Characterising the astrophysical S-factor for $^{12}C + ^{12}C$ fusion with wave-packet dynamics

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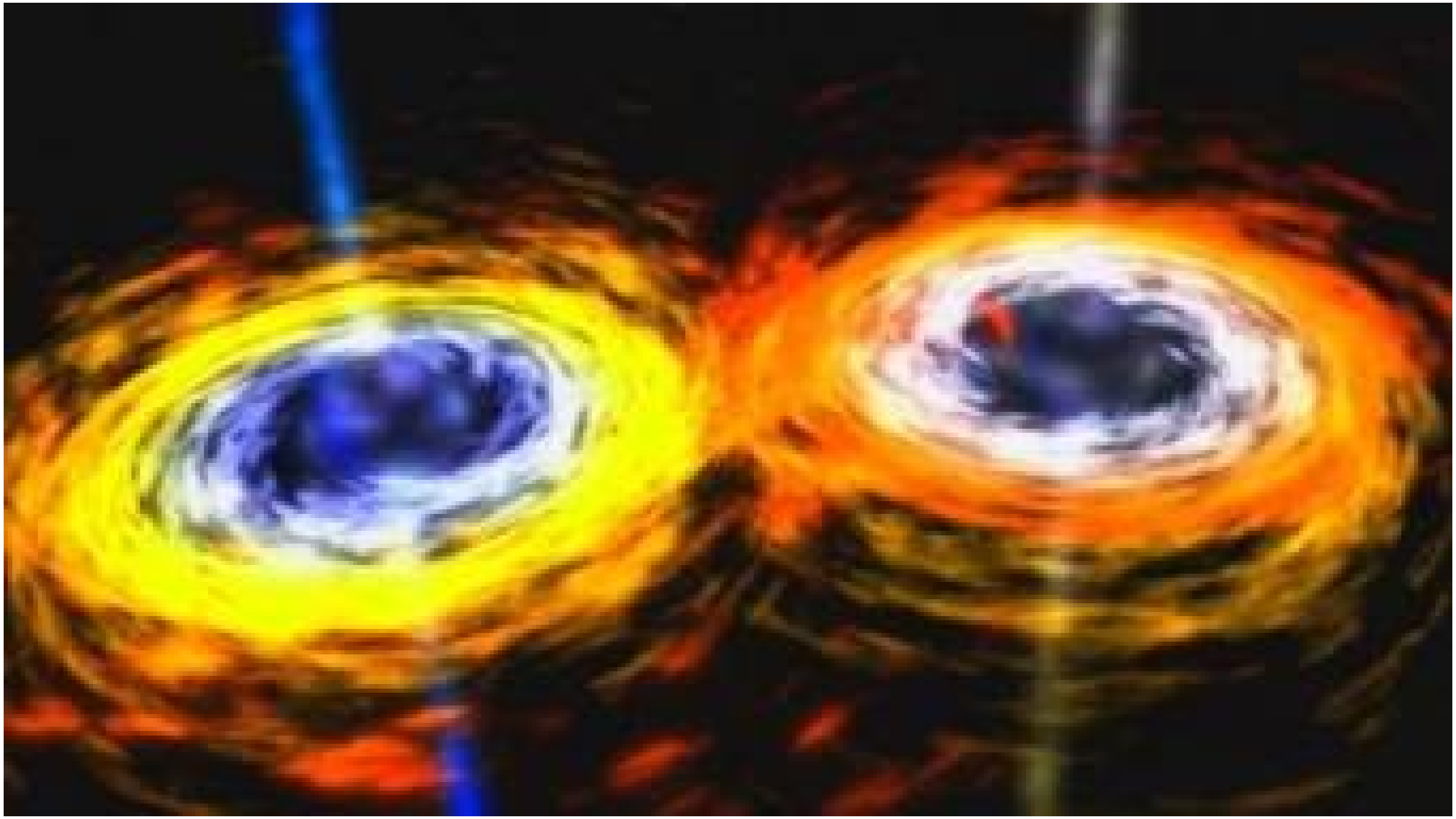
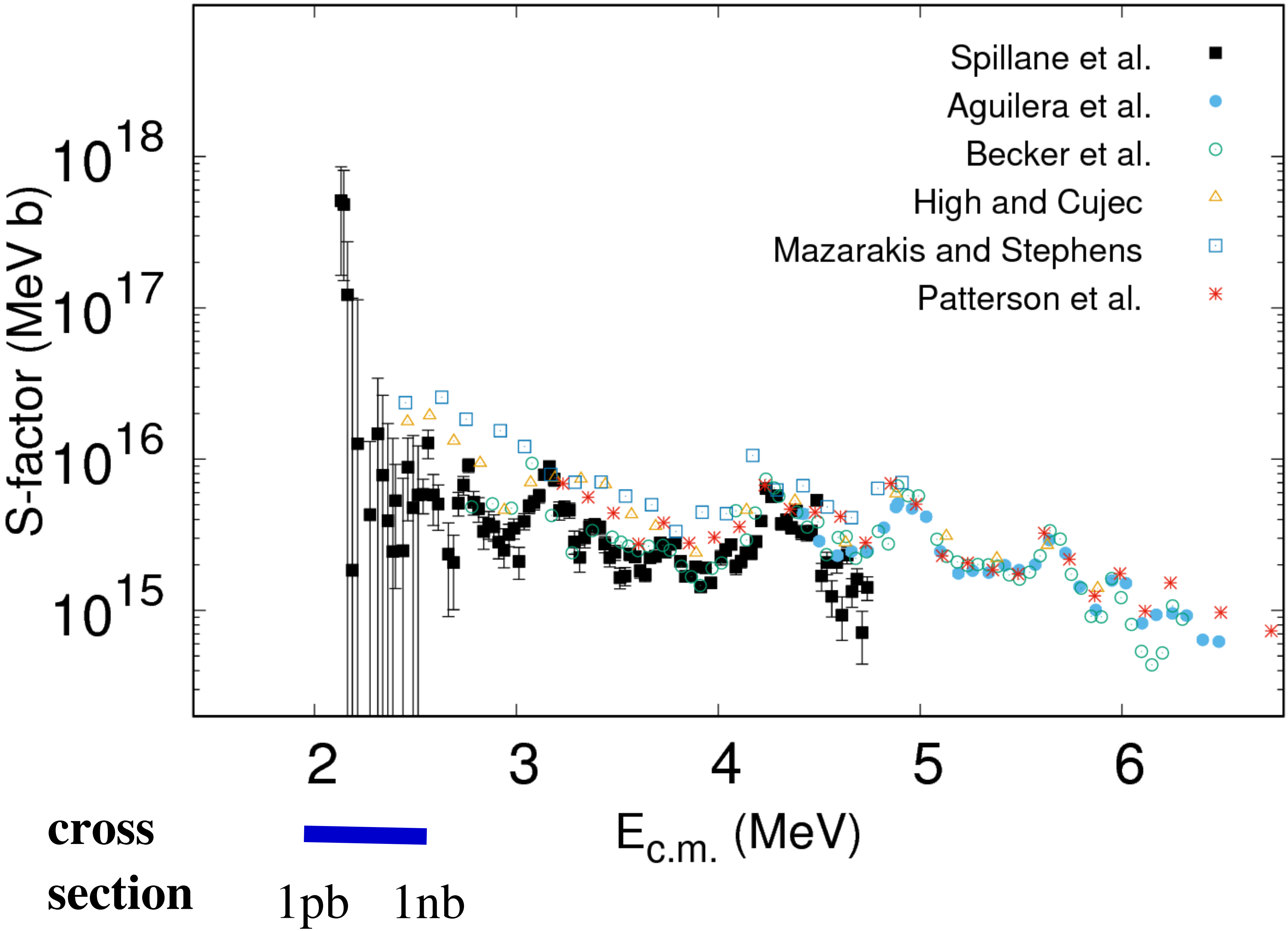
How do two ^{12}C nuclei fuse at sub-barrier energies?



