Particle-vibration coupling effect on β-decay of magic nuclei

Yifei Niu

INFN, Milano, Italy
Institute of Fluid Physics, CAEP, China

April 19, 2015

Collaborators:

Zhongming Niu: Anhui University, China
Gianluca Colò: Università degli Studi and INFN, Milano, Italy
Enrico Vigezzi: INFN, Milano, Italy

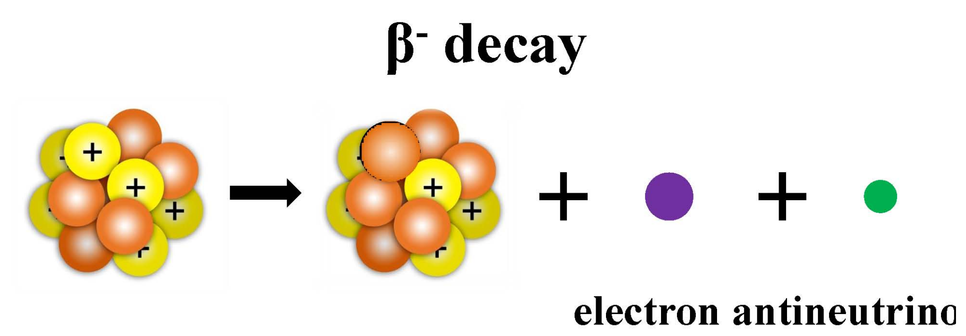
OUTLINE

1. Introduction
2. Theory framework
3. Results and discussions
4. Summary and Perspectives

Introduction — *Nuclear β-decay*

Nuclear β^- decay

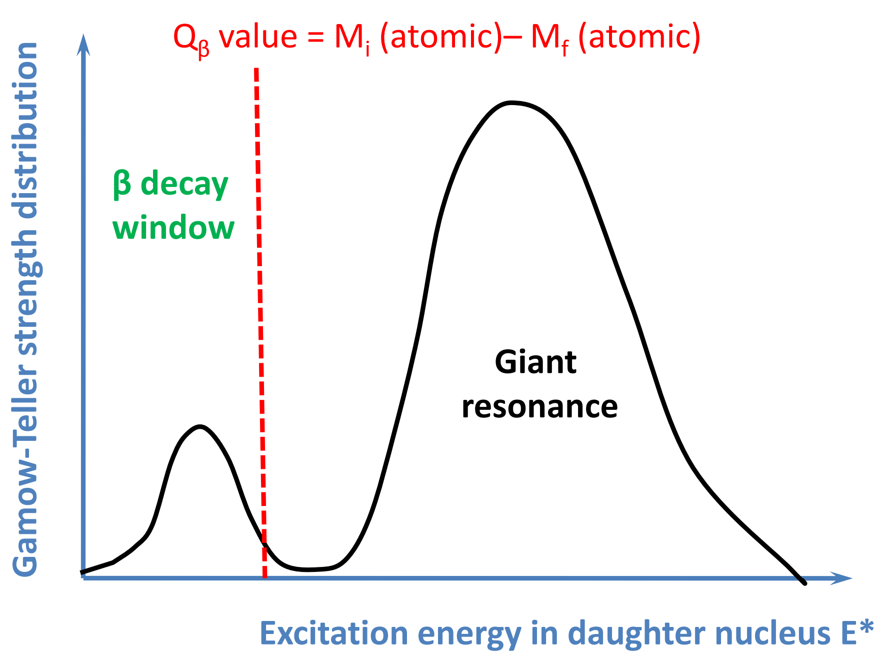
$${}_{N}X_{Z}^{A} \rightarrow {}_{N-1} X_{Z+1}^{A} + e^- + \bar{\nu}_e$$



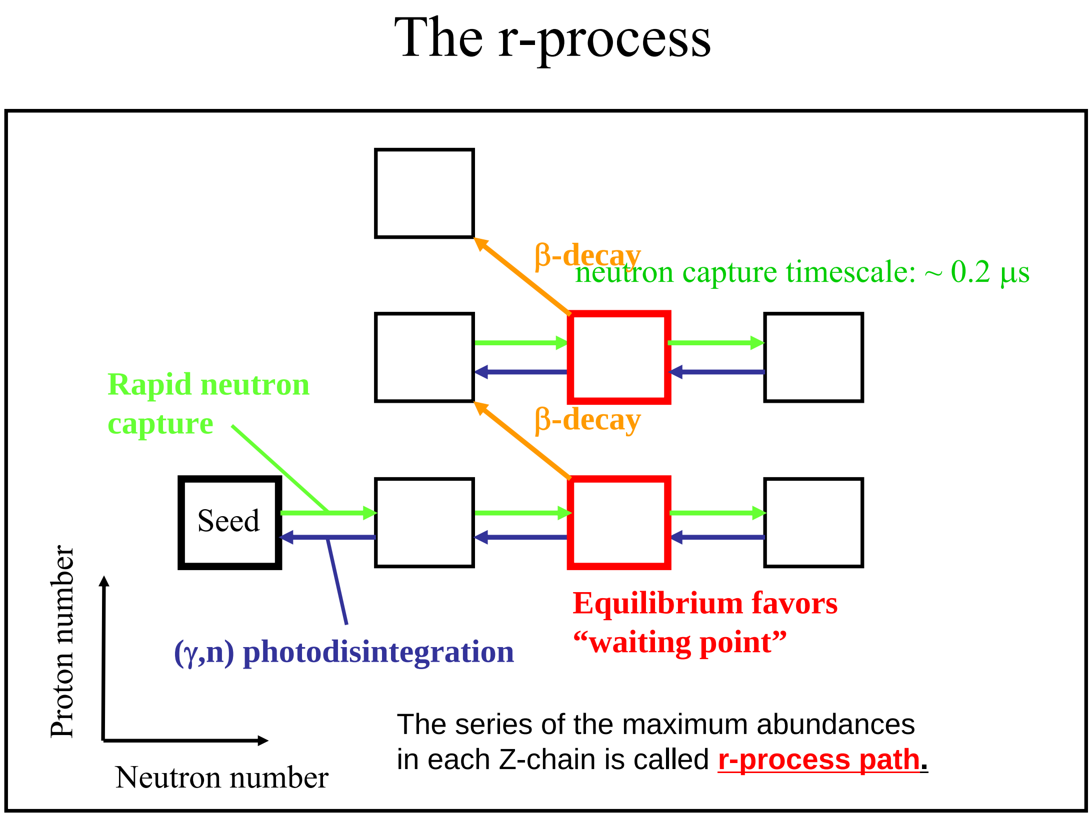
- mainly determined by GT transition
- energy conservation

$$M_i = M_f + E^* + T_e + E_\nu$$

- GT strength distribution
  - ✓ spin-orbit splitting
  - ✓ spin-isospin channel of nuclear effective interaction
- β-decay and GTR measurements complement each other



Introduction — *Nuclear β-decay*



β-decay and r-process

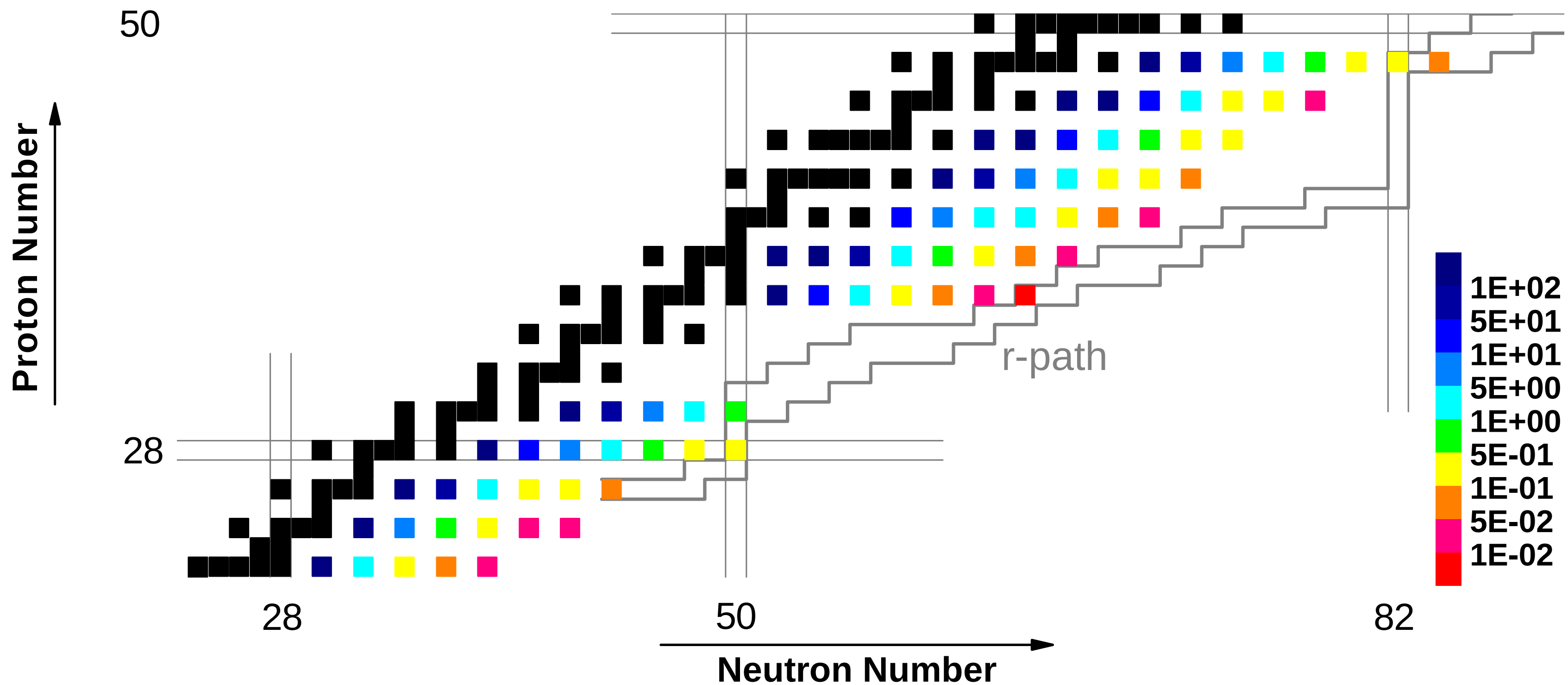
- r-process: produce the heavy elements beyond iron
- β-decay half-life:

