Unexpected chiral dynamics from lattice QCD calculations of nuclear systems

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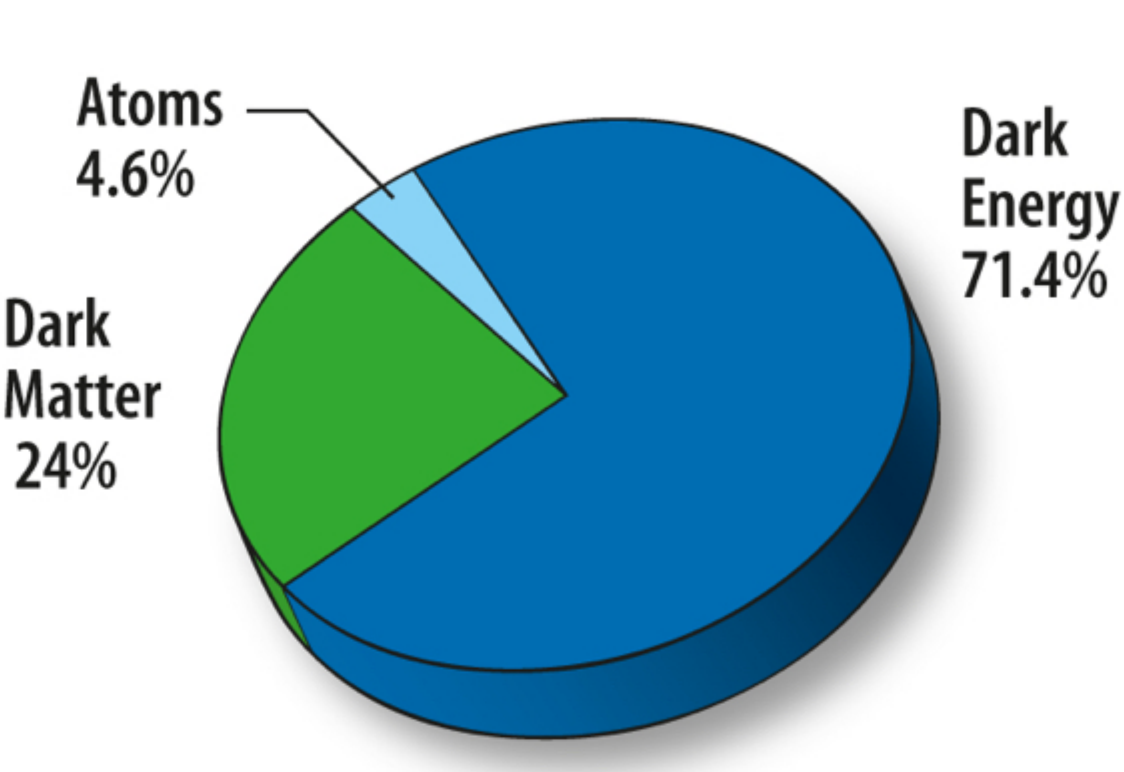
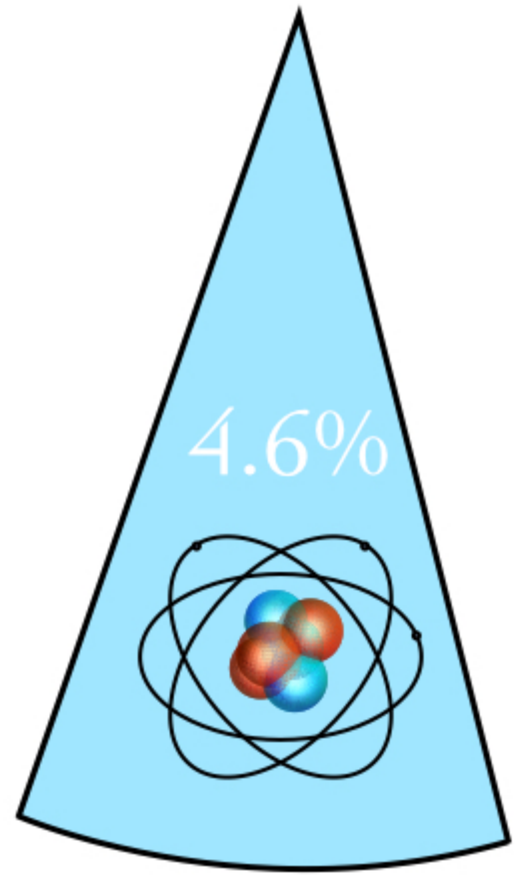
Pisa — Chiral Dynamics 6/30/2015

Outline

* Motivation
* Lattice QCD
* Knobs in Lattice QCD
* Extracting interactions
* Nuclear results
* Summary

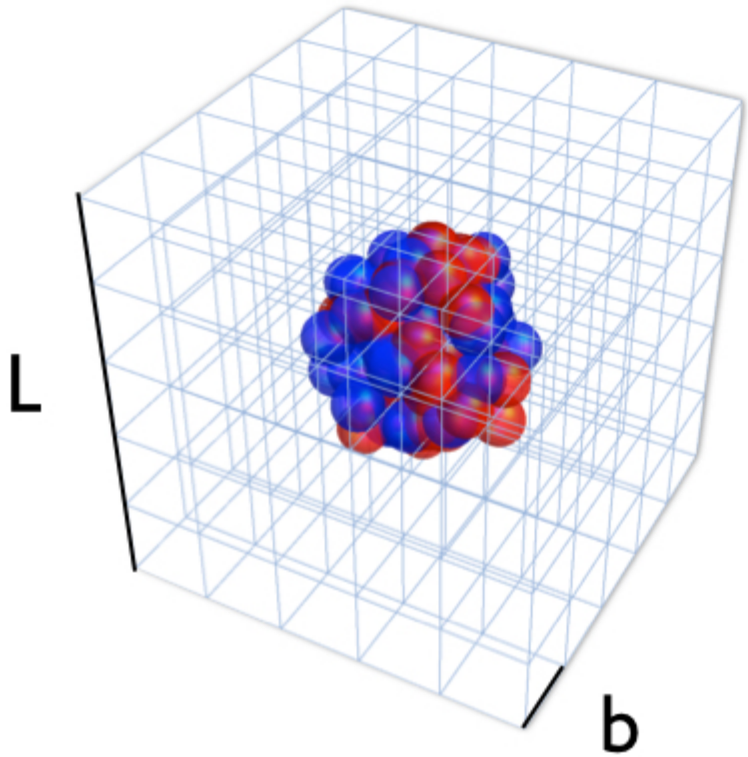
Motivation





This talk is about ongoing efforts to understand the observed 4.6% — the physics of atomic nuclei— quantitatively from first principles

Lattice QCD = QCD on a grid or lattice



volume: $M_\pi L \gg 1$

infrared cutoff

lattice spacing: $b \ll M_N^{-1}$

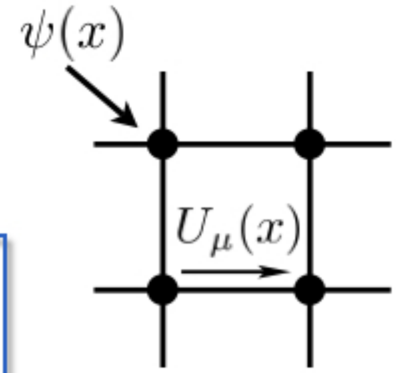
ultraviolet cutoff

Can use Effective Field Theory to extrapolate in L and b!

[Symanzik (1983), Gasser and Leutwyler (1988)]

Systematic uncertainties from lattice artifacts are controlled

QCD path integral with Montecarlo



$$\langle \mathcal{O} \rangle \sim \int dU_\mu \, d\bar{\psi} \, d\psi \; \mathcal{O}\left(U, \psi, \bar{\psi}\right) \; e^{-S_g(U) - \bar{\psi} D(U) \psi}$$

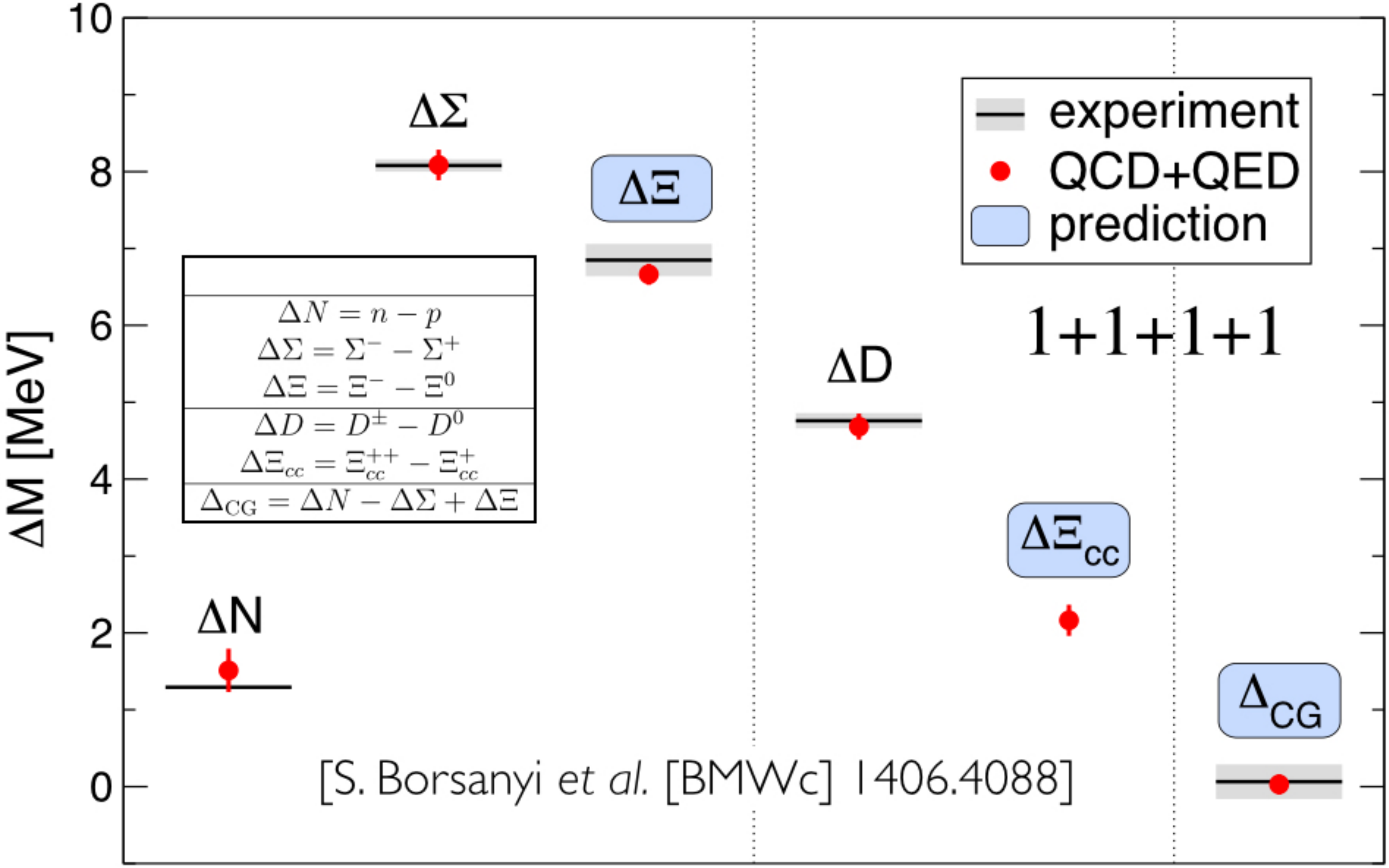
propagators

N gauge configurations

$$\langle \mathcal{O} \rangle \sim \int dU_\mu \, \mathcal{O}(D(U)^{-1}) \;\; \det\left(f(U)\right) \;\; e^{-S_g(U)} \; = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(D(U_i)^{-1})$$

Estimate of $\mathcal{O}$ with $\sigma_{\mathcal{O}} \sim 1/\sqrt{N}$

State of the Art: QCD+QED



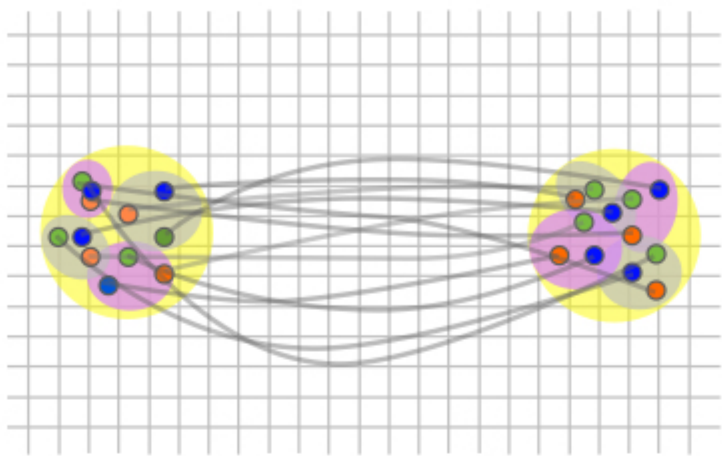
Lattice QCD for nuclear physics is expensive

- Signal/noise (sign problem) and statistics

  [Lepage (1989)]

- Number of contractions

  [Detmold,Orginos(2012),Doi,Endres(2012)]



★ Fundamental parameters as knobs:

α_s α_e m_u m_d m_s

Nuclear fine-tunings!

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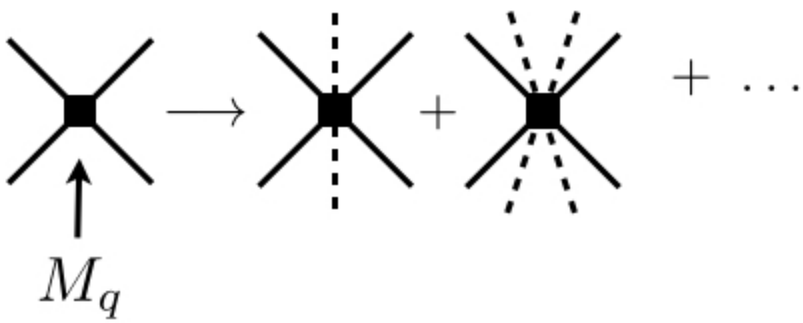
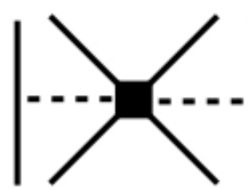
Nuclear fine-tunings!

- Calculation of nuclear forces requires these knobs!





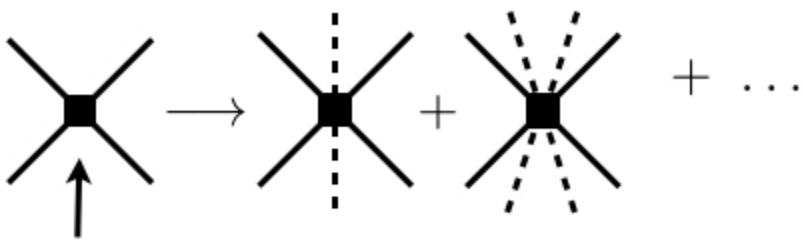
Four-body force

Fundamental parameters as knobs:

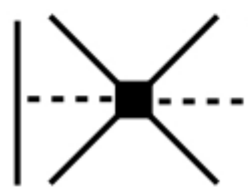
α_s α_e m_u m_d m_s m_c

Nuclear fine-tunings!

- Calculation of nuclear forces requires these knobs!



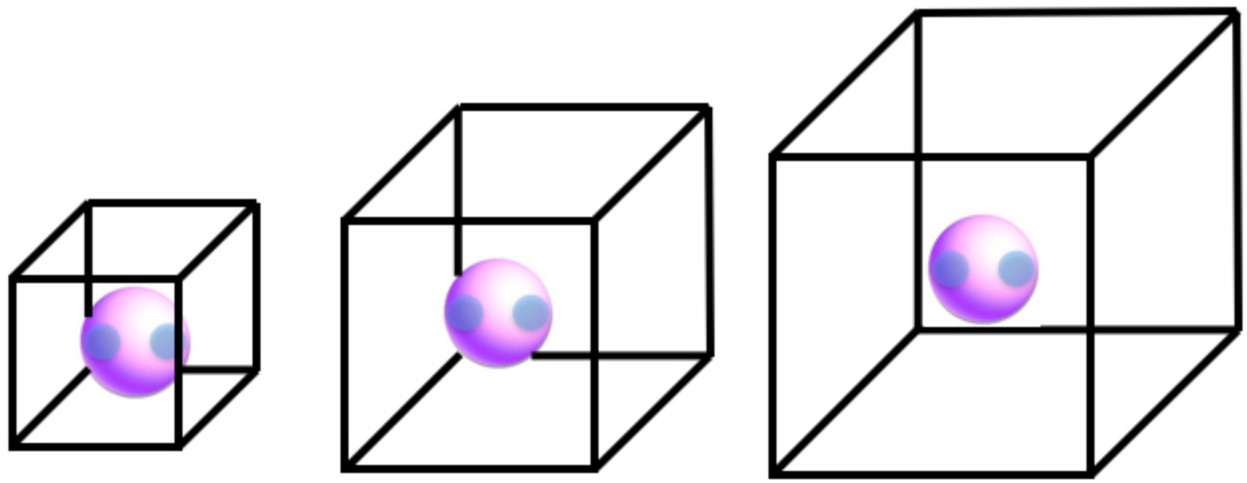
M_q



Four-body force

- Interactions of nuclei with dark matter

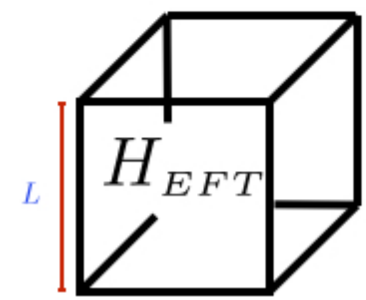
Lattice size as a knob



- Calculation of interactions requires this knob!