Picture taken from BBC News

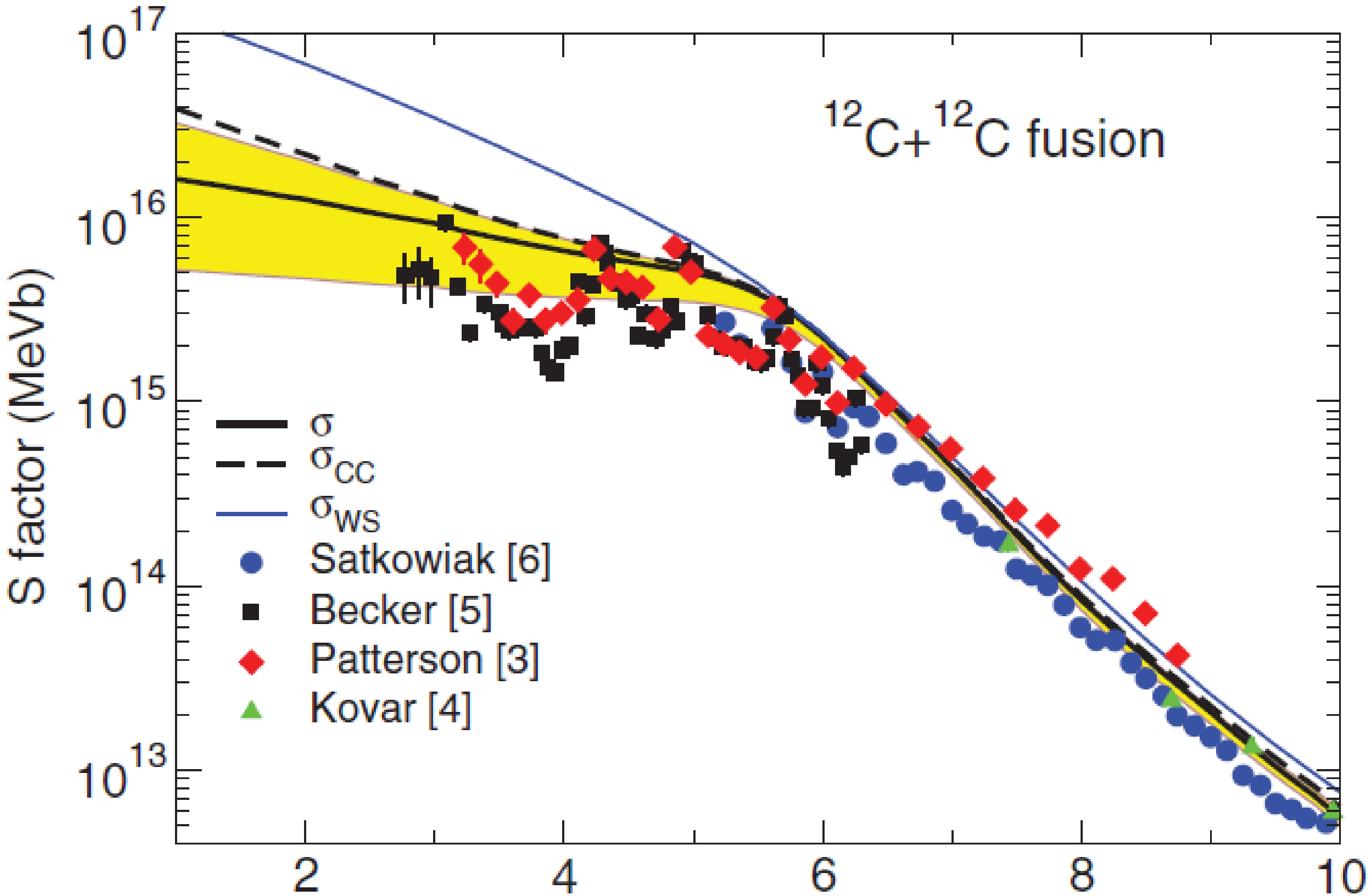
AD-T & Wiescher, Physical Review C **97** (2018) 055802

Astrophysical S-Factor Excitation Function for $^{12}C + ^{12}C$



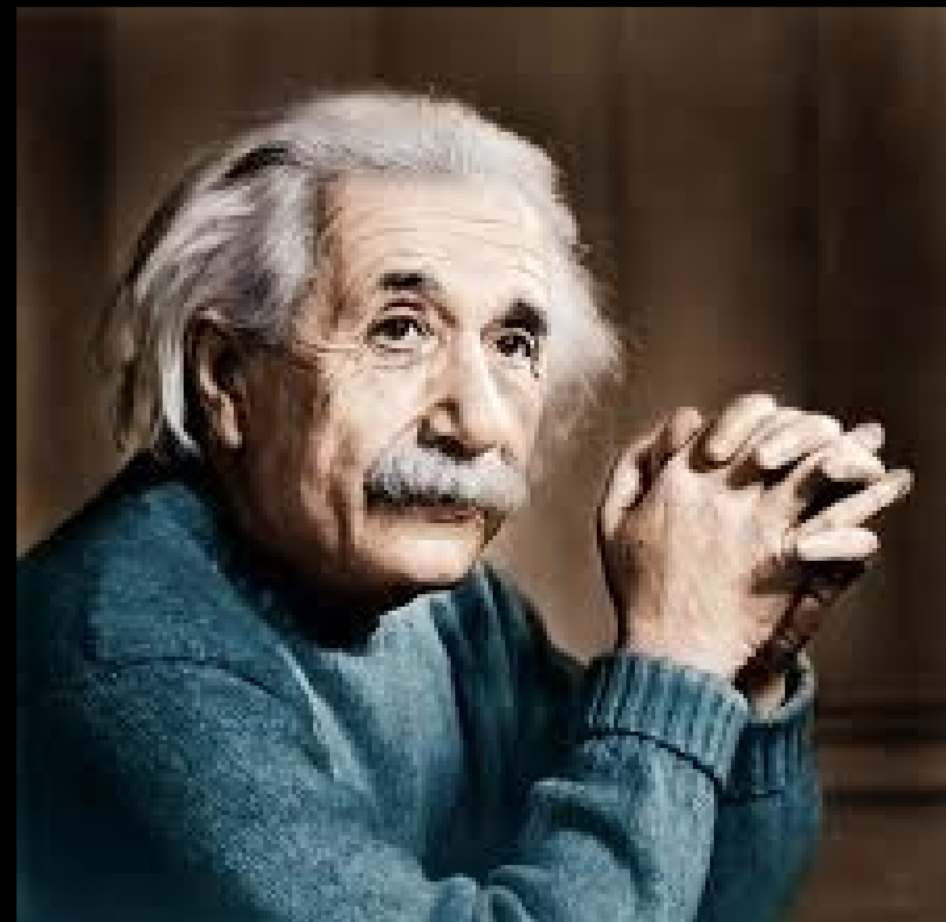
Coupled-Channels Calculations for $^{12}C + {}^{12}C$

Jiang, Esbensen et al., PRL **110** (2013) 072701



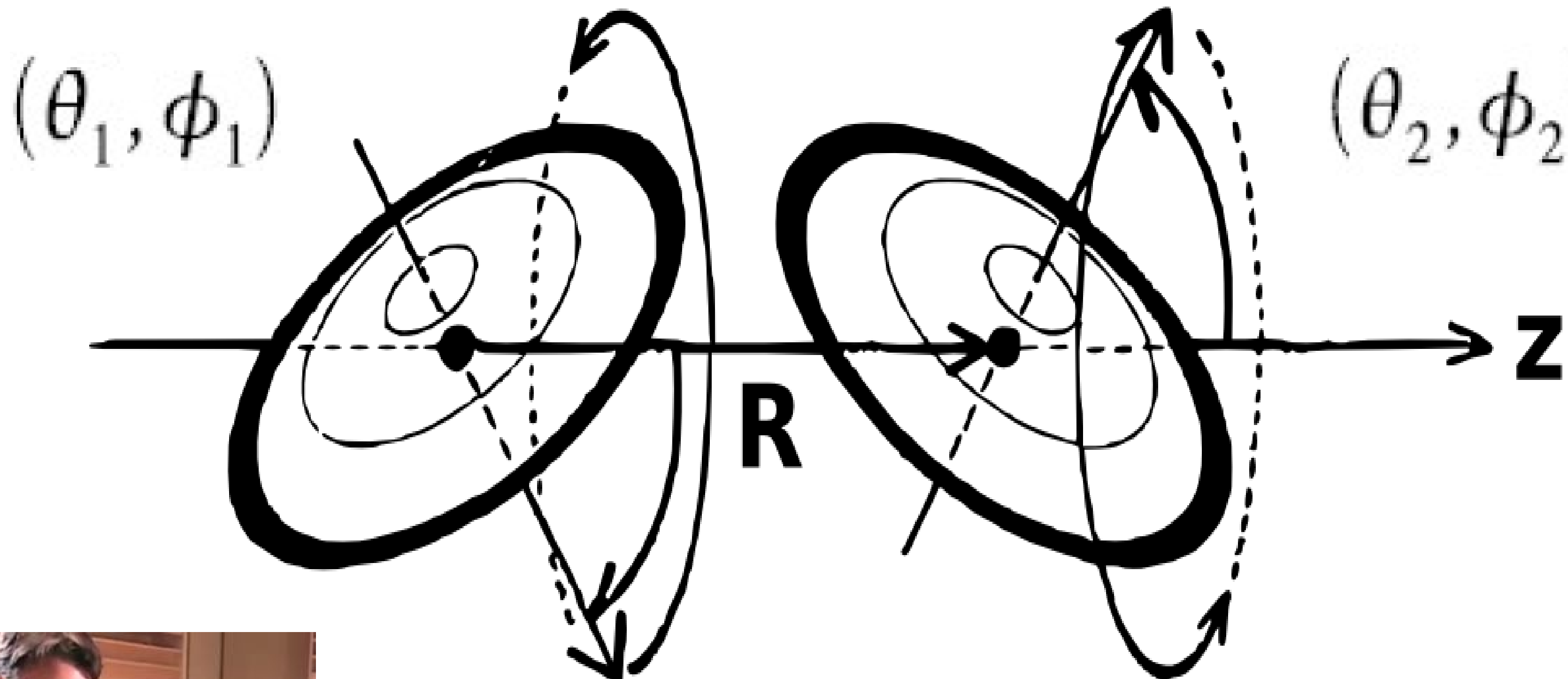
Ec.m. (MeV)