  govern the abundance flow from neighbouring isotopic chains

  $\Rightarrow$ set the time-scale of r-process

  ($\sim$ the sum of β-decay half-lives of nuclei in the r-process path)

Introduction — *Experimental half-lives*



— Z. Niu, Y. Niu, et al., PLB 723, 172, 2013

- ✓ development of radioactive ion-beam facilities ⇒ important advances
  ^{78}Ni and around *Hosmer, et al., PRL 94, 112501, 2005; Xu, et al., PRL 113, 032505, 2014*
  very neutron rich Kr to Tc isotopes *Nishimura, et al., PRL 106, 052502, 2011*
- ✓ most neutron rich nuclei relevant for r-process: out of experimental reach

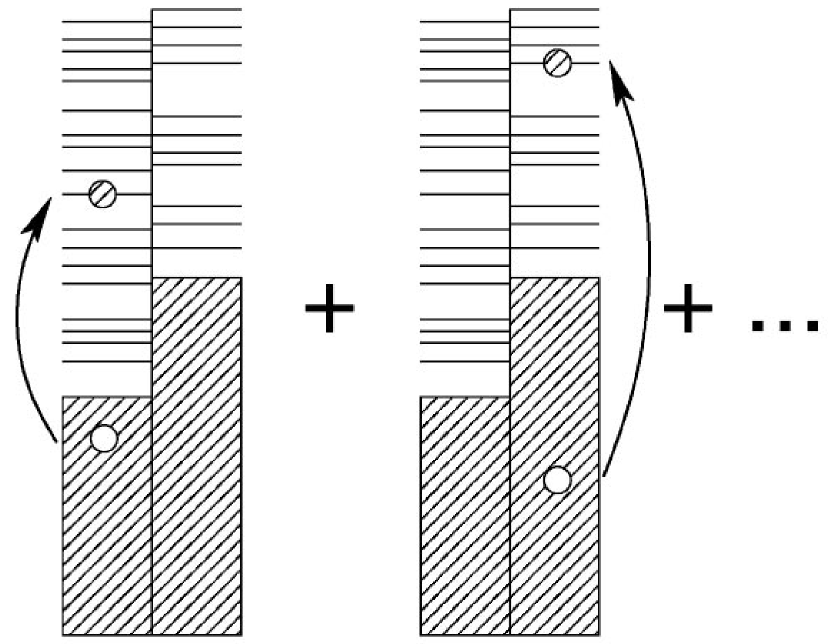
Introduction — *Theoretical investigations*

- ♢ ab-initio approach: for very light nuclei *Barrett, et al. PPNP 69, 131, 2013*
- ♢ shell model: up to $A = 40 - 50$ or around magic regions assuming a frozen core

  Langanke, et al., ADNDT 79, 1, 2001; Suzuki, et al., PRC 85, 015802, 2012; Li, et al., JPG 41, 105102, 2014
- ♢ Random Phase Approximation (RPA)

  ✓ non-self-consistent
  - Quasi-particle RPA (QRPA) based on FRDM *Moller, et al., ADNDT 66, 131, 1997*
  - DF3 + CQRPA *Borzov, PRC 67, 025802, 2003*

  ✓ self-consistent
  - QRPA based on Skyrme density functional *Engel, et al., PRC 60, 014302, 1999*
  - QRPA based on covariant density functional
    RHB + QRPA *Nikšić, et al., PRC 71, 014308, 2005; Marketin, et al., PRC 75, 024304, 2007*
    RHFB + QRPA *Niu, et al., PLB 723, 172, 2013*

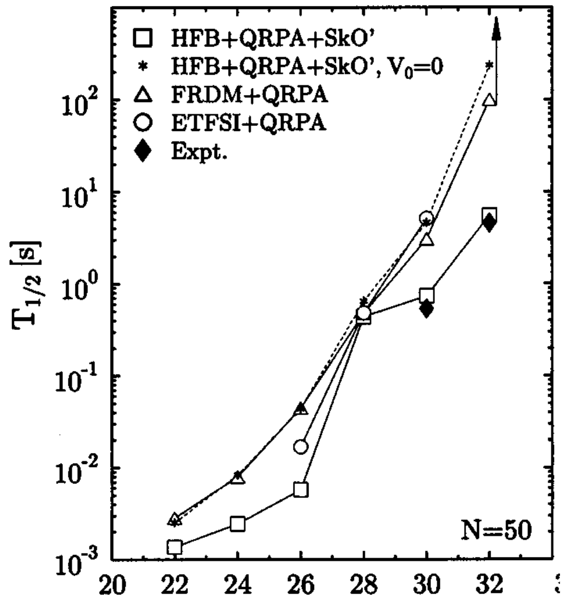


RPA

Introduction — *Problems with RPA description*

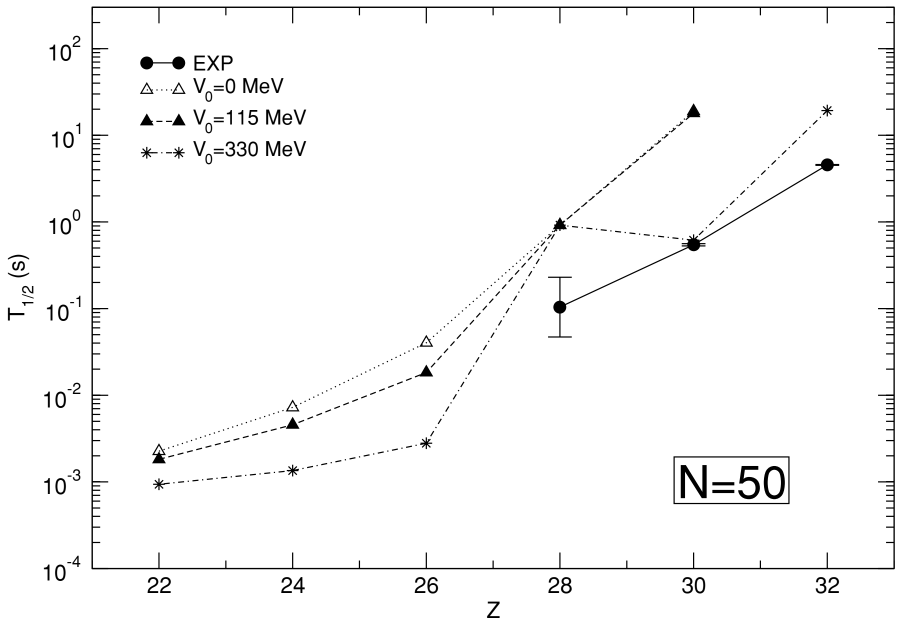
◇ Skyrme HFB+QRPA

Engel, et al., PRC 60, 014302, 1999



◇ RHB+QRPA

Nikšić, et al., PRC 71, 014308, 2005



self-consistent QRPA

- ✓ Half-lives are systematically overestimated
- ✓ Half-lives can be reduced with the inclusion of attractive isoscalar pn pairing
- ✓ The isoscalar pn pairing has little or no effect on closed-shell nuclei, like ^{78}Ni.

