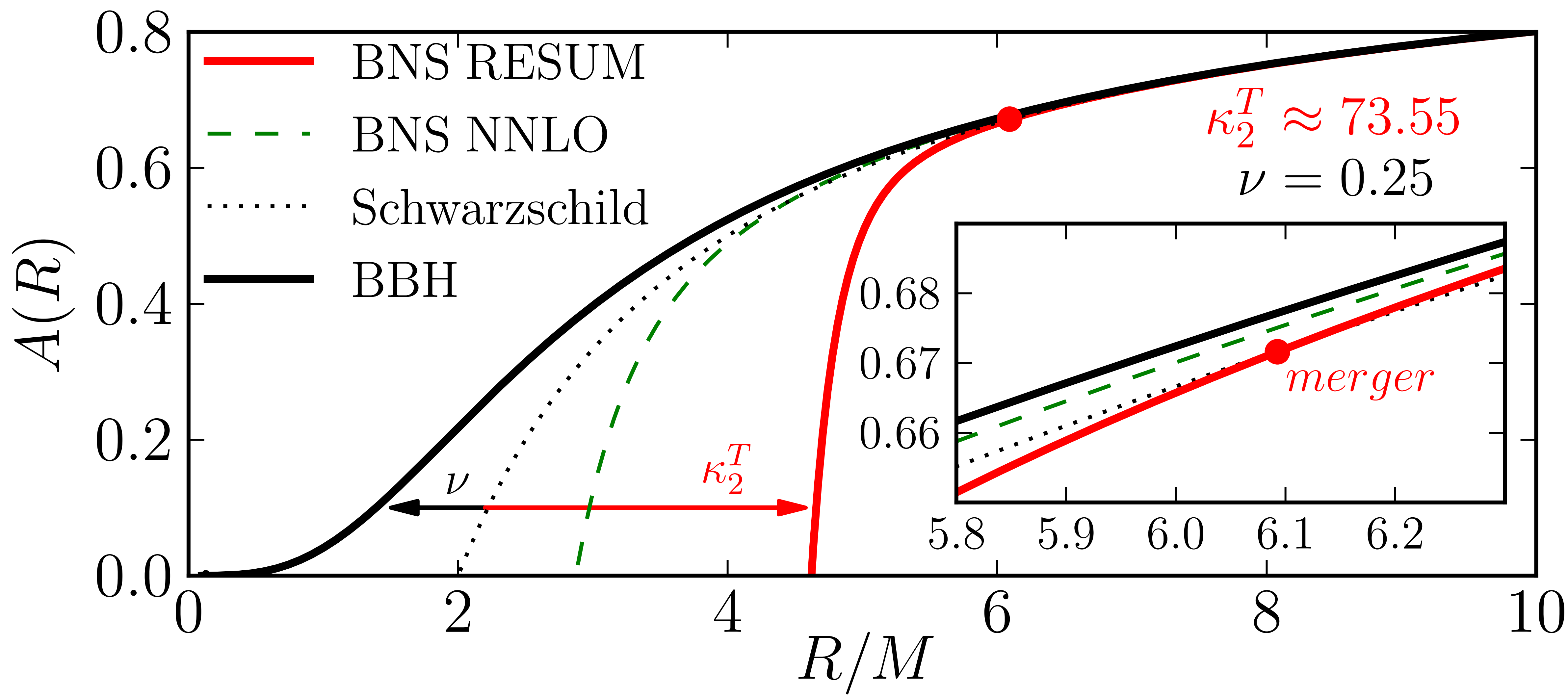
Presence of a pole: potential strongly attractive @ mrg

$$A_T^{(+)}(u;\nu) \equiv -\sum_{\ell=2}^{4}\left[\kappa_A^{(\ell)} u^{2\ell+2}\hat{A}_A^{(\ell^+)} + (A \leftrightarrow B)\right]$$

$$\hat{A}_A^{(2^+)}(u) = 1 + \frac{3u^2}{1-r_{\rm LR}u} + \frac{X_A\tilde{A}_1^{(2^+)1\rm SF}}{(1-r_{\rm LR}u)^{7/2}} + \frac{X_A^2\tilde{A}_2^{(2^+)2\rm SF}}{(1-r_{\rm LR}u)^p}$$

$p = 4$

Because of analytic considerations (BD 2015)



EOB (orbital) interaction potential

$$A(u) = A^0(u) + A^{\rm tidal}$$

$$A^{\rm tidal} = \sum_{\ell\geq 2} -\kappa_\ell^T u^{2\ell+2} \hat{A}_\ell^{\rm tidal}(u)$$ "Newtonian" (LO) part x PN corrections (NLO, NNLO, ...)

pointmass part

$$A_0 = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\right)\nu u^4 + \nu[a_5^c(\nu) + a_5^{\ln}\ln u]u^5 + \nu[a_6^c(\nu) + a_6^{\ln} u]u^6$$

