Bounding the space of S-matrices:
Rebooting the Bootstrap

Andrea L. Guerrieri

"Joint Rome Seminar", 21st of December, 2017

S-Matrix Bootstrap IV: flavored amplitudes and chiral symmetry [work in progress]

with J. Penedones, J. Toledo, P. Vieira

unesp
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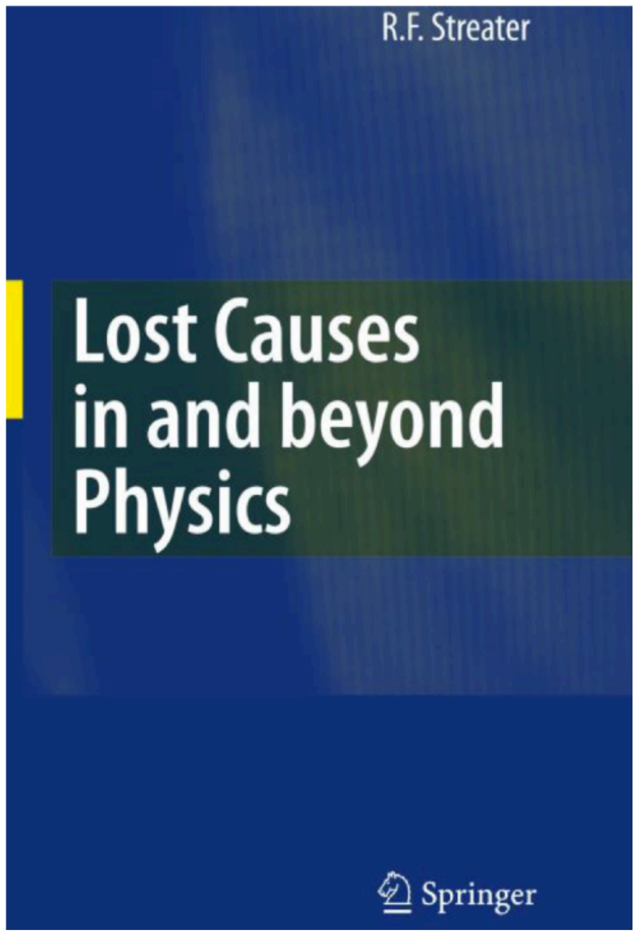
"JÚLIO DE MESQUITA FILHO"

ICTP SAIFR

Simons Collaboration on
The Nonperturbative Bootstrap

Why doing this?



"It may be flogging a dead horse, but I will mention this subject, and why it failed."

Quantum field theory is hard

Question: What is the spectrum and coupling of a strongly coupled QFT?

1) Ratio of glueball masses in YM in 3+1 dimensions

2) Ising with $T \neq T_c$ and $h \neq 0$ even in 2d

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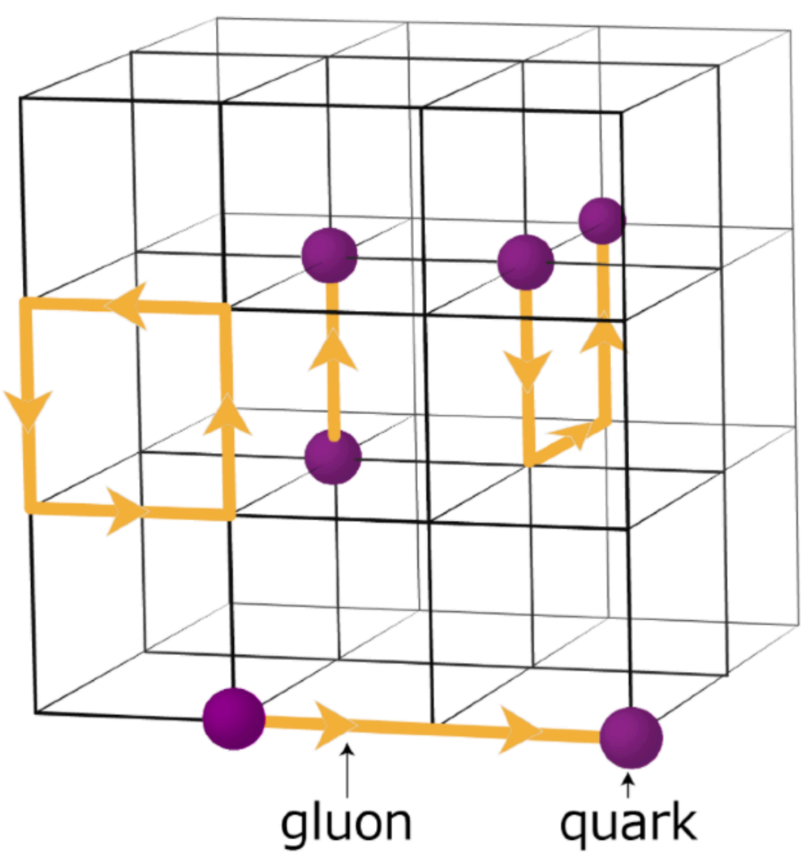

Lattice

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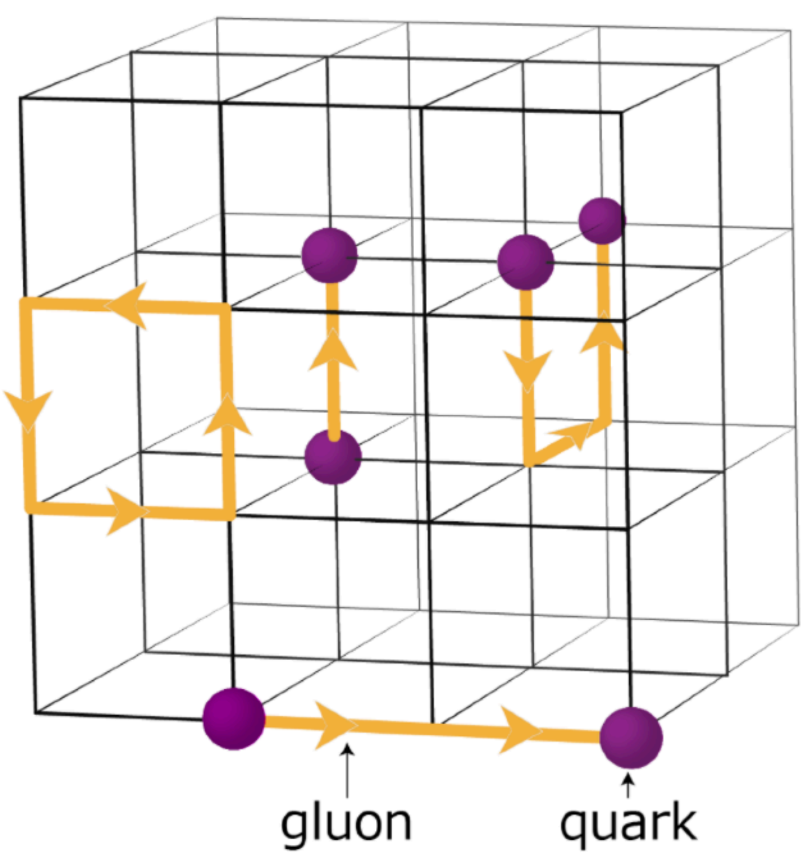
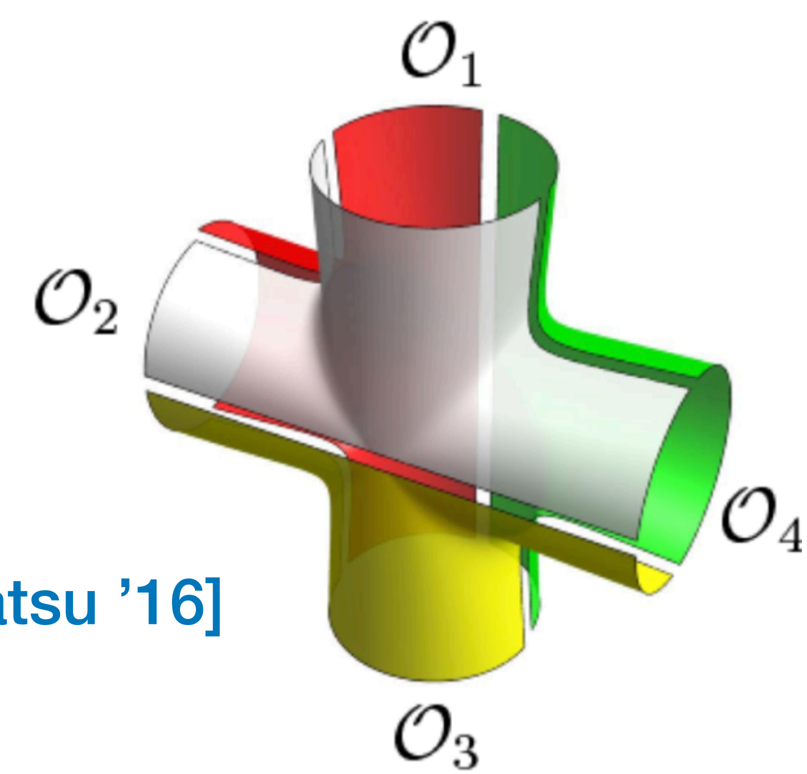
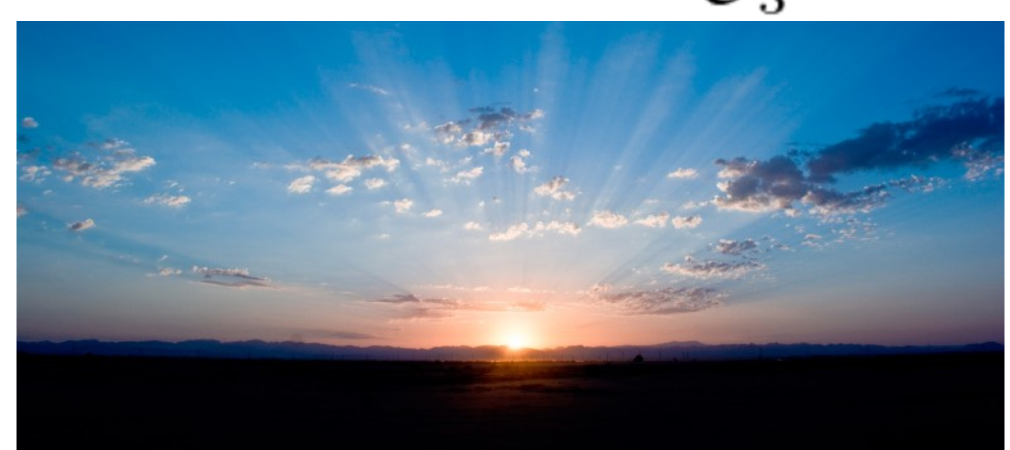


Lattice

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle =$$



[Fleury, Komatsu '16]



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Question: What is the spectrum and coupling of a strongly coupled QFT?

