A variational characterizaton of Higgs fields on gluon bundles

We show that the Lie derivative of gluon fields is parametrized by Higgs fields defined by the kernel of a gauge-natural Jacobi morphism.

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Framework

- $\mathbf{P} \to \mathbf{X}$ principal bundle with structure group $\mathbf{G}$
  $L_k(\mathbf{X})$ is the bundle of k–frames in $\mathbf{X}$
- $r \leq k$ the *gauge-natural prolongation of* $\mathbf{P}$
  $\mathbf{W}^{(r,k)}\mathbf{P} \doteq J_r\mathbf{P} \times_{\mathbf{X}} L_k(\mathbf{X})$
  principal bundle over $\mathbf{X}$ with structure group $\mathbf{W}^{(r,k)}_n\mathbf{G}$
  (semidirect product of $GL_k(n)$ on $\mathbf{G}^r_n$)
  $GL_k(n)$ group of k–frames in $I\!R^n$
  $\mathbf{G}^r_n$ the space of (r,n)-velocities on $\mathbf{G}$.
- Let $\mathbf{F}$ be a manifold and $\zeta : \mathbf{W}^{(r,k)}_n\mathbf{G} \times \mathbf{F} \to \mathbf{F}$ be a left action of $\mathbf{W}^{(r,k)}_n\mathbf{G}$ on $\mathbf{F}$.
- associated *gauge-natural bundle* of order (r,k): $\mathbf{Y}_\zeta \doteq \mathbf{W}^{(r,k)}\mathbf{P} \times_\zeta \mathbf{F}$.

Main results

- $\mathbf{P}$ be a principal bundle with structure group $\mathbf{G}$, *e.g.* $SU(3) \times SU(2) \times U(1)$.
- coupling with gravity: structure bundle of the theory is the fibered product $\mathbf{\Sigma} = \bar{\Sigma} \times_{\mathbf{X}} \mathbf{P}$, where $\bar{\Sigma}$ be the principal spin bundle with structure group $Spin(1,3)$..
- Action on a spinor matter manifold $V = \mathbb{C}^k$
  $\phi : Spin(1,3) \times SU(3) \times SU(2) \times U(1) \times V$
  given by a choice of Dirac matrices for each component of the spinor field.
- corresponding Lagrangian
  $\lambda = \bar{\psi}(i\gamma_\mu D^\mu - m)\psi - \frac{1}{4}(\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \mathcal{F}^A_{\mu\nu}\mathcal{F}^{\mu\nu}_A + \mathcal{F}^a_{\mu\nu}\mathcal{F}^{\mu\nu}_a)$
  The bundle of principal connections on $\mathbf{\Sigma}$ is a gauge-natural bundle associated with the gauge-natural prolongation $W^{(1,1)}\mathbf{\Sigma}$.
- $\mathfrak{k}$ the Lie algebra of generalized Jacobi vector fields generating associated *canonical* conserved quantities
- **responsible of the canonical variational breaking of symmetry and of the generation of Higgs fields**
- $\mathfrak{h}$ the Lie algebra of right-invariant vertical vector fields on $W^{(1,1)}\mathbf{\Sigma}$
- The Lie algebra $\mathfrak{k}$ is characterized as a Lie subalgebra of $\mathfrak{h}$
- split reductive structure $\mathfrak{h} = \mathfrak{k} \oplus Im\,\mathcal{J}$, $[\mathfrak{k}, Im\,\mathcal{J}] = Im\,\mathcal{J}$
- for each $\mathbf{p} \in W^{(1,1)}\Sigma$
  denote $\mathcal{S} \doteq \mathfrak{h}_{\mathbf{p}}$, $\mathcal{R} \doteq \mathfrak{k}_{\mathbf{p}}$ and $\mathcal{V} \doteq Im\,\mathcal{J}_{\mathbf{p}}$
- reductive Lie algebra decomposition $\mathcal{S} = \mathcal{R} \oplus \mathcal{V}$, with $[\mathcal{R}, \mathcal{V}] = \mathcal{V}$.
  $\mathcal{S}$ is the Lie algebra of the Lie group $W^{(1,1)}_4\mathbf{G}$.
- There exists an isomorphism between $\mathcal{V} \doteq Im\mathcal{J}_{\mathbf{p}}$ and $T\mathbf{X}$ so that $\mathcal{V}$ turns out to be the image of an horizontal subspace.
  we caracterize a principal bundle $\mathbf{S} \to \mathbf{X}$, with $dim\mathbf{S} = dim\,\mathcal{S}$ and such that $\mathbf{X} = \mathbf{S}/\mathbf{R}$, where $\mathbf{R}$ is a Lie group of the Lie algebra $\mathcal{R}$ and $\mathcal{R} = T_{\mathbf{q}}\mathbf{S}/\mathbf{R}$;
- *the principal subbundle* $\mathbf{S} \subset \mathbf{W}^{(1,1)}\mathbf{\Sigma}$ *is then a* **reduced principal bundle**.

