



The Higgsploding Universe

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A Standard Model Tale

Before the discovery of the Higgs boson – (Yang-Mills theories)

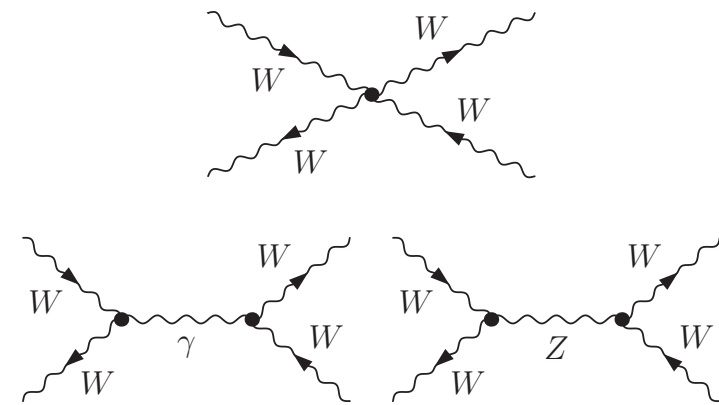
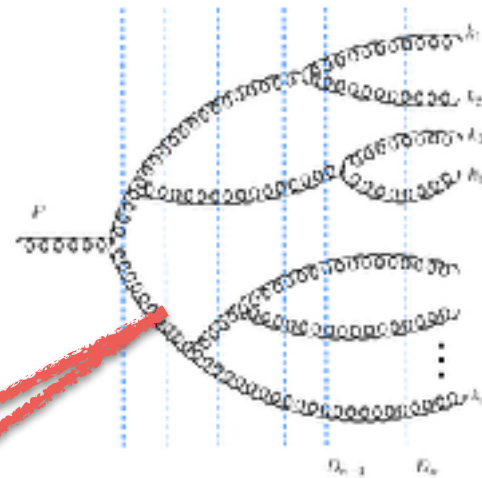
Situation at tree-level

Multiplicity

Gluon amplitudes for
 $n \times \text{gluon} \rightarrow m \times \text{gluon}$
for given helicity and
color structure strong
cancellations between
Feynman diagrams

[Parke, Taylor '86]

$m \text{ gluon} \rightarrow m! \text{ Feyn. diags}$
but not $m!$ growth for
Amplitude value



Kinetic Energy

Perturbative unitarity
violated at high energies

model inconsistent
(at high energies)

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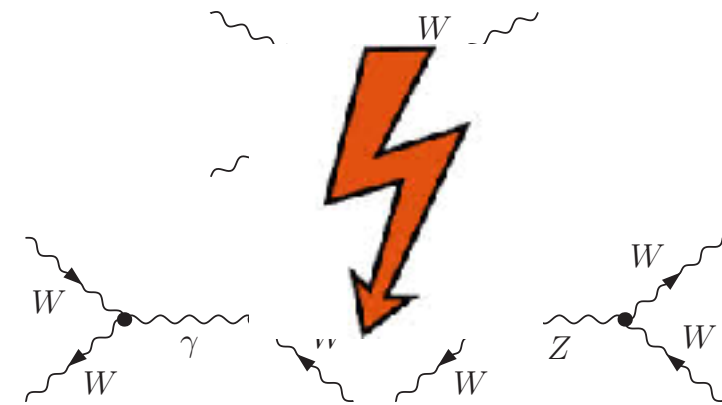
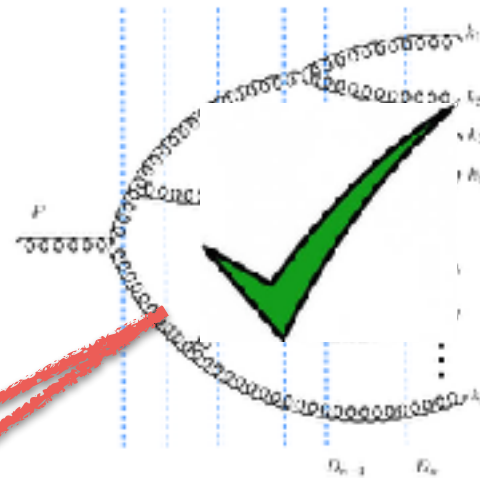
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After the discovery of the Higgs boson – **complete Standard Model**

Situation at tree-level

Multiplicity

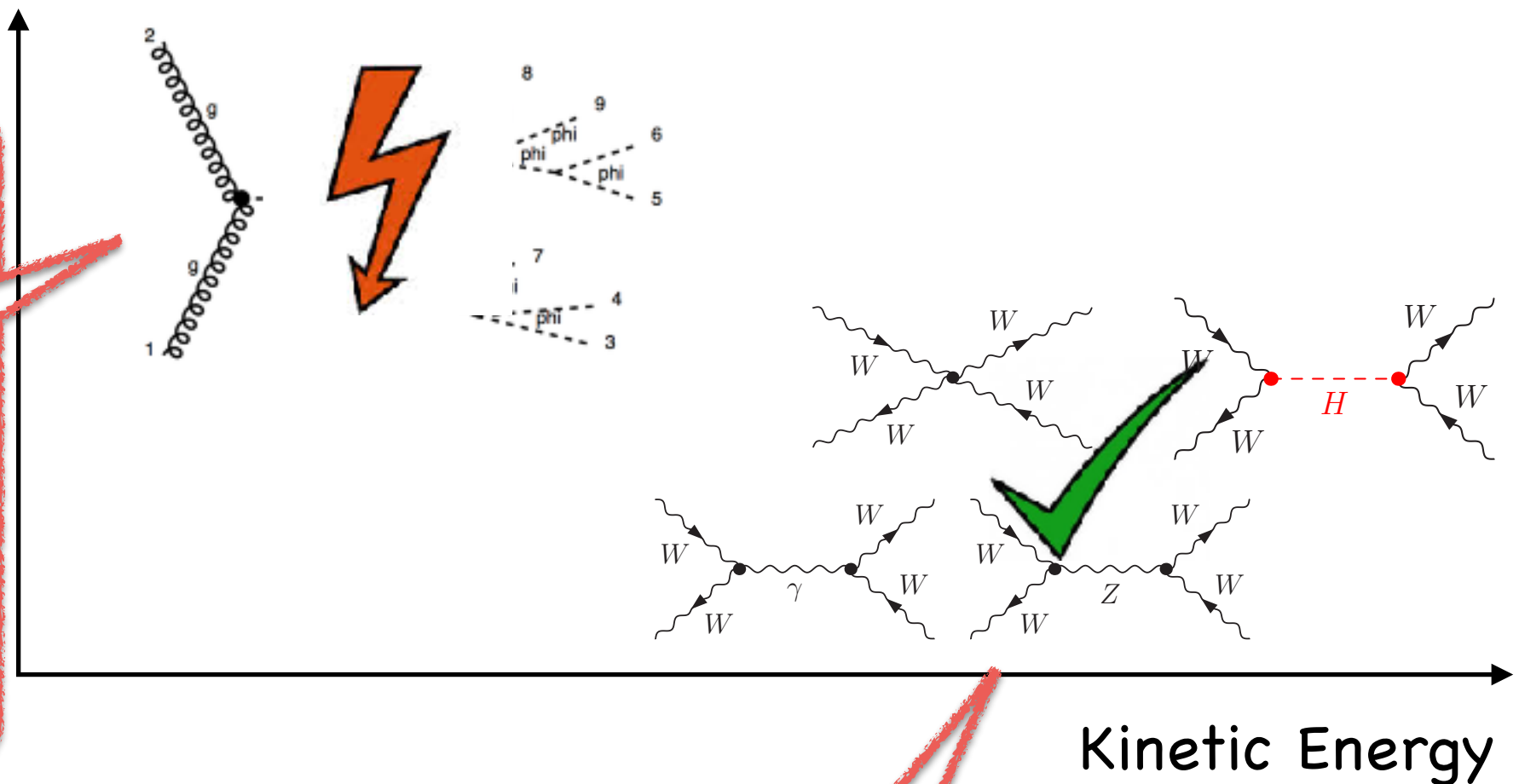
Lack of symmetry
for elementary
scalar particle

$$h^* \rightarrow n \times h$$

n Higgs $\rightarrow n!$ Feyn. diags

and $n!$ growth for
Amplitude value

[Brown '92] [Voloshin '92]



Perturbative unitarity
restored at high energies

model consistent/calculable
(to high energies)

+ Hierarchy problem (Loop level)



Calculation of $1^* \rightarrow n$ amplitudes

Assume Lagrangian

$$\mathcal{L}_\rho(\phi) = \frac{1}{2} (\partial\phi)^2 - \frac{1}{2} M^2 \phi^2 - \frac{1}{4} \lambda \phi^4 + \rho \phi$$

The amplitude is calculated using the LSZ reduction technique [Brown '92]

$$\langle n | \phi(x) | 0 \rangle = \lim_{\rho \rightarrow 0} \left[\prod_{j=1}^n \lim_{p_j^2 \rightarrow M^2} \int d^4 x_j e^{i p_j \cdot x_j} (M^2 - p_j^2) \frac{\delta}{\delta \rho(x_j)} \right] \langle 0_{\text{out}} | \phi(x) | 0_{\text{in}} \rangle_\rho$$

where the tree-level approximation is obtained via $\langle 0_{\text{out}} | \phi(x) | 0_{\text{in}} \rangle_\rho \longrightarrow \phi_{\text{cl}}(x)$

and $\phi_{\text{cl}}(x)$ is a solution to the classical field equation

IDEA: Fill whole phase-space with particles, i.e. produce all particles at mass threshold

with $\vec{p}_j = 0$ $p_j^\mu = (\omega, \vec{0})$ and $\rho(x) = \rho(t) = \rho_0(\omega) e^{i\omega t}$

Here QFT \rightarrow time-dep QM:

$$(M^2 - p_j^2) \frac{\delta}{\delta \rho(x_j)} \longrightarrow (M^2 - \omega^2) \frac{\delta}{\delta \rho(t_j)} = \frac{\delta}{\delta z(t_j)}$$

$$z(t) := \frac{\rho_0(\omega) e^{i\omega t}}{M^2 - \omega^2 - i\epsilon} := z_0 e^{i\omega t}, \quad z_0 = \text{finite const}$$

Hence, the generating function of tree amplitudes on multi-particle thresholds is a classical solution to the Euler-Lagrange equation. It solves an ordinary differential equation with no source term

$$d_t^2 \phi + M^2 \phi + \lambda \phi^3 = 0$$

$h^* \rightarrow nh$ relies on outgoing particles
thus, only positive freq. modes present

with $\phi_{\text{cl}}(t) = z(t) + \sum_{n=2}^{\infty} d_n z(t)^n$, $z := z_0 e^{iMt}$ (initial condition)

The coefficients d_n determine the actual amplitudes by differentiation w.r.t. z

$$\mathcal{A}_{h^* \rightarrow n \times h} = \left(\frac{\partial}{\partial z} \right)^n h_{\text{cl}} \Big|_{z=0} = n! d_n = n! (2v)^{1-n} \quad \text{Factorial growth!}$$

$$\phi_{\text{cl}}(t) = \frac{z(t)}{1 - \frac{\lambda}{8M^2} z(t)^2}$$

$$\mathcal{A}_{1 \rightarrow n} = n! \left(\frac{\lambda}{8M^2} \right)^{\frac{n-1}{2}}$$

Same findings by [Voloshin '92] [Argyres, Kleiss, Papadopoulos '92]
[Libanov, Rubakov, Son, Troitski '94]

Several generalisations of this approach:

- Higgs like, ie. ϕ^4 with vev:

[Brown '92]

$$\mathcal{L}(h) = \frac{1}{2} (\partial h)^2 - \frac{\lambda}{4} (h^2 - v^2)^2 \quad \longrightarrow \quad \mathcal{A}_{1 \rightarrow n} = \left(\frac{\partial}{\partial z} \right)^n h_{\text{cl}} \Big|_{z=0} = n! (2v)^{1-n}$$

- Gauge-Higgs theory:

[Khoze '14]

Higgs process $\mathcal{A}(h \rightarrow n \times h + m \times Z_L) = (2v)^{1-n-m} n! m! d(n, m)$

Z process $\mathcal{A}(Z_L \rightarrow n \times h + (m+1) \times Z_L) = \frac{1}{(2v)^{n+m}} n! (m+1)! a(n, m)$

- Go beyond mass threshold (needs space-dep sol.):

DGL: $-(\partial^\mu \partial_\mu + M_h^2) \varphi = 3\lambda v \varphi^2 + \lambda \varphi^3$ $\varepsilon = \frac{1}{n M_h} E_n^{\text{kin}} = \frac{1}{n} \frac{1}{2M_h^2} \sum_{i=1}^n \vec{p}_i^2$

$$\downarrow$$

$$\mathcal{A}_n(p_1 \dots p_n) = n! (2v)^{1-n} \left(1 - \frac{7}{6} n \varepsilon - \frac{1}{6} \frac{n}{n-1} \varepsilon + \mathcal{O}(\varepsilon^2) \right)$$

[Argyres, Kleiss, Papadopoulos '92] [Libanov, Rubakov, Son, Troitski '94]

How about loops?

Usual criticism: need to include loops to render cross section finite.

Keep in mind, we calculate exclusive rate of massive internal and outgoing particles -> **no mass-divergencies and observable IR-safe**

Loop corrections calculated by expanding around classical field $\phi(x) = \phi_0(x) + \phi_q(x)$

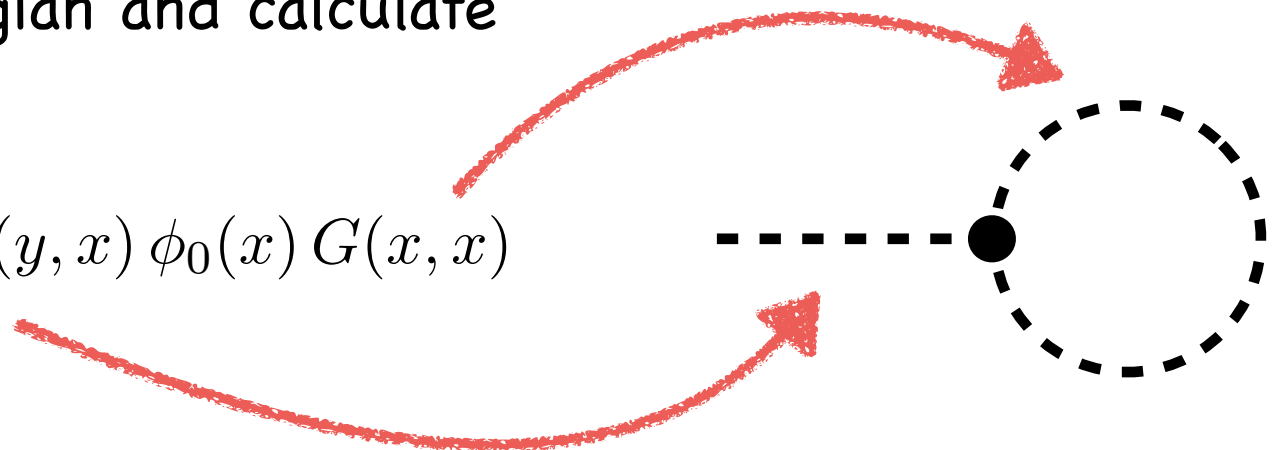
Euclidean Lagrangian becomes $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_q)^2 + \frac{1}{2}(m^2 + 3\lambda\phi_0^2)\phi_q^2 + \lambda\phi_0\phi_q^3 + \frac{\lambda}{4}\phi_q^4$.