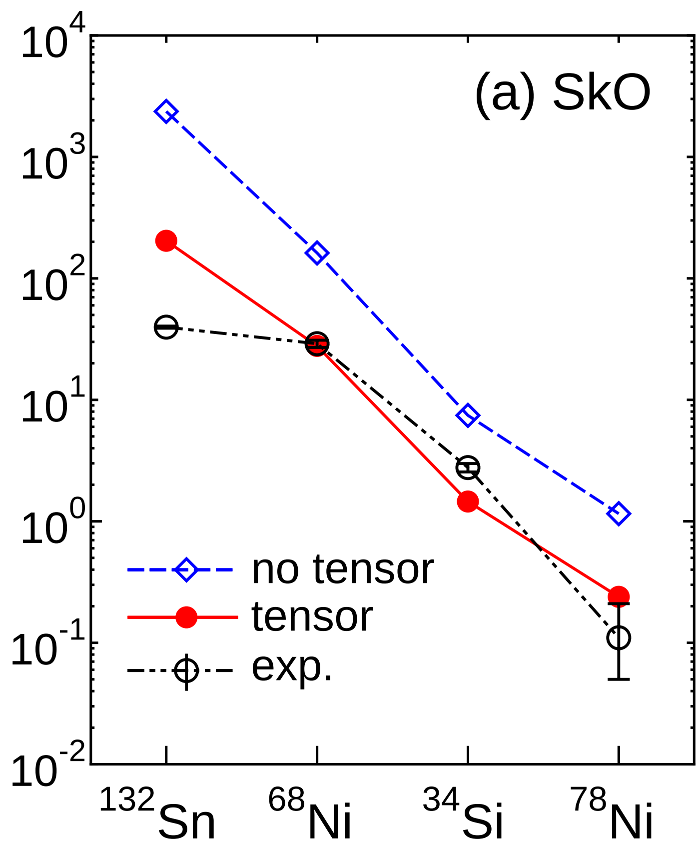
Introduction — *Possible solutions*

- Possible solution: tensor force

  Minato, et al., PRL 110, 122501, 2013

  Skyrme RPA + tensor

  β-decay half-lives are reduced



- *Another possible solution*:

  Skyrme RPA + particle-vibration coupling (PVC)

  Niu, et al., PRC 85, 034314, 2012

RPA+PVC

- ✓ Include correlations beyond 1p-1h configurations
- ✓ The downward shift of excitation energy and spreading of the GT strength is found.
- ✓ It is expected the PVC could help with reducing β-decay half-life.

Advantage

- ✓ Without introducing new parameters

Introduction — *RPA+PVC: more than half-life*

◇ Spreading Width

✓ (Q)RPA: coherent superposition of 1p-1h states

✓ Spreading width $\Gamma^{\downarrow}$

energy and angular momentum of coherent vibrations
$\Rightarrow$ more complicated states of 2p-2h, 3p-3h $\cdots$ character

◇ To describe the spreading width

✓ Second RPA *Drozdz, et al., PR 197, 1, 1990*

configuration space 1p-1h, 2p-2h

✓ RPA + PVC (particle-vibration coupling)
configuration space 1p-1h, 1p-1h $\otimes$ phonon

- relativistic functional

  Litvinova, et al., PLB 730, 307, 2014; Marketin, et al., PRC 706, 477, 2012

- skyrme functional *Colo, et al., PRC 50, 1496, 1994*

Introduction — *In this work*

Goal

- To develop self-consistent skyrme RPA + PVC model by including the whole two-body interaction in the PVC vertex
- To investigate the particle-vibration coupling effects on nuclear β-decay in magic nuclei

  ✓ the spreading width and fragmentation of GT strength

  ✓ the β-decay half-life

Theory framework — β-decay half-life

Half-life calculated in the allowed GT approximation

$$T_{1/2} = \frac{D}{g_A^2 \int^{Q_\beta} S(E) f(Z,\omega) dE}, \quad (1)$$

- $D = 6163.4$ s and $g_A = 1$
- $S(E)$: GT strength distribution with respect to daughter nucleus
- integrated phase volume

$$f(Z,\omega_0) = \frac{1}{(m_e c^2)^5} \int_{m_e c^2}^{\omega_0} p_e E_e (\omega_0 - E_e)^2 F_0(Z+1, E_e) dE_e. \quad (2)$$

p_e: momentum; E_e: energy; $F_0(Z+1, E_e)$: Fermi function of the electron

$\omega_0 = Q_\beta + m_e c^2 - E$

Theory framework — *Strength function from RPA+PVC*

The nuclear excitation configuration space is divided into three subspaces: Q_1, Q_2, P.

- Q_1: 1p-1h within the set $|i\rangle$ — obtained by solving the HF equation
- P: 1p-1h where the particle is in an unbound state $\Rightarrow$ *Escaping Width*
- Q_2: "doorway states" $|N\rangle$: 1p-1h $\otimes$ phonon $\Rightarrow$ *Spreading Width*

Strength function

$$S(\omega) = \sum_n \frac{1}{\sigma_n\sqrt{2\pi}} \mathrm{e}^{-\frac{(\omega-\Omega_n)^2}{2\sigma_n^2}} B_n, \quad \sigma_n = \left(\frac{\Gamma_n(\omega)}{2} + \Delta\right)/\sqrt{2\ln 2}, \tag{3}$$

where

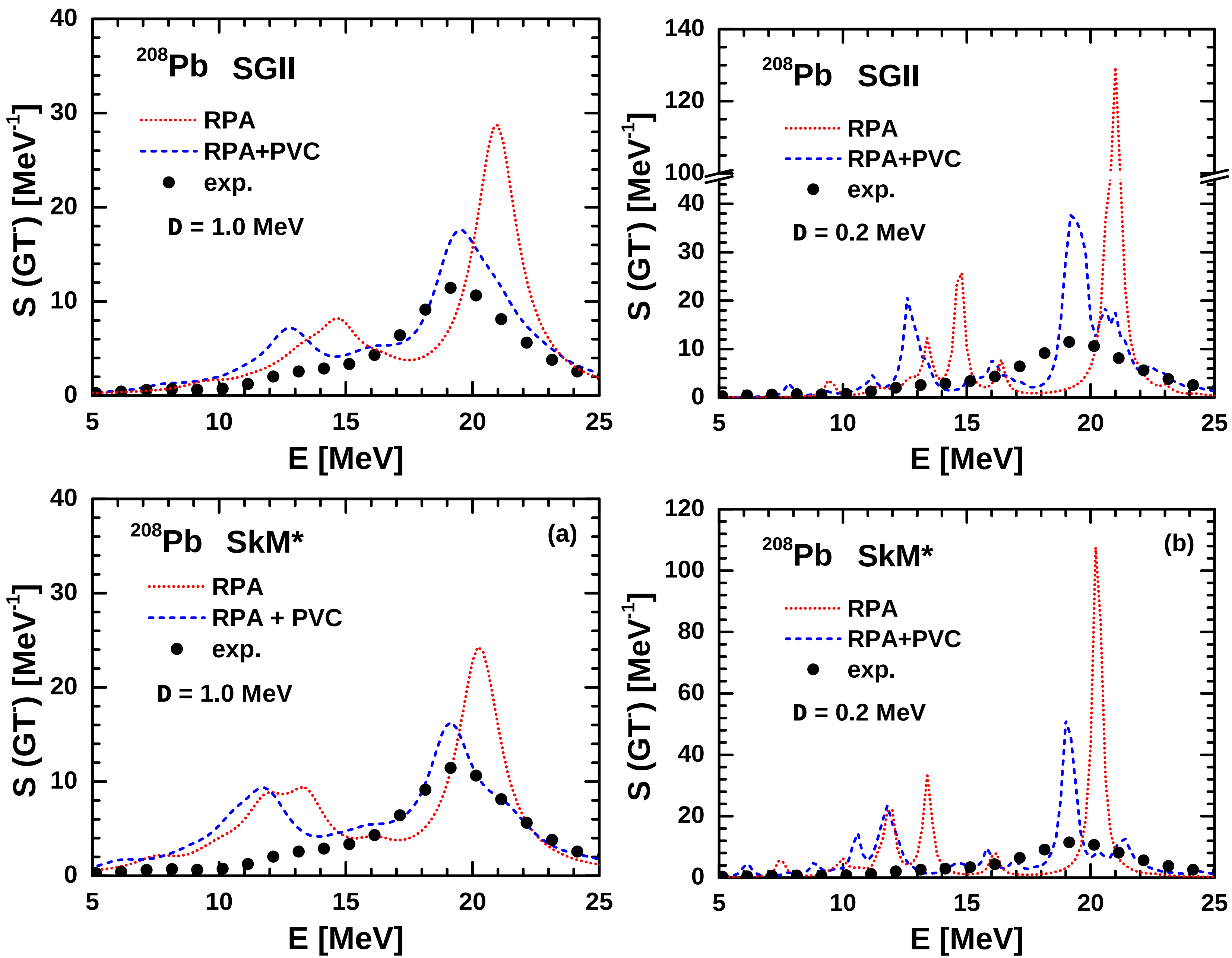
$$\Omega_n(\omega) = E_{\mathrm{RPA}} + \mathrm{Re}[(\Sigma)_{\mathrm{GT,GT}}(\omega)] \tag{4}$$