(omit now the orders of a gauge-natural prolongation)

- The Lie group $\mathbf{R}$ of the Lie algebra $\mathcal{R}$ is in particular a closed subgroup of $W\mathbf{G}$
- We have the composite fiber bundle
  $W\mathbf{\Sigma} \to W\mathbf{\Sigma}/\mathbf{R} \to \mathbf{X}$,
  $W\mathbf{\Sigma}/\mathbf{R} = W\mathbf{\Sigma} \times_{W\mathbf{G}} W\mathbf{G}/\mathbf{R} \to \mathbf{X}$ is a gauge-natural bundle functorially associated with $W\mathbf{\Sigma} \times W\mathbf{G}/\mathbf{R} \to \mathbf{X}$ by the right action of $W\mathbf{G}$.
- The left action of $W\mathbf{G}$ on $W\mathbf{G}/\mathbf{R}$ is in accordance with the reductive Lie algebra decomposition.

Variational (gluon) Cartan connection

- Definition:
  We call a global section $h : \mathbf{X} \to W\mathbf{\Sigma}/\mathbf{R}$ a *gluon gauge-natural Higgs field.*
- ω principal connection on $W\mathbf{\Sigma}$;
  $\bar{\omega}$ principal connection on the principal bundle $\mathbf{S}$ defines the splitting $T_{\mathbf{p}}\mathbf{S} \simeq_{\bar{\omega}} \mathcal{R} \oplus \hat{H}_{\mathbf{p}}$, $\mathbf{p} \in \mathbf{S}$.
- It is defined a principal Cartan connection of type $\mathcal{S}/\mathcal{R}$, such that $\hat{\omega}|_{V\mathbf{S}} = \bar{\omega}$.
- It is a connection on $W\mathbf{\Sigma} = \mathbf{S} \times_{\mathbf{R}} W\mathbf{G} \to \mathbf{X}$, thus a Cartan connection on $\mathbf{S} \to \mathbf{X}$ with values in $\mathcal{S}$, the Lie algebra of the gauge-natural structure group of the theory;
  it splits into the $\mathcal{R}$-component which is a principal connection form on the $\mathcal{R}$-manifold $\mathbf{S}$, and the $\mathcal{V}$-component which is a displacement form
- *A gluon gauge-natural Higgs field is therefore a global section of the Cartan horizontal bundle* $\hat{H}_p$*, with* $\mathbf{p} \in \mathbf{S}$, it is related with the displacement form defined by the $\mathcal{V}$-component of the Cartan connection $\hat{\omega}$ above.
- Each global section h of $W\Sigma/\mathbf{R} \to \mathbf{X}$ thus defines a vertical covariant differential.
- The Lie derivative of fields is constrained and it is parametrized by gluon Higgs fields h characterized by $\mathfrak{K}$.

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