NEUTRAL PION MATTER IN STRONG MAGNETIC FIELDS

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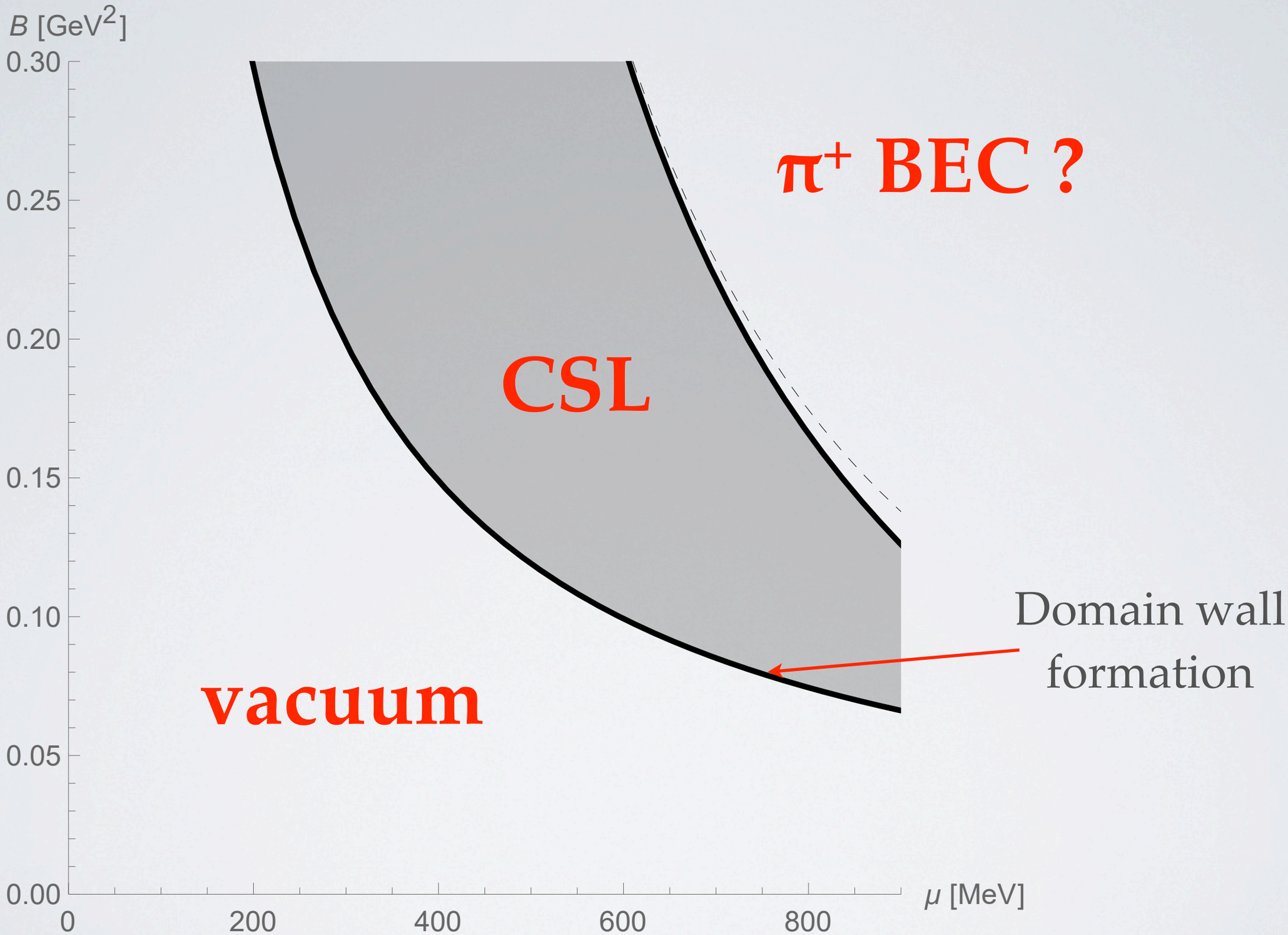
CSL PHASE DIAGRAM



Conversion factor: $1\ \mathrm{GeV}^2 \approx 1.7 \times 10^{20}\ \mathrm{G}$

TB, Yamamoto, JHEP **04** (2017)