After promoting classical solution ϕ_0 to quantum expectation value $\langle\phi\rangle = \phi_0 + \langle\phi_q\rangle$

Individual amplitudes calculated via gen. functional $\langle n|\phi|0\rangle = \left(\frac{\partial}{\partial z_0}\right)^n (\phi_0 + \langle\phi_q\rangle)|_{z_0=0}$

Use Feynman rules of Eucl. Lagrangian and calculate

$$\langle\phi_q(y)\rangle_{1\text{-loop}} = (-3\lambda) \int d^4x G(y,x) \phi_0(x) G(x,x)$$

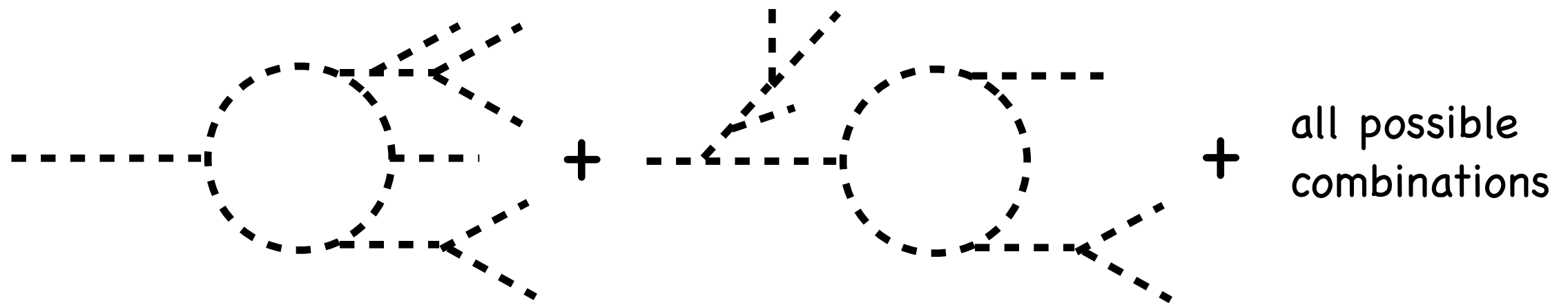


You will find for the combined tree + 1-loop
generating functional

[Smith '92]
[Voloshin '92]

$$\phi_{0+1}(t) = \frac{z(t)}{1 - (\bar{\lambda}/8\bar{m}^2)z(t)^2} \left(1 - \frac{3\lambda}{4} F \frac{(\lambda/8m^2)^2 z(t)^4}{(1 - (\lambda/8m^2)z(t)^2)^2} \right)$$

Now follow Brown's program to build



One obtains for scalar loops

$$A_n = n! (2v)^{1-n} \left[1 + n(n-1) \frac{\sqrt{3}\lambda}{8\pi} + O(\lambda^2) \right] \quad \text{for} \quad \lambda n \ll 1$$

and including fermion loops it is argued cancellations can occur [Voloshin '17]

$$A_n \rightarrow A_n \times \left[1 + (-1)^{2r} C(r) n^{4r-4} \lambda \right] \quad \text{with} \quad r = m_t / \sqrt{2\lambda v^2}$$

(exponentiate for $n\lambda > 1$)? in SM subleading to scalar loops

In non-rel. limit the LO cross section for n-Higgs production scales like:

$$\sigma_n \propto \exp \left[\frac{1}{\lambda} F_{\text{h.g.}}(\lambda n, \varepsilon) \right] \quad \text{with} \quad \frac{1}{\lambda} F_{\text{h.g.}}(\lambda n, \varepsilon) = \frac{\lambda n}{\lambda} (f_0(\lambda n) + f(\varepsilon))$$

for a scalar theory with SSB: $f_0(\lambda n) = \log \frac{\lambda n}{4} - 1$ at tree level

[Libanov, Rubakov, Son, Troitsky '94] $f(\varepsilon) \rightarrow \frac{3}{2} \left(\log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon$ for $\varepsilon \ll 1$

However, leading loop contributions can be resummed (only valid when $n\lambda < 1$):

Resummed 1-loop contribution:

$$\mathcal{A}_{1 \rightarrow n} = \mathcal{A}_{1 \rightarrow n}^{\text{tree}} \times \exp [B \lambda n^2 + \mathcal{O}(\lambda n)] \quad \text{with} \quad B = \frac{\sqrt{3}}{4\pi}$$

$$f_0(\lambda n) = \underbrace{\log \frac{\lambda n}{4} - 1}_{\text{tree}} + \underbrace{\lambda n \frac{\sqrt{3}}{4\pi}}_{\text{loop}} \quad \text{significant loop enhancement}$$

Higher loops expected to scale $\left(\frac{n\lambda}{4\pi} \right)^{\# \text{Loop}}$

$$f(\varepsilon) \rightarrow \frac{3}{2} \left(\log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{23}{12} \varepsilon \quad \text{for} \quad \varepsilon \ll 1$$

[Smith '92, Voloshin '92]

kinematics

[Voloshin '17]

From amplitudes to cross sections


$$\sigma_{n,m} = \int d\Phi_{n,m} \frac{1}{n! m!} |\mathcal{A}_{h^* \rightarrow n \times h + m \times Z_L}|^2 \quad \times \quad \text{flux factor}$$

$$\int d\Phi_n = (2\pi)^4 \delta^{(4)}(P_{\text{in}} - \sum_{j=1}^n p_j) \prod_{j=1}^n \int \frac{d^3 p_j}{(2\pi)^3 2p_j^0}$$

Bose statistics factors for n identical Higgs and m identical long. Vec.

Integration with $n\varepsilon_h$ fixed $\Phi_n \simeq \frac{1}{\sqrt{n}} \left(\frac{M_h^2}{2} \right)^n \exp \left[\frac{3n}{2} \left(\log \frac{\varepsilon_h}{3\pi} + 1 \right) + \frac{n\varepsilon_h}{4} + \mathcal{O}(n\varepsilon_h^2) \right]$

factorial growth (Stirling formula)

 $\sigma_{n,m} \sim \exp \left[2 \log(\kappa^m d(n, m)) + n \log \frac{\lambda n}{4} + m \log \frac{\lambda m}{4} \right.$

$$\left. + \frac{n}{2} \left(3 \log \frac{\varepsilon_h}{3\pi} + 1 \right) + \frac{m}{2} \left(3 \log \frac{\varepsilon_V}{3\pi} + 1 \right) - \frac{25}{12} n \varepsilon_h - 3.15 m \varepsilon_V + \mathcal{O}(n\varepsilon_h^2 + m\varepsilon_V^2) \right]$$

kinematic (phase space)
suppression

For $n\lambda > 1$ loops overpower tree result,
how about semi-classical approach?

[Son '95]

- Multiparticle decay rates Γ_n can be calculated using semi-classical method

➔ intrinsically non-perturbative method

➔ no reference to perturbation theory

- Path-integral calculated in deepest descend method, where

$$\lambda \rightarrow 0, \quad n \rightarrow \infty, \quad \text{with } \lambda n = \text{fixed}, \quad \varepsilon = \text{fixed}.$$

- Semi-classical calculation in regime where $\lambda n = \text{fixed} \ll 1$, $\varepsilon = \text{fixed} \ll 1$,
reproduces tree-level perturbative result for non-relativistic final states

Remarkably this semi-classical calculation also reproduces the 1-loop resummed calculation in this limit

Semi-classical calculation for rate $R(1 \rightarrow nh, E)$

[Son '95]

- Semi-classical calculation is applicable and more relevant for non-perturbative regime of Higgs production, where

$$\lambda n = \text{fixed} \gg 1, \quad \varepsilon = \text{fixed} \ll 1.$$

- This calculation was carried out with result given by [Khoze '17]

$$\mathcal{R}_n(\lambda; n, \varepsilon) = \exp \left[\frac{\lambda n}{\lambda} \left(\log \frac{\lambda n}{4} + 0.85 \sqrt{\lambda n} + \frac{1}{2} + \frac{3}{2} \log \frac{\varepsilon}{3\pi} - \frac{25}{12} \varepsilon \right) \right],$$

higher orders are suppressed by powers of $\mathcal{O}(1/\sqrt{\lambda n})$ and powers of ε

The main idea of the semi-classical setup

[Son '95]

- $R_n(E)$ is the probability rate for the local operator $\mathcal{O}(0)$ to create n particles of energy E from the vacuum

$$R_n(E) = \int \frac{1}{n!} d\Phi_n \langle 0 | \mathcal{O}^\dagger S^\dagger P_E | n \rangle \langle n | P_E S \mathcal{O} | 0 \rangle$$

P_E is the projection operator on states with fixed energy E

$$\mathcal{O} = e^{jh(0)}$$

and the limit $j \rightarrow 0$ is taken in the calculation of the probability rates

$$R_n(E) = \lim_{j \rightarrow 0} \int \frac{1}{n!} d\Phi_n \langle 0 | e^{jh(0)\dagger} S^\dagger P_E | n \rangle \langle n | P_E S e^{jh(0)} | 0 \rangle$$

Note: non dynamical (non-propagating) initial state $\mathcal{O} | 0 \rangle$

The semi-classical deepest descent limit:

$$\lambda \rightarrow 0, \quad n \rightarrow \infty, \quad \text{with } \lambda n = \text{fixed}, \quad \varepsilon = \text{fixed}.$$

Evaluate the path integral in this double-scaling limit.
 n enters via the coherent state formalism.

The main idea of the semi-classical setup

[Son '95]

1. Solve the classical equations without source term

$$\frac{\delta S}{\delta h(x)} = 0$$

by finding a complex-valued solution $h(x)$ with a point-like singularity at the origin and regular everywhere else in Minkowski space

2. Impose the initial and final-time boundary conditions

$$\lim_{t \rightarrow -\infty} h(x) = v + \int \frac{d^3 k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} a_{\mathbf{k}} e^{ik_{\mu} x^{\mu}}$$

$$\lim_{t \rightarrow +\infty} h(x) = v + \int \frac{d^3 k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \left(b_{\mathbf{k}} e^{\omega_{\mathbf{k}} T - \theta} e^{-ik_{\mu} x^{\mu}} + b_{\mathbf{k}}^* e^{ik_{\mu} x^{\mu}} \right)$$

(T and θ are parameters arising from projection onto final states with definite E and n)

The main idea of the semi-classical setup

[Son '95]

3. Compute the energy and the particle numbers using the $t \rightarrow +\infty$ asymptotics of $h(x)$

$$E = \int d^3k \, \omega_{\mathbf{k}} b_{\mathbf{k}}^* b_{\mathbf{k}} e^{\omega_{\mathbf{k}} T - \theta}, \quad n = \int d^3k \, b_{\mathbf{k}}^* b_{\mathbf{k}} e^{\omega_{\mathbf{k}} T - \theta}.$$

At $t \rightarrow -\infty$ the energy and the particle number are vanishing.

The energy is conserved by regular solutions and changes continuously from 0 to E at the singularity at $t=0$.

4. Eliminate the T and θ parameters in favour of E and n using the expressions above. Finally compute the function $W(E, n)$

$$W(E, n) = ET - n\theta - 2\text{Im}S[h]$$

and thus determine the semiclassical rate $R_n(E) = \exp[W(E, n)]$

Thus we have computed the rate R in the large λn limit:

using the semi-classical approach and the thin-wall approximation

Explosive growth of $1 \rightarrow n$ process

$$\mathcal{R}_n(s) := \frac{1}{2M_h^2} \int d\Pi_n |\mathcal{M}(1 \rightarrow n)|^2 \quad \varepsilon = \frac{\sqrt{s} - nM_h}{nM_h}$$

where

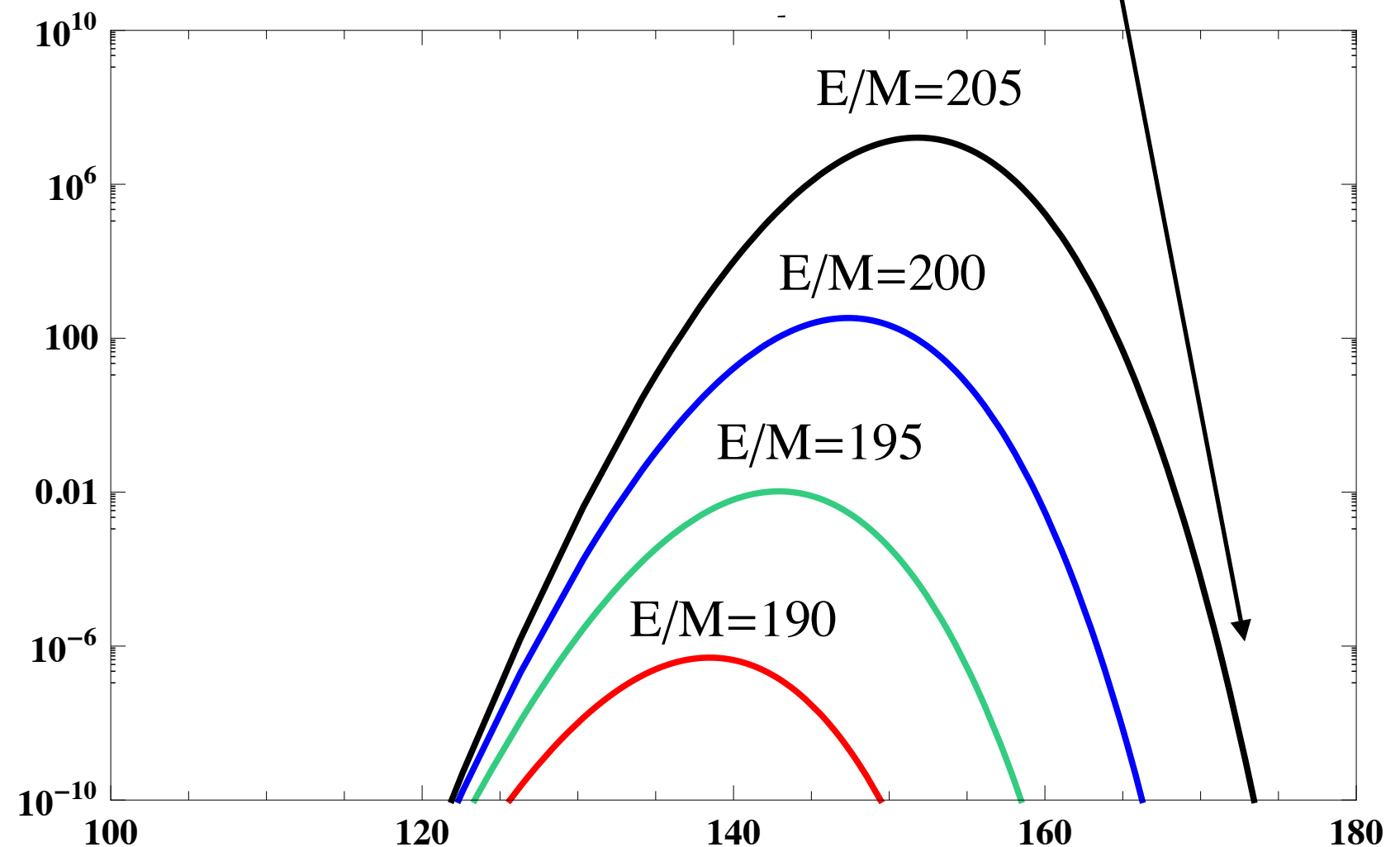
$$\mathcal{R} = \exp \left[\frac{\lambda n}{\lambda} \left(\log \frac{\lambda n}{4} + 3.02 \sqrt{\frac{\lambda n}{4\pi}} - 1 + \frac{3}{2} \left(\log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \right) \right]$$

