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QCD sum rules for 70 - plet baryons

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1. Magnetic moments of the 70-plet octet in terms of the D and F quantities characteristic for octet coupling.

2. Non-relativistic limit of the F and D descrition.

3. Borel QCD sum rules for the 70 -plet octet baryons.

4. Comparison with some other models.

1. Introduction

Magnetic moments of the octet baryons $1/2^-$ attracted much attention recently. They were analyzed in the various quark model formalism as chiral models with configuration mixing Dahyia-Gupta 2003. Recently the lattice QCD analysis was performed in Frank Lee-Alexandru 2010.

QCD sum rules also were constructed for the $1/2^-$ baryons Kondo-Morimatsu-Nishikawa-Kanada-En'yo 2007, Aliev-Savci 2014, Aliev-VZ 2015.

QCD sum rules for baryons are written mainly for the octet $1/2^+$ baryons which enter the 56-plet of the SU(6) symmetry – the group-theoretical basis of the nonrelativistic quark model (NRQM).

The interpolating baryon currents are analogue of the corresponding baryon wave functions in NRQM and can be reduced to them up to a factor Ioffe-Belyaev 1984. We have shown in several works that QCD sum rules for baryons $1/2^+$ exhibit the unitary symmetry pattern in the sense that all the results can be written in terms of only two functions of the D and F type characteristic for octet couplings Coleman-Glashow 1979.

Although NRQM has no behavior in terms of F and D couplings, we have shown years ago that the description in terms of F and D can be achieved in the quark-diquark model Gelmi-Lepshokov-VZ 1985.

2. QCD sum rules and F and D pattern

We have shown earlier that QCD sum rules for the magnetic moments of the octet baryons have the same unitary symmetry pattern.

Polarization operator

$$\Pi_B = i \int d^4x e^{ipx} \langle 0|T\{\eta_B(x)\bar{\eta}_B(0)\}|0\rangle_\gamma \tag{1}$$

where T is the time ordering operator, γ means external electromagnetic field while η_B is the interpolating current with the quantum numbers of the B baryon could written as

$$\Pi_{\Sigma^0} = e_u \Pi_1(u, d, s) + e_d \Pi_1(d, u, s) + e_s \Pi_2(u, d, s),$$

which is analogue of

$$\mu(\Sigma^0(ud, s)) = (e_u + e_d)F + e_s(F - D).$$

We take a little old-fashioned QCD sum rule for the baryon magnetic moments (Pasupathy1987)

$$\frac{1}{4}\tilde{\beta}^2_\Sigma[\mu(\Sigma^0) + AM^2]e^{-m^2_\Sigma/M^2} = (e_u + e_d)\{\frac{M^6}{4L^{4/9}} - \frac{a^2L^{4/9}}{144}[2 - (\kappa - 2\xi)]$$
$$-\frac{\chi a^2}{24L^{4/27}}(M^2 - \frac{m_0^2}{8L^{4/9}}) + \frac{bM^2}{96L^{4/9}}\} + e_s\{-\frac{a^2L^{4/9}}{24} + \frac{bM^2}{192L^{4/9}}\}$$
$$= e_u\Pi_1(u, d, s; M^2) + e_d\Pi_1(d, u, s; M^2) + e_s\Pi_2(u, d, s; M^2),$$

Here M^2 is Borel parameter, a, b, $\chi = -3.3$ etc being vev, while L=ln$(M^2/\Lambda$
ln$(\mu^2/\Lambda^2_{QCD}$ with Λ_{QCD} and normalization point μ.

$$\frac{1}{12}\tilde{\beta}^2_\Lambda[\mu(\Lambda) + A'M^2]e^{-m^2_\Lambda/M^2} = (e_u + e_d + 4e_s)\{\frac{M^6}{4L^{4/9}} - \frac{a^2L^{4/9}}{144}\times \quad (2)$$

$$[2 - (\kappa - 2\xi)] - \frac{\chi a^2}{24L^{4/27}}(M^2 - \frac{m_0^2}{8L^{4/9}}) + \frac{bM^2}{96L^{4/9}}\}$$

$$+(2e_u + 2e_d - e_s)\{-\frac{a^2L^{4/9}}{24} + \frac{bM^2}{192L^{4/9}}\}.$$

These QCD sum rules exhibit unitary symmetry pattern.