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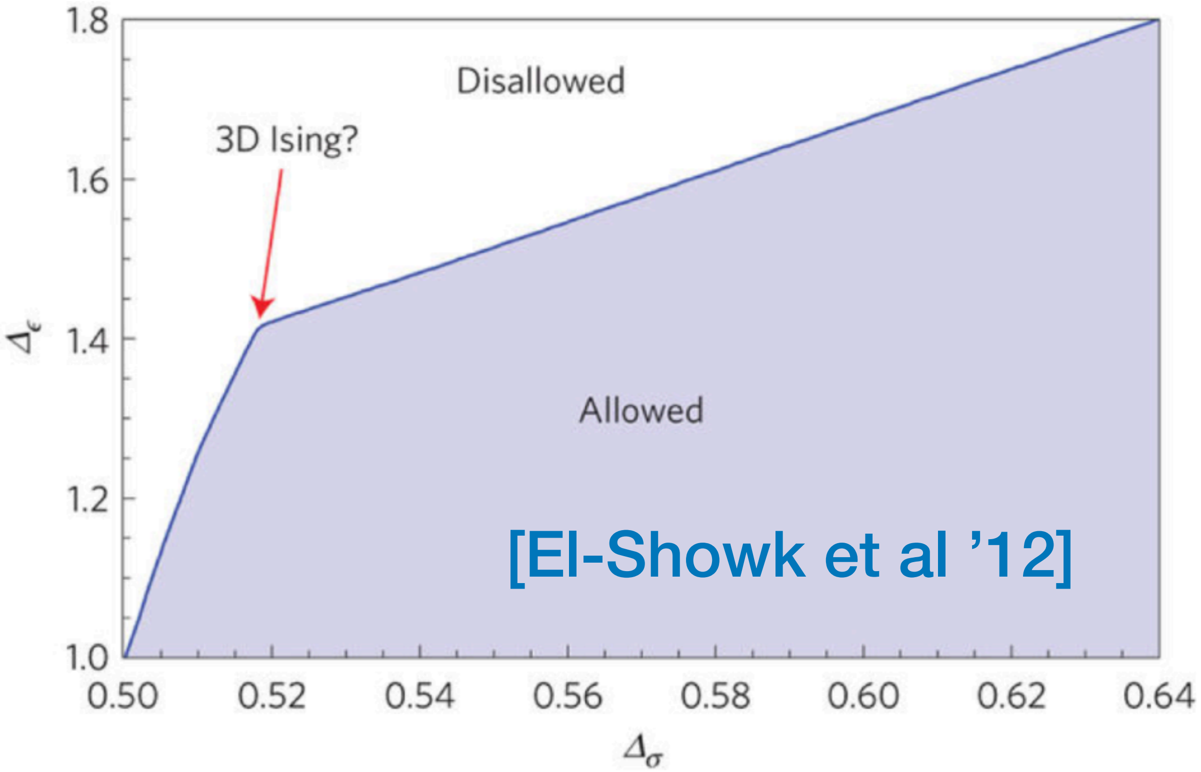
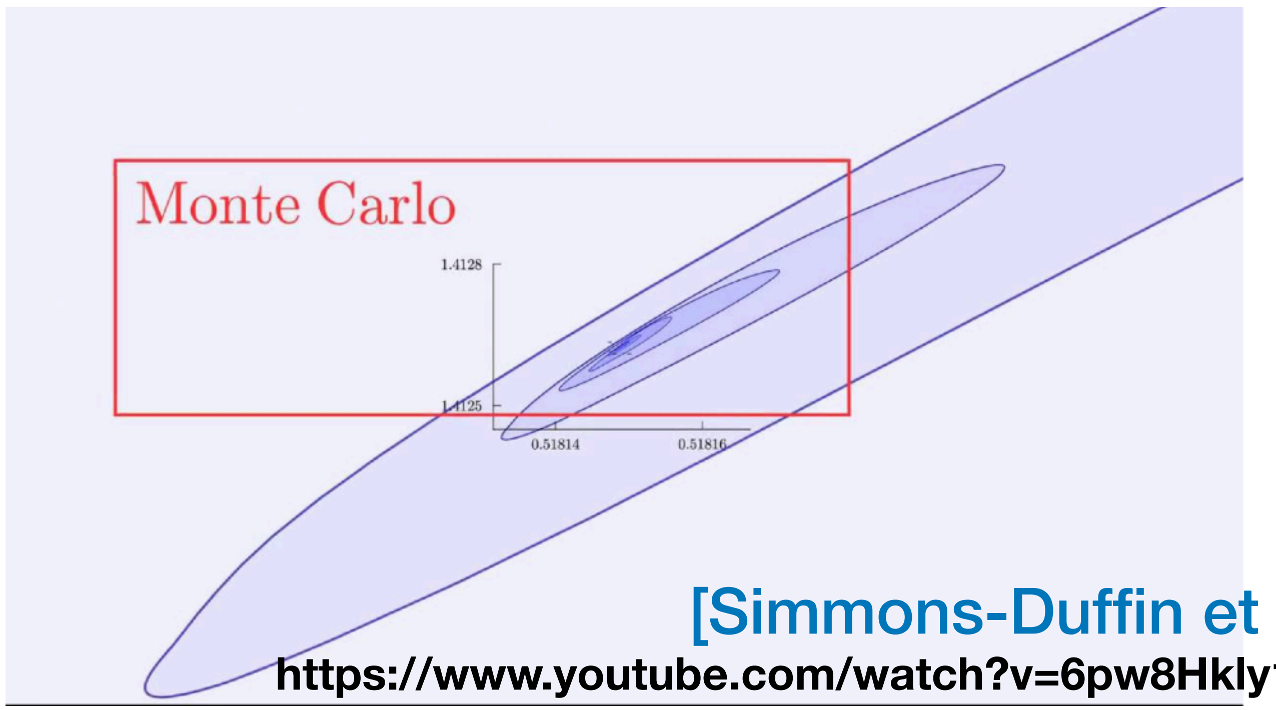
Humble question: Are there bounds on the spectrum and coupling of a QFT?



[El-Showk et al '12]



[Simmons-Duffin et al]
https://www.youtube.com/watch?v=6pw8Hkly1yo

The (New) Bootstrap Manifesto

To constrain and (maybe) determine quantum field theories from first principles

Unitarity

Causality (analyticity)

Crossing symmetry

Old and New Lessons

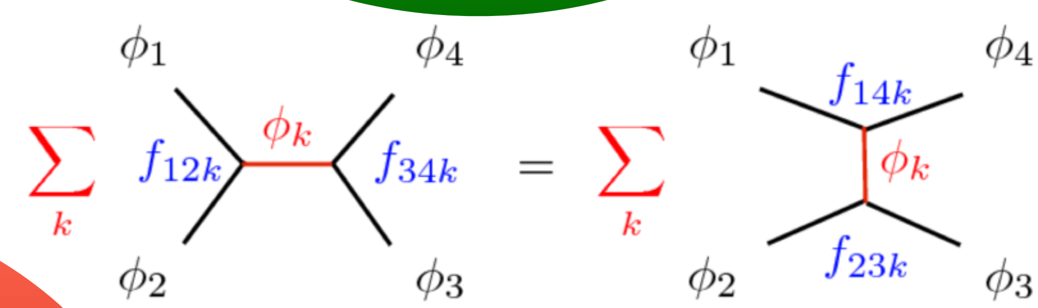
Lehmann, Symanzik, Zimmermann
Froissart, Gribov, Martin, Regge,
Chew, Frautschi,Mandelstam,
Veneziano, Lovelace, Shapiro, Virasoro,…

The analytic S-matrix

Ferrara, Gatto, Grillo, Parisi,
Belavin, Polyakov, Zamolodchikov,
Rattazzi, Rychkov, Vichi, Tonni,…

