Numerical studies on hydrogen-filled capillary discharges with focus on active plasma lens applications

Emanuele Brentegani

Supervisors: Enrica Chiadroni, Stefano Atzeni

SAPIENZA
UNIVERSITÀ DI ROMA

INFN
Istituto Nazionale di Fisica Nucleare
LABORATORI NAZIONALI DI FRASCATI

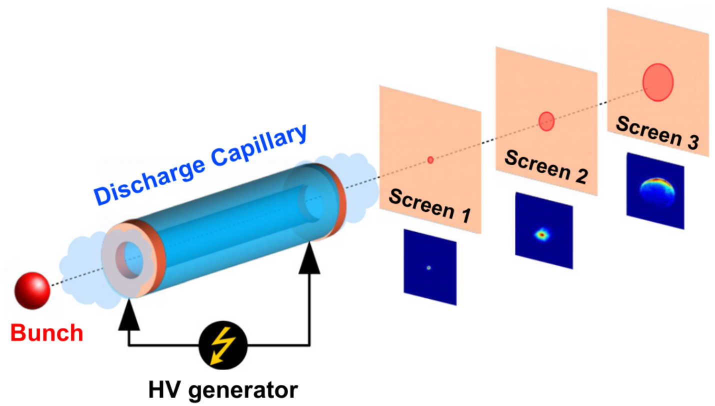
SPARC LAB

Contents

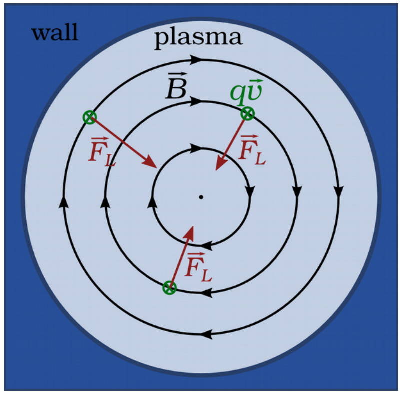
1. Introduction on active plasma lenses: working principle

What is an active plasma lens*?

It is a device which can focus an electron beam, thanks to an azimuthal magnetic field generated by a discharge current induced in a gas-filled capillary



Scheme of principle of an active plasma lens [*courtesy of R. Pompili]*



Schematic transverse view of an active plasma lens

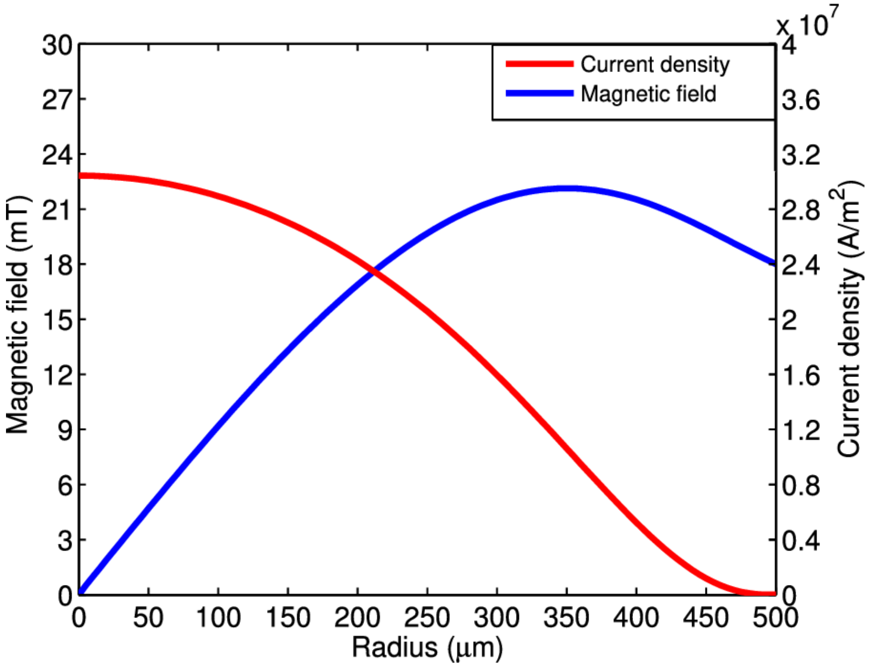
- Interesting features:
  - Magnetic field gradients even higher than those achievable in permanent magnet quadrupoles, up to several kT/m
  - Compact and capable of focusing a beam in both transverse planes
  - The focusing strength, K, scales as 1/γ

*W.K.H. Panofsky and W.R. Baker, Rev. Sci. Instr. 21, 445 (1950)

1. Introduction on active plasma lenses: aberrations

However, transverse **non-linearity of the azimuthal magnetic field** produces **aberrations** which cause an increase of both emittance and minimum achievable spot size.