3. Octet magnetic moments in 56-plet

We now return back in order to deduce D and F pattern in NRQM. The 56-plet wave functions of the octet baryons could be written as (for the proton)

$$\sqrt{18}|p\rangle = |2u_1u_1d_2 - u_1d_1u_2 - d_1u_1u_2 + \quad (3)$$

$$2u_1d_2u_1 - u_1u_2d_1 - d_1u_2u_1 + 2d_2u_1u_1 - u_2u_1d_1 - u_2d_1u_1\rangle.$$

(Subindices 1,2 mean spin up (down).)

In the framework of the quark-diquark model in which the boson quantum or photon distinguishes between the interaction with the diquark (qq) of two similar quarks and that with the single quark q'

we arrive with the NRQM wave functions to the unitary model result in terms of the F and D coupling constants. It arises to the statement that we need to introduce 4 different matrix elements

$$\langle \mathbf{q}_\uparrow \mathbf{q}_\uparrow, \mathbf{q}'_\downarrow | \hat{\omega}_\mathbf{q} | \mathbf{q}_\uparrow \mathbf{q}_\uparrow, \mathbf{q}'_\downarrow \rangle = \mathbf{w}^\mathbf{q}_{\uparrow\uparrow}, \qquad \langle \mathbf{q}_\uparrow \mathbf{q}_\downarrow, \mathbf{q}'_\uparrow | \hat{\omega}_\mathbf{q} | \mathbf{q}_\uparrow \mathbf{q}_\downarrow, \mathbf{q}'_\uparrow \rangle = \mathbf{w}^\mathbf{q}_{\uparrow\downarrow} \tag{4}$$

$$\langle \mathbf{q}_\uparrow \mathbf{q}_\uparrow, \mathbf{q}'_\downarrow | \hat{\omega}_{\mathbf{q}'} | \mathbf{q}_\uparrow \mathbf{q}_\uparrow, \mathbf{q}'_\downarrow \rangle = \mathbf{v}^{\mathbf{q}'}_{\uparrow\uparrow}, \qquad \langle \mathbf{q}_\uparrow \mathbf{q}_\downarrow, \mathbf{q}'_\uparrow | \hat{\omega}_{\mathbf{q}'} | \mathbf{q}_\uparrow \mathbf{q}_\downarrow, \mathbf{q}'_\uparrow \rangle = \mathbf{v}^{\mathbf{q}'}_{\uparrow\downarrow}.$$

With the modified magnetic operator $\hat{\mathbf{e}}_\mathbf{q} \hat{\mathbf{w}}_\mathbf{q} \sigma^\mathbf{q}_\mathbf{z}$

$$\mu(\mathbf{\Sigma^0}(\mathbf{ud}, \mathbf{s})) = (\mathbf{e_u} + \mathbf{e_d}) \frac{\mathbf{2}}{\mathbf{3}} \mathbf{w}_{\uparrow\uparrow}$$

$$-\mathbf{e_s} \frac{\mathbf{1}}{\mathbf{3}} (\mathbf{2v}_{\uparrow\uparrow} - \mathbf{v}_{\uparrow\downarrow}) = (\mathbf{e_u} + \mathbf{e_d})\mathbf{F} + \mathbf{e_s}(\mathbf{F} - \mathbf{D}), \tag{5}$$

$$\mu(\mathbf{\Lambda}(\mathbf{u}, \mathbf{d}, \mathbf{s})) = (\mathbf{e_u} + \mathbf{e_d})(\mathbf{F} - \frac{\mathbf{2}}{\mathbf{3}}\mathbf{D}) + \mathbf{e_s}(\mathbf{F} + \frac{\mathbf{1}}{\mathbf{3}}\mathbf{D}) \tag{6}$$

with redefinitions $\frac{2}{3}\mathbf{w}_{\uparrow\uparrow} = \mathbf{F}, \quad \mathbf{w}_{\uparrow\downarrow} = \mathbf{D}$, $(\mathbf{2v}_{\uparrow\uparrow} - \mathbf{v}_{\uparrow\downarrow}) = \mathbf{3}(\mathbf{D} - \mathbf{F})$.

Thus we have shown the model reveal the unitary symmetry pattern

4. Magnetic moments in 70-plet

These results open us the way to treat 70-plet octet 1/2 baryons But existing QCD sum rules for the octet $1/2^-$ baryons hardly could be reduced to the NRQM results of the 70-plet - the usual place for $1/2^-$ baryons.

