CMB as a source of gravitons and light pseudo-scalar particles

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Introduction to GWs

- Gravitational waves (GWs) are thought to be space-time perturbations which travels outward from the source with the speed of light.
- The idea of a gravitational field traveling through spaces dates back to **Laplace** (1779), **Poincaré** and **Maxwell**.
- **Albert Einstein** in 1916 was the first person to formulate the theory of GWs based on the just then published theory of general relativity.
- However, the nature of the GWs too many times has been doubted, criticized and their existence has been several times questioned including Einstein himself.

- Einstein to his friend **Max Born** (1936)
  Together with a young collaborator, I arrived at the interesting result that gravitational waves do not exist, though they had been assumed a certainty to the first approximation. This shows that the non-linear general relativistic field equations can tell us more or, rather, limit us more than we have believed up to now (Born 1971, p. 125)
- This episode is one of several ones which doubted the existence of GWs. Sir **Arthur Eddington** in 1922 wrote that GWs travel with "the speed of thought".
- **Nathan Rosen** in 1955 putted forward the hypothesis that GWs do not carry energy which casted further doubts on the wave phenomena in gravitation theory.

On what consist linearized GR?

The starting point toward a linearized theory of gravity are Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

The metric tensor is in general expanded as follows

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu} + O(h^2), \qquad |h_{\mu\nu}| \ll 1.$$

The linearized Ricci tensor is given by

$$R_{\mu\nu} \simeq \frac{1}{2}(\partial_\mu \partial^\alpha h_{\alpha\nu} + \partial^\beta \partial_\nu h_{\mu\beta} - \Box h_{\mu\nu} - \partial_\mu \partial_\nu h).$$

and the Ricci scalar is given by

$$R \simeq \partial^\alpha \partial^\beta h_{\alpha\beta} - \Box h$$

Linearized Einstein equations

Linearized Einstein equations

$$\Box h_{\mu\nu} = -16\pi G (T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T)$$

The energy-momentum tensor of GWs is shown to be

$$t_{\mu\nu} = \frac{1}{32\pi G}\langle \partial_\mu h_{\alpha\beta}\, \partial_\nu h^{\alpha\beta} \rangle.$$

The total luminosity emitted by a source is $L \propto \langle \dot{h}_{ij}^{TT}\, \dot{h}_{ij}^{TT} \rangle$ and reads

$$L_{\mathrm{quad}} = \frac{G}{5}\, \langle \dddot{Q}_{ij} \dddot{Q}_{kl} \rangle_{\mathrm{ret}}$$

where

$$Q^{ij} \equiv \int d^3x\, \rho(\mathbf{x}, t) \left(x^i x^j - \frac{1}{3} r^2 \delta^{ij} \right)$$

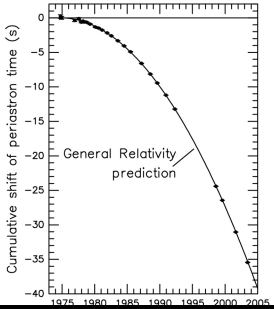
- The question if GWs do carry energy is important not only within the theory of GR but also in the field of quantum gravity.
- At the Chapel Hill conference (1957) "The Role Of Gravitation in Physics" **Felix Pirani** showed based on equations of geodesic deviation how particles in the path of the wave were moved about relative to each other in the metric of the wave.
- Sticky bead argument by **Richard Feynman**: a passing gravitational wave should in principle cause a bead on a stick (oriented parallel to the direction of propagation of the wave) to slide back and forth, thus heating the bead and the stick by friction. So, the energy of the GWs has been converted into heat and consequently GWs should carry energy!