and

$$\Gamma_n(\omega) = -2\mathrm{Im}[(\Sigma)_{\mathrm{GT,GT}}(\omega)]. \tag{5}$$

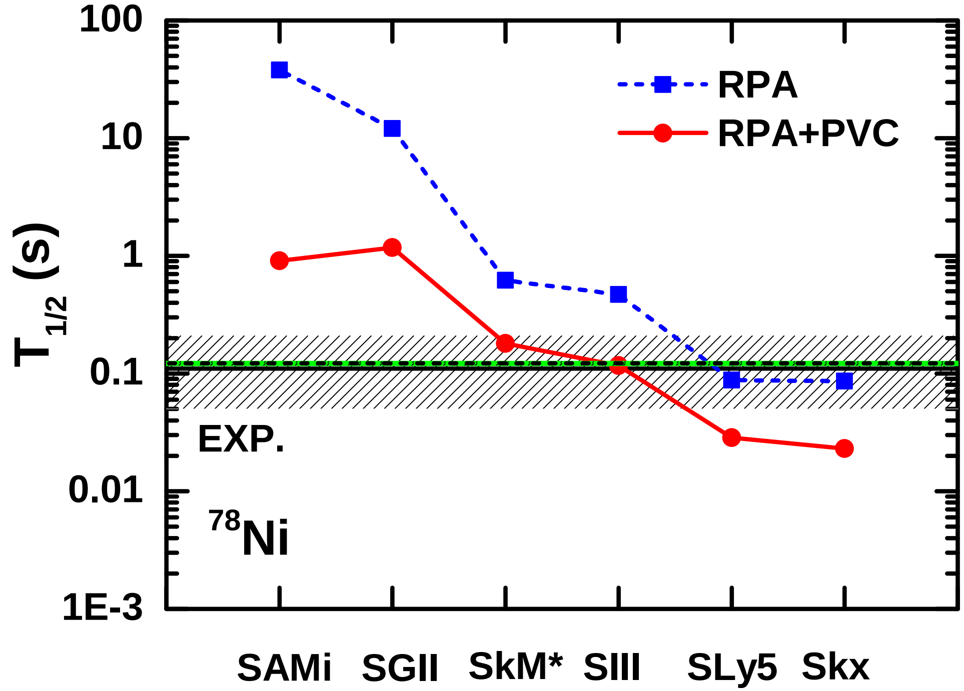
$\Sigma_{\mathrm{GT,GT}}(\omega)$ is the self-energy of the GT state.

Results and discussions — ^{208}*Pb Gamow-Teller strength distribution*



Y. Niu, G. Colò, and E. Vigezzi, PRC 90, 054328 (2014)

Results and discussions — *PVC effects on β-decay half-life*

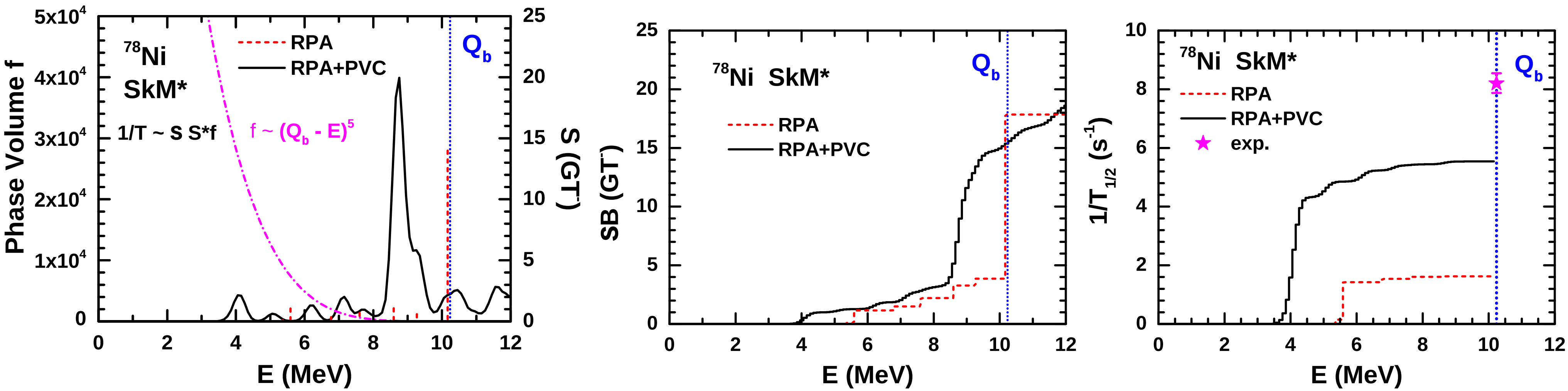


Exp. : Hosmer, et al., PRL 94, 112501, 2005; Xu, et al., PRL 113, 032505, 2014.

- ^{78}Ni: bottle-neck nucleus in r-process
- Skyrme interactions are not well constrained in spin-isospin channel
- PVC reduces half-lives for all interactions.
  Reduction factor R=42 (SAMi), 10 (SGII), 4 (SkM*, SIII, SLy5, Skx)
- SkM*: reproduce well both GTR and β-decay half-life

Y. Niu, Z. Niu, G. Colò, and E. Vigezzi, PRL 114, 142501 (2015)

Results and discussions — *How PVC reduces the half-life*



Exp. : Xu, et al., PRL 113, 032505, 2014

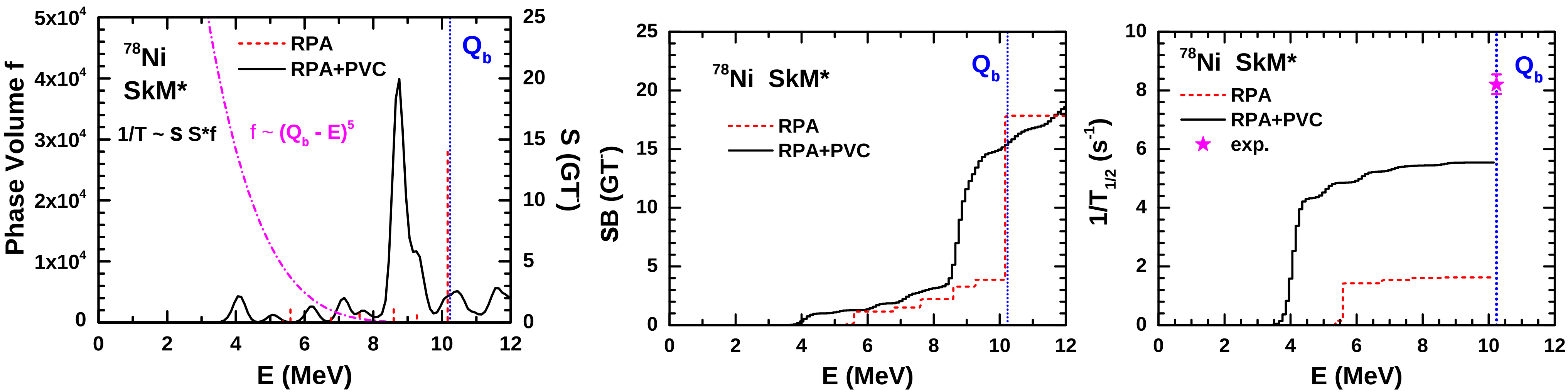
Half life

$$T_{1/2} = \frac{D}{g_A^2 \int^{Q_\beta} S(E) f(Z,\omega) dE}, \tag{6}$$

Phase volume

$$f(Z,\omega_0) = \frac{1}{(m_e c^2)^5} \int_{m_e c^2}^{\omega_0} p_e E_e (\omega_0 - E_e)^2 F_0(Z+1, E_e) dE_e. \tag{7}$$