The N^{*+} of the octet of the 70-plet have quark wave function Dahyia

$$\sqrt{\mathbf{18}}|\mathbf{N}^*\rangle = |\mathbf{2u_1u_1d_2 - u_1d_1u_2 - d_1u_1u_2 +} \tag{7}$$

$$\mathbf{2d_1u_2u_1 - u_1u_2d_1 - u_1d_2u_1 + 2u_2d_1u_1 - us_2u_1d_1 - d_2u_1u_1}\rangle.$$

Magnetic moment of the $\mathbf{N}^*$ with the magnetic operator $\mathbf{\hat{e}_q\sigma_z^q}$ would be

$$\mu(\mathbf{N}^*(\mathbf{uu},\mathbf{d})) = \frac{\mathbf{2}}{\mathbf{3}}\mu_{\mathbf{u}} + \frac{\mathbf{1}}{\mathbf{3}}\mu_{\mathbf{d}}. \tag{8}$$

Instead for the $\mathbf{\Sigma}^*(\mathbf{ud},\mathbf{s})$ it and $\mathbf{\Lambda}^*$ yields

$$\mu(\mathbf{\Sigma}^{\mathbf{0}*}(\mathbf{ud},\mathbf{s})) = \mu(\mathbf{\Lambda}^*) = \frac{\mathbf{1}}{\mathbf{3}}\mu_{\mathbf{u}} + \frac{\mathbf{1}}{\mathbf{3}}\mu_{\mathbf{d}} + \frac{\mathbf{1}}{\mathbf{3}}\mu_{\mathbf{s}} \tag{9}$$

.

CONTRARY to the usually quoted results the NRQM $\mathbf{\Sigma}^*\mathbf{\Lambda}^*$ 70-plet transition magnetic moment should be zero !

Now let us arrive at F and D description suitable for the QCD sum rules for the 70-plet with the correct results in the non-relativistic limit. We proceed with the wave function of the octet of the 70-plet

$$\mu(\mathbf{N}^{*+}) = \mathbf{e_u}\frac{\mathbf{1}}{\mathbf{6}}(\mathbf{8w_{\uparrow\uparrow} + 4w_{\uparrow\downarrow}}) + \mathbf{e_d}\frac{\mathbf{1}}{\mathbf{6}}(\mathbf{4v_{\uparrow\uparrow} + 2v_{\uparrow\downarrow}}). \tag{10}$$

With $\frac{\mathbf{2}}{\mathbf{3}}\mathbf{w_{\uparrow\uparrow} = F}$, $\quad \mathbf{w_{\uparrow\downarrow} = D}$, $\quad (\mathbf{2v_{\uparrow\uparrow} - v_{\uparrow\downarrow}}) = \mathbf{3}(\mathbf{D - F})$ and $\mathbf{v_{\uparrow\uparrow} = D}$, $\quad \mathbf{v_{\uparrow\downarrow} = 3F - D}$ and performing calculations for 70-plet we get

$$\mu(\mathbf{\Sigma}^{*0}) = (\mathbf{e_u} + \mathbf{e_d})\frac{\mathbf{1}}{\mathbf{2}}\mathbf{F} + \mathbf{e_s}(\mathbf{2F} - \mathbf{D}).$$

$$\mu(\mathbf{\Lambda}^{*}) = (\mathbf{e_u} + \mathbf{e_d})\frac{\mathbf{1}}{\mathbf{6}}(\mathbf{9F} - \mathbf{4D}) + \mathbf{e_s}\frac{\mathbf{1}}{\mathbf{3}}\mathbf{D}.$$

In the limit $\mathbf{F} = \mathbf{2/3D}$, $\mathbf{D} = \mathbf{1}$ and $\mathbf{e_q} \to \mu_\mathbf{q}$ we return to NRQM. Putting quark electric charges we get unitary symmetry results in analogy with the 56-plet :

$$\mu(\mathbf{N}^{*+}) = \mu(\mathbf{\Sigma}^{*+}) = \frac{\mathbf{1}}{\mathbf{3}}\mathbf{D}, \qquad \mu(\mathbf{N}^{*0}) = \mu(\mathbf{\Xi}^{*0}) = \mathbf{F} - \frac{\mathbf{2}}{\mathbf{3}}\mathbf{D}, \tag{11}$$

$$\mu(\mathbf{\Sigma}^{*-}) = \mu(\mathbf{\Xi}^{*-}) = -\mathbf{F} + \frac{\mathbf{1}}{\mathbf{3}}\mathbf{D}, \quad \mu(\mathbf{\Sigma}^{*0}) = -\mu(\mathbf{\Lambda}^{*}) = -\frac{\mathbf{1}}{\mathbf{2}}(\mathbf{F} - \frac{\mathbf{2}}{\mathbf{3}}\mathbf{D}),$$

$$\mu(\mathbf{\Sigma}^{*0}\mathbf{\Lambda}^{*}) = -\frac{\mathbf{3}}{\mathbf{2}}(\mathbf{F} - \frac{\mathbf{2}}{\mathbf{3}}\mathbf{D}).$$

