# Charmed Mesons and Charmonia: Three-Meson Strong Couplings


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## Couplings, form factors & decay constants

By use of a relativistic dispersion approach relying on the constituent-quark model, we study the strong couplings $g_{PP'V}$ and $g_{PV'V}$ for vector mesons $V$, with polarization vectors $\varepsilon_\mu$, and pseudoscalar mesons $P$, in the amplitudes

$$\langle P'(p_2)\,V(q)|P(p_1)\rangle = -\frac{g_{PP'V}}{2}\,(p_1+p_2)^\mu\,\varepsilon^*_\mu(q)\ ,$$
$$\langle V'(p_2)\,V(q)|P(p_1)\rangle = -g_{PV'V}\,\epsilon_{\varepsilon^*(q)\,\varepsilon^*(p_2)\,p_1\,p_2}\ , \qquad q\equiv p_1-p_2\ .$$

Of particular interest to us will be the case when there is, at least, one of the charmonia $J/\psi$ or $\eta_c$ among this meson triple. These strong couplings enter into the residues of poles in corresponding transition form factors for $q^2>0$ arising from intermediate meson states. The form factors relevant for us are

$$\langle P'(p_2)|\bar q_2\,\gamma_\mu\,q_1|P(p_1)\rangle = F_+^{P\to P'}(q^2)\,(p_1+p_2)_\mu+\cdots\ ,$$
$$\langle V(p_2)|\bar q_2\,\gamma_\mu\,q_1|P(p_1)\rangle = \frac{2\,V^{P\to V}(q^2)}{M_P+M_V}\,\epsilon_{\mu\,\varepsilon^*(p_2)\,p_1\,p_2}\ ,$$
$$\langle V(p_2)|\bar q_2\,\gamma_\mu\,\gamma_5\,q_1|P(p_1)\rangle = \mathrm{i}\,q_\mu\,(\varepsilon^*(p_2)\,p_1)\,\frac{2\,M_V}{q^2}\,A_0^{P\to V}(q^2)+\cdots\ .$$

With the decay constants of the vector and pseudoscalar mesons $f_V$ and $f_P$, defined in terms of the meson-to-vacuum transition amplitude of the vector quark current $\bar{q}_2\,\gamma_\mu\,q_1$ or axial-vector quark current $\bar{q}_2\,\gamma_\mu\,\gamma_5\,q_1$ according to

$$\langle 0|\bar{q}_1\,\gamma_\mu\,q_2|V(q)\rangle = f_V\,M_V\,\varepsilon_\mu(q)\ , \qquad \langle 0|\bar{q}_1\,\gamma_\mu\,\gamma_5\,q_2|P(q)\rangle = \mathrm{i}\,f_P\,q_\mu\ ,$$

the poles, at pseudoscalar and vector resonances $P_R$ and $V_R$, are of the form

$$F_+^{P\to P'}(q^2) = \frac{g_{PP'V_R}\,f_{V_R}}{2\,M_{V_R}}\,\frac{1}{1-\frac{q^2}{M_{V_R}^2}} + \cdots\ ,$$

$$V^{P\to V}(q^2) = \frac{(M_V+M_P)\,g_{PVV_R}\,f_{V_R}}{2\,M_{V_R}}\,\frac{1}{1-\frac{q^2}{M_{V_R}^2}} + \cdots\ ,$$

$$A_0^{P\to V}(q^2) = \frac{g_{PP_RV}\,f_{P_R}}{2\,M_V}\,\frac{1}{1-\frac{q^2}{M_{P_R}^2}} + \cdots\ .$$

# Quark-model-based dispersion approach

We compute the form factors $F_+^{P\to P'}(q^2)$, $V^{P\to V}(q^2)$, and $A_0^{P\to V}(q^2)$ within the framework of a relativistic constituent-quark picture [1]. To this end, we must relate the currents defining the form factors to their constituent-quark ($Q$) counterparts: this task is easily accomplished for heavy-quark currents,

$$\bar{q}_1\,\gamma_\mu\,q_2 = g_V\,\bar{Q}_1\,\gamma_\mu\,Q_2 + \cdots\ , \qquad \bar{q}_1\,\gamma_\mu\,\gamma_5\,q_2 = g_A\,\bar{Q}_1\,\gamma_\mu\,\gamma_5\,Q_2 + \cdots\ ,$$