Results and discussions — *How PVC reduces the half-life*

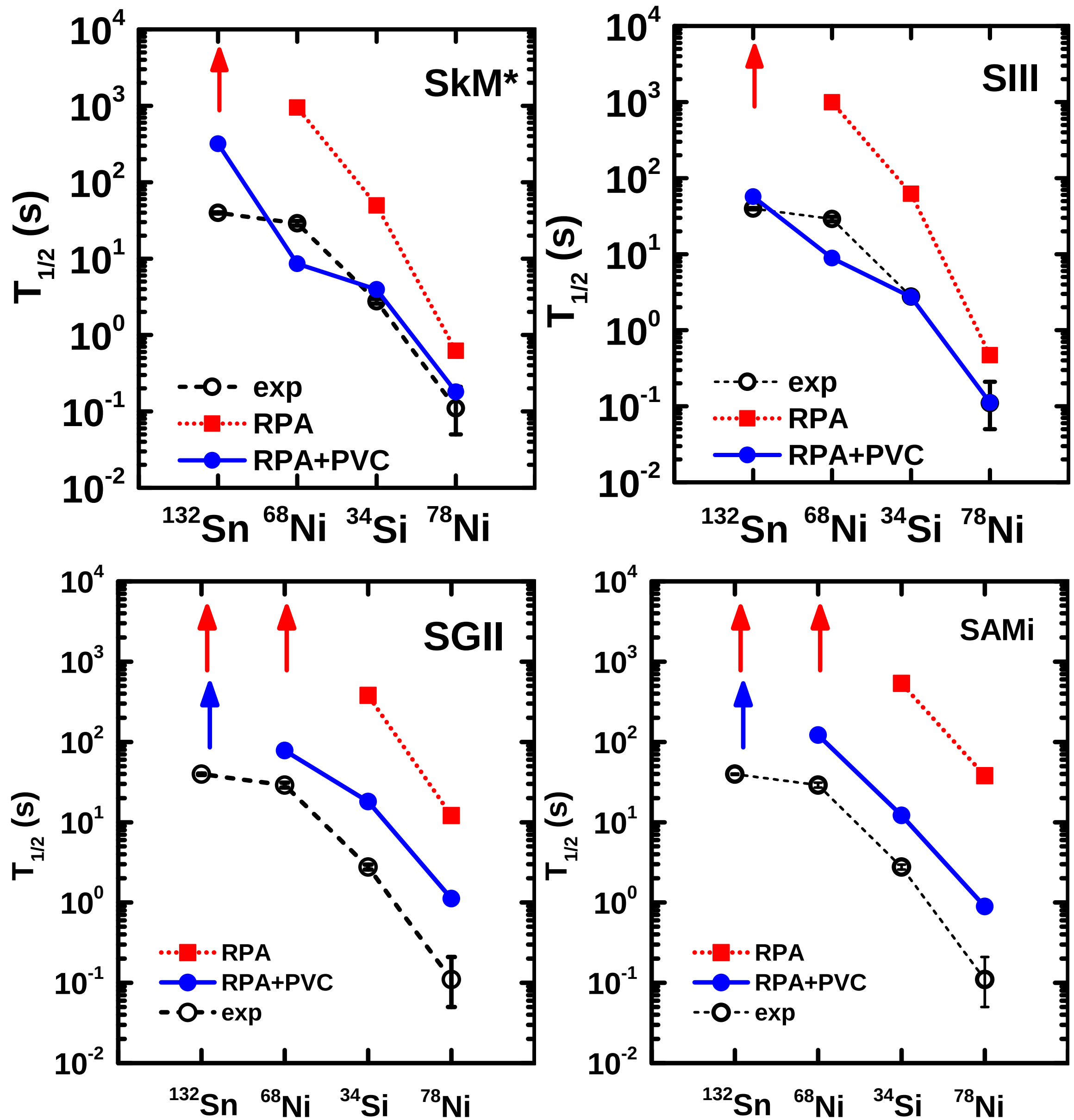


Exp. : Xu, et al., PRL 113, 032505, 2014

- With the inclusion of PVC, the RPA energy is shifted downwards by about 2 MeV.
- The strength of each peak is basically kept conserved as the RPA case.
- Due to the big increase in phase volume, the contribution to the half-life also changes a lot from RPA to PVC.

Although the PVC doesn't change the strength of each peak, it reduces the half-life dramatically by shifting downwards the excitation energy.

Results and discussions — β-decay half-life



Summary and Perspectives

The self-consistent Skyrme RPA + PVC model is developed and used for the calculation of the Gamow-Teller transitions and β-decay half-lives in magic nuclei.

- Coupling with phonons is relevant to producing a more realistic strength distribution characterized by a spreading width. As a result, very good agreement with experiment is obtained for ^{208}Pb.
- Coupling with phonons shifts energy downwards by 1-2 MeV, and hence increases the decay phase space. As a consequence, the β-decay half-life is reduced, and reproduces the experimental data very well.

Perspectives:

- apply in other weak-interaction processes like electron capture.
- include the pairing correlations for open-shell nuclei
- include temperature effect

Acknowledgements

THANK YOU !

Theory framework — *Effective Hamiltonian*

The nuclear excitation configuration space is divided into three subspaces: Q_1, Q_2, P.

- Q_1: 1p-1h within the set $|i\rangle$ — obtained by solving the HF equation
- P: 1p-1h where the particle is in an unbound state $\Rightarrow$ *Escaping Width*
- Q_2: "doorway states" $|N\rangle$: 1p-1h $\otimes$ phonon $\Rightarrow$ *Spreading Width*

In order to work inside the space Q_1, the effective Hamiltonian is taken

$$\begin{aligned}\mathcal{H}(\omega) &= Q_1HQ_1 + W^{\uparrow}(\omega) + W^{\downarrow}(\omega) \\ &= Q_1HQ_1 + Q_1HP\frac{1}{\omega - PHP + i\epsilon}PHQ_1 + Q_1HQ_2\frac{1}{\omega - Q_2HQ_2 + i\epsilon}Q_2HQ_1. \quad (8)\end{aligned}$$

In this work, only the spreading width will be considered, i.e., the effective Hamiltonian

$$\mathcal{H}(\omega) = Q_1HQ_1 + W^{\downarrow}(\omega) \quad (9)$$

will be solved.

Theory framework — *Eigenvalue equation for effective Hamiltonian*

The creator $\mathcal{O}_\nu^\dagger$ of state $|\nu\rangle$, solutions of effective Hamiltonian, is

$$\mathcal{O}_\nu^\dagger = \sum_{\omega_n>0} F_n^{(\nu)} O_n^\dagger - \bar{F}_n^{(\nu)} \bar{O}_n^\dagger, \tag{10}$$

where $O_n^\dagger$ and $\bar{O}_n^\dagger$ are creation operators of RPA states $|n\rangle$ with energy ω_n, and $|\bar{n}\rangle$ with energy $-\omega_n$.

The eigenvalue equation for the effective Hamiltonian is

$$[\mathcal{H}, \mathcal{O}_\nu^\dagger] = (\Omega_\nu - i\frac{\Gamma_\nu}{2})\mathcal{O}_\nu^\dagger. \tag{11}$$

The matrix form is

$$\begin{pmatrix} \mathcal{D} + \mathcal{A}_1(\omega) & \mathcal{A}_2(\omega) \\ -\mathcal{A}_3(\omega) & -\mathcal{D} - \mathcal{A}_4(\omega) \end{pmatrix} \begin{pmatrix} F^{(\nu)} \\ \bar{F}^{(\nu)} \end{pmatrix} = (\Omega_\nu - i\frac{\Gamma_\nu}{2}) \begin{pmatrix} F^{(\nu)} \\ \bar{F}^{(\nu)} \end{pmatrix}, \tag{12}$$

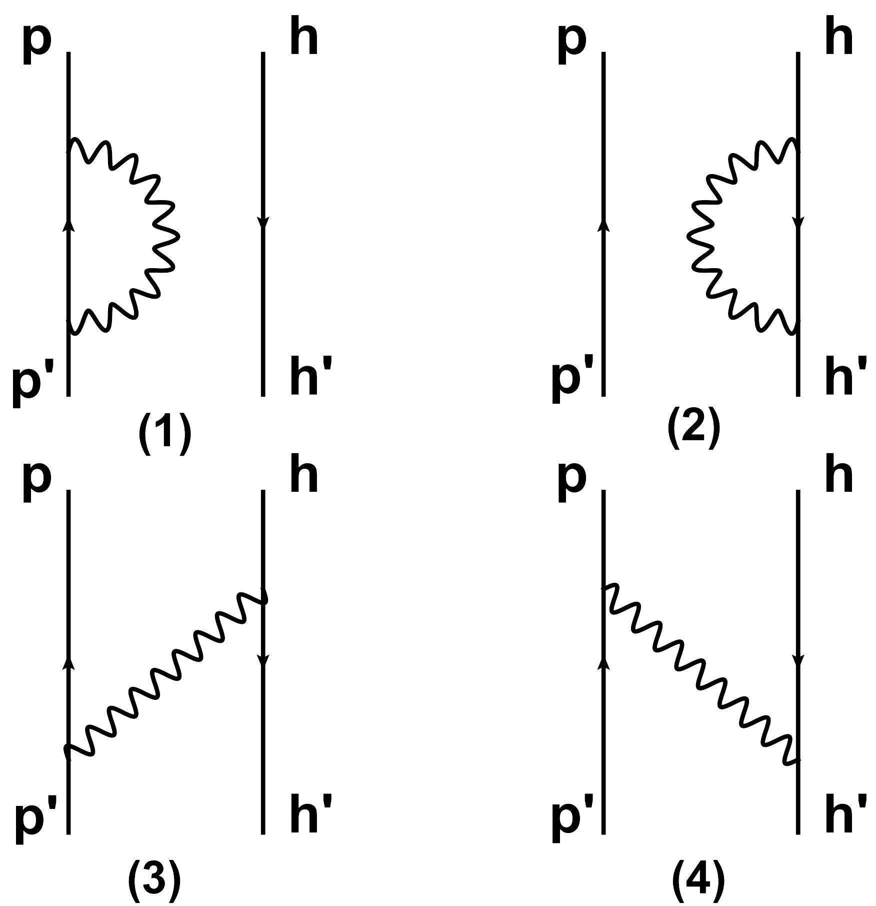
where $\mathcal{D}$ is a diagonal matrix with the RPA eigenvalues, and the $\mathcal{A}_i$ matrices contain the spreading contributions.

