Heavy-Meson Decay Constants: QCD Sum-Rule Glance at Isospin Breaking

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Hadron features from QCD sum-rule POV

QCD sum rules are relations between hadron features and QCD parameters resulting from the evaluation of correlators of interpolating currents at both hadronic level and QCD level by insertion of complete sets of hadron states, exploitation of Wilson's operator product expansion, Borel transformation, and the quark–hadron duality assumption beyond some effective threshold.

In order to both raise the accuracy of sum-rule extractions and estimate the systematic errors [1], we developed [2] a modified formalism which takes into account the dependence of its effective threshold on the Borel parameter(s). This enabled an improved prediction of heavy-meson decay constants [3], in particular, the finding $f_{B^*_{(s)}} < f_{B_{(s)}}$ [4] later on confirmed by lattice QCD [5].

QCD sum rules encode in analytic form the dependences of some features of the given hadron under study on the parameters of QCD. This enables us to discuss the impact of the isospin breaking effected by the slight difference in the masses of the light u and d quarks on the decay constants of heavy–light mesons, by working out the decay-constant's relation to this light mass. For convenience, we'll focus to the case of infinitely large Borel-mass parameter.

Correlation functions and decay constants

Within this study, let us consider pseudoscalar (P_q) and vector (V_q) mesons (generically labeled $M_q \equiv P_q, V_q$) of mass M_{M_q}, composed of a heavy quark $Q = b, c$ and a light quark $q = u, d$, of masses m_Q and m_q, resp. In terms of suitable interpolating quark-current operators, the axial-vector (A) current $A_\mu(x) = \bar{q}(x)\,\gamma_\mu\,\gamma_5\,Q(x)$ and the vector (V) current $V_\mu(x) = \bar{q}(x)\,\gamma_\mu\,Q(x)$, the decay constants f_{M_q} of these mesons (with momentum p) are defined by

$$\langle 0|\,A_\mu(0)\,|P_q(p)\rangle = \mathrm{i}\,f_{P_q}\,p_\mu\ , \qquad \langle 0|\,V_\mu(0)\,|V_q(p)\rangle = f_{V_q}\,M_{V_q}\,\varepsilon_\mu(p)\ ,$$

with the vector meson's polarization vector $\varepsilon_\mu(p)$. The decay constants may be inferred from the correlators of two such currents $J_\mu(x) = A_\mu(x), V_\mu(x)$:

$$\Pi^{(J)}_{\mu\nu}(p) = \mathrm{i}\int \mathrm{d}^4x\,\exp(\mathrm{i}\,p\,x)\,\langle 0|\,T\big(J_\mu(x)\,J^\dagger_\nu(0)\big)\,|0\rangle\ , \qquad J = \mathrm{A}, \mathrm{V}\ .$$

We derive f_{M_q} for P_q from the coefficients of the Lorentz structure $p_\mu\,p_\nu$ and for V_q from the coefficients in front of the Lorentz structure $p_\mu\,p_\nu - p^2\,g_{\mu\nu}$. In the limit of infinitely large Borel-mass variable, the QCD sum rule found for f_{M_q} involves a perturbatively accessible spectral density $\rho_J(s, m_Q, m_q, \alpha_\mathrm{s})$, which is but the operator product expansion's lowest term, and the effective threshold $s_\mathrm{eff}(m_Q, m_q) \approx (m_Q + \Delta + \eta\,m_q)^2$, with its parameter Δ fixing the value of f_{M_q} in the chiral limit and η parametrizing the deviation therefrom:

$$f^2_{M_q} = \int\limits_{(m_Q+m_q)^2}^{s_\mathrm{eff}(m_Q,m_q)} \mathrm{d}s\,\rho_J(s, m_Q, m_q, \alpha_\mathrm{s})\ , \qquad M_q = P_q, V_q\ , \qquad J = \mathrm{A}, \mathrm{V}\ .$$