Conformal bootstrap

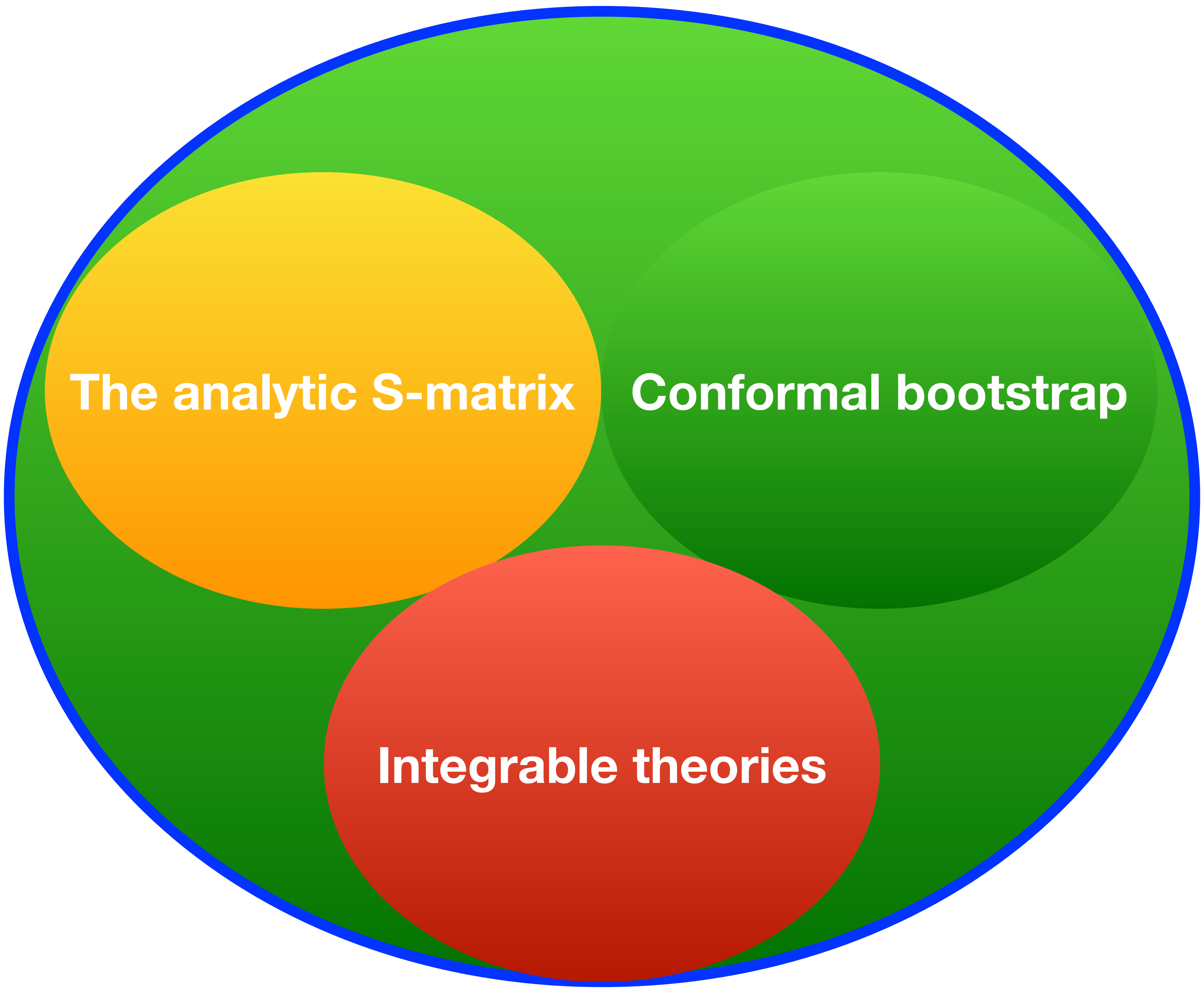
$$A(s,t) = \int_0^1 x^{-\alpha(s)-1}(1-x)^{-\alpha(t)-1}dx = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$$



Integrable theories

Bethe, Yang, Baxter, Faddeev,
Onsager, Zamolodchikov^2,
Vieira, Basso, Komatsu, Fleury,…

Old and New Lessons



Plan of the talk

2 to 2 scattering in 1+1 dimensions

Ex 1: trilinear coupling with a bound state

2 to 2 scattering in 3+1 dimensions

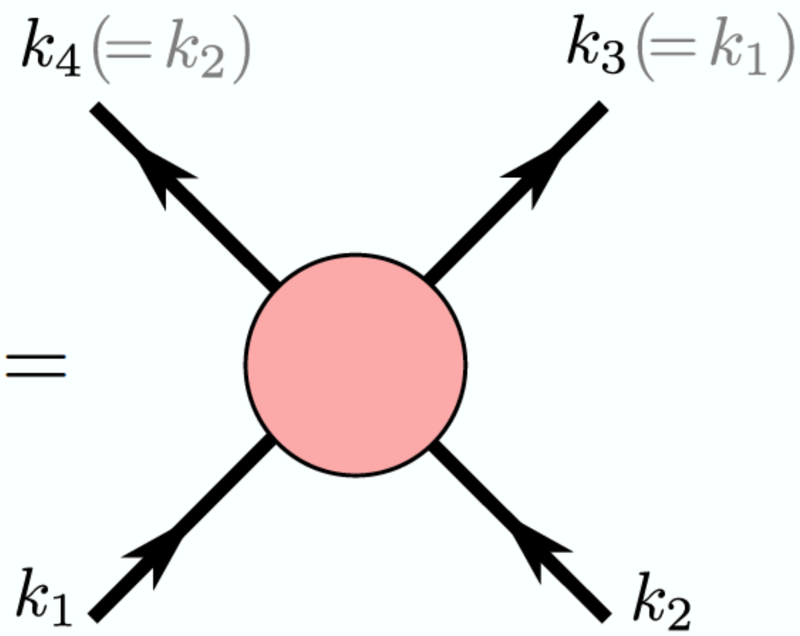
Ex 2: trilinear coupling with a J=0 bound state

Ex 3: quartic coupling with flavor symmetry

Conclusions and Hopes

2 to 2 scattering in 1+1 dimensions

$S(s) =$



$$k_i^2 = m^2$$

$$s \equiv (k_1 + k_2)^2$$

$$t \equiv (k_2 - k_3)^2 \;\; = 4m^2 - s$$

$$u \equiv (k_3 - k_1)^2 = 0$$

2 to 2 scattering in 1+1 dimensions

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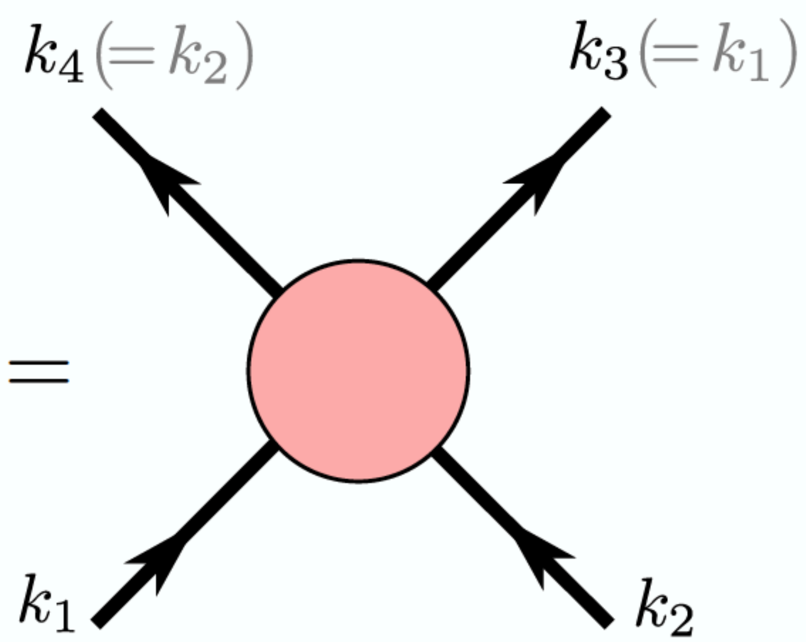

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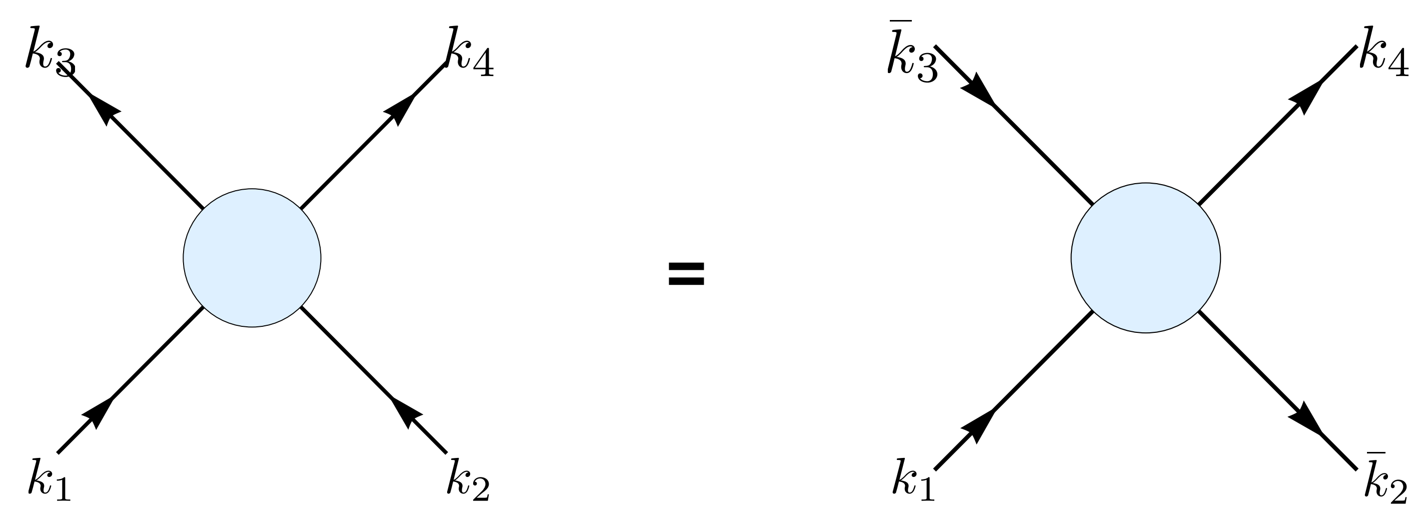
Crossing symmetry

$$S(s) = S(4m^2 - s)$$



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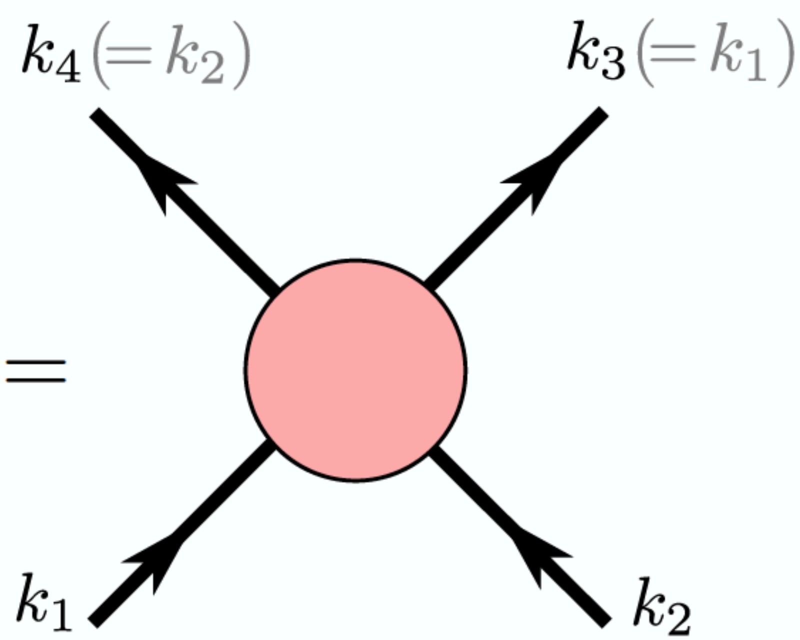