1PN 2PN 3PN 4PN

5PN term "informed" by NR

$$a_6^c(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.3804$$

$$A^{\rm resum}(u;\nu,a_6^c) = P_5^1[A_0(u;\nu,a_6^c)]$$

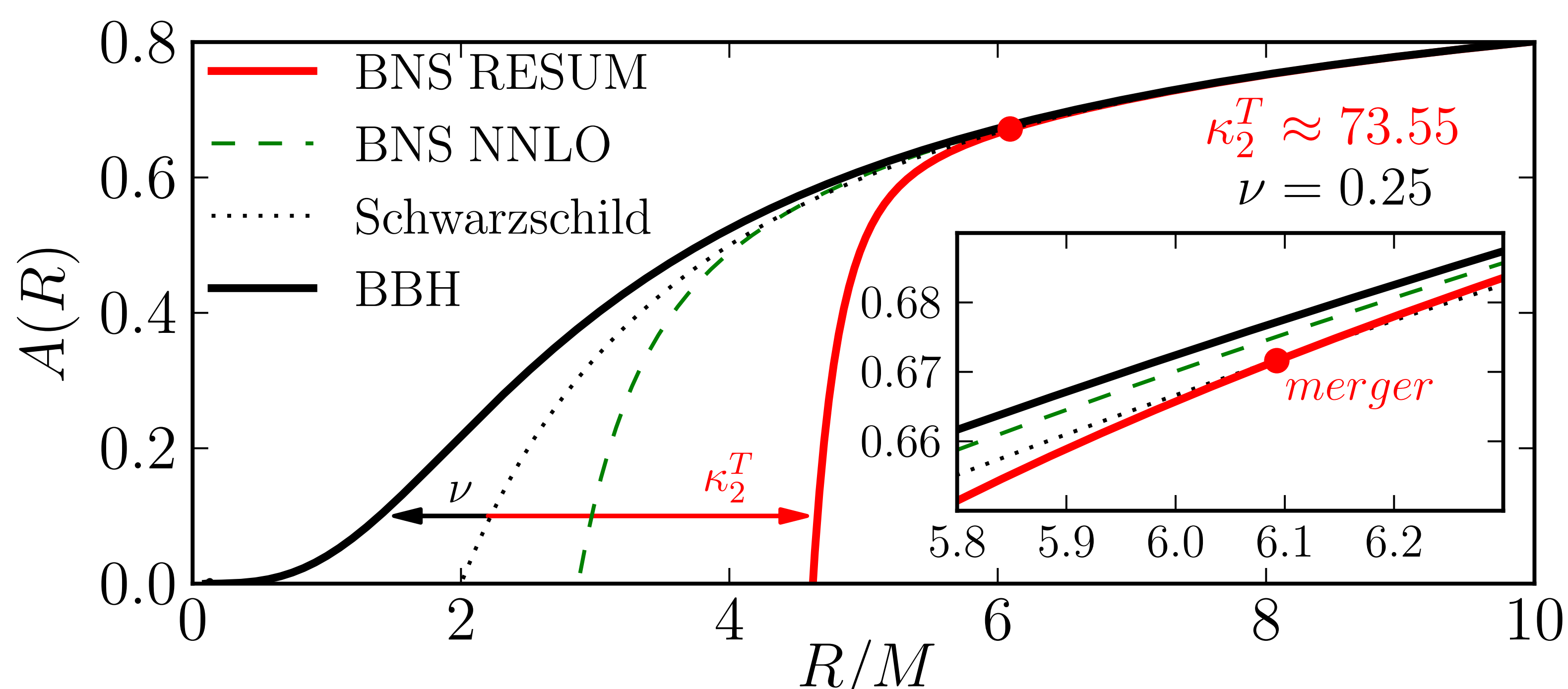
tidal part

$$A_T^{(+)}(u;\nu) \equiv -\sum_{\ell=2}^{4}\left[\kappa_A^{(\ell)} u^{2\ell+2}\hat{A}_A^{(\ell^+)} + (A \leftrightarrow B)\right]$$

$$\kappa_A^{(\ell)} = 2\frac{X_B}{X_A}\left(\frac{X_A}{C_A}\right)^{2\ell+1} k_\ell^A$$

