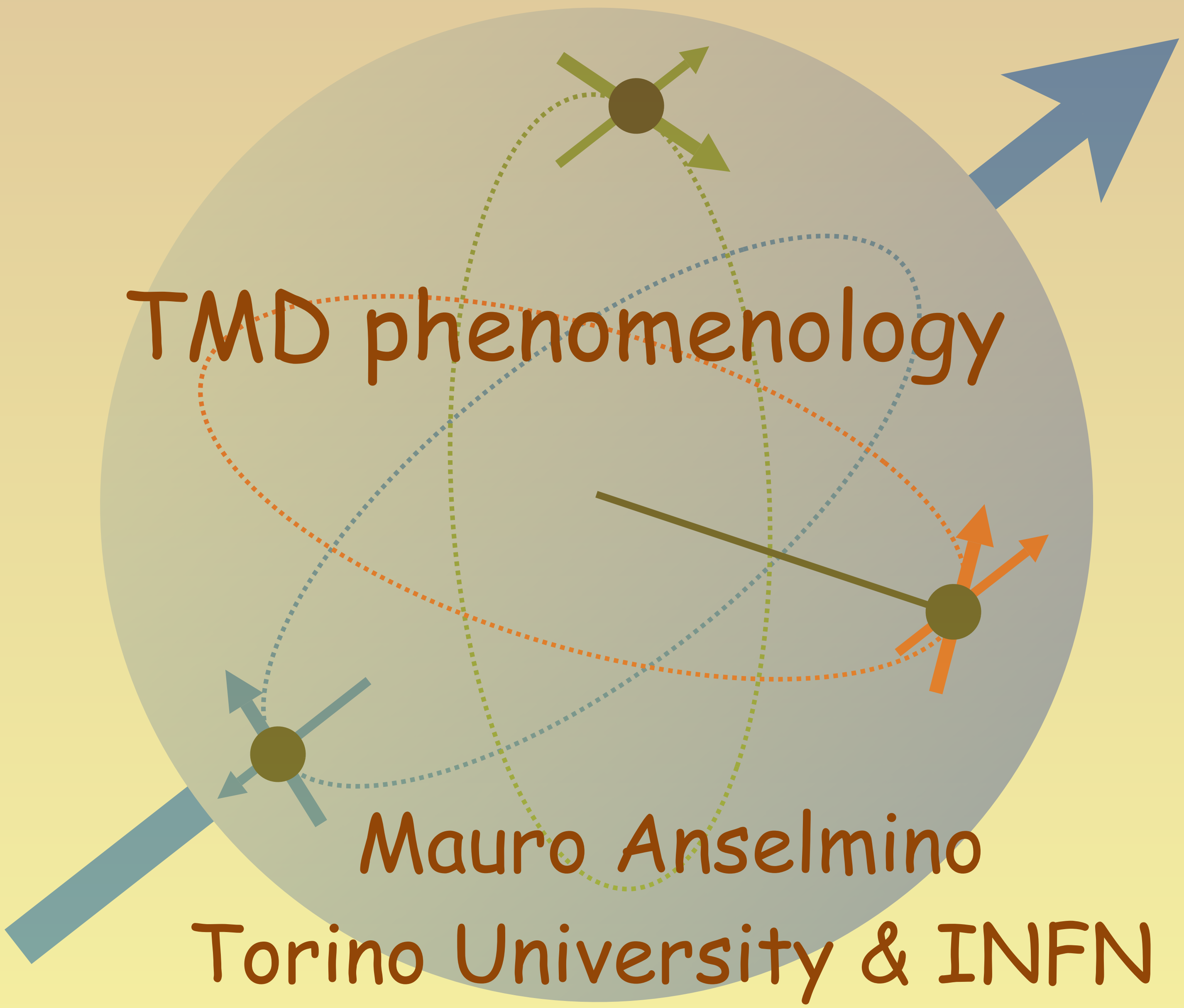
Light Cone 2015

INFN Frascati National Laboratories September 21-25, 2015

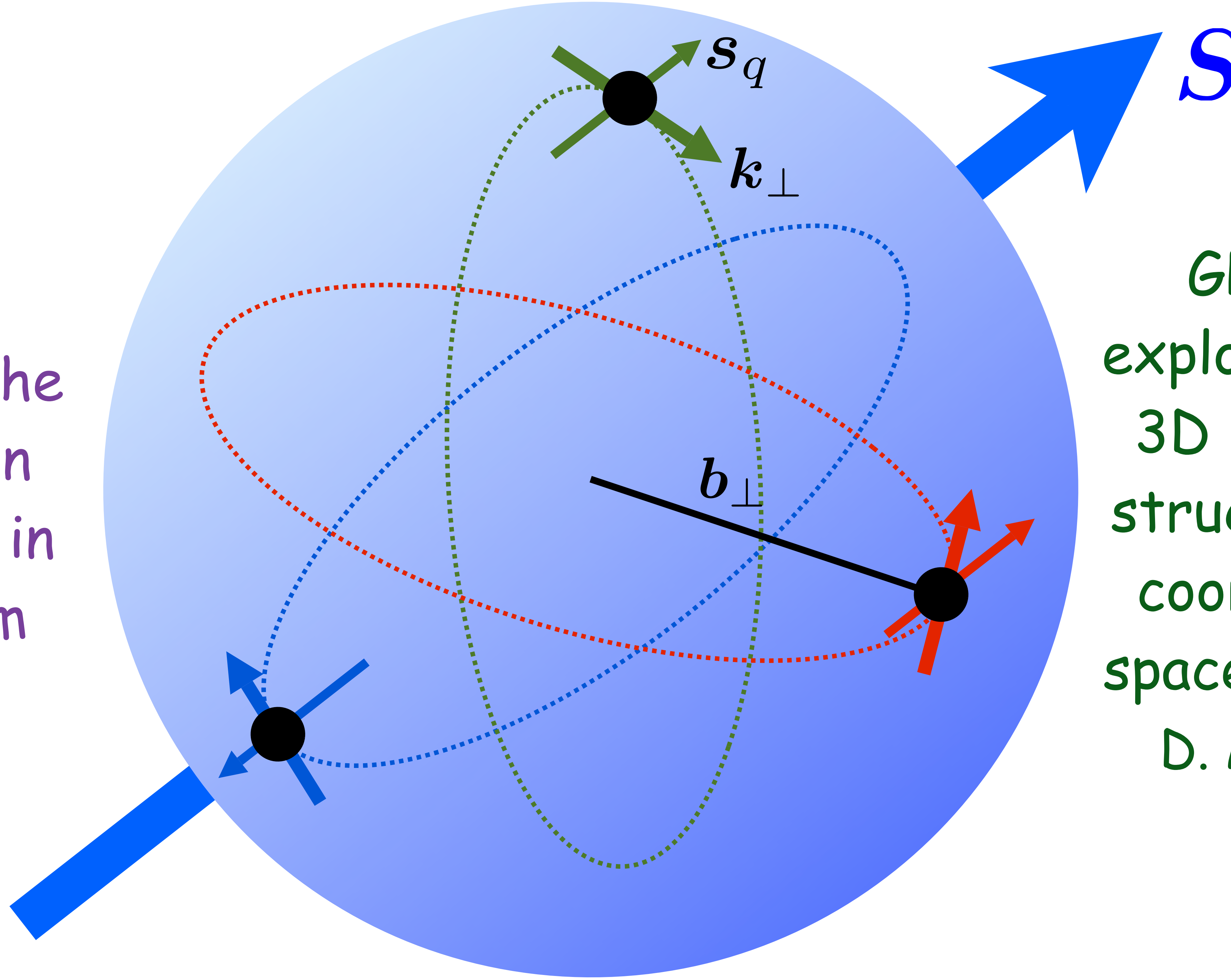


TMD phenomenology

Mauro Anselmino
Torino University & INFN

September 21, 2015

TMDs =
exploring the
3D nucleon
structure, in
momentum
space



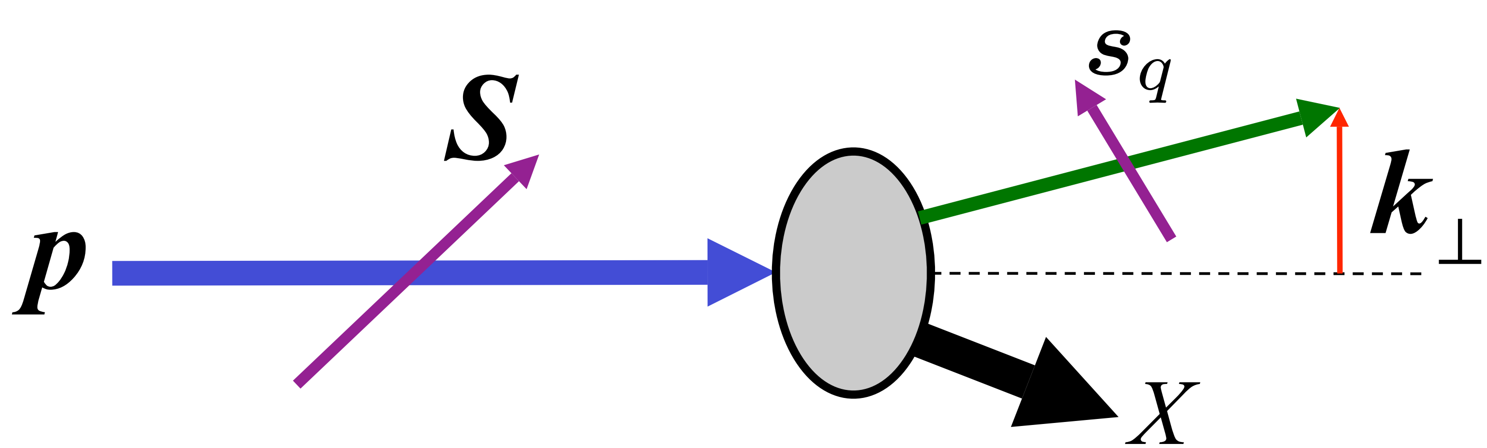
GPDs =
exploring the
3D nucleon
structure, in
coordinate
space (talk by
D. Müller)

the nucleon is still a very mysterious object .....
and the most abundant piece of matter in the Universe

simple parton model

TMDs = Transverse Momentum Dependent
Parton Distribution Functions (TMD-PDF) or
Transverse Momentum Dependent
Fragmentation Functions (TMD-FF)

TMD-PDFs give the number density of partons, with their intrinsic motion and spin, inside a fast moving proton, with its spin.



$\boldsymbol{S}\cdot(\boldsymbol{p}\times\boldsymbol{k}_\perp)$ $\quad$ $\boldsymbol{s}_q\cdot(\boldsymbol{p}\times\boldsymbol{k}_\perp)$ $\quad$ $\boldsymbol{S}\cdot\boldsymbol{s}_q$ $\quad\cdots$

"Sivers effect" "Boer-Mulders effect"

there are 8 independent TMD-PDFs

$f_1^q(x, \boldsymbol{k}_\perp^2)$ unpolarized quarks in unpolarized protons
unintegrated unpolarized distribution

$g_{1L}^q(x, \boldsymbol{k}_\perp^2)$ correlate s_L of quark with S_L of proton
unintegrated helicity distribution

$h_{1T}^q(x, \boldsymbol{k}_\perp^2)$ correlate s_T of quark with S_T of proton
unintegrated transversity distribution

only these survive in the collinear limit

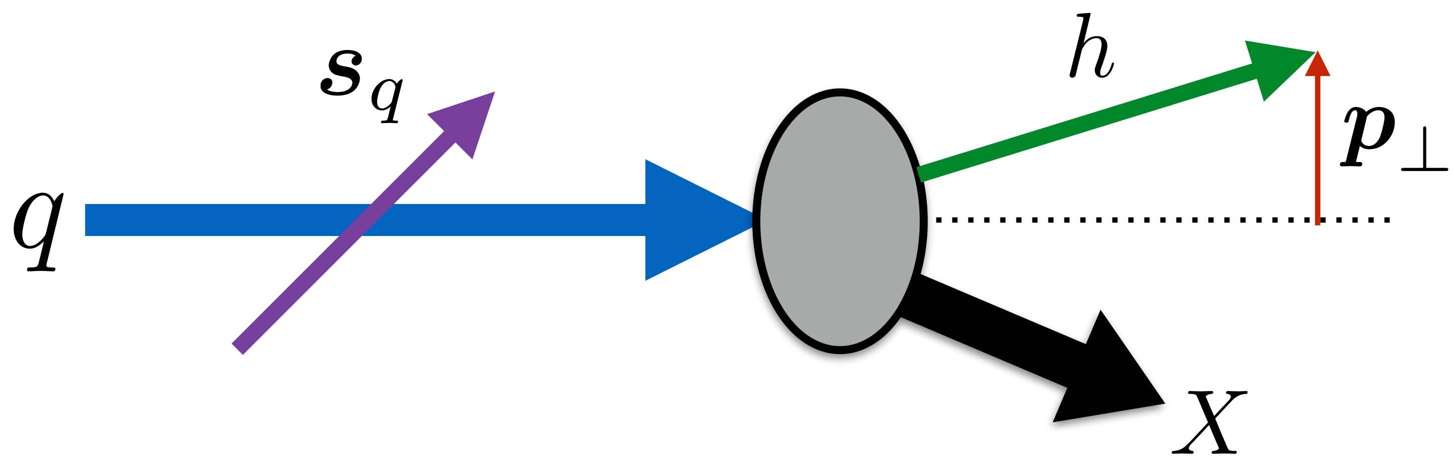
$f_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$ correlate $k_\perp$ of quark with S_T of proton (Sivers)

$h_1^{\perp q}(x, \boldsymbol{k}_\perp^2)$ correlate $k_\perp$ and s_T of quark (Boer-Mulders)

$g_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$ $h_{1L}^{\perp q}(x, \boldsymbol{k}_\perp^2)$ $h_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$

different double-spin correlations

TMD-FFs give the number density of hadrons, with their momentum, originated in the fragmentation of a fast moving parton, with its spin.



$\boldsymbol{s}_q \cdot (\boldsymbol{p}_q \times \boldsymbol{p}_\perp)$ "Collins effect"

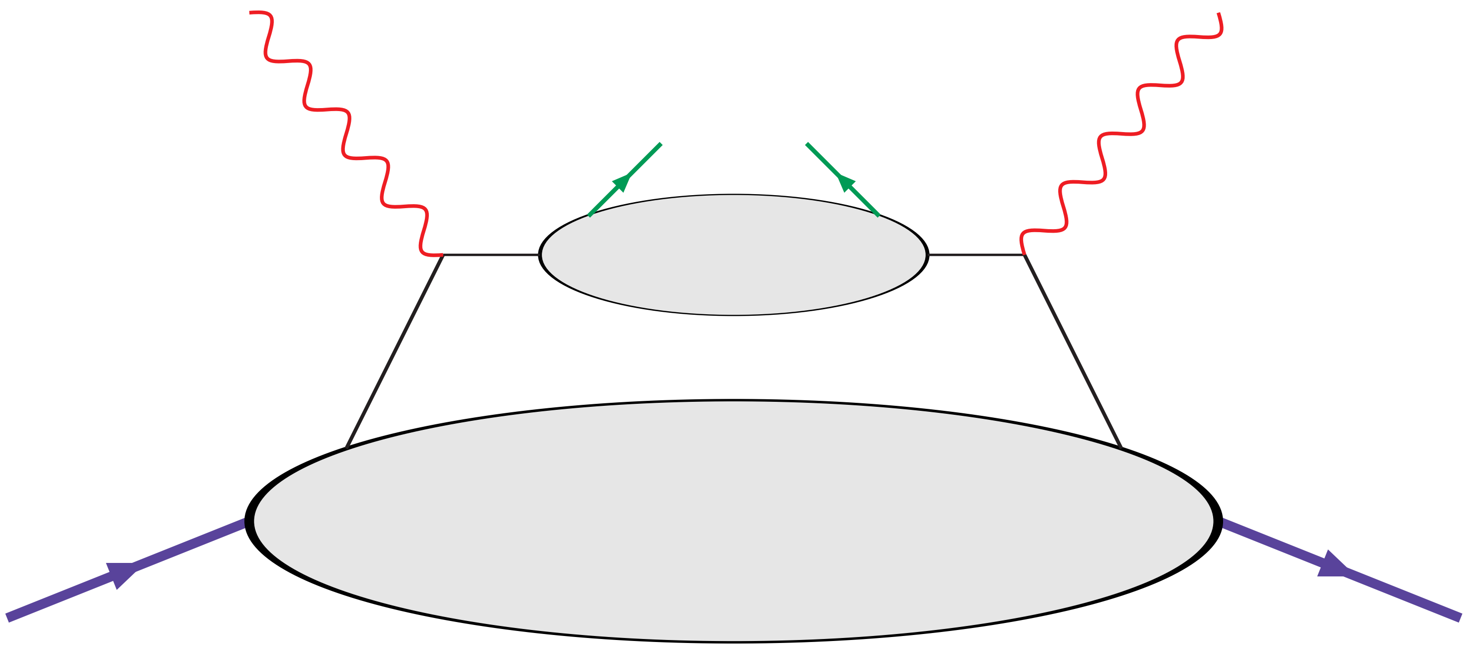
there are 2 independent TMD-FFs for spinless hadrons

$D_1^q(z, \boldsymbol{p}_\perp^2)$ unpolarized hadrons in unpolarized quarks unintegrated fragmentation function

$H_1^{\perp q}(z, \boldsymbol{p}_\perp^2)$ correlate $p_\perp$ of hadron with s_T of quark (Collins)

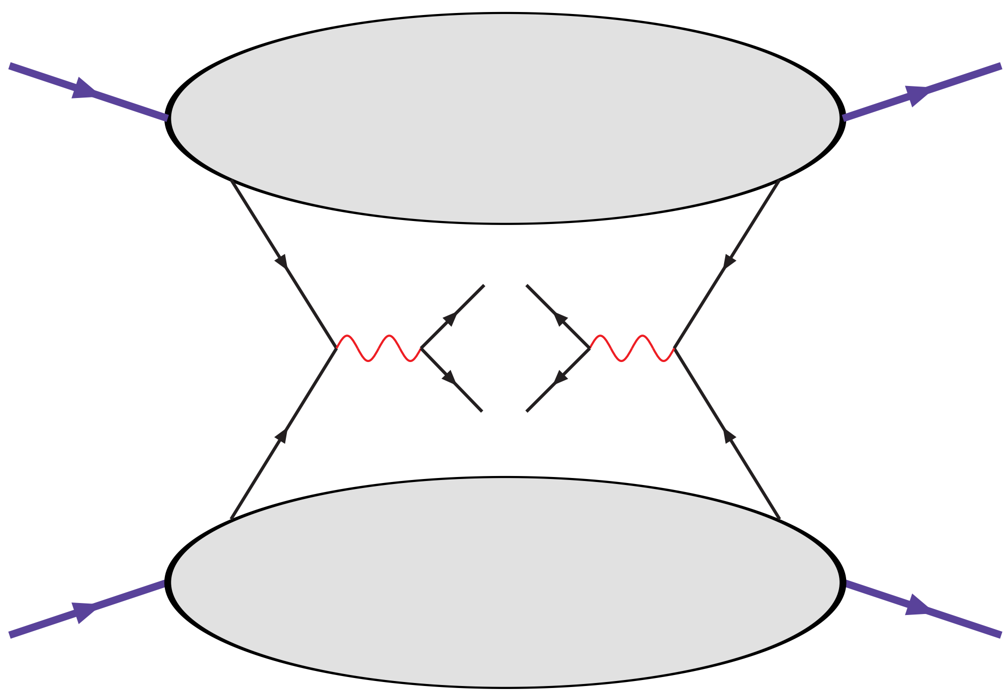
how to "measure" TMDs?