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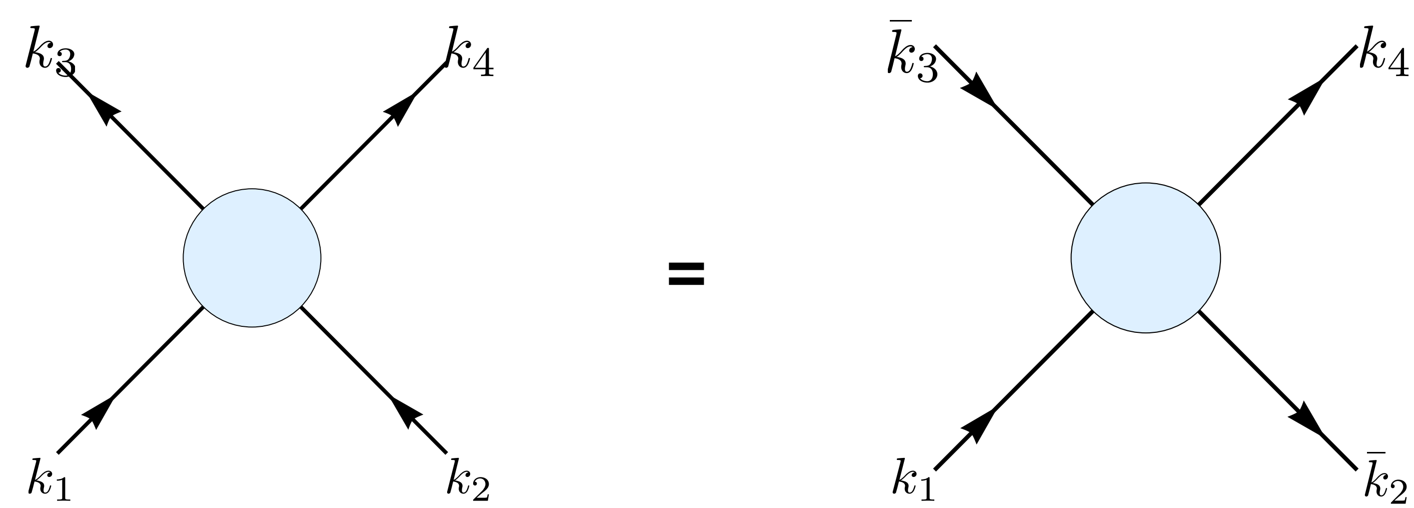
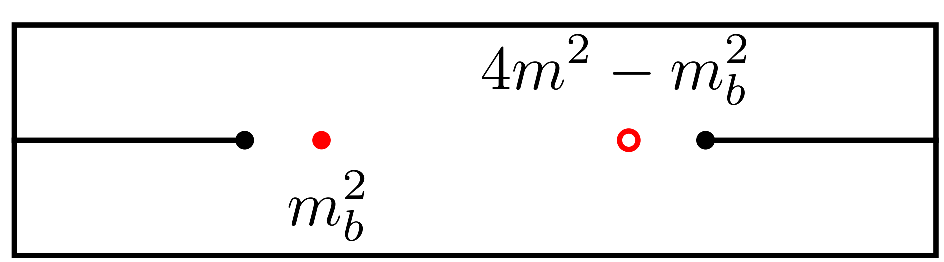
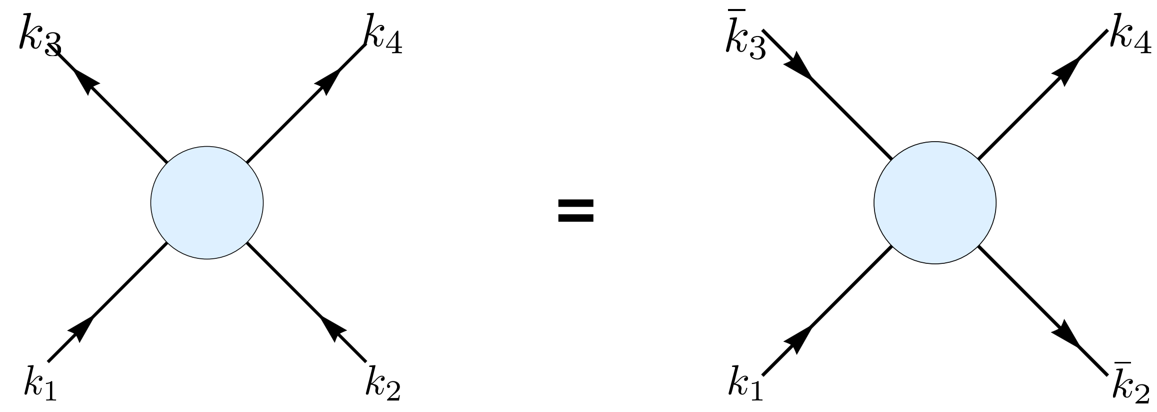


Analytic structure



Constraints and Question

Crossing symmetry



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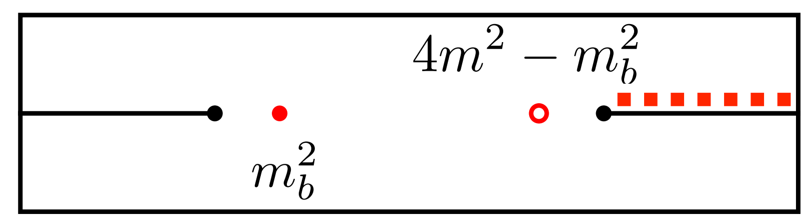

Real analyticity

$$S(s^*) = [S(s)]^*$$

Constraints and Question

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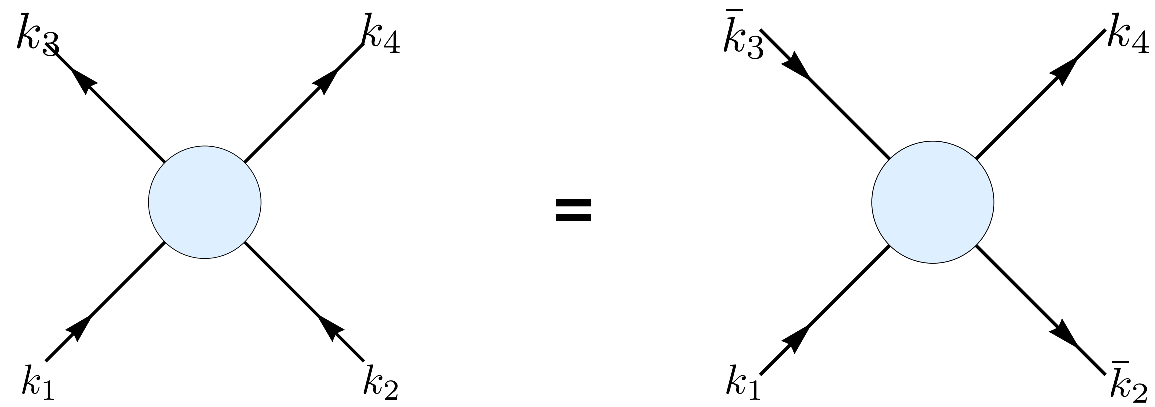

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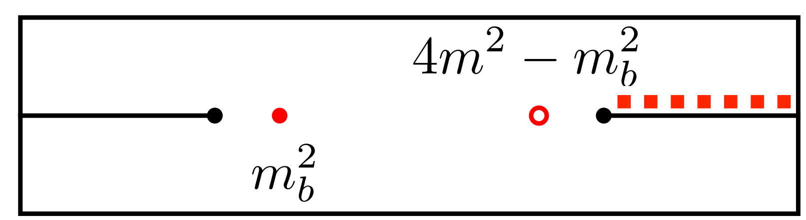

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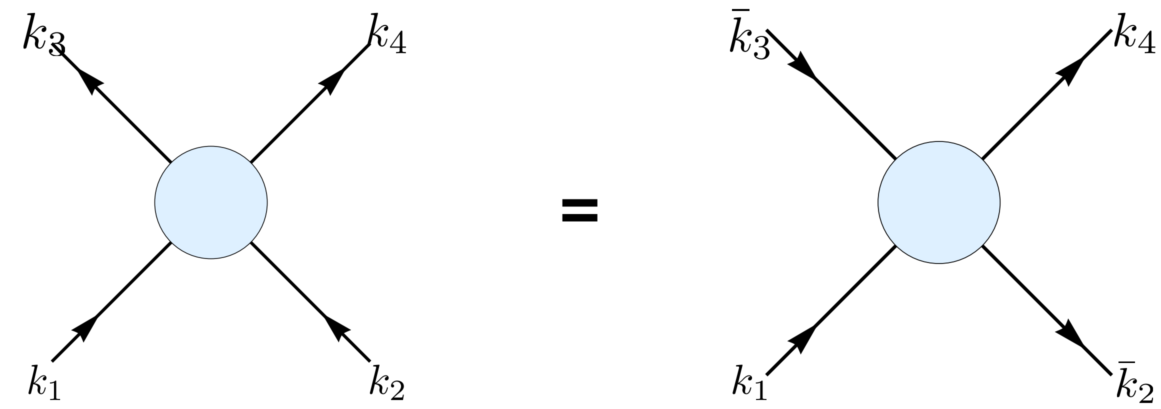