$$p\cot\delta = \frac{1}{\pi L}\mathcal{S}(\tilde{p})$$

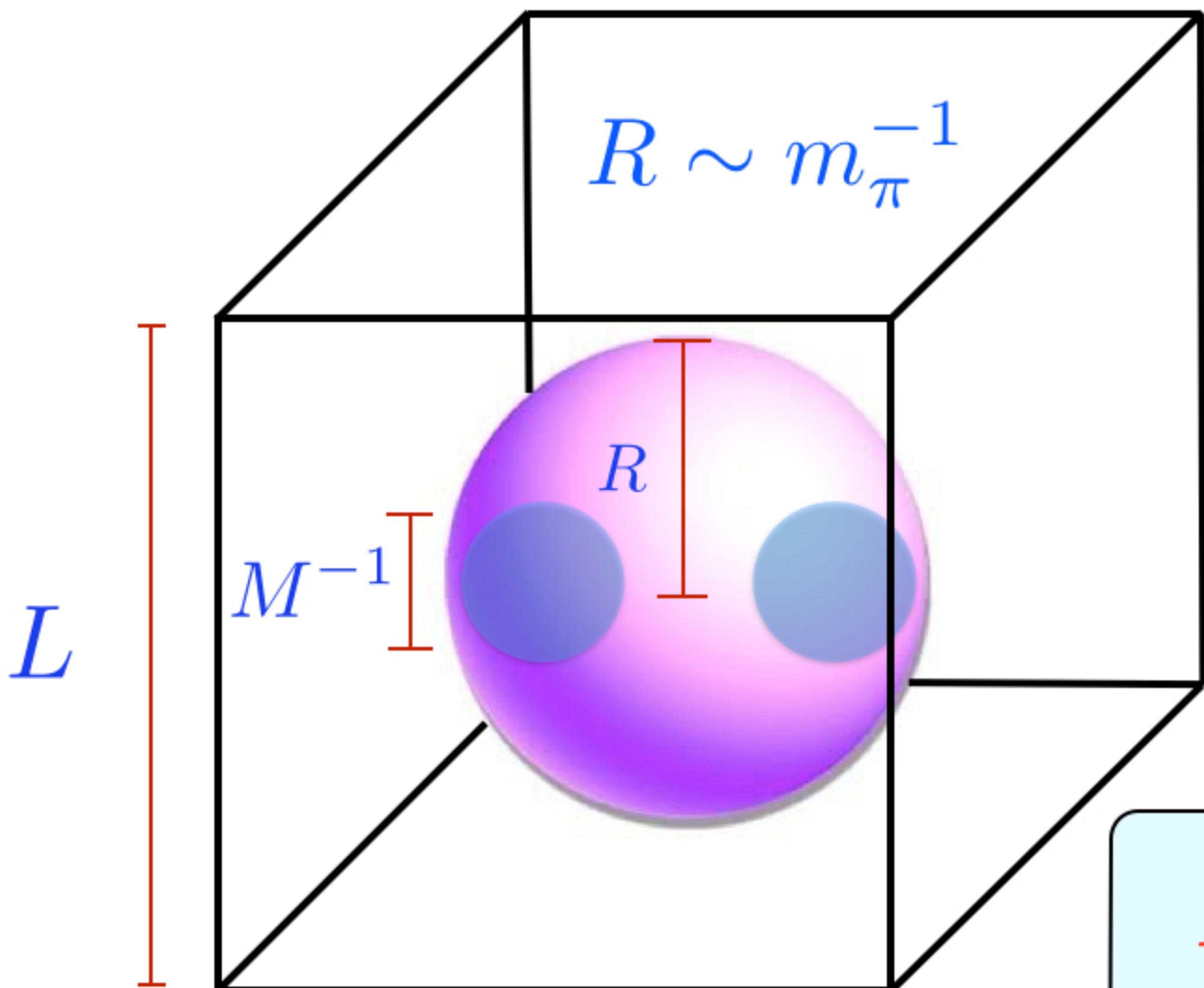
Lüscher Quantization Condition



EFT in a Box

Lüscher Quantization Condition

[Luescher (1990)]

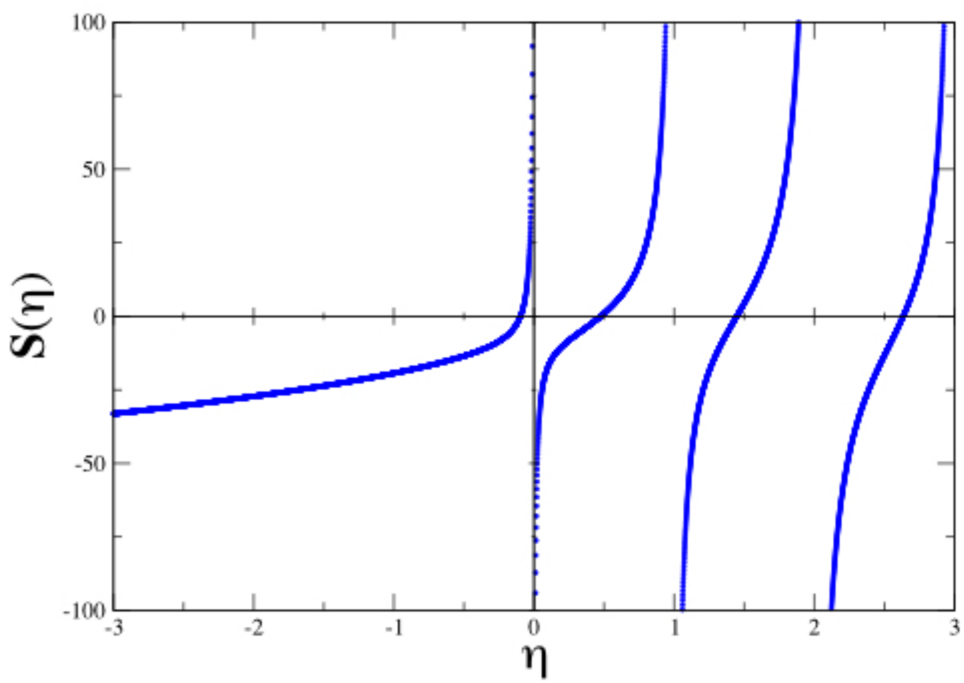


$$R < L/2$$

✓ S-wave quantization condition:

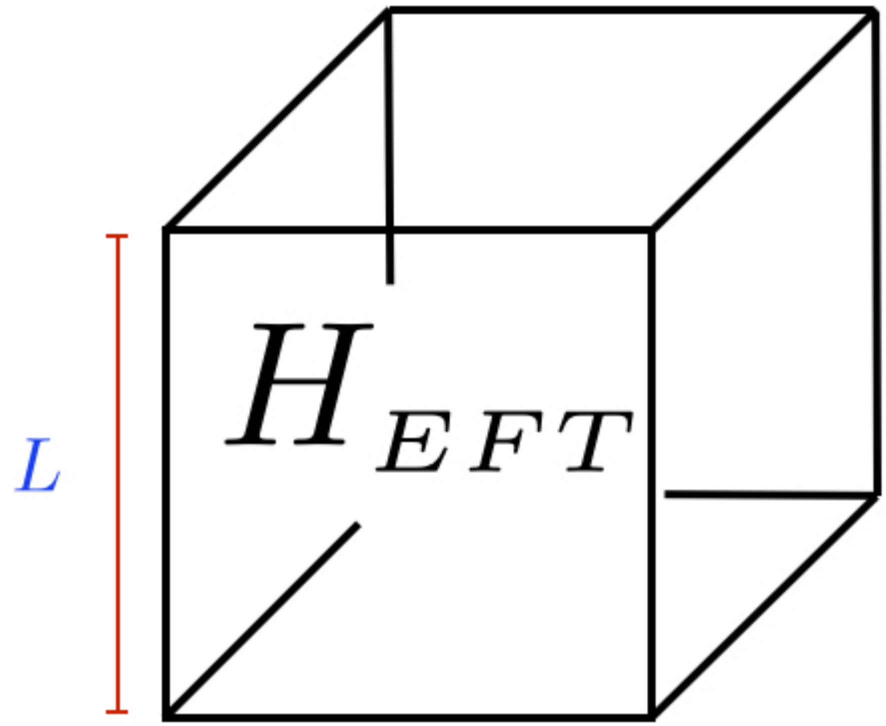
$$\boxed{p \cot \delta = \frac{1}{\pi L} \mathcal{S}(\tilde{p})} + \mathcal{O}\left(e^{-m_\pi L}\right)$$

Valid in QFT up to inelastic threshold



$$\mathcal{S}(x) \equiv \sum_{\mathbf{n}}^{\Lambda_n} \frac{1}{|\mathbf{n}|^2 - x^2} - 4\pi\Lambda_n$$

EFT in a Box



Effective field theory Hamiltonian in a finite volume with periodic BCs

$$V(\mathbf{r}) = V(\mathbf{r} + \mathbf{m}L)$$

$$\psi(\mathbf{r}) = \psi(\mathbf{r} + \mathbf{m}L)$$

$$V(\mathbf{r}) \;=\; \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{r}}\, \tilde{V}(\mathbf{k}) \;\rightarrow\; V_L(\mathbf{r}) \;=\; \frac{1}{L^3}\sum_{\mathbf{n}} e^{i2\pi\mathbf{n}\cdot\mathbf{r}/L}\, \tilde{V}(\frac{2\pi}{L}\mathbf{n})$$

$$\psi(\mathbf{r}) \;=\; \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{r}}\, \tilde{\psi}(\mathbf{k}) \;\rightarrow\; \psi_L(\mathbf{r}) \;=\; \frac{1}{L^3}\sum_{\mathbf{n}} e^{i2\pi\mathbf{n}\cdot\mathbf{r}/L}\, \tilde{\psi}_L(\frac{2\pi}{L}\mathbf{n})$$

3-dimensional Schrödinger equation in finite volume

$$\frac{-\hbar^2}{2\mu}\boldsymbol{\nabla}^2\psi_L(\mathbf{r}) \;+\; V_L(\mathbf{r})\psi_L(\mathbf{r}) \;=\; E\;\psi_L(\mathbf{r})$$

3-dimensional Schrödinger equation in finite volume

$$\frac{-\hbar^2}{2\mu}\boldsymbol{\nabla}^2\psi_L(\mathbf{r}) \;+\; V_L(\mathbf{r})\psi_L(\mathbf{r}) \;=\; E\;\psi_L(\mathbf{r})$$

↓

$$\frac{\hbar^2}{2\mu}\left(\frac{2\pi}{L}\right)^2 |\mathbf{n}|^2\;\tilde{\psi}_L(\frac{2\pi}{L}\mathbf{n}) \;+\; \sum_{\overline{\mathbf{n}}}\;\tilde{V}(\frac{2\pi}{L}\left(\mathbf{n}-\overline{\mathbf{n}}\right))\;\tilde{\psi}_L(\frac{2\pi}{L}\overline{\mathbf{n}}) \;=\; E_L\;\tilde{\psi}_L(\frac{2\pi}{L}\mathbf{n})$$

3-dimensional Schrödinger equation in finite volume

$$\frac{-\hbar^2}{2\mu}\boldsymbol{\nabla}^2\psi_L(\mathbf{r}) \;+\; V_L(\mathbf{r})\psi_L(\mathbf{r}) \;=\; E\;\psi_L(\mathbf{r})$$

$$\downarrow$$

$$\frac{\hbar^2}{2\mu}\left(\frac{2\pi}{L}\right)^2 |\mathbf{n}|^2\;\tilde{\psi}_L(\frac{2\pi}{L}\mathbf{n}) \;+\; \sum_{\overline{\mathbf{n}}}\tilde{V}(\frac{2\pi}{L}(\mathbf{n}-\overline{\mathbf{n}}))\;\tilde{\psi}_L(\frac{2\pi}{L}\overline{\mathbf{n}}) \;=\; E_L\;\tilde{\psi}_L(\frac{2\pi}{L}\mathbf{n})$$

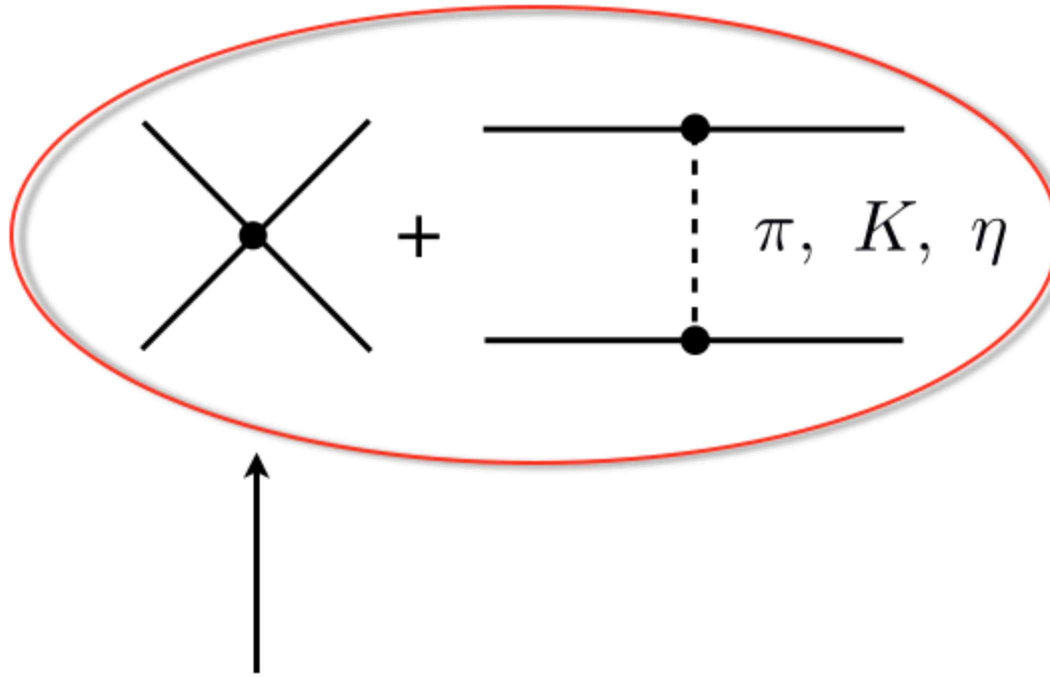
$$\hat{H}_{\mathbf{n},\mathbf{n}'} \;=\; \frac{2\pi^2\hbar^2}{\mu L^2}\;|\mathbf{n}|^2\;\delta_{\mathbf{n},\mathbf{n}'} \;+\; \tilde{V}(\frac{2\pi}{L}(\mathbf{n}-\mathbf{n}'))$$

Diagonalize large symmetric matrix

E.g. Match BB levels to Effective Field Theory

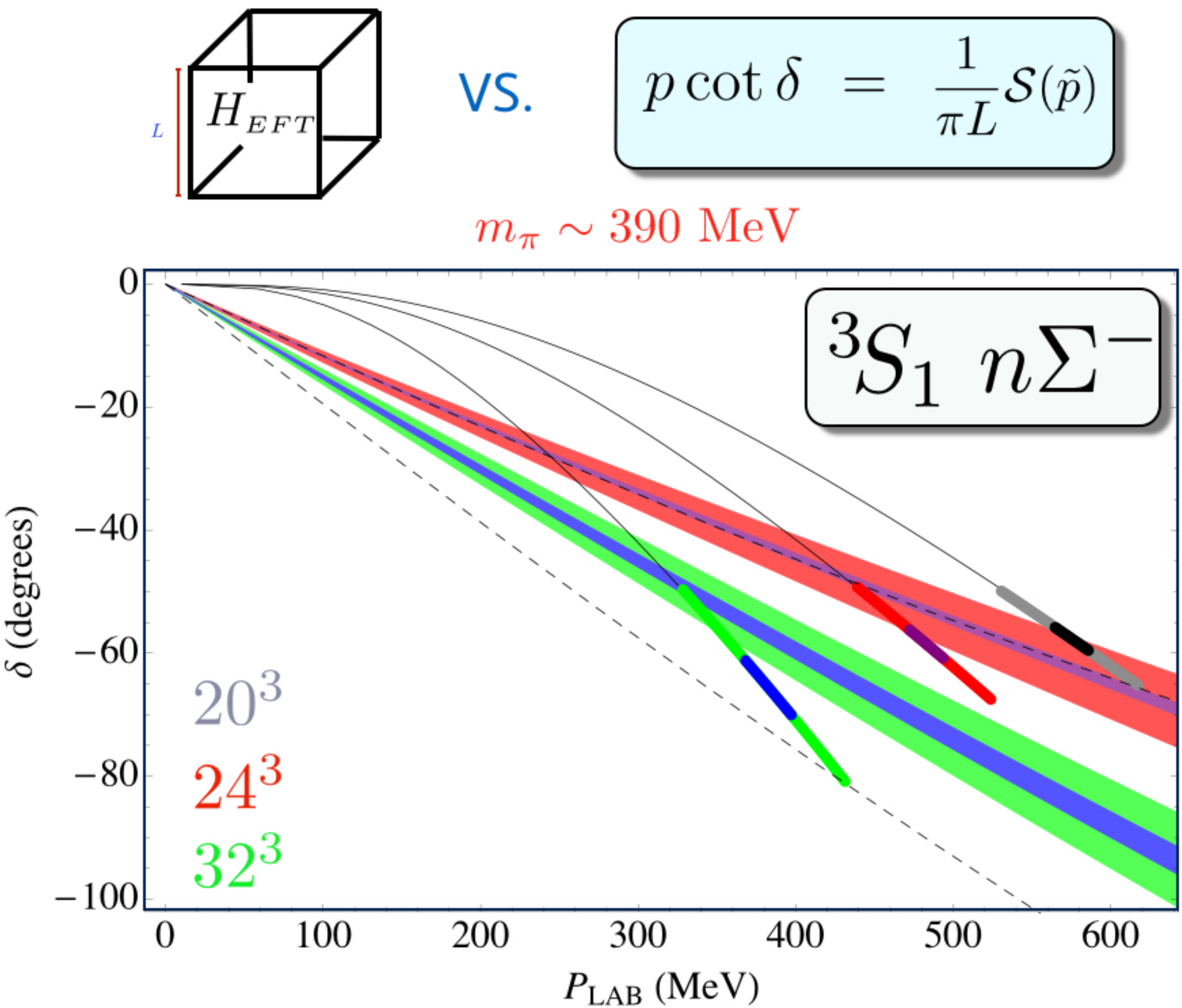
LO BB potential:

[Weinberg (1990)]



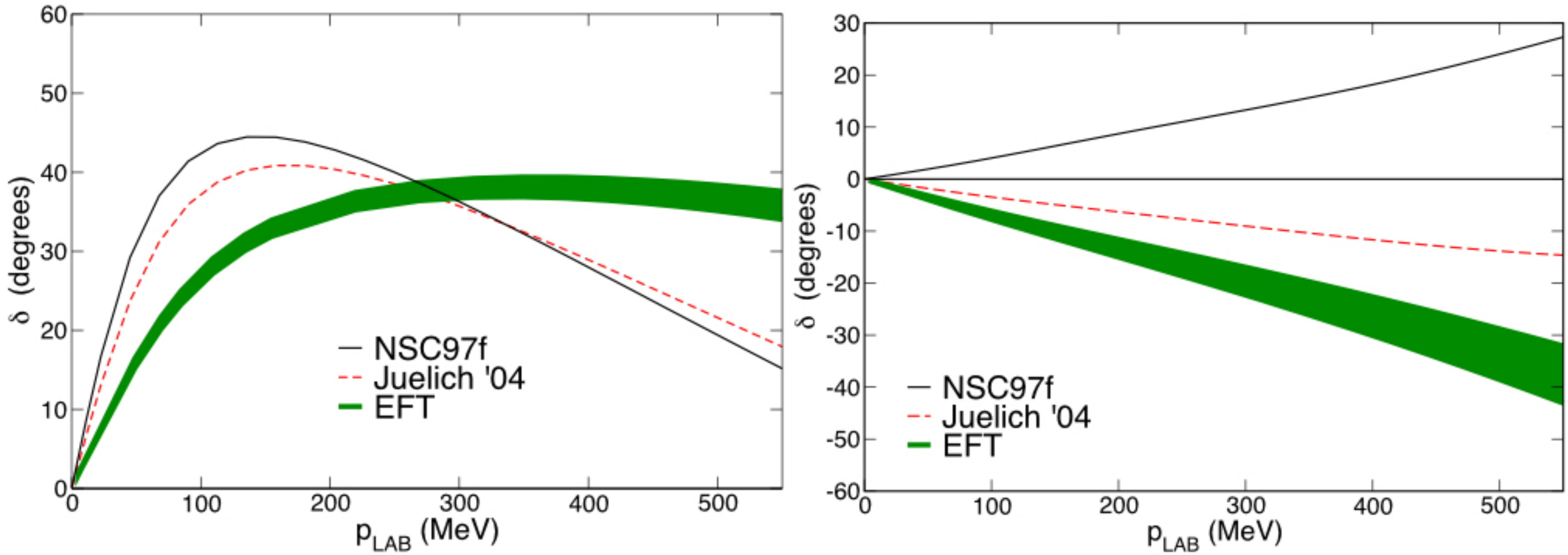
Fit coupling to match energy levels

Now we have LO potential at ALL pion masses!