needs processes which relate physical observables
to parton intrinsic motion



SIDIS

$\ell\, N \to \ell\, h\, X$



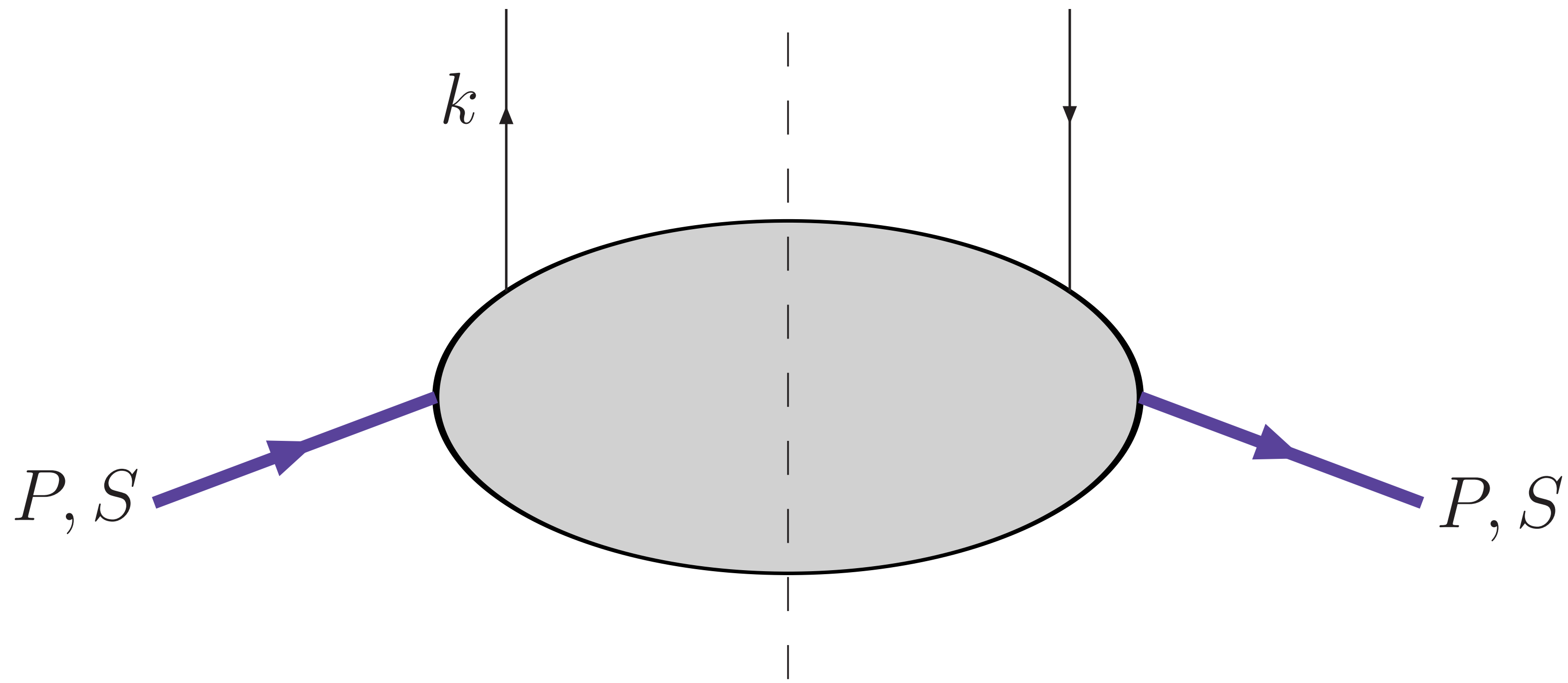
Drell-Yan processes

$p\, N \to \ell^+ \ell^- X$

a similar diagram for $e^+ e^- \to h_1\, h_2\, X$

and, possibly, for $p\, N \to h\, X$

The nucleon correlator, in collinear configuration: 3 distribution functions



$$\Phi_{ij}(k;P,S) = \sum_X \int \frac{d^3\boldsymbol{P}_X}{(2\pi)^3\, 2E_X} (2\pi)^4\, \delta^4(P-k-P_X) \langle PS|\overline{\Psi}_j(0)|X\rangle\langle X|\Psi_i(0)|PS\rangle$$

$$= \int d^4\xi\, e^{ik\cdot\xi} \langle PS|\overline{\Psi}_j(0)\Psi_i(\xi)|PS\rangle$$

$$\Phi(x,S) = \frac{1}{2}\left[f_1(x) \not{n}_+ + S_L\, g_{1L}(x) \gamma^5 \not{n}_+ + h_{1T}\, i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu\right]$$

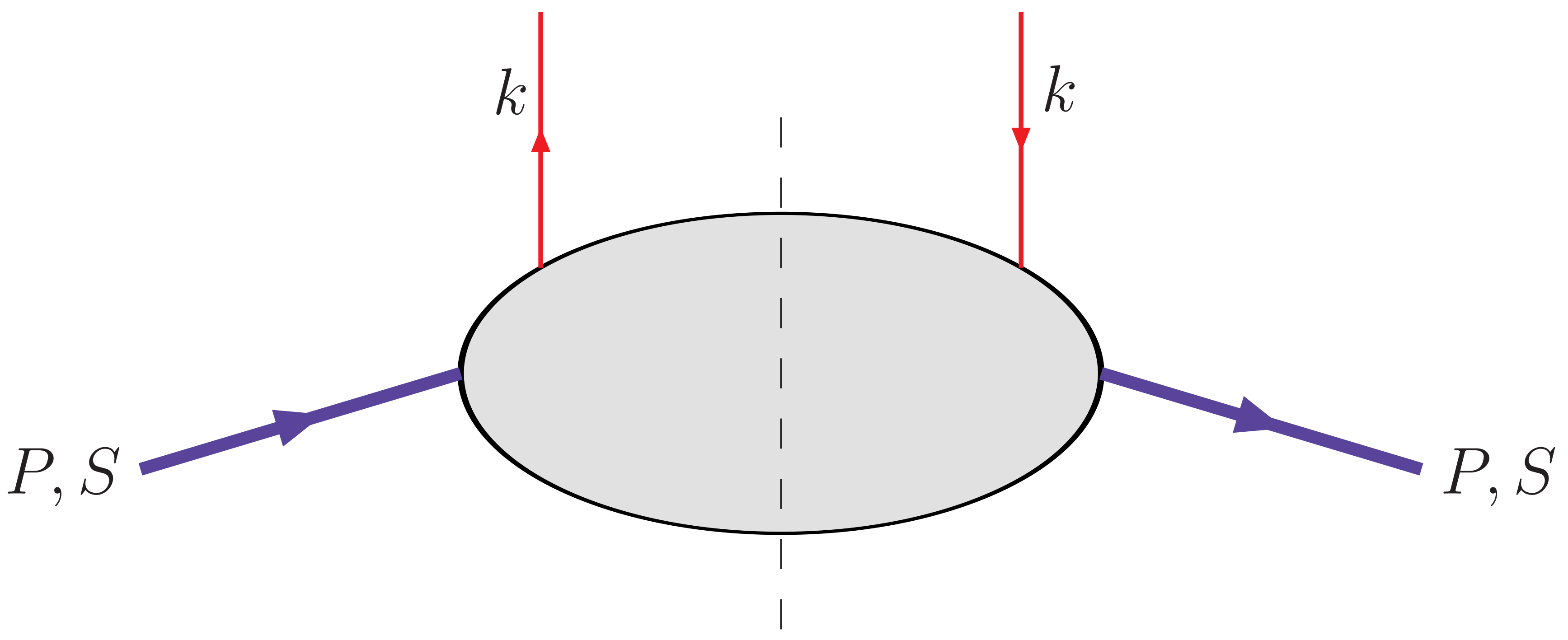
$f_1(x)$: q $\quad$ $g_{1L}(x)$: Δq $\quad$ h_{1T}: Δ_Tq

TMD-PDFs: the leading-twist correlator, with intrinsic $k_\perp$, contains 8 independent functions

$$\Phi(x, \boldsymbol{k}_\perp) = \frac{1}{2}\Big[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \left(S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp \right) \gamma^5 \not{n}_+$$

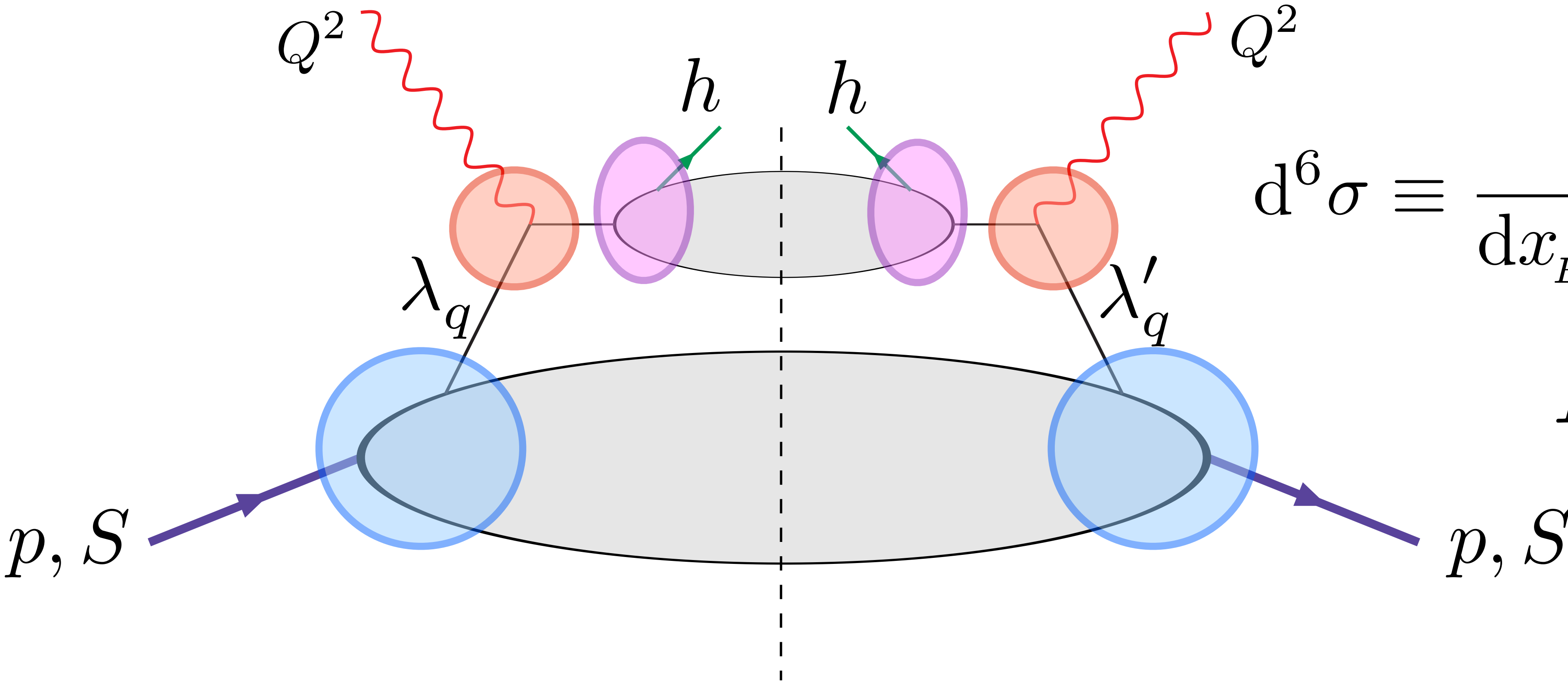
$$+ h_{1T} i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \left(S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp \right) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M}$$

$$+ h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \Big]$$



with partonic interpretation

TMDs in SIDIS



$$d^6\sigma \equiv \frac{d^6\sigma^{\ell p^\uparrow \to \ell h X}}{dx_B \, dQ^2 \, dz_h \, d^2\boldsymbol{P}_T \, d\phi_S}$$

$$\boldsymbol{P}_T = \boldsymbol{p}_\perp + z\boldsymbol{k}_\perp$$

TMD factorization holds at large Q^2, and $P_T \approx k_\perp \approx \Lambda_{QCD}$

Two scales: $P_T \ll Q^2$

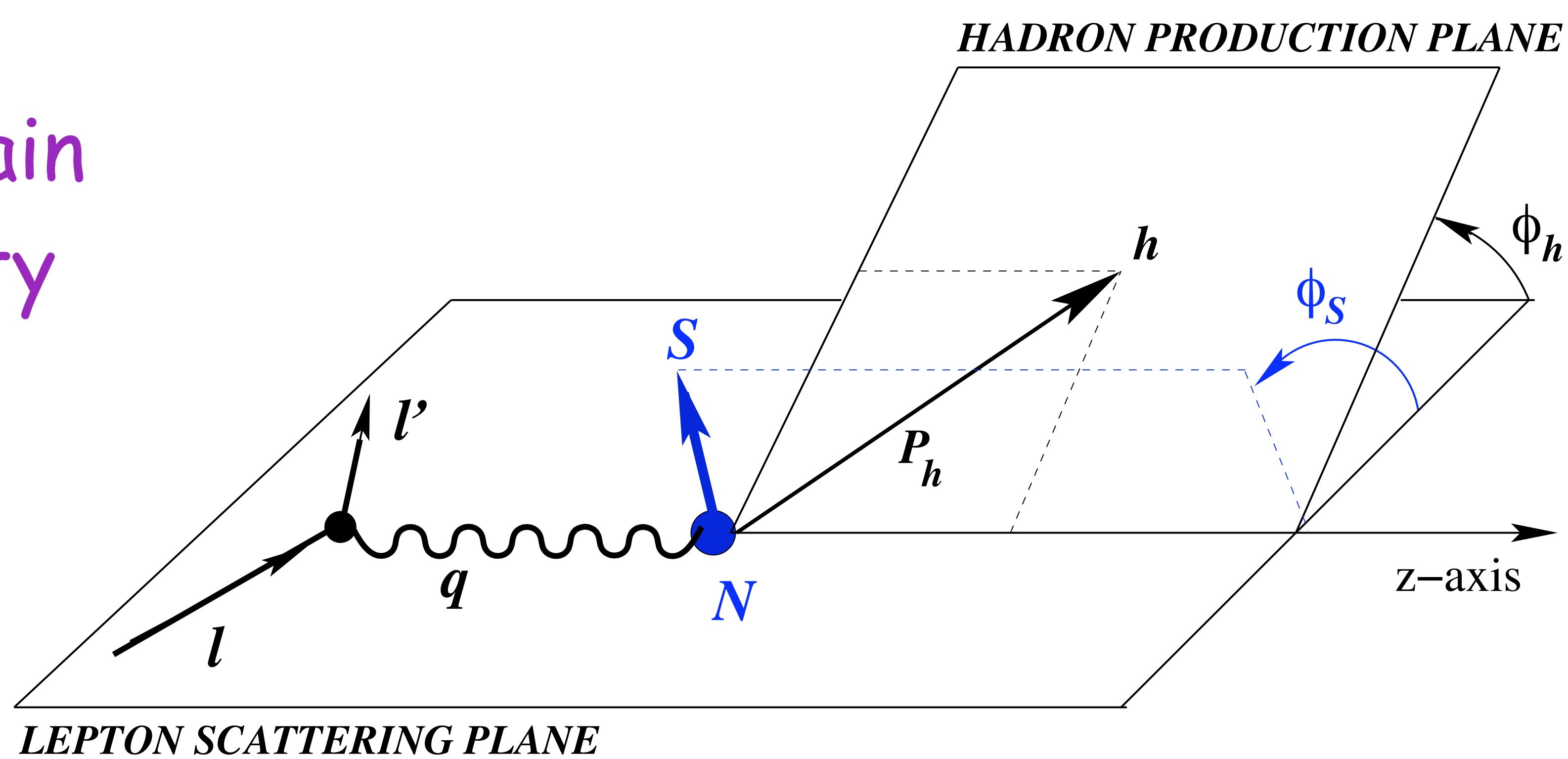
TMD-PDFs — hard scattering — TMD-FFs

$$d\sigma^{\ell p \to \ell h X} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes d\hat{\sigma}^{\ell q \to \ell q}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)$$

(Collins, Soper, Ji, J.P. Ma, Yuan, Qiu, Vogelsang, Collins, Metz...)