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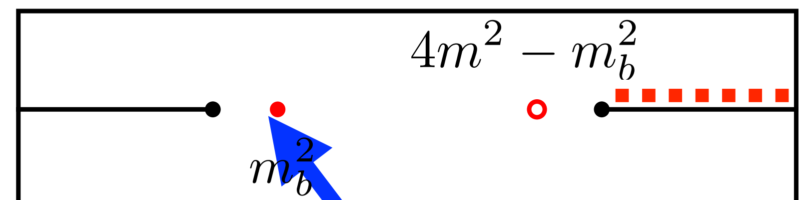

Real analyticity

$$S(s^*) = [S(s)]^*$$

Unitarity

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Question: $\max\ g_b^2$?

$$S(s) \sim \frac{g_b^2}{s - m_b^2}$$

Analytic Solution

[Creutz '72]
[Symanzik '61]

$$S_{opt}(s) = \frac{\sqrt{s(4m^2 - s)} + \sqrt{m_b^2(4m^2 - m_b^2)}}{\sqrt{s(4m^2 - s)} - \sqrt{m_b^2(4m^2 - m_b^2)}} \equiv [m_b](s)$$

CDD factor

[Castillejo, Dalitz, Dyson]

Pole at $s = m_b^2 > 2$

No particle production $|S_{opt}(s)|^2 = 1, \quad s > 4m^2$

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Proof

$$h(s) := \frac{S(s)}{[m_b](s)} \quad \Longrightarrow$$

$h(s)$ *Analytic in the plane minus the cut*

$|h(s)| \leq 1$ *Bounded on the boundary*

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Maximum modulus Principle $\Downarrow$

$$|h(m_b^2)| = \left| \frac{g^2}{\mathrm{Res}_{s=m_b^2}[m_b](s)} \right| \leq 1$$

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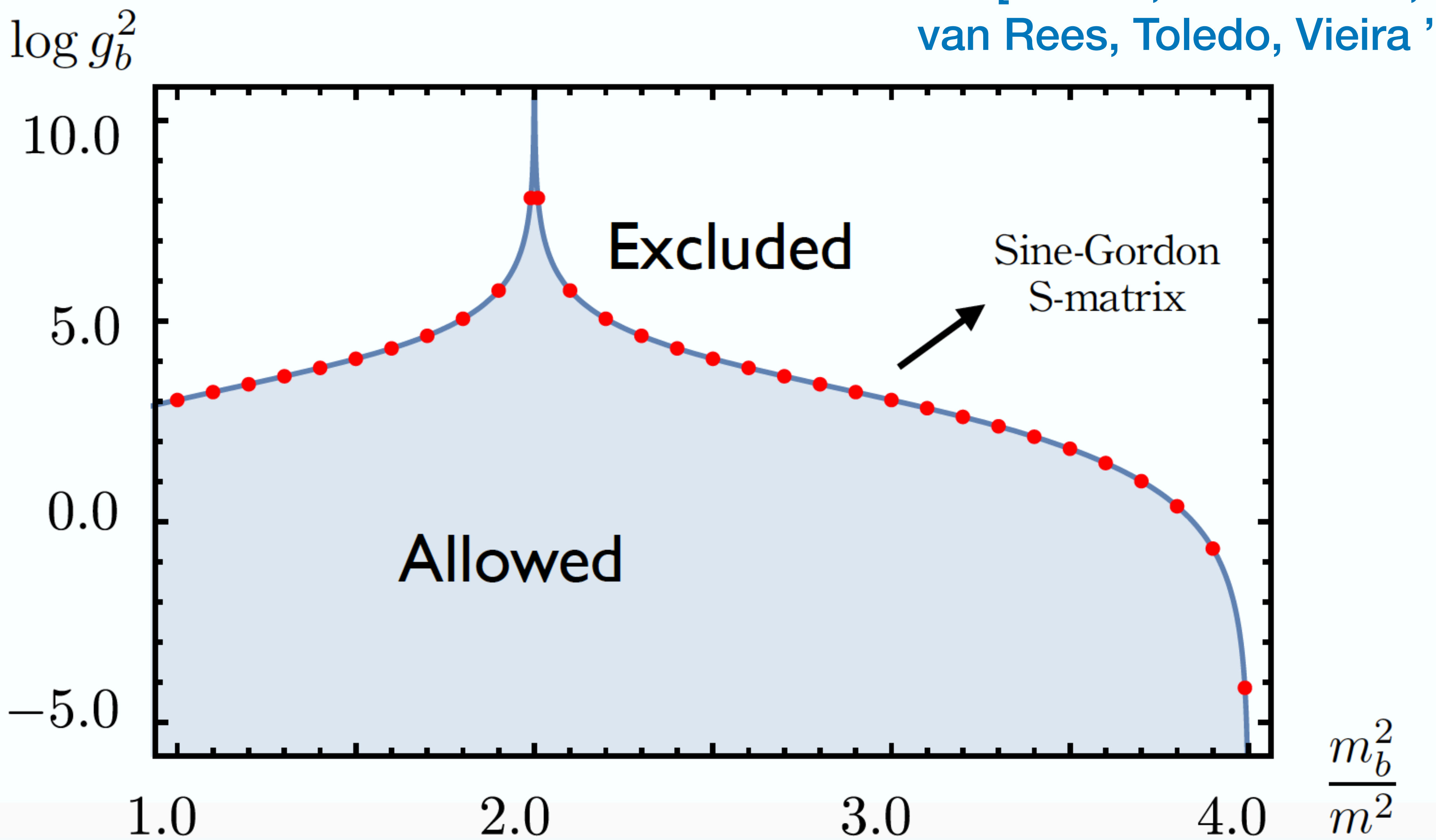
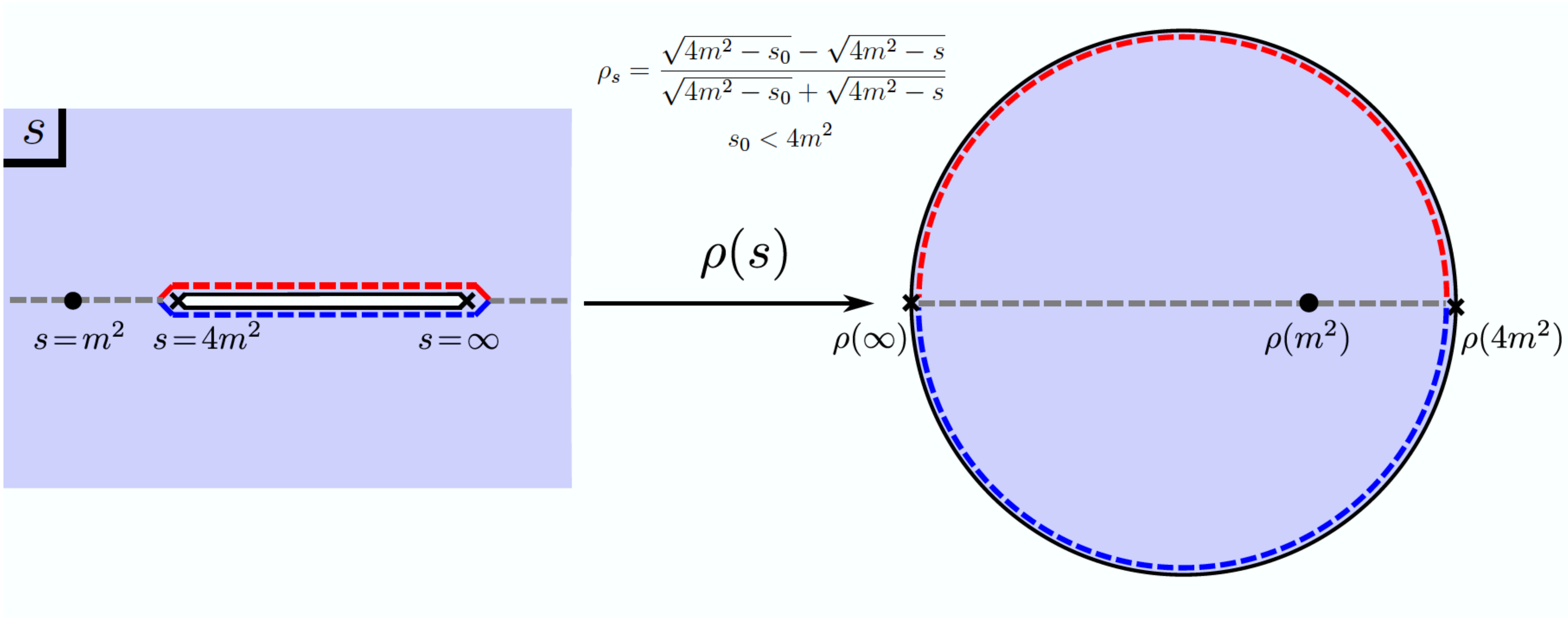
[Paulos, Penedones, van Rees, Toledo, Vieira '16]



Numerical approach



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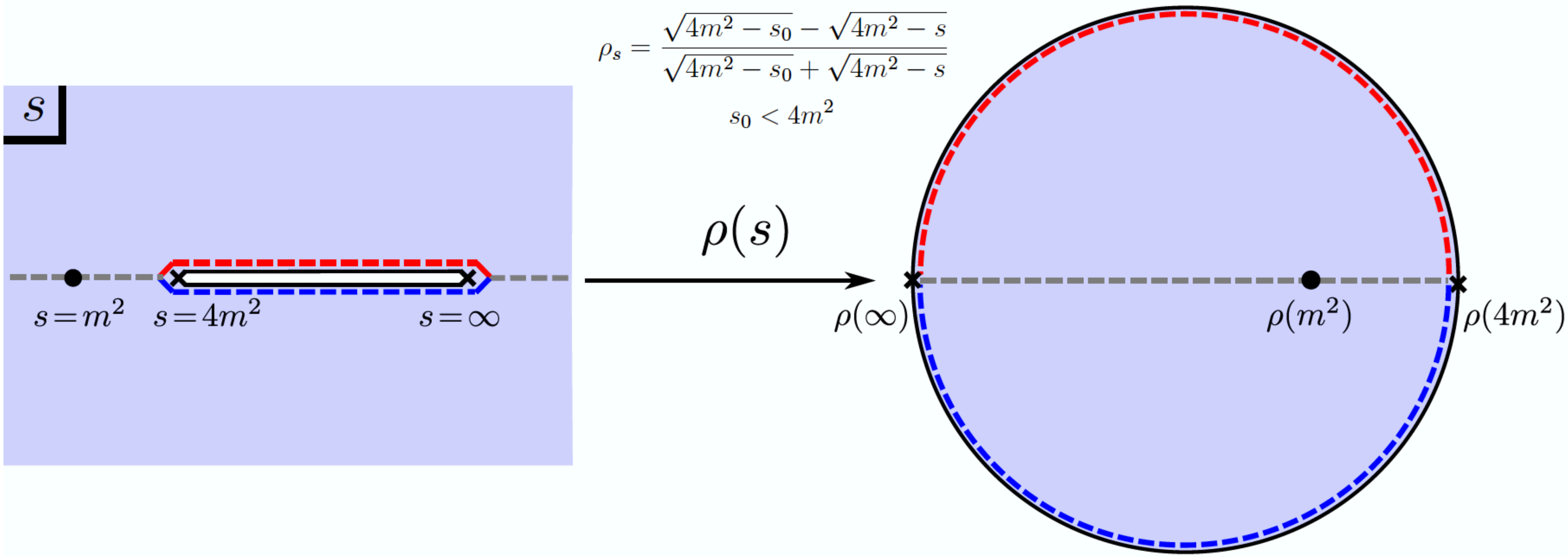