Energy loss of the binary system PSR B1913+16

For two point masses m_1 and m_2 in elliptic orbit with reduced mass $\mu = m_1 m_2/(m_1 + m_2)$, the period time derivative is

$$-\left\langle \frac{dP_b}{dt} \right\rangle_{\rm quad} = \frac{192\pi}{5} \frac{G\mu(m_1+m_2)^{3/2}}{a^{5/2}(1-e^2)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right),$$

where $\dot{P}_b^{\rm exp} = -2.4056(41)\cdot 10^{-12}$ and $P_b^{\rm GR} = -2.40242(2)\cdot 10^{-12}$



Other binary systems Stairs *et al.*'02 and Kramer *et al.*'06

PK parameter	Observed	GR expectation	Ratio
$\dot{P}_b$	1.252(17)	1.24787(13)	1.003(14)
γ (ms)	0.3856(26)	0.38418(22)	1.0036(68)
s	0.99974(−39,+16)	0.99987(−48,+13)	0.99987(50)
r (μs)	6.21(33)	6.153(26)	1.009(55)

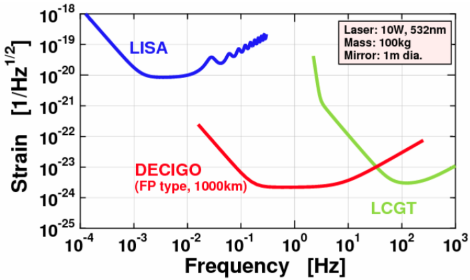
Timing parameter	PSR J0737−3039A	PSR J0737−3039B
Right Ascension α	$07^h37^m51^s.24927(3)$	–
Declination δ	$-30°39'40''.7195(5)$	–
Proper motion in the RA direction (mas yr^{-1})	−3.3(4)	–
Proper motion in Declination (mas yr^{-1})	2.6(5)	–
Parallax, π (mas)	3(2)	–
Spin frequency ν (Hz)	44.054069392744(2)	0.36056035506(1)
Spin frequency derivative $\dot{\nu}$ (s^{-2})	$-3.4156(1)\times10^{-15}$	$-0.116(1)\times10^{-15}$
Timing Epoch (MJD)	53156.0	53156.0
Dispersion measure DM (cm^{-3}pc)	48.920(5)	–
Orbital period P_b (day)	0.10225156248(5)	–
Eccentricity e	0.0877775(9)	–
Projected semi-major axis $x = (a/c)\sin i$ (s)	1.415032(1)	1.5161(16)
Longitude of periastron ω (deg)	87.0331(8)	87.0331 + 180.0
Epoch of periastron T_0 (MJD)	53155.9074280(2)	–
Advance of periastron $\dot{\omega}$ (deg/yr)	16.89947(68)	[16.96(5)]
Gravitational redshift parameter γ (ms)	0.3856(26)	–
Shapiro delay parameter s	0.99974(−39, +16)	–
Shapiro delay parameter r (μs)	6.21(33)	–
Orbital period derivative $\dot{P}_b$	$-1.252(17)\times10^{-12}$	–
Timing data span (MJD)	52760 – 53736	52760 – 53736
Number of time offsets fitted	10	12
RMS timing residual σ (μsec)	54	2169
Total proper motion (mas yr^{-1})	4.2(4)	
Distance d(DM) (pc)	~ 500	
Distance $d(\pi)$ (pc)	200 – 1000	
Transverse velocity (d = 500 pc) (km s^{-1})	10(1)	
Orbital inclination angle (deg)	88.69(-76,+50)	
Mass function ($M_\odot$)	0.29096571(87)	0.3579(11)
Mass ratio, R	1.0714(11)	
Total system mass ($M_\odot$)	2.58708(16)	
Neutron star mass ($m_\odot$)	1.3381(7)	1.2489(7)

Table 3. Orbital parameters of PSR B1534+12 in the DD and DDGR models[a]