- In a discharge the current density distribution is mainly dependent on the plasma electrical resistivity
- The plasma resistivity mainly depends on its temperature: $\eta_{\mathrm{plasma}} \propto T^{-3/2}$
- The plasma tends to be warmer in the middle of the channel, due to the cooling effect of the capillary walls
- Current density concentrates near the axis
- Non satisfactory magnetic field profiles

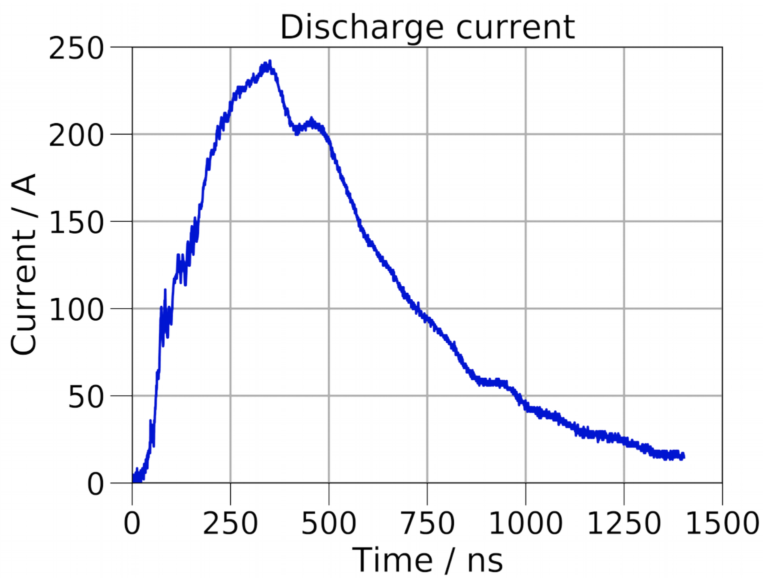


Example of expected transverse distribution of current density and magnetic field in a capillary discharge (with a current of 45A).
[reprinted from Appl. Phys. Lett. 110, 104101 (2017)]

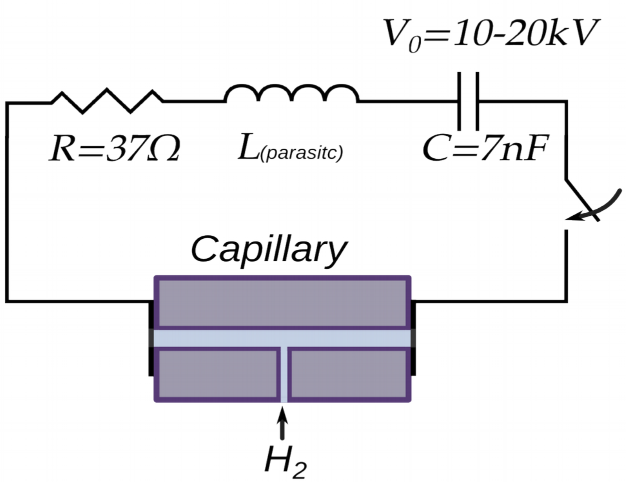
2. Real device

- Made of printed plastic or sapphire (with external support of printed plastic)
- Filled with hydrogen (≈10^{-7}g/cm^3)
- Typical dimensions:
  - Diameter of the aperture: 1-2 mm
  - Length: 1-3 cm

Capillary, made of printed plastic



Example of time profile of the current during a discharge

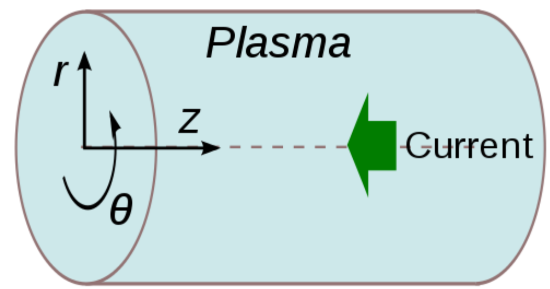


Scheme of the electric circuit powering the gas discharge.

3. Simulations: 1D case

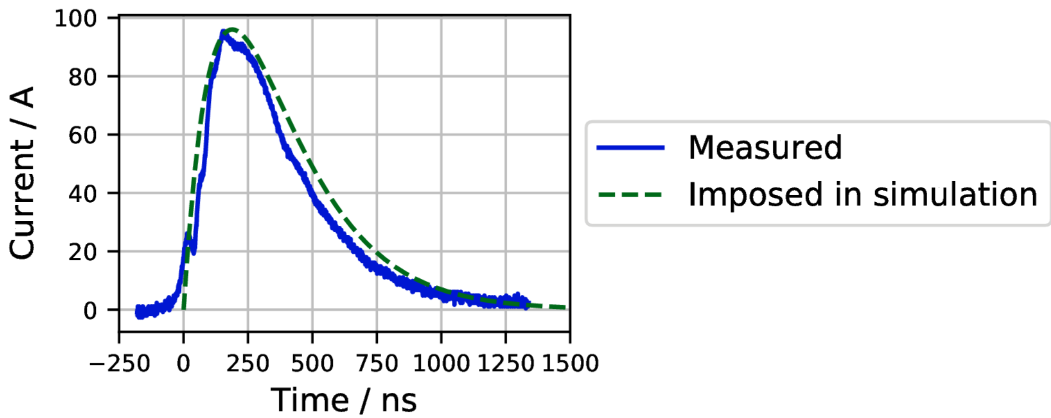
Some 1D simulations for a case of our interest have been performed by N. A. Bobrova with a dissipative MHD code *:

- Time evolution of the plasma in 1D axis-symmetric approximation, **quantities evolve only in radial direction and in time**



$$\frac{\partial}{\partial \theta} \equiv 0, \quad \frac{\partial}{\partial z} \equiv 0$$

- **The time profile of the current is imposed,** in order to emulate the measured current

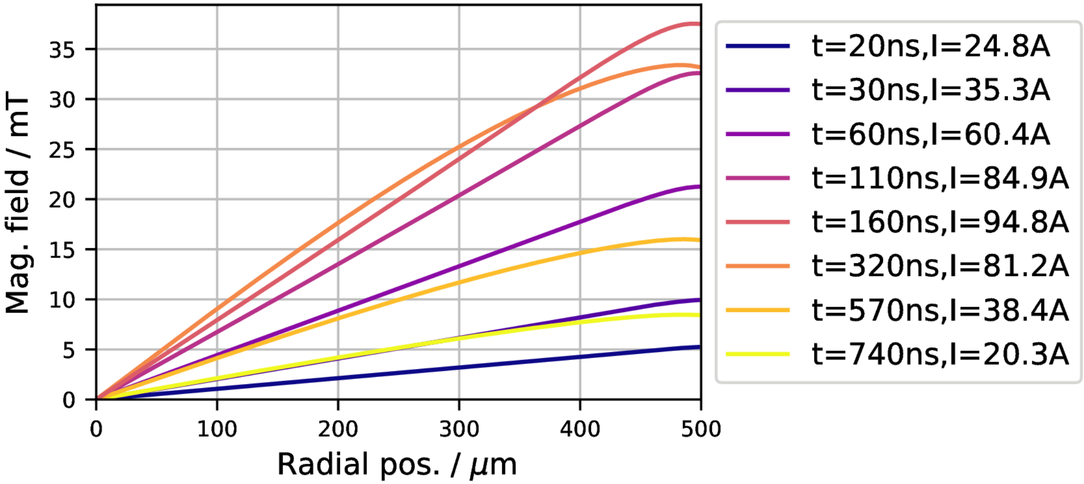


- A capillary of radius 0.5mm was simulated, with an initial gas density of $6.4 \cdot 10^{16}$ H_2 particles per cm^{-3}

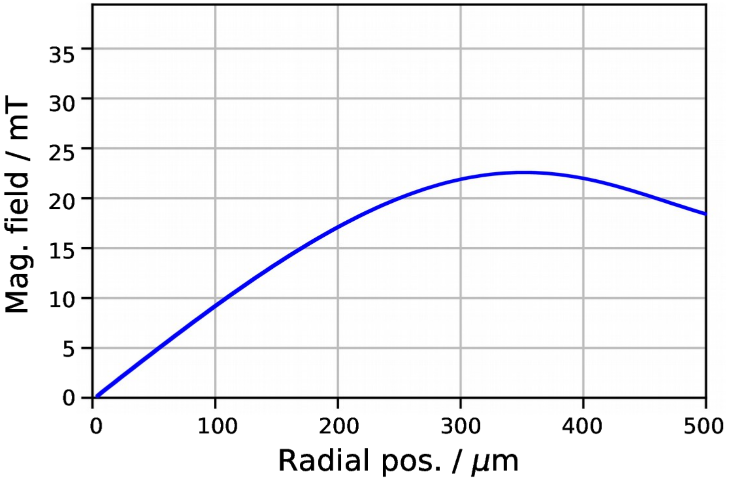
*N. Bobrova et al., Phys. Rev. E65, 016407 (2001)

3. Simulations: 1D case

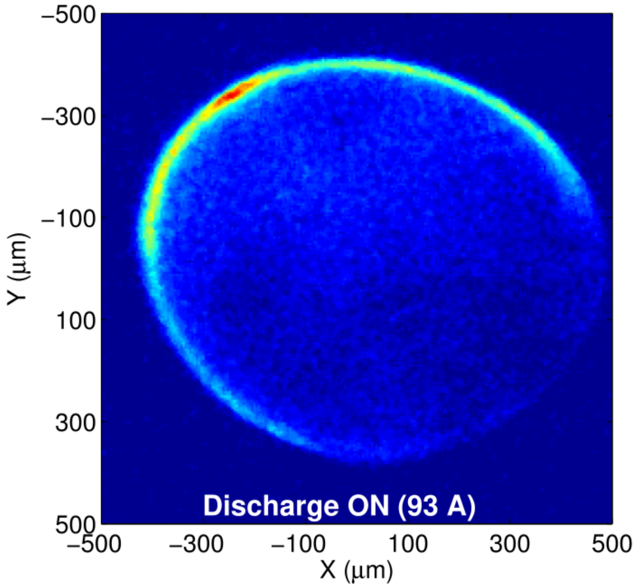
- The time evolution of the magnetic field has been computed, and compared to what expected and measured



Computed magnetic field at different times in 1D simulation.