Theory framework — *Spreading terms*

The matrix element of the spreading term $W^{\downarrow}$ in the basis of p-h configurations from Q_1 space is

$$W^{\downarrow}_{ph,p'h'}(\omega) = \sum_N \frac{\langle ph|V|N\rangle\langle N|V|p'h'\rangle}{\omega - \omega_N}, \quad (13)$$

which is a sum of the following four diagrams.



Theory framework — *Strength function*

$$S(\omega) = -\frac{1}{\pi}\mathrm{Im}\sum_{\nu}\langle 0|\hat{O}_{\mathrm{GT}\pm}|\nu\rangle^2 \frac{1}{\omega - \Omega_\nu + i(\frac{\Gamma_\nu}{2} + \Delta)} \quad (14)$$

Diagonal Approximation

We could take Lorentzian form

$$S(\omega) = \sum_n \frac{1}{\pi}\frac{\frac{\Gamma_n(\omega)}{2} + \Delta}{(\omega - \Omega_n(\omega))^2 + (\frac{\Gamma_n(\omega)}{2} + \Delta)^2} B_n, \quad B_n = |\langle 0|\hat{O}_{\mathrm{GT}-}|n\rangle|^2, \quad (15)$$

or the Gaussian form of strength distribution

$$S(\omega) = \sum_n \frac{1}{\sigma_n\sqrt{2\pi}} \mathrm{e}^{-\frac{(\omega-\Omega_n)^2}{2\sigma_n^2}} B_n, \quad \sigma_n = (\frac{\Gamma_n(\omega)}{2} + \Delta)/\sqrt{2\ln 2}, \quad (16)$$

where

$$\Omega_n(\omega) = E_{\mathrm{RPA}} + \mathrm{Re}[(\mathcal{A}_1)_{\mathrm{GTR,GTR}}(\omega)] \quad (17)$$

and

$$\Gamma_n(\omega) = -2\mathrm{Im}[(\mathcal{A}_1)_{\mathrm{GTR,GTR}}(\omega)]. \quad (18)$$

Appendix — *Spreading terms*

These four diagrams are expressed by

$$W^{\downarrow J}(1) = \delta_{hh'}\delta_{j_p j_{p'}} \sum_{p'',nL} \frac{1}{\omega - (\omega_n + \epsilon_{p''} - \epsilon_h) + i\Delta} \frac{\langle n_p j_p||V||j_{p''}, nL\rangle \langle n_{p'} j_{p'}||V||j_{p''}, nL\rangle^*}{\hat{j}_p^2} \quad (19)$$

$$W^{\downarrow J}(2) = \delta_{pp'}\delta_{j_h j_{h'}} \sum_{h'',nL} \frac{1}{\omega - (\omega_n - \epsilon_{h''} + \epsilon_p) + i\Delta} \frac{\langle n_h j_h||V||j_{h''}, nL\rangle \langle n_{h'} j_{h'}||V||j_{h''}, nL\rangle^*}{\hat{j}_h^2} \quad (20)$$

$$W^{\downarrow J}(3) = \sum_{nL} \frac{(-)^{j_p - j_{h'} + J + L}}{\omega - (\omega_n + \epsilon_p - \epsilon_{h'}) + i\Delta} \begin{Bmatrix} j_p & j_h & J \\ j_{h'} & j_{p'} & L \end{Bmatrix} \langle j_{p'}||V||j_p, nL\rangle \langle j_{h'}||V||j_h, nL\rangle \quad (21)$$

$$W^{\downarrow J}(4) = \sum_{nL} \frac{(-)^{j_{p'} - j_h + J + L}}{\omega - (\omega_n + \epsilon_{p'} - \epsilon_h) + i\Delta} \begin{Bmatrix} j_p & j_h & J \\ j_{h'} & j_{p'} & L \end{Bmatrix} \langle j_p||V||j_{p'}, nL\rangle \langle j_h||V||j_{h'}, nL\rangle \quad (22)$$

The evaluation of the reduced matrix element

$$\langle i||V||j, nL\rangle = \sqrt{2L+1} \sum_{ph} X_{ph}^{nL} V_L(ihjp) + (-)^{L + j_h - j_p} Y_{ph}^{nL} V_L(ipjh), \quad (23)$$

where V_L is the p-h coupled matrix element,

$$V_L(ihjp) = \sum_{\text{all } m} (-)^{j_j - m_j + j_h - m_h} \langle j_i m_i j_j - m_j|LM\rangle \langle j_p m_p j_h - m_h|LM\rangle \langle j_i m_i, j_h m_h|V|j_j m_j, j_p m_p\rangle. \quad (24)$$

Appendix — *Spreading terms in RPA basis*

To transform the spreading terms from p-h basis of Q_1 to RPA basis, we get the matrices elements $\mathcal{A}_i$.

$$(\mathcal{A}_1)^{\rm spr}_{mn} = \sum_{ph,p'h'} W^{\downarrow}_{ph,p'h'}(\omega) X^{(m)}_{ph} X^{(n)}_{p'h'} + W^{\downarrow *}_{ph,p'h'}(-\omega) Y^{(m)}_{ph} Y^{(n)}_{p'h'}, \qquad (25)$$

$$(\mathcal{A}_2)^{\rm spr}_{mn} = \sum_{ph,p'h'} W^{\downarrow}_{ph,p'h'}(\omega) X^{(m)}_{ph} Y^{(n)}_{p'h'} + W^{\downarrow *}_{ph,p'h'}(-\omega) Y^{(m)}_{ph} X^{(n)}_{p'h'} \qquad (26)$$

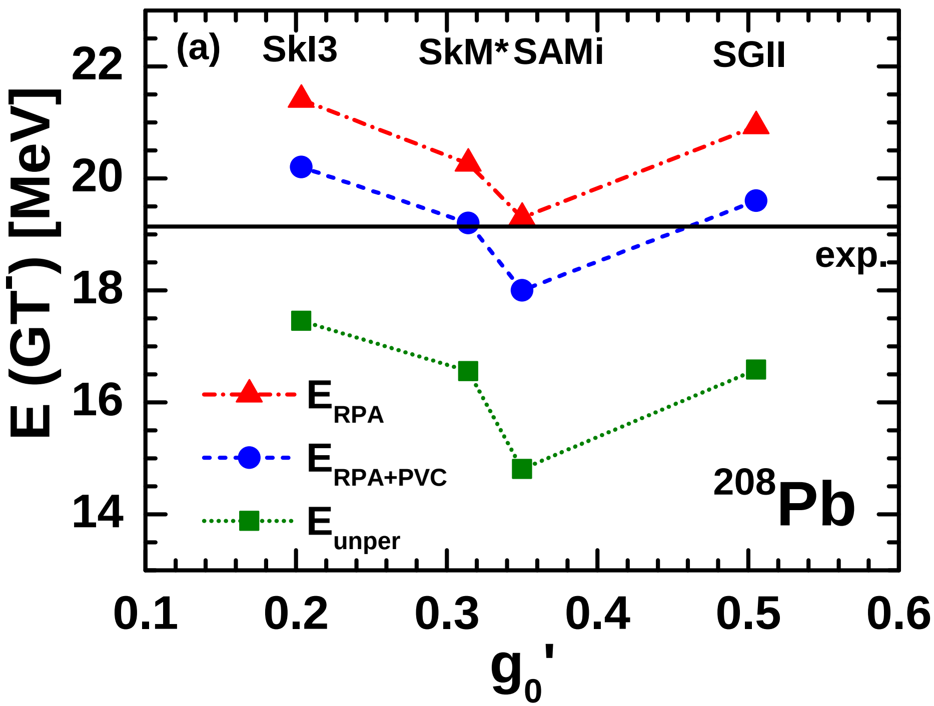
$$(\mathcal{A}_3)^{\rm spr}_{mn} = \sum_{ph,p'h'} W^{\downarrow}_{ph,p'h'}(\omega) Y^{(m)}_{ph} X^{(n)}_{p'h'} + W^{\downarrow *}_{ph,p'h'}(-\omega) X^{(m)}_{ph} Y^{(n)}_{p'h'}, \qquad (27)$$