by introducing form factors $g_V$ and $g_A$, but not so simple for light quarks [2]. Numerically, we employ [3], for the constituent-quark masses and couplings, $m_d = m_u = 0.23$ GeV, $m_s = 0.35$ GeV, $m_c = 1.45$ GeV and $g_V = g_A = 1$. The relativistic dispersion formalism allows us to represent the quantities of interest by integrals, over invariant masses of intermediate quark–antiquark states, of spectral densities derived from (one-loop) Feynman diagrams and wave functions of the involved pseudoscalar or vector mesons [4], of the form

$$\phi_{P,V}(s) = \frac{\pi}{\sqrt{2}}\,\frac{\sqrt{s^2-(m_1^2-m^2)^2}}{\sqrt{s-(m_1-m)^2}}\,\frac{w_{P,V}(k^2)}{s^{3/4}}\ ,$$

$$k^2 = \frac{(s-m_1^2-m^2)^2-4\,m_1^2\,m^2}{4\,s}\ , \qquad \int \mathrm{d}k\,k^2\,w_{P,V}^2(k^2) = 1\ .$$

For the radial meson wave functions $w_{P,V}(k^2)$, we assume Gaussian shapes:

$$w_{P,V}(k^2) \propto \exp\left(-\frac{k^2}{2\,\beta_{P,V}^2}\right) .$$

The decay constants $f_{P,V}$ become spectral integrals of densities $\rho_{P,V}(s)$ and the form factors $F_+^{P\to P'}(q^2)$, $V^{P\to V}(q^2)$ and $A_0^{P\to V}(q^2)$, generically labelled $\mathcal{F}(q^2)$, double dispersion integrals of double spectral densities $\Delta(s_1, s_2, q^2)$:

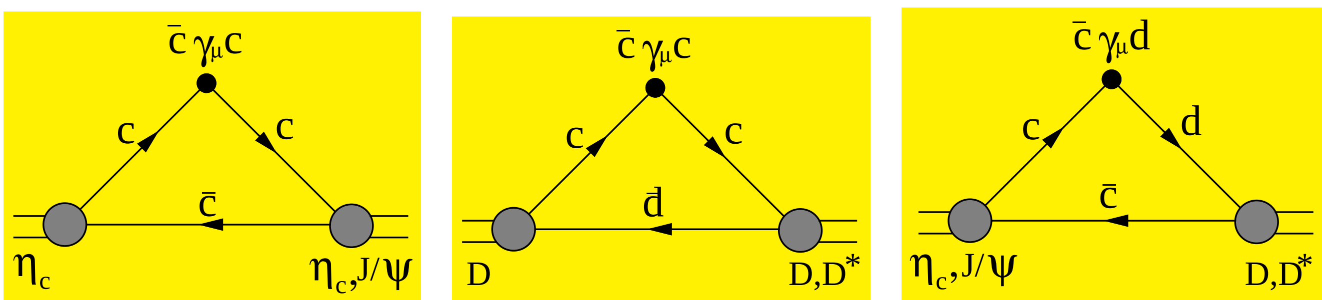
$$f_{P,V} = \int \mathrm{d}s\, \phi_{P,V}(s)\, \rho_{P,V}(s) \ ,$$

$$\mathcal{F}(q^2) = \int \mathrm{d}s_1\, \phi_1(s_1) \int \mathrm{d}s_2\, \phi_2(s_2)\, \Delta(s_1, s_2, q^2) \ .$$

$D_{(s)}^{(*)}$ and $c\bar{c}$ meson masses $M$, decay constants $f$ and slope parameters $\beta$ [5]:

| Meson | $D$ | $D^*$ | $D_s$ | $D_s^*$ | $\eta_c$ | $J/\psi$ |
|---|---|---|---|---|---|---|
| $M$ (GeV) | 1.87 | 2.010 | 1.97 | 2.11 | 2.980 | 3.097 |
| $f$ (MeV) | $206 \pm 8$ | $260 \pm 10$ | $248 \pm 2.5$ | $311 \pm 9$ | $394.7 \pm 2.4$ | $405 \pm 7$ |
| $\beta$ (GeV) | 0.475 | 0.48 | 0.545 | 0.54 | 0.77 | 0.68 |

Feynman graphs for transitions induced by quark vector currents $\bar{Q}_1\, \gamma_\mu\, Q_2$:




All form factors $\mathcal{F}$, computed off their resonances $R$, can be interpolated by

$$\mathcal{F}(q^2) = \frac{\mathcal{F}(0)}{\left(1 - \frac{q^2}{M_R^2}\right)\left(1 - \frac{\sigma_1\, q^2}{M_R^2} + \frac{\sigma_2\, q^4}{M_R^4}\right)} \ , \qquad \mathrm{Res}\, \mathcal{F}(M_R^2) = \frac{\mathcal{F}(0)}{1 - \sigma_1 + \sigma_2} \ .$$

From the residues, involving the meson masses, decay constants, and strong couplings, the latter are derived, by combined fits if present more than once.