H_{EFT}
L
vs.
$p\cot\delta \;=\; \frac{1}{\pi L}\mathcal{S}(\tilde{p})$
$m_\pi \sim 390$ MeV
$^3S_1\ n\Sigma^-$
20^3
24^3
32^3
δ (degrees)
$P_{\rm LAB}$ (MeV)
0
−20
−40
−60
−80
−100
0
100
200
300
400
500
600

$^1S_0\ n\Sigma^-$ $^3S_1\ n\Sigma^-$

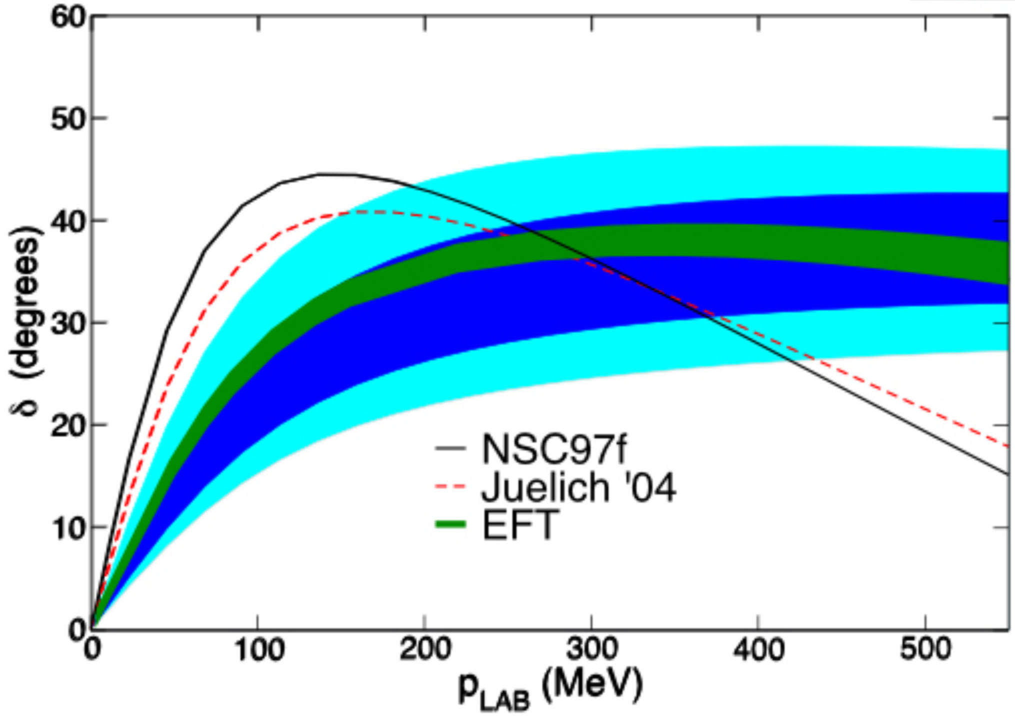


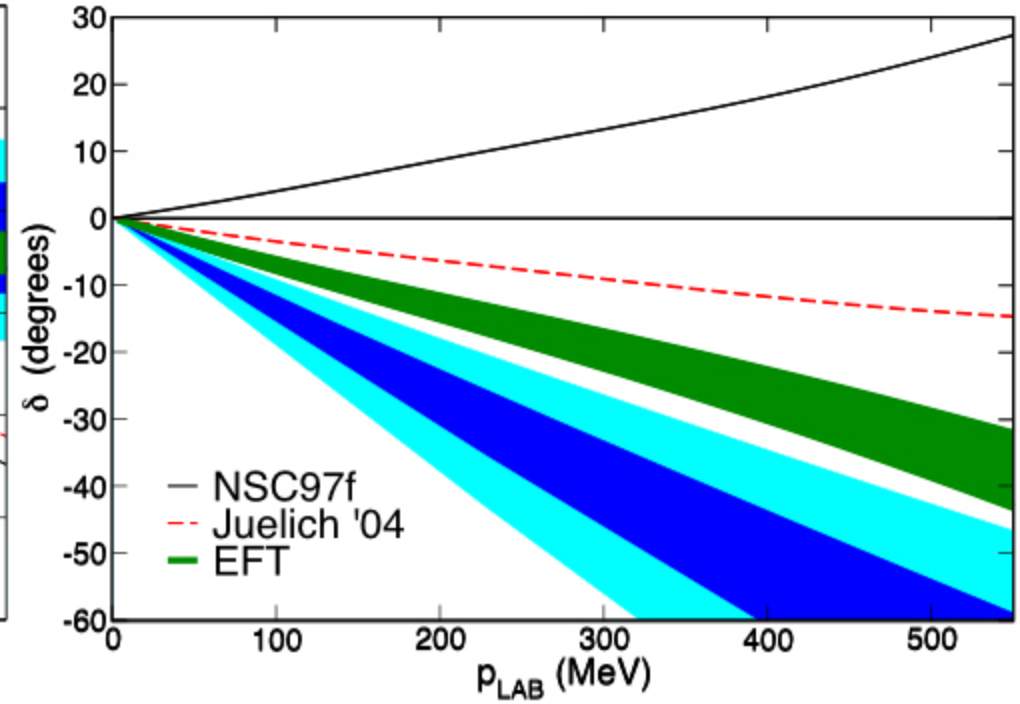
[Haidenbauer, Meissner,(1990)]

$^{1}S_{0}\ n\Sigma^{-}$

NPLQCD

$^{3}S_{1}\ n\Sigma^{-}$



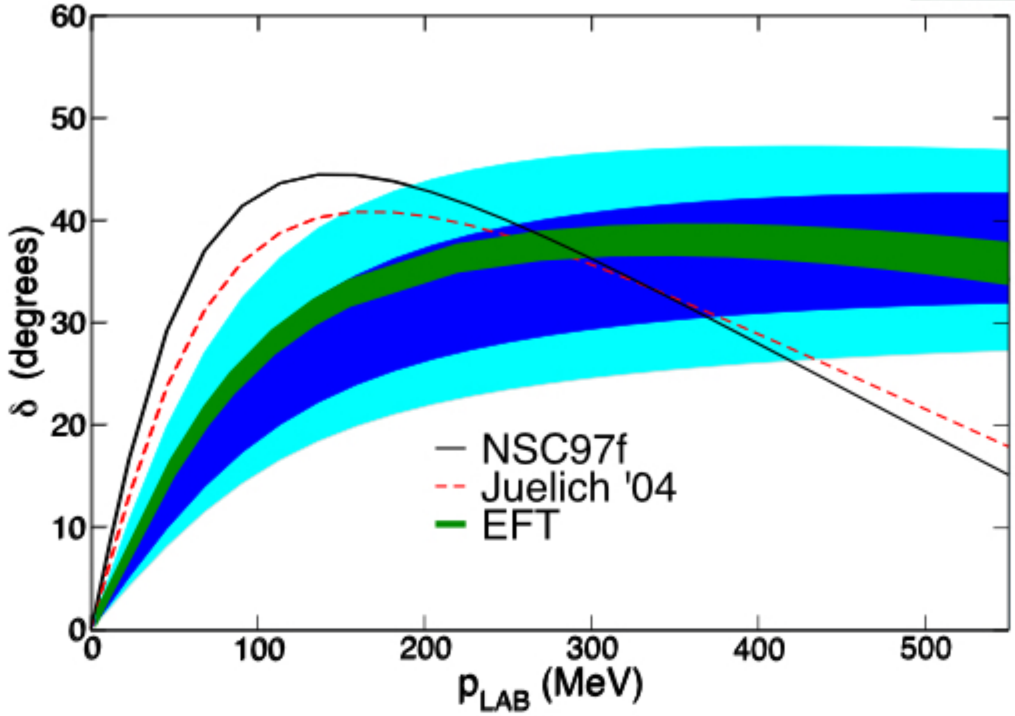


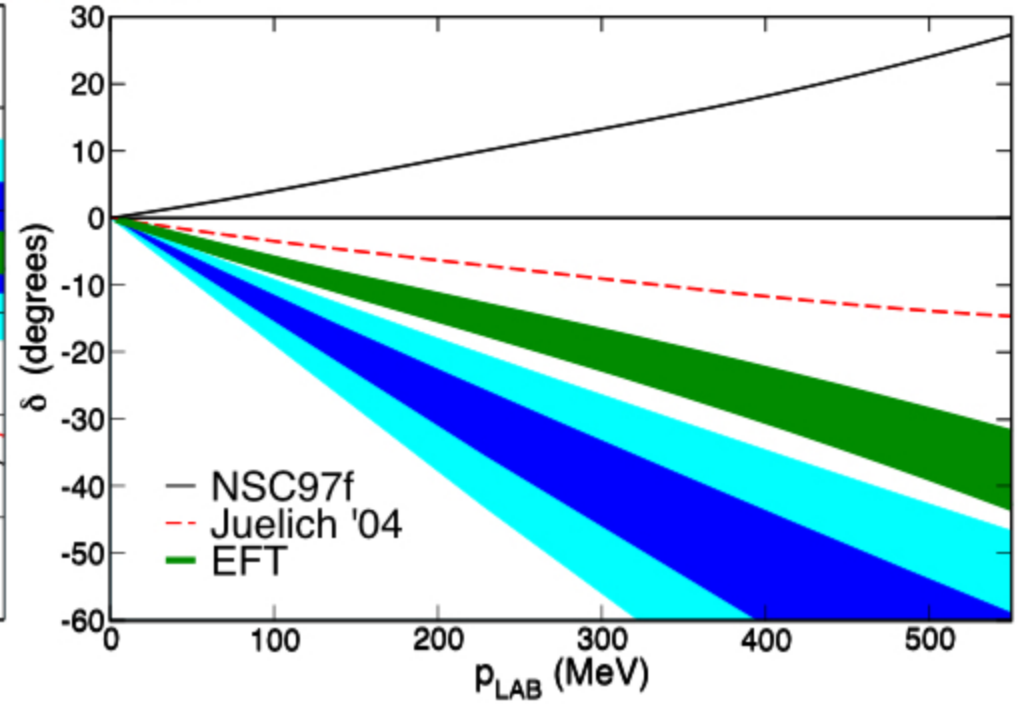
[Haidenbauer, Meissner,(1990)]

$^1S_0 \ n\Sigma^-$

NPLQCD

$^3S_1 \ n\Sigma^-$



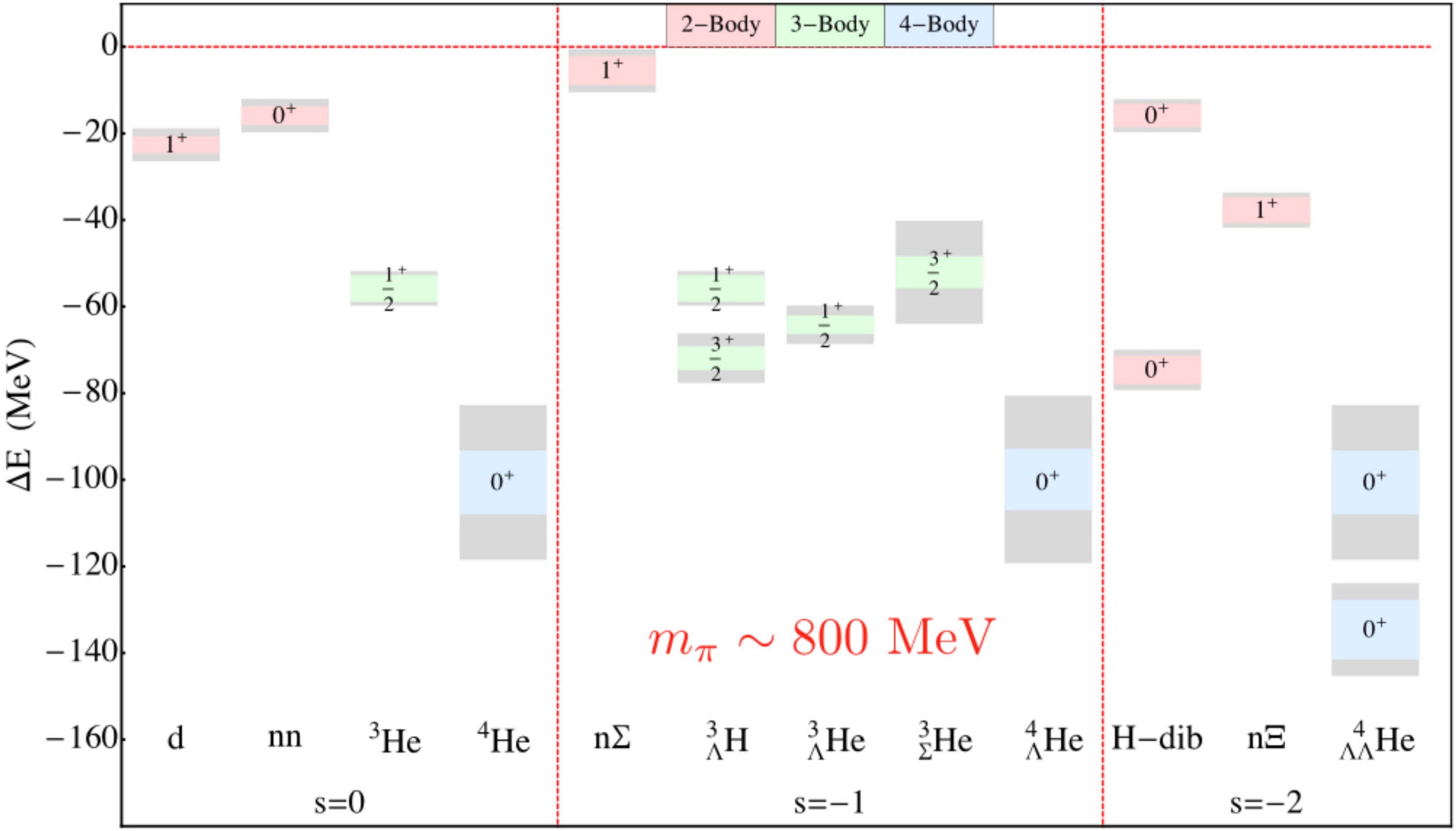


[Haidenbauer, Meissner,(1990)]

- First Lattice QCD predictions for nuclear physics
- Relevant for dense matter energy shift of the hyperon

(Hyper)Nuclei in the SU(3) Limit

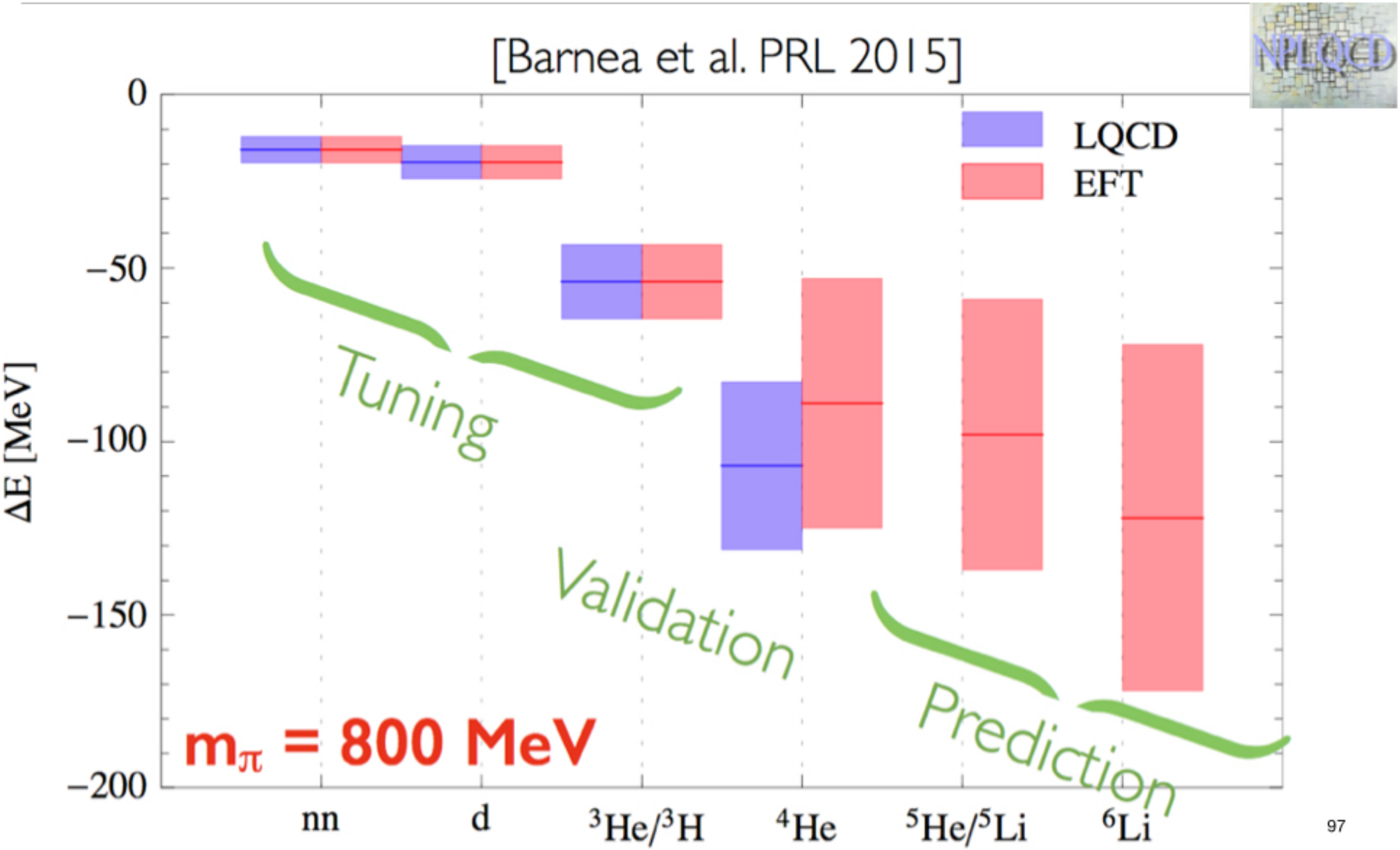
[Savage||]



Nuclear structure + LQCD

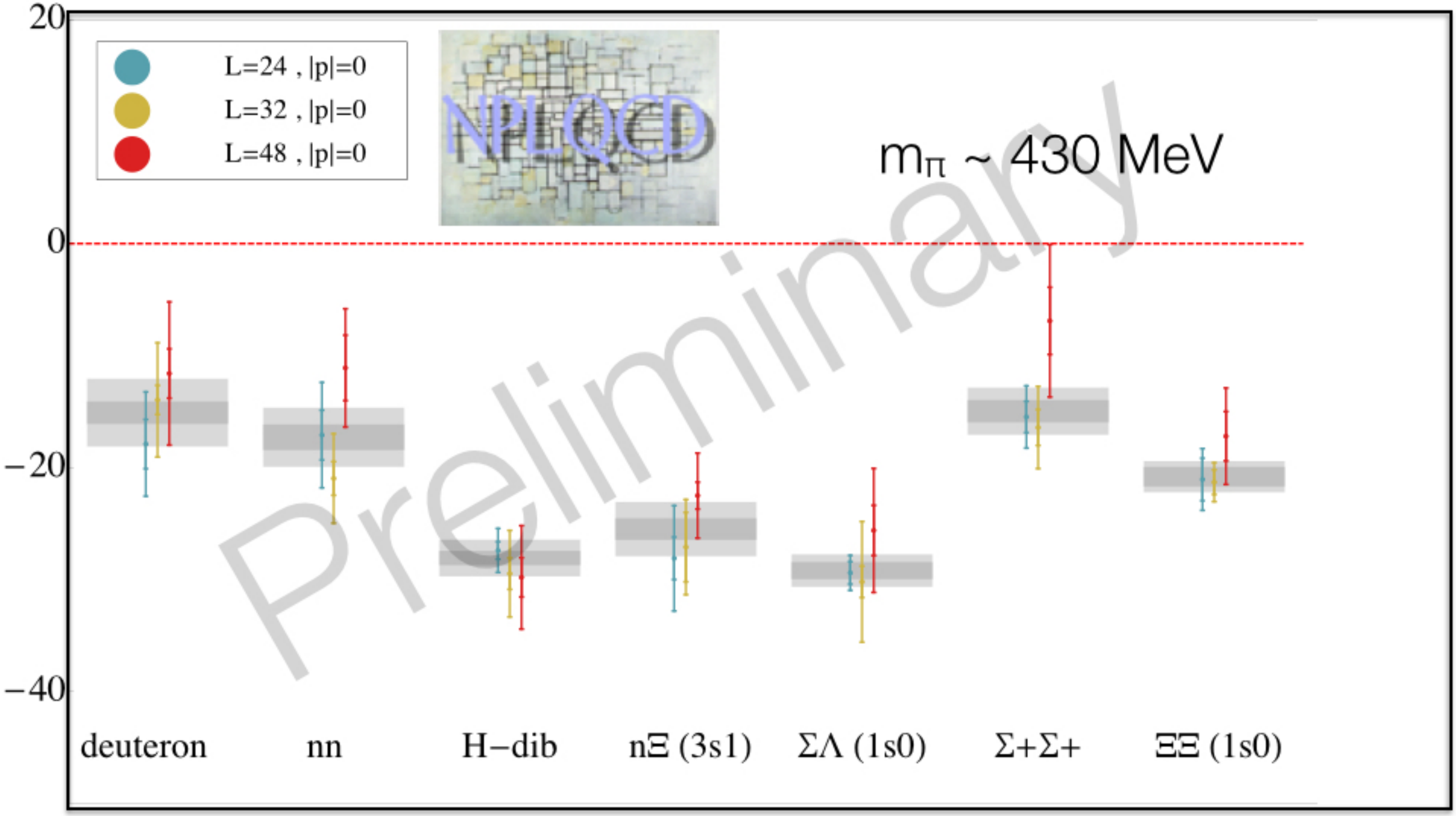
[Contessi||]

[Barnea et al. PRL 2015]

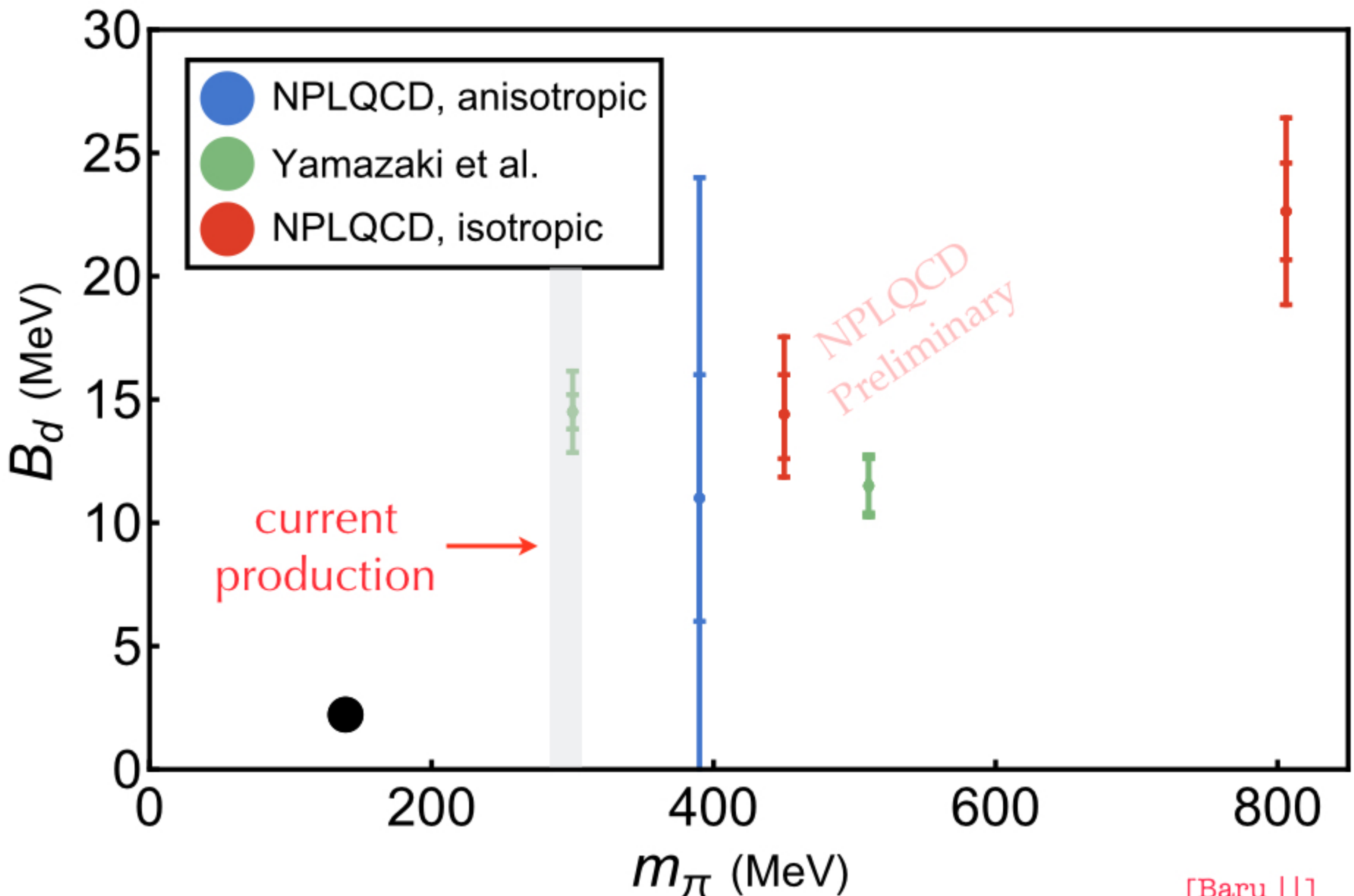


(Hyper)Nuclei in 2+1

two-body systems:

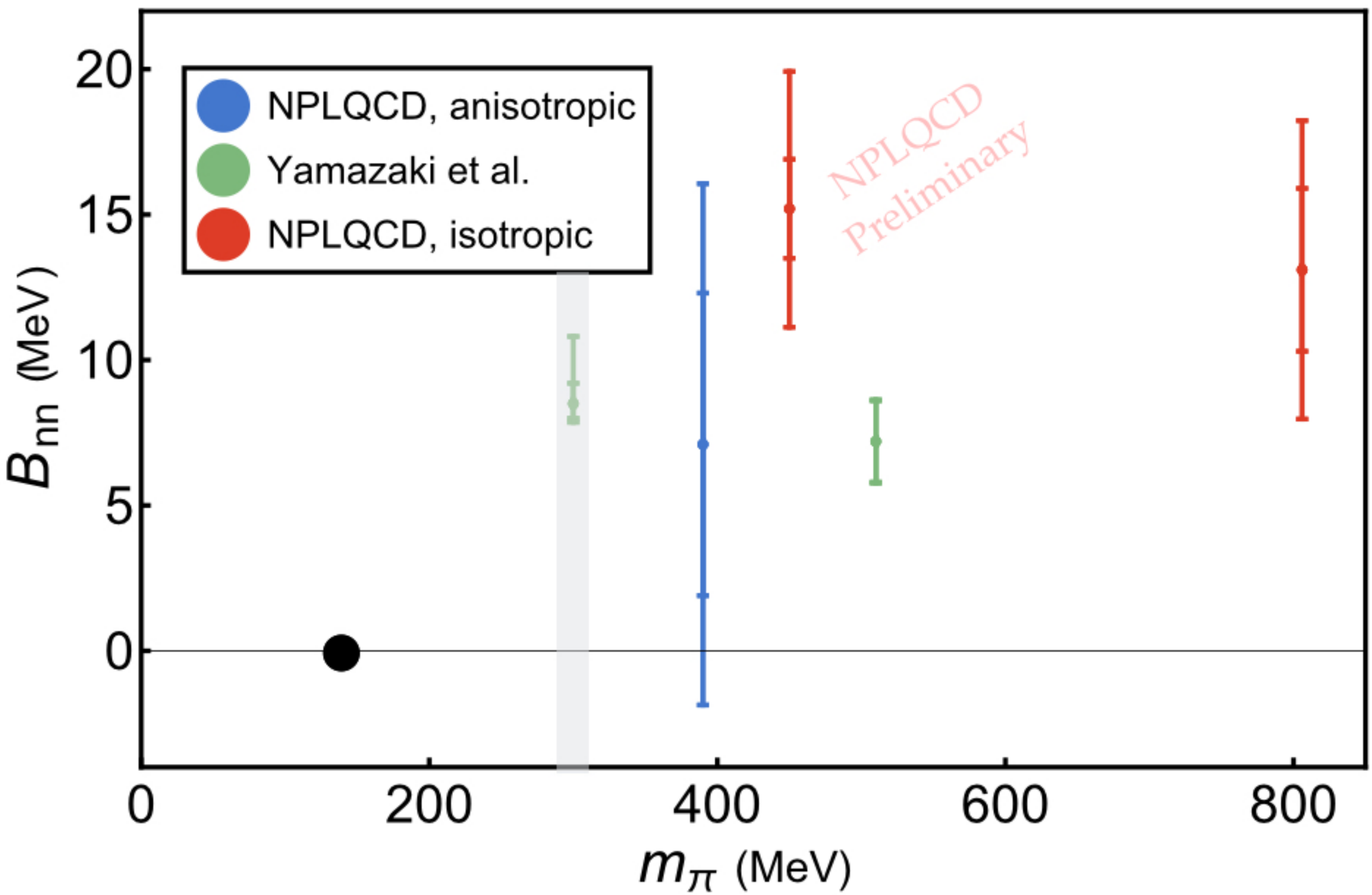


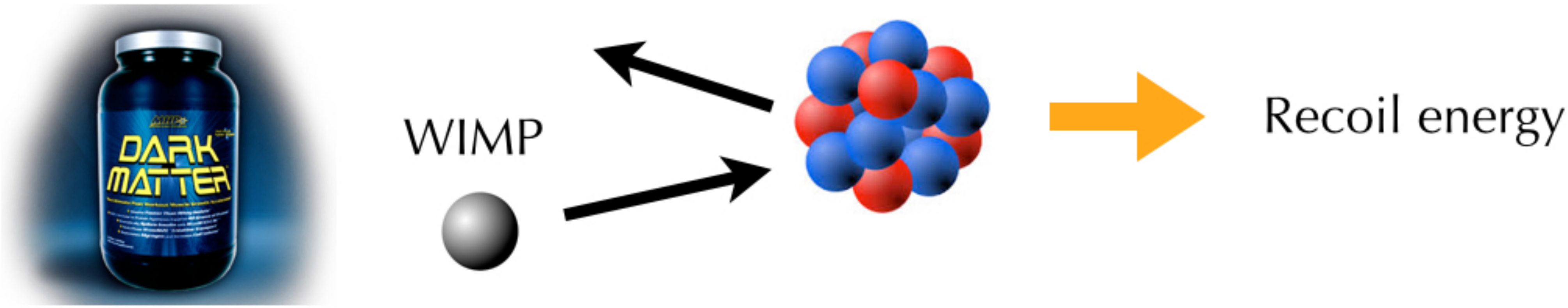
Deuteron binding energy from LQCD

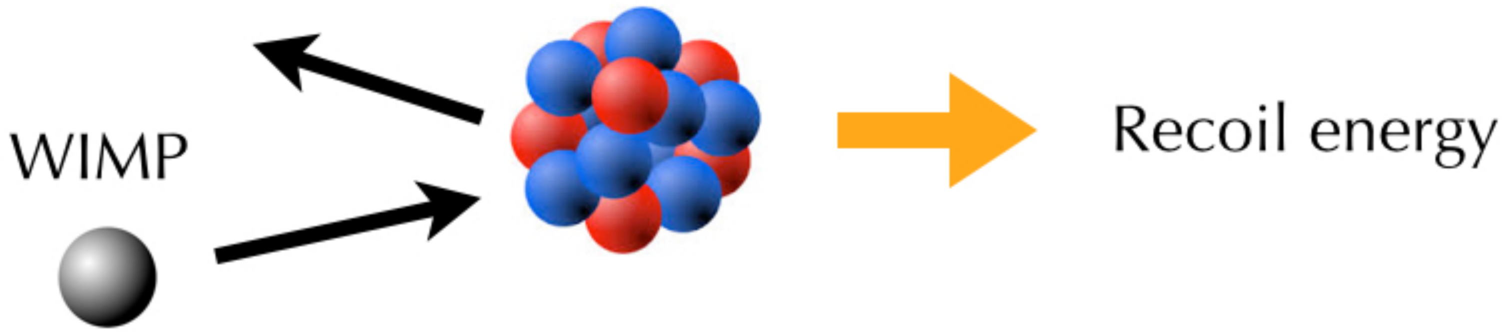


[Baru ||]

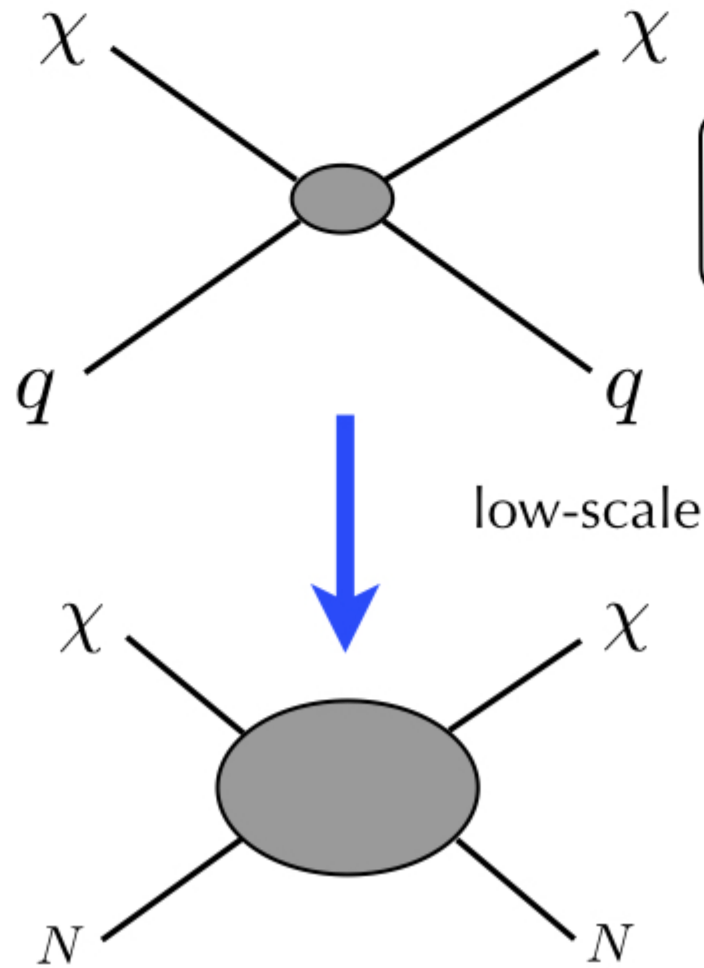
Dineutron binding energy from LQCD




DARK
MATTER
WIMP
Recoil energy



Spin-independent WIMP-quark interactions



$$\mathcal{L} = G_F \, \overline{\chi}\chi \sum_q a_S^{(q)} \, \overline{q}q$$

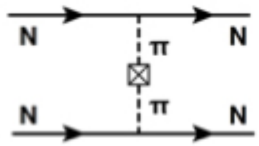
dim-3 operator transforms like quark masses

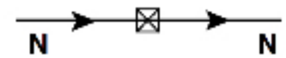
$$\sim G_F \, \bar{\chi}\chi \, \langle N|\bar{q}q|N\rangle$$

sigma term

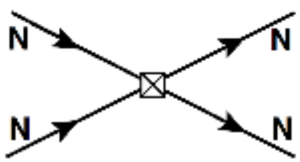
WIMP-QCD EFT

$$\mathcal{L} \;=\; \frac{G_F}{2}\,\overline{\chi}\chi\,\left[(a_S^{(u)}+a_S^{(d)})\overline{q}q \;+\; (a_S^{(u)}-a_S^{(d)})\overline{q}\tau^3 q \;+\; a_S^{(s)}\overline{s}s \;+\; \ldots\right]$$





$$\mathcal{L} \;\to\; G_F\,\overline{\chi}\chi\,\Bigg(\frac{1}{4}\langle 0|\overline{q}q|0\rangle\,\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] + \frac{1}{4}\langle N|\overline{q}q|N\rangle N^\dagger N\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] - \frac{1}{4}\langle N|\overline{q}\tau^3 q|N\rangle\left(N^\dagger N\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \;-\; 4N^\dagger a_{S,\xi}N\right) \;+\; \ldots\Bigg)$$

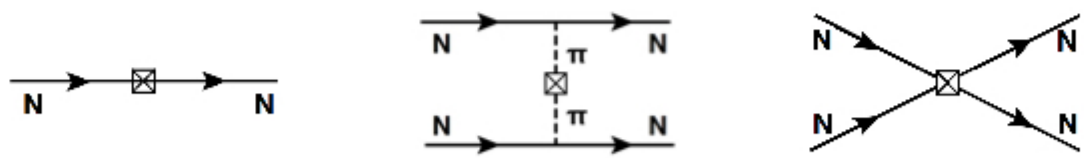


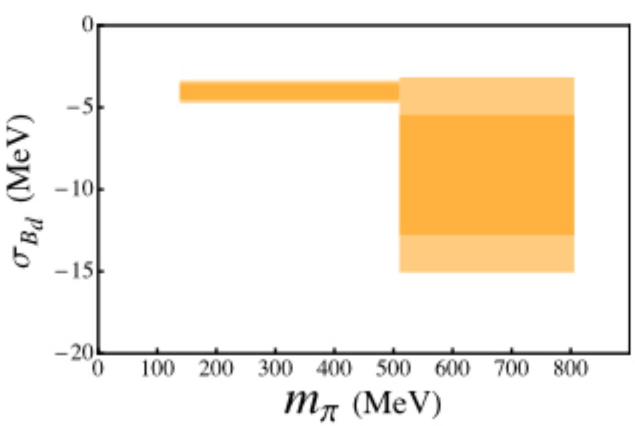
$$-G_F\overline{\chi}\chi\left(D_{S,1}\left(N^\dagger N\right)^2\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \;+\; D_{S,2}N^\dagger N N^\dagger a_{S,\xi}N + D_{T,1}\left(N^\dagger\sigma^a N\right)^2\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \;+\; D_{T,2}N^\dagger\sigma^a N N^\dagger\sigma^a a_{S,\xi}N\right)$$

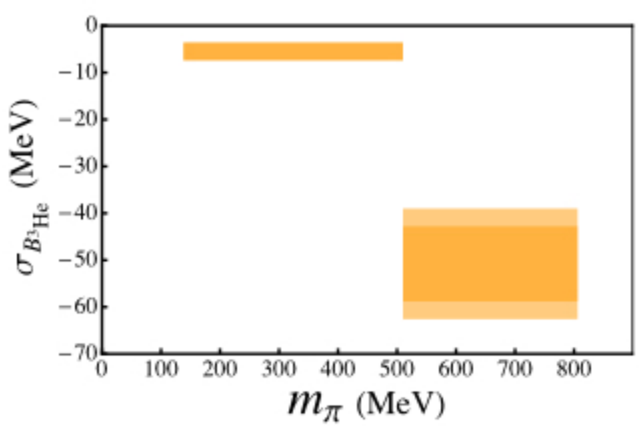
Nuclear sigma terms from binding energies

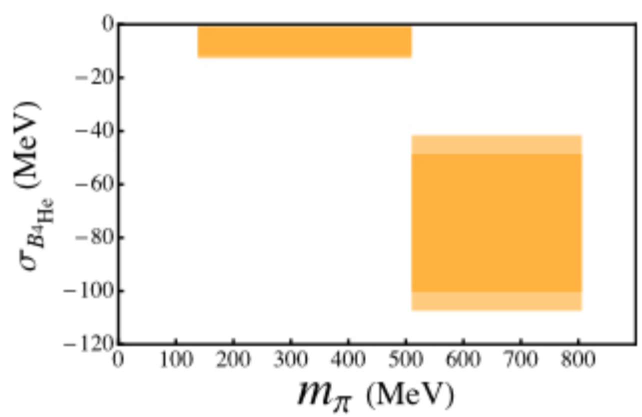
$$\sigma_{Z,N} = \overline{m}\langle Z,N(\mathrm{gs})|\ \overline{u}u + \overline{d}d\ |Z,N(\mathrm{gs})\rangle = \overline{m}\frac{d}{d\overline{m}}E_{Z,N}^{(gs)}$$

$$= \left[\,1 + \mathcal{O}\left(m_\pi^2\right)\,\right]\frac{m_\pi}{2}\frac{d}{dm_\pi}E_{Z,N}^{(\mathrm{gs})}$$

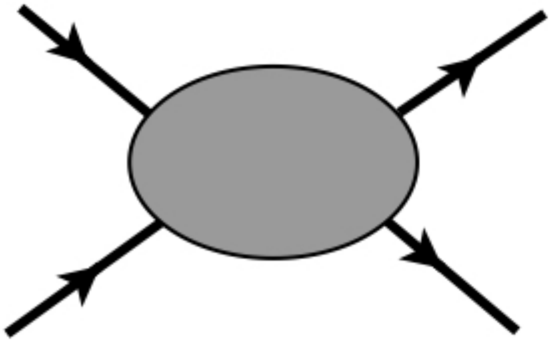








Nucleon-nucleon scattering



$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r |\mathbf{k}|^2 + P |\mathbf{k}|^4 + \mathcal{O}\left(|\mathbf{k}|^6\right)$$

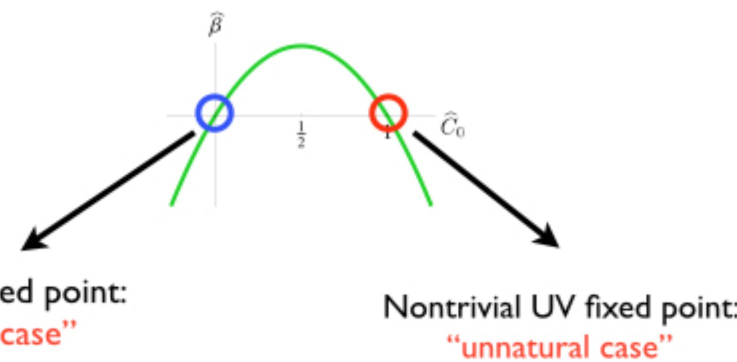
effective range: range of interaction

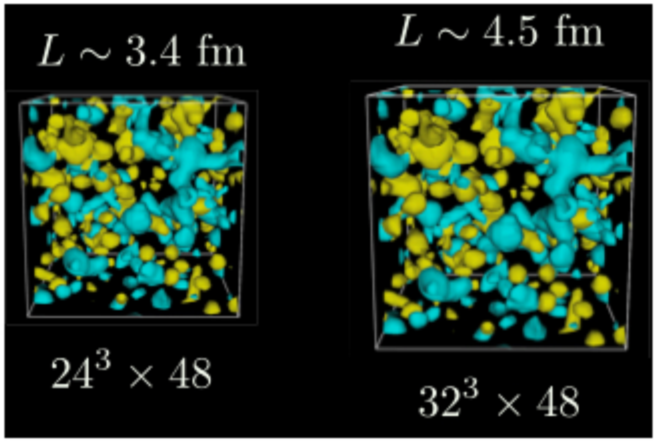
scattering length: unbounded

EXPERIMENT:

$a^{(^1S_0)}$	$= -23.71$ fm	$a^{(^3S_1)}$	$= 5.43$ fm
$r^{(^1S_0)}$	$= 2.73$ fm	$r^{(^3S_1)}$	$= 1.75$ fm

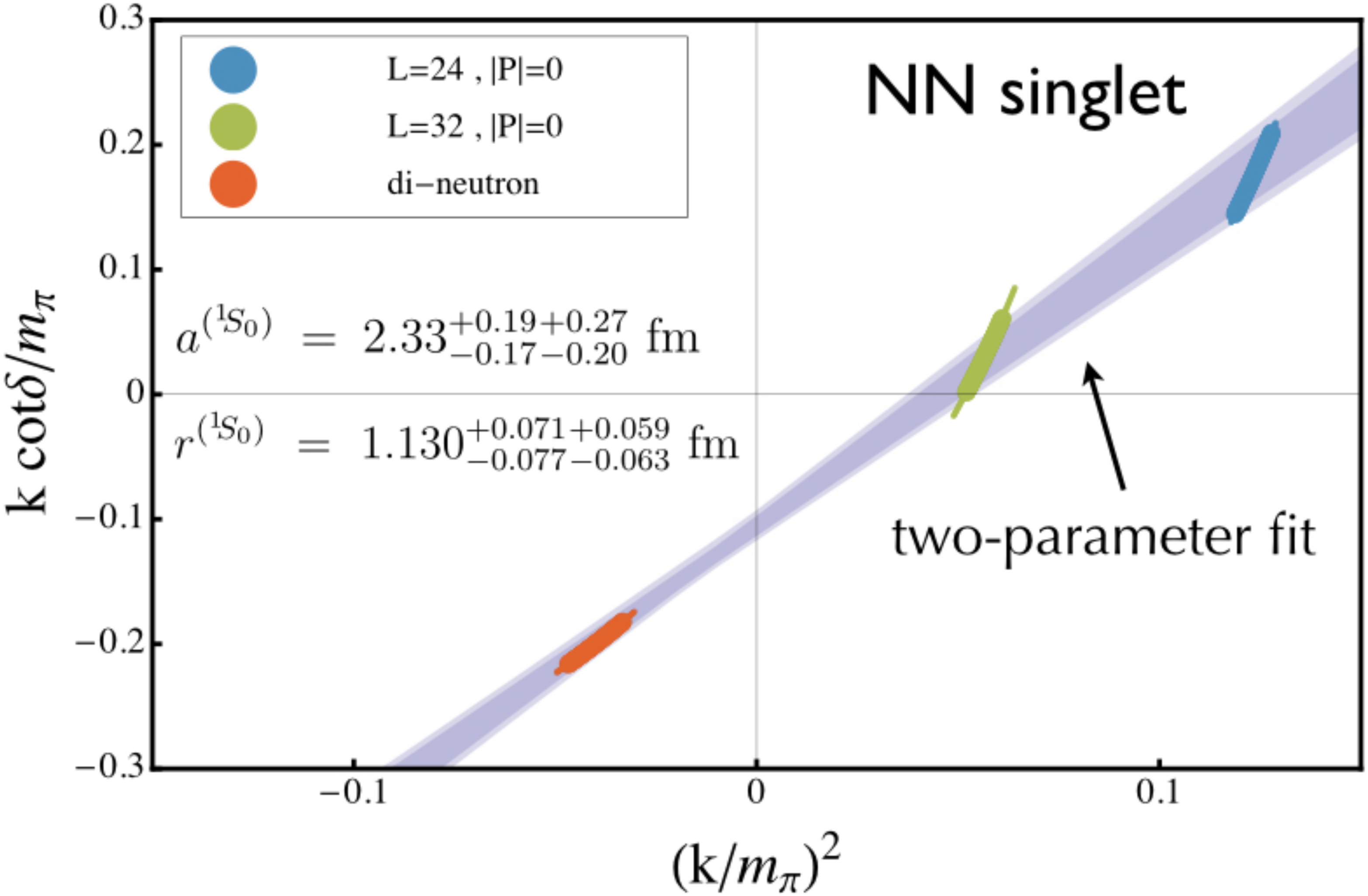
$$a^{(^1S_0)} \gg \Lambda_{QCD}^{-1}$$



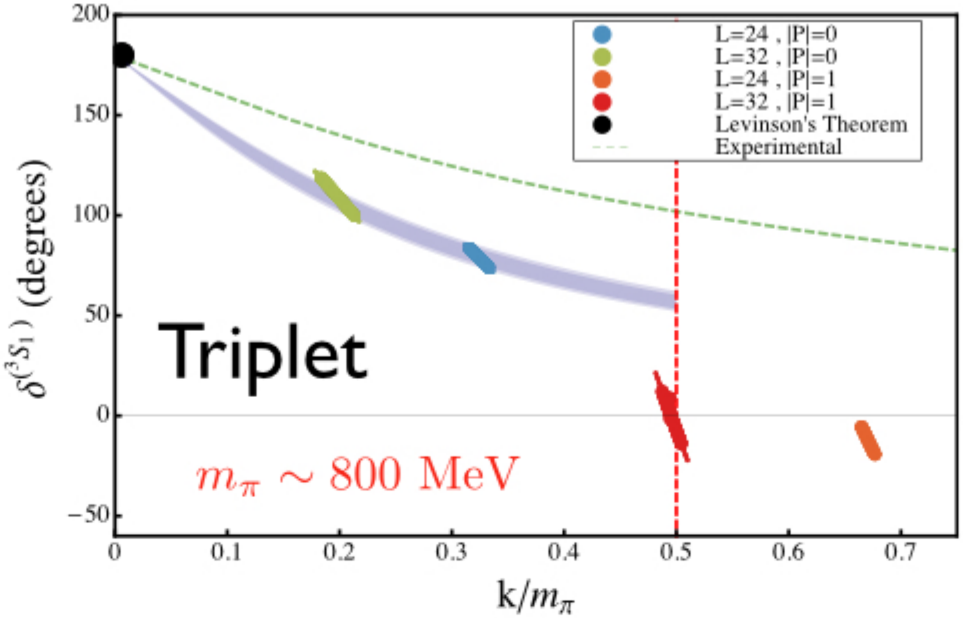


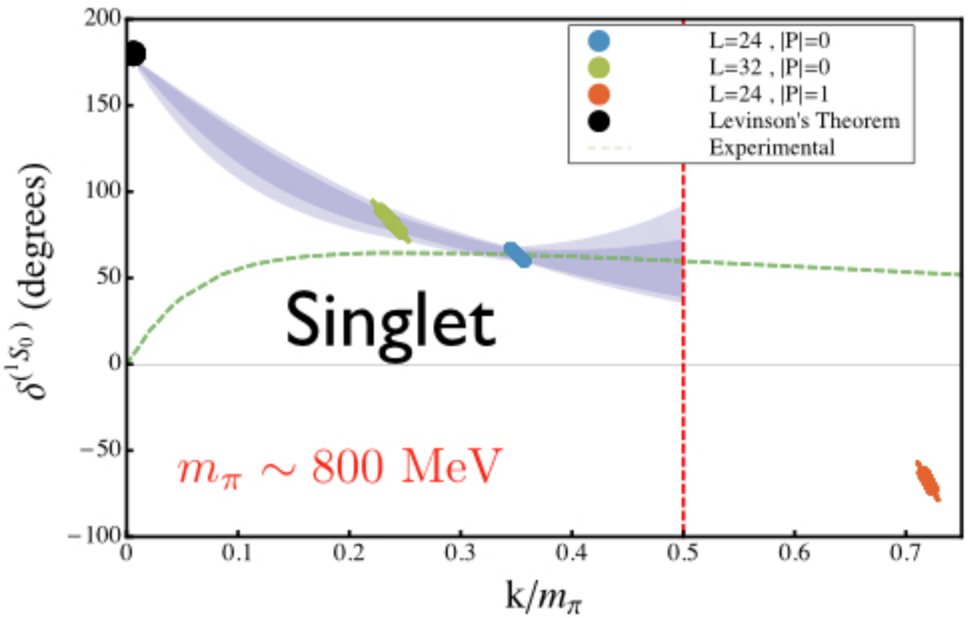
$$p \cot \delta \;=\; \frac{1}{\pi L} \mathcal{S}(\tilde{p})$$

$m_\pi \sim 800$ MeV

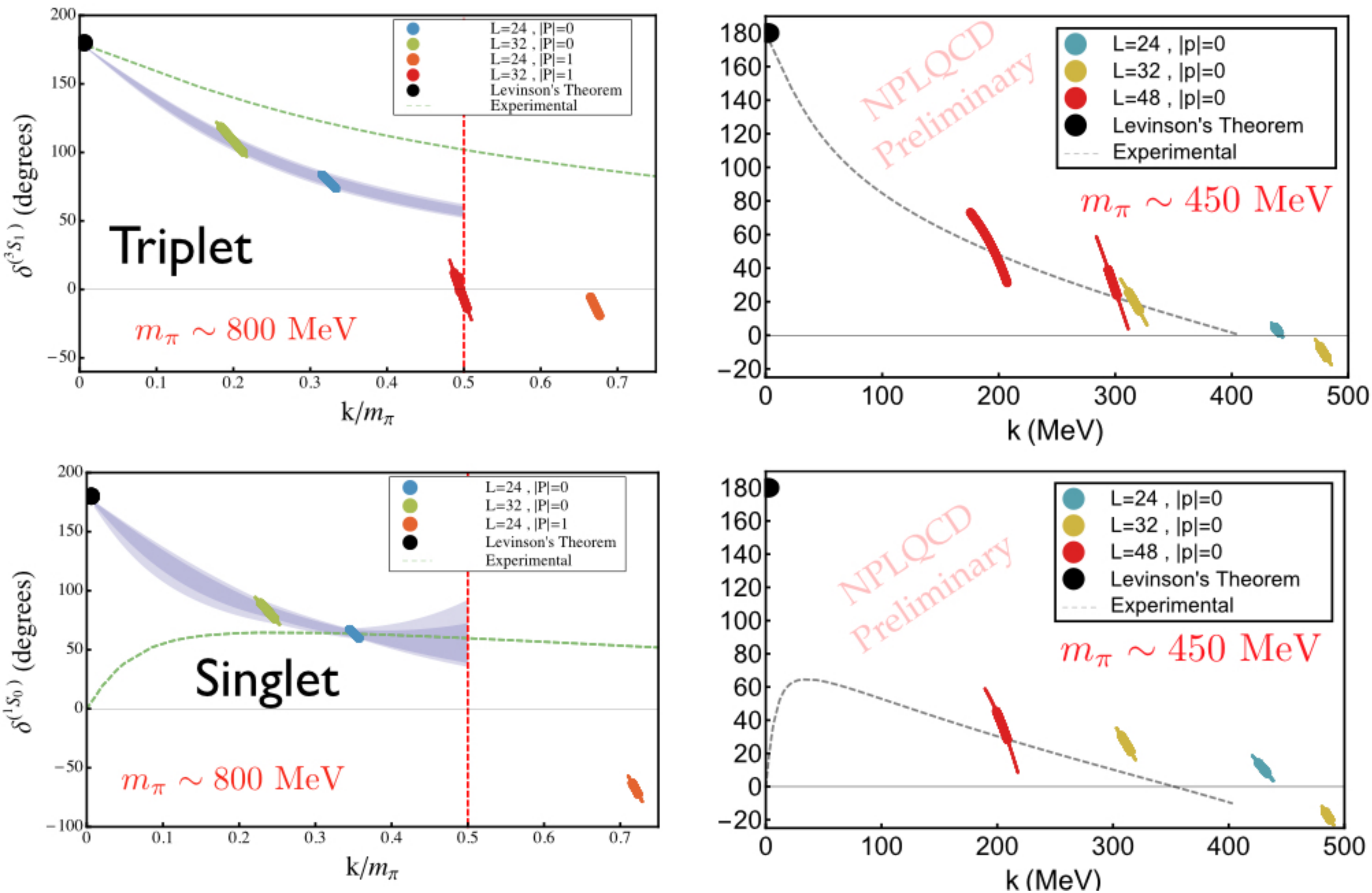


NN S-wave phase shifts

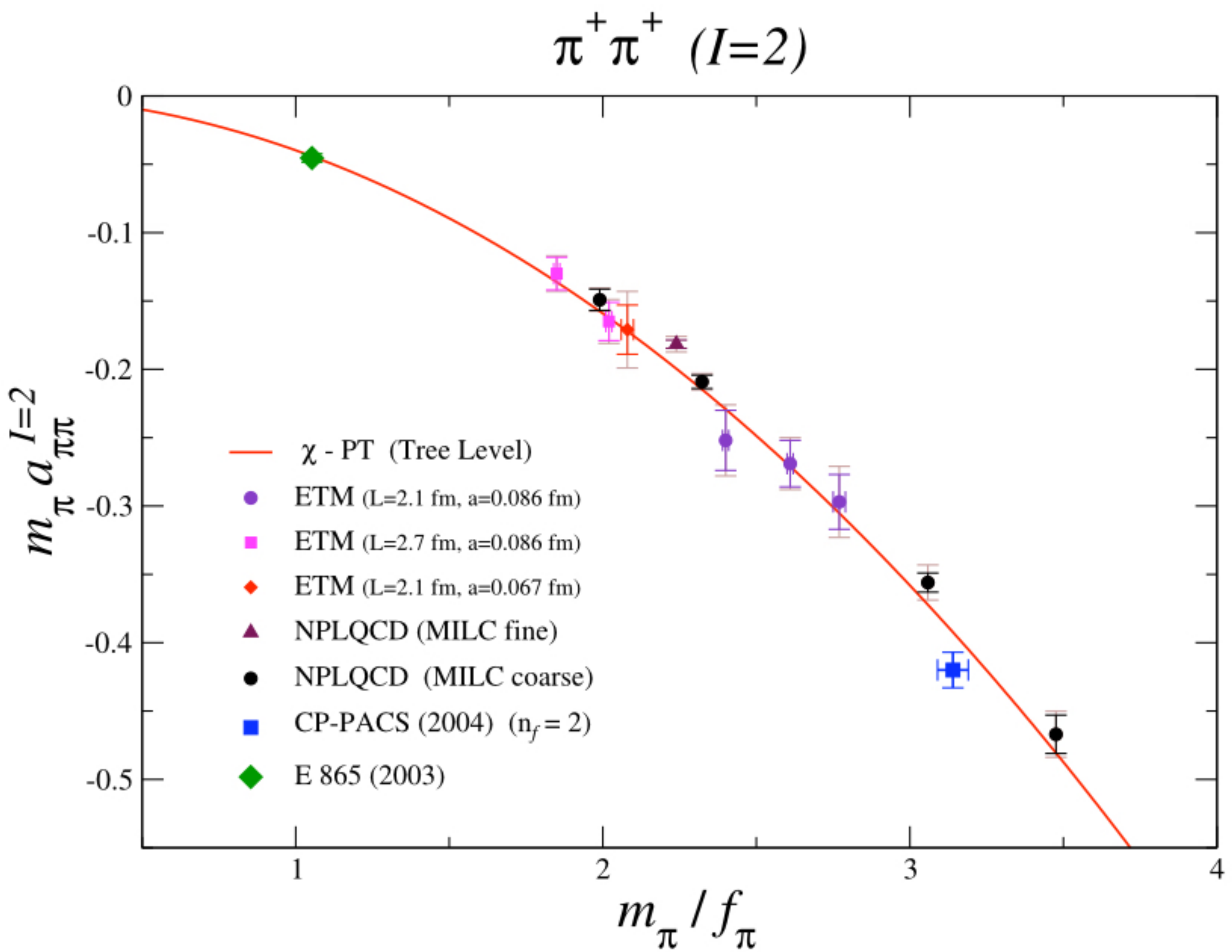




NN S-wave phase shifts



Some unexpected chiral dynamics from LQCD



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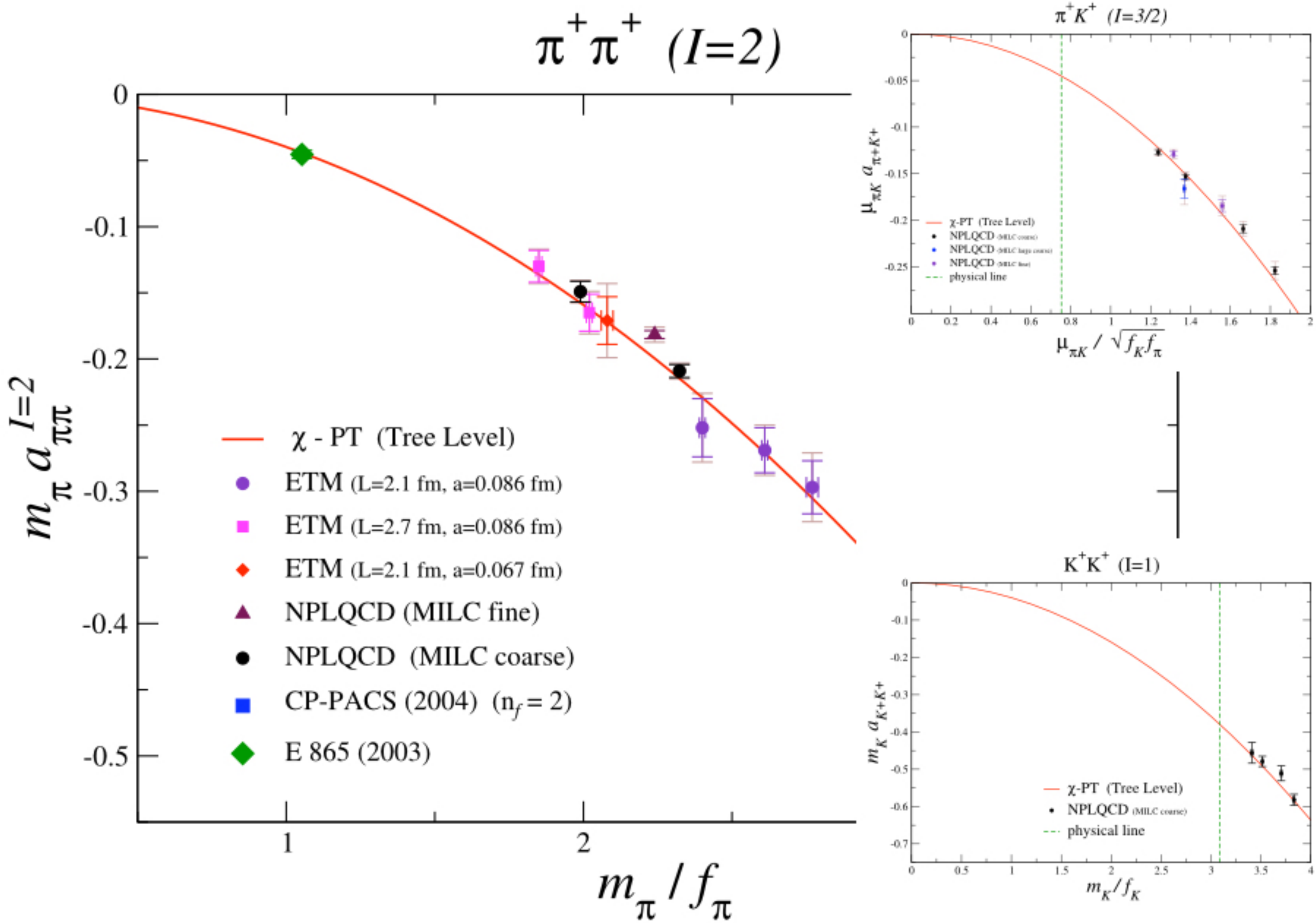
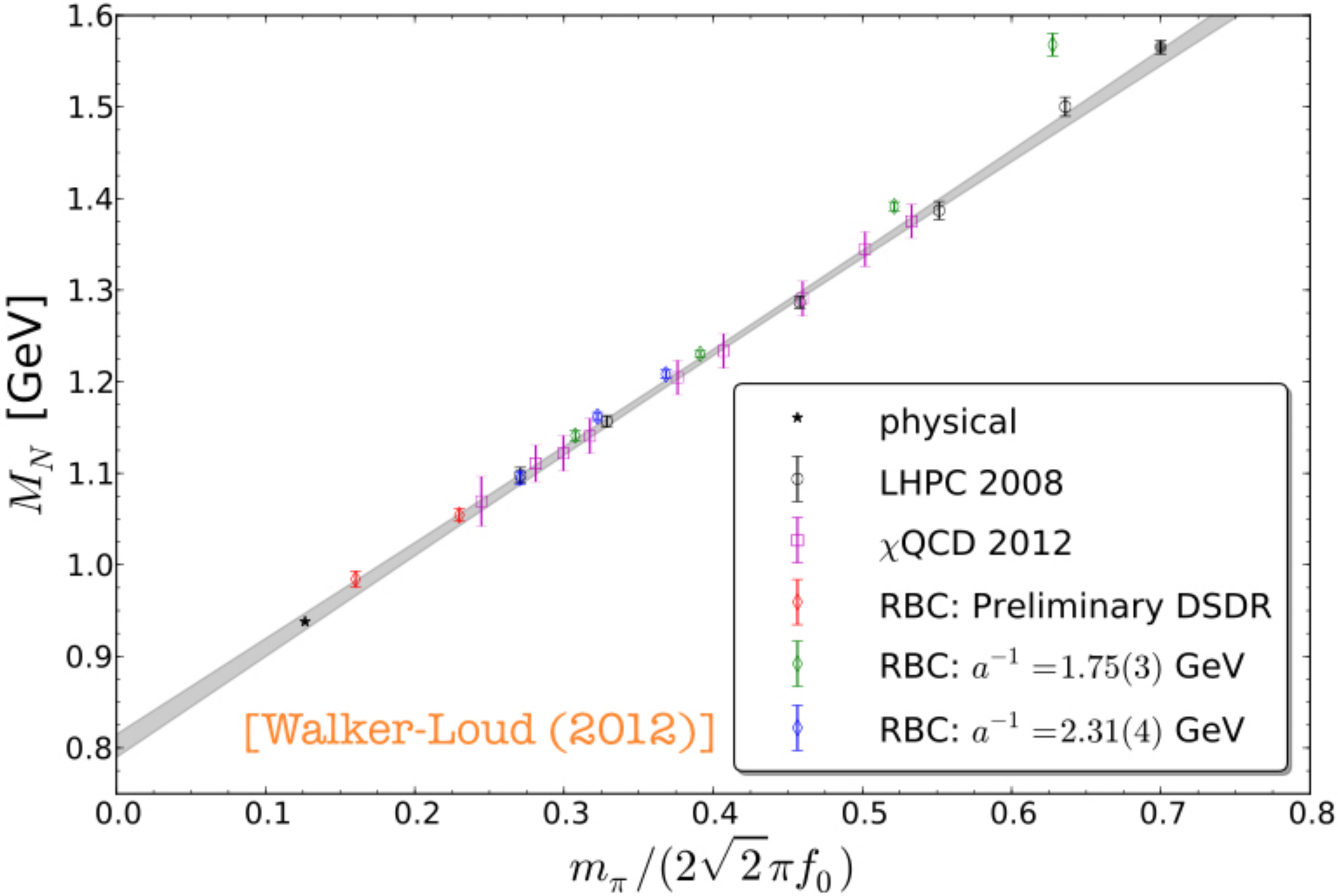
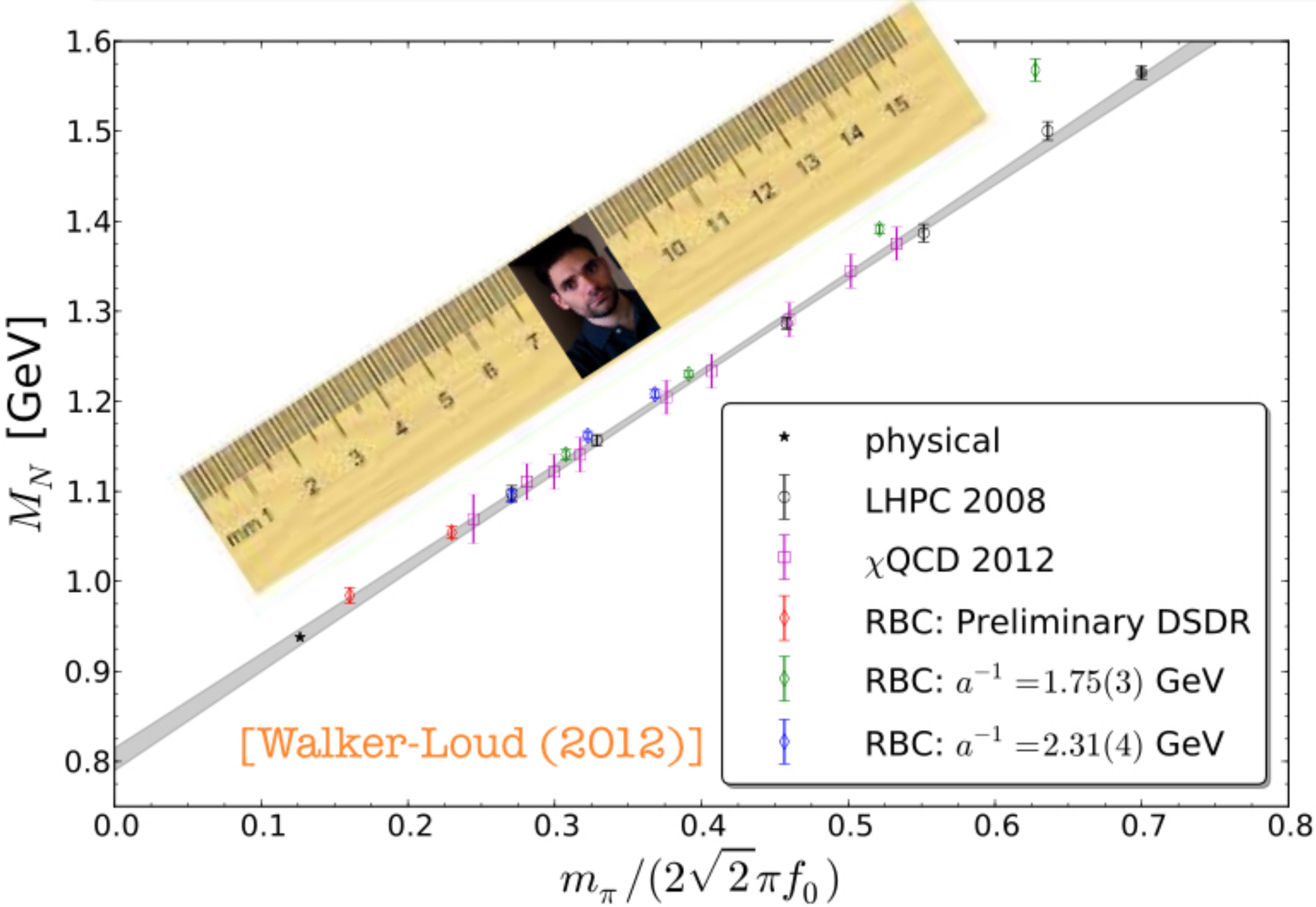


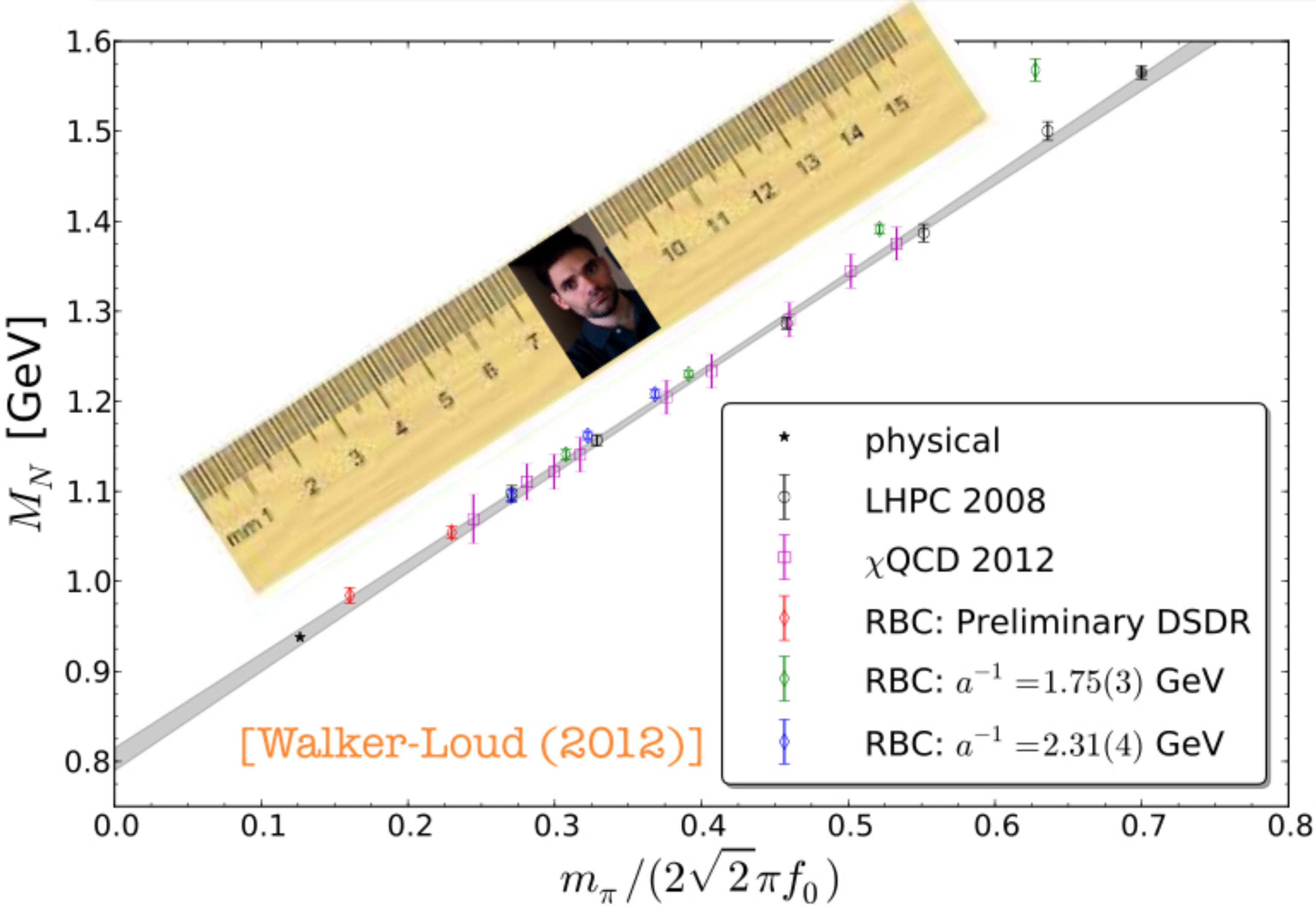
Nucleon mass



Nucleon mass

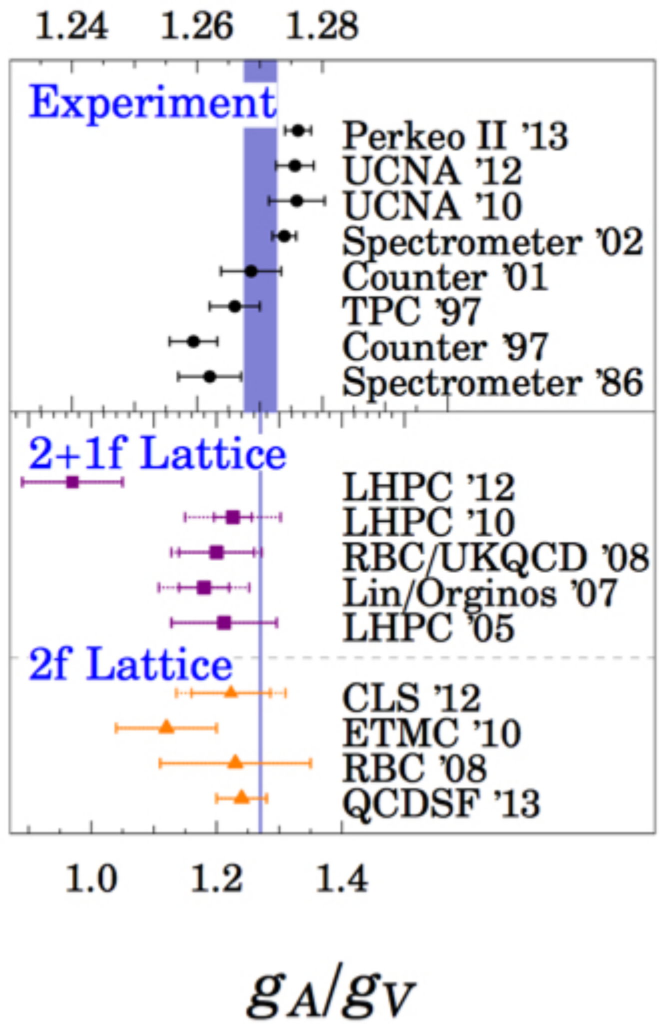


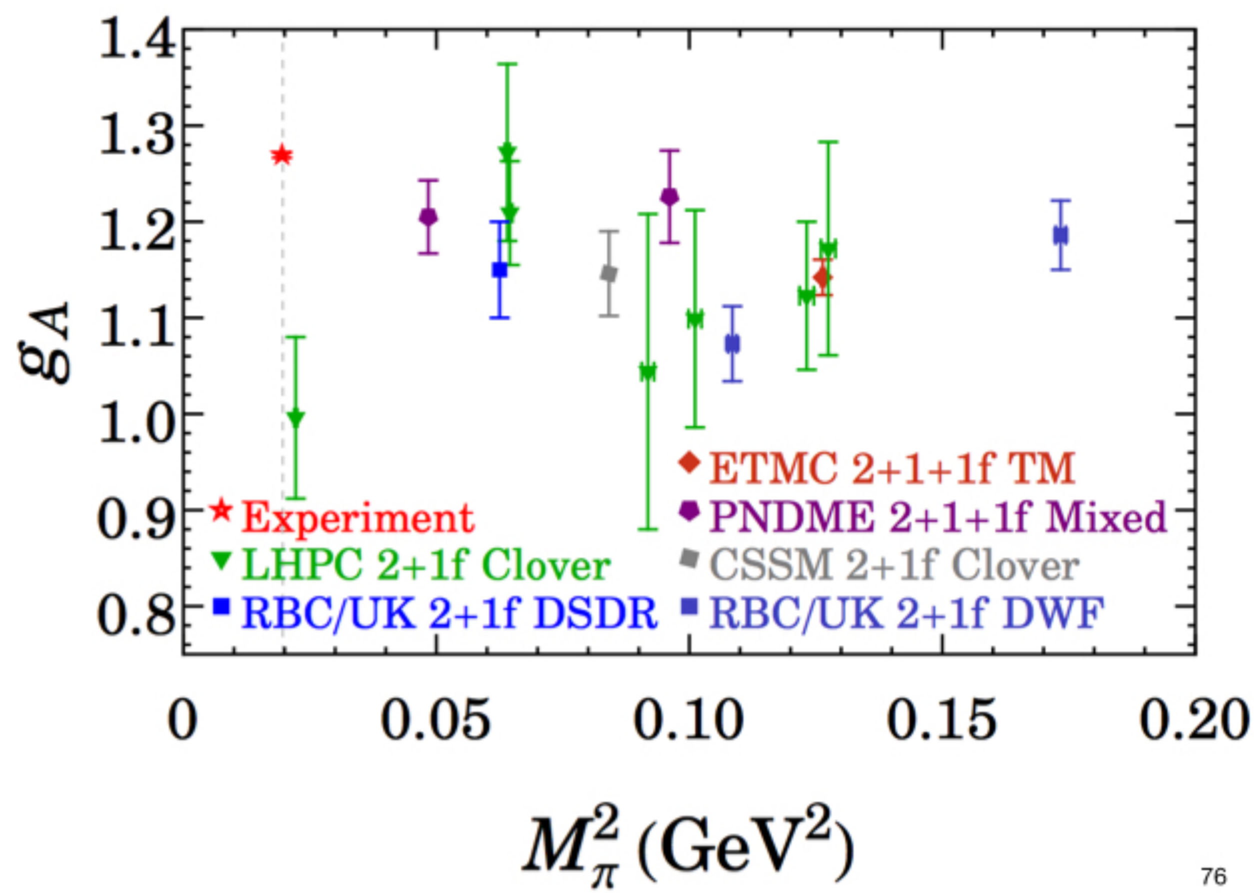
Nucleon mass



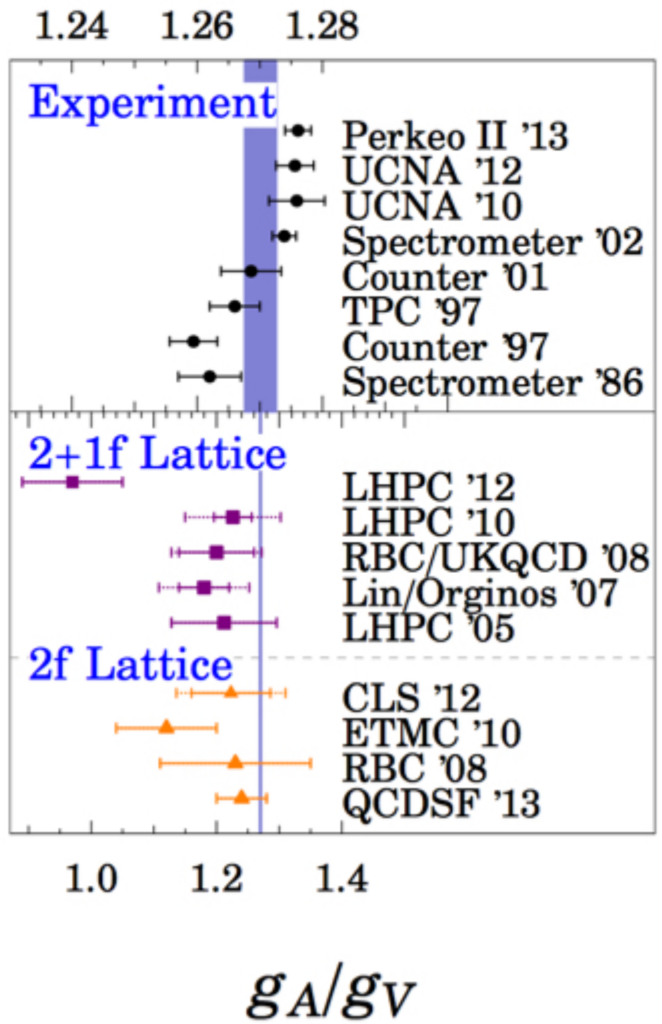
Good model: $M_N[\text{MeV}] = 800 + m_\pi$

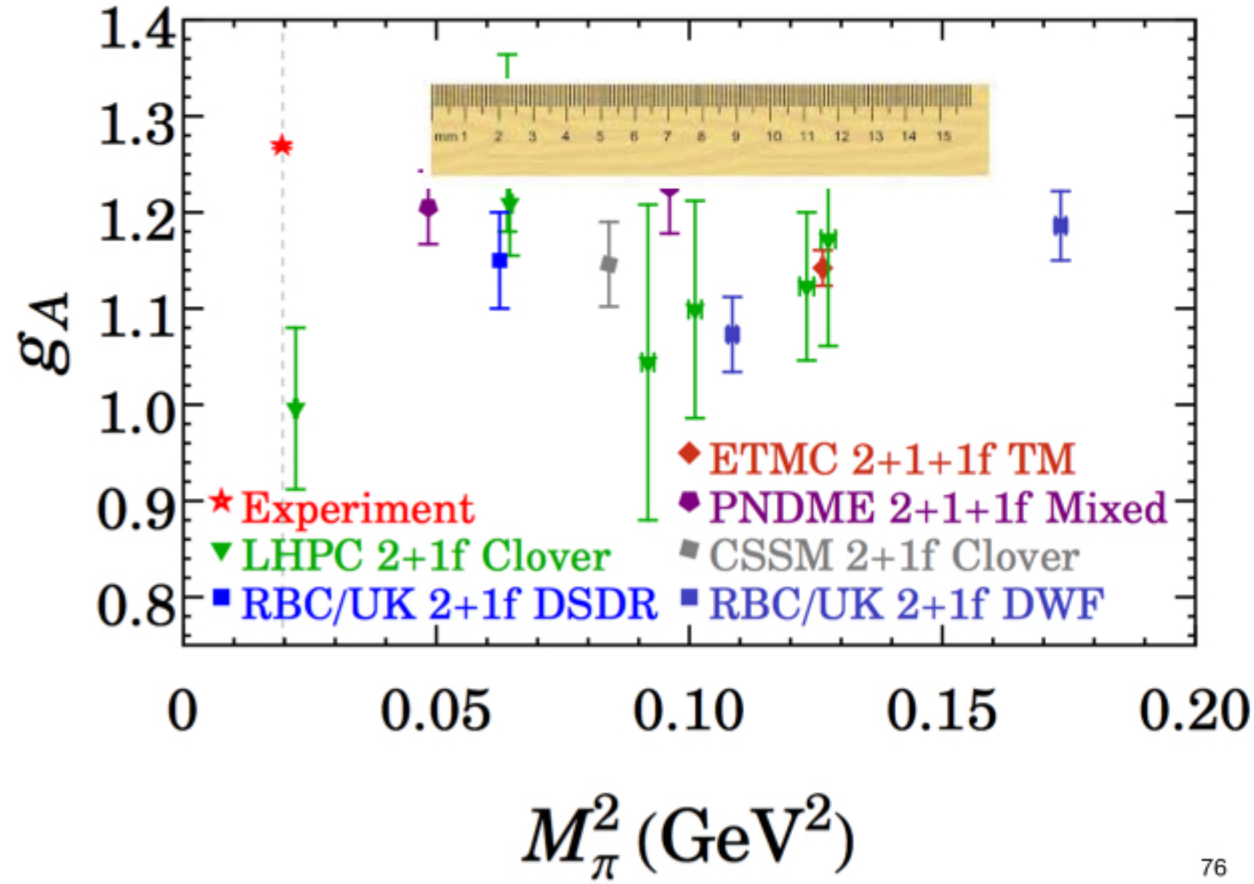
Nucleon axial charge





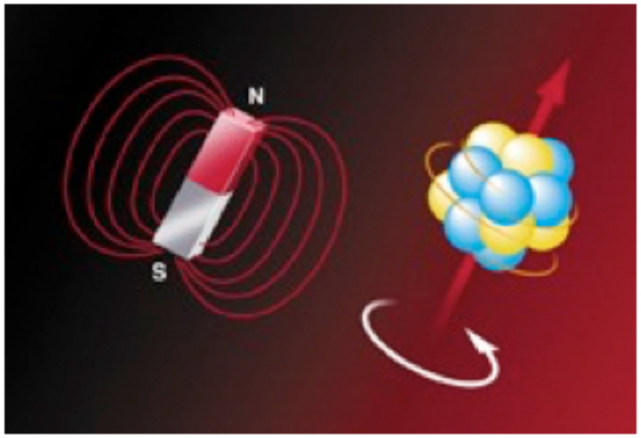
Nucleon axial charge





Nuclear structure: magnetic moments

[Savage | |]



$U_Q(1)$ phase

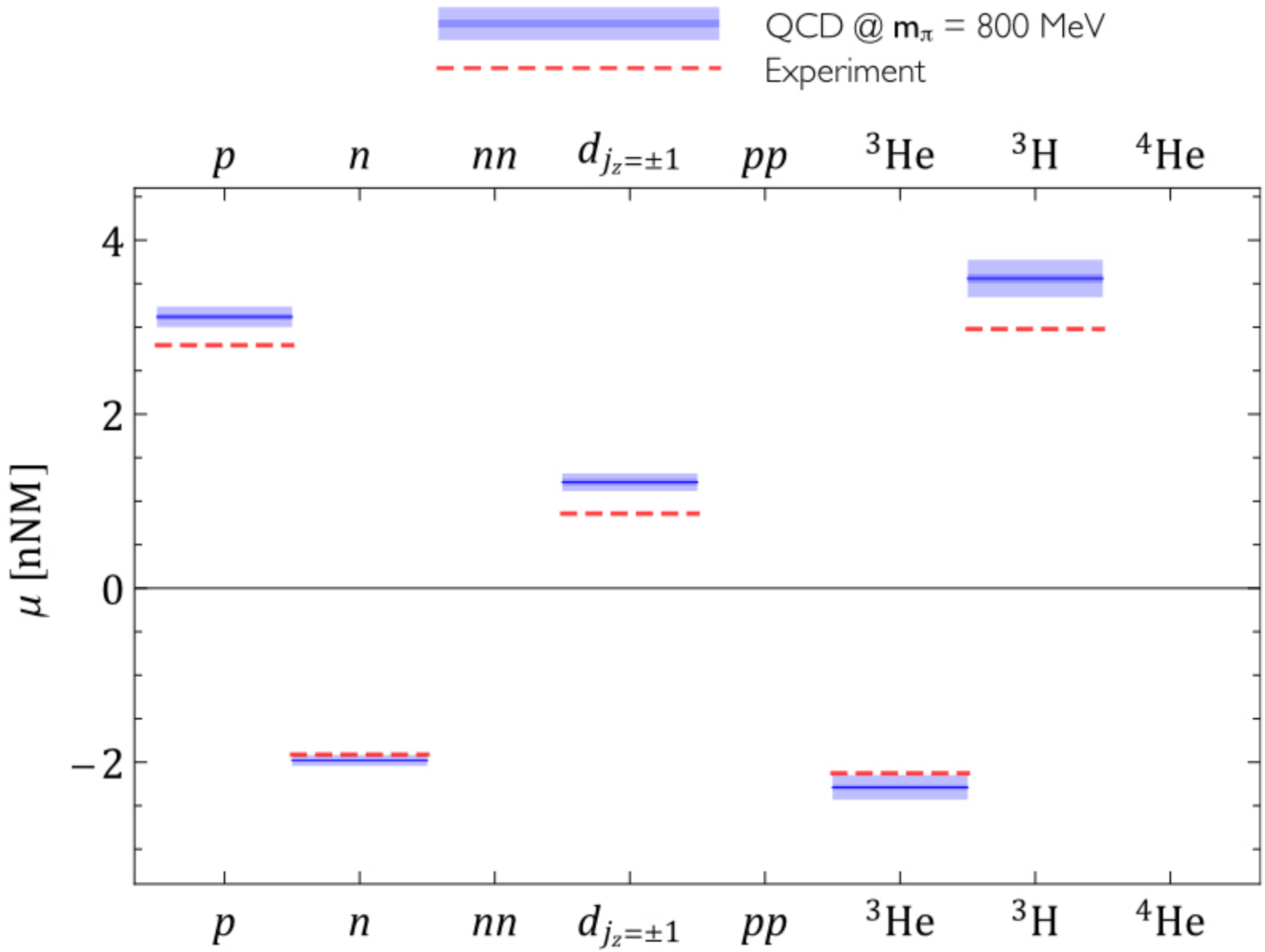
$$U_\mu(x) = e^{i\frac{6\pi Q_q \tilde{n}}{L^2} x_1 \delta_{\mu,2}} \times e^{-i\frac{6\pi Q_q \tilde{n}}{L} x_2 \delta_{\mu,1} \delta_{x_1,L-1}}$$

- Hadronic and nuclear correlation functions are modified in the presence of external fields. For example, E&M field gives:

$$E_{h;j_z}(\mathbf{B}) = \sqrt{M_h^2 + P_\parallel^2 + (2n_L+1)|Q_h e\mathbf{B}|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

Landau level

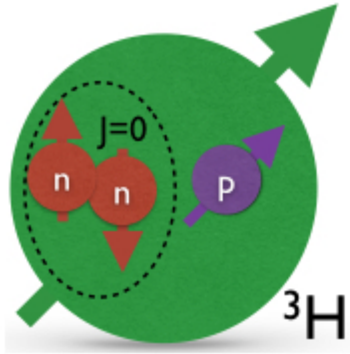
- Can extract magnetic moments, polarizabilities, …
- Extendable to external axial fields, etc.



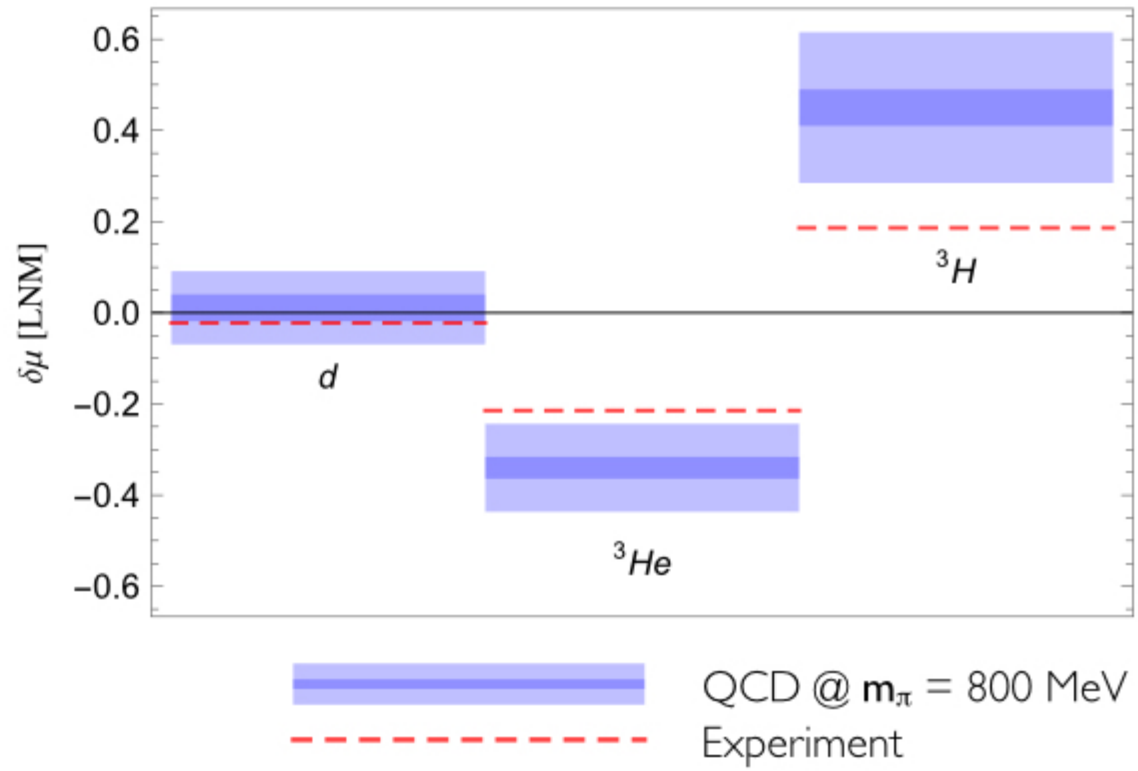
✦ Almost no quark mass dependence in units of $\dfrac{e}{2M(m_\pi)}$

Nuclei as groupings of nucleons: shell model!

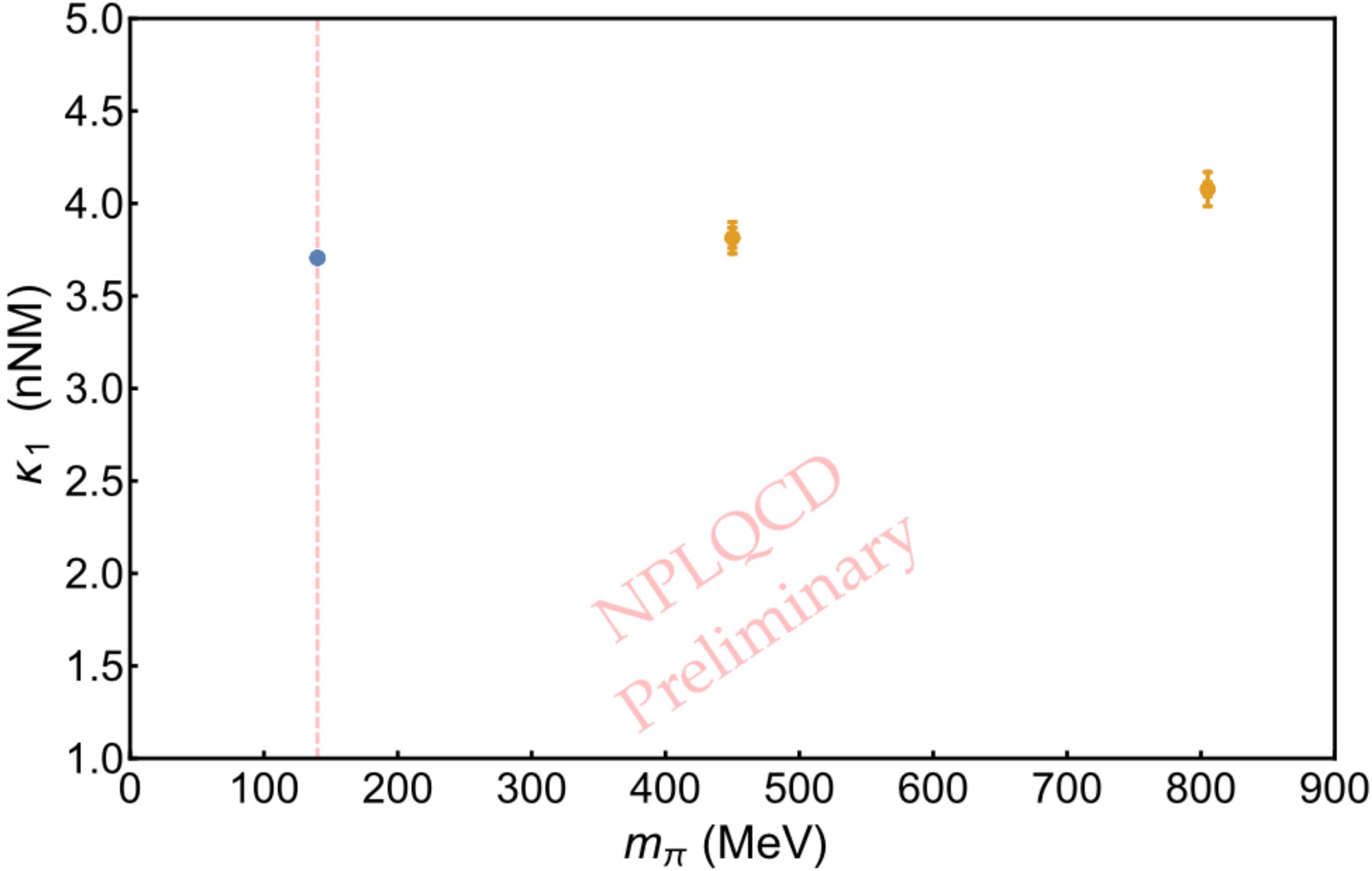
$$\mu_{^3\mathrm{H}} = \mu_p$$
$$\mu_{^3\mathrm{He}} = \mu_n$$
$$\mu_d = \mu_p + \mu_n$$



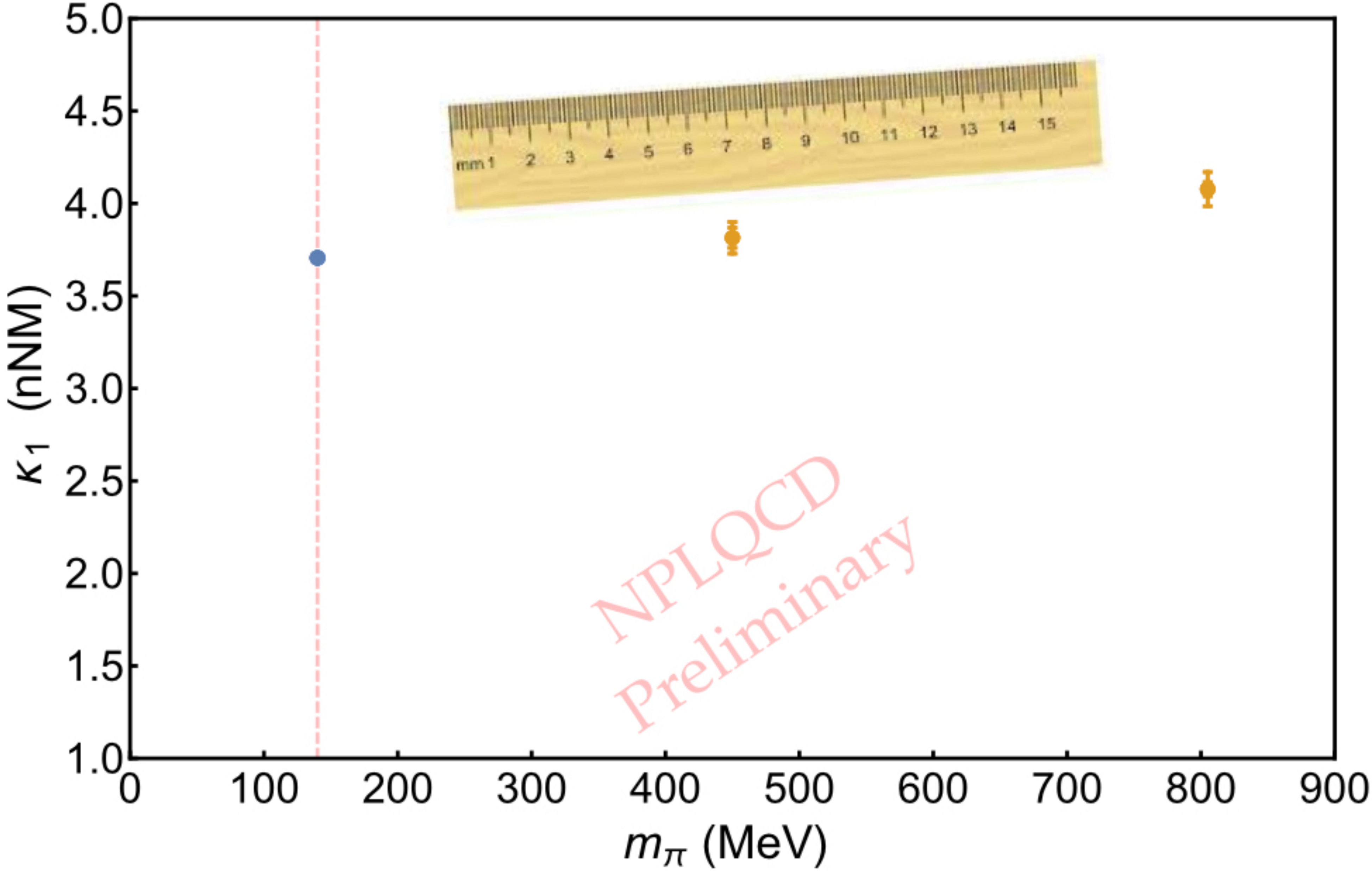
Difference between nuclear magnetic moments and shell model predictions



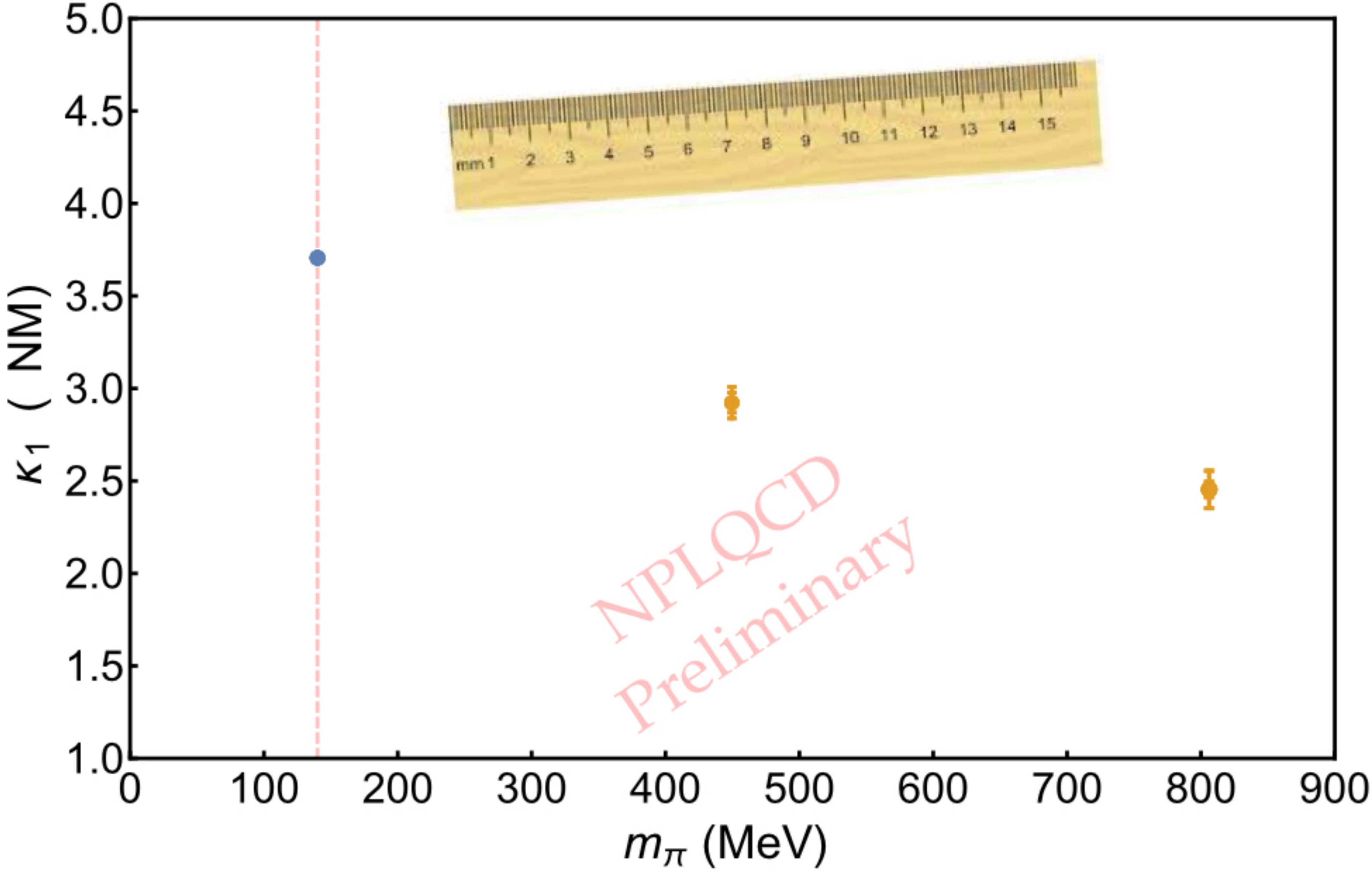
Nucleon isovector magnetic moment



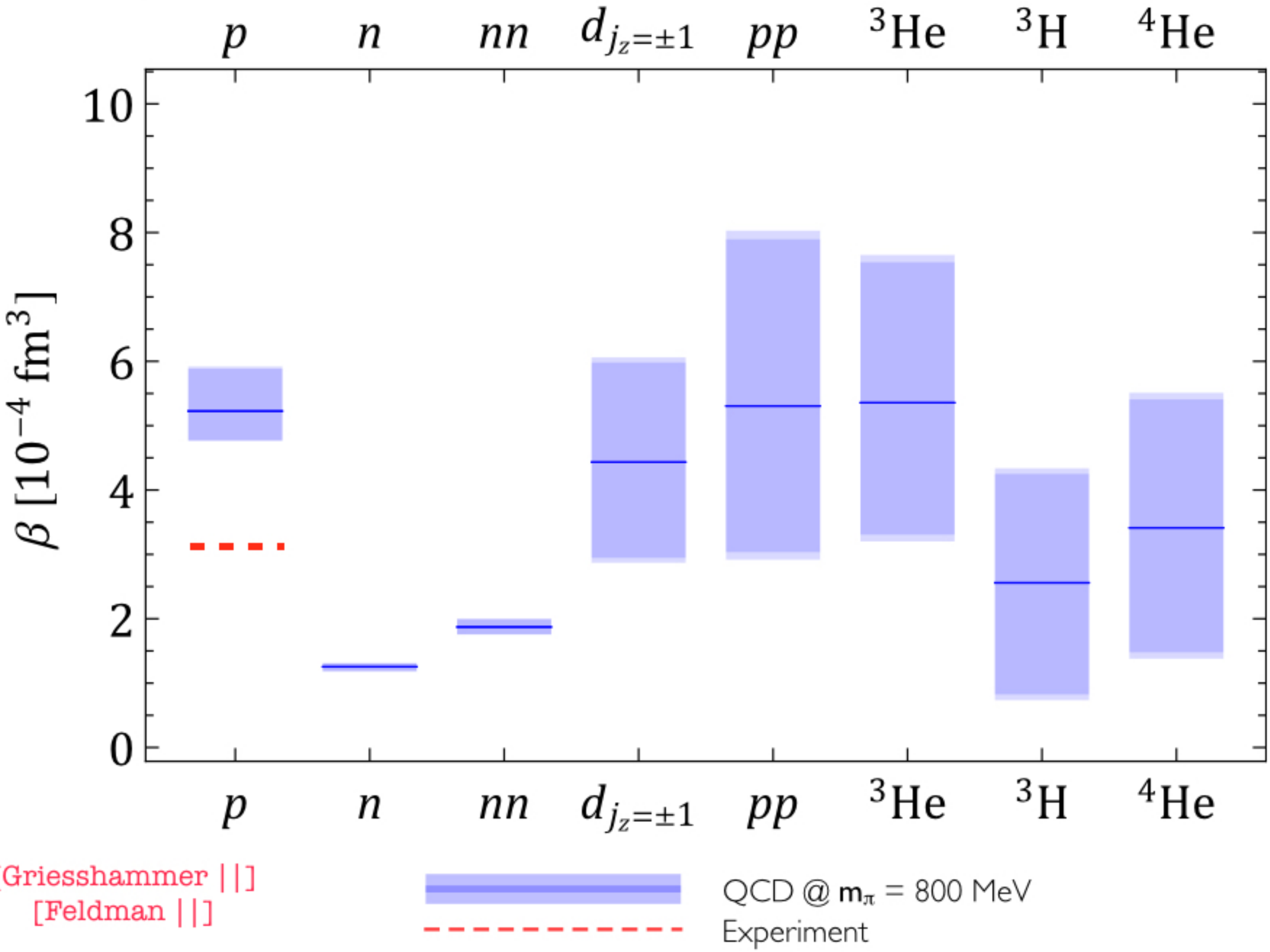
Nucleon isovector magnetic moment



Nucleon isovector magnetic moment

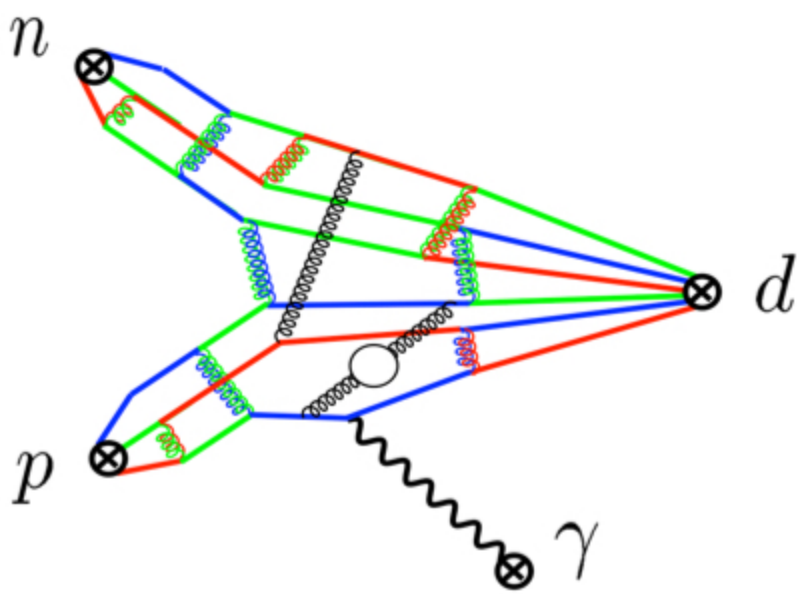


Nuclear structure: polarizabilities



[Griesshammer ||]
[Feldman ||]

Nuclear reaction: $np \to d\gamma$



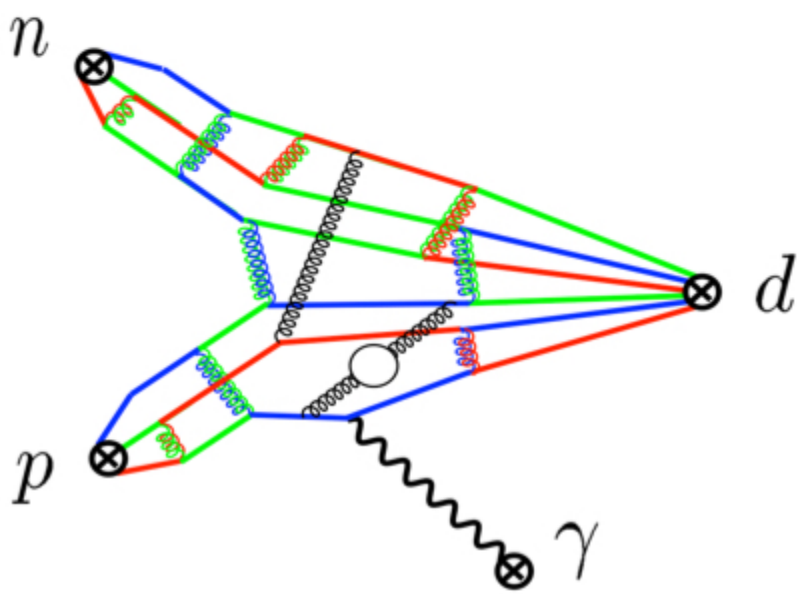
[Detmold and Savage (2004)]

$$\left[p\cot\delta_1 - \frac{S_+ + S_-}{2\pi L}\right]\left[p\cot\delta_3 - \frac{S_+ + S_-}{2\pi L}\right] = \left[\frac{|e\mathbf{B}|l_1}{2} + \frac{S_+ - S_-}{2\pi L}\right]^2$$

$$S_\pm \equiv S\left(\frac{L^2}{4\pi^2}(p^2 \pm |e\mathbf{B}|\kappa_1)\right)$$

$$\Delta E_{^3S_1,^1S_0} = \mp Z_d^2(\kappa_1 + \gamma_0 l_1)\frac{|e\mathbf{B}|}{M} + \ldots = \mp(\kappa_1 + \overline{L}_1)\frac{|e\mathbf{B}|}{M} + \ldots$$

Nuclear reaction: $np \to d\gamma$



[Detmold and Savage (2004)]

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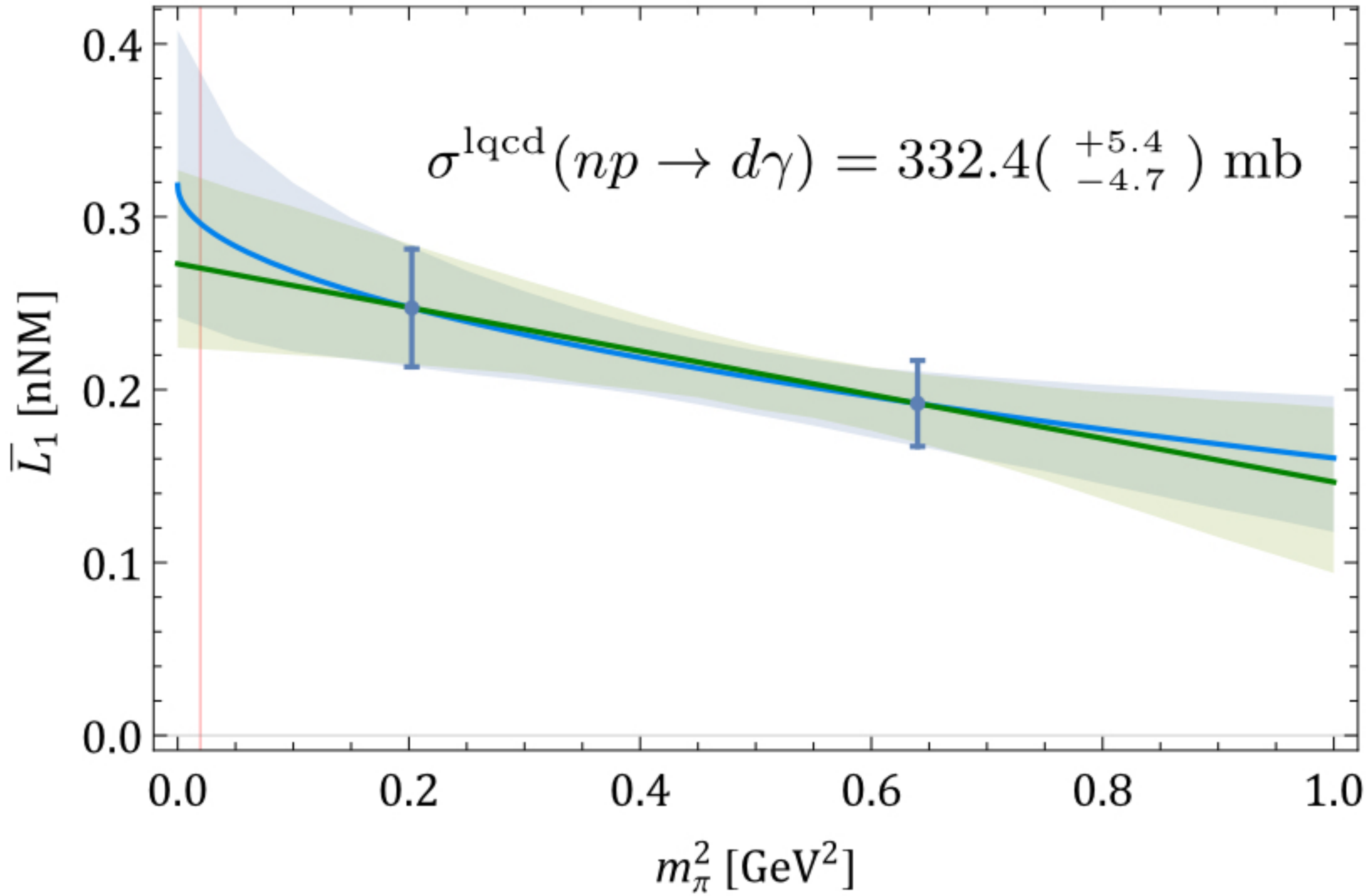
$np \to d\gamma$



$$\sigma^{\rm expt}(np \to d\gamma) = 334.2(0.5)\ \text{mb}$$

- Remarkable progress is being made in understanding the visible matter in the Universe from first principles using lattice QCD.

- Quark mass dependence of many nuclear observables is unexpected from a chiral perturbation theory perspective.

- Background field method is proving remarkably successful: static electromagnetic properties of light nuclei are being determined and the first postdiction of a nuclear reaction now exists.

USQCD
US Lattice Quantum Chromodynamics
NSF
Jefferson Lab
Exploring the Nature of Matter
SciDAC
Scientific Discovery through Advanced Computing


NPLQCD