“WE CANNOT SOLVE OUR PROBLEMS WITH THE SAME THINKING WE USED WHEN WE CREATED THEM”



The $^{12}C + ^{12}C$ Molecular Structure

Greiner, Park & Scheid, in Nuclear Molecules, World Scientific, 1994



Quadrupole deformation of ^{12}C: ~ **- 0.5**

How does this molecular structure affect low-energy fusion?

Wave-Packet Dynamics

D.J. Tannor, Quantum Mechanics: a Time-Dependent Perspective, USB, 2007

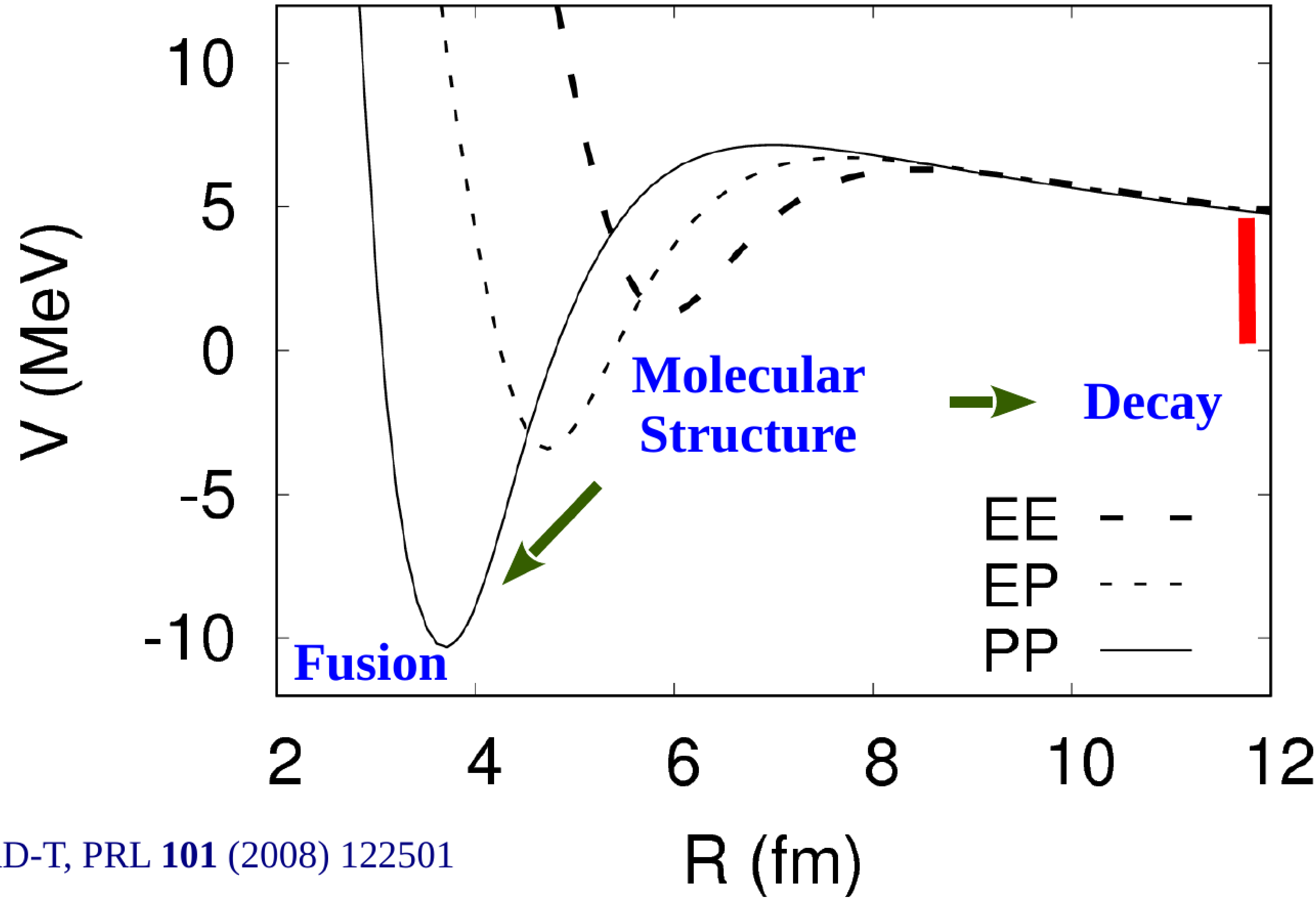
- **Preparation:** the initial state $\Psi(t=0)$

- **Time propagation:** $\Psi(0) \longrightarrow \Psi(t)$,
  guided by the operator, $\exp(-i\,\hat{H}\,t/\hbar)$

  $\hat{H}$ is the model Hamiltonian

- **Analysis:** extraction of probabilities from the time-dependent wave function

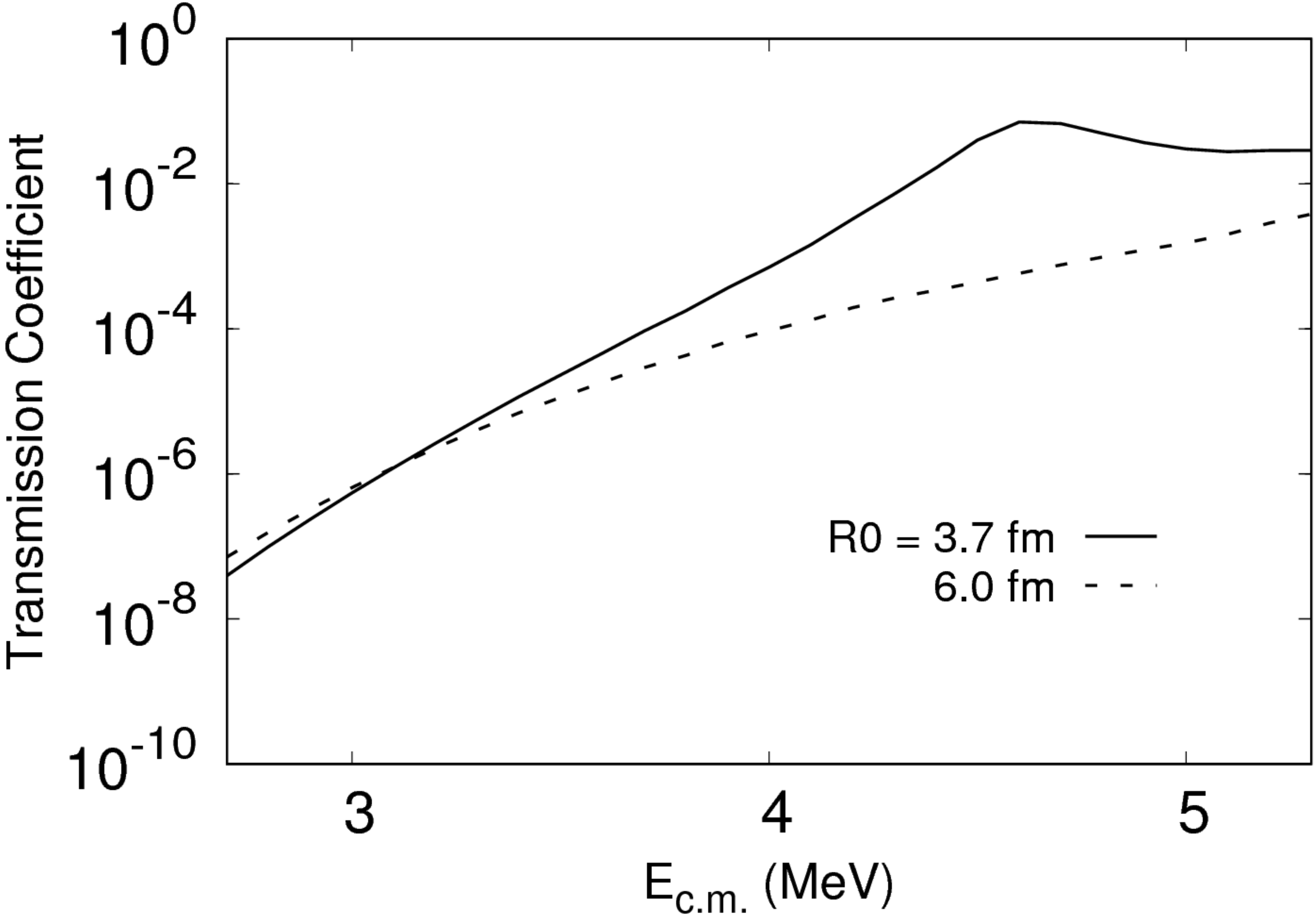
Collective Potential-Energy Landscape for $^{12}C + ^{12}C$