Magnetic field expected after 450ns from the start of the discharge from analysis of Sparc electron beam*.



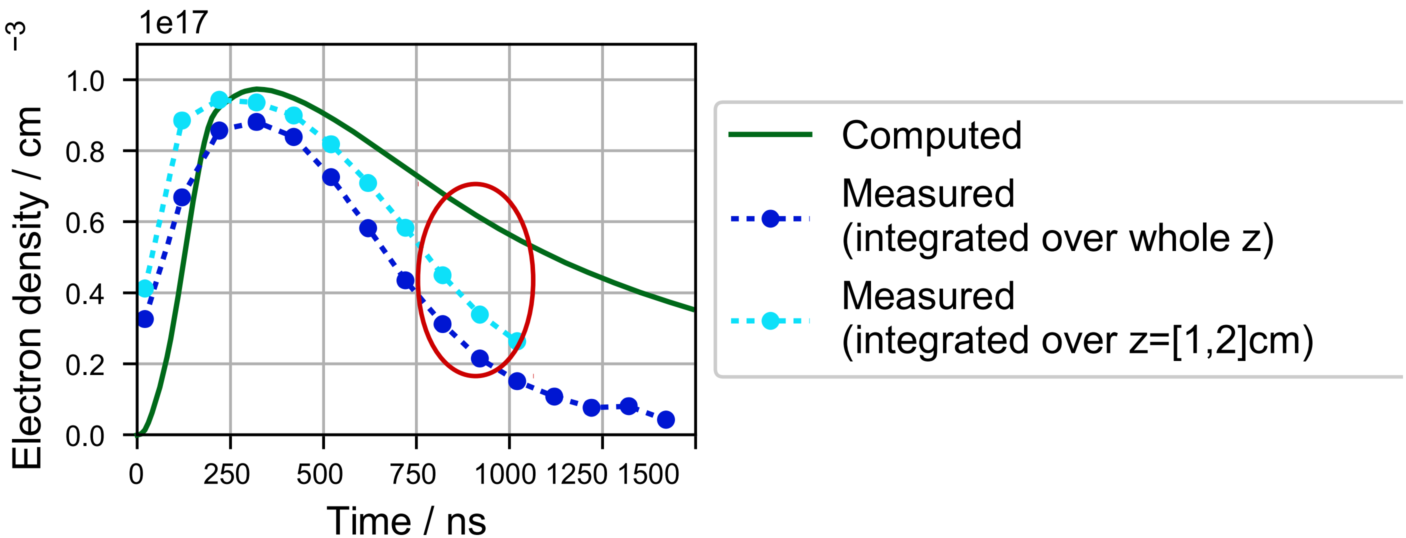
Beam spot 20cm downstream the active lens, operating at 93A*.

- It seems that the focusing properties of the active lens do not match what foreseen by the simulations, field quality is too good

*R. Pompili et al., Appl. Phys. Lett. 110,104101 (2017)

3. Simulations: 1D case

- The time evolution of the average electron density has been computed and compared to the experimental value



- There is a discrepancy in the decay of the electron density
- The difference may be due to gas outflow from the capillary (not accounted for in 1D model)
- With simple analytical calculations, it is possible to estimate the time scale of the gas outflow from the capillary*:

$$\rho = \rho_0 e^{-t/\tau}, \quad \tau \approx L_{cap}/c_s \approx 1\mu\text{s}$$

- To take into account gas outflow, one would **need** (at least) **bi-dimensional simulations**

*N. Bobrova et al., Phys. Rev. E65, 016407 (2001)

3. Simulations: model for 2D simulations

- A fluid model is employed $\leftarrow$ *Kinetic approach is computationally impractical*

- The plasma is studied as a hot gas which evolves because of the thermal pressure (**hydrodynamic** approximation)
- Main source of heat is the **ohmic dissipation**
- A divergence-free current density is computed with a time-varying static potential (**static current flow** case)

$$\begin{cases} \frac{\partial}{\partial t}\rho + \nabla \cdot (\rho \vec{v}) = 0 \\ \frac{\partial}{\partial t}(\rho \vec{v}) + \nabla \cdot (\rho \vec{v} \vec{v}) = -\nabla p \\ \frac{\partial}{\partial t} E + \nabla \cdot (E \vec{v}) = -\nabla \cdot (p \vec{v}) + \nabla \cdot (\kappa \nabla T) + \eta \|\vec{J}\|^2 \\ \nabla \cdot \vec{J} = 0, \ \vec{J} = -\frac{\nabla V}{\eta} \end{cases}$$