LOW-ENERGY EFT FOR QCD

Gasser, Leutwyler, Ann. Phys. **158** (1984)

- Symmetry-breaking pattern:

$$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \to \mathrm{SU}(2)_\mathrm{V}$$

- Parameterization of the coset space:

$$[\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}]/\mathrm{SU}(2)_\mathrm{V} \quad \Longrightarrow \quad \Sigma \in \mathrm{SU}(2)$$

- Leading-order effective Lagrangian:

$$\mathscr{L} = \frac{f_\pi^2}{4}\left[\mathrm{tr}(D_\mu \Sigma^\dagger D^\mu \Sigma) + m_\pi^2\, \mathrm{tr}(\Sigma + \Sigma^\dagger)\right]$$

Explicit symmetry breaking

WESS–ZUMINO–WITTEN TERM

Anomalous coupling of pions has nontrivial consequences:

- Pion fields can carry baryon number!
- Neutral pions can couple to electromagnetic fields!

$$S_{\mathrm{WZW}} = -\int d^4x \left(A^{\mathrm{B}}_{\mu} - \frac{1}{2} A_{\mu} \right) j^{\mu}_{\mathrm{B}}$$

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Baryon number gauge field

Electromagnetic gauge field

Goldstone–Wilczek current

$$j^\mu_{\mathrm{B}} = -\frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\left[(\Sigma D_\nu \Sigma^\dagger)(\Sigma D_\alpha \Sigma^\dagger)(\Sigma D_\beta \Sigma^\dagger) + \frac{3i}{4} F_{\nu\alpha}\tau_3 (\Sigma D_\beta \Sigma^\dagger + D_\beta \Sigma^\dagger \Sigma)\right]$$

Son, Zhitnitsky, PRD **70** (2004)
Son, Stephanov, PRD **77** (2008)

NEUTRAL PION BACKGROUND

$$\mathscr{L} = \frac{f_\pi^2}{4}\left[\mathrm{tr}(D_\mu \Sigma^\dagger D^\mu \Sigma) + m_\pi^2\,\mathrm{tr}(\Sigma + \Sigma^\dagger)\right] + \mathscr{L}_{\mathrm{WZW}}$$

$$\Sigma = e^{i\tau_3\phi}$$

Son, Stephanov, PRD **77** (2008)

$$\mathscr{H} = \frac{f_\pi^2}{2}(\boldsymbol{\nabla}\phi)^2 + m_\pi^2 f_\pi^2(1-\cos\phi) - \frac{\mu}{4\pi^2}\boldsymbol{B}\cdot\boldsymbol{\nabla}\phi$$

Favors one-dimensional modulation in direction of $\boldsymbol{B}$!