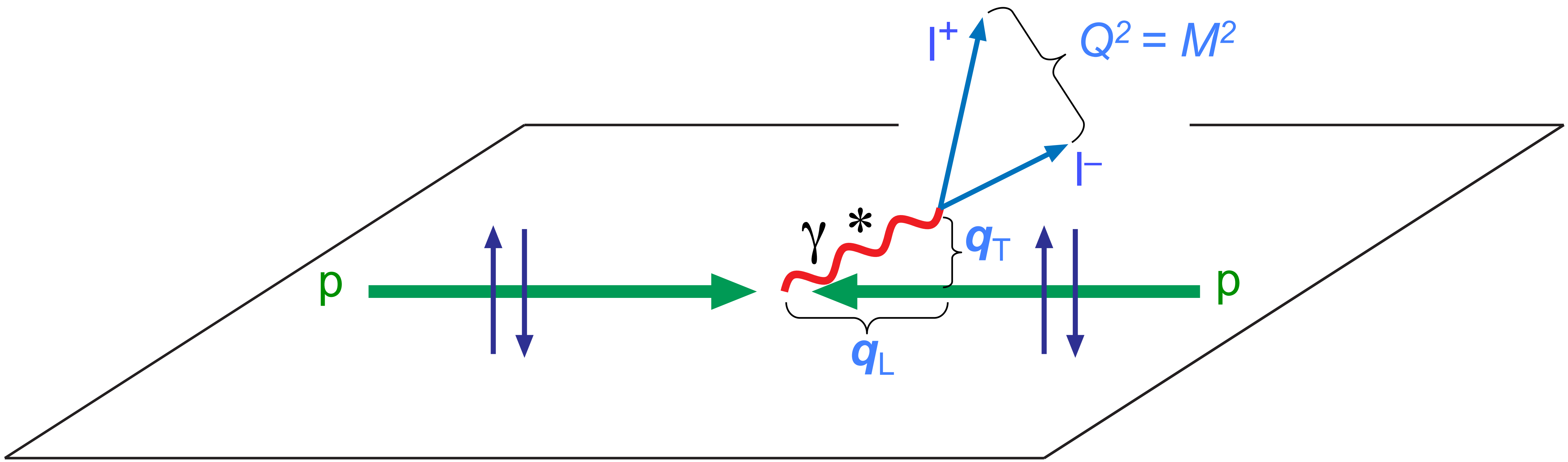
$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} = \; & F_{UU} + \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\cos\phi\, F_{UU}^{\cos\phi} + \lambda\,\frac{1}{Q}\sin\phi\, F_{LU}^{\sin\phi} \\
& + S_L \left\{ \sin(2\phi)\, F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\sin\phi\, F_{UL}^{\sin\phi} + \lambda \left[F_{LL} + \frac{1}{Q}\cos\phi\, F_{LL}^{\cos\phi} \right] \right\} \\
& + S_T \Big\{ \underbrace{\sin(\phi-\phi_S)\, F_{UT}^{\sin(\phi-\phi_S)}}_{\text{Sivers}} + \underbrace{\sin(\phi+\phi_S)\, F_{UT}^{\sin(\phi+\phi_S)}}_{\text{Collins}} + \sin(3\phi-\phi_S)\, F_{UT}^{\sin(3\phi-\phi_S)} \\
& + \frac{1}{Q}\left[\sin(2\phi-\phi_S)\, F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\, F_{UT}^{\sin\phi_S} \right] \\
& + \lambda \left[\cos(\phi-\phi_S)\, F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q}\left(\cos\phi_S\, F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S)\, F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \Big\}
\end{aligned}
$$

the $F^{(\cdots)}_{S_B\, S_T}$ contain the TMDs; plenty of Spin Asymmetries



TMDs in Drell-Yan processes

COMPASS, RHIC, Fermilab, NICA, AFTER...



factorization holds, two scales, M^2, and $q_T \ll M$

$$d\sigma^{D-Y} = \sum_a f_q(x_1, \boldsymbol{k}_{\perp 1}; Q^2) \otimes f_{\bar{q}}(x_2, \boldsymbol{k}_{\perp 2}; Q^2)\, d\hat{\sigma}^{q\bar{q}\to\ell^+\ell^-}$$

direct product of TMDs, no fragmentation process

Case of one polarized nucleon only

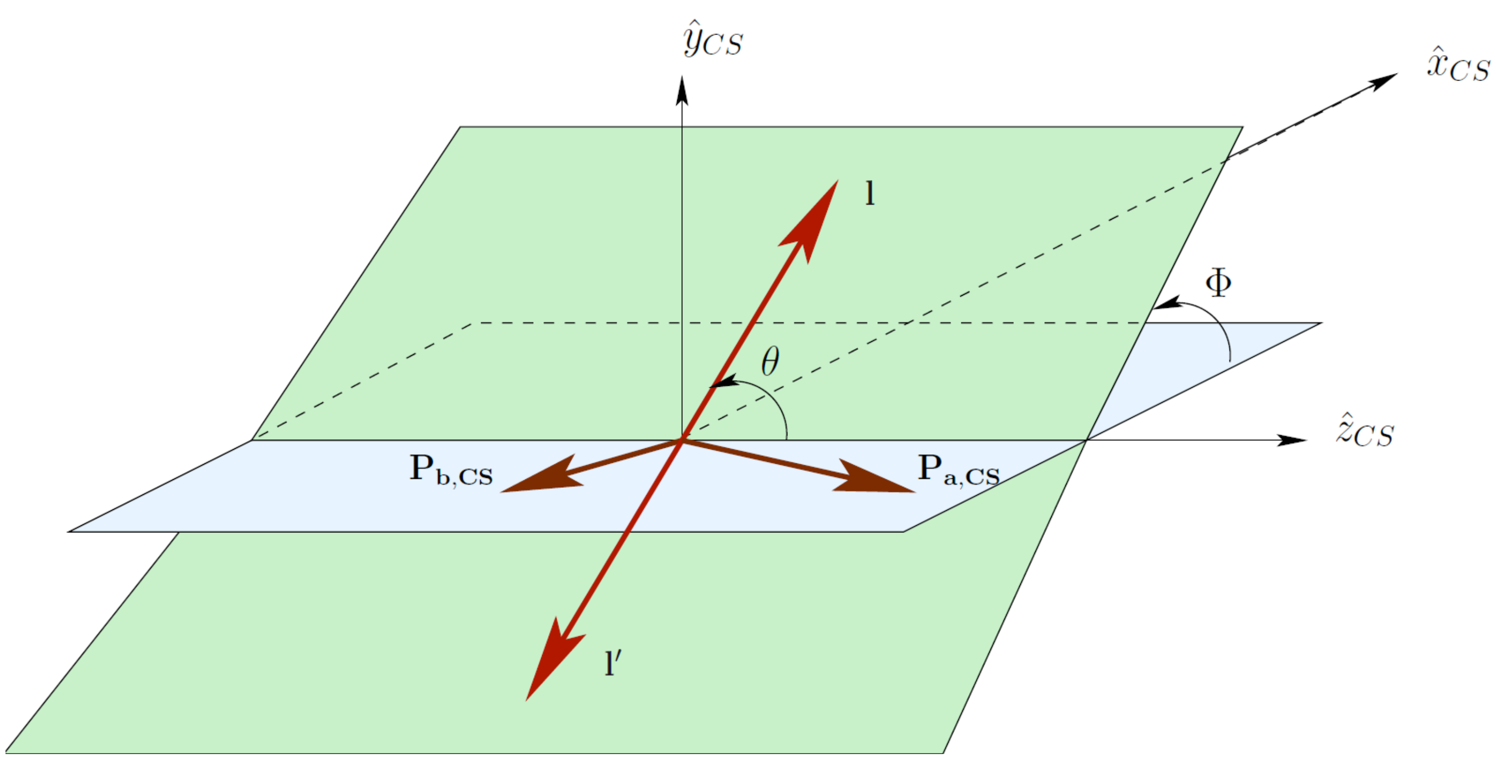
$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\Omega} = \frac{\alpha^2}{\Phi\,q^2}\Big\{(1+\cos^2\theta)\,F_U^1 + (1-\cos^2\theta)\,F_U^2 + \sin 2\theta\cos\phi\,F_U^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_U^{\cos 2\phi}$$

$$+\; S_L\Big(\sin 2\theta\sin\phi\,F_L^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_L^{\sin 2\phi}\Big)$$

$$+\; S_T\Big[\Big(F_T^{\sin\phi_S} + cos^2\theta\,\tilde{F}_T^{\sin\phi_S}\Big)\sin\phi_S + \sin 2\theta\Big(\sin(\phi+\phi_S)\,F_T^{\sin(\phi+\phi_S)}$$

$$+\; \sin(\phi-\phi_S)\,F_T^{\sin(\phi-\phi_S)}\Big)$$

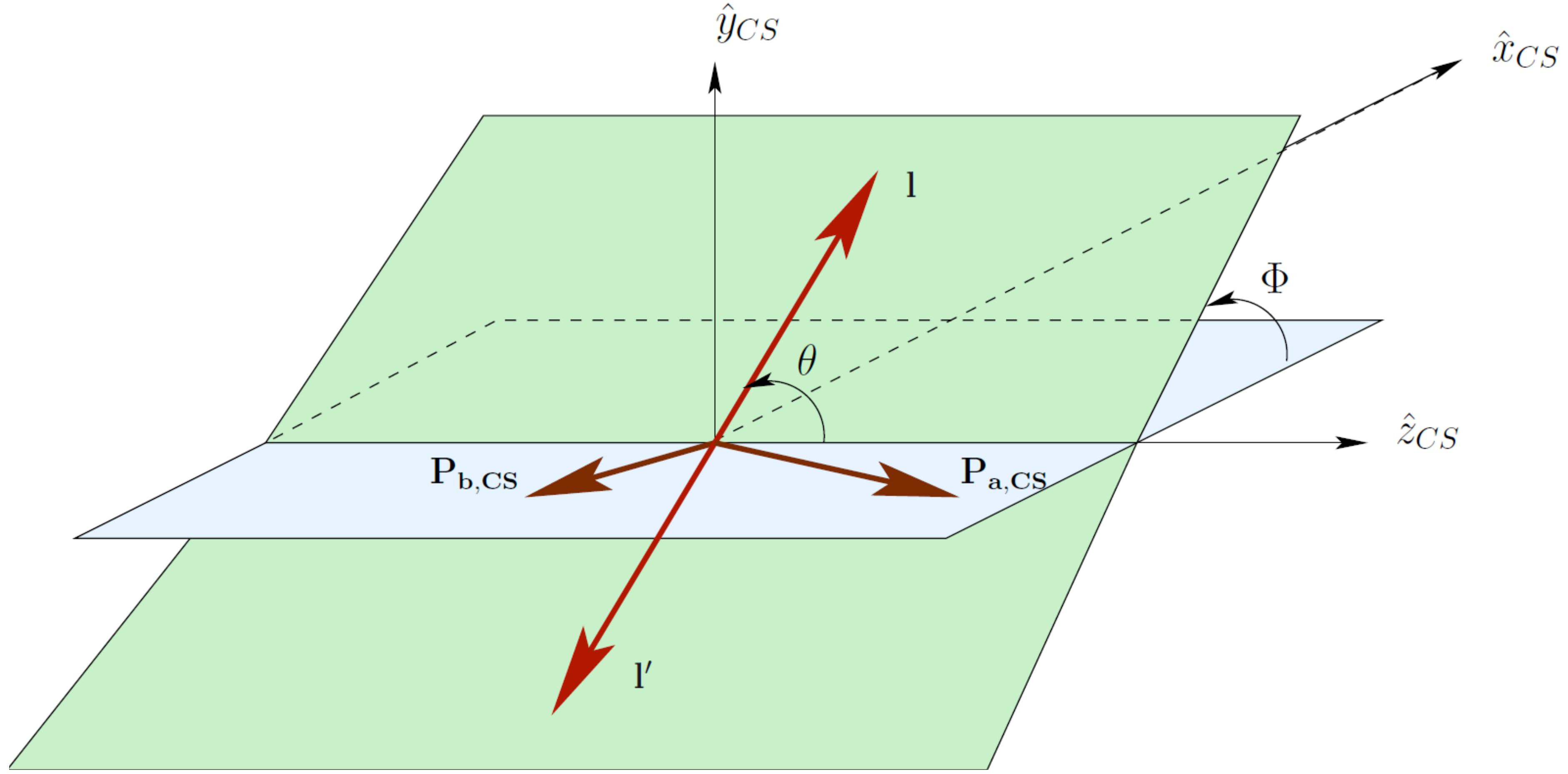
$$+\; \sin^2\theta\Big(\sin(2\phi+\phi_S)\,F_T^{\sin(2\phi+\phi_S)} + \sin(2\phi-\phi_S)\,F_T^{\sin(2\phi-\phi_S)}\Big)\Big]\Big\}$$



Collins-Soper frame

Unpolarized cross section already very interesting

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1+\lambda\cos^2\theta+\mu\,\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$

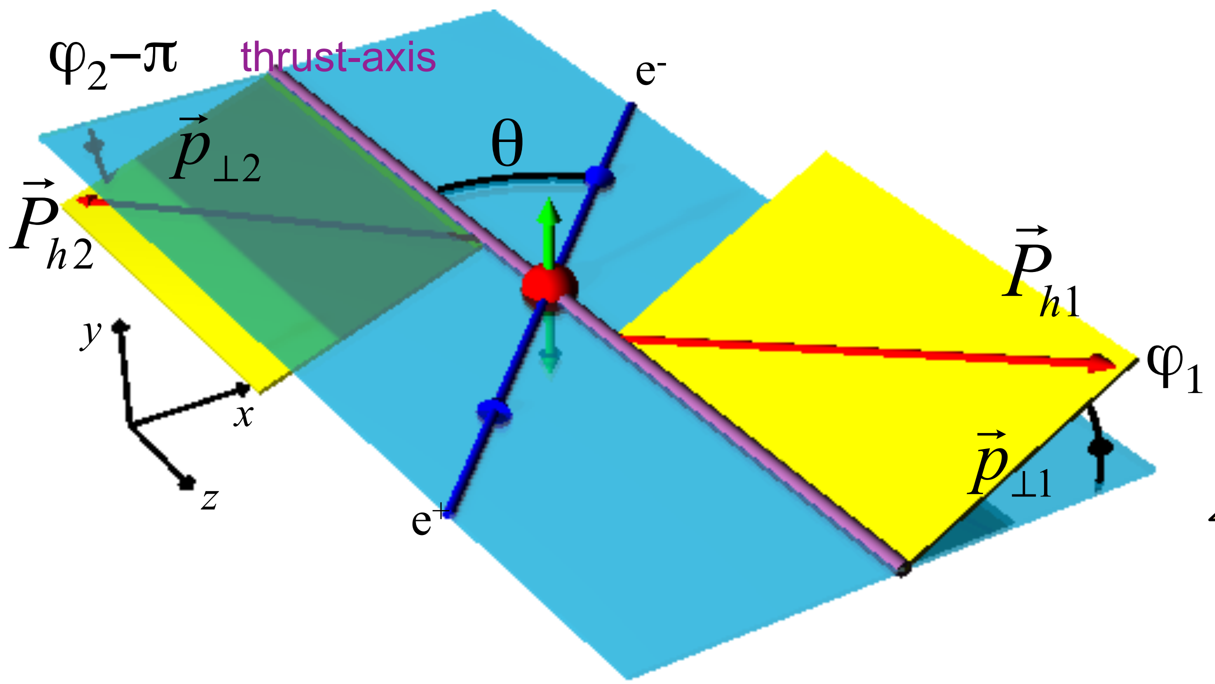


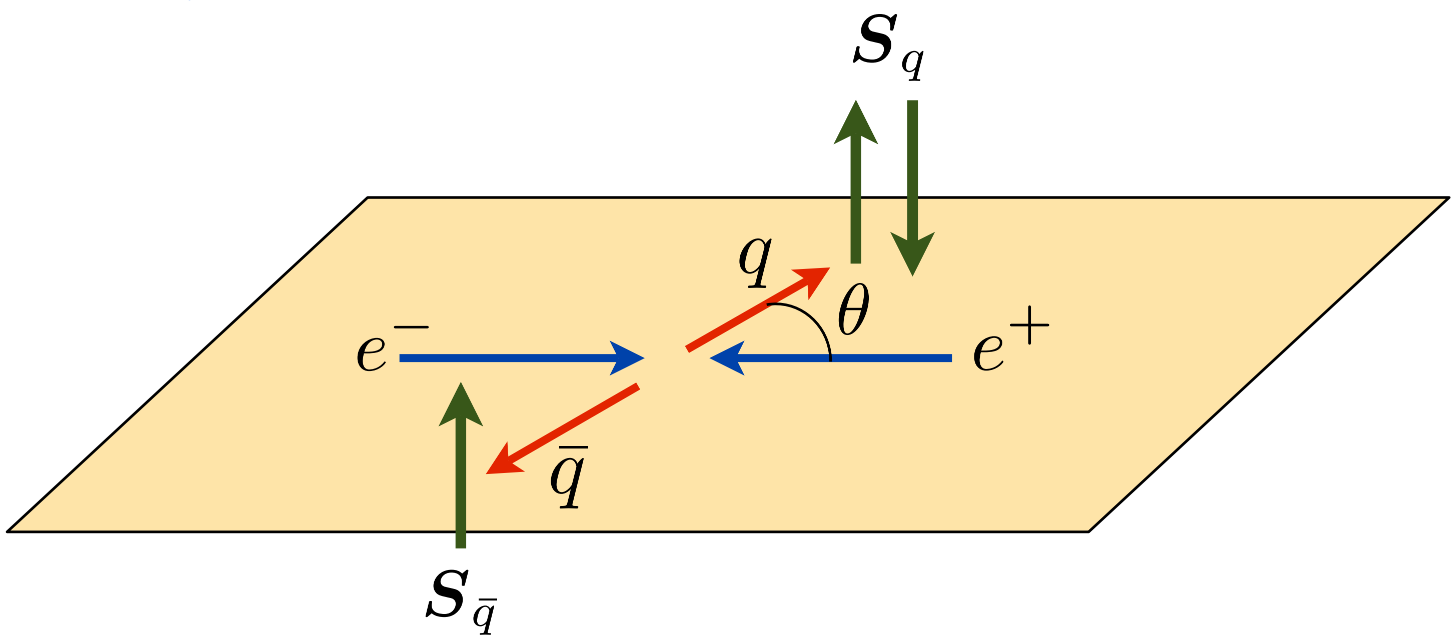
Collins-Soper frame

naive collinear parton model: $\lambda = 1 \quad \mu = \nu = 0$

Collins function from e⁺e⁻ processes

Belle, BaBar, BES-III





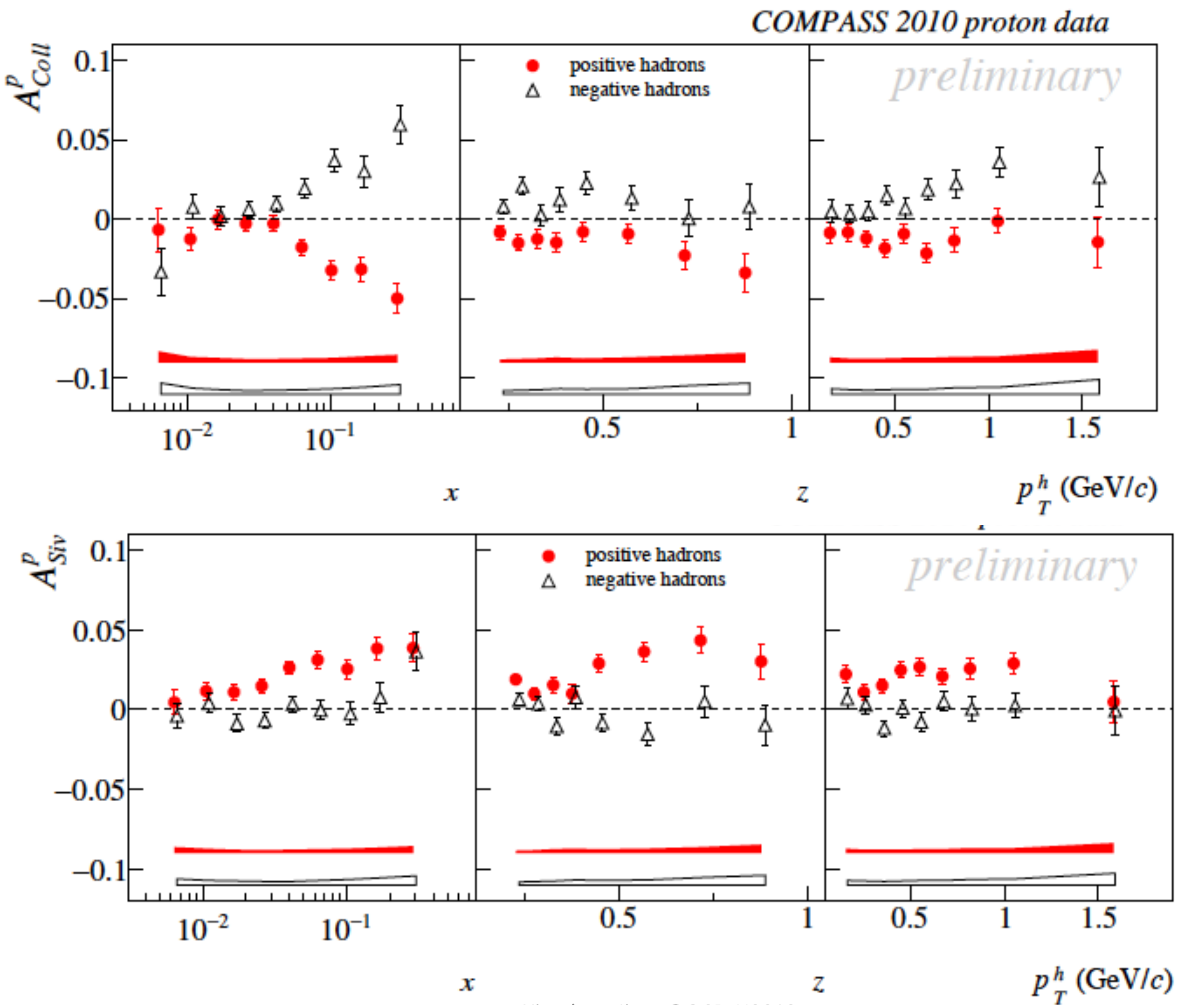
$$\frac{\mathrm{d}\sigma^{e^+e^-\to q^\uparrow \bar{q}^\uparrow}}{\mathrm{d}\cos\theta} = \frac{3\pi\alpha^2}{4s}\, e_q^2\, \cos^2\theta \qquad \frac{\mathrm{d}\sigma^{e^+e^-\to q^\downarrow \bar{q}^\uparrow}}{\mathrm{d}\cos\theta} = \frac{3\pi\alpha^2}{4s}\, e_q^2$$

$$A_{12}(z_1, z_2, \theta, \varphi_1+\varphi_2) \equiv \frac{1}{\langle d\sigma\rangle} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1+\varphi_2)}$$

$$= 1 + \frac{1}{4}\,\frac{\sin^2\theta}{1+\cos^2\theta}\,\cos(\varphi_1+\varphi_2) \times \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