ρ: mass density
$\vec{v}$: fluid velocity
p: thermal pressure
E: total energy density
κ: thermal conductivity
T: plasma temperature
$\vec{J}$: current density
V: electric potential
η: electrical resistivity

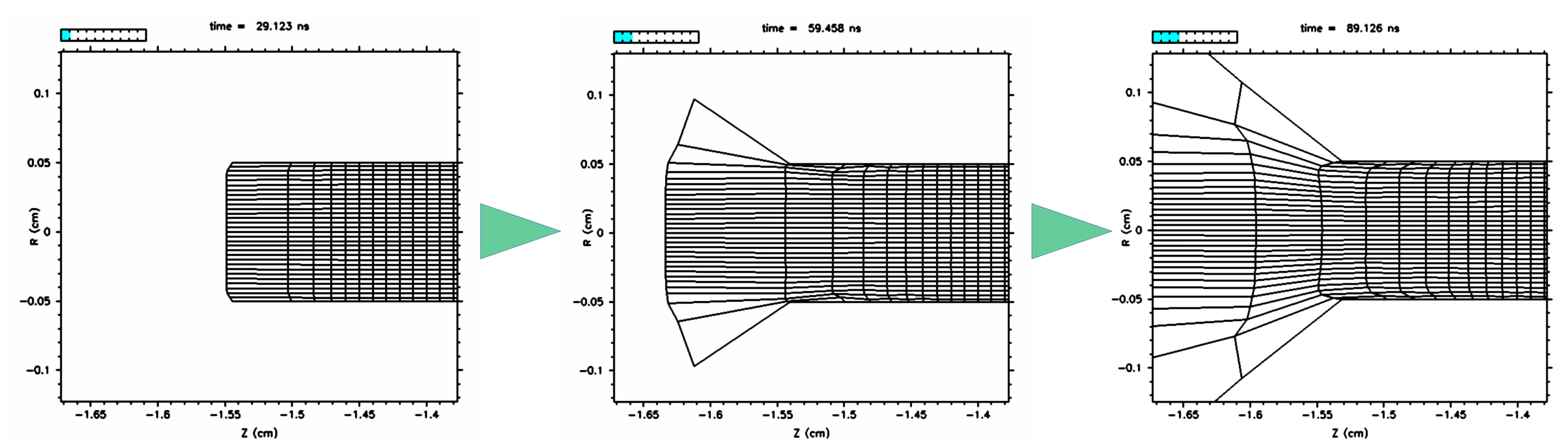
Further remarks:
- For now, Lorentz force on the fluid is neglected
- No self consistent magnetic field is present $\rightarrow$ skin effect cannot be seen

Additional details:

- The capillary has a circular cross section → **2D axial symmetry** is employed
- An ionization model exploiting a "**local thermodynamic equilibrium**" approximation is suitable:

$$Z(z,r,t) = f(T(z,r,t), \rho(z,r,t)), \quad Z\text{: ionization degree}$$

- **Lagrangian approach** (the mesh moves to follow the matter):
  - **advantage**: no need to mesh the whole domain, as it would be in Eulerian approach
  - **disadvantage**: *grid pathologies* have to be controlled

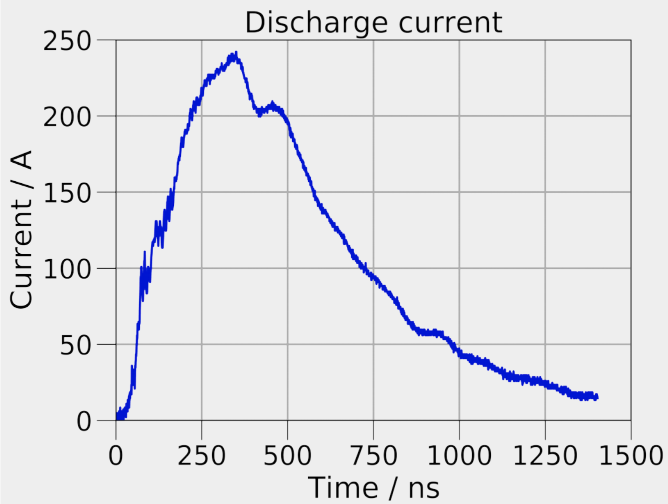


Typical evolution of the mesh in a hydrodynamic simulation (with DUED)