$\lambda n \gg 1$ small ε $n \rightarrow n_{\max} \Rightarrow \varepsilon \rightarrow 0$

- Massive energy dependence

- Very narrow - resonance-like - peak in n

R



$$n < n_{\max} = E/M_h$$

Thus we have computed the rate R in the large λn limit:

using the semi-classical approach and the thin-wall approximation

Explosive growth of $1 \rightarrow n$ process $\mathcal{R}_n(s) := \frac{1}{2M_h^2} \int d\Pi_n |\mathcal{M}(1 \rightarrow n)|^2$

where $\mathcal{R} = \exp \left[\frac{\lambda n}{\lambda} \left(\log \frac{\lambda n}{4} + 3.02 \sqrt{\frac{\lambda n}{4\pi}} - 1 + \frac{3}{2} \left(\log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \right) \right]$

energy low



energy beyond threshold



Schwinger-Dyson-propagator and optical theorem


SD propagator, valid in perturbative and non-perturbative QFT

$$\Delta(p) = \int d^4x e^{ip \cdot x} \langle 0 | T (\phi(x) \phi(0)) | 0 \rangle = \frac{i}{p^2 - m_0^2 - \Sigma(p^2) + i\epsilon}$$

where $-i\Sigma(p^2) = \sum \text{-(1PI)}$ and the physical (pole) mass is $m^2 = m_0^2 + \Sigma(m^2)$

with the renormalisation constant $Z_\phi = \left(1 - \frac{d\Sigma}{dp^2} \Big|_{p^2=m^2}\right)^{-1}$ we define the renorm. quantities

$$\begin{aligned}\Delta_R(p) &= Z_\phi^{(-1)} \Delta(p), \\ \Sigma_R(p) &= Z_\phi (\Sigma(p^2) - \Sigma(m^2) - \Sigma'(m^2)(p^2 - m^2))\end{aligned}$$

 renormalised propagator $\Delta_R(p) = \frac{i}{p^2 - m^2 - \Sigma_R(p^2) + i\epsilon}$

Schwinger-Dyson-propagator and optical theorem

[Khoze, MS '17]

The optical theorem now relates the $1^* \rightarrow nh$ amplitudes with the imaginary part of the self-energy (valid to all orders)

$$- \operatorname{Im} \Sigma_R(p^2) = m \Gamma(p^2) \quad \longleftrightarrow \quad - \operatorname{Im} \left(\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \right) = m \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$$

$$\text{where } \Gamma(s) = \sum_{n=2}^{\infty} \Gamma_n(s) \quad \text{and} \quad \Gamma_n(s) = \frac{1}{2m} \int \frac{d\Phi_n}{n!} |\mathcal{M}(1 \rightarrow n)|^2$$

$$\text{and thus } \Delta_R(p) = \frac{i}{p^2 - m^2 - \operatorname{Re} \Sigma_R(p^2) + im\Gamma(p^2) + i\epsilon}$$

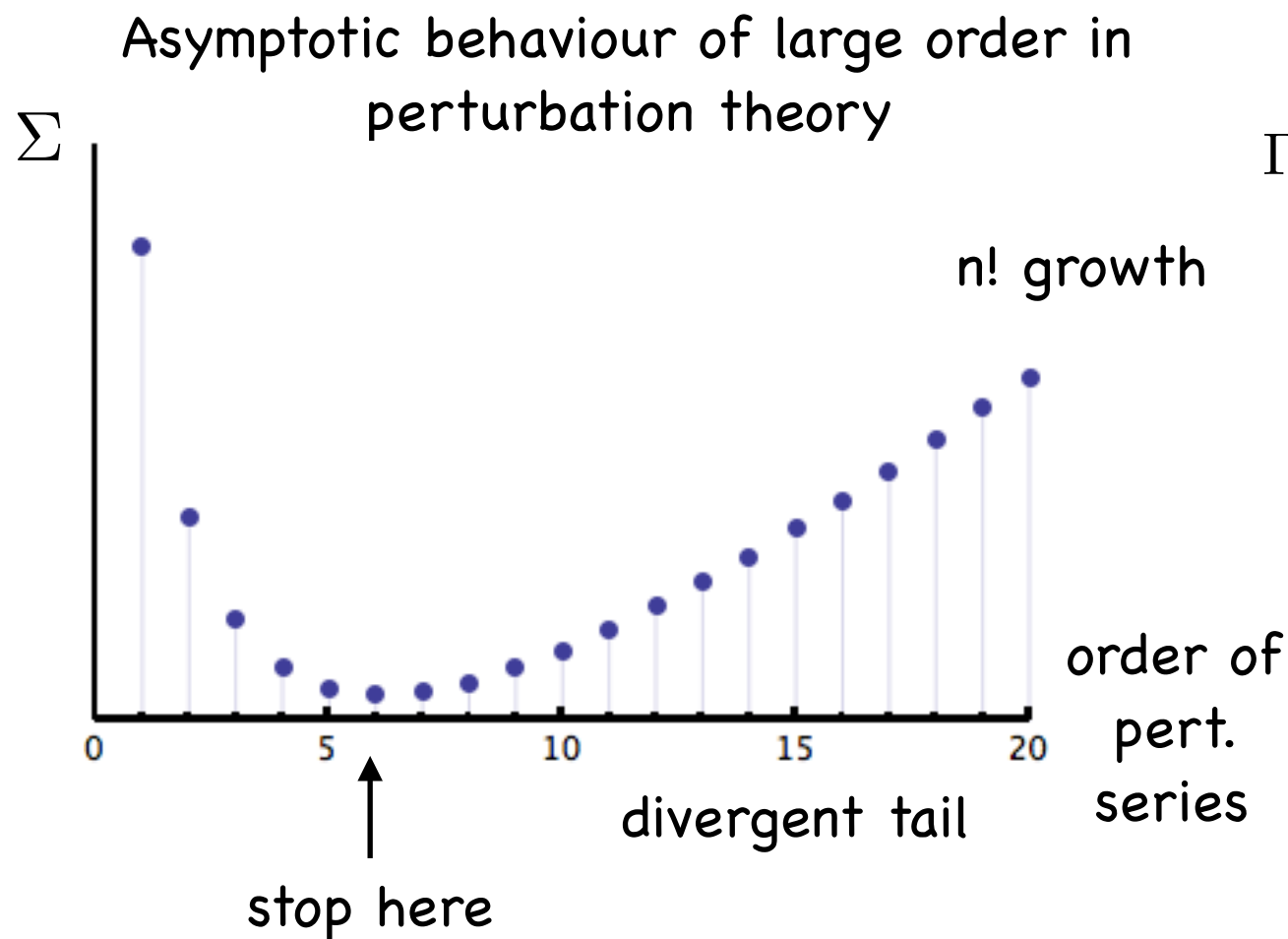
No information as
perturbation theory breaks
down for many loops, but no
physical reason to explode
and not possible to cancel
imaginary part

Higgsplodes



“It is just like asymptotic perturbative series!
We always expected it to break down!”

Except, its not...



$$\Sigma = \sum_{n=1}^{\infty} \Sigma_n \rightarrow \infty$$

[Dyson '52]

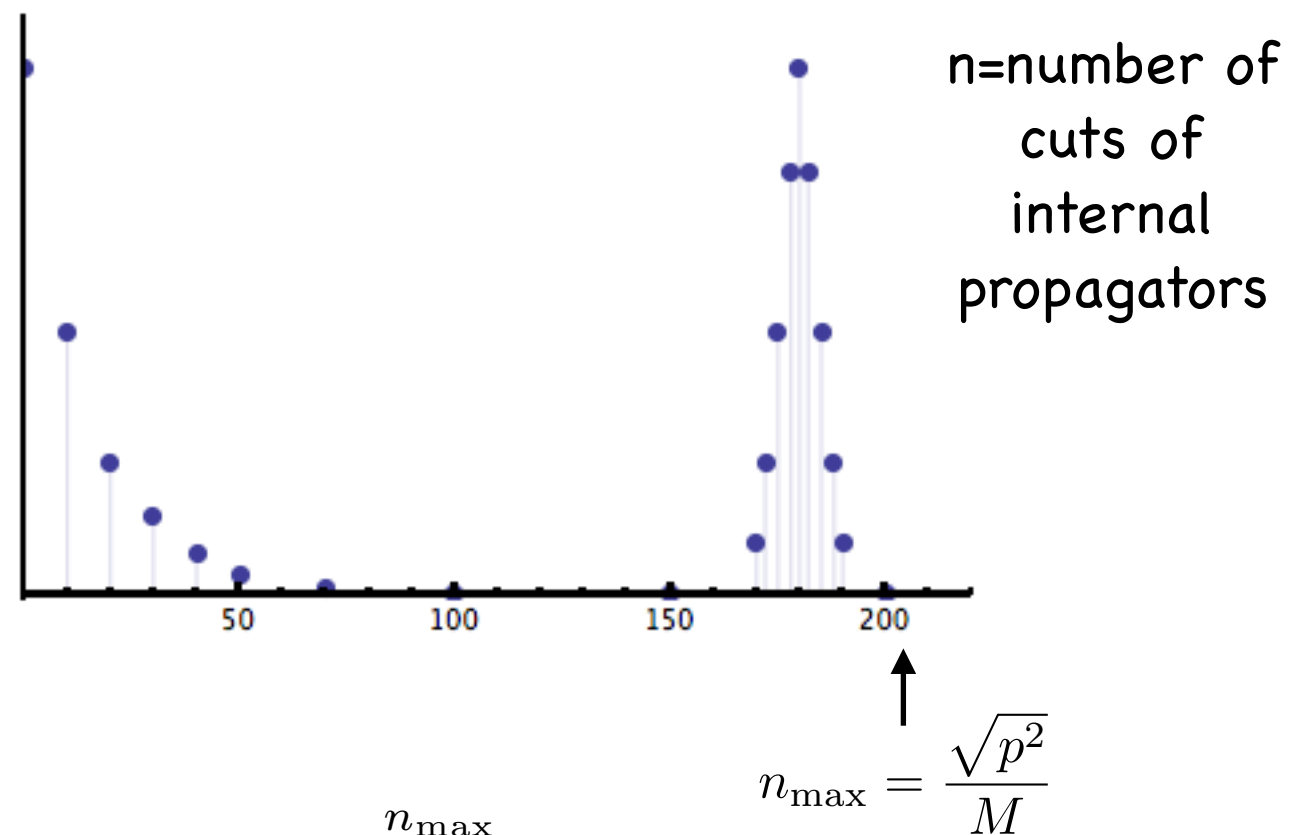
e.g. [Broadhurst, Kreimer '99]

Frascati

Weird Theoretical Ideas

Higgspllosion

$\Gamma(p^2) \sim \text{Im } \Sigma(p^2)$



$$\text{Im } \Sigma(p^2) \simeq \sum_{n=1}^{n_{\text{max}}} \Gamma_n = \text{finite sum}$$

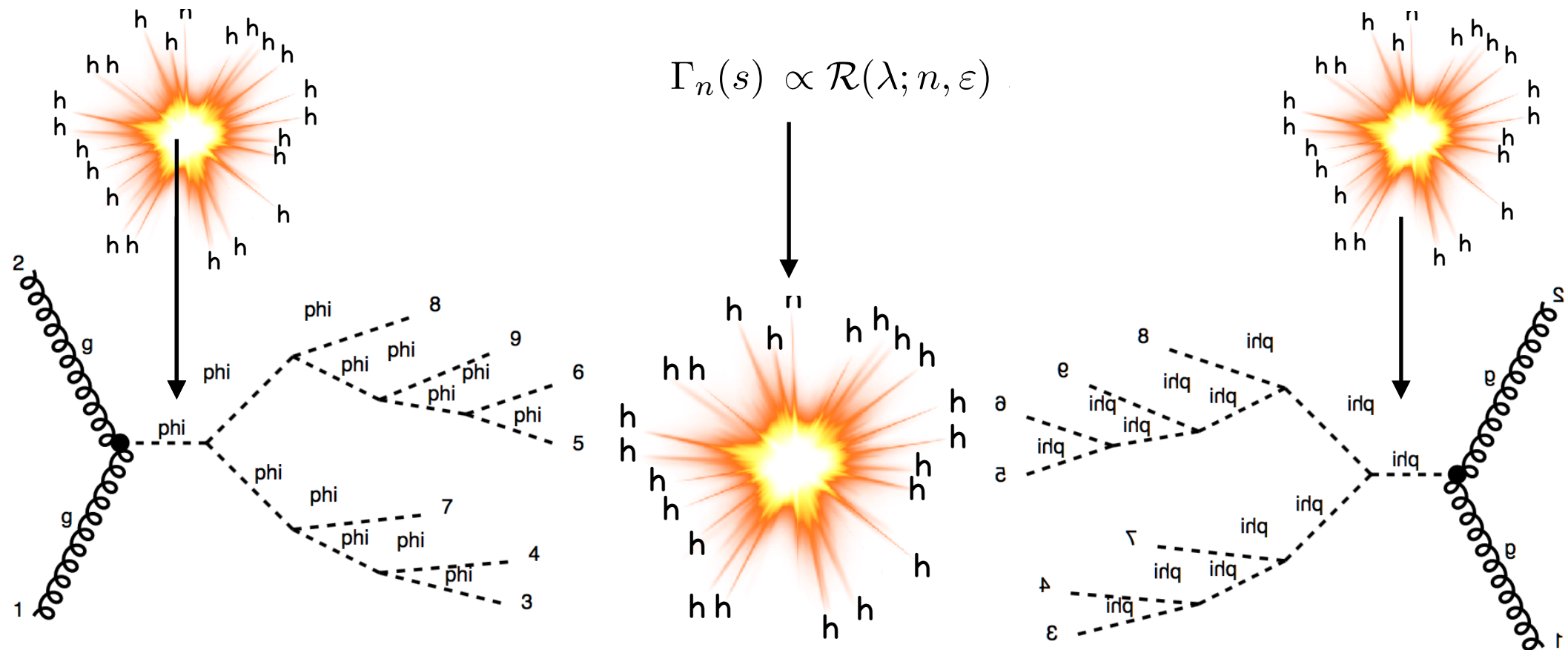
Not the same types of beasts

Higgspersion

[Khoze, MS '17]