	DD model	DDGR model
Orbital period, P_b (d)	0.420737299122(10)	0.420737299123(10)
Projected semi-major axis, x (s)	3.729464(2)	3.7294641(4)
Eccentricity, e	0.2736775(3)	0.27367740(14)
Longitude of periastron, ω (deg)	274.57679(5)	274.57680(4)
Epoch of periastron, T_0 (MJD)	50260.92493075(4)	50260.92493075(4)
Advance of periastron, $\dot{\omega}$ (deg yr^{-1})	1.755789(9)	*1.7557896*
Gravitational redshift, γ (ms)	2.070(2)	*2.069*
Orbital period derivative, $(\dot{P}_b)^{obs}$ (10^{-12})	−0.137(3)	*−0.1924*
Shape of Shapiro delay, s	0.975(7)	*0.9751*
Range of Shapiro delay, r (μs)	6.7(1.0)	*6.626*
Derivative of x, $|\dot{x}|$ (10^{-12})	< 0.68	< 0.015
Derivative of e, $|\dot{e}|$ (10^{-15} s^{-1})	< 3	< 3
Total mass, $M = m_1 + m_2$ ($M_\odot$)	...	2.678428(18)
Companion mass, m_2 ($M_\odot$)	...	1.3452(10)
Excess $\dot{P}_b$ (10^{-12})	...	0.055(3)

GWs detectors

The goal of DECIGO is to detect various sources of gravitational waves mainly between 0.1 Hz and 10 Hz. After 3 years of correlations in orbit, its aim is to reach a $h_f \sim 10^{-25}$ Hz$^{-1/2}$. Other ambitious program for DECIGO is to reach a $h_f \sim 10^{-27}$ Hz$^{-1/2}$ after 5 years of data correlation **Seto (2001)**. This project is dubbed the Ultimate DECIGO.



Bounds on stochastic background of GWs

- In general a stochastic background of GWs is characterized by its density parameter, $\Omega_{\rm gw} = \rho_{\rm gw}/\rho_c$, $\rho_c = 1.878 \cdot 10^{-29} h_0^2$ g/cm^3.
- CMB observations can constrain the number density of gravitons at the post recombination epoch.
- CMB constrain GWs in the frequency range $3 \cdot 10^{-18}$ Hz $< f < 10^{-16}$ Hz.
- The density parameter in GWs from CMB angular constrains

$$\Omega_{\rm gw}(f) < 10^{-10} \left(H_0/f\right)^2$$

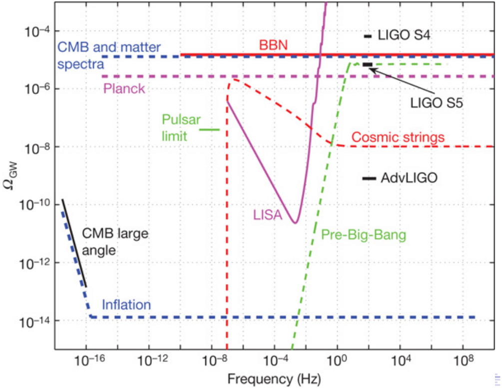
- BBN bound constrain the number of extra degrees of freedom at BBN. In the case of gravitons

$$h_0^2 \Omega_{\rm gw}(t_0) \leq 5.7 \cdot 10^{-6} (N - N_\nu)$$

GW production by different models, Abbot *et al*'09

- For example inflation produce an almost flat spectrum of GWs in the frequency range 10^{-16} Hz $< f < 10^{9}$ Hz,

$$h_0^2\Omega_{\rm gw}(f) \simeq 8.75 \cdot 10^{-6}(H/m_{\rm Pl})^2$$



Introduction to axions: strong CP problem

- QCD is the framework of physics which describes the strong interaction between quarks, gluons etc.
- the most general form of the Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \bar{q}(i\gamma_\mu D^\mu - M_q)q - \frac{\alpha_s}{8\pi} \bar{\theta} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

- the fundamental parameters of QCD are $-\pi < \bar{\theta} < \pi$, M_q quark mass matrix, and strong coupling constant α_s
- the P, T and CP violating term is

$$\mathcal{L}_\theta = \frac{\alpha_s}{8\pi} \bar{\theta} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