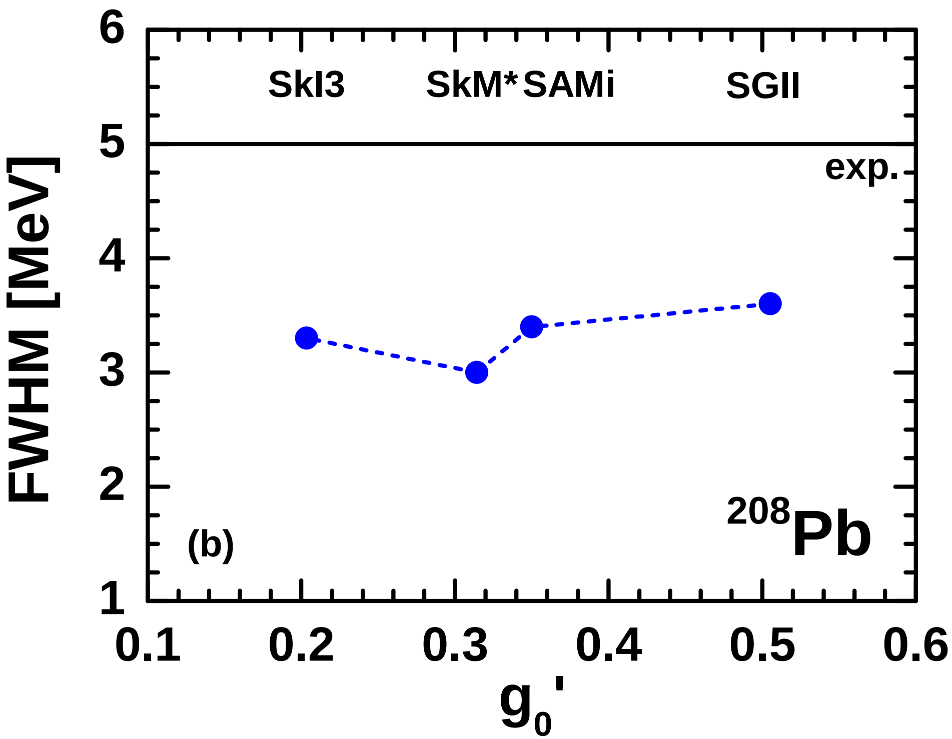
$$(\mathcal{A}_4)^{\rm spr}_{mn} = \sum_{ph,p'h'} W^{\downarrow}_{ph,p'h'}(\omega) Y^{(m)}_{ph} Y^{(n)}_{p'h'} + W^{\downarrow *}_{ph,p'h'}(-\omega) X^{(m)}_{ph} X^{(n)}_{p'h'}. \qquad (28)$$

Numerical details

- HF equation: solved in coordinate space $R_{box} = 21$ fm, $dr = 0.1\ fm$.
- RPA configuration space: $\epsilon_p \leq 100$ MeV
- Phonons: $0^+, 1^-, 2^+, 3^-, 4^+, 5^-, 6^+$
- Phonon energy $\omega \leq 20$ MeV
- Phonon strength (isoscalar or isovector) $\geq 5\%$
- intermediate particle in the door-way state: $\epsilon_p \leq 100$ MeV

Results and discussions — *PVC effects on GTR with different interactions*



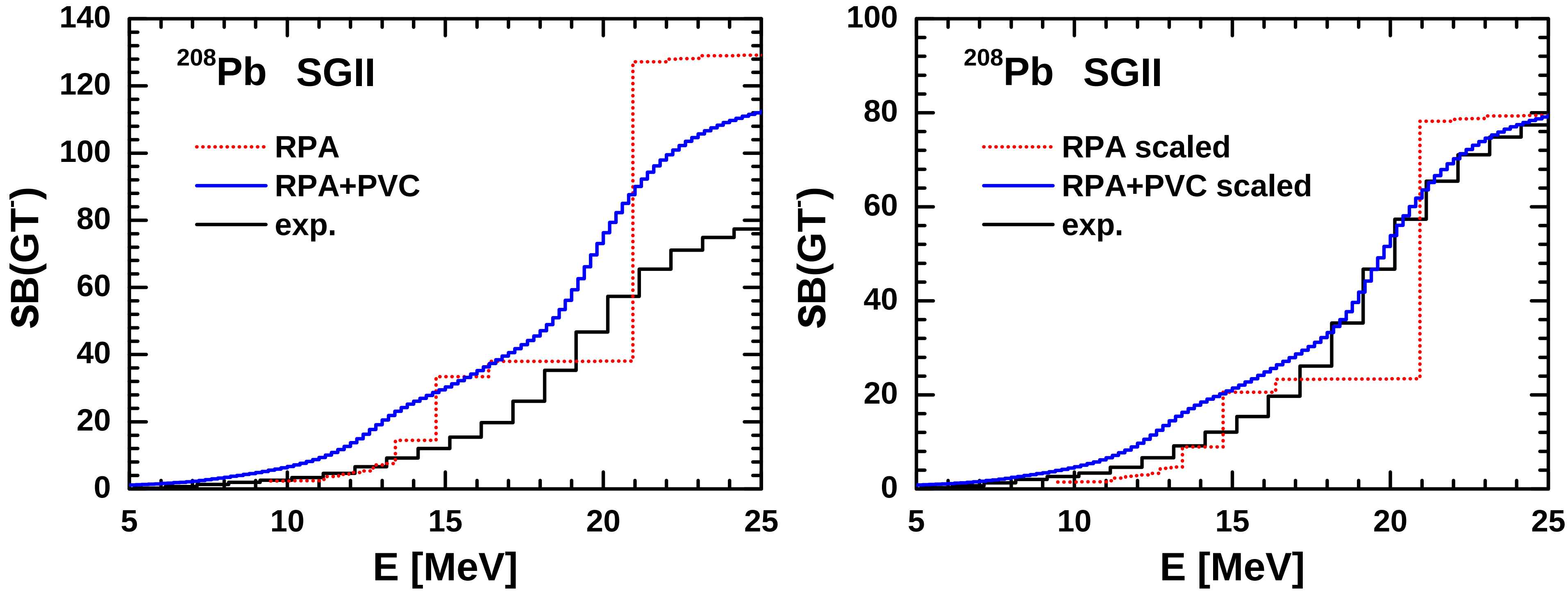


PVC Effects

weakly dependent on the interactions

- peak energy is shifted downwards by 1.2 MeV
- A width of $\sim$ 3.5 MeV is acquired

Results and discussions — ^{208}*Pb Gamow-Teller cumulative sum*



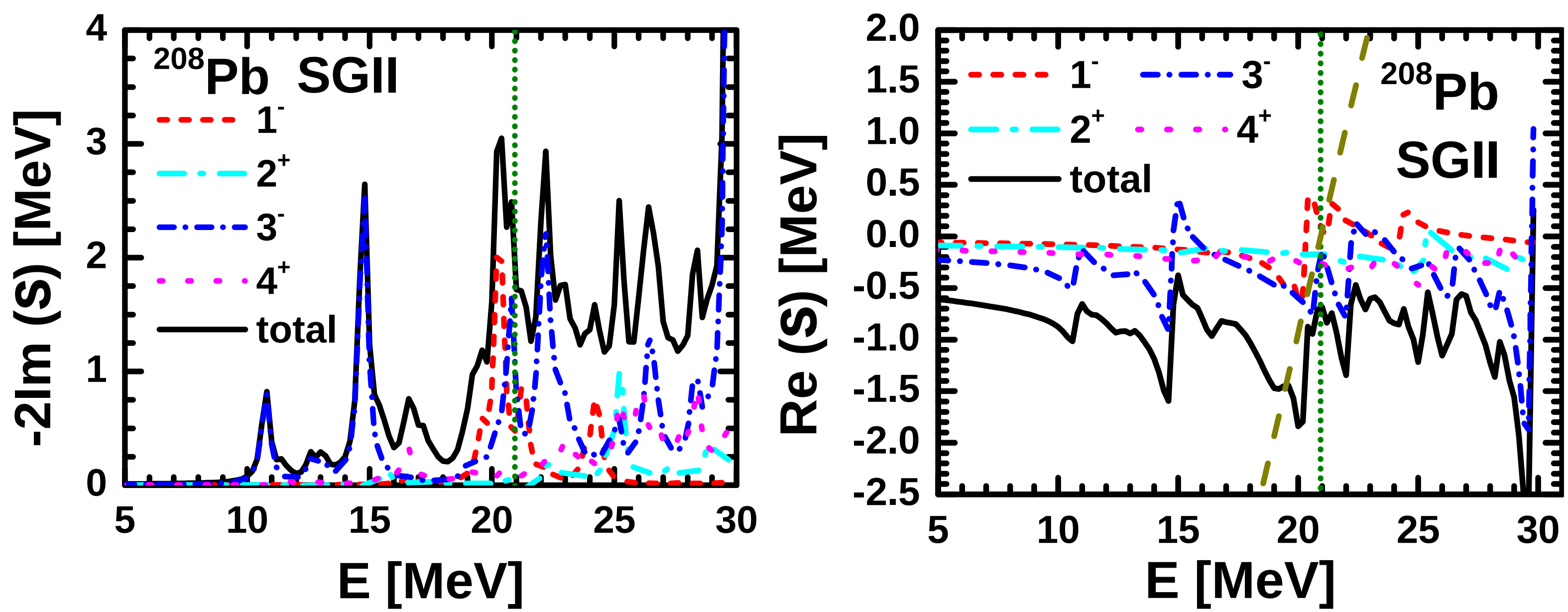
	RPA	RPA+PVC
Ikeda sum rule	99.99%	95.2% (97.3% for $\Delta = 0.2$ MeV)
Above 25 MeV	3%	15%