Previous calculations neglected 'width'/self-energy contribution to scalar propagator

$$\mathcal{M}_{gg \rightarrow h^*} \times \frac{i}{p^2 - m_h^2 - \text{Re}\tilde{\Sigma}(p^2) + im_h\Gamma(p^2)} \times \mathcal{M}_{h^* \rightarrow n \times h}$$



$$\sigma_{gg \rightarrow n \times h}^{\Delta} \sim y_t^2 \frac{m_t^2}{m_h} \log^4 \left(\frac{m_t}{\sqrt{s}} \right) \times \frac{1}{(s - \text{Re}\Sigma(s))^2 + m_h^2 \Gamma^2(s)} \times \Gamma_n(s)$$

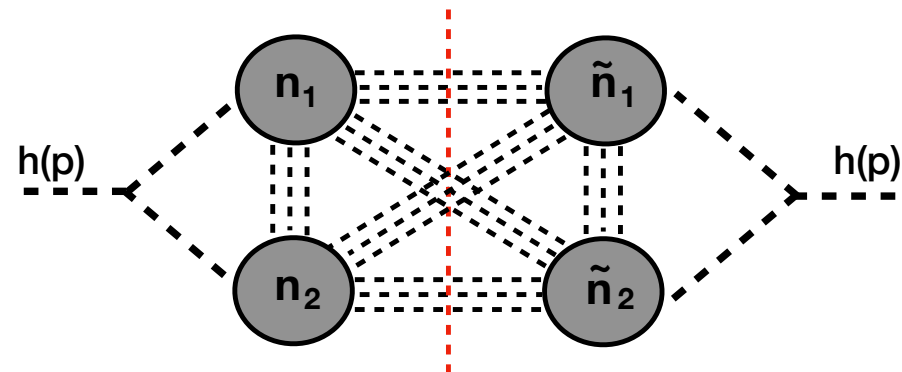
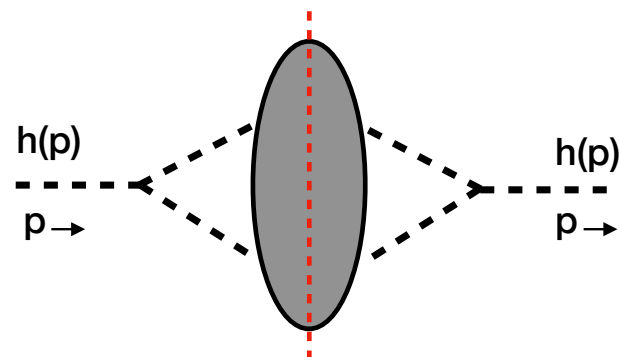
-> no violation of perturbative unitarity for large multiplicities

Higgspersion in loops

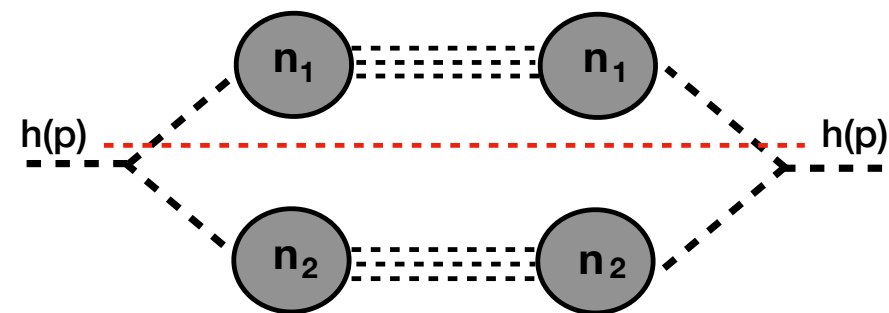
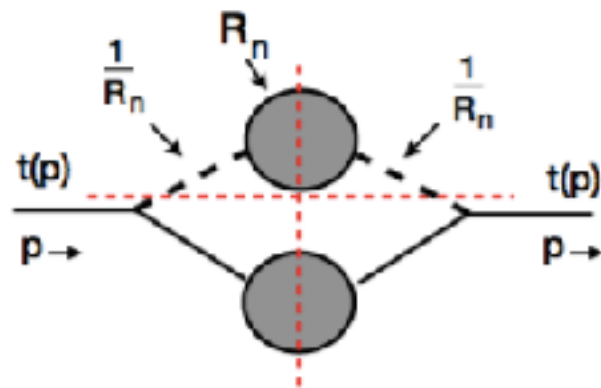
Continuous resummation of the SD propagator does not shut down imaginary part.
You need to consider the self-energy as one object.

$$\text{Im } \Sigma_n \sim \frac{1}{n!} (\mathcal{A}_n)^2 \sim \begin{cases} \frac{1}{n!} \times n! \times n! \sim n! & : \text{all terms included} \\ \frac{1}{n!} \times n! \sim 1 & : \text{no interference terms.} \end{cases}$$

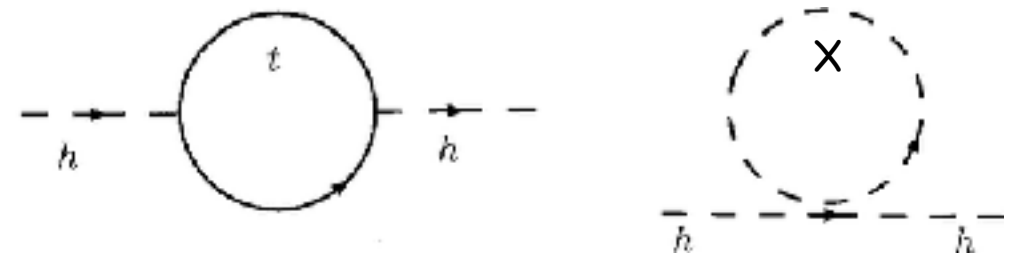
dominant contributions:



subleading contributions:



$$\Delta M_h^2 \sim \lambda_P \int \frac{d^4 p}{16\pi^4} \frac{1}{M_X^2 - p^2 + i \text{Im} \Sigma_X(p^2)}$$



$$= \lambda_P \int \frac{d^4 p}{16\pi^4} \left(\frac{M_X^2 - p^2}{(M_X^2 - p^2)^2 + (\text{Im} \Sigma_X(p^2))^2} - \frac{i \text{Im} \Sigma_X(p^2)}{(M_X^2 - p^2)^2 + (\text{Im} \Sigma_X(p^2))^2} \right)$$

Due to Higgspllosion the multi-particle contribution to the width of X explode at $p^2 = s_*$ where $\sqrt{s_*} \simeq \mathcal{O}(25)\text{TeV}$

→ It provides a sharp UV cut-off in the integral, possibly at $s_* \ll M_X^2$

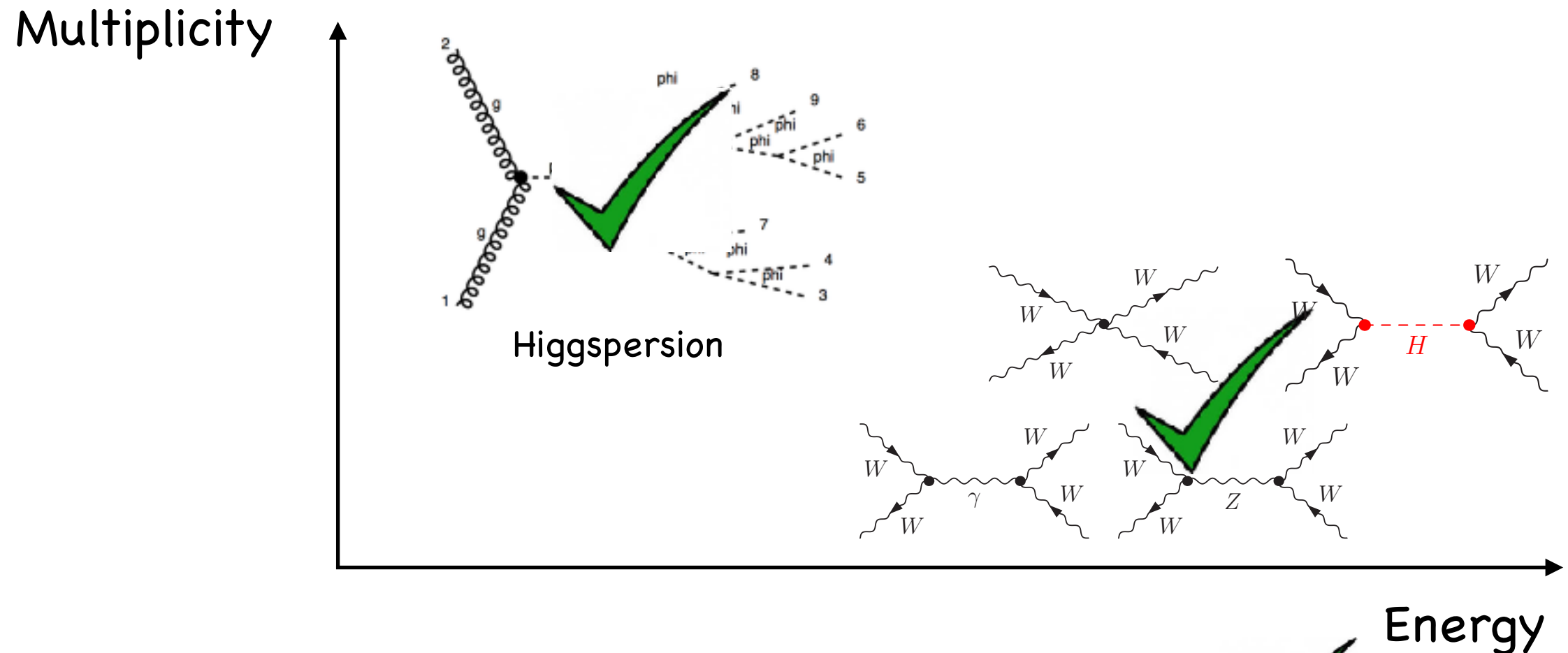
Hence, the contribution to the Higgs mass amounts to

$$\Delta M_h^2 \propto \lambda_P \frac{s_*}{M_X^2} s_* \ll \lambda_P M_X^2$$

and thus mends the Hierarchy problem by $\left(\frac{\sqrt{s_*}}{M_X}\right)^4 \simeq \left(\frac{25 \text{ TeV}}{M_X}\right)^4$

If Higgspllosion is not a mathematical artefact but realised in nature:

Situation at tree-level



+ Hierarchy problem (Loop level)

Higgspllosion



SM heals itself, retains self-consistency
to very high energies and multiplicities

Consequences of Higgspllosion

[Khoze, MS '17]

- SM has new physical scale

(close analogy to Sphaleron)

$$E_* = C \frac{m_h}{\lambda} \quad \text{with } C = \text{const.}$$

$$M_{\text{sph}} = \text{const} \frac{m_W}{\alpha_w}$$

Scaling behaviour of propagator:

$$\Delta(x) := \langle 0 | T(\phi(x) \phi(0)) | 0 \rangle \sim \begin{cases} m^2 e^{-m|x|} & : \text{ for } |x| \gg 1/m \\ 1/|x|^2 & : \text{ for } 1/E_* \ll |x| \ll 1/m , \\ E_*^2 & : \text{ for } |x| \lesssim 1/E_* \end{cases}$$

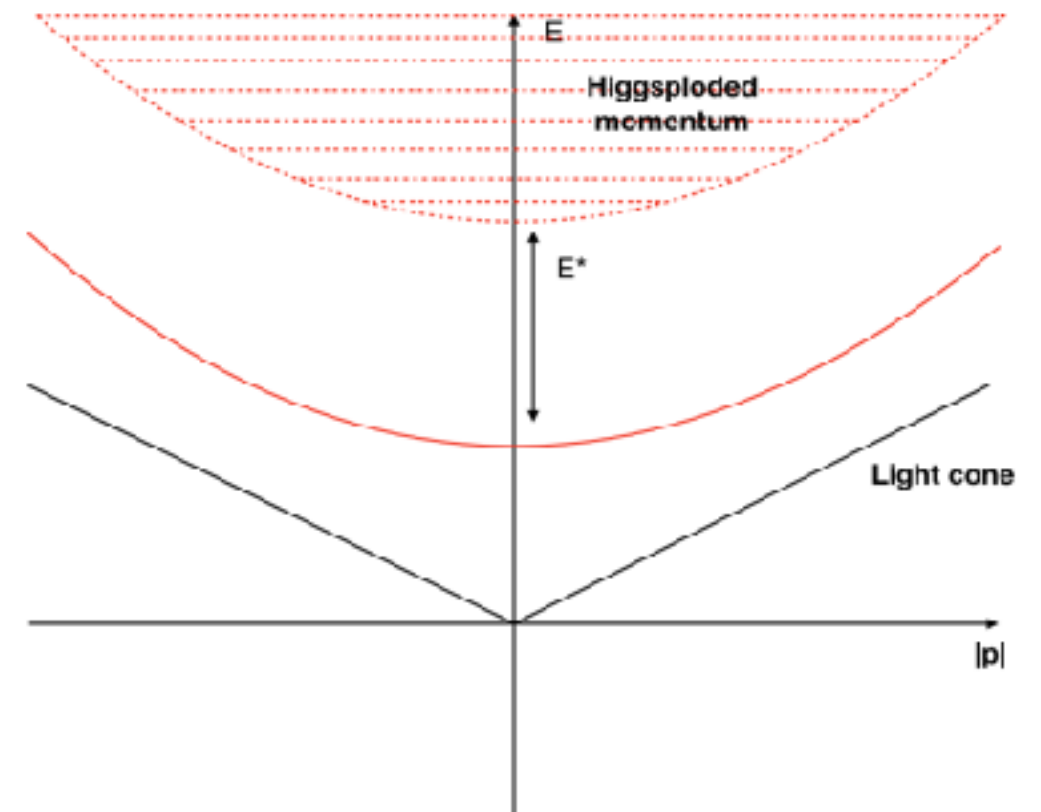
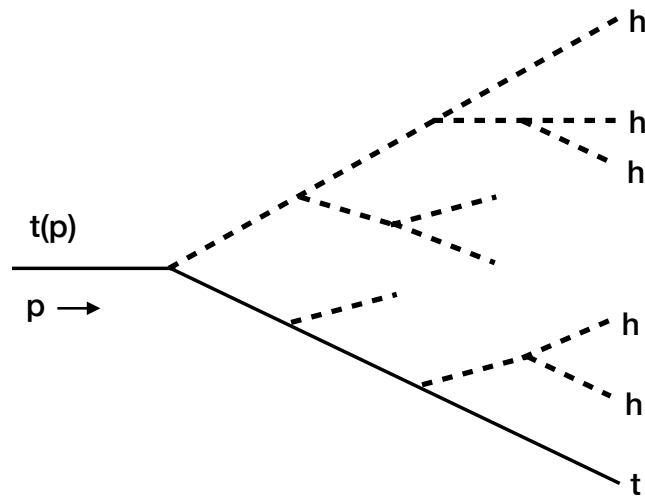
for $|x| \lesssim 1/E_*$ one enters the Higgspllosion regime

Effect calculable on the lattice?