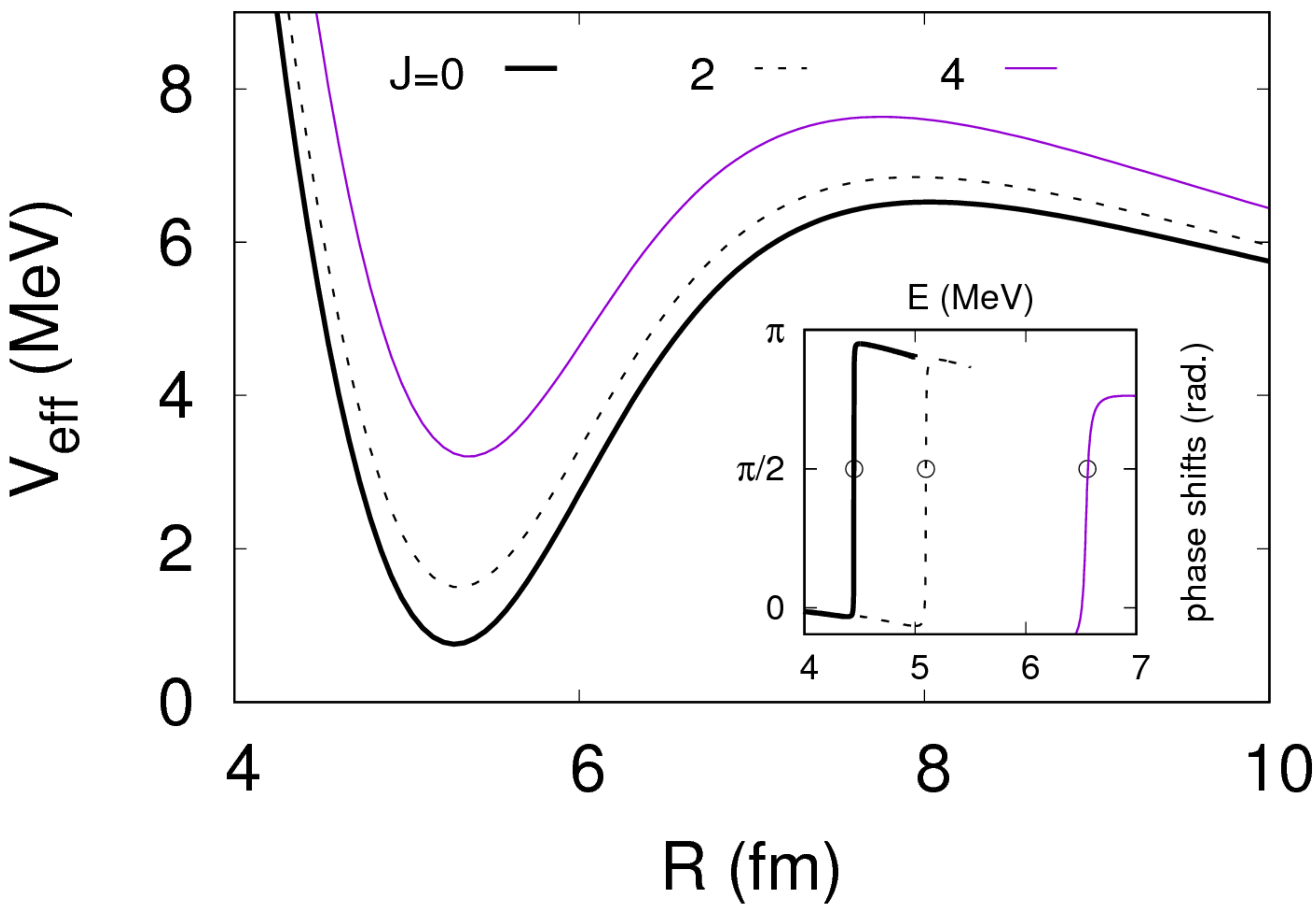
AD-T, PRL **101** (2008) 122501

Moeller & Iwamoto, NPA **575** (1994) 381

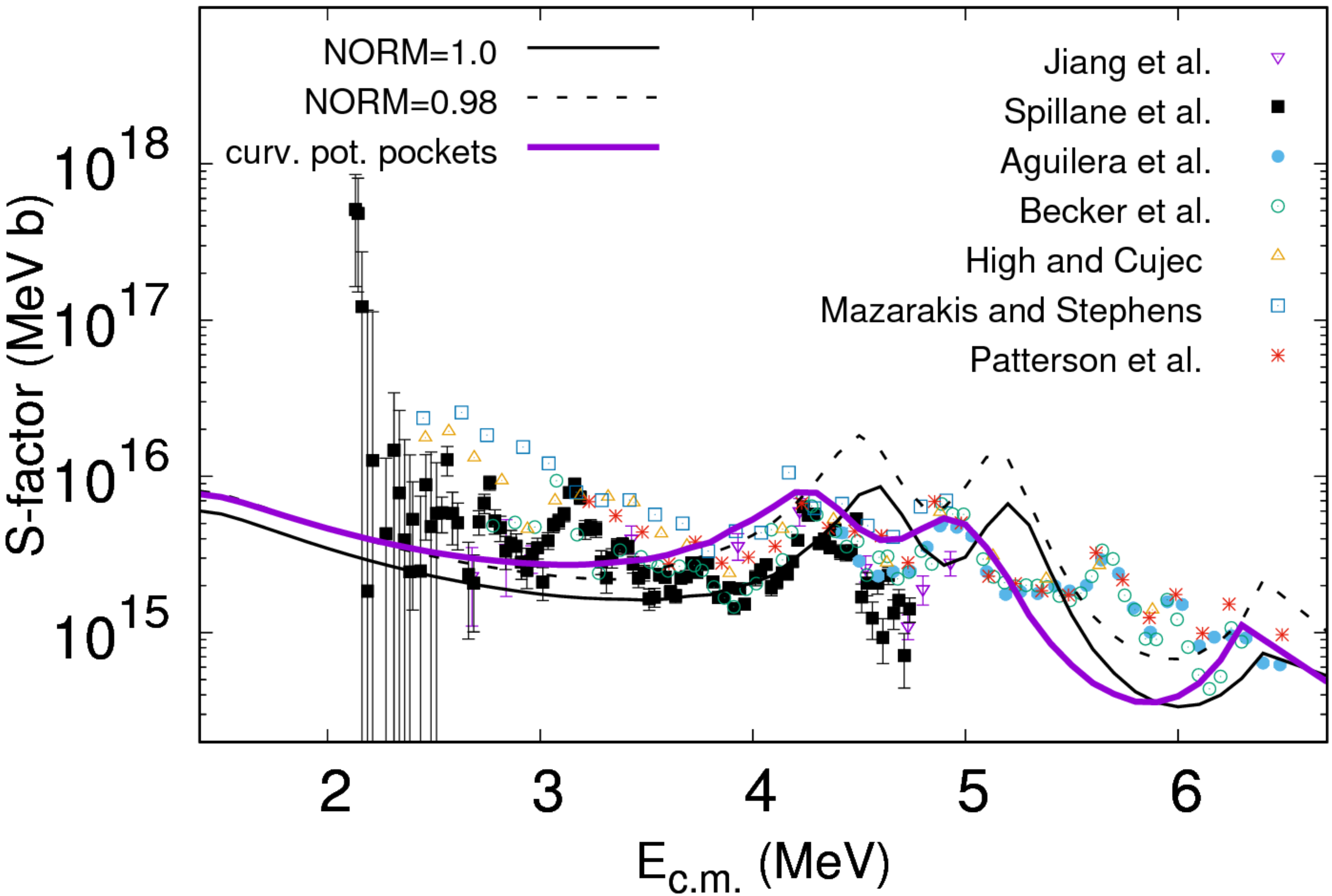
Role of the imaginary fusion potential in the transmission coefficient



Phase shift analysis of the effective potentials for molecular configurations



Astrophysical S-Factor for $^{12}C + ^{12}C$ Fusion



Conclusions

AD-T & Wiescher, Physical Review C **97** (2018) 055802

- The **fusion imaginary potential** for specific dinuclear configurations is crucial for the appearance of resonances.