Strong CP problem

- The theta term is $G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} = -4\mathbf{E}^a \cdot \mathbf{B}^a$ where $\mathbf{E}^a$ and $\mathbf{B}^a$ are the colored electric and magnetic fields.
- It is odd under P, T and leads to CP violating term in flavor conserving interactions.
- the CP violating Lagrangian induces electric dipole moments in baryons (for example neutrons) which have not been observed.
- theoretically (**Callan, Curtis, Dashen and Gross, 1976, Baluni, 1979**)

$$d_n(\bar{\theta}) \simeq e\,\bar{\theta}\frac{m_u m_d}{(m_u + m_d)m_n^2} \simeq 10^{-16}\bar{\theta}\,e\,\mathrm{cm}$$

- experimentally $d_n < 2.9 \times 10^{-26} e$ cm which implies $\bar{\theta} \lesssim 10^{-10}$ **Baker et. al., 2006**
- strong CP problem: why is $\bar{\theta}$ so small?

Peccei-Quinn mechanism

- An elegant solution was proposed by **R. Peccei and H. Quinn, 1977** by introducing a global chiral $U(1)$ symmetry called the Peccei-Quinn symmetry $U(1)_{PQ}$
- as a consequence a new physical field called axion $a(x)$ is postulated which vanish $\bar{\theta}$ dynamically (**Peccei and Quinn, 1977, Wilczek, 1978, Weinberg, 1978**)

FRANK WILCZEK

2004 Nobel Laureate
Herman Feshbach Professor of Physics
Massachusetts Institute of Technology

Peccei-Quinn mechanism

- the axion is the pseudo Nambu Goldstone boson of the broken $U(1)_{PQ}$
- The new field $a(x)$ couples to gluons as

$$\mathcal{L}_{aG\tilde{G}} = \frac{\alpha_S}{8\pi f_a} aG\tilde{G},$$

  where f_a is the axion decay constant.
- the effective interaction Lagrangian becomes

$$\mathcal{L}_{aG\tilde{G}} = \left(\bar{\theta} + \frac{a}{f_a}\right) \frac{\alpha_S}{8\pi} G\tilde{G}$$

- Non perturbative QCD effects induce a potential $V(a)$ for the axion field whose minimum is at $\langle a \rangle = -\bar{\theta} f_a$
- therefore the $\bar{\theta}$ term is cancelled in the QCD Lagrangian and CP is restored!

QCD axion

- axions acquire an "effective mass" through their mixing with scalar mesons (π^0) and gluons
- considering for example at first approximation only up (u) and down (d) quarks (**Bardeen and Tye 1978**)

$$m_a = \frac{f_\pi m_\pi}{f_a} \frac{\sqrt{m_u m_d}}{m_u + m_d} \simeq 6\,\mathrm{eV}\left(\frac{10^6\mathrm{GeV}}{f_a}\right)$$

where $m_\pi = 135$ MeV, $f_\pi \simeq 92$ MeV and $m_u/m_d = 0.3 - 0.6$.

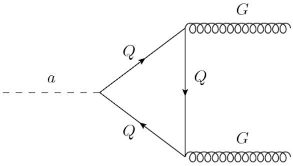
- originally the $U(1)_{PQ}$ symmetry breaking was linked with the electro-weak symmetry breaking $f_a \sim E_{EW} = 246$ GeV.
- the original Peccei-Quinn mechanism was ruled out because the axion was not found in experiments

Invisible axion

- In order to save the Peccei-Quinn mechanism and bypassing the experimental limits, f_a must be raised by several orders of magnitude.
- the axion is introduced as a phase in the Higgs sector and in the effective lagrangian always appear (a/f_a).
- increasing f_a lowers the axion mass and coupling to the SM particles $\rightarrow$ the Peccei-Quinn remains untouched.
- two important implementations of this idea, have proposed: **Kim-Shifman-Vainshtein-Zakharov (KSVZ) axion model 1979-80** and **Dine-Fischler-Sdrenicki-Zhitnitsky (DFSZ) axion model, 1980**.
- for example in the KSVZ model are introduced a complex scalar field Φ i.e. a $SU(2) \times U(1)$ singlet and a new massless fermion field Ψ