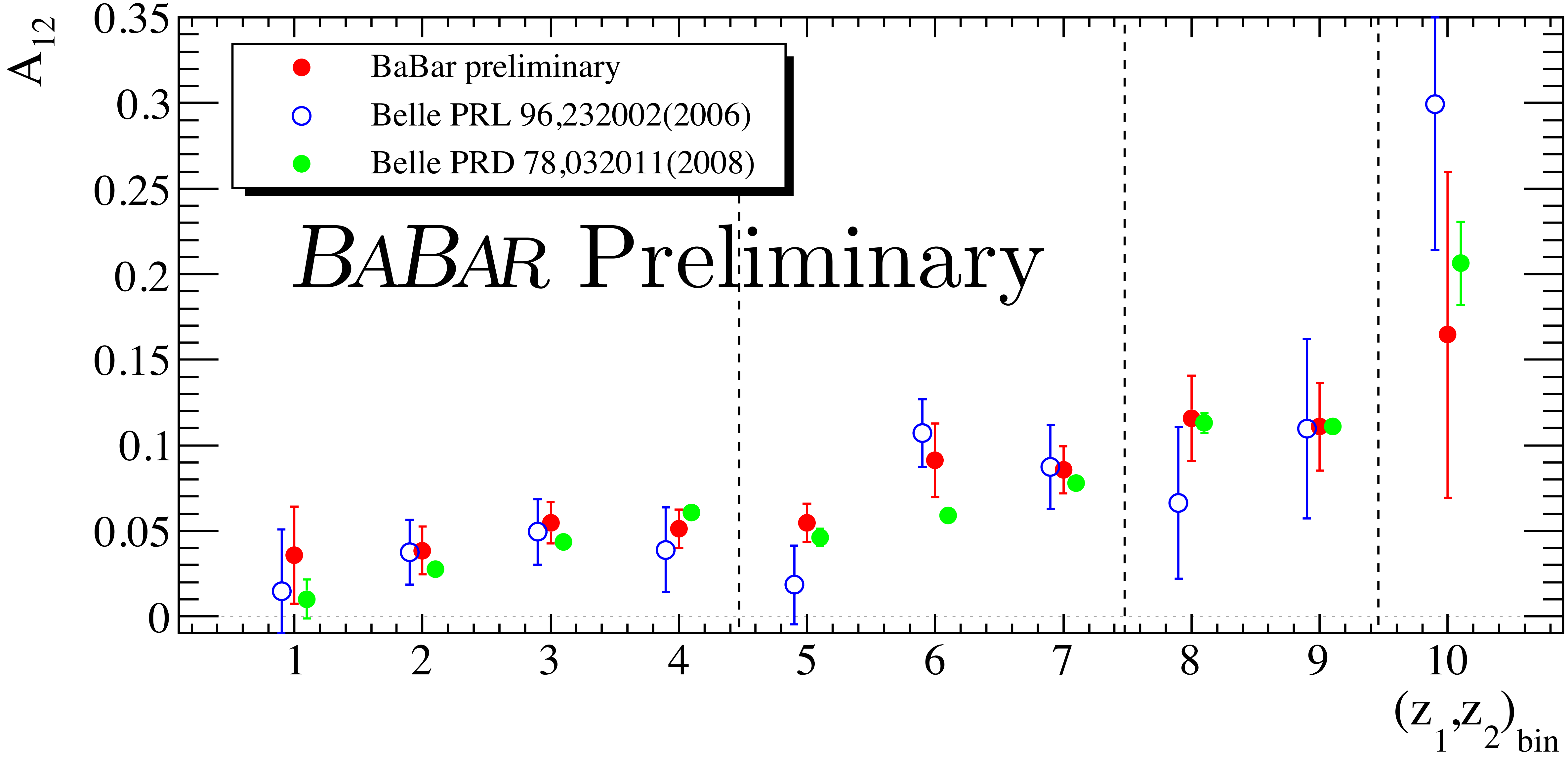
another similar asymmetry can be measured, A_0

Experimental results:
clear evidence for Sivers and Collins effects from SIDIS data (HERMES, COMPASS, JLab)
(talks by L. Pappalardo, H. Avakian, F. Bradamante, P. Rossi…)

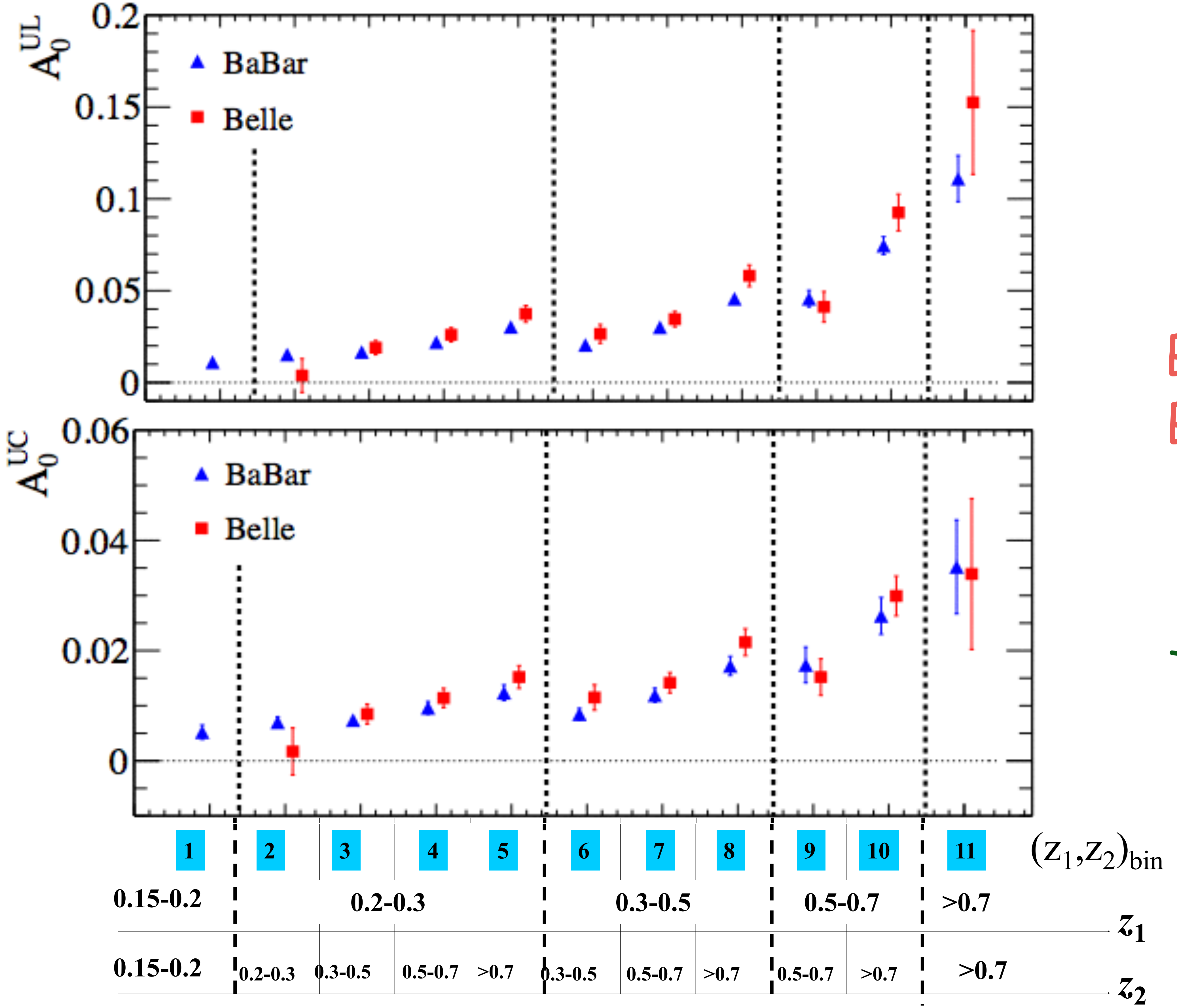


independent evidence for Collins effect from e^+e^- data at Belle, BaBar and BES-III

$$A_{12}(z_1, z_2) \sim \Delta^N D_{h_1/q^\uparrow}(z_1) \otimes \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)$$



I. Garzia, arXiv:1201.4678

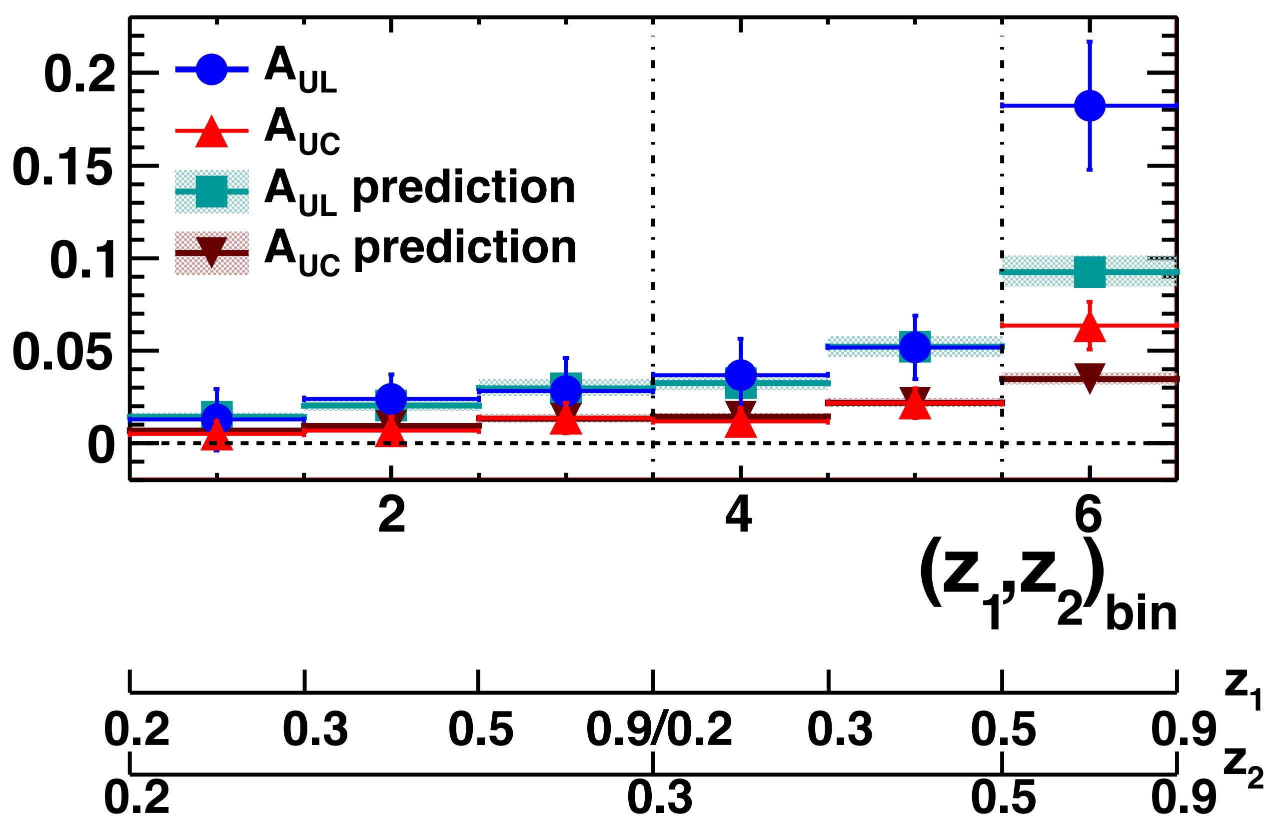


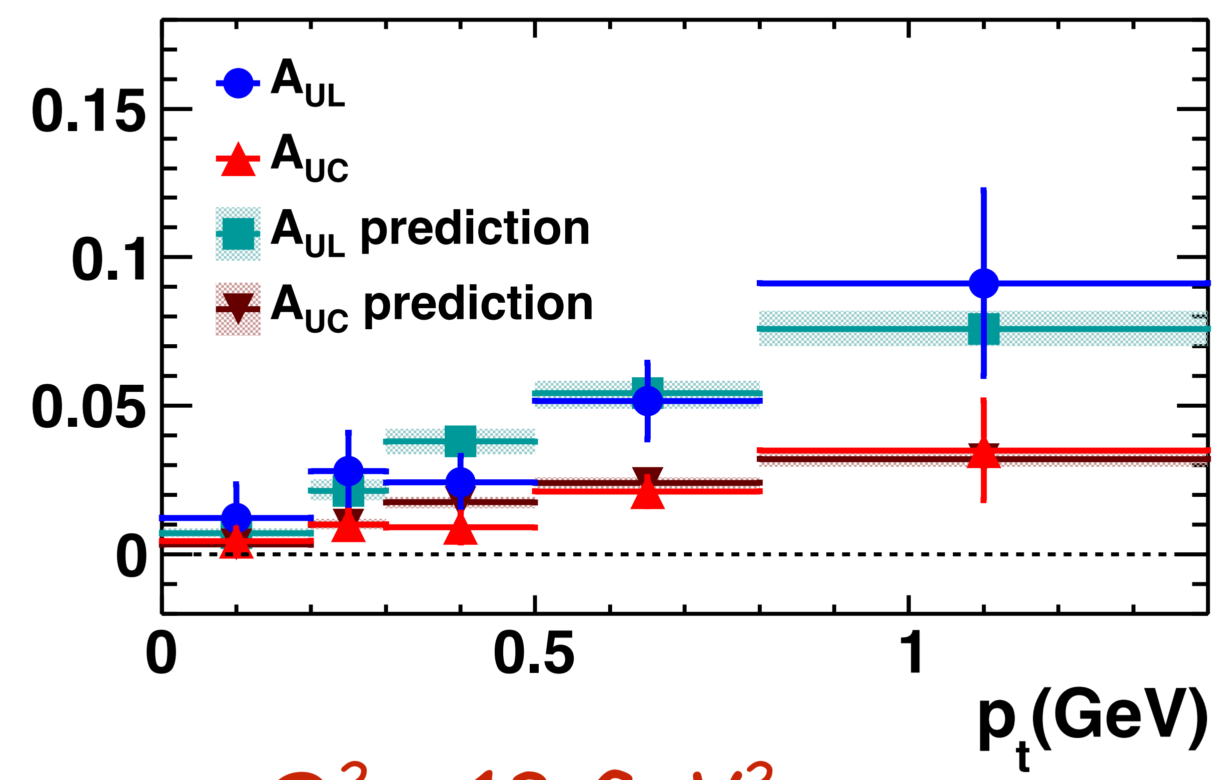
BaBar and Belle data on A_0

(I. Garzia talk at TMDe2015)

a similar asymmetry just measured by BES-III

(arXiv 1507:06824)





$Q^2 = 13\ GeV^2$

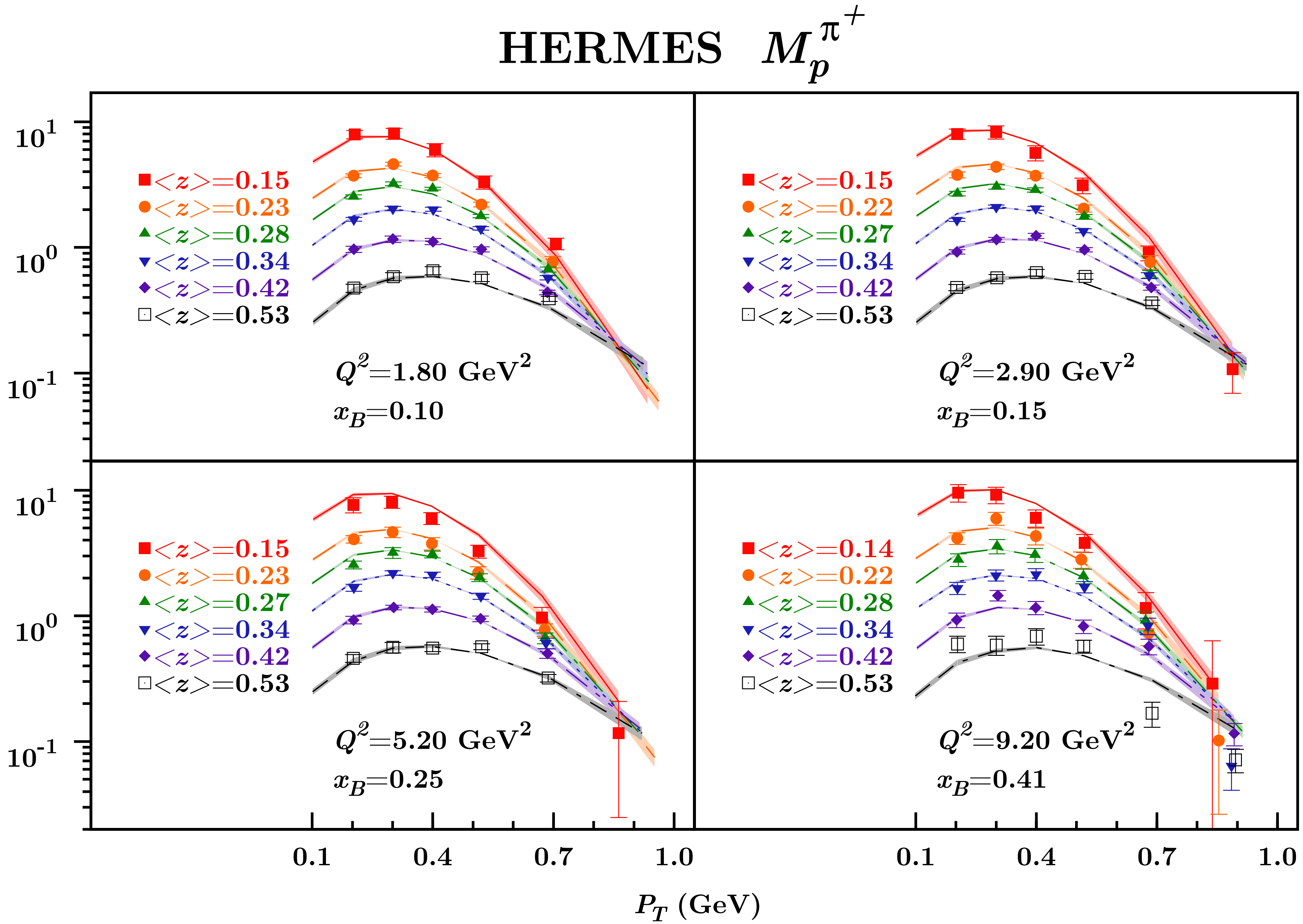
Collins effect clearly observed both in SIDIS and e+e- processes, by several Collaborations