Consequences of Higgspllosion

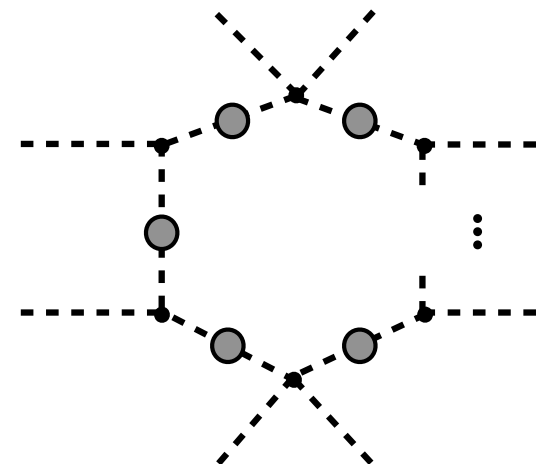
[Khoze, MS '17]

- All particles Higgsplode if **virtual** enough
e.q. top quark, Z boson and even graviton Higgsplodes



- As all virtual particles Higgsplode, all virtual corrections are regulated

In full analogy to [Polchinski '84]



Consequences of Higgspllosion

- As all loop-diagrams are regulated, i.e. quantum fluctuations are exponentially suppressed, the Standard Model develops an asymptotic fix point.

→ Classical/Deterministic theory

→ From high scale, quantum fluctuations are emergent phenomenon

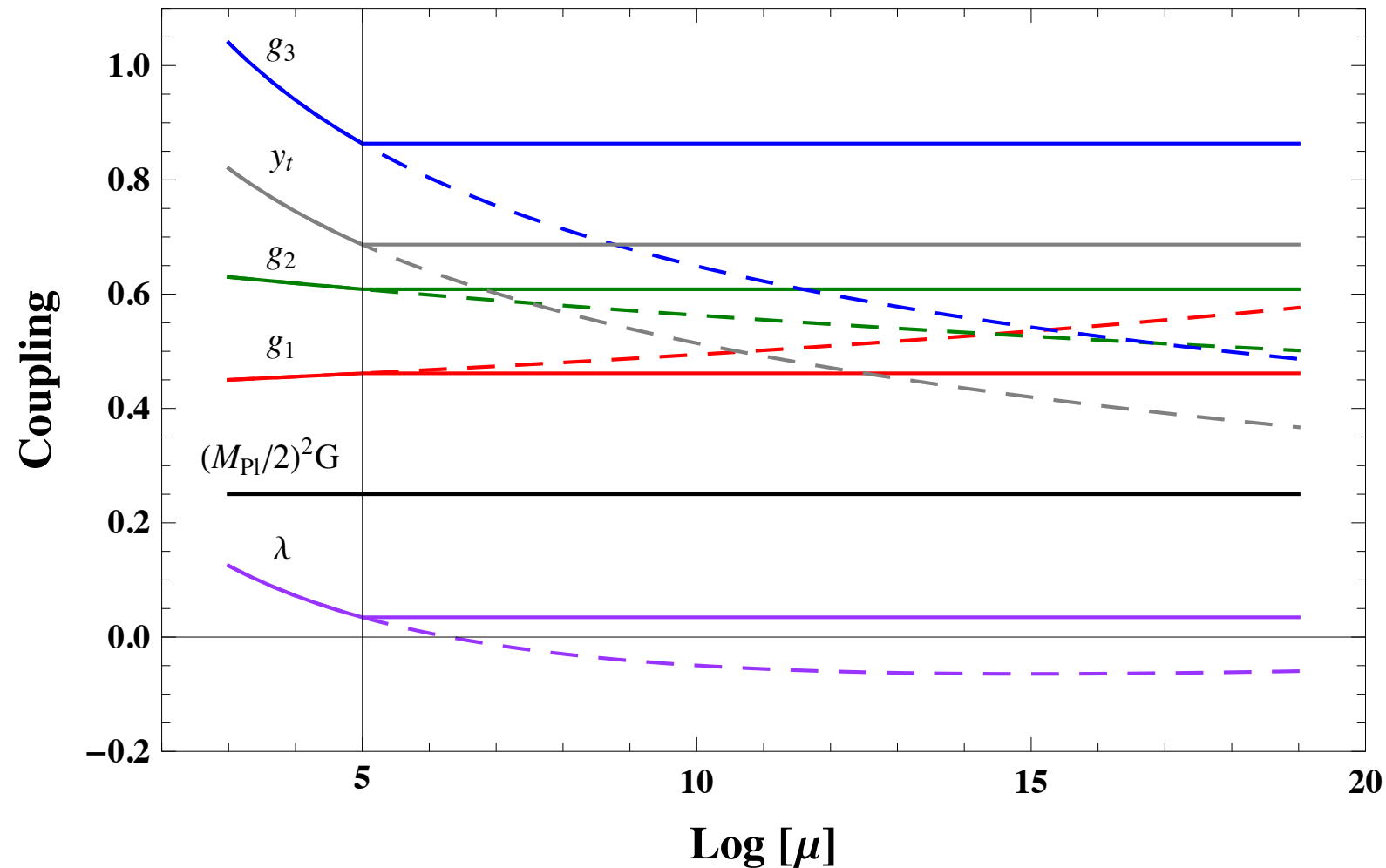
- SM is embedded into asymptotically safe theory (see talk by R. Percacci)

→ Graviton Higgsplodes as well, as do all quantum corrections

→ Allows to combine QFT and Gravity

Consequences of Higgspllosion

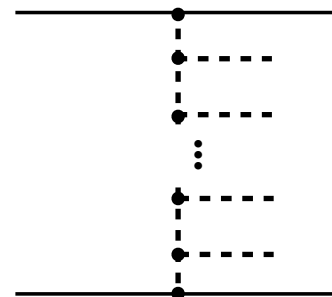
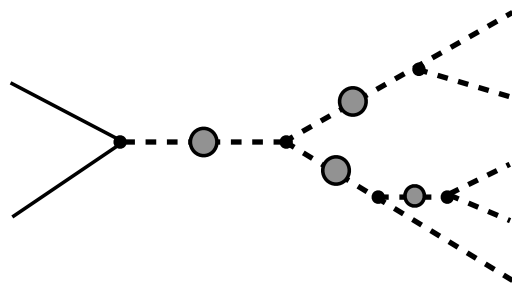
Running of couplings in presence of Higgspllosion



- ➡ Higgs self-coupling doesn't turn negative
- ➡ Electroweak potential remains stable
- ➡ No Landau poles for U(1) and Yukawas

Consequences of Higgspllosion

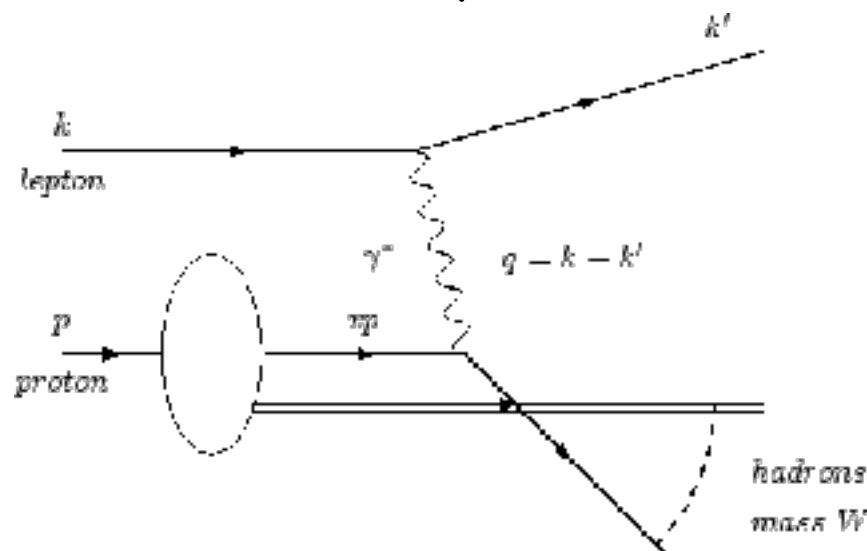
- High-energy scatterings are significantly modified, i.e. virtual s-channel particles are Higgsperised



At high energies
only ladder diagrams
important

In analogy to
Reggeon picture

example DIS



To probe smaller and
smaller structure the
photon needs to be more
and more virtual

Higgspllosion sets cut-off



theory with
'minimal-length'

dynamical
mechanism for
classicalization

(see talk by G. Dvali)

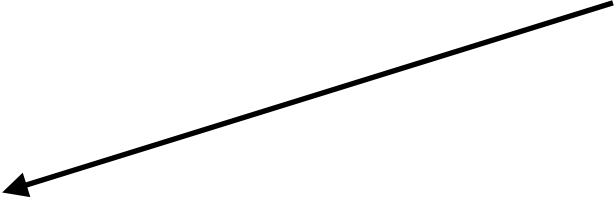
Questions on implications:

[Khoze, MS '17]

Is Inflation excluded/affected by Higgspllosion?

Not necessarily... for example singlet field S non-minimally coupled to gravity

Take Lagrangian in Jordan frame


$$\mathcal{L} = \sqrt{-g} \left[-\frac{M_{Pl} + \xi_s S^2}{2} R + \partial_\mu H^\dagger \partial^\mu H + (\partial_\mu S)^2 - V(H, S) \right]$$

the scalar potential is $V(H, S) = -\mu_h H^\dagger H + \lambda_h (H^\dagger H)^2 - \frac{1}{2} \mu_S^2 S^2 + \frac{1}{4} \lambda_S S^4 + \frac{1}{2} \lambda_{Sh} H^\dagger H S^2$

During Inflation Higgs mass in Inflaton background large $M_h \simeq \sqrt{\frac{\lambda_{Sh}}{2}} S(x) \simeq \frac{M_{Pl}}{\sqrt{\xi_s}}$



No phase space for S to Higgsplode



Picture changes fundamentally during reheating



Questions on implications:

Is the existence of Axions (light scalars) irreconcilable with Higgspllosion?

QCD-Axion provides predictive framework to address this question

[Grilli di Cortona, Hardy, Vega, Villadoro '16]

$$m_a \simeq \frac{5.7 \cdot 10^{15} \text{eV}}{f_a} \quad \text{and} \quad \lambda_a \equiv \left. \frac{\partial^4 V(a)}{\partial a^4} \right|_{a=0} \simeq -0.346 \frac{m_a^2}{f_a^2}$$

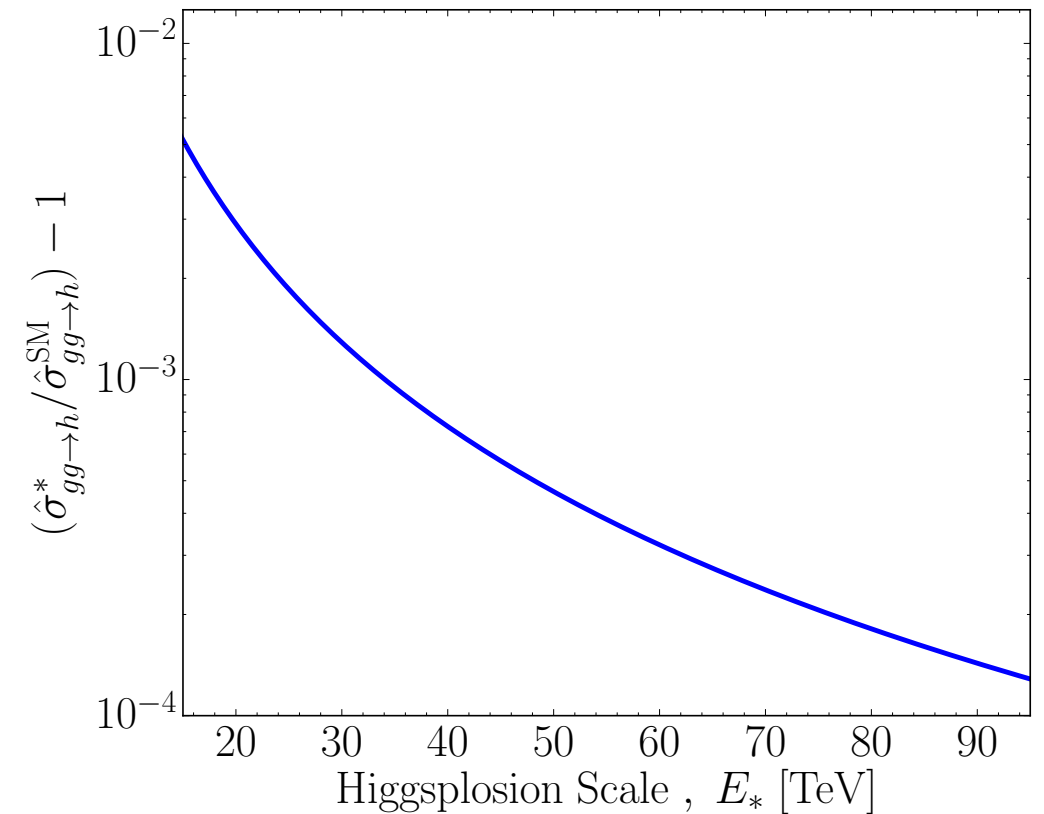
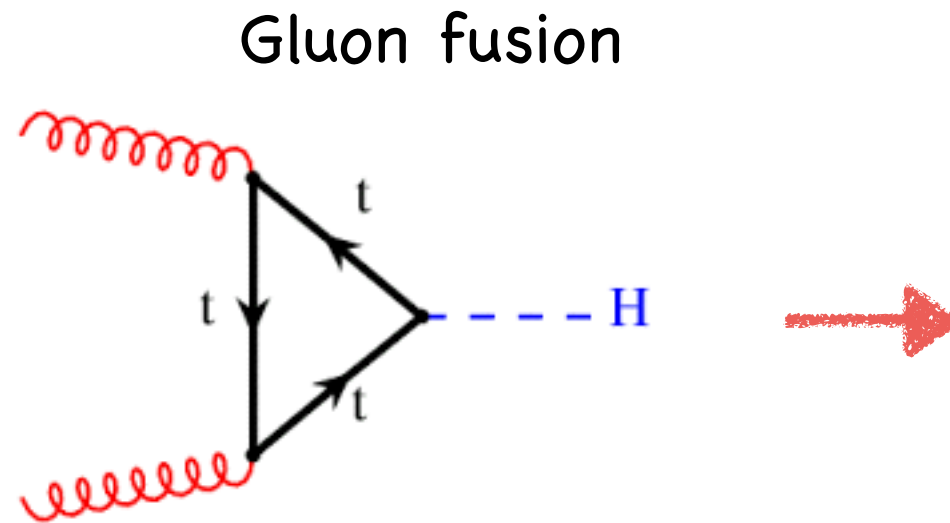
 Axionpllosion scale $E_*^{\text{Axion}} \simeq 60 \frac{f_a^2}{m_a}$  limit $f_a \gtrsim 2.1 \text{ GeV}$

current experimental limit $f_a \gtrsim 10^8 - 10^{17} \text{ GeV}$

 If scalars are very weakly coupled they will not trigger X-pllosion

Can we discover Higgspllosion?