3. Simulations: preliminary results of 2D simulations

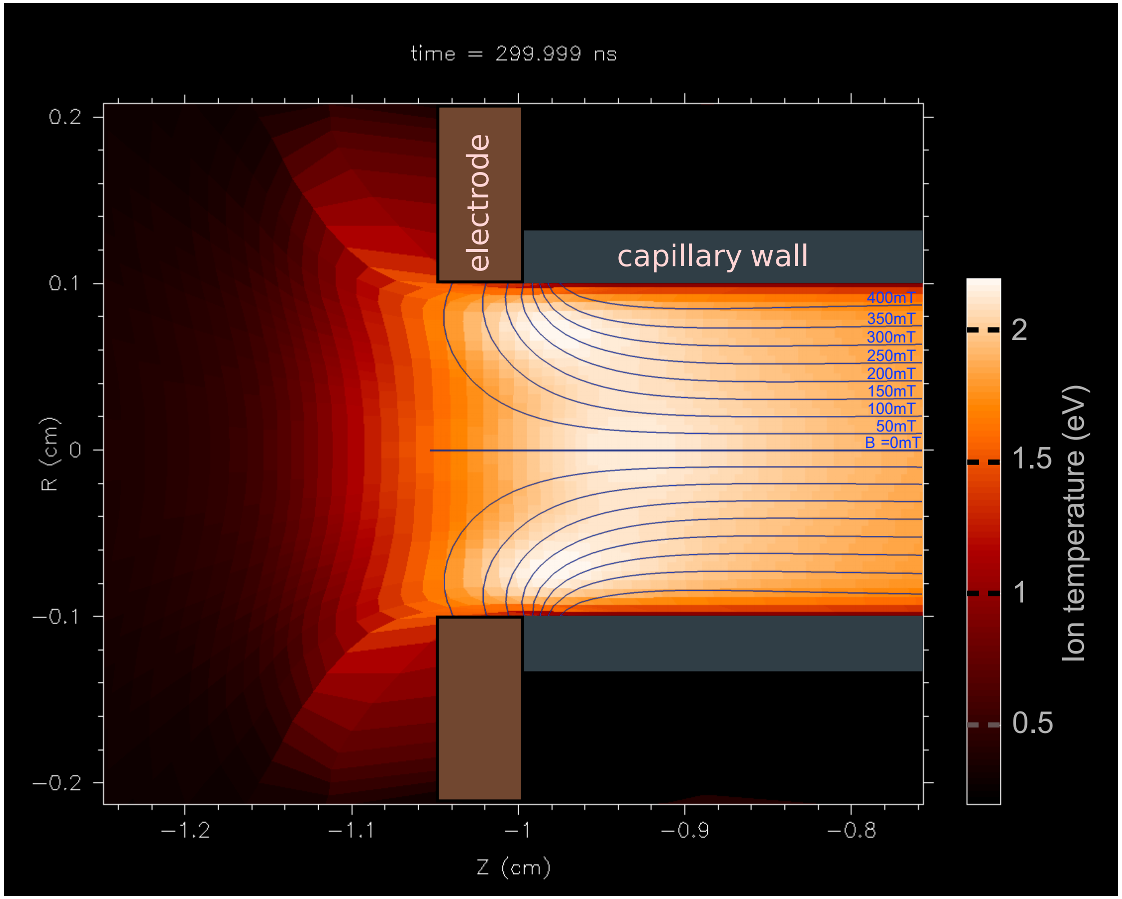
A suitably modified, reduced version of the code **DUED***, that implements the above model, has been used to simulate the discharge in a realistic capillary

Discharge configuration:

Capillary diameter	2mm
Capillary length	2cm
Initial gas density	$2 \cdot 10^{-7}$ g/cm^3
Initial gas temperature	9000K
Current profile	

*Atzeni et al., Comput. Phys. Commun. 169 (2005) 153

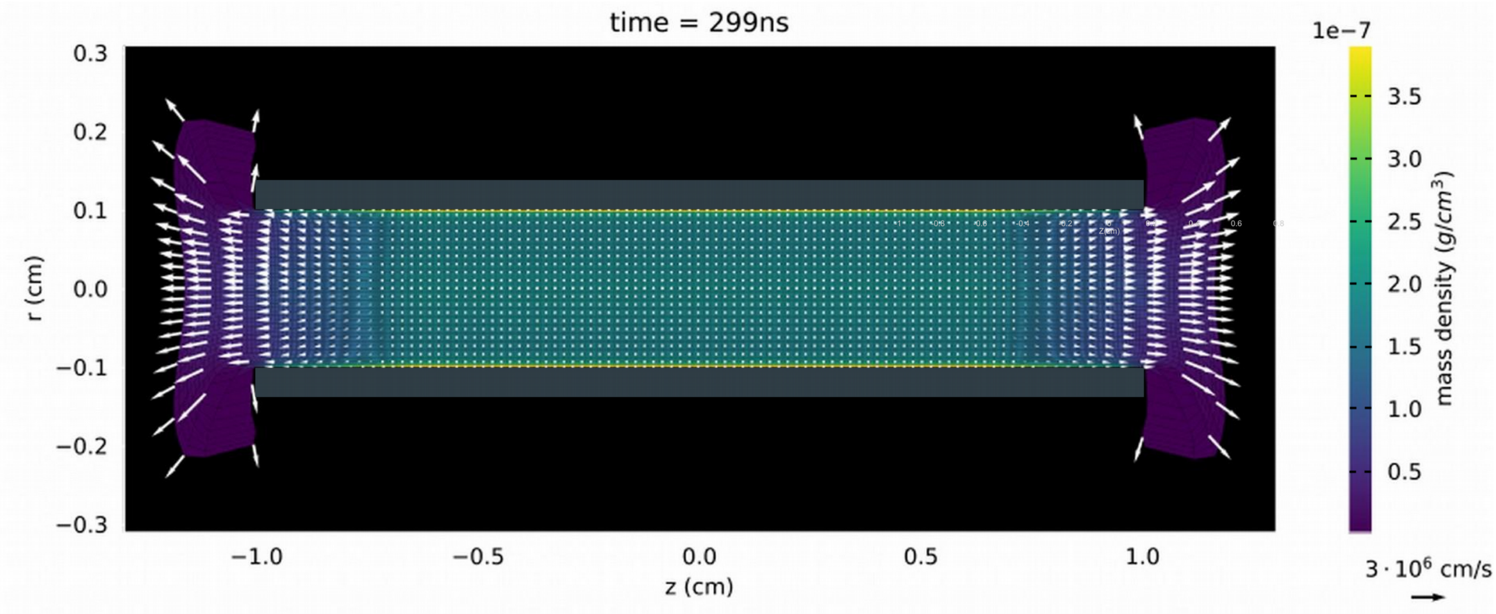
3. Simulations: preliminary results of 2D simulations