TMD extraction from data - first phase

(simple parameterisation, no TMD evolution, limited number of parameters, ...) (talks by O. Gonzalez, A. Bacchetta)

unpolarised TMDs - fit of SIDIS multiplicities

(M.A, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (2014) 005)



clear support for a gaussian distribution

$$\frac{d^2 n^h(x_{\scriptscriptstyle B}, Q^2, z_h, P_T)}{dz_h\, dP_T^2} = \frac{1}{2P_T} M_n^h(x_{\scriptscriptstyle B}, Q^2, z_h, P_T) = \frac{\pi \sum_q e_q^2\, f_{q/p}(x_{\scriptscriptstyle B})\, D_{h/q}(z_h)}{\sum_q e_q^2\, f_{q/p}(x_{\scriptscriptstyle B})} \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\,\langle k_\perp^2\rangle$$

$$f_{q/p}(x, k_\perp) = f_{q/p}(x)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

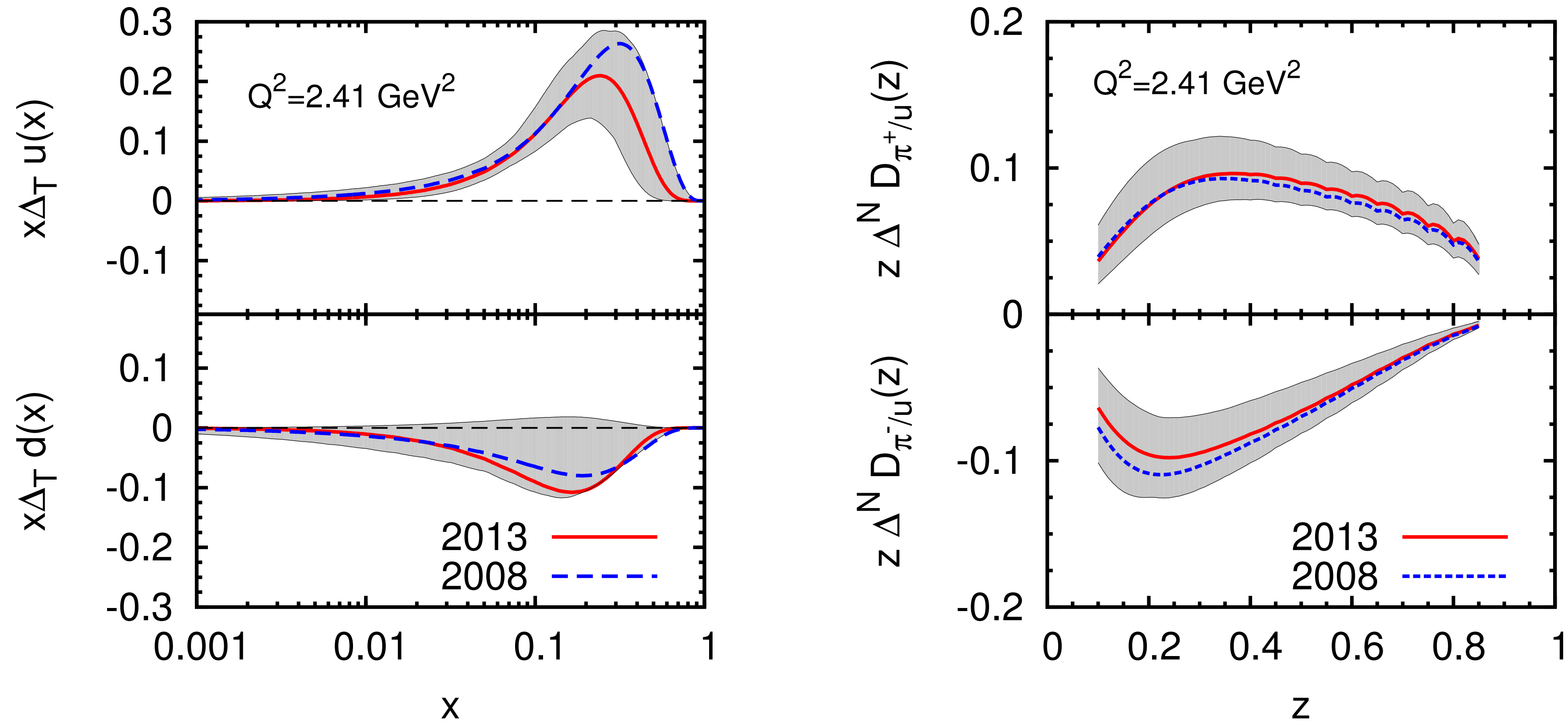
$$D_{h/q}(z, p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

$$\langle k_\perp^2\rangle = 0.57 \qquad\qquad \langle p_\perp^2\rangle = 0.12$$

a similar analysis performed by Signori, Bacchetta, Radici, Schnell, JHEP 1311 (2013) 194; it also assumes gaussian behaviour

TMD extraction: transversity and Collins functions - first phase

M. A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, PRD 87 (2013) 094019



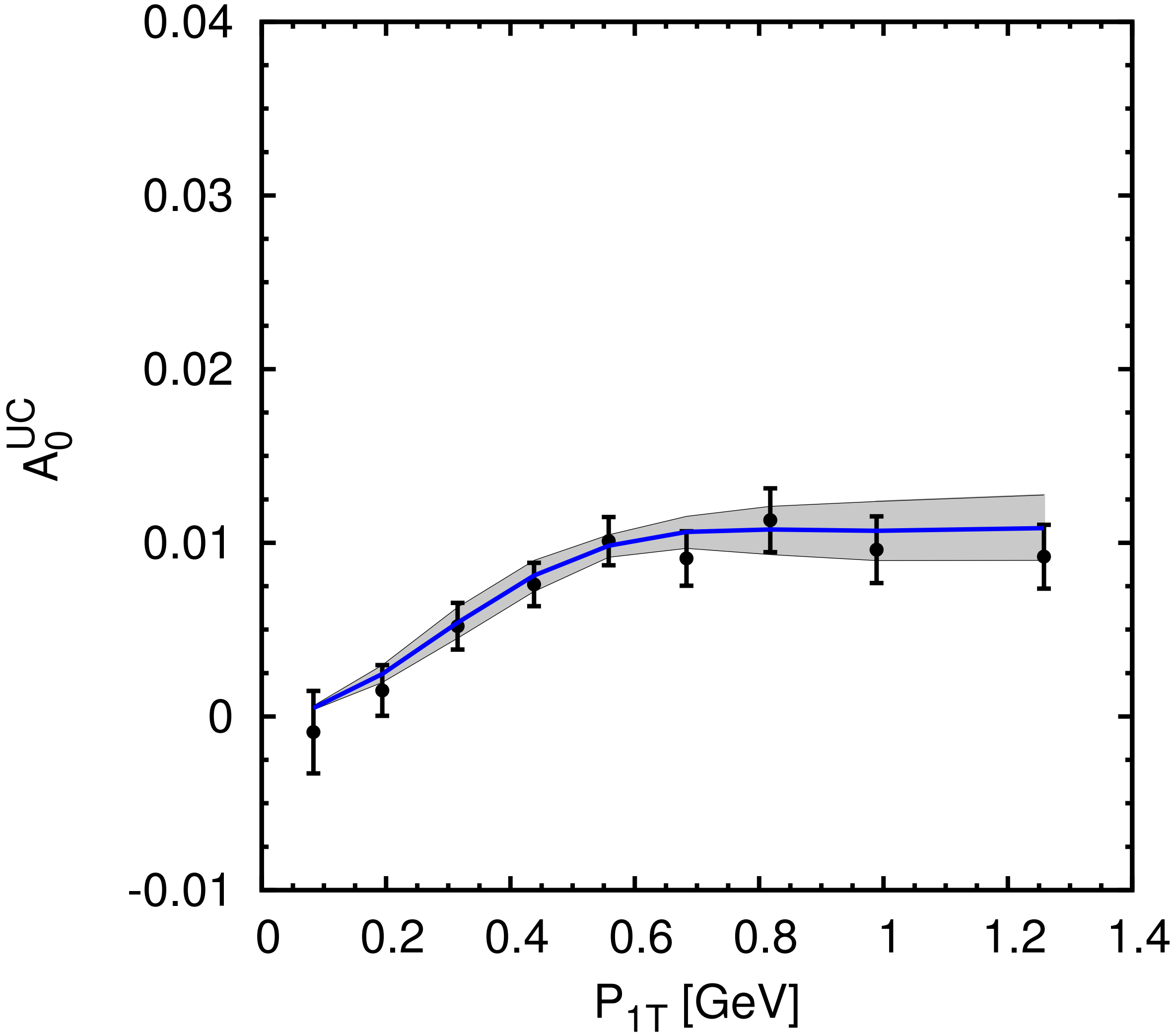
$$\Delta_T q(x,k_\perp) \;=\; \frac{1}{2}\,\mathcal{N}_q^T(x)\,[f_{q/p}(x)+\Delta q(x)]\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T}$$

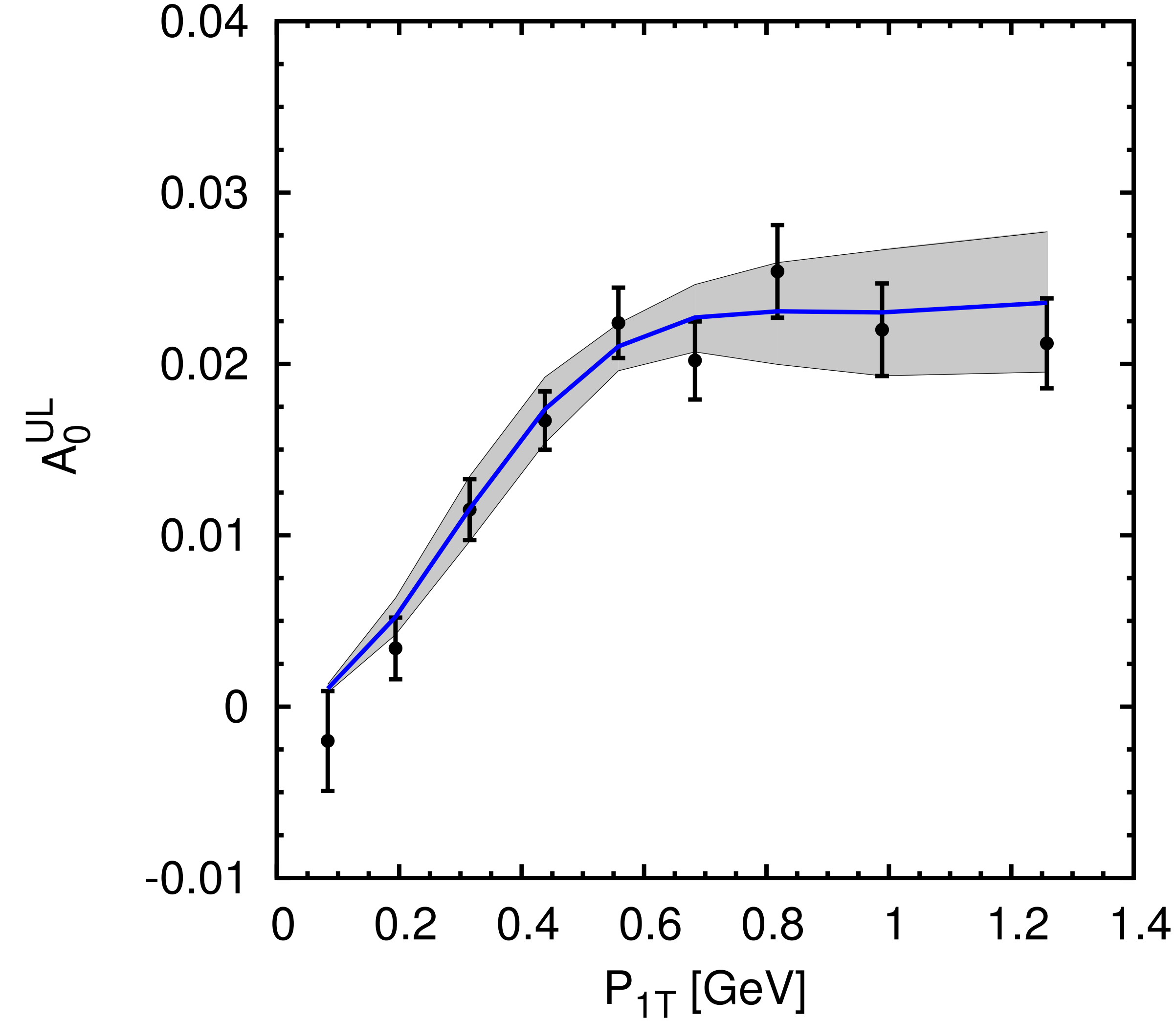
$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) \;=\; 2\,\mathcal{N}_q^C(z)\,D_{h/q}(z)\,h(p_\perp)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

SIDIS and e+e- data, simple parameterization, no TMD evolution, agreement with extraction using di-hadron FF

(recent papers by Bacchetta, Courtoy, Guagnelli, Radici, JHEP 1505 (2015) 123; Kang, Prokudin, Sun, Yuan, Phys. Rev. D91 (2015) 071501; arXiv:1505.05589)

recent BaBar data on the $p_\perp$ dependence of the Collins function (first direct measurement)





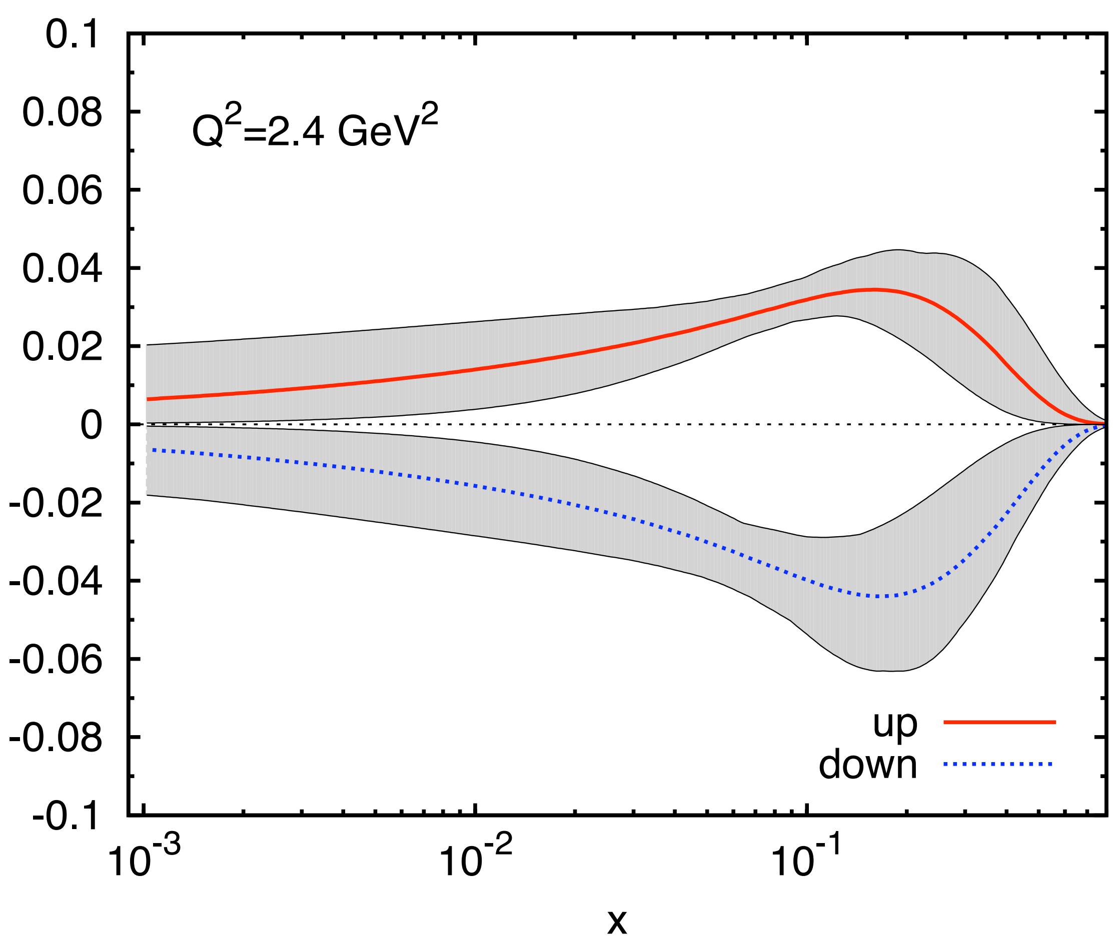
gaussian $p_\perp$ dependence of Collins functions

(M.A., Boglione, D'Alesio, Gonzalez, Melis, Murgia, Prokudin, in preparation)

extraction of u and d Sivers functions - first phase

M.A, M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin
(in agreement with several other groups)

$x\,\Delta^N f_q^{(1)}(x,Q)$



$$\Delta^N f_q^{(1)}(x,Q) = \int d^2\boldsymbol{k}_\perp \frac{k_\perp}{4M_p}\,\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) = -f_{1T}^{\perp(1)q}(x,Q)$$

parameterization of the Sivers function:

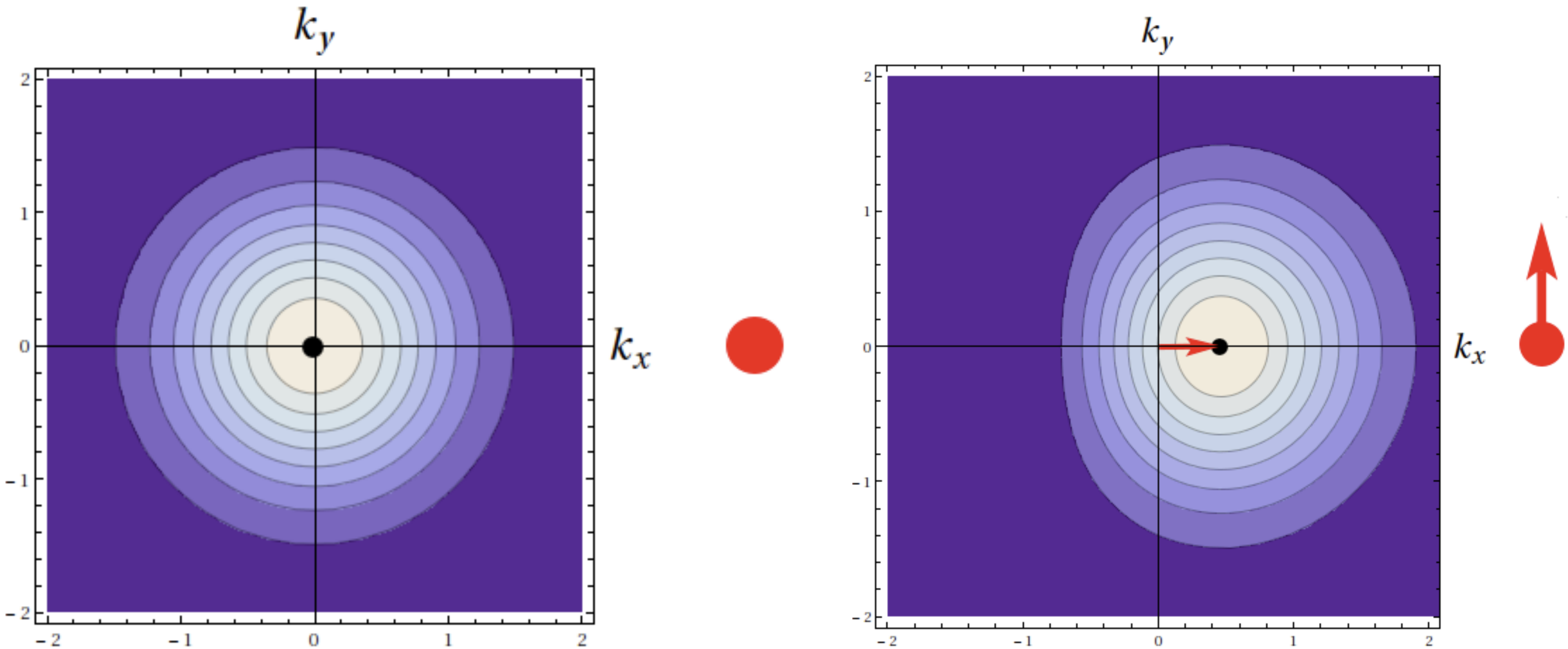
$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) = 2\,\mathcal{N}(x)\,h(k_\perp)\,f_q(x,Q)\,\frac{1}{\pi\langle k_\perp^2\rangle}e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

Q^2 evolution only taken into account in the collinear part (usual PDF)

Sivers effects induces distortions in the parton distribution

$$\hat{f}_{q/p^\uparrow}(x, \boldsymbol{k}_\perp, S\,\hat{\boldsymbol{j}}; Q) = \hat{f}_{q/p}(x, k_\perp; Q) - \hat{f}_{1T}^{\perp q}(x, k_\perp; Q)\,\frac{k_\perp^x}{M_p}$$

$\boldsymbol{S} = 0$ u quark $\boldsymbol{S} = S\,\hat{\boldsymbol{j}}$



courtesy of Alexei Prokudin

Sivers function and angular momentum

Ji's sum rule

forward limit of GPDs

$$J^q = \frac{1}{2}\int_0^1 dx\, x\left[H^q(x,0,0) + E^q(x,0,0)\right]$$

usual PDF $q(x)$

cannot be measured directly

anomalous magnetic moments

$$\kappa^p = \int_0^1 \frac{dx}{3}\left[2E^{u_v}(x,0,0) - E^{d_v}(x,0,0) - E^{s_v}(x,0,0)\right]$$

$$\kappa^n = \int_0^1 \frac{dx}{3}\left[2E^{d_v}(x,0,0) - E^{u_v}(x,0,0) - E^{s_v}(x,0,0)\right]$$

$$(E^{q_v} = E^q - E^{\bar{q}})$$

Sivers function and angular momentum

assume

$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)E^a(x,0,0;Q_L^2)$$

$$f_{1T}^{\perp(0)a}(x,Q) = \int d^2\boldsymbol{k}_\perp\, \widehat{f}_{1T}^{\perp a}(x,k_\perp;Q)$$

L(x) = lensing function
(unknown, can be computed in models)

parameterise Sivers and lensing functions

fit SIDIS and magnetic moment data

obtain E^q and estimate total angular momentum

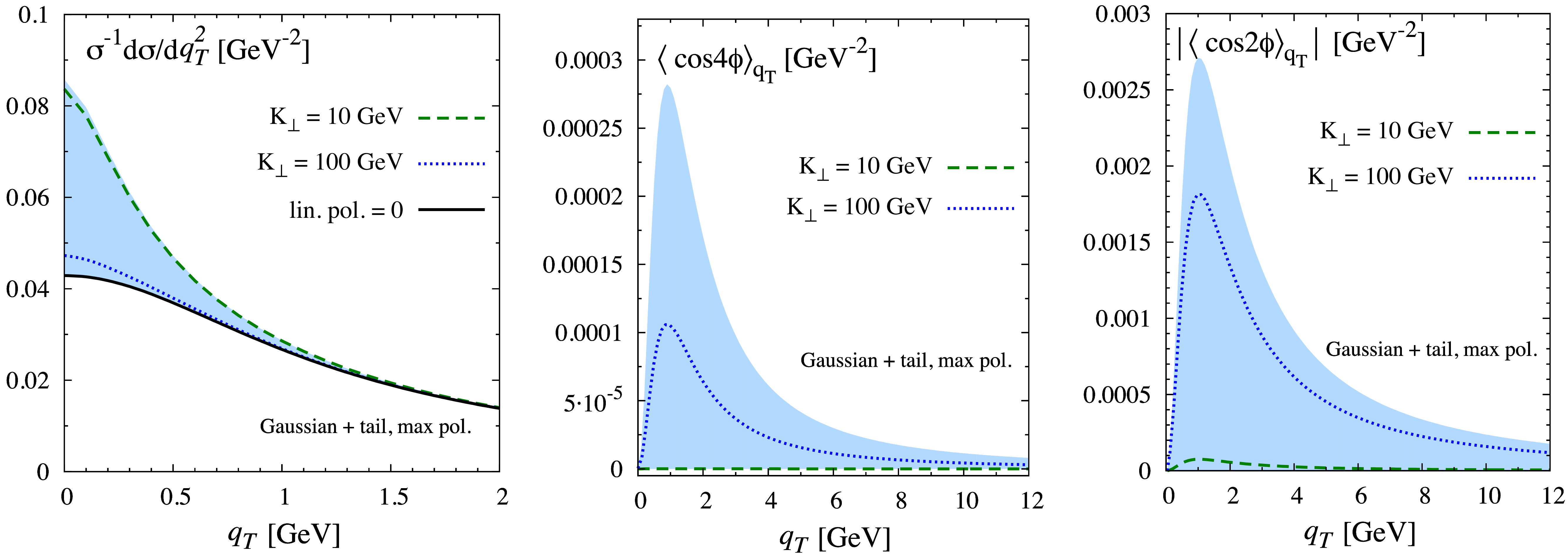
results at $Q^2 = 4$ GeV^2: $J^u \approx 0.23$, $J^{q\neq u} \approx 0$

Bacchetta, Radici, PRL 107 (2011) 212001

Talks by C. Lorcé and M. Burkardt for Wigner distribution and orbital angular momentum

TMDs at LHC - linearly polarised gluons in unpolarized protons

(talks by M. Echevarria, A. Signori)

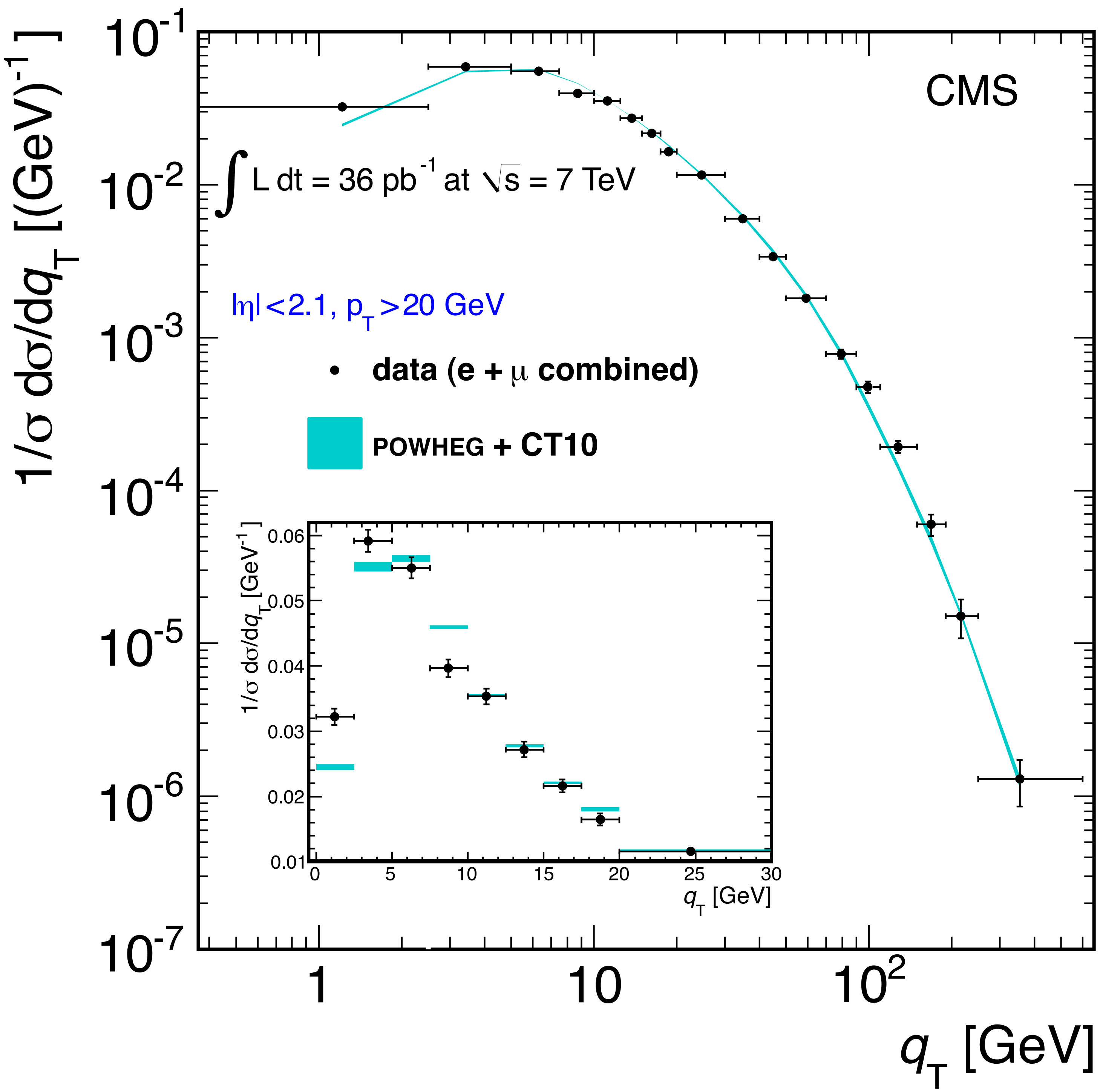


$$p(P_A) + p(P_B) \to H(K_H) + \text{jet}(K_\text{j}) + X$$

$$\boldsymbol{K}_\perp = (\boldsymbol{K}_{H\perp} - \boldsymbol{K}_{\text{j}\perp})/2 \qquad \boldsymbol{q}_T = \boldsymbol{K}_{H\perp} + \boldsymbol{K}_{\text{j}\perp}$$

Boer, Pisano, Phys. Rev. D91 (2015) 7, 074024

Z-boson transverse momentum q_T spectrum in pp collisions at the LHC



The small q_T region cannot be explained by usual collinear PDF factorization: needs TMD-PDFs

Phys. Rev. D85 (2012) 032002

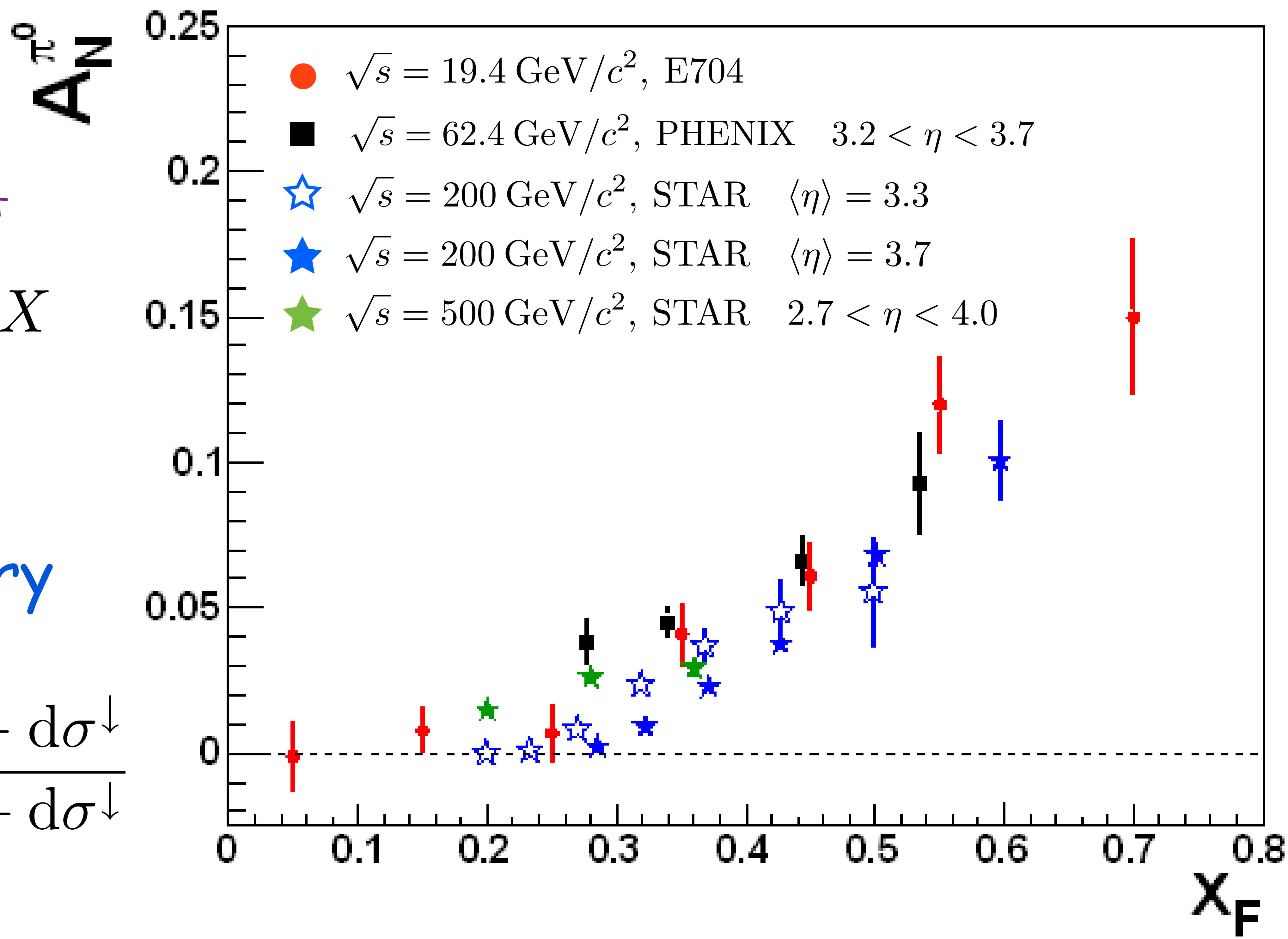
other measured evidence of the Sivers and Collins effects

large P_T

$$p^{\uparrow}\, p \rightarrow \pi\, X$$

Single Spin Asymmetry

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$



TMDs and QCD - TMD evolution

study of the QCD evolution of TMDs and TMD factorisation in rapid development

Collins-Soper-Sterman resummation - NP B250 (1985) 199

Idilbi, Ji, Ma, Yuan - PL B597, 299 (2004); PR D70 (2004) 074021

Ji, Ma, Yuan - PL B597 (2004) 299; PR. D71 (2005) 034005

Collins, "Foundations of perturbative QCD", Cambridge University Press (2011)

Aybat, Rogers, PR D83 (2011) 114042

Aybat, Collins, Qiu, Rogers, PR D85 (2012) 034043

Echevarria, Idilbi, Schafer, Scimemi, arXiv:1208.1281

Echevarria, Idilbi, Scimemi, JHEP 1207 (2012) 002

Aybat, Prokudin, Rogers, PRL 108 (2012) 242003

Anselmino, Boglione, Melis, PR D86 (2012) 014028

Aidala, Field, Gamberg, Rogers, PR D89 (2014) 094002

Echevarria, Idilbi, Kang, Vitev, PR D89 (2014) 074013

Bacchetta, Prokudin, NP B875 (2013) 536

Godbole, Misra, Mukherjee, Raswoot, PR D88 (2013) 014029

Boer, Lorcé, Pisano, Zhou, arXiv:1504.04332 (2015)

Boglione, Gonzalez, Melis, Prokudin, JHEP 1502 (2015) 095

Kang, Prokudin, Sun, Yuan, arXiv:1505.05589

+ many more authors…

different TMD evolution schemes and different implementation within the same scheme

dedicated workshops, QCD Evolution 2011, 2012, 2013, 2014, 2015

see, "Transverse momentum dependent (TMD) parton distribution functions: status and prospects", arXiv: 1507.05267 (from "Resummation, Evolution, Factorization", Antwerp 2014)

dedicated tools:

TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions

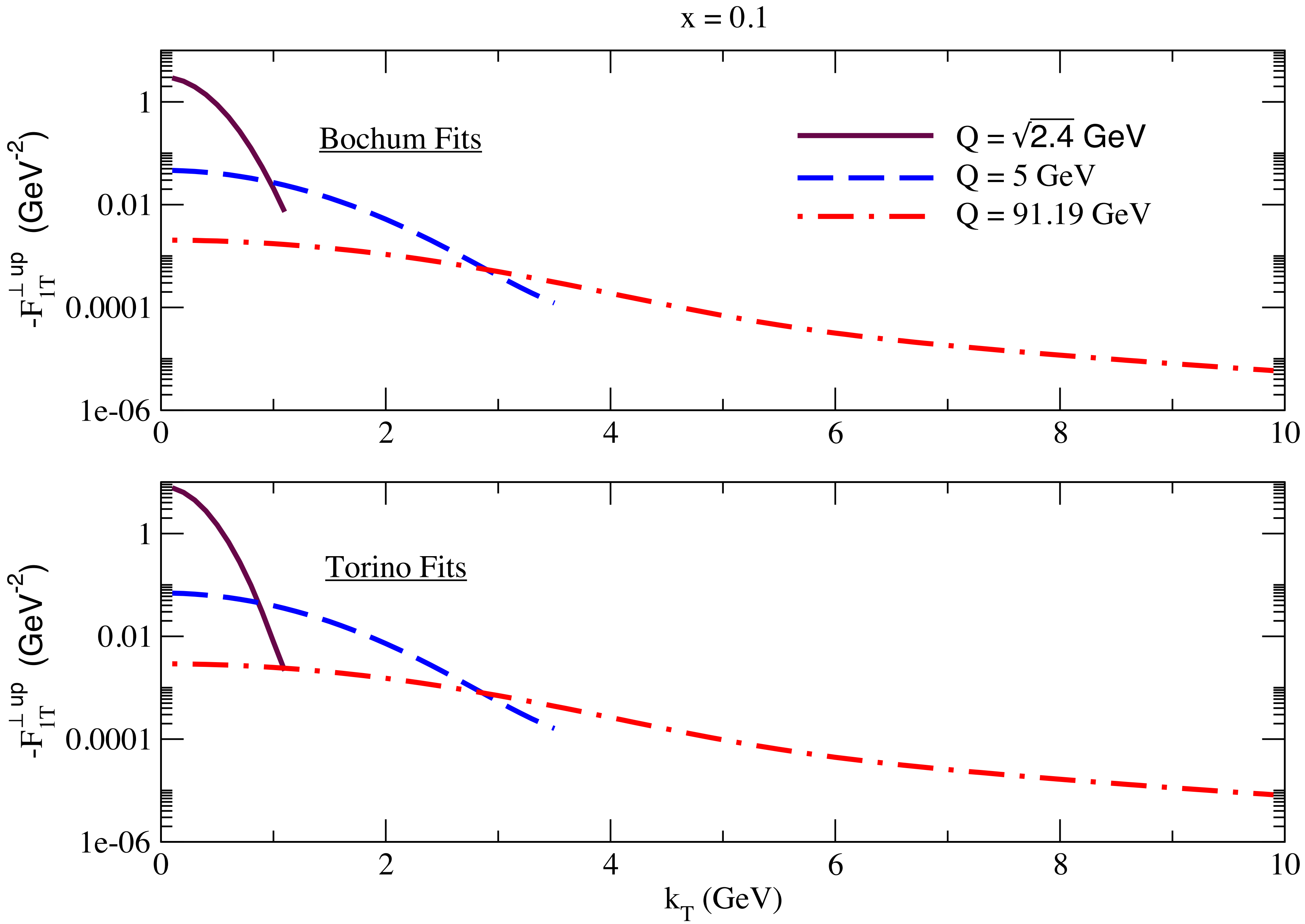
Hautmann, Jung, Kramer, Mulders, Nocera, Rogers, Signori

TMD phenomenology - phase 2

how does gluon emission affect the transverse motion?

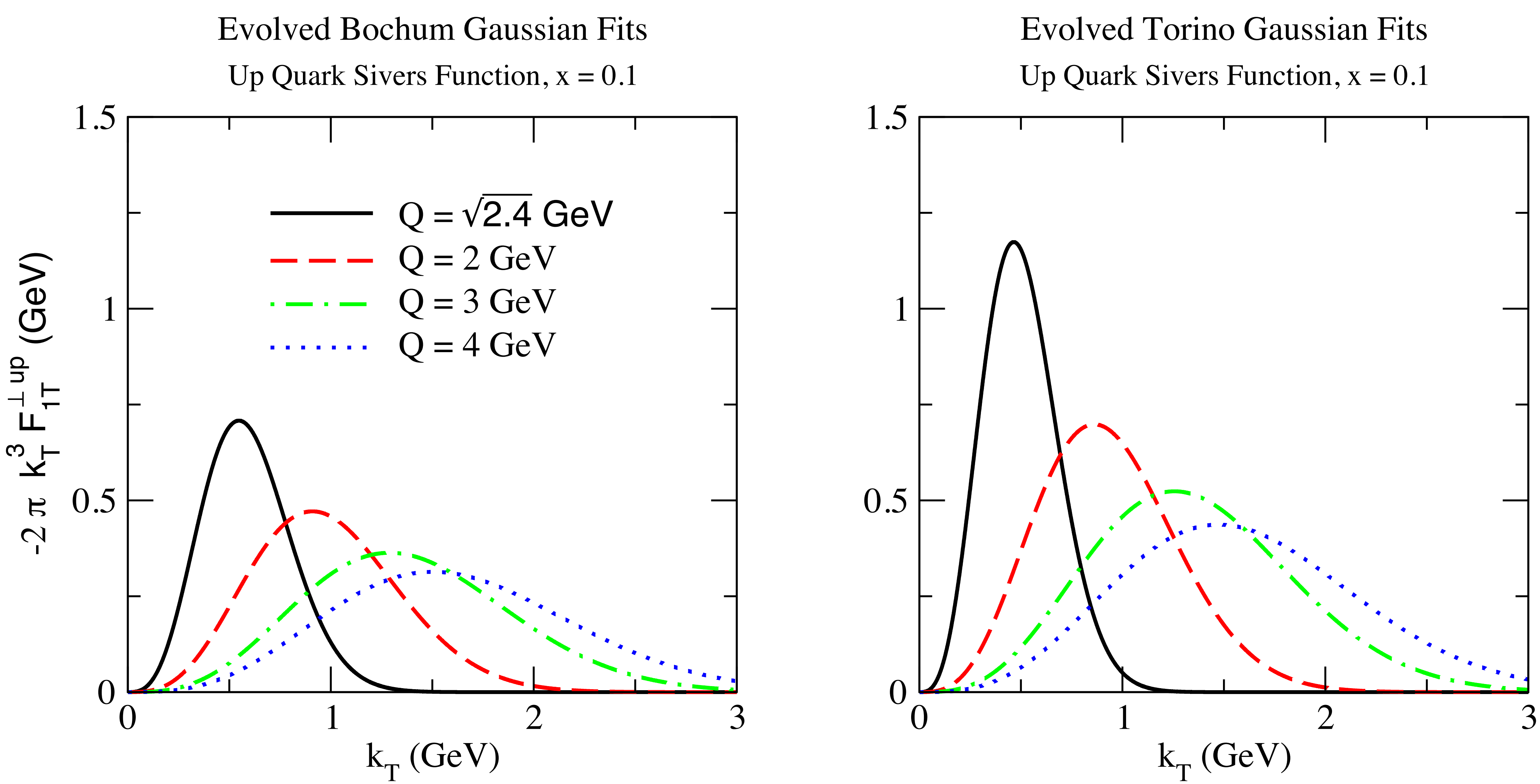
a few selected results

TMD evolution of up quark Sivers function



Aybat, Collins, Qiu, Rogers, Phys. Rev. D85 (2012) 034043

TMD evolution of up quark Sivers function

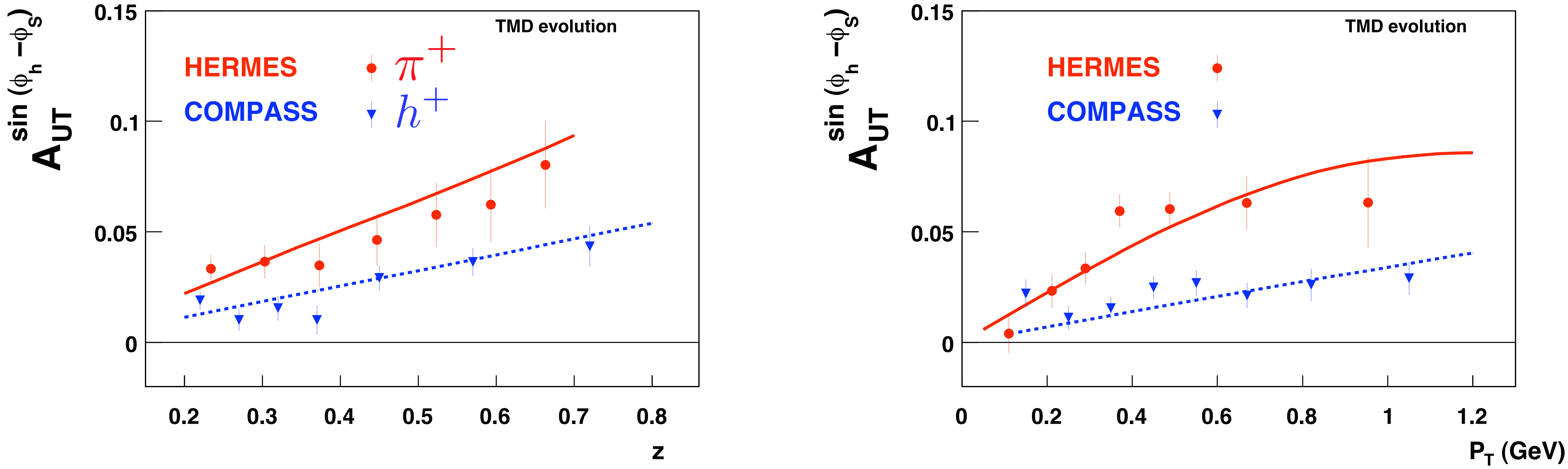


Aybat, Collins, Qiu, Rogers, Phys.Rev. D85 (2012) 034043

TMD evolution of Sivers function studied also by
Echevarria, Idilbi, Kang, Vitev, Phys. Rev. D89 (2014) 074013

first phenomenological applications to data

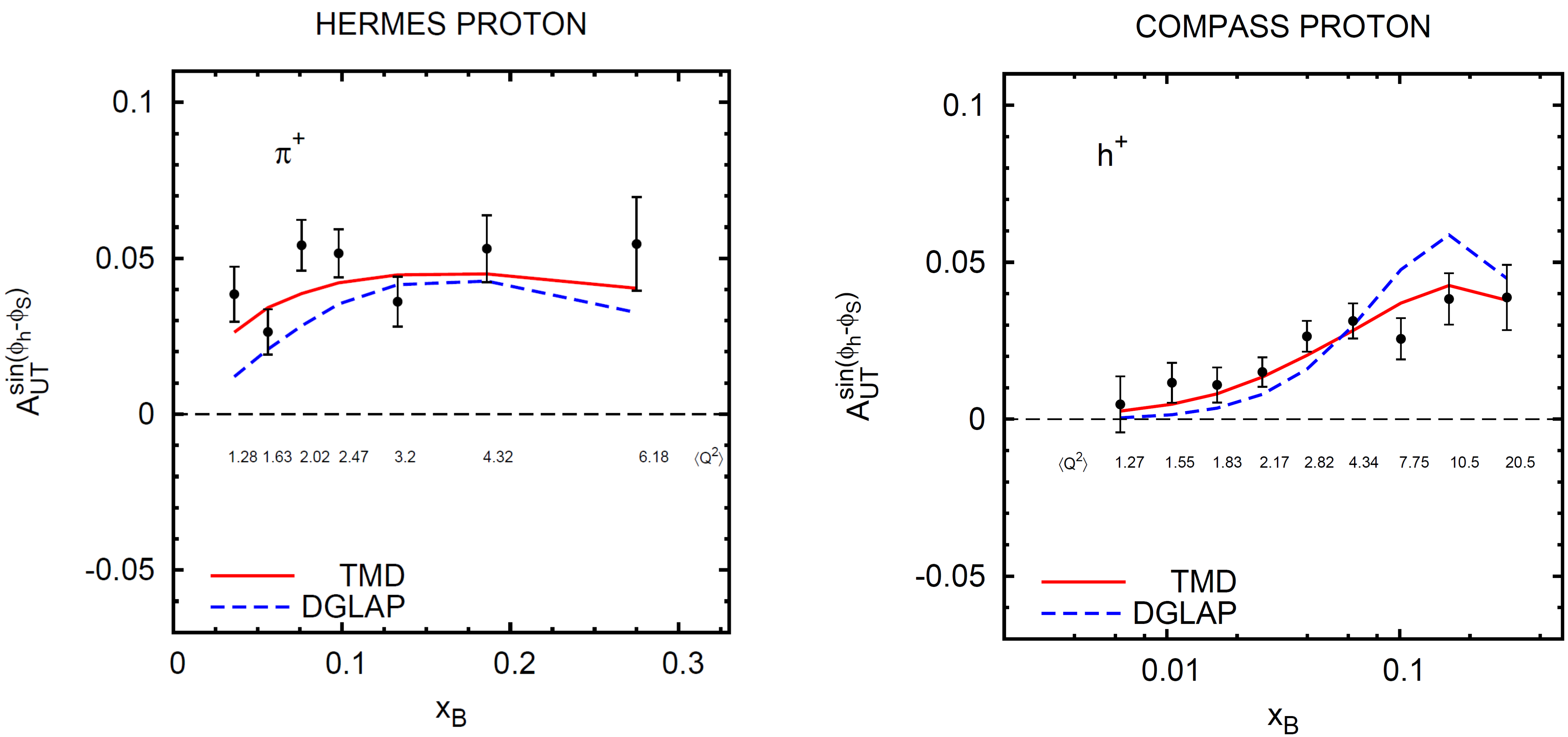
Aybat, Prokudin, Rogers, PRL 108 (2012) 242003



existing fits (red line, Torino) of HERMES data at $\langle Q^2 \rangle = 2.4$ GeV^2, extrapolated with TMD evolution up to $\langle Q^2 \rangle = 3.8$ GeV^2 and compared with COMPASS data (dashed line)

fit of SIDIS data with a specific TMD evolution

M.A., M. Boglione, S. Melis, PR D86 (2012) 014028; arXiv:1204.1239



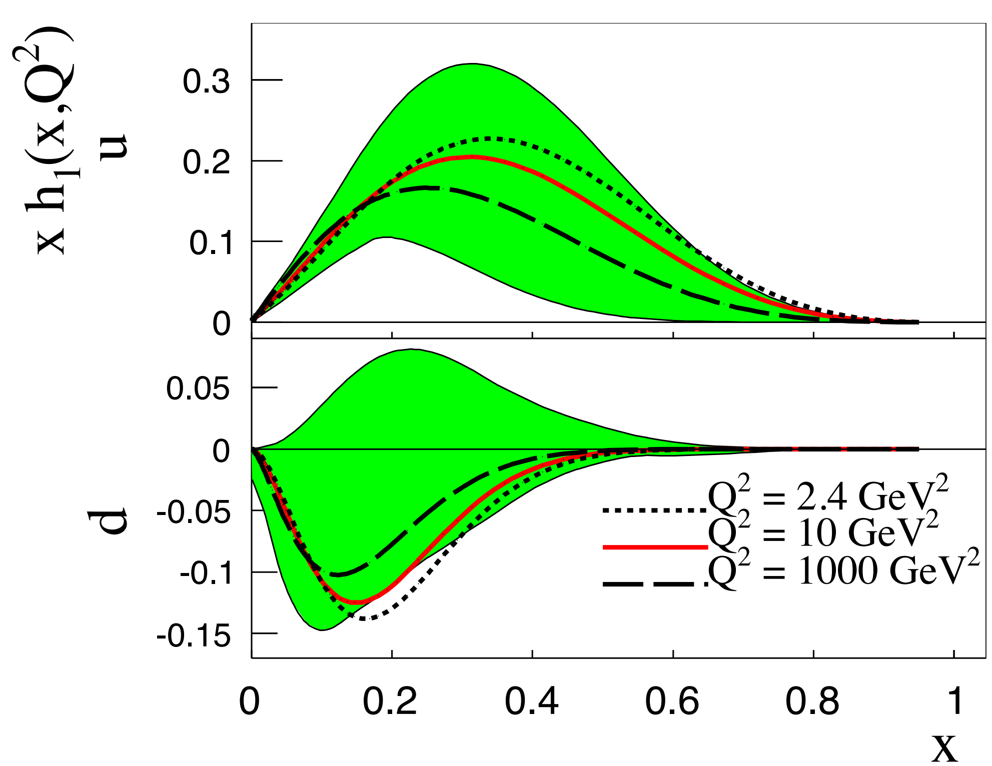
large $x_B \Rightarrow$ large Q^2

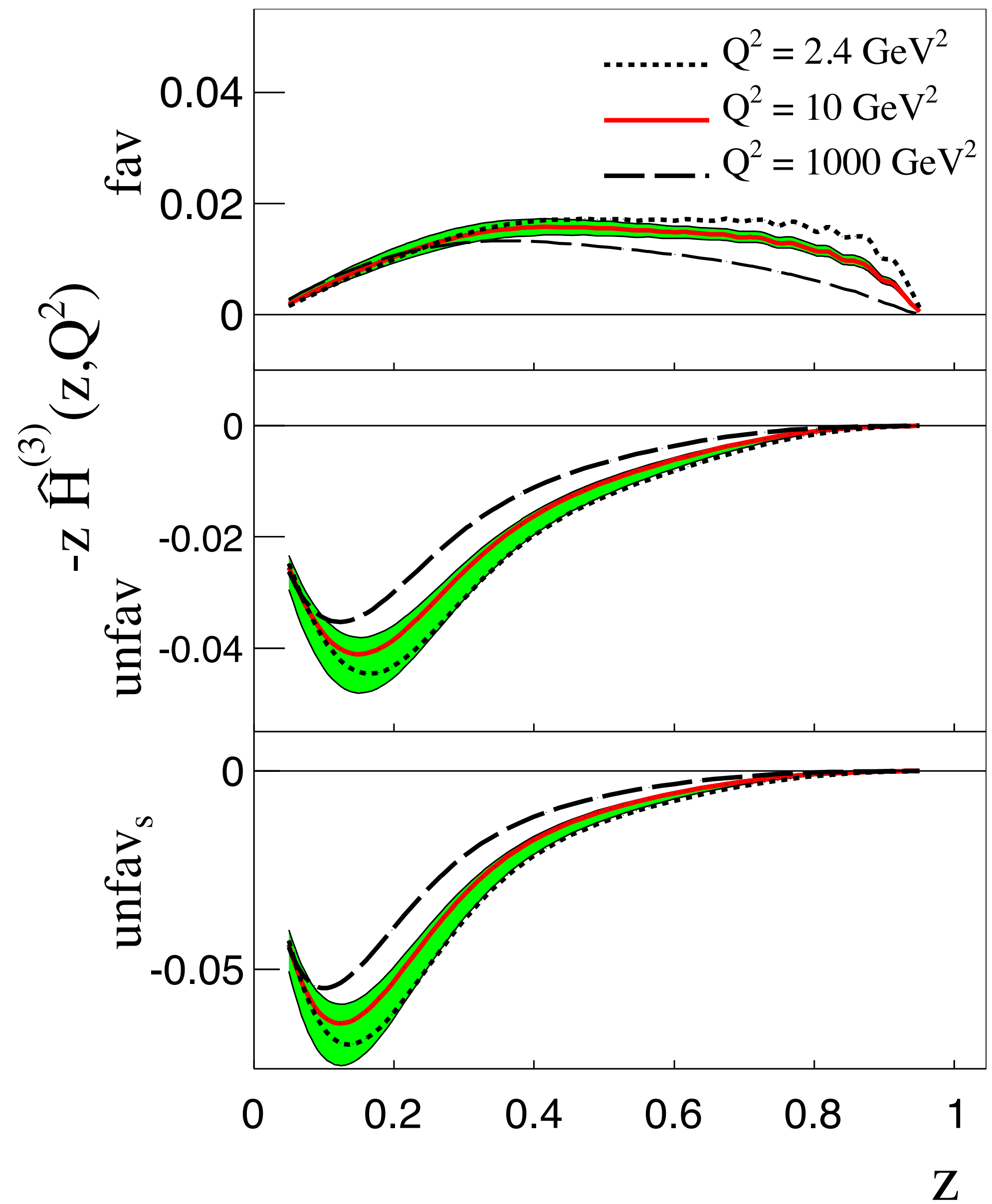
TMD evolution fits better the large Q^2 data

Extraction of transversity and Collins functions with TMD evolution

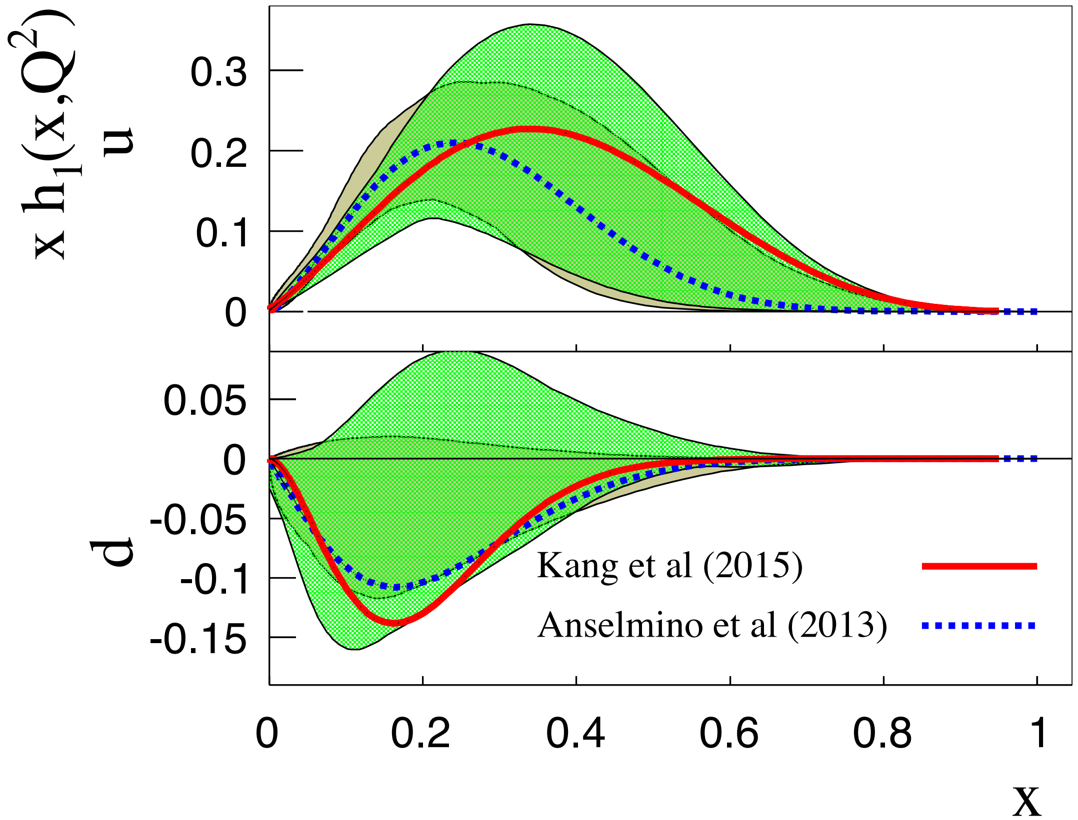
(Kang, Prokudin, Sun, Yuan, arXiv:1505.05589)

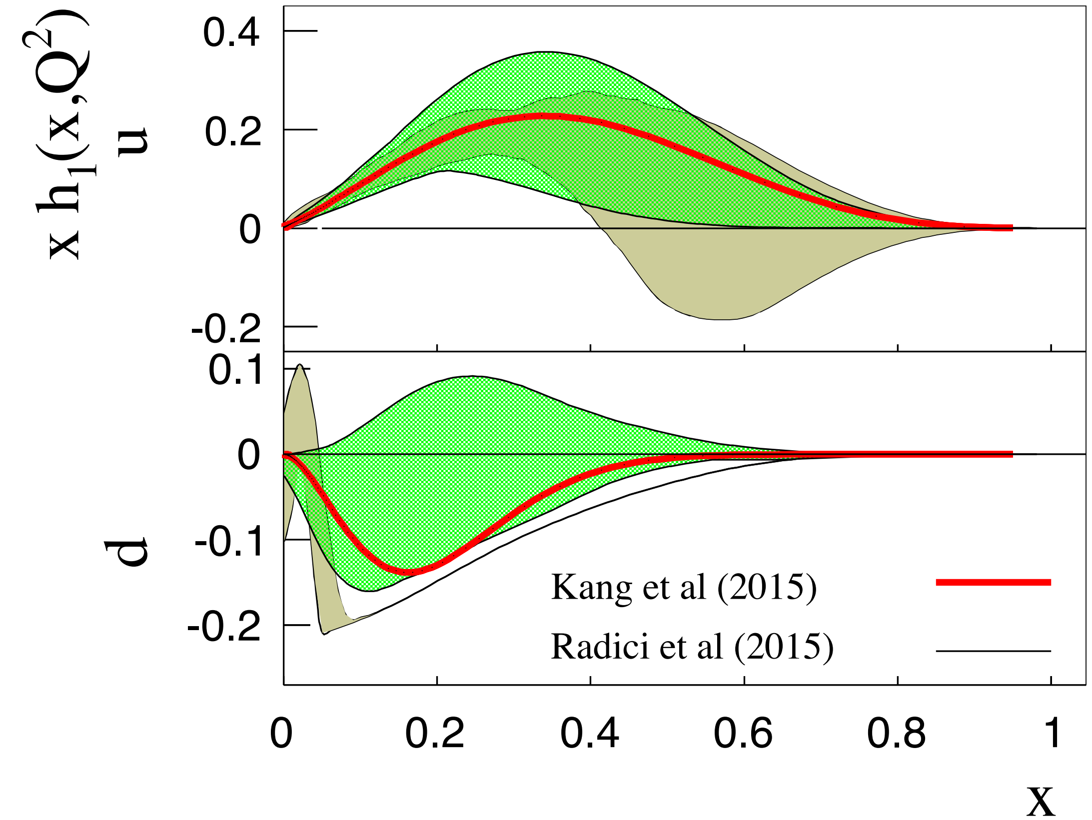
transversity distributions

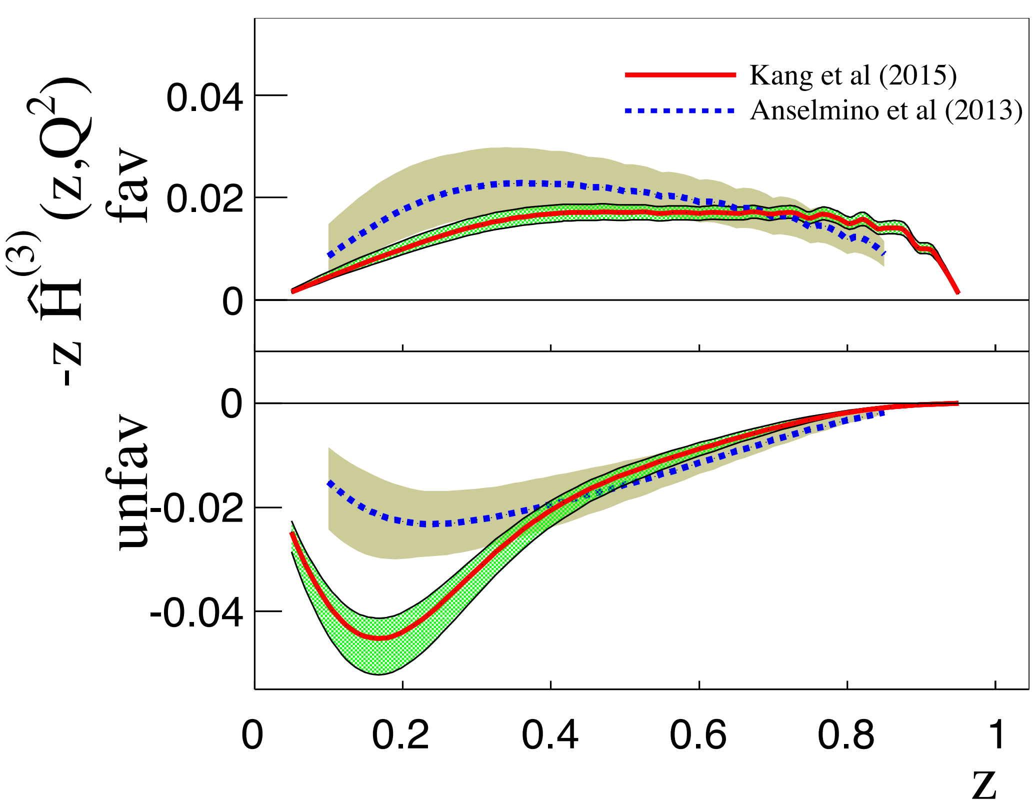




moment of Collins functions





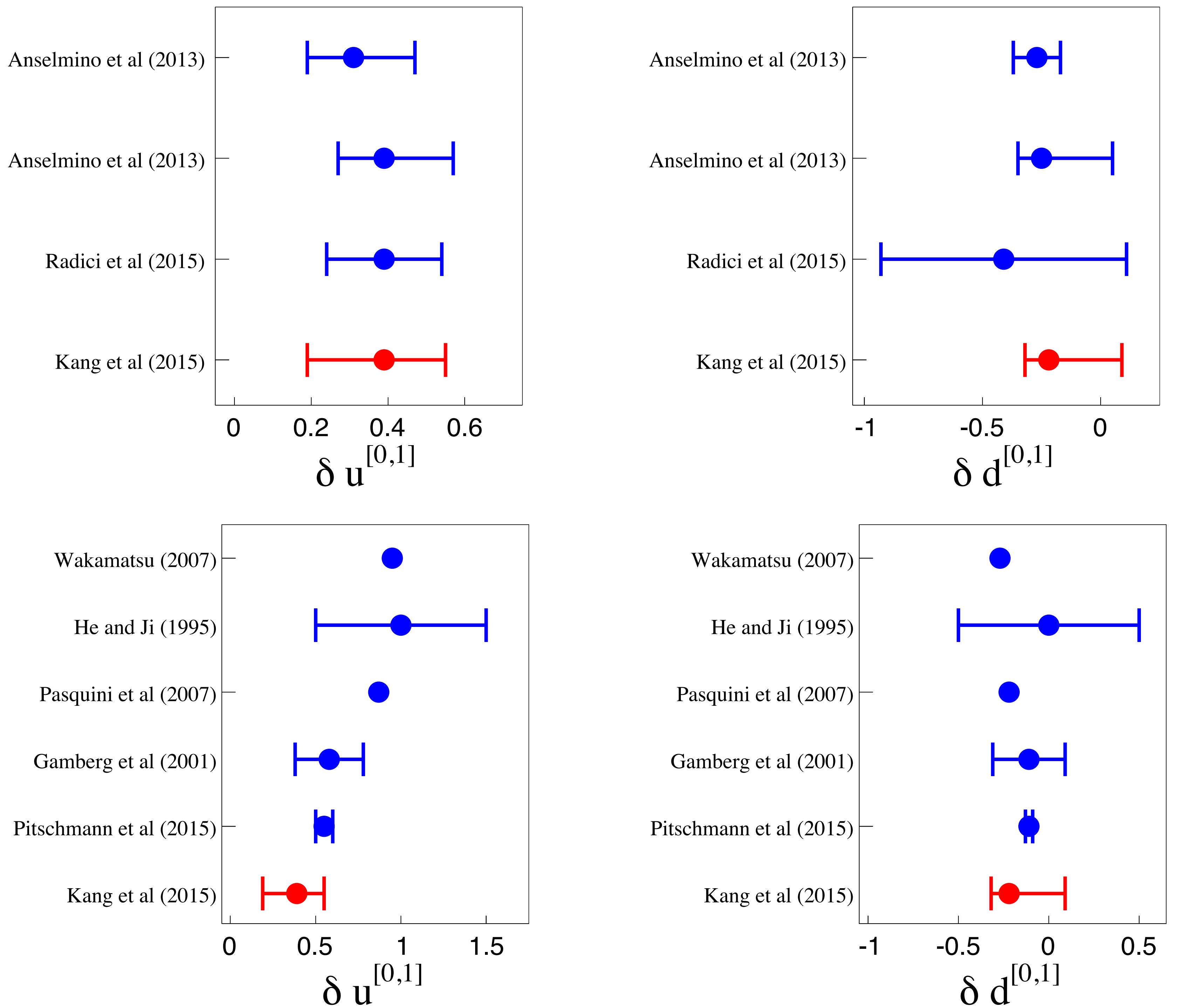


comparison with phase 1 extraction, Q^2 = 2.4 GeV^2

(Kang, Prokudin, Sun, Yuan, arXiv:1505.05589)

(talks by O. Gonzalez, A. Bacchetta)

comparison of tensor charges from different extractions and models, at $Q^2 = 10$ GeV^2

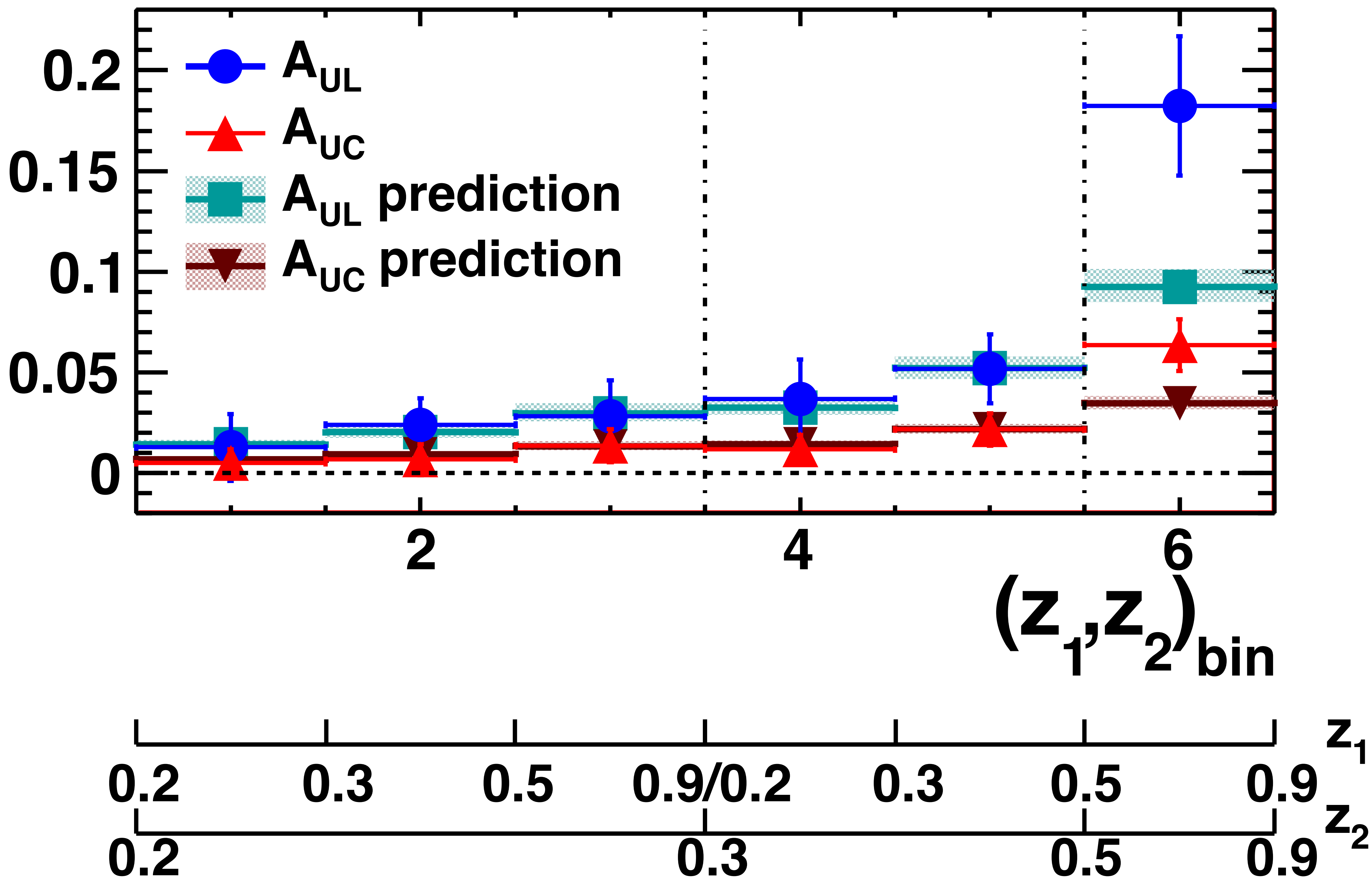


$$\delta q = \int_0^1 dx\, [\Delta_T q(x) - \Delta_T \bar{q}(x)]$$

predictions for BES-III e^+e^- Collins asymmetry A_0 in excellent agreement with data, Q^2 = 13 GeV^2

(some difficulties without TMD evolution)

(Kang, Prokudin, Sun, Yuan, arXiv:1505.05589)



Conclusions

Sivers and Collins effects are well established, many transverse spin asymmetries resulting from them. Sivers function and orbital angular momentum?

Evidence for gaussian $k_\perp$ and $p_\perp$ dependence of unpolarised TMD-PDFs and TMD-FFs

Gluon TMDs deserve special attention; they might play a role at LHC

Much progress in studies of TMD factorisation and TMD evolution; phenomenological implementation in progress

Combined data from SIDIS, Drell-Yan, e+e-, with theoretical modelling, should lead to a true 3D imaging of the proton

waiting for JLab 12, new COMPASS results, future facilities....
(talks by F. Bradamante, P. Rossi)