- $E = 25$ MeV : exp. strength/RPA+PVC strength $\sim$ 71%

Appendix — ^{208}Pb Self-Energy

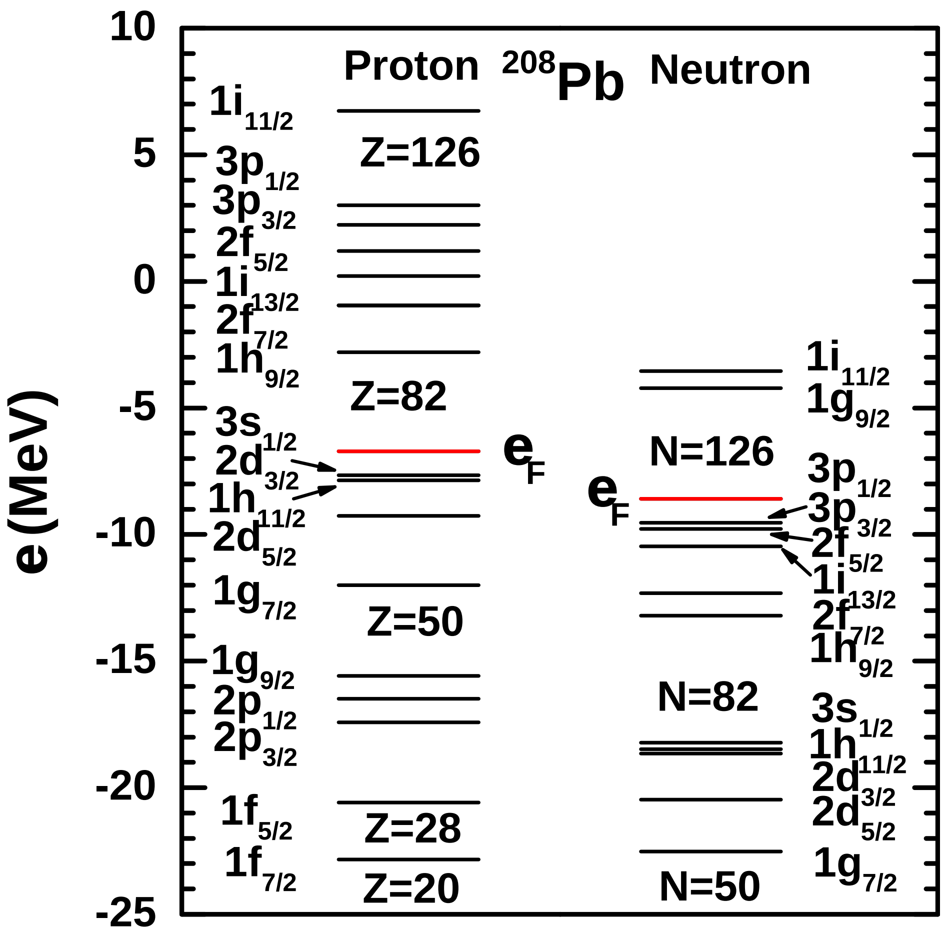
$$\Sigma = (\mathcal{A}_1)^{\rm spr}_{GTR,GTR} = \sum_{ph,p'h'} W^{\downarrow}_{ph,p'h'}(\omega) X^{(GTR)}_{ph} X^{(GTR)}_{p'h'} + W^{\downarrow *}_{ph,p'h'}(-\omega) Y^{(GTR)}_{ph} Y^{(GTR)}_{p'h'}$$

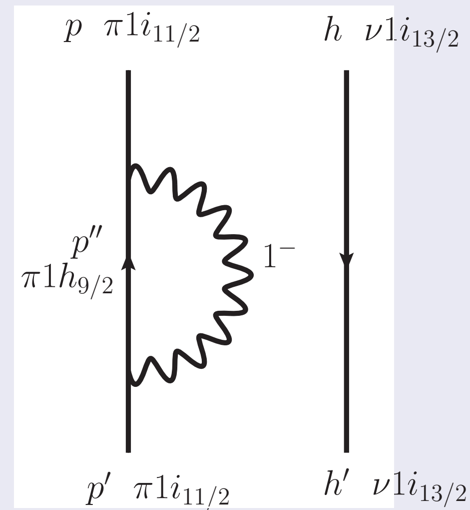
$Re(\Sigma)(E_{GTR})$: energy shift $\quad -2Im(\Sigma)(E_{GTR})$: width



- $2^+, 4^+$ phonons (isoscalar): not important $\Leftarrow$ cancellation betwwen W_1, W_2 and W_3, W_4 diagrams
- $1^-, 3^-$ phonons (isovector): most important phonons $\Leftarrow$ no cancellation betwwen W_1, W_2 and W_3, W_4 diagrams

Appendix — ^{208}Pb Why are 1^- phonons important?





Energy denominator

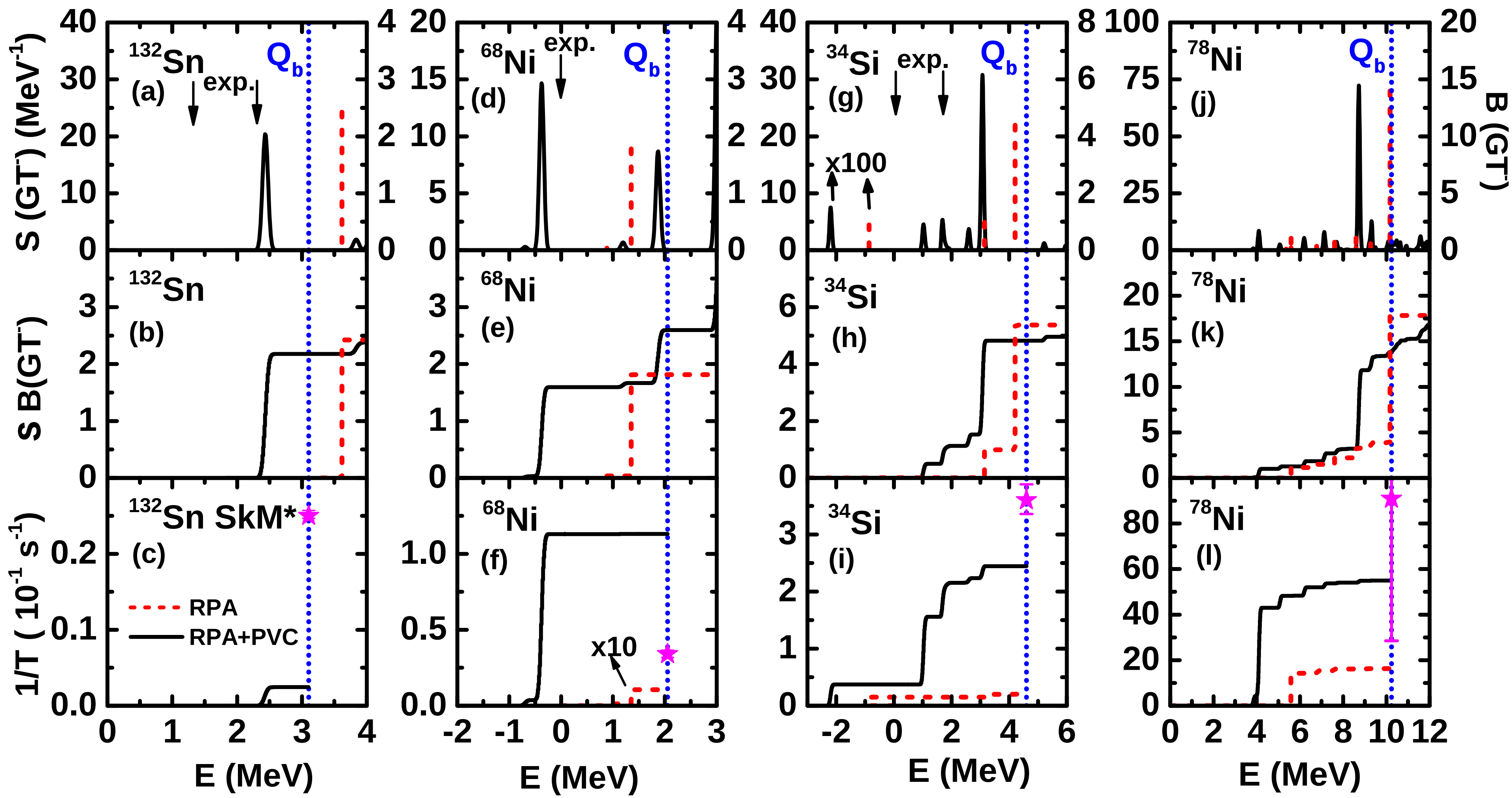
$$\frac{1}{E_{GTR} - (\omega_{nL} + \epsilon_{p''} - \epsilon_h) + i\Delta}$$

$$= \frac{1}{\epsilon_p - \epsilon_h + \Delta E - (\omega_{nL} + \epsilon_{p''} - \epsilon_h) + i\Delta}$$

$$= \frac{1}{\epsilon_p - \epsilon_{p''} + \Delta E - \omega_{nL} + i\Delta}$$

1^- phonon with $\omega_{1^-} = 12.63$ MeV (GDR) is important!

Results and discussions — *How PVC reduces the half-life*



Exp. : AME2012, Chinese Physics C 36, 1603 (2012)

Y. Niu, Z. Niu, G. Colò, and E. Vigezzi, PRL 114, 142501 (2015)