- **Three resonant structures** are revealed in the calculations, reproducing similar structures in the experimental data.

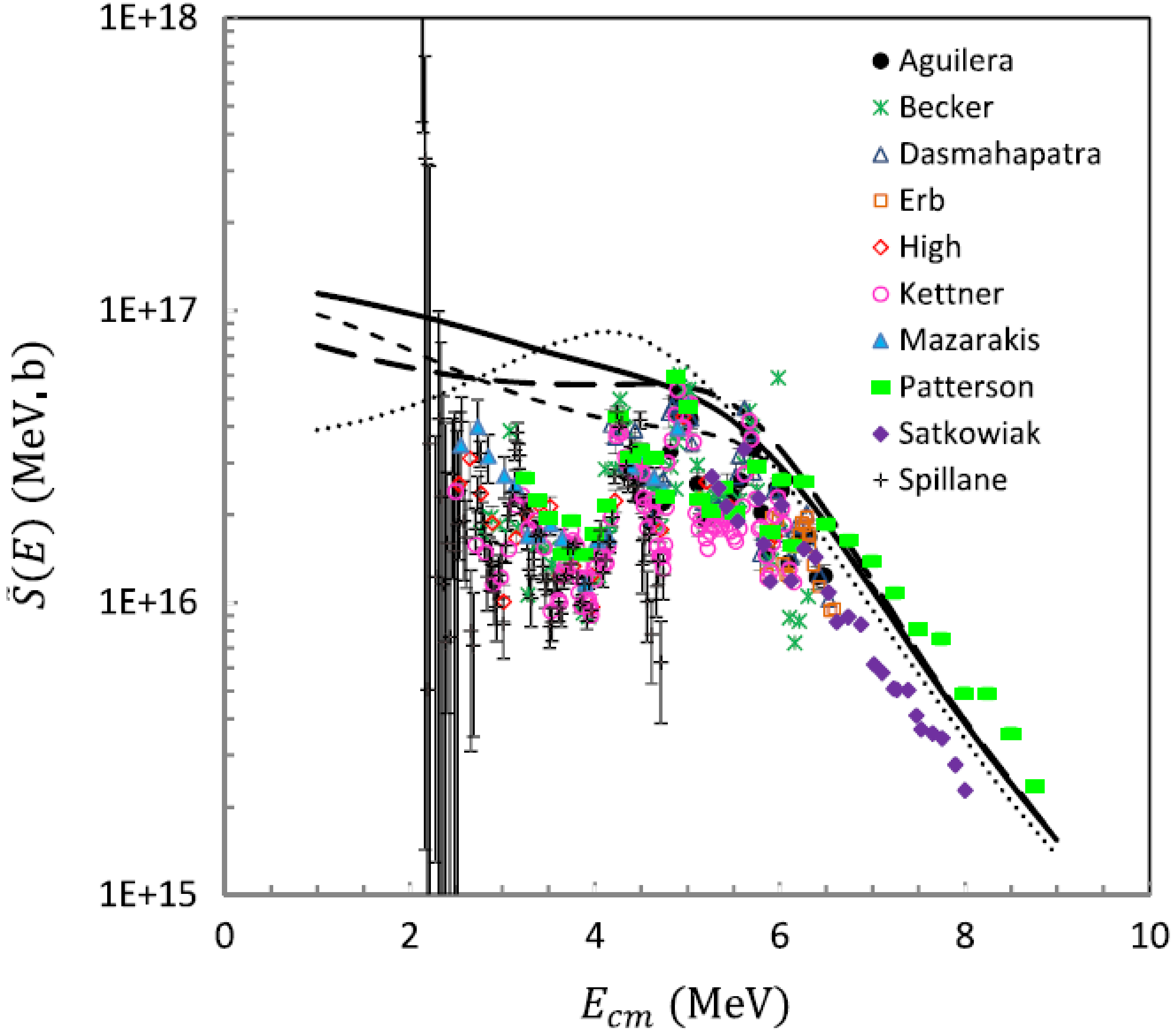
- Fusion cross sections **monotonically** decline towards stellar energies.

- **Resonant structures** in the experimental data that are not explained may be due to **cluster effects** in the nuclear molecule.

EXTRA SLIDES

Coupled-Channels Calculations for ^{12}C + ^{12}C

Assuncao & Descouvemont, PLB **723** (2013) 355



Fusion Cross Section & Astrophysical S-Factor

$$S(E) = \sigma(E)\, E\, \exp(2\pi\eta)$$

Structural factor [MeV barn]

Fusion cross section [barn = 10^{-28} m^2]

$$\eta = \left(\frac{\mu}{2}\right)^{1/2} \frac{Z_1 Z_2 e^2}{\hbar E^{1/2}}$$

Sommerfeld parameter

S(E) represents the fusion cross section free of Coulomb suppression, which is adequate for extrapolation towards stellar energies

Kinetic-Energy of Two Deformed Colliding Nuclei

Gatti *et al.*, JCP **123** (2005) 174311

$$\frac{2\hat{T}}{\hbar^2} = -\frac{1}{\mu}\frac{\partial^2}{\partial R^2} + \left(\frac{1}{I_1} + \frac{1}{\mu R^2}\right)\hat{j}_1^2 + \left(\frac{1}{I_2} + \frac{1}{\mu R^2}\right)\hat{j}_2^2$$

$$+ \frac{1}{\mu R^2}\left[\hat{j}_{1,+}\hat{j}_{2,-} + \hat{j}_{1,-}\hat{j}_{2,+} + J(J+1)\right.$$

$$\left. - 2k_1^2 - 2k_1k_2 - 2k_2^2\right] - \boxed{\frac{C_+(J,K)}{\mu R^2}(\hat{j}_{1,+} + \hat{j}_{2,+})}$$

$$- \boxed{\frac{C_-(J,K)}{\mu R^2}(\hat{j}_{1,-} + \hat{j}_{2,-})}$$

Coriolis interaction

μ is the reduced mass for the radial motion,
I_i is the ^{12}C rotational inertia,
J is the total angular momentum with projection $K = k_1 + k_2$,
$C_\pm(J,K) = \sqrt{J(J+1) - K(K\pm 1)}$,
$\hat{j}_i^2 = -\frac{1}{\sin\theta_i}\frac{\partial}{\partial\theta_i}\sin\theta_i\frac{\partial}{\partial\theta_i} + \frac{k_i^2}{\sin^2\theta_i}$,
$\hat{j}_{i,\pm} = \pm\frac{\partial}{\partial\theta_i} - k_i\cot\theta_i$, with $k_i \rightarrow k_i \pm 1$.

Initial state $\Psi(t=0)$: the ^{12}C nuclei are well separated

$$\Psi_0(R,\theta_1,k_1,\theta_2,k_2) = \chi_0(R)\,\psi_0(\theta_1,k_1,\theta_2,k_2),$$

Radial motion

Internal rotational motion

$$\chi_0(R) = (\sqrt{\pi}\sigma)^{-1/2}\,\exp\left[-\frac{(R-R_0)^2}{2\sigma^2}\right] e^{iP_0(R-R_0)},$$

$$\begin{aligned}\psi_0(\theta_1,k_1,\theta_2,k_2) = & \left[\zeta_{j_1,m_1}(\theta_1,k_1)\zeta_{j_2,m_2}(\theta_2,k_2)\right.\\ & \left.+(-1)^J\zeta_{j_2,-m_2}(\theta_1,k_1)\zeta_{j_1,-m_1}(\theta_2,k_2)\right]\\ & /\sqrt{2+2\,\delta_{j_1,j_2}\delta_{m_1,-m_2}},\end{aligned}$$

where $\zeta_{j,m}(\theta,k) = \sqrt{\frac{(2j+1)(j-m)!}{2\,(j+m)!}}\,P_j^m(\cos\theta)\,\delta_{km}$,
and P_j^m are associated Legendre functions.