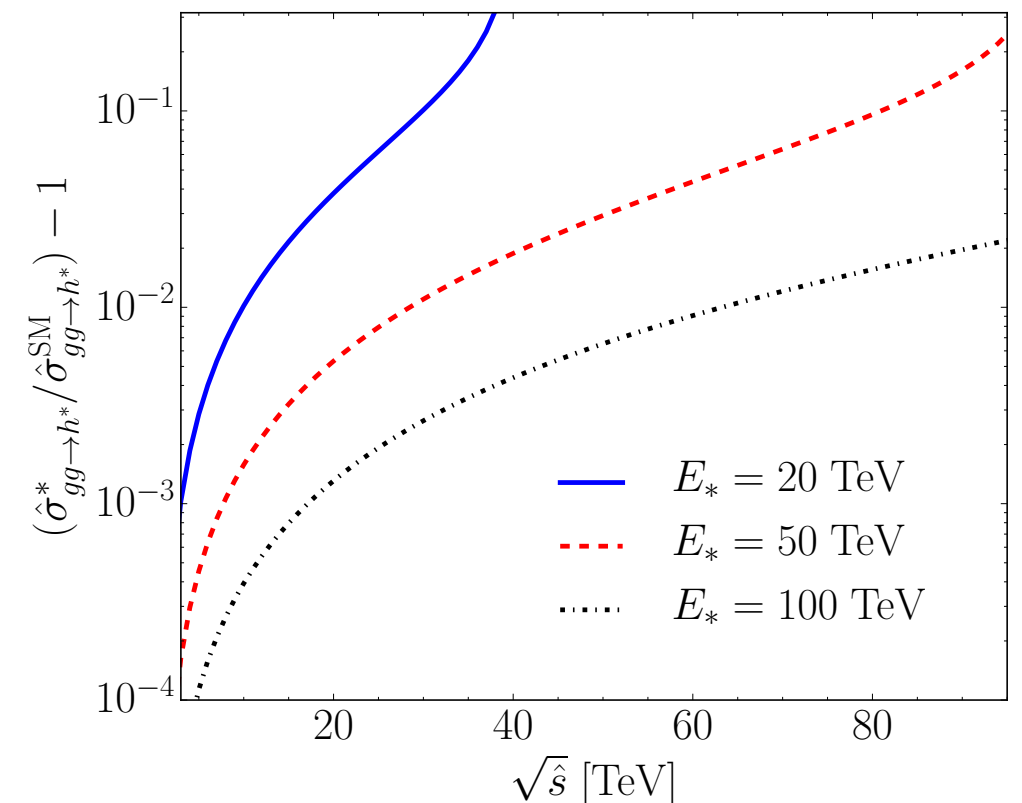
Modification in loop corrections:



Gluon-fusion for different s-hat or mH

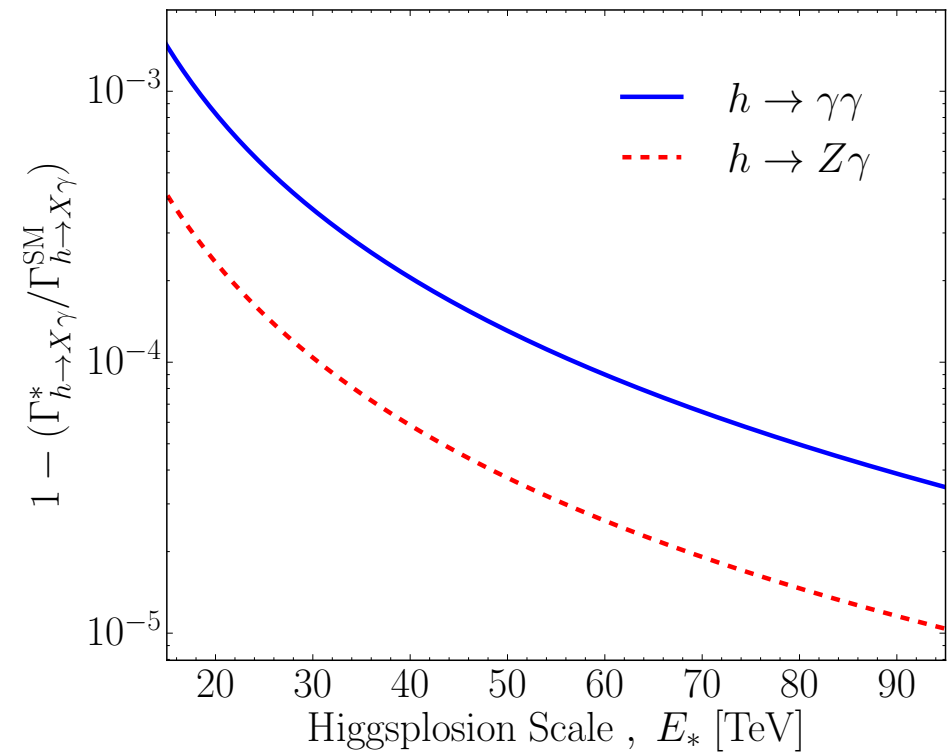
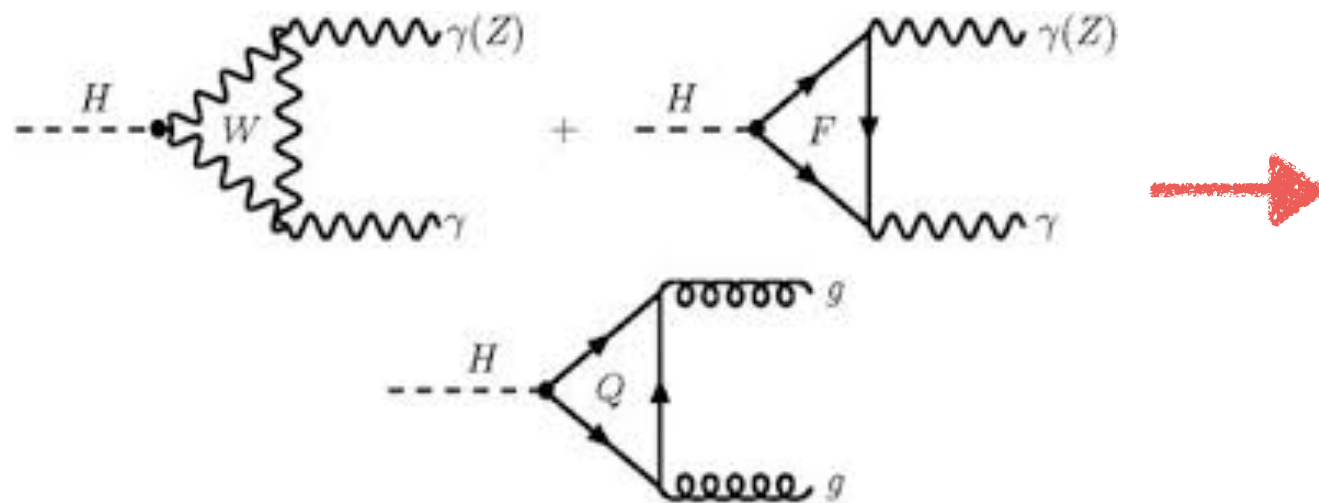
O(1) effects when $\sqrt{\hat{s}} \sim 2E_*$

[Khoze, Reiness, MS, Waite '17]

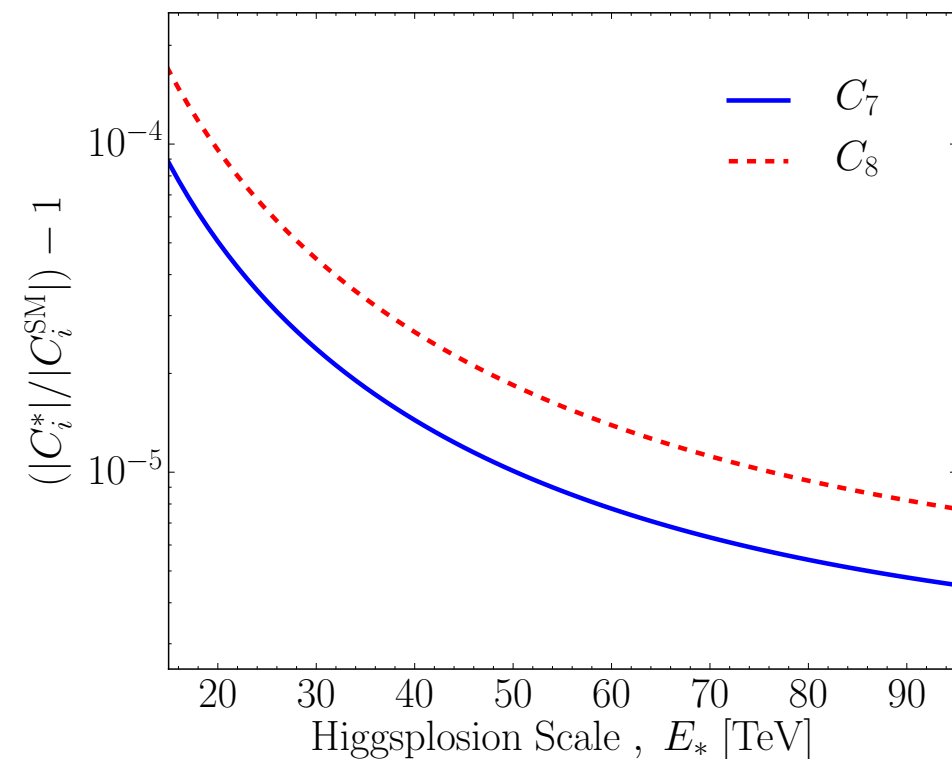
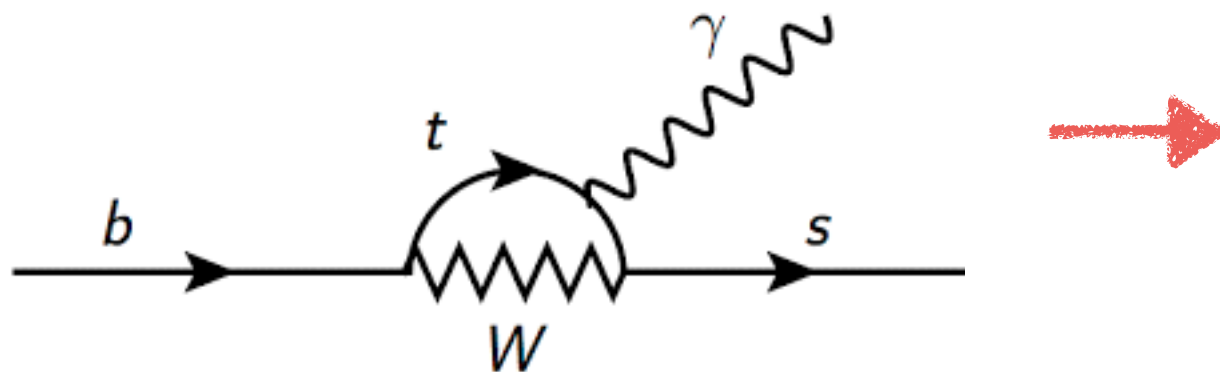


Can we discover Higgspllosion?

Loop-induced Higgs decays



$b \rightarrow s$ gamma



Can we discover Higgspllosion?

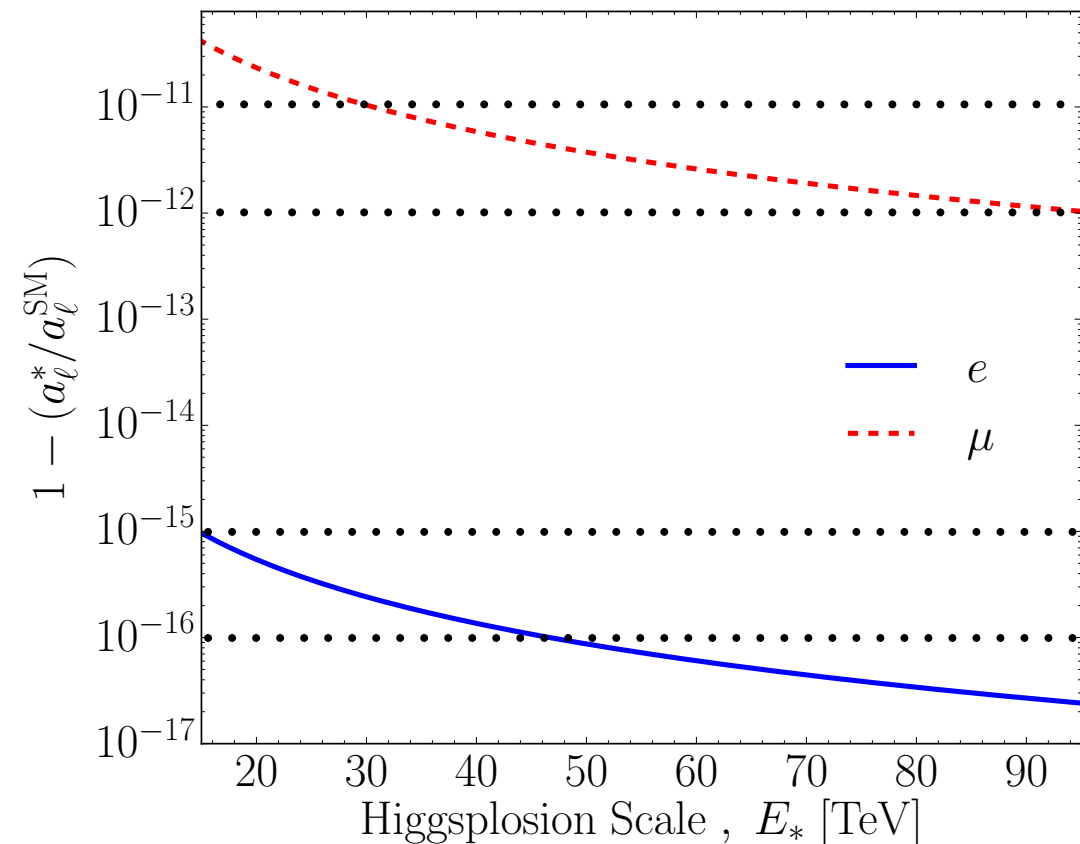
Anomalous magnetic moment of the muon and electron