Ansatz $$S(s,t) = \frac{g_b^2}{s-m_b^2} + \frac{g_b^2}{t-m_b^2} + \sum_{a,b=0} c_{(a,b)} \rho_s^a \rho_t^b$$

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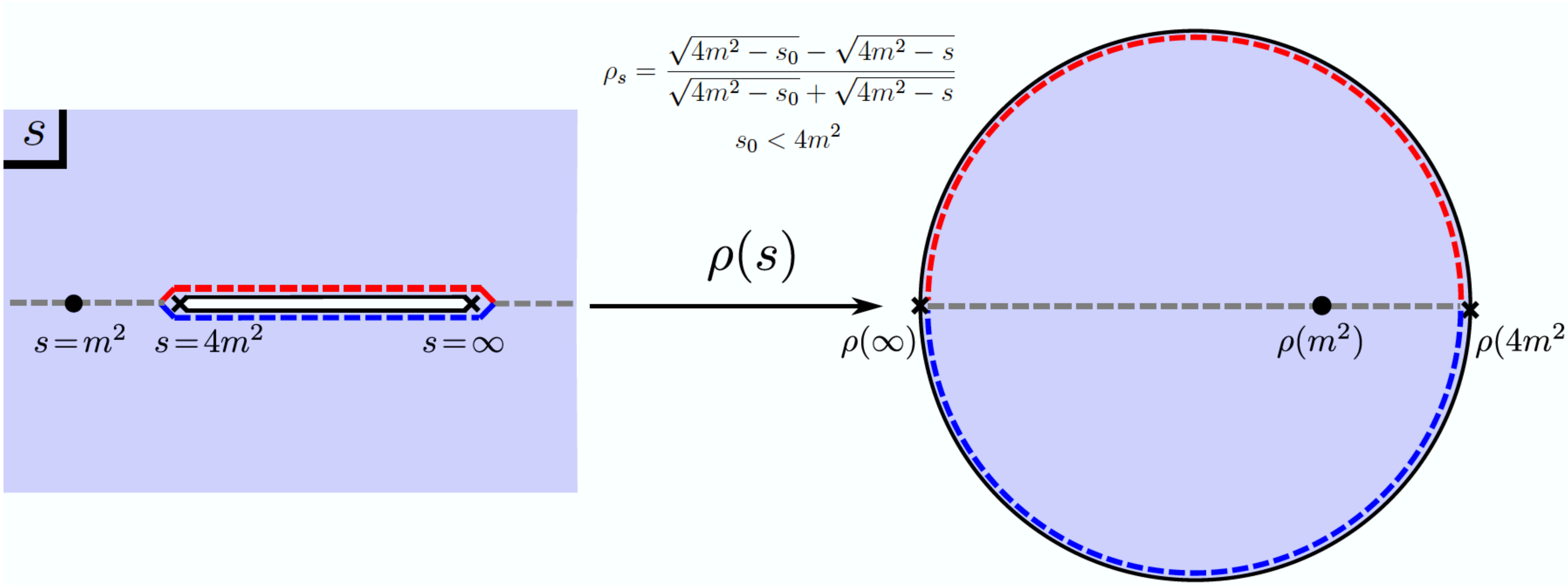

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Analyticity and Crossing automatic

Unitarity gives quadratic constraints $$|S(s,4m^2-s)|^2 \le 1, \quad s > 4m^2$$

Numerical approach

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Numerics: Truncate to a finite number of variables and quadratic constraints

$$a + b \leq N_{max}, \quad \{g_b^2, c_{(a,b)}\} \quad \text{at } s = s_1, s_2, \ldots s_M$$

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[Simmons-Duffin '15]

Use Semidefinite Programming (SDPB) to maximize g_b^2 subject to the unitarity constraints. The analytic solution is obtained for $N_{max} \to \infty$

2 to 2 scattering in d+1 dimensions

$$\langle p_3, p_4 | S | p_1, p_2 \rangle = \mathbb{I} + i(2\pi)^{d+1} \delta^{(d+1)}(p_1 + p_2 - p_3 - p_4) M(s, t, u)$$

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Partial waves

$$S_\ell(s) = 1 + i\frac{(s-4m^2)^{\frac{d-2}{2}}}{\sqrt{s}}\int_{-1}^{1} dx(1-x^2)^{\frac{d-3}{2}} P_\ell^{(d)}(x)M(s,x)$$

$$t = \frac{1}{2}(x-1)(s-4)$$

$$u = -\frac{1}{2}(x+1)(s-4)$$

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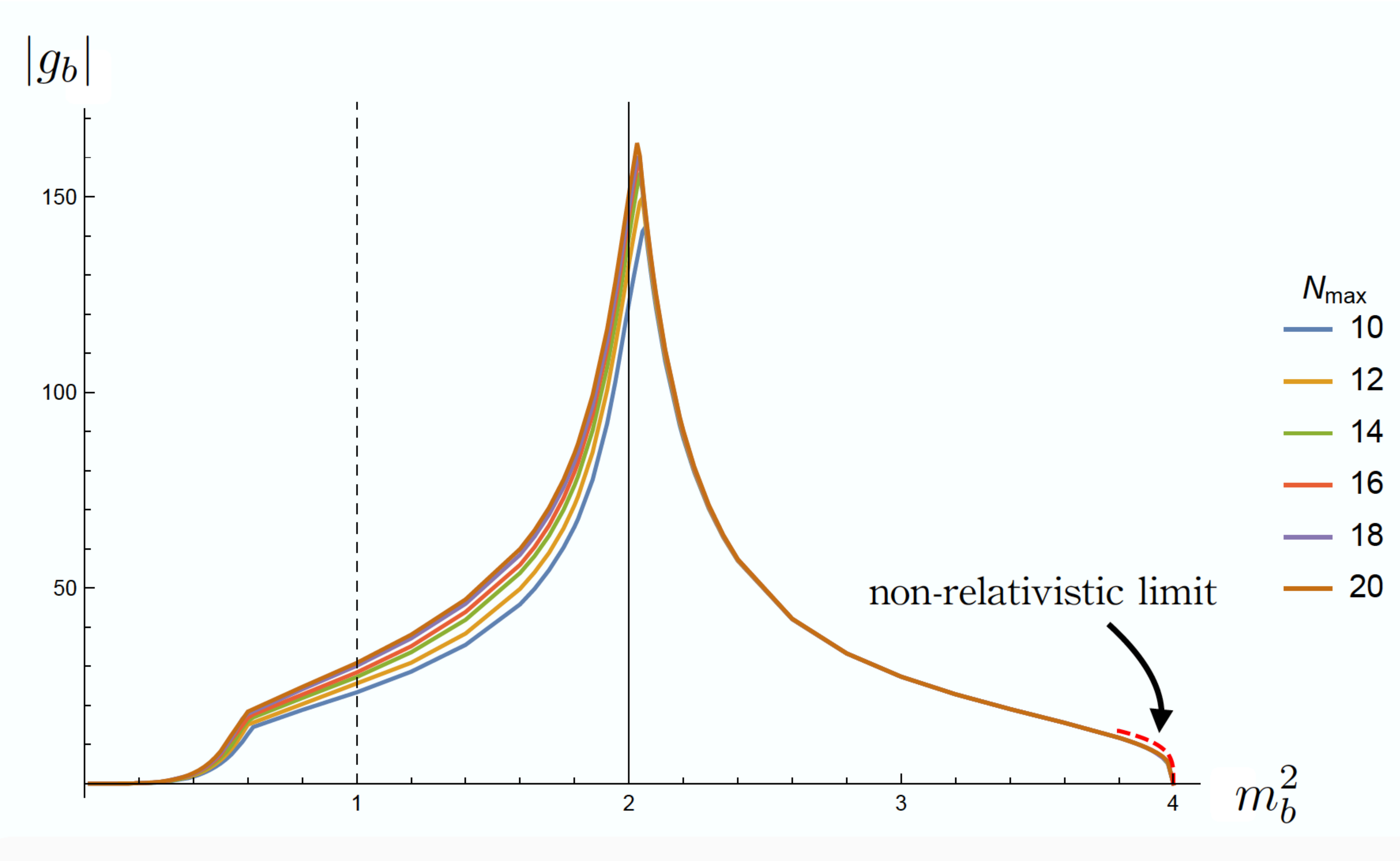
Unitarity

$$|S_\ell(s)|^2 \leq 1, \quad s > 4m^2, \ell = 0, 2, 4, \ldots, \ell_{max}$$

Quadratic Constraints on the variables $\{g_b^2, \alpha_{(a,b,c)}\}, \quad a + b + c \leq N_{max}$

Maximal cubic coupling in 3+1 dimensions



[Paulos, Penedones,
van Rees, Toledo, Vieira '16]

Adding some flavor

Scattering of particles with flavor symmetry SO(N) (ISOSPIN)

$$M(s,t,u) = \delta^{ab}\delta^{cd}A(s|t,u) + \delta^{ac}\delta^{bd}A(t|u,s) + \delta^{ad}\delta^{bc}A(u|s,t)$$

Mandelstam double dispersion relations: $A(s|t,u) = A_1(s,t) + A_1(s,u) + A_2(t,u)$

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Mandelstam double dispersion relations: $A(s|t,u) = A_1(s,t) + A_1(s,u) + A_2(t,u)$

$$A(s|t,u) = \alpha_0\left(\frac{1}{\rho_t - 1} + \frac{1}{\rho_u - 1}\right) + \frac{\beta_0}{\rho_s - 1} + \gamma_0 + \sum_{\{a,b\}\neq\{0,0\}} \alpha_{0,b,c}\, \rho_{t,u}^{(b,c)} + \sum_{\{a,b,c\}\neq\{0,0,0\}} \beta_{a,b,c}\rho_s^a \rho_{t,u}^{(b,c)}$$

Improving Convergence

Automatic Crossing and Analyticity, but Unitarity?