The spectral densities may be found in form of expansions in powers of both light-quark mass m_q and strong coupling $\alpha_\mathrm{s}(\mu)$, at renormalization point μ:

$$\rho_J(s, m_Q, m_q, \alpha_\mathrm{s}) = R_J(s, m_Q, \alpha_\mathrm{s}) + m_q\,r_J(s, m_Q, \alpha_\mathrm{s}) + O(m_q^2)\ ,$$

$$R_J(s, m_Q, \alpha_\mathrm{s}) = R^{(0)}_J(s, m_Q) + \frac{\alpha_\mathrm{s}(\mu)}{\pi}\,R^{(1)}_J(s, m_Q) + O(\alpha_\mathrm{s}^2)\ ,$$

$$r_J(s, m_Q, \alpha_\mathrm{s}) = r^{(0)}_J(s, m_Q) + \frac{\alpha_\mathrm{s}(\mu)}{\pi}\,r^{(1)}_J(s, m_Q) + O(\alpha_\mathrm{s}^2)\ .$$

Accordingly, the expansion of any f_{M_q} QCD sum rule in powers of m_q reads

$$f_{M_q}^2 = \int_{m_Q^2}^{(m_Q+\Delta)^2} \mathrm{d}s\, R_J(s, m_Q, \alpha_s) + m_q \int_{m_Q^2}^{(m_Q+\Delta)^2} \mathrm{d}s\, r_J(s, m_Q, \alpha_s)$$
$$+ 2\,\eta\, m_q\, (m_Q + \Delta)\, R_J(s, m_Q, \alpha_s)|_{s=(m_Q+\Delta)^2} + O(m_q^2) \ .$$

We infer the required spectral densities from Ref. [6] for P_q and from Ref. [7] for V_q, and obtain, in the heavy-quark case $\Delta \ll m_Q$, as isospin breaking [8]

$$\frac{f_{M_d}^2 - f_{M_u}^2}{m_d - m_u} = \frac{N_c}{2\,\pi^2}\,\frac{\Delta^2}{m_Q}\,(1 + 2\,\eta) + O(\alpha_s) + O\left(\frac{1}{m_Q^2}\right) , \qquad M_q \equiv P_q, V_q \ .$$

The m_q dependence of the relevant spectral densities is known up to $O(\alpha_s)$.

Summary, findings, conclusions, insights [8]

The only free parameters left, Δ and η, are fixed by comparison: for Δ, with isospin-symmetric f_{M_q} predictions by earlier QCD sum-rule analyses [3] and lattice QCD and, for η, with predictions of the decay constants of the heavy strange pseudoscalar or vector mesons, by QCD sum rules and lattice QCD. The truncation of the perturbative expansions to a finite order in α_s induces an unphysical moderate dependence of our numerical results on the scale μ.

Isospin effects on decay constants of pseudoscalar and vector heavy mesons:

Meson M_q	μ [GeV]	η	$\frac{f_{M_d} - f_{M_u}}{m_d - m_u}(\mu)$	$f_{M_d} - f_{M_u}$ [MeV]
B	3.5	1.5	0.426	0.933
	4.5	1.4	0.437	0.911
B^*	3.5	1.08	0.39	0.846
	4.5	1.07	0.40	0.833
D	2	1.65	0.46	1.1
	3	1.65	0.459	1.0
D^*	2	2.1	0.65	1.62
	3	2.3	0.69	1.55

The difference of the d- and u-type decay constants must be proportional to the isospin-breaking quark-mass difference; the differences' ratio is positive:

$$\frac{f_{M_d} - f_{M_u}}{m_d - m_u} > 0 \ .$$

Hence, for the isospin-breaking-betraying differences of the decay constants of nonstrange bottom and charmed pseudoscalar and vectors mesons we get

$$f_{B^0} - f_{B^+} = 0.92 \pm 0.02 \text{ MeV} \ , \qquad f_{B^{*0}} - f_{B^{*+}} = 0.84 \pm 0.02 \text{ MeV} \ ,$$
$$f_{D^+} - f_{D^0} = 1.05 \pm 0.02 \text{ MeV} \ , \qquad f_{D^{*+}} - f_{D^{*0}} = 1.60 \pm 0.05 \text{ MeV} \ ,$$

with uncertainties of the order of just a few percent: such predictions will be refined if taking advantage of adequate results of chiral perturbation theory. In the B-meson case, our findings are smaller than those by lattice QCD [9].

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