Particular of the plasma temperature (colored map) and azimuthal magnetic field (contour lines) in proximity of the left electrode at 300ns from the start of the discharge.

- It is possible to compute the magnetic field as post-processing
- Maps of relevant quantities can be obtained
- The temperature reached by the plasma seems to be in qualitative agreement with what expected

3. Simulations: preliminary results of 2D simulations



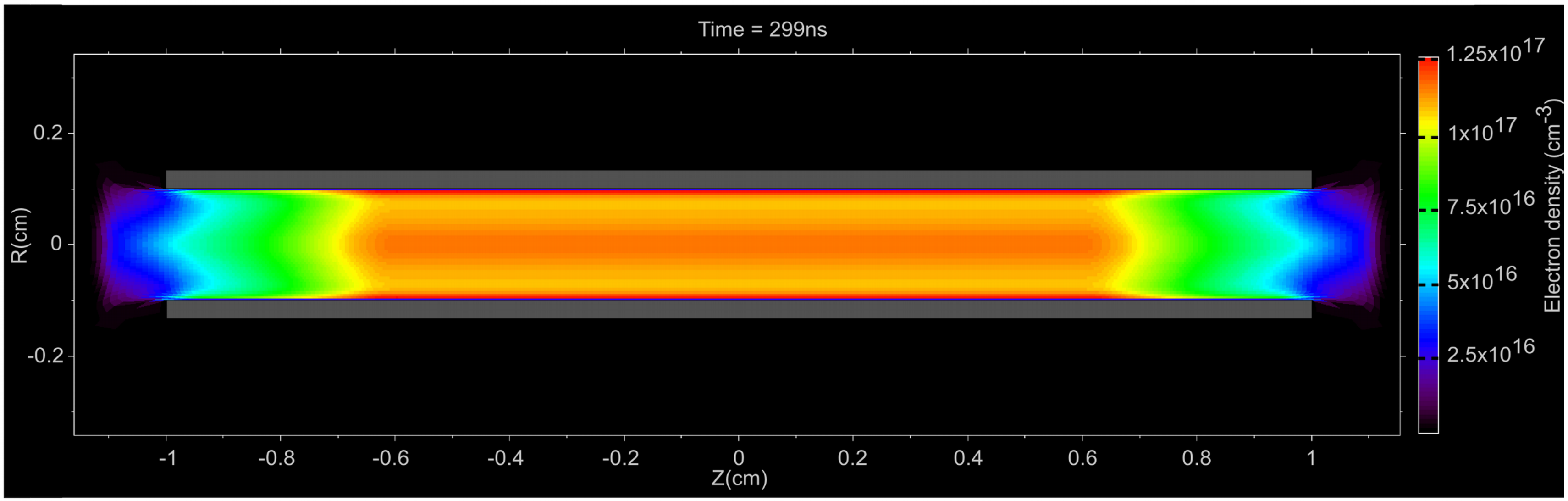
Mass density map and velocity field of the gas flowing from the capillary at 300ns from the start of the discharge.

- The outflow of hot gas from the extremities of the capillary can be observed
- The bulk velocity of the gas near the electrodes ($3 \cdot 10^6$ cm/s) is in accordance with the ion acoustic velocity (free expansion of plasma):

$$c_\mathrm{s} = \sqrt{\frac{k_\mathrm{B} T}{m_{\mathrm{H}^+}}} \approx 1 \cdot 10^6 \mathrm{cm/s}, \quad (T = 11600\mathrm{K})$$

3. Simulations: preliminary results of 2D simulations

- It is possible to compare the electron density computed by the code with the one that we can measure with the Stark broadening technique



Electron density map at 300ns from the start of the discharge.