Time Propagation of the Wave Function

$$|\Psi_J(t)\rangle = \underbrace{e^{-i\,\hat{H}\,t/\hbar}}_{\text{evolution operator}}\,|\Psi_J(0)\rangle$$

The evolution operator is represented as a convergent series of modified Chebyshev polynomials

Tannor, Quantum Mechanics from a Time-Dependent Perspective, USB, 2007

Power Spectrum of the Wave Function

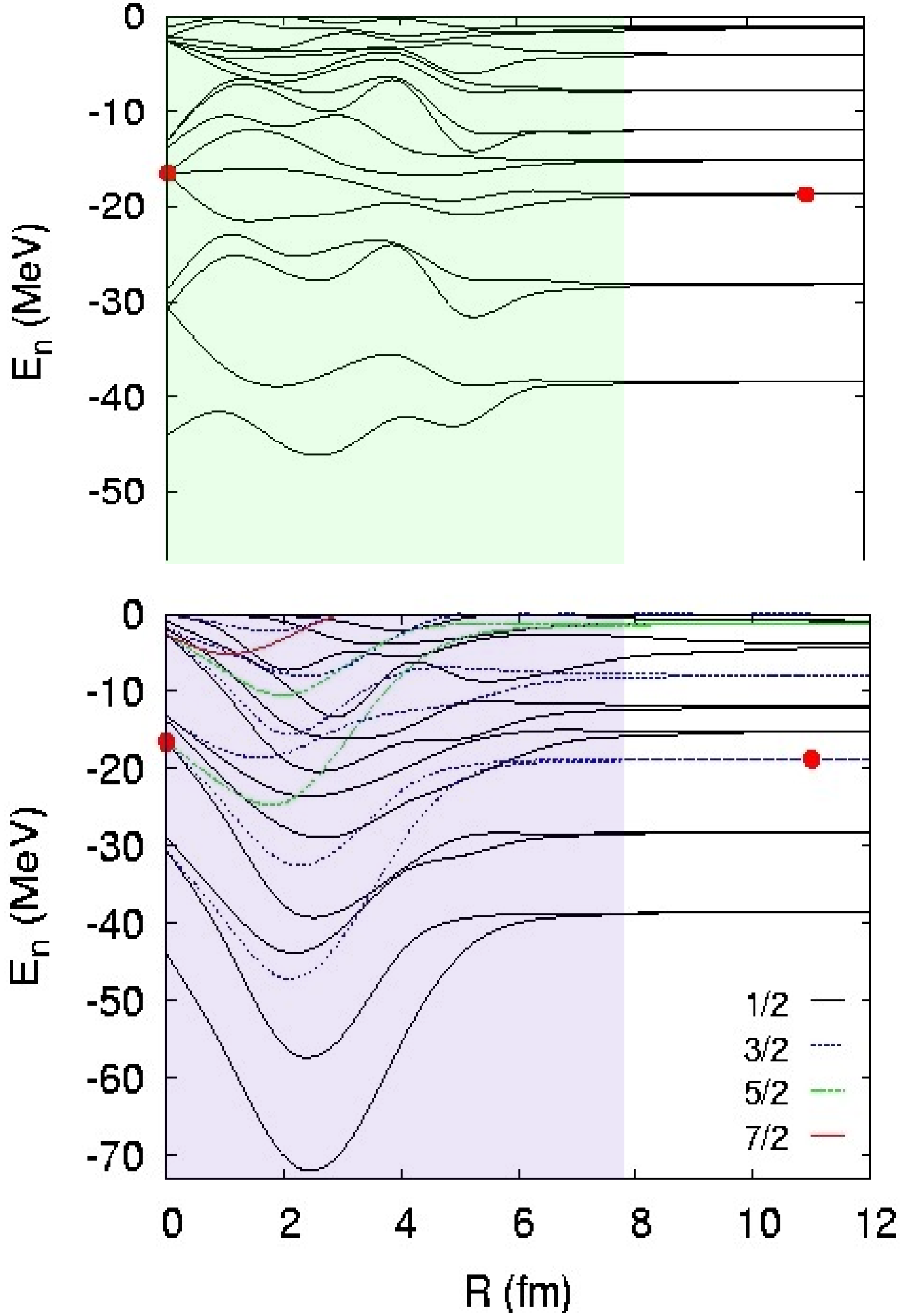
$$\mathcal{P}(E) = \langle \Psi(t) | \underbrace{\delta(E - \hat{H})}_{\text{Energy projector}} | \Psi(t) \rangle$$

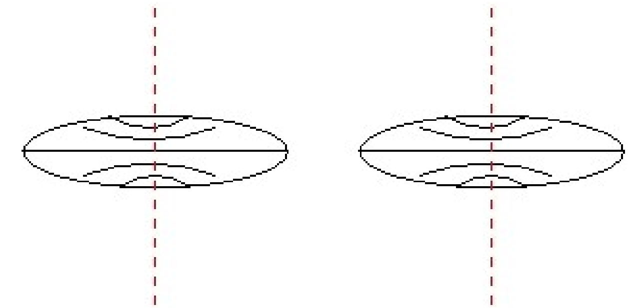
Reflection & Transmission Coefficients

$$\mathcal{R}(E) = \frac{\mathcal{P}^{final}(E)}{\mathcal{P}^{initial}(E)}$$

$$T(E) = 1 - \mathcal{R}(E)$$

Sensitivity of Molecular Shell Structure to the ^{12}C Alignment

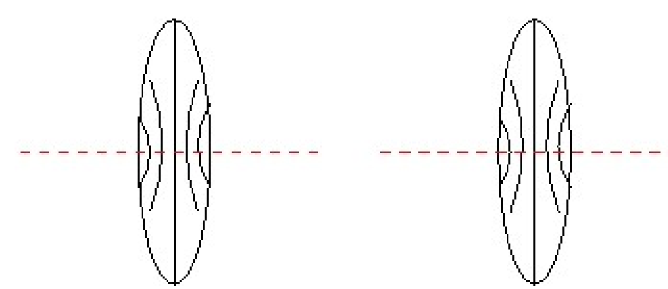




$$V = \sum_{s=1}^{2} e^{-i\mathbf{R}_s\hat{k}}\,\hat{U}(\Omega_s)\,V_s\,\hat{U}^{-1}(\Omega_s)\,e^{i\mathbf{R}_s\hat{k}}$$

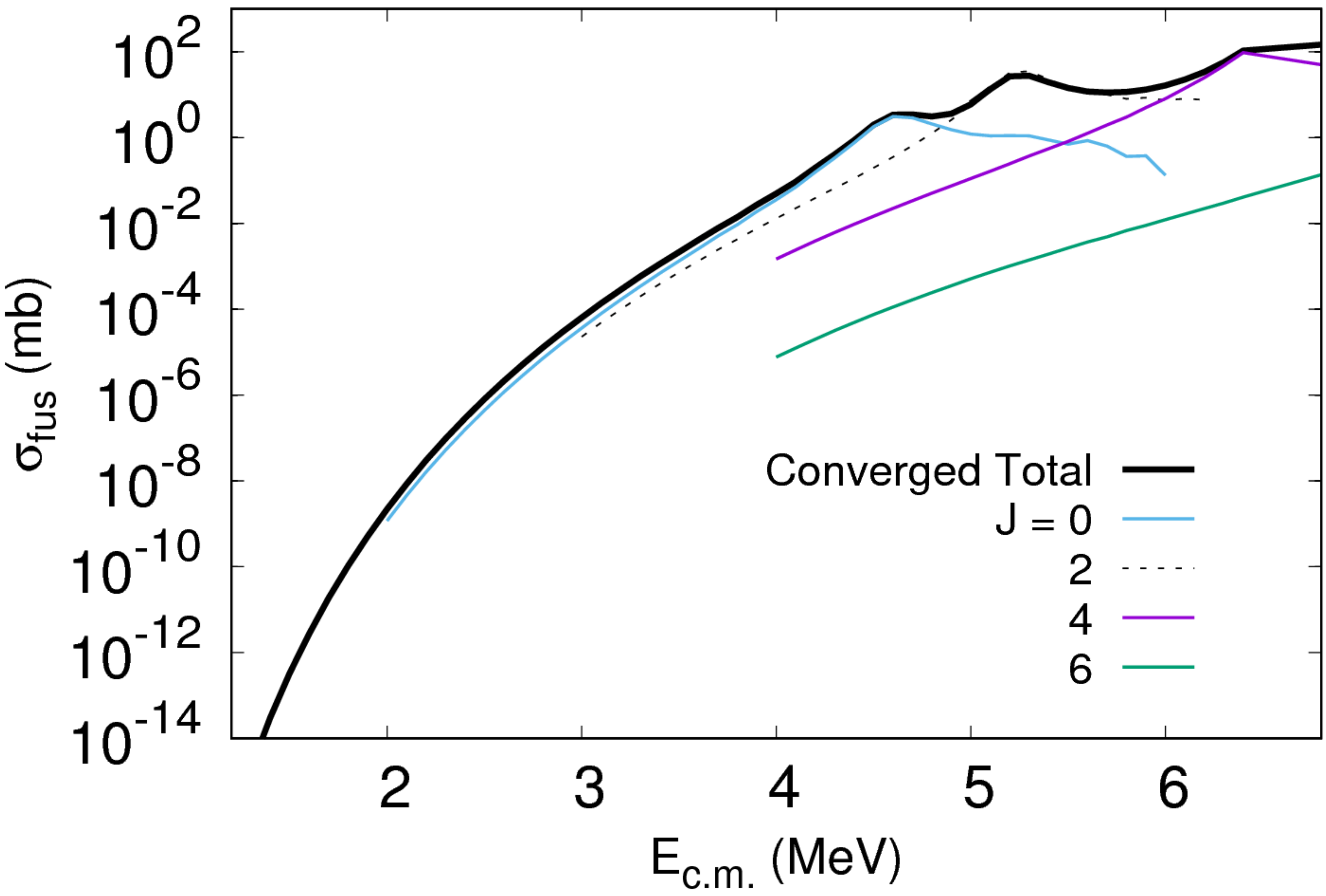
$$V_s \approx \sum_{\nu\mu}^{N} |s\nu\rangle\, V^s_{\nu\mu}\, \langle s\mu|$$