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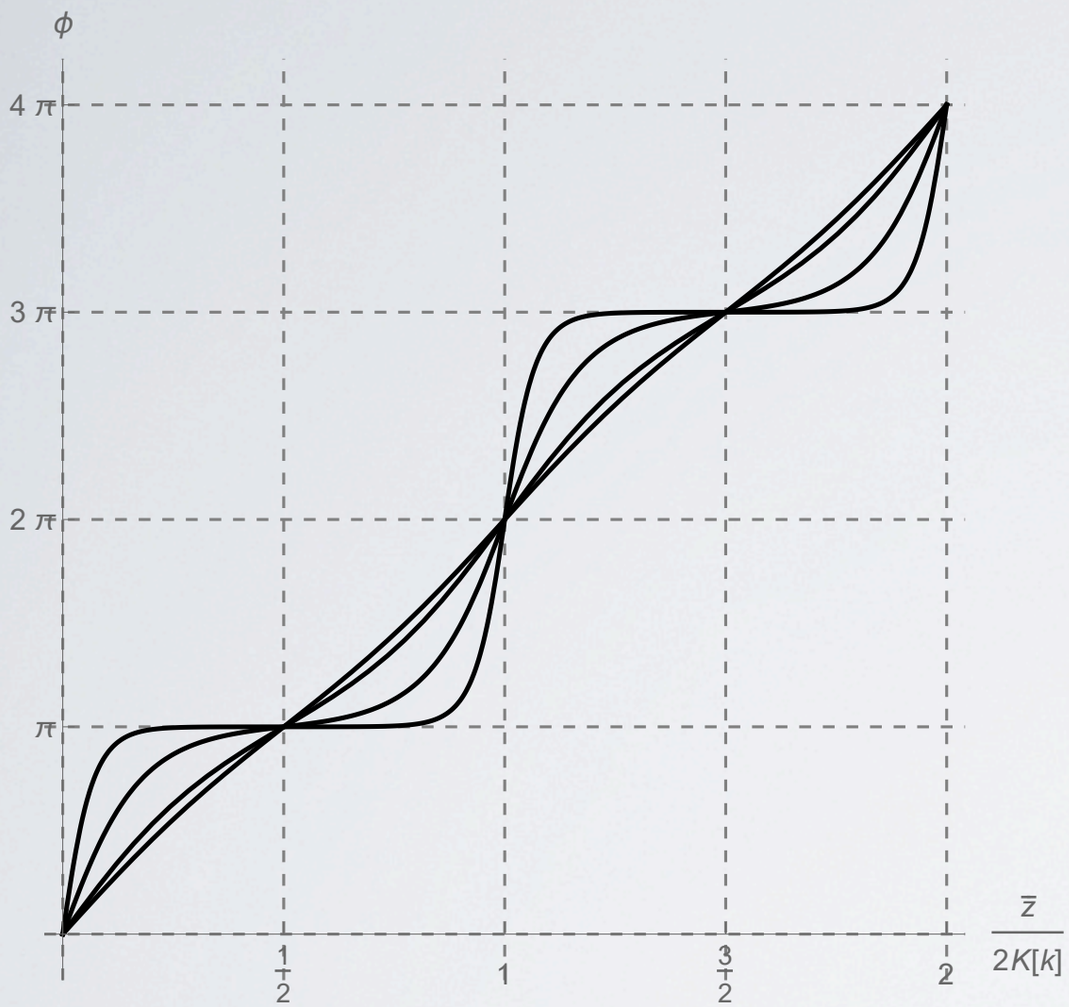
$$\cos\frac{\phi(\bar{z})}{2} = \mathrm{sn}(\bar{z}, k), \qquad \bar{z} = \frac{z m_\pi}{k}$$

Jacobi elliptic function

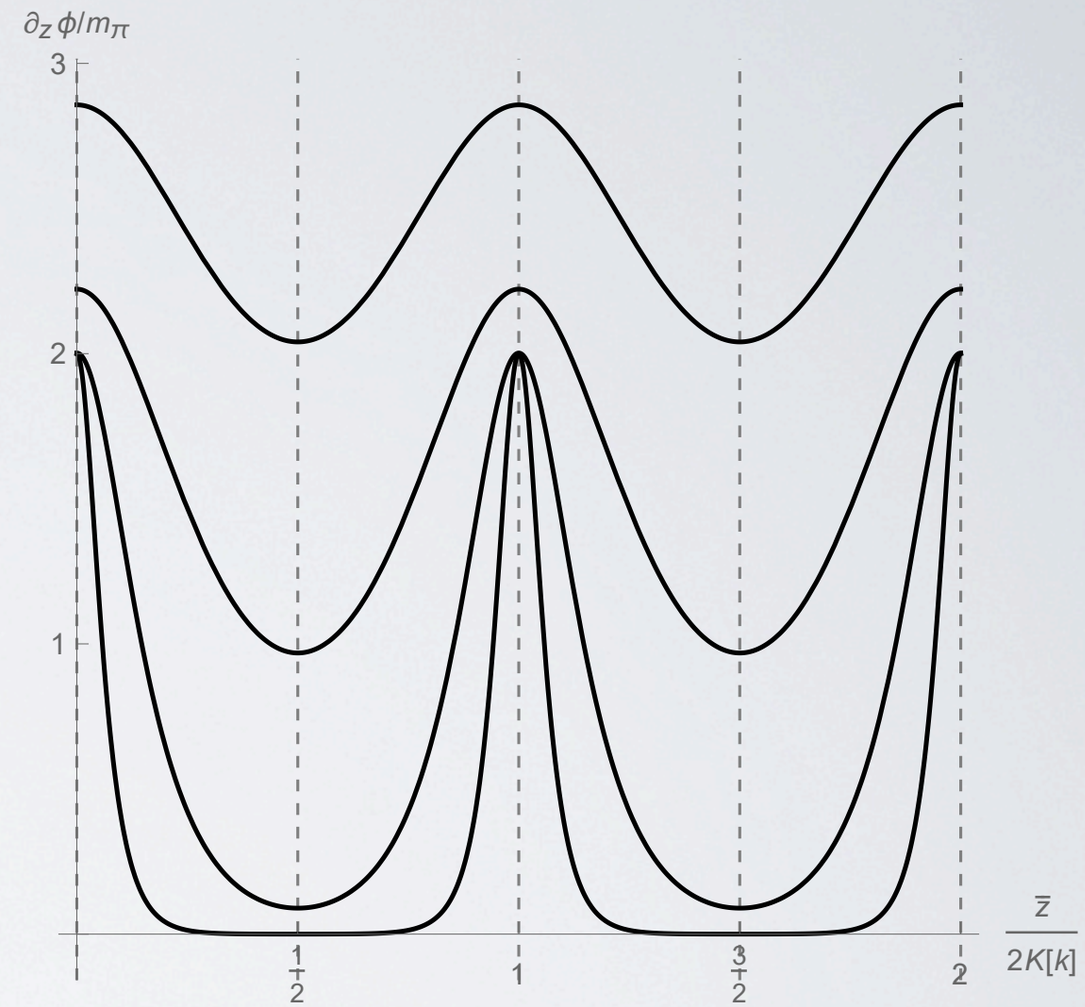
Elliptic modulus

CHIRAL SOLITON LATTICE

The solution is a periodic lattice of topological solitons!



Phase of the solution



Topological charge density

$$n_{\mathrm{B}}(z) = \frac{B}{4\pi^2}\partial_z\phi(z)$$

$$m(z) = \frac{\mu}{4\pi^2}\partial_z\phi(z)$$

Kishine, Ovchinnikov,
Solid State Phys. **66** (2015)

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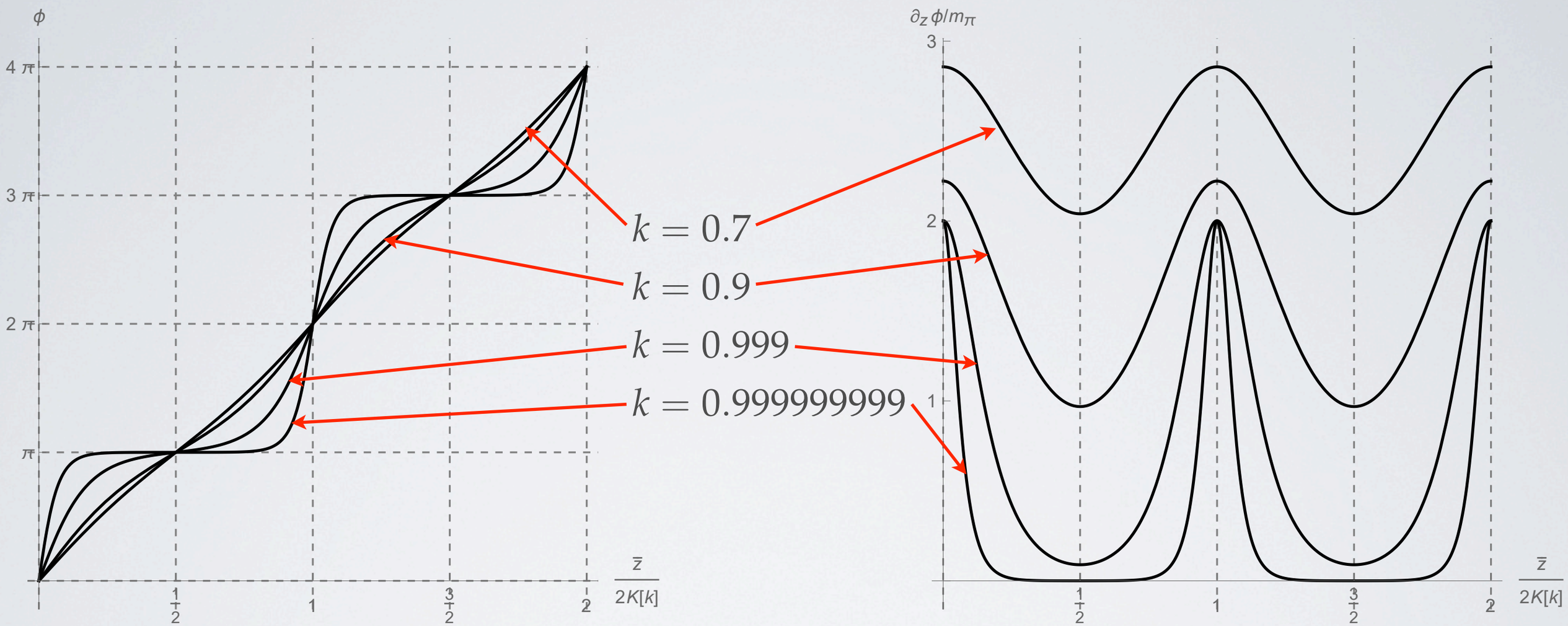

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CRITICAL MAGNETIC FIELD

CSL is favored over QCD vacuum and nuclear matter above certain critical value of magnetic field.

$$\frac{E(k)}{k} = \frac{\mu B}{16\pi m_\pi f_\pi^2} \quad \Longrightarrow \quad B_{\rm CSL} = \frac{16\pi m_\pi f_\pi^2}{\mu}$$

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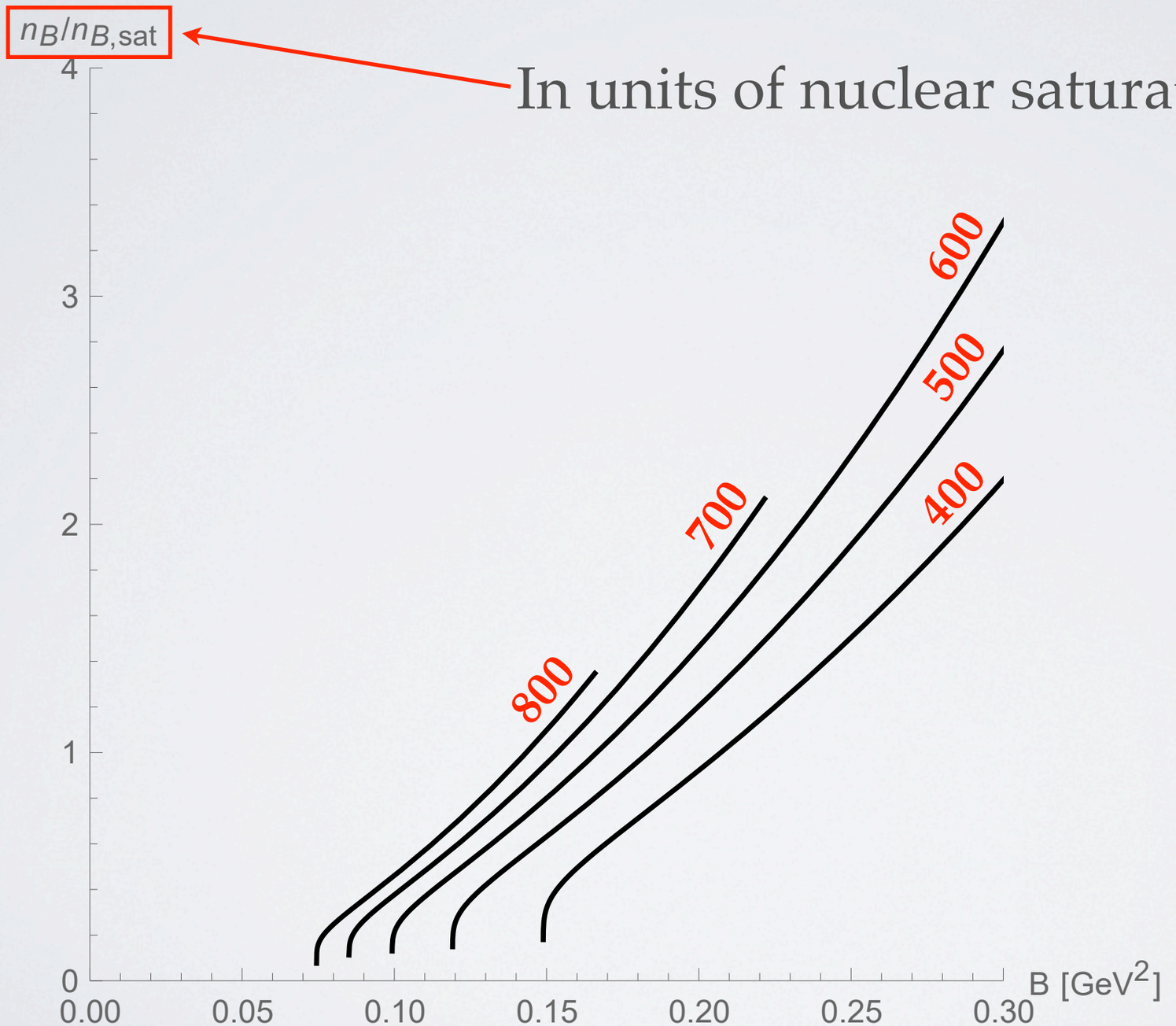
$$\left.\begin{array}{r} m_\pi \approx 140 \text{ MeV} \\ f_\pi \approx 92 \text{ MeV} \\ \boxed{\mu = 900 \text{ MeV}} \end{array}\right\} \quad \Longrightarrow \quad B_{\rm CSL} \approx 0.066 \text{ GeV}^2$$

Conversion factor: $1 \text{ GeV}^2 \approx 1.7 \times 10^{20}$ G

Such fields can occur in the cores of neutron stars!

BARYON NUMBER DENSITY

What is the corresponding average baryon number density?



In units of nuclear saturation density!

CHARGED PION FLUCTUATIONS

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$$\Sigma = e^{i\tau_3\phi} e^{\frac{i}{f_\pi}\vec{\tau}\cdot\vec{\pi}}$$

Expand to second order in the field fluctuations

$$\mathscr{L}_{\mathrm{bilin}} = \frac{1}{2}(\partial_\mu\vec{\pi})^2 + (A^\mu - \partial^\mu\phi)(\pi_1\partial_\mu\pi_2 - \pi_2\partial_\mu\pi_1) + \frac{1}{2}A^\mu A_\mu(\pi_1^2 + \pi_2^2) - \frac{1}{2}m_\pi^2\vec{\pi}^2\cos\phi$$

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- CSL background acts as a chemical potential on charged pions!
- Chiral limit: easily solvable.
- General case: Lamé equation (n=2), solvable with some effort.

Li, Kusnezov, Iachello, JPA **33** (2000)
Finkel, González-López, Rodríguez, JPA **33** (2000)

CHARGED PION SPECTRUM

- Chiral limit:

$$\omega = \sqrt{p_z^2 - \frac{\mu B p_z}{2\pi^2 f_\pi^2} + (2n+1)B}$$

$$\min \omega^2 = B - \left(\frac{\mu B}{4\pi^2 f_\pi^2}\right)^2$$

- General case:

$$\min \omega^2 = B - \frac{m_\pi^2}{k^2}\left(2 - k^2 + 2\sqrt{1 - k^2 + k^4}\right)$$

The bottom of the lowest Landau level reaches zero in sufficiently strong magnetic fields!

COUPLING IN ELECTROMAGNETIC FIELD

- Excitation spectrum in the chiral limit:

$$\omega_1(\boldsymbol{p}) = \frac{|\boldsymbol{p}|}{CB}\sqrt{\boldsymbol{p}_\perp^2 + \frac{\boldsymbol{p}_\parallel^2}{\mu^2 C^2}} + \mathcal{O}(\boldsymbol{p}^2)$$

$$\omega_2(\boldsymbol{p}) = \mu C^2 B + \mathcal{O}(\boldsymbol{p}^2)$$

$$\omega_3(\boldsymbol{p}) = CB + \mathcal{O}(\boldsymbol{p}^2)$$

$$C = \frac{1}{4\pi^2 f_\pi}$$

- Only one gapless mode, with a quadratic dispersion!
- Pressure due to the gapless mode:

$$P = \frac{\zeta(\frac{5}{2})}{16\pi^{3/2}}(CB)^{3/2}T^{5/2}\int_0^\pi \frac{\sin\theta\, d\theta}{(\sin^2\theta + \frac{1}{\mu^2 C^2}\cos^2\theta)^{3/4}}$$

TB, Kadam, JHEP **03** (2017)

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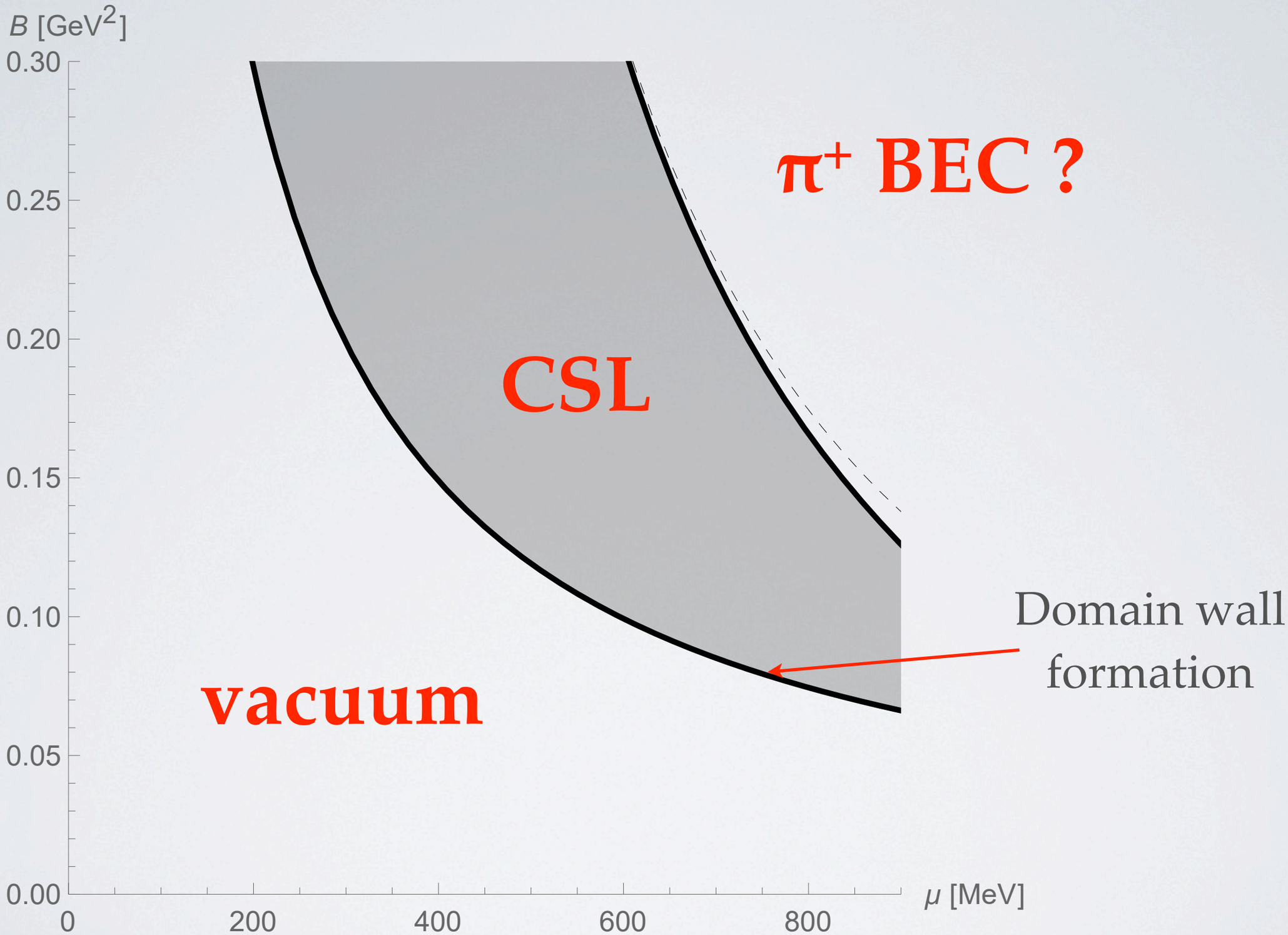

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