Invisible axion

- It can be shown that in the KSVZ model, axions interact with gluons through the effective term $\mathcal{L} \propto (a/f_a)G\tilde{G}$



- in a similar fashion as in the case of axion-gluon coupling, axions can couple to two photons through the interaction term

$$\mathcal{L}_{aF\tilde{F}} = -\frac{g_{a\gamma\gamma}}{4} aF\tilde{F}$$

- the coupling $g_{a\gamma\gamma}$ gets contribution from the coupling of heavy quarks to the electroweak gauge bosons and by the mixing of axions with mesons

Enlarging the family: axion-like particles

- several axion models exist, providing different coupling constants f_a, C_f to the SM particles (KSVZ, DFSZ etc.)
- PQ mechanism works for every value of f_a but the PQ-scale is speculative
- experimental searches need to look for f_a or m_a and coupling coefficients which enters the Lagrangian C_f, C_γ where $g_{a\gamma\gamma} = (\alpha/2\pi f_a)C_\gamma$
- the most important axion direct search is the two-photon coupling, **Sikivie 1983**
- however even the two-photon coupling is difficult because axions are weakly coupled in the allowed range of f_a or $g_{a\gamma\gamma}$
- other impostors, namely pseudo-scalar particles can mimic the axion-photon interaction, **Masso and Toldra 1996**

$$\mathcal{L} = -(1/4)g_{\phi\gamma}F\tilde{F}\phi$$

Astrophysical and cosmological limits on axions and ALPs

- cosmological limits from: Cold DM, hot DM and topological defect decay
- laboratory limits from: ADMX, IAXO, CAST, Telescope etc.
- astrophysical limits: SN 1987A, HB stars, white dwarfs cooling etc.

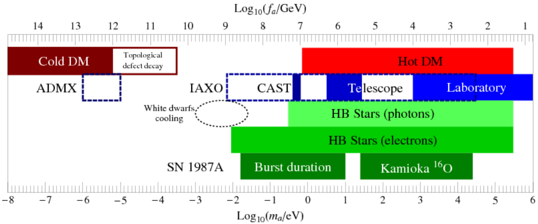


Figure : limits on axion mass and f_a, *D. Cadamuro '12*

Astrophysical and cosmological limits on axions

- Laboratory experiments (no evidence of axion coupling to fermions or nucleons), $m_a \lesssim 0.6$ KeV
- Telescope region: no observation of photons by axion decay ($a \to \gamma\gamma$) in the spectrum of galaxies and EBL
- HB stars the axion production channel inside stars leads to fast consumption of fuel$\to$ fast aging

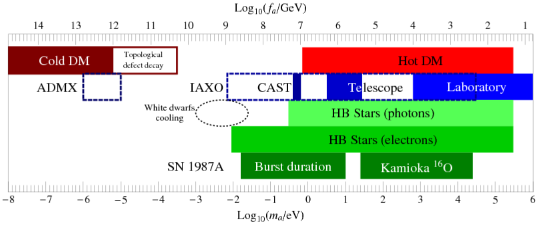
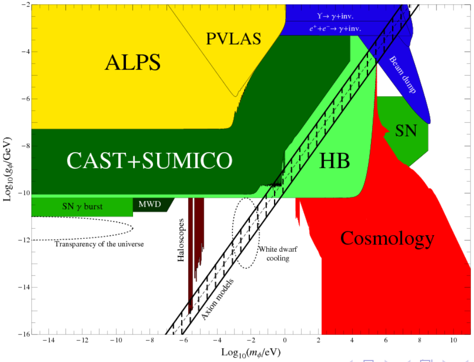