Potential Separable Expansion Method

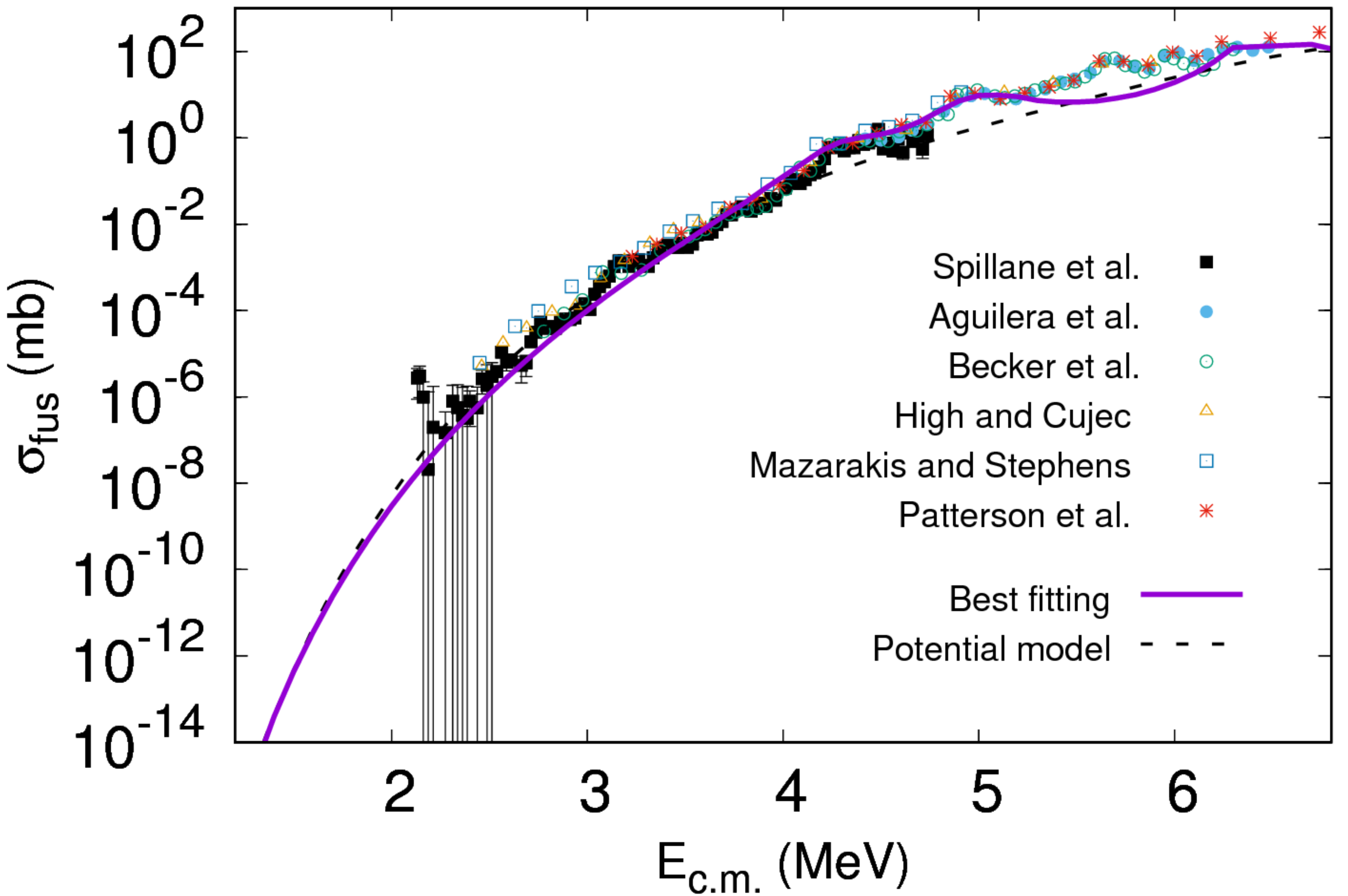


AD-T, PRL **101** (2008) 122501

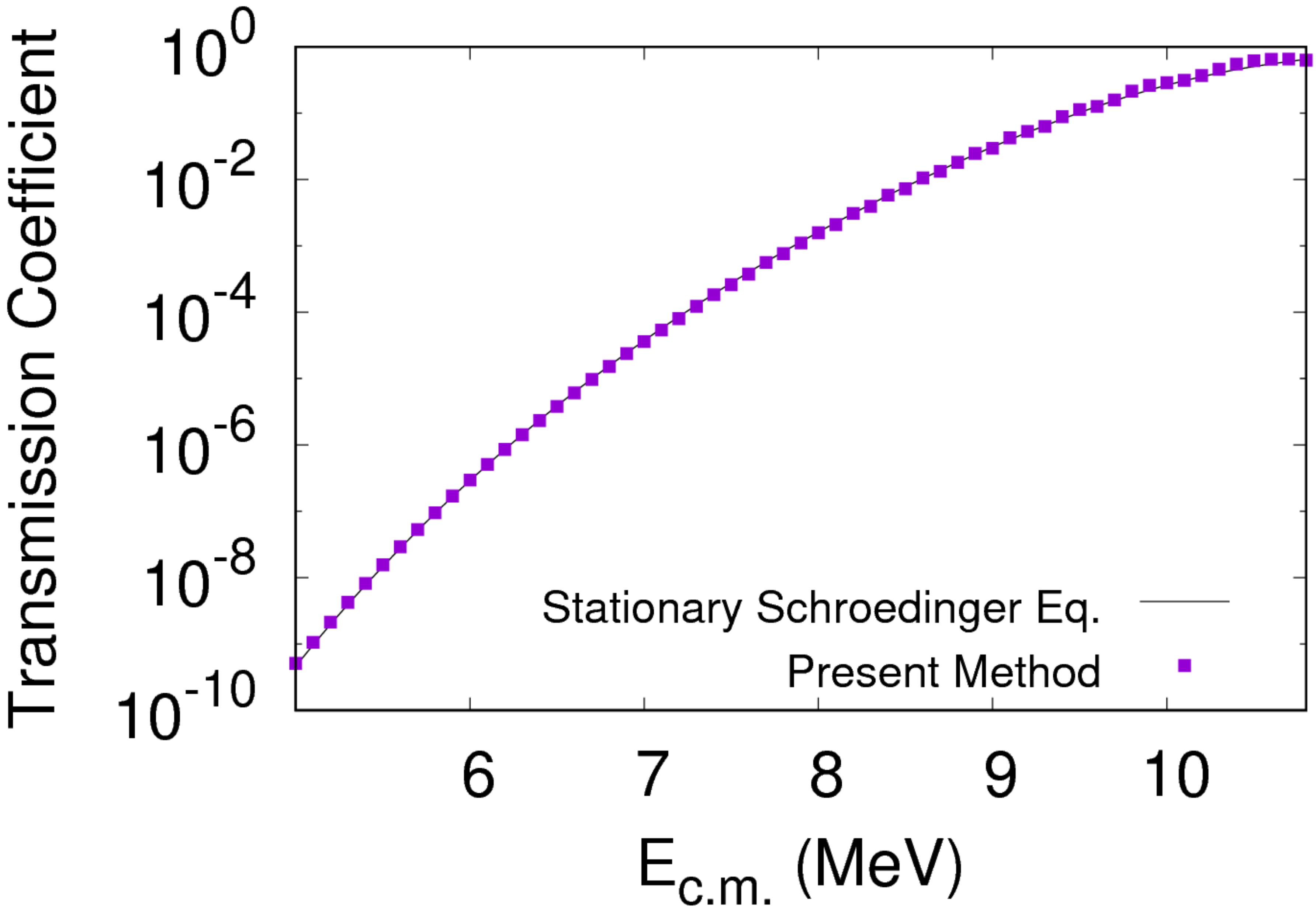
Fusion Excitation Function for $^{12}C + ^{12}C$