Love numbers

$$X_{A,B} \equiv m_{A,B}/M$$



WAVEFORM

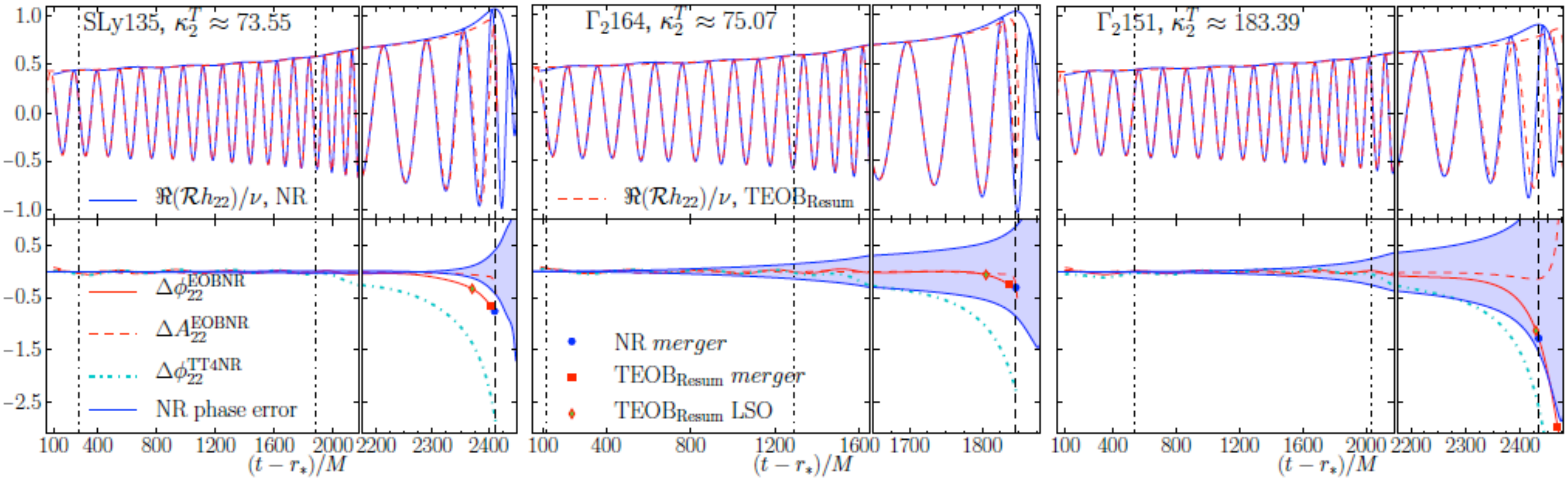


FIG. 3: Phasing and amplitude comparison (versus NR retarded time) between TEOB$_{\rm Resum}$, NR and the phasing of TT4 for three representative models. Waves are aligned on a time window (vertical dot-dashed lines) corresponding to $I_\omega \approx (0.04, 0.06)$. The markers in the bottom panels indicate: the crossing of the TEOB$_{\rm Resum}$ LSO radius; NR (also with a dashed vertical line) and EOB merger moments.

Name	EOS	κ_2^T	$r_{\rm LR}$	$\mathcal{C}_{A,B}$	$M_{A,B}[M_\odot]$	$M_{\rm ADM}^0[M_\odot]$	$\mathcal{J}_{\rm ADM}^0[M_\odot^2]$	$\Delta\phi_{\rm NRmrg}^{\rm TT4}$	$\Delta\phi_{\rm NRmrg}^{\rm TEOB_{NNLO}}$	$\Delta\phi_{\rm NRmrg}^{\rm TEOB_{Resum}}$	$\delta\phi_{\rm NRmrg}^{\rm NR}$
2B135	2B	23.9121	3.253	0.2049	1.34997	2.67762	7.66256	−1.25	−0.19	+0.57[a]	±4.20
SLy135	SLy	73.5450	3.701	0.17381	1.35000	2.67760	7.65780	−2.75	−1.79	−0.75	±0.40
$\Gamma_2 164$	$\Gamma = 2$	75.0671	3.728	0.15999	1.64388	3.25902	11.11313	−2.29	−1.36	−0.31	±0.90
$\Gamma_2 151$	$\Gamma = 2$	183.3911	4.160	0.13999	1.51484	3.00497	9.71561	−2.60	−1.92	−1.27	±1.20
H4135	H4	210.5866	4.211	0.14710	1.35003	2.67768	7.66315	−3.02	−2.43	−1.88	±1.04
MS1b135	MS1b	289.8034	4.381	0.14218	1.35001	2.67769	7.66517	−3.25	−2.84	−2.45	±3.01

S. Bernuzzi, A. Nagar, T. Dietrich & T. Damour, PRL 114 (2015), 161103

Nagar, Bernuzzi, Del Pozzo et al.: spin (in preparation)

SUMMARY

1. Italian-driven (INFN Torino) effort of having NR-informed, EOB-based waveform models for DA purposes. BBH & BNS. Spin aligned. Higher modes.

2. TEOBResumS: spin-aligned waveform model for BBH and NS. It can be used for measuring the EOS through the measure of the tidal polarizability constants.

3. Compatibility (within NR errors) between such EOBNR model and state-of-the art NR data over mass ratio and spin

4. BNS waveforms are very mature, and different avatars exists that are being used for DA purposes to measure the EOS