Figure : limits on axion mass and f_a, *D. Cadamuro '12*

Astrophysical and cosmological limits on ALPs

- The KSVZ axion model for $0.6 \lesssim C_\gamma \lesssim 6$ is shown.
- SN γ burst: transparency of SN core to ALP propagation
- ALPS is based in the light shining through the wall experiment at DESY



$\gamma - g$ mixing

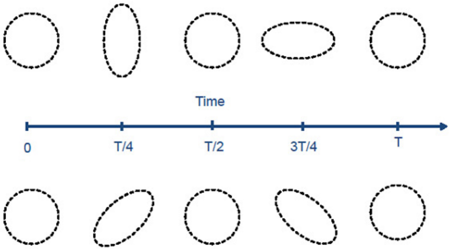
- electromagnetic waves(photons) can transforms into gravitational waves(gravitons) in the presence of a constant external magnetic field, **Gertsenshtein 1962, Lupanov 1967.**
- the reverse process $g \to \gamma$ was considered by **Mitskevich 1969, Boccaletti, De Sabbata, Fortini and Gualdi 1970, Zel'dovich 1973** etc.
- for an extended region of a magnetic field there are oscillations of gravitons into photons and vice-verse in complete analogy with neutrino oscillations.
- An elegant approach to the graviton to photon conversion was derived by, **Raffelt and Stodolsky (1988)**.

$g - \gamma$ mixing

- an electromagnetic wave with vectors **E** and **B** when crosses a static external magnetic field $\mathbf{B}_{ext}$, generates a time varying energy momentum tensor

$$T_{ij} \propto B_i B_j^{ext}$$

- a gravitational wave, h_{ij}, transversing a static external magnetic field $\mathbf{B}_{ext}$ generates distortion in space which stretches the external field $|h_{ij}|\mathbf{B}_{ext}$

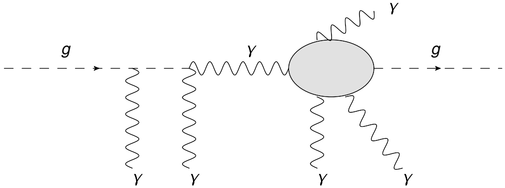


How things work: quantitative description

- The starting point is the action of the graviton-photon system

$$\mathcal{S} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}R - \frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{\alpha^2}{90m_e^4}\int \mathrm{d}^4x\sqrt{-g}[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(\tilde{F}_{\mu\nu}F^{\mu\nu})^2]$$

- the third term include non linear QED effects (**Heisemberg and Euler'36, Schwinger'51**)



- we expand the true metric tensor around the flat Minkowski space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + k h_{\mu\nu}(\mathbf{x}, t), \quad k = (32\pi G)^{1/2}$$

- the equation of motion for the fields $h_{\mu\nu}$ and A^{ν} are in the WKB limit ($\lambda_p \ll \lambda_B$)

$$\left[(\omega + i\partial_{\mathbf{x}})\mathbf{I} + \begin{bmatrix} \omega(n-1)_{\lambda} & B_T/m_{Pl} \\ B_T/m_{Pl} & 0 \end{bmatrix}\right] \begin{bmatrix} A_{\lambda}(\mathbf{x}) \\ h_{\lambda}(\mathbf{x}) \end{bmatrix} = 0\,,$$

- the medium gives an effective mass to the photon

$$\omega_{pl}^2 = 4\pi\alpha n_e/m_e$$

- the QED refraction index is (**S. Adler'71, E. Brezin'71**)

$$n_{\times,+} = 1 + \frac{\alpha}{4\pi}\left(\frac{B_T}{B_c}\right)^2 \left[\left(\frac{14}{45}\right)_{\times}, \left(\frac{8}{45}\right)_{+}\right].$$

where $B_c = m_e^2/e = 4.41 \times 10^{13}$ G

Mixing angle and transition probability

- the diagonalized matrix $\mathcal{M}'$ has the entries

$$\mathcal{M}' = \begin{bmatrix} m'_1 & 0 \\ 0 & m'_2 \end{bmatrix} \qquad m'_{1,2} = \frac{m_\lambda}{2} \pm \frac{m_\lambda}{2\cos\theta}$$