Fusion Excitation Function for $^{12}C + ^{12}C$



Transmission coefficients for $^{16}O + ^{16}O$ central collisions

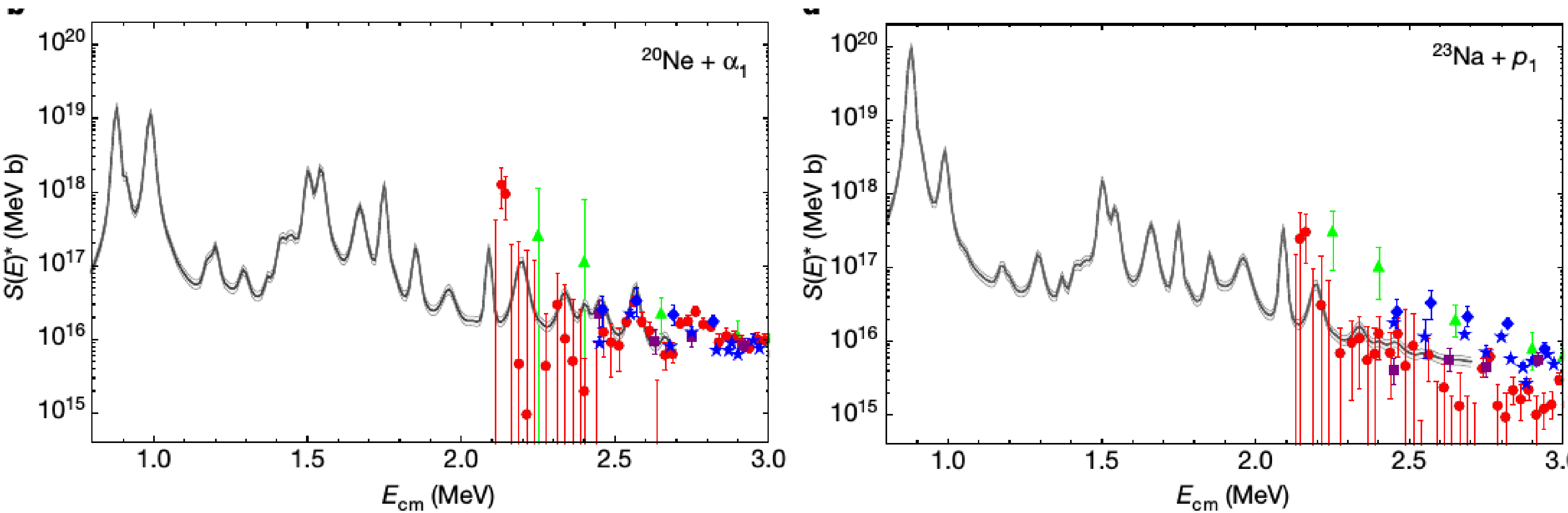


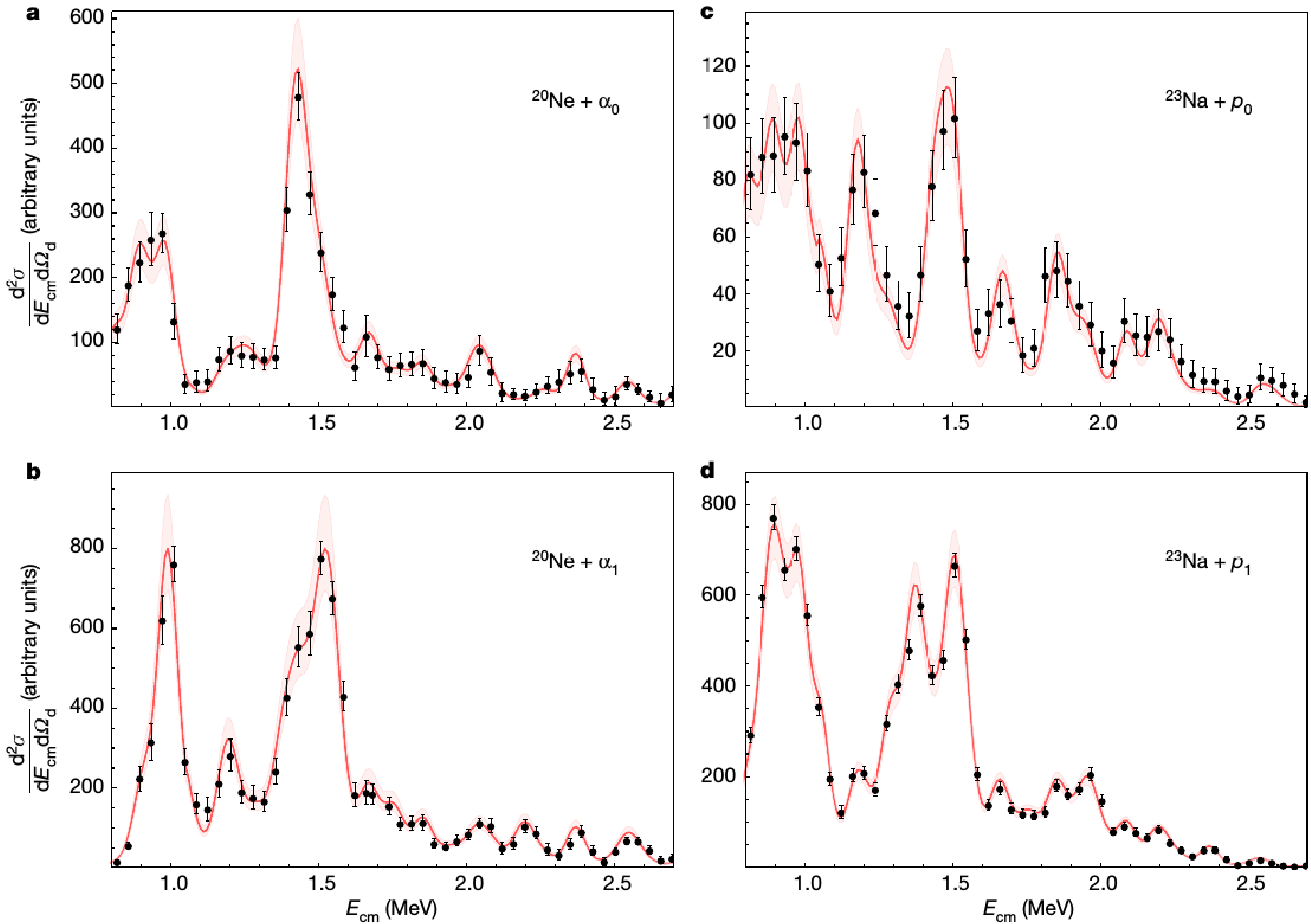
LETTER

https://doi.org/10.1038/s41586-018-0149-4

An increase in the $^{12}C + ^{12}C$ fusion rate from resonances at astrophysical energies

A. Tumino[1,2]*, C. Spitaleri[2,3], M. La Cognata[2], S. Cherubini[2,3], G. L. Guardo[2,4], M. Gulino[1,2], S. Hayakawa[2,5], I. Indelicato[2], L. Lamia[2,3], H. Petrascu[4], R. G. Pizzone[2], S. M. R. Puglia[2], G. G. Rapisarda[2], S. Romano[2,3], M. L. Sergi[2], R. Spartá[2] & L. Trache[4]





a
b
c
d
^{20}Ne + α_0
^{20}Ne + α_1
^{23}Na + p_0
^{23}Na + p_1
$\frac{d^2\sigma}{dE_{cm}d\Omega_d}$ (arbitrary units)
E_{cm} (MeV)