# Strong couplings: $\eta_c\eta_c J/\psi$ and $\eta_c J/\psi J/\psi$

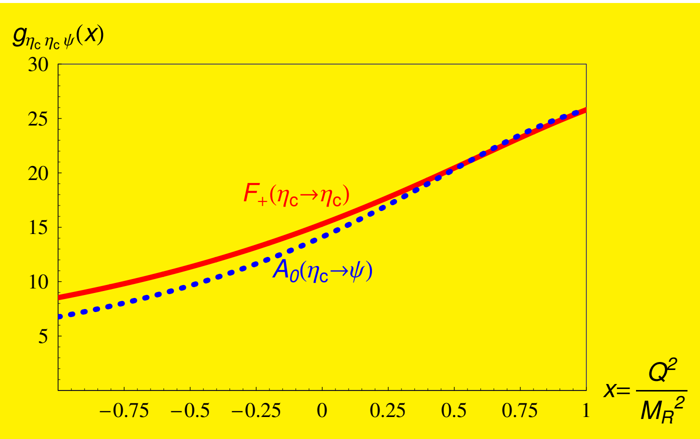
As one example representative for the general situation, consider the strong coupling $g_{\eta_c\eta_c\psi}$ of two pseudoscalar $\eta_c$ mesons and one vector $J/\psi$ meson [6] by realizing that this quantity enters in and therefore can be extracted from

- the residue $\text{Res}\, F_+^{\eta_c\to\eta_c}(M_\psi^2)$ of the pole at $q^2 = M_\psi^2$ of the form factor $F_+^{\eta_c\to\eta_c}(q^2)$ for the ("elastic") transition $\eta_c \to \eta_c$ enabled by the current $\bar{c}\,\gamma_\mu\, c$ coupling with decay constant $f_\psi$ to the vector resonance $J/\psi$ and
- the residue $\text{Res}\, A_0^{\eta_c\to\psi}(M_{\eta_c}^2)$ of the pole at $q^2 = M_{\eta_c}^2$ of the form factor $A_0^{\eta_c\to\psi}(q^2)$ for the transition $\eta_c \to J/\psi$ induced by the current $\bar{c}\,\gamma_\mu\,\gamma_5\, c$ that couples, with decay constant $f_{\eta_c}$, to the pseudoscalar resonance $\eta_c$:

$$\text{Res}\, F_+^{\eta_c\to\eta_c}(M_\psi^2) = g_{\eta_c\eta_c\psi}\,\frac{f_\psi}{2\,M_\psi}\ , \qquad \text{Res}\, A_0^{\eta_c\to\psi}(M_{\eta_c}^2) = g_{\eta_c\eta_c\psi}\,\frac{f_{\eta_c}}{2\,M_\psi}\ .$$

Upon determination of the meson wave-function parameters $\beta$ by requiring the dispersion representation of the decay constants $f_{P,V}$ to reproduce their known values, the strong couplings may be calculated, individually for each transition of interest, from the spectral representation of its associated form factor: our couplings' off-resonance behaviour exhibits excellent agreement.

Behaviour of the off-shell strong coupling $g_{\eta_c\eta_c\psi}(x)$ as a function of $x \equiv \frac{q^2}{M_R^2}$, $R = J/\psi, \eta_c$, from the two transitions $\eta_c \to \eta_c$ (red) and $\eta_c \to J/\psi$ (blue):




A combined fit with the four parameters $g_{\eta_c\eta_c\psi}$, $A_0^{\eta_c\to\psi}(0)$, $\sigma_1^{\mathcal{F}}$ then yields [6]

$$g_{\eta_c\eta_c\psi} = 25.8 \pm 1.7\ .$$

Along a similar route, we find [6] for the strong coupling of one pseudoscalar $\eta_c$ meson and two vector $J/\psi$ mesons, $g_{\eta_c\psi\psi}$, entering in only one transition,

$$g_{\eta_c\psi\psi} = (10.6 \pm 1.5)\ \text{GeV}^{-1}\ .$$

# Strong couplings: $\eta_c$ and $J/\psi$ to $D$ and $D^*$

Allowing also for currents with a $d$ or $s$ quark (and merging strong-coupling multiple occurrences, all of them showing nearly perfect concord), we get [6]