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}$$

anomalous magnetic moment given
by $F_2(0)$

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{theory}} \simeq 2.90 \cdot 10^{-9}$$

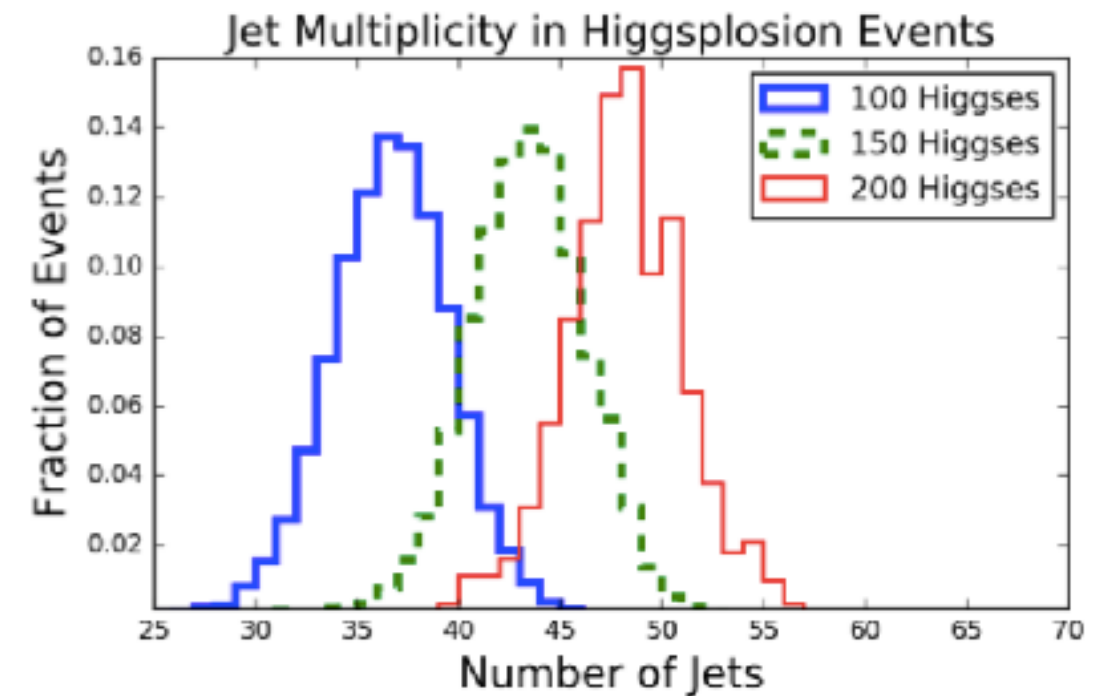
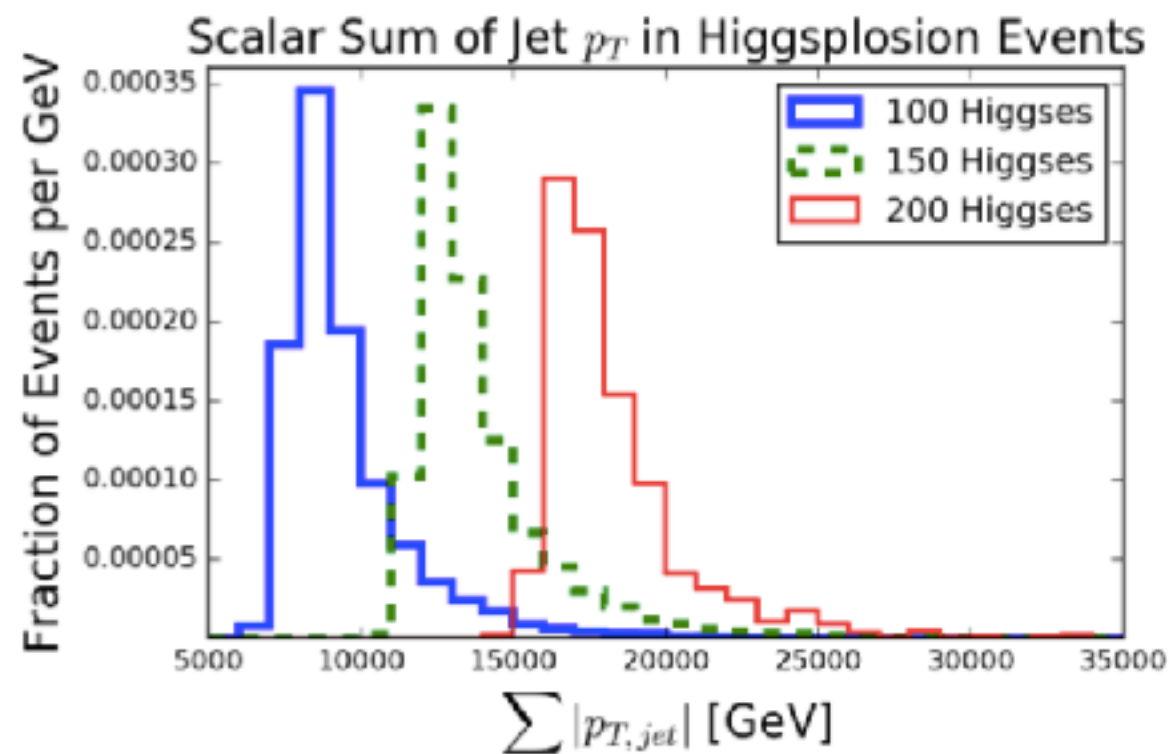
$$a_e^{\text{exp}} = 11596521807.3(2.8) \cdot 10^{-13}$$



Problem: all corrections scale like \hat{s}/E_*^2

Prospects of direct observation of Higgslosion

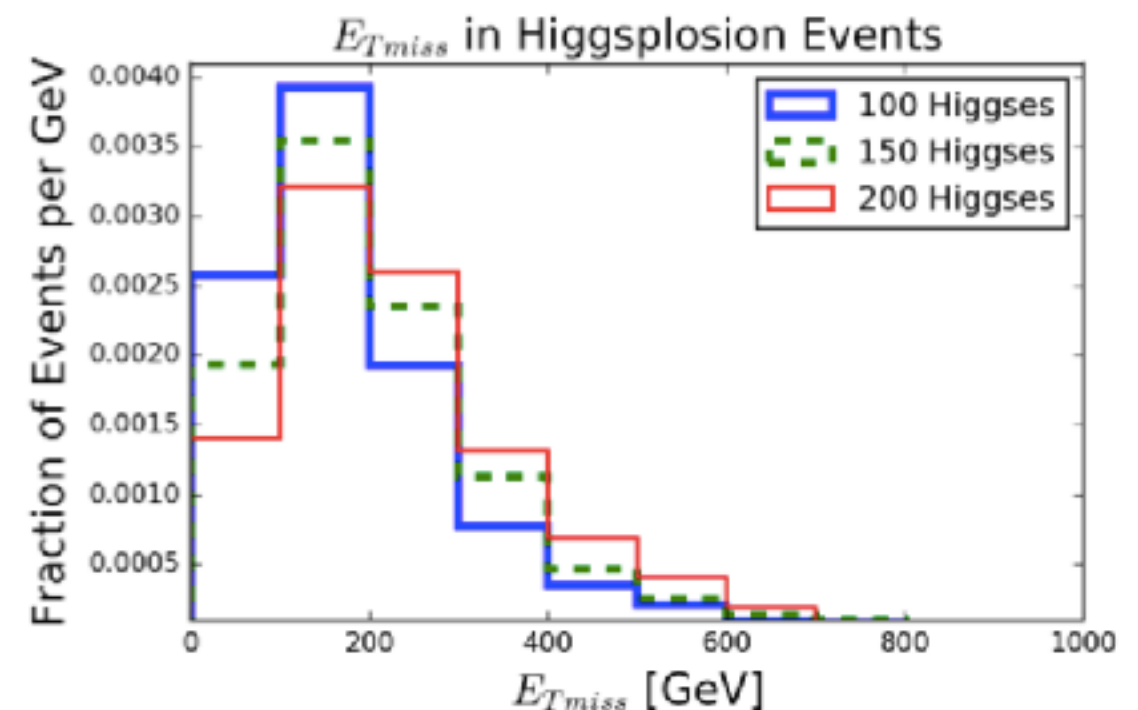
Collider observables for Higgslosion production



see [Gainer '17]

If Higgslosion is realised, the
detector lights up like a
Christmas tree

→ discovery with few events possible



Prospects of direct observation of Higgspllosion

Partonic gluon-fusion cross section:

$$\sigma_{gg \rightarrow n \times h}^{\Delta} \sim y_t^2 m_t^2 \log^4 \left(\frac{m_t}{\sqrt{s}} \right) \times \frac{\mathcal{R}_n(s)}{\left(1 - \frac{m_h^2}{s}\right)^2 + \mathcal{R}^2(s)}$$

$$\sigma_{gg \rightarrow n \times h} \sim \begin{cases} \mathcal{R} & : \text{ for } \sqrt{s} \leq E_* \text{ where } \mathcal{R} \lesssim 1 \\ 1/\mathcal{R} \rightarrow 0 & : \text{ for } \sqrt{s} \geq E_* \text{ where } \mathcal{R} \gg 1 \end{cases} \quad \text{asymptotic large energy behaviour}$$

there is smooth-spot for energy into hard process, or subsequent emissions of jets

$$\sigma_{gg \rightarrow n \times h} \sim \begin{cases} \mathcal{R} & : \text{ for } \sqrt{s} \ll E_* \text{ where } \mathcal{R} \ll 1 \\ 1 & : \text{ for } \sqrt{s} \geq E_* \text{ where } \mathcal{R} \sim 1 \end{cases} \quad \text{maximum probability for hard process}$$

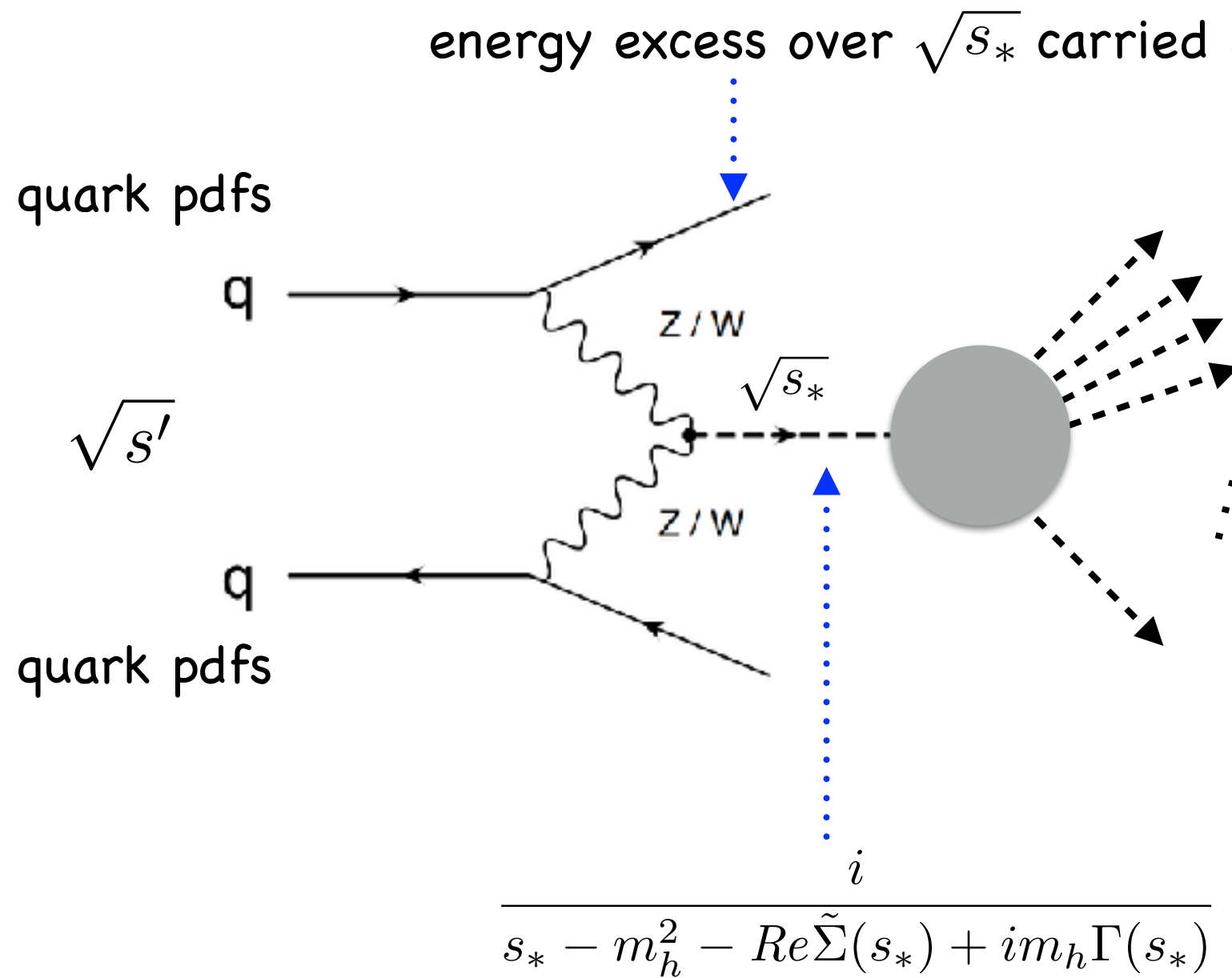
Partonic process want to stay 'on resonance', where $\sum_n \mathcal{R}_n(s) = \mathcal{R}(s) \sim 1$



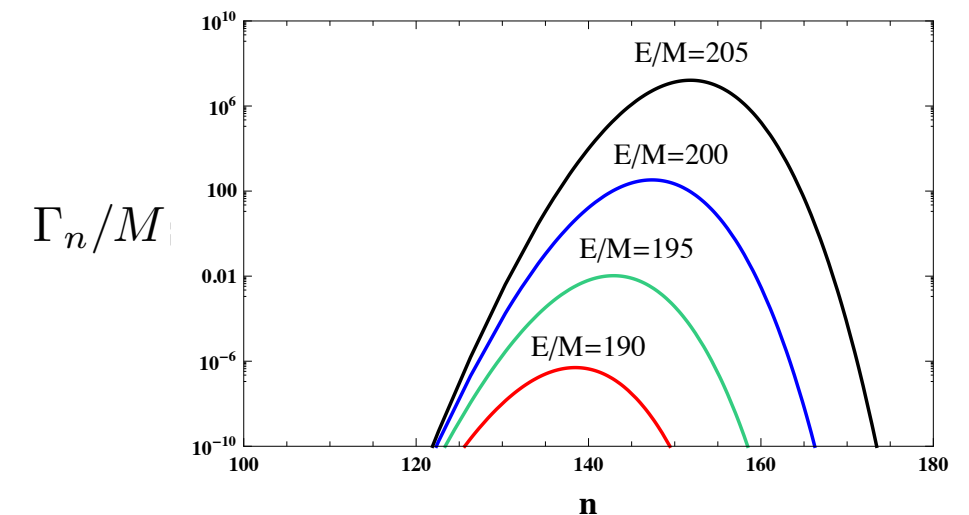
adjust x in hadron collision or emit excessive energy via jets