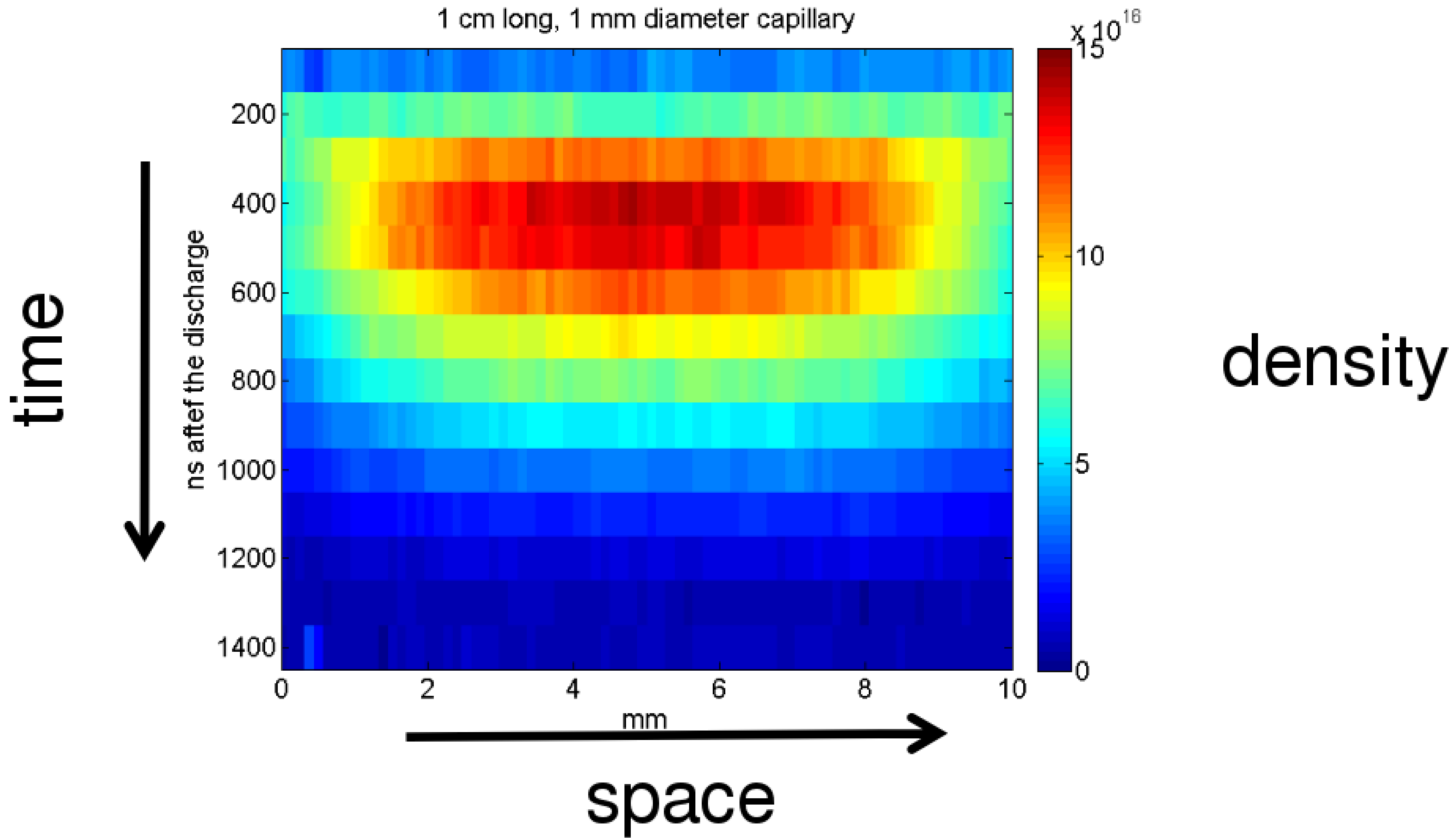
- Longitudinally resolved electron density measurements is implemented at SPARC_LAB *, this will allow for a first validation of the simulation results

*F. Filippi et al, J. Instrum, 11(09), C09015 (2016)

5. Conclusions and outlook

- I have shown the working principle of an active plasma lens
  - Active plasma lenses are promising technologies
  - Non-uniform discharge current can lead to aberrations in the lens
- I have commented the results of 1D simulations for a case of our interest
  - Plasma outflow cannot be considered in 1D models
  - Predictions on the transverse magnetic field from 1D simulations are not satisfying
- The need for 2D simulations is being addressed
  - Hydrodynamic model with joule heating driven by a static current flow
  - Ionization degree computed with local thermodynamic equilibrium approximation
  - Lagrangian approach
- Future steps:
  - Thorough comparison of the computed longitudinal electron density profiles with the measured ones
  - Optimization of the design parameters in order to improve the focal properties of the lens

Appendix



Picture and data inside is courtesy of F. Filippi