5. QCD sum rules for the 70-plet

Now we can proceed with the QCD sum rules Pasupathy1987 which take the form for the 70-plet

$$\frac{1}{4}\tilde{\beta}^2_{\Sigma^*}[\mu(\Sigma^{*0}) + A'M^2]e^{-m^2_{\Sigma^*}/M^2} = \frac{1}{2}(e_u + e_d + 4e_s)\times$$

$$\{\frac{M^6}{4L^{4/9}} - \frac{a^2L^{4/9}}{144}[2-(\kappa-2\xi)] - \frac{\chi a^2}{24L^{4/27}}(M^2 - \frac{m_0^2}{8L^{4/9}})$$
$$+\frac{bM^2}{96L^{4/9}}\} - e_s\{-\frac{a^2L^{4/9}}{24} + \frac{bM^2}{192L^{4/9}}\}$$

and for Λ term

$$\frac{1}{12}\tilde{\beta}^2_{\Lambda^*}[\mu(\Lambda^*) + A'M^2]e^{-m^2_{\Lambda^*}/M^2}$$

$$= (e_u + e_d)\frac{3}{2}\{\frac{M^6}{4L^{4/9}} - \frac{a^2L^{4/9}}{144}[2-(\kappa-2\xi)] - \frac{\chi a^2}{24L^{4/27}}(M^2 - \frac{m_0^2}{8L^{4/9}}) +$$
$$\frac{bM^2}{96L^{4/9}}\} - (2e_u + 2e_d - e_s)\{-\frac{a^2L^{4/9}}{24} + \frac{bM^2}{192L^{4/9}}\}$$

One can see that sum rules have changed drastically.

$\mu(\mathbf{B})$	**56-plet PRD33**	**This Work 70-plet**	**$1/2^-$ Lattice**	**NCQM**	**χCQM $\mu^{\mathbf{S}}_{\mathbf{val}}$**	**$1/2^-$ Aliev**	70-plet **F=2.5 D=4.5**
$\mu(\mathbf{p})$	**2.72**	**0.83**	**-1.8**	**1.894**	**1.411**	**1.4**	**1.5**
$\mu(\mathbf{n})$	**-1.65**	**0.22**	**-1.0**	**-1.284**	**-1.192**	**-0.54**	**-0.5**
$\mu(\mathbf{\Sigma^+})$	**2.52**	**0.70**	**-0.6**	**1.814**	**1.297**	**1.8**	**1.5**
$\mu(\mathbf{\Sigma^-})$	**-1.13**	**-1.13**	**1.0**	**-0.689**	**-0.333**	**-1.1**	**-1.0**
$\mu(\mathbf{\Sigma^0})$		**0.11**	**0.1**	**0.820**	**0.739**	**0.4**	**0.25**
$\mu(\mathbf{\Lambda})$	**-0.50**	**-0.11**	**-0.1**			**-0.26**	**-0.25**
$\mu(\mathbf{\Xi^0})$	**-0.89**	**-0.43**	**-0.5**	**-0.990**	**-1.00**	**-0.55**	**-0.5**
$\mu(\mathbf{\Xi^-})$	**-1.18**	**-1.18**	**0.8**	**-0.315**	**-0.027**	**-1.2**	**-1.0**

Table 1: **Magnetic moments of 70-plet and 56-plet baryons**

Upon using 70-plet formulas we described exactly magnetic moments of p, n, $\Sigma^{\pm}$, for Λ and Σ^0 results are less impressive and are of the same sign contrary to Lattice ones although of the right order of magnitude. So results of the lattice of calculations of of Frank Lee-Alexandru indicate that $1/2^-$ octet seems to belong to the 70-plet.

The recent work Aliev-Savci seems to correspond to 70-plet in terms of D and F (see the last column of the Table 1). We reproduce very accurately the results of Aliev-Savci with D^*=4.5, F^*=1.5.

6. Conclusion

Octet baryons in the 70-plet are analyzed in a way similar to those of the 56-plet.

Magnetic moments are written in terms of the D and F quantities characteristic for octet coupling.

The main formulas for the magnetic moments are written in such a way as to obtain the NRQM results as well as unitary symmetry ones.

Borel QCD sum rules are constructed for the magnetic moments of the 70-plet octet.