Prospects of direct observation of Higgsboson

Vector boson fusion at high-energy pp colliders (FCC)

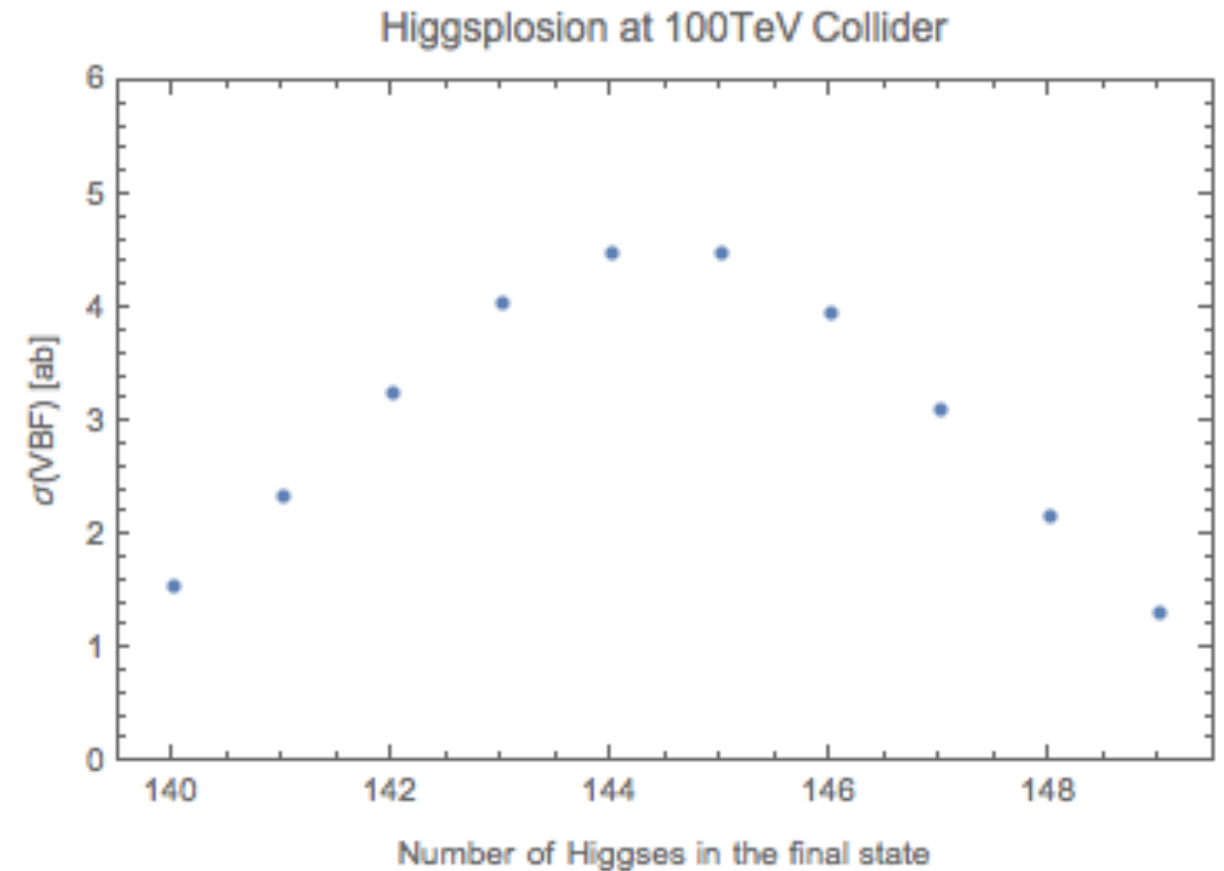
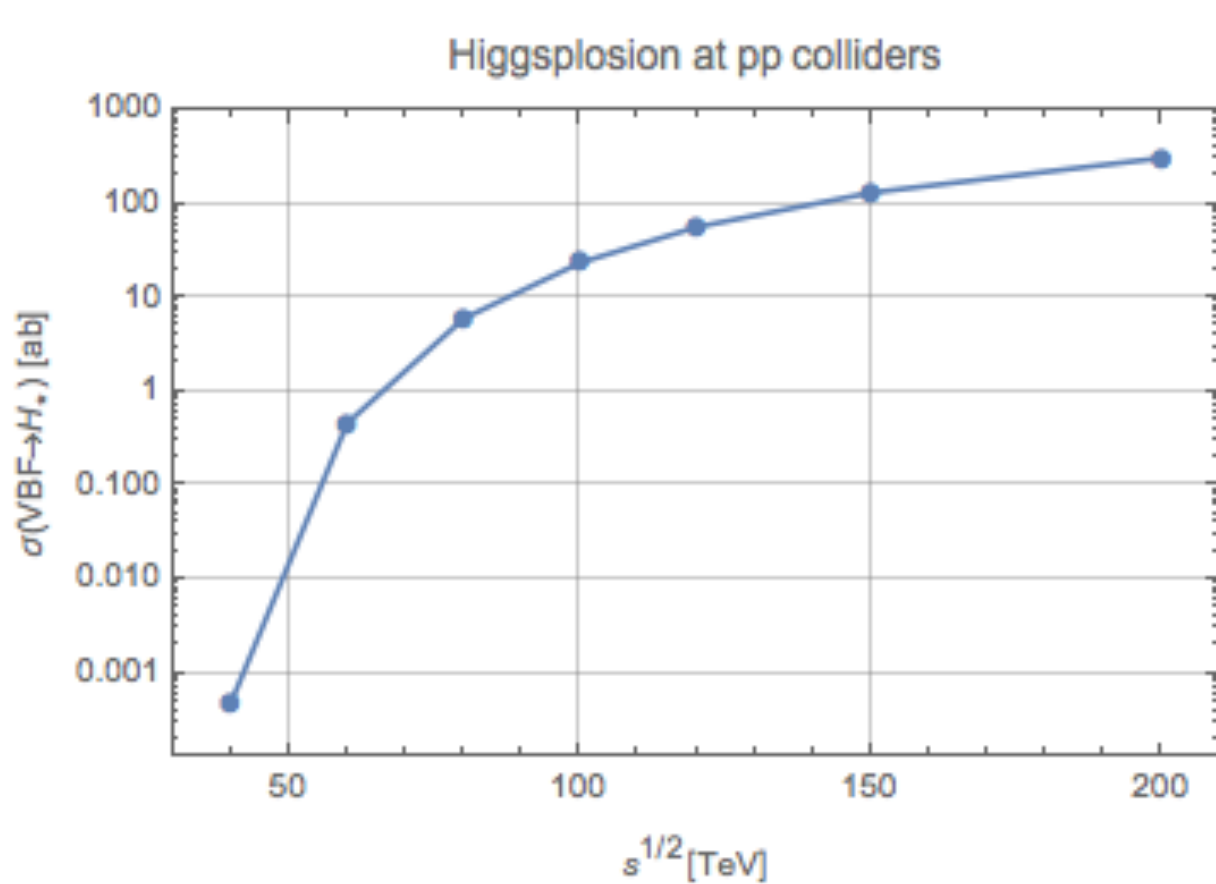


n non-relativistic Higgses
Higgsboson at $\sqrt{s_*}$



Propagator with Higgsboson at $\sqrt{s_*}$

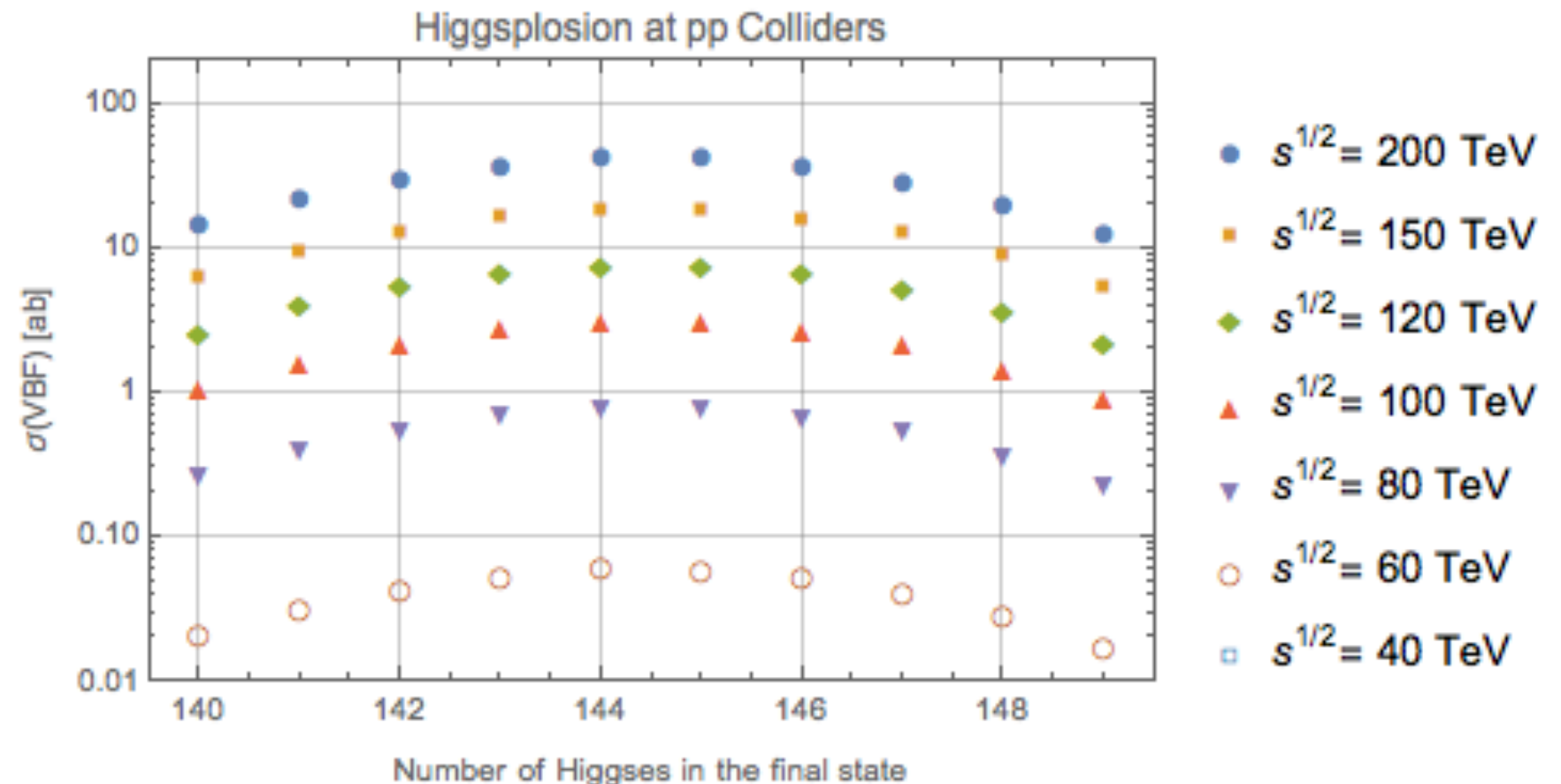
Vector boson fusion at high-energy pp colliders (FCC)

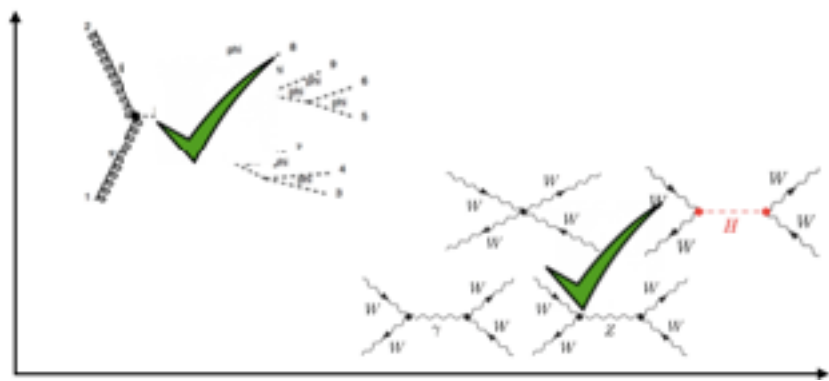


using $p_{t\text{jet}} > 40$ GeV

[Khoze, Scholtz, MS in prep]

preliminary: no Higgs
decays into SM dofs
included;
& no vector bosons in
final states yet





Train of thought:



$h^* \rightarrow n h$ shows factorial growth in classical, 1-loop resummed and semi-classical calculations

optical theorem
(all orders)



If $\Gamma_n(p^2)$ for any n explodes $\Gamma_{\text{tot}}(p^2)$ explodes



Imaginary part of self-energies explode



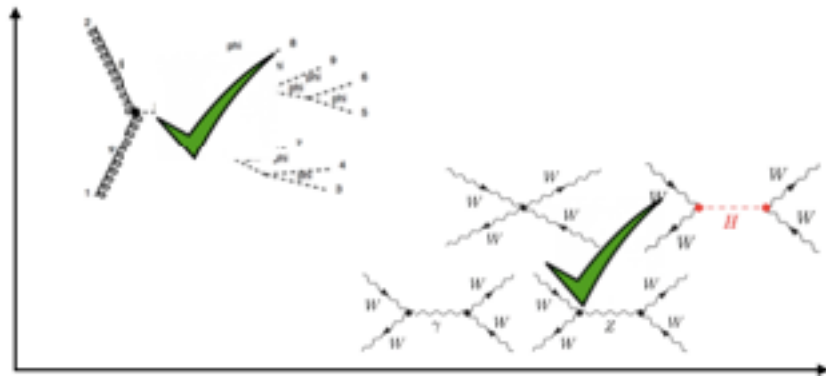
Real part can't cancel imaginary part of self-energy



All 2-point and n -point functions shut down beyond Higgsplosion scale



New physical scale in SM, no Unitarity violation, no Hierarchy problem, asymptotically safe theory, stable vacuum, minimal-length theory



Summary



If Higgspllosion realised it will have spectacular consequences, i.e. many pieces fall into place

The SM has a new energy scale one can test

Currently, idea relies on 20th century QFT

Experimental tests

- > build $O(100)$ TeV collider
- > much improved low energy measurements, e.g. $g-2$

Theoretical tests

- > Lattice calculation
- > Improved semi-classical approach?