- the mixing angle reads $(m_{\gamma g} = B_T / m_{Pl})$

$$\frac{1}{2}\tan(2\theta) = \frac{m_{\gamma g}}{m_2}$$

- the probability for a photon with polarization state A_+ to transform into a graviton after traveling a path z is

$$P_{\gamma g} = |\langle h_+(z)|A_+(0)\rangle|^2 = \sin^2(2\theta)\sin^2(m_{g\gamma} z/2)$$

Coherence breaking of photons

- During the graviton-photon oscillation, photons can scatter with the surrounding medium. For photons with energies $\omega < m_e$ the only dominant process that causes a damping in the oscillation process is due to Thompson scattering

$$\Gamma_\gamma = \sigma_T n_e, \qquad \sigma_T = 6.65 \cdot 10^{-25}\,\mathrm{cm}^2$$

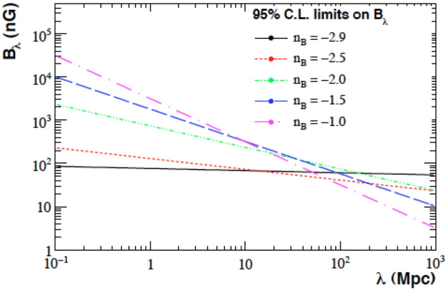
- the wave function approximation is not accurate on describing the oscillation process in the case when the oscillation length is greater than the mean free path, $l_{osc} \gg l_{free}$
- the system becomes open and the total Hamiltonian is not hermitian.
- when there is a loss of coherence a density matrix description is needed

Large scale magnetic field

- for photons to convert into gravitons is necessary a background magnetic field (better homogeneous but not necessary)
- Theoretical models and some observational effects suggest the presence of homogeneous magnetic field in intergalactic space, extragalactic space and on large scales i.e. Hubble horizon.
- We are interested on large scale homogeneous magnetic fields, namely comparable with horizon scale $H_0^{-1} \simeq 10^{28}$ cm
- Propagation of photons can be modeled in two ways: single domain magnetic field or many domains with random direction of the field **B**
- a commonly used assumption in cosmology is conservation of magnetic field flux, namely $B(t) = B_i(a_i/a)^2$

Large scale magnetic field

- From CMB angular anisotropy, $B \lesssim 3 \cdot 10^{-9}$ G on length scale $\lambda_B \sim$ Mpc (**Paoletti and Finelli '12**)
- Faraday rotation of the CMB polarization, $B \lesssim 10^{-8} - 10^{-6}$ G for $\lambda_B \sim 10^3$ Mpc (**Kahniashvili, Maravin and Kosowsky '09**)



Eq. of motions for the density operator: photon-graviton

- We need also to take into account the expansion of the Universe on $\gamma - g$ oscillation
- We need to write equations of motions in the FRW metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}x_i\mathrm{d}x_j$$

- in the FRW metric the von Neumann equation reads

$$iHa\frac{d\rho}{da} = [M, \rho] - i\{\Gamma, \rho\}$$

- the equations of motions are

$$\begin{aligned}
\rho'_{\gamma\gamma} &= (-2m_{g\gamma}I - \Gamma_\gamma\,\rho_{\gamma\gamma})/(Ha), \\
\rho'_{gg} &= 2m_{g\gamma}I/(Ha), \\
R' &= (mI - \Gamma_\gamma R/2)/(Ha), \\
I' &= (-mR - \Gamma_\gamma I/2 - m_{g\gamma}(\rho_{gg} - \rho_{\gamma\gamma}))/(Ha)
\end{aligned}$$