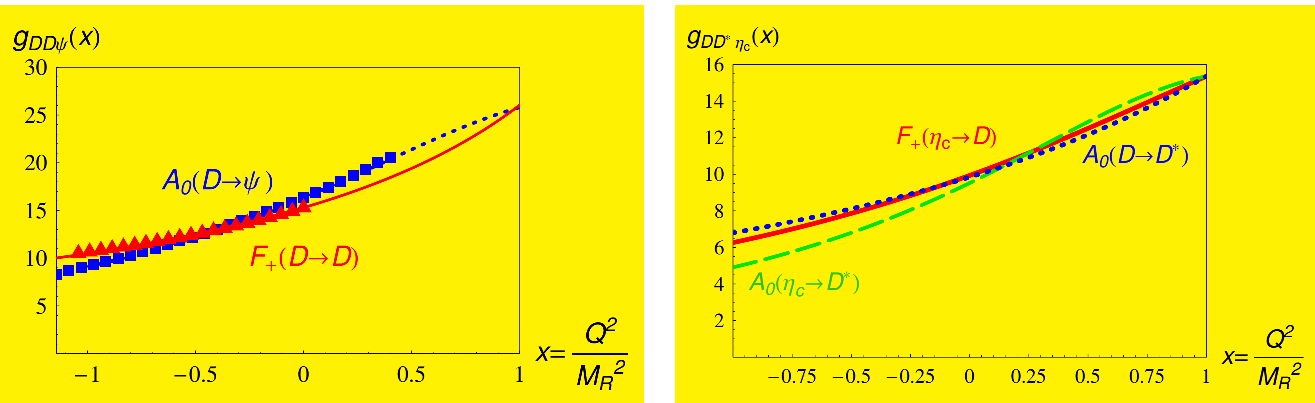
- for the strong $J/\psi$ or $\eta_c$ couplings to non-strange charmed mesons $D^{(*)}$

$$\begin{aligned} g_{DD\psi} &= 26.04 \pm 1.43\ , \\ g_{DD^*\psi} &= (10.7 \pm 0.4)\ \text{GeV}^{-1}\ , \\ g_{DD^*\eta_c} &= 15.51 \pm 0.45\ , \\ g_{D^*D^*\eta_c} &= (9.76 \pm 0.32)\ \text{GeV}^{-1}\ , \end{aligned}$$

- and for the strong $J/\psi$ or $\eta_c$ couplings to strange charmed mesons $D_s^{(*)}$

$$\begin{aligned} g_{D_sD_s\psi} &= 23.83 \pm 0.78\ , \\ g_{D_sD_s^*\psi} &= (9.6 \pm 0.8)\ \text{GeV}^{-1}\ , \\ g_{D_sD_s^*\eta_c} &= 14.15 \pm 0.52\ , \\ g_{D_s^*D_s^*\eta_c} &= (8.27 \pm 0.37)\ \text{GeV}^{-1}\ . \end{aligned}$$

Dependence of the off-shell strong couplings $g_{D\hat{D}\psi}(x) = \frac{2\,M_\psi}{f_D}\,\hat{x}\,A_0^{D\to\psi}(q^2)$ (left, blue), $g_{DD\hat{\psi}}(x) = \frac{2\,M_\psi}{f_\psi}\,\hat{x}\,F_+^{D\to D}(q^2)$ (left, red), $\hat{x} \equiv 1 - x$, $g_{D\hat{D}^*\psi}(x)$ (right, red), $g_{DD^*\hat{\psi}}(x)$ (right, blue) and $g_{\hat{D}D^*\psi}(x)$ (right, green) on $x \equiv \frac{q^2}{M_R^2}$:



# Observations, comparison and conclusions

- Successful extrapolations of all interpolated results for strong couplings derived at $q^2 < 0$ confirm the presence of the poles expected for $q^2 > 0$.
- Concerning $SU(3)$ breaking, the net outcome of replacing a $d$ quark by the $s$ quark is a reduction of the affected strong coupling by about 10%.
- Despite undeniable similarities of the approaches, our $D_{(s)}^{(*)}$ couplings [6] are more than twice as large as the results arising from QCD sum rules.

Comparison of our strong-coupling predictions with QCD sum-rule results:

| Coupling | $g_{DD\psi}$ | $g_{DD^*\psi}$ (GeV$^{-1}$) | $g_{D_sD_s\psi}$ | $g_{D_sD_s^*\psi}$ (GeV$^{-1}$) |
|---|---|---|---|---|
| This work | $26.04 \pm 1.43$ | $10.7 \pm 0.4$ | $23.83 \pm 0.78$ | $9.6 \pm 0.8$ |
| Sum rules | $11.6 \pm 1.8$ [7] | $4.0 \pm 0.6$ [7] | $11.96 \pm 1.34$ [8] | $4.30 \pm 1.53$ [9] |

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