Adding some flavor

$$A(s|t,u) = \alpha_0 \left(\frac{1}{\rho_t - 1} + \frac{1}{\rho_u - 1} \right) + \frac{\beta_0}{\rho_s - 1} + \gamma_0 + \sum_{\{a,b\} \neq \{0,0\}} \alpha_{0,b,c}\, \rho_{t,u}^{(b,c)} + \sum_{\{a,b,c\} \neq \{0,0,0\}} \beta_{a,b,c} \rho_s^a \rho_{t,u}^{(b,c)}$$

Automatic Crossing and Analyticity, but Unitarity?

$$T^0(s,t,u) = N_f A(s|t,u) + A(t|u,s) + A(u|s,t)$$

$$T^1(s,t,u) = A(t|u,s) - A(u|s,t)$$

$$T^2(s,t,u) = A(t|u,s) + A(u|s,t)$$

Semidefinite programming OK!

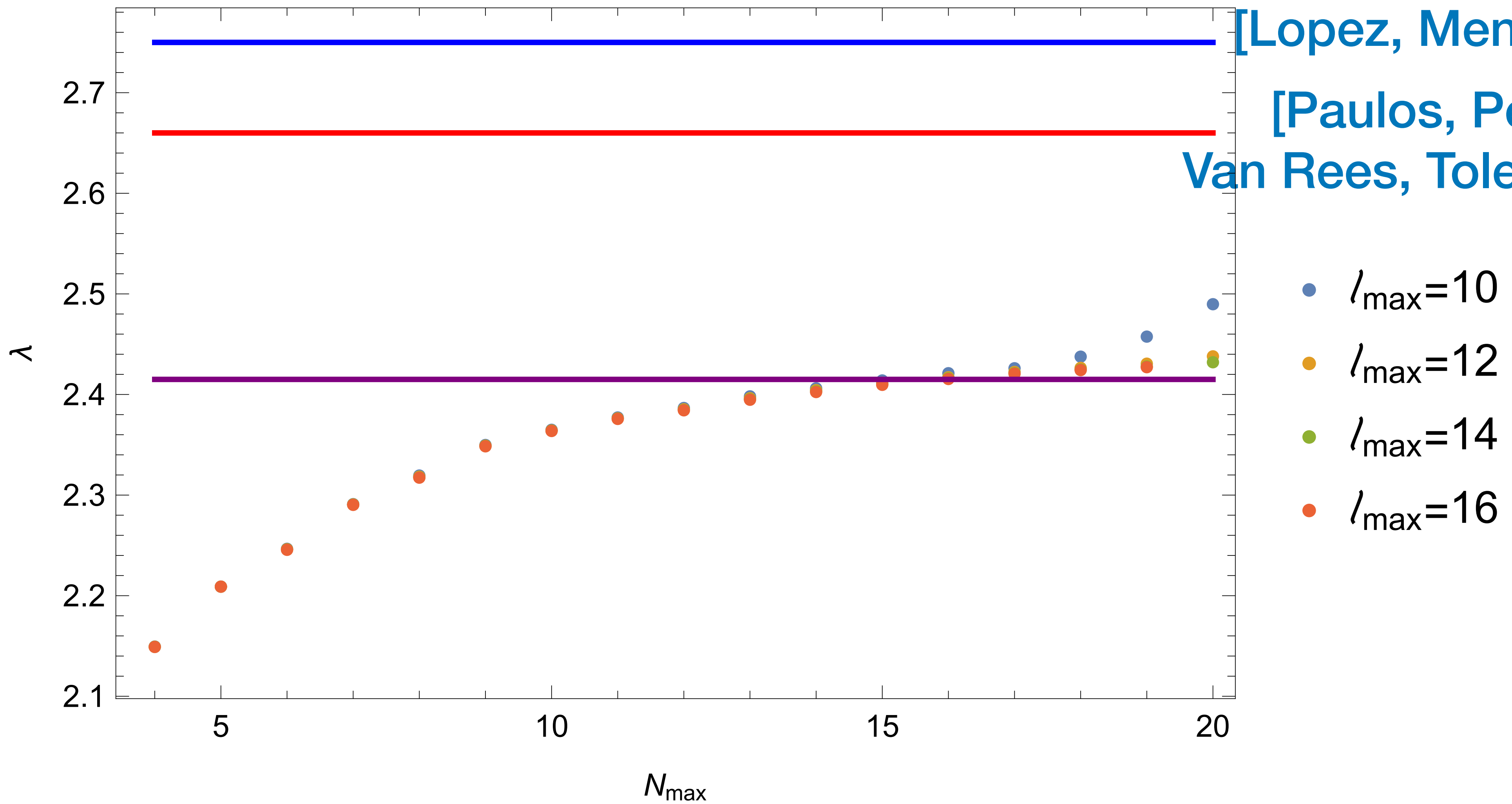
$$\boxed{|T_\ell^I(s)|^2 \leq 1, \quad s > 4m^2}$$

Roy Equations approach inverted: diagonal in the unitarity channels, complicated crossing pattern.

Maximal Quartic Coupling

Question: $\max \lambda, \quad \lambda = \frac{1}{32\pi} M^{11,11}\left(s = t = u = \frac{4}{3}m^2\right)$

$SO(3)$ symmetry



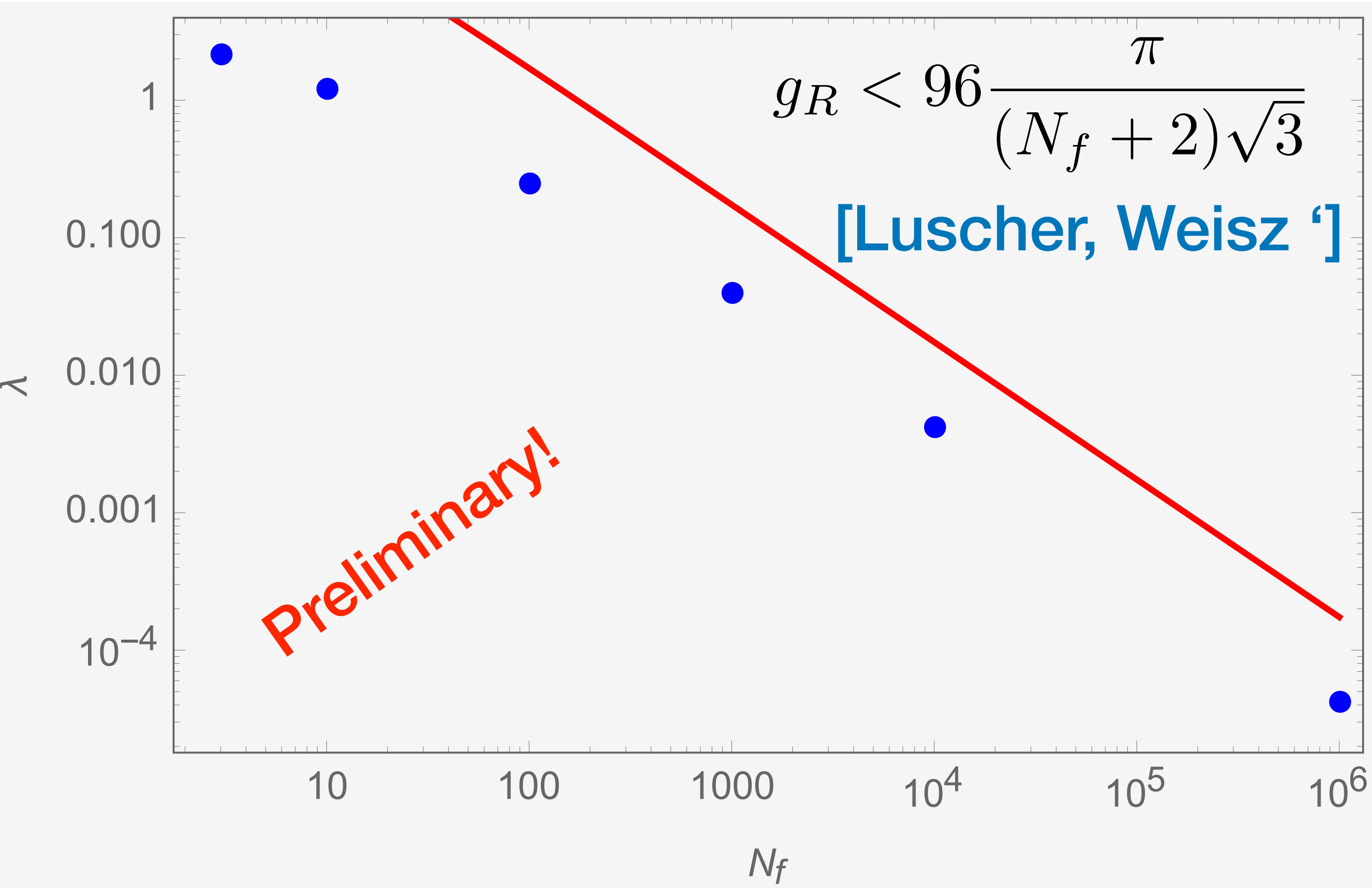
[Lopez, Mennessier '77]

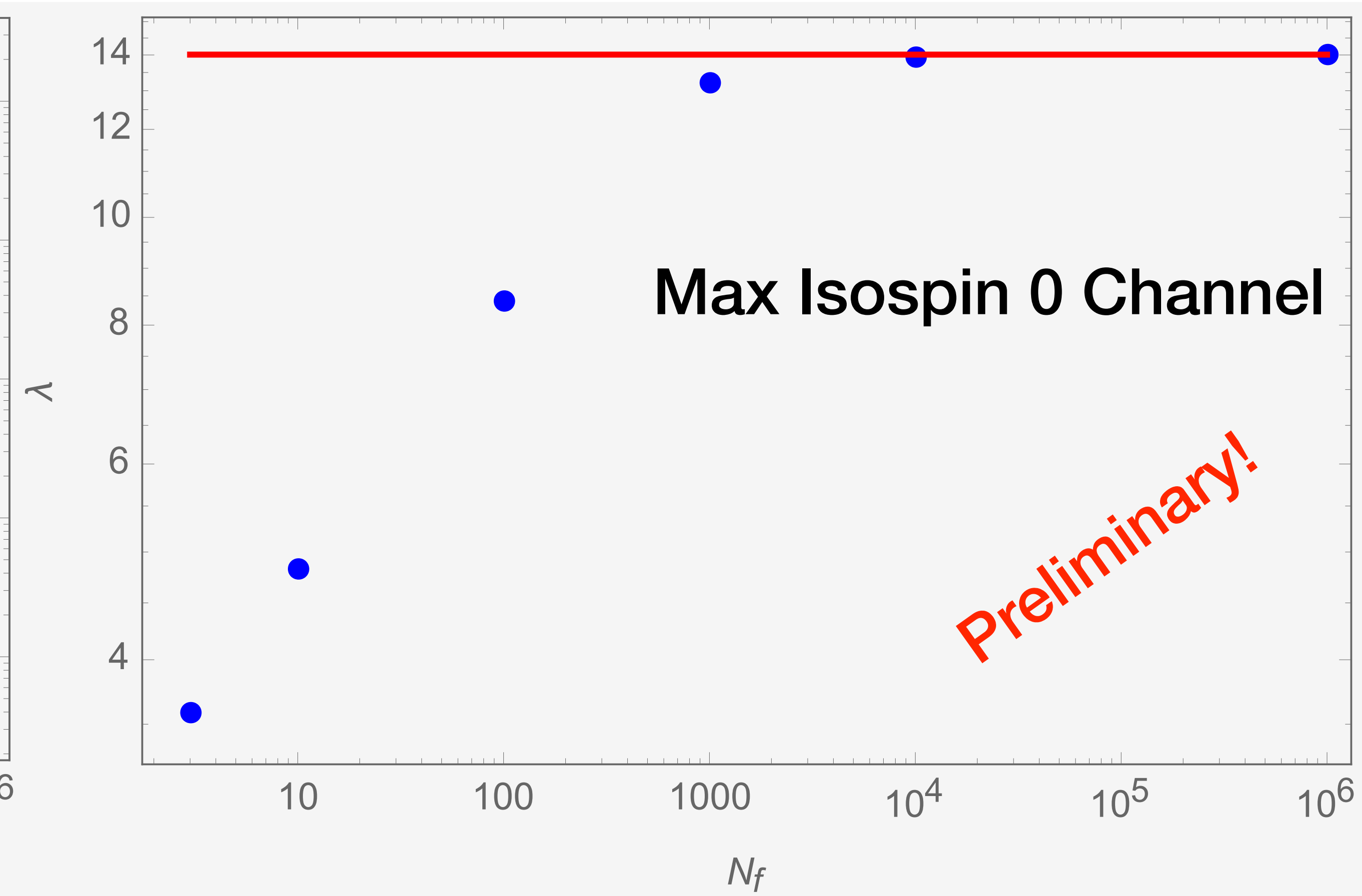
[Paulos, Penedones, Van Rees, Toledo, Vieira '17]

[ALG, Penedones, Toledo, Vieira]

Large Nf limit

Found expected suppression with 1/Nf.





The isospin 0 channel ~ O(1), only coupling surviving for large Nf.

[ALG, Penedones, Toledo, Vieira]

Conclusions and Hopes

Lower bounds, Froissart-like bounds, phase shifts, scattering lengths, coupled channels, resonances, spinning bound states, spinning amplitudes, Regge behavior, multiparticle amplitudes, non-lightest particle scattering, etc…

Chiral Symmetry breaking, zeroes and pions

[ALG, Penedones, Toledo, Vieira]

Resonance prediction, feasible

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Bounds on New Physics couplings

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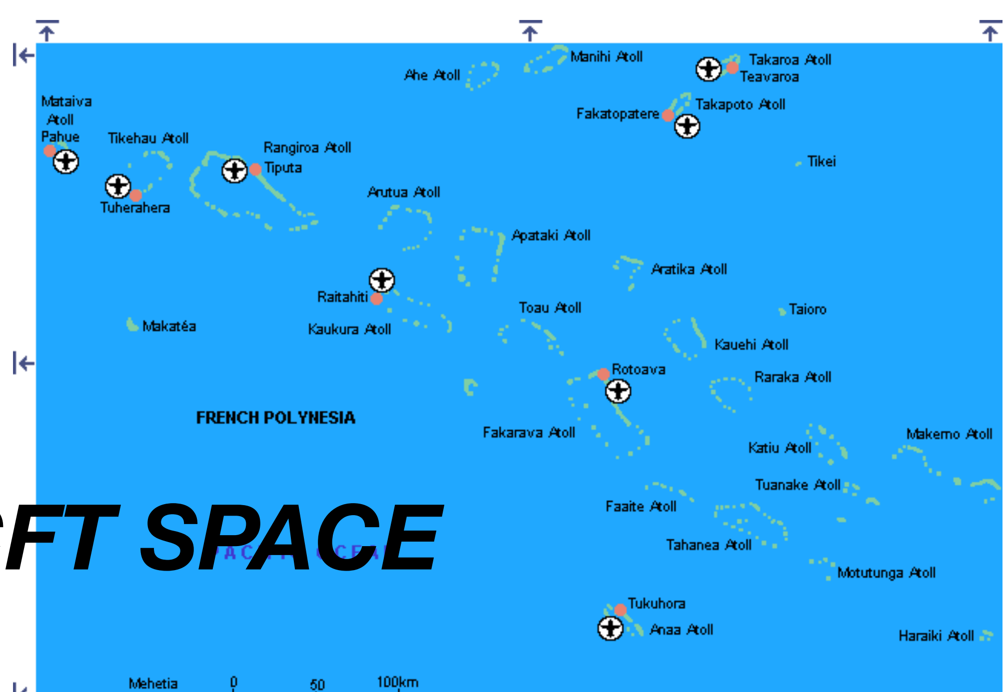
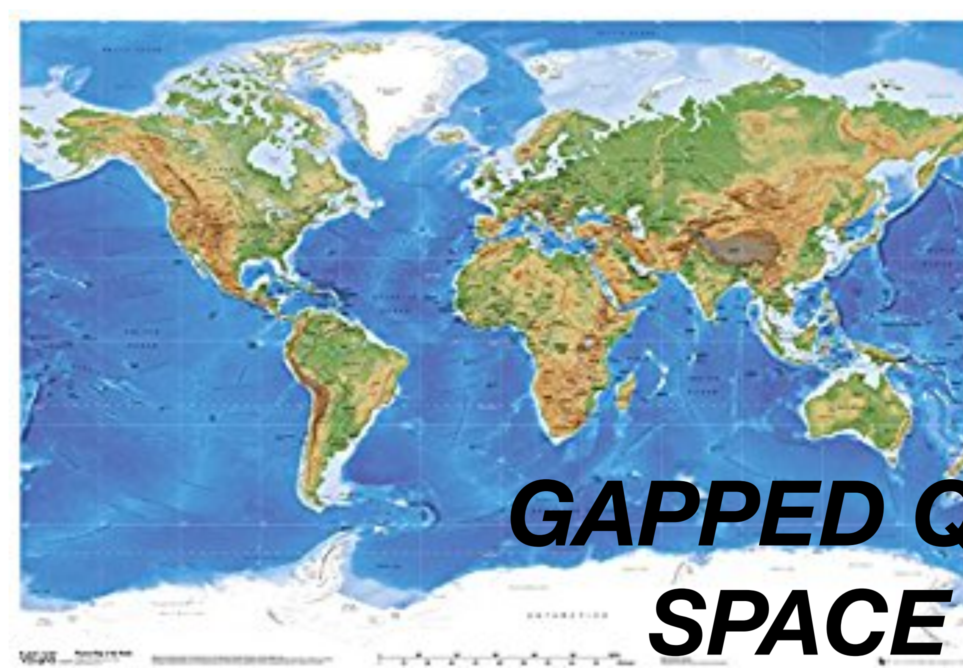
[ALG, Penedones, Toledo, Vieira]

Bounds on New Physics couplings



CFT SPACE



GAPPED QFT SPACE

Chiral zeroes from PT

The Physics of (quasi)-Goldstones is captured by the chiral effective Lagrangian.

$$L_2 = \frac{1}{4}\mathrm{tr}(\partial_\mu\phi\partial^\mu\phi) - \frac{1}{2}Bm\,\mathrm{tr}(\phi^2) + \frac{1}{24F^2}(\mathrm{tr}([\phi,\partial_\mu\phi]\phi\partial^\mu\phi) + B\mathrm{tr}(M\phi^4)) + \mathcal{O}(F^{-4})$$

$$T_0 \equiv T_0^{(0)} = \frac{2s - m_\pi^2}{16\pi F^2} \longrightarrow s_0 = \frac{m_\pi^2}{2}$$

$$T_1 \equiv T_1^{(1)} = \frac{s - 4m_\pi^2}{48\pi F^2} \longrightarrow s_1 = 4m_\pi^2$$

$$T_2 \equiv T_2^{(0)} = \frac{-s + 2m_\pi^2}{16\pi F^2} \longrightarrow s_2 = 2m_\pi^2$$