Eq. of motions for the density operator: photon-graviton

- In order to solve the system of differential equations one needs $m_{\gamma g}(a)$, $m(a)$ and $\Gamma_\gamma(a)$. Complicated expressions! Also are needed the initial conditions, $\rho_{gg}(t_i) = R(t_i) = I(t_i) = 0$ and $\rho_{\gamma\gamma}(t_i) = 1/2$
- In all equations is present the Hubble parameter which for the ΛCDM model is

$$H(T) = H_0 \left[\Omega_\Lambda + \Omega_M \left(\frac{T}{T_0}\right)^3 + \Omega_R \left(\frac{T}{T_0}\right)^4\right]^{1/2}$$

- Planck collaboration gives $h_0^2 \Omega_M = 0.12$, $h_0 = 0.67$, and $\Omega_\Lambda = 0.68$. The contribution to the energy density of relativistic species comes from both photons and neutrinos, $\Omega_R = 4.15 \times 10^{-5} h_0^{-2}$

Hydrogen ionization history

- In the case of mixing with gravitons the most important period is at post recombination
- It is very important to know the fraction of free electrons in that period or simply the hydrogen ionization fraction $X_e(T)$
- We use the three-level approximation for the hydrogen atom (1s, 2s, 2p)
- In this approximation $X_e(T)$ satisfies the following equation **Peebles 1968**

$$\frac{\mathrm{d}X_e}{\mathrm{d}T} = \frac{\alpha n_B}{HT}\left(1 + \frac{\beta}{\Gamma_{2s} + 8\pi/\lambda_\alpha^3 n_B(1 - X_e)}\right)^{-1}\left(\frac{SX_e^2 + X_e - 1}{S}\right),$$

$\Gamma_{2s} = 8.22458\ \mathrm{s}^{-1}$ is the photon decay rate of 2s state,
$\lambda_\alpha = 1215.682 \times 10^{-8}$ cm is wavelength of Layman α photons

Hydrogen ionization history

- the function $S(T)$ enters Saha equation $X_e[1 + SX_e] = 1$ and is given by

$$S(T) = 1.747 \times 10^{-22} e^{157894/T} \left(\frac{T}{1\text{K}}\right)^{3/2} (\Omega_B h_0^2),$$

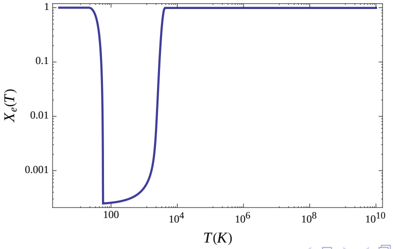
where $\Omega_B h_0^2 = 0.022$ is the baryon density parameter. The functions $\alpha(T)$-the case B coefficient and $\beta(T)$ are given respectively by **Pequignot et al. '91 and Hummer '94**:

$$\alpha(T) = \frac{1.4377 \times 10^{-10} \left(\frac{T}{1\text{K}}\right)^{-0.6166}}{1 + 5.085 \times 10^{-3} \left(\frac{T}{1\text{K}}\right)^{0.53}} \quad \text{cm}^3\,\text{s}^{-1},$$

$$\beta(T) = 2.4147 \times^{15} \left(\frac{T}{1\text{K}}\right)^{3/2} e^{-39474/T} \alpha(T) \quad \text{cm}^{-3}.$$

Hydrogen ionization history

- For $T > 4226$ K hydrogen is completely ionized
- The solution for $X_e(T)$ for $T < 4226$ K is valid until the period of re-ionization of the Universe.
- Universe re-ionization epoch is the most mysterious in the whole its evolution
- Re-ionization started at $z \sim 20$ and was completed at $z \sim 7$ **Dunkley et al (WMAP Collaboration) '09**



Solution of Eq. motion photon-graviton (DE '13)

- CMB photons non-resonantly oscillate into gravitons
- Equation for the density operator $\hat{\rho}$ are highly stiff
- however as far as $l_{\rm osc} \ll H^{-1}$, $I' \simeq R' \simeq 0$ (steady state approximation)
- I and R can be expressed as a function of $\rho_{\gamma\gamma}$ and ρ_{gg} and obtain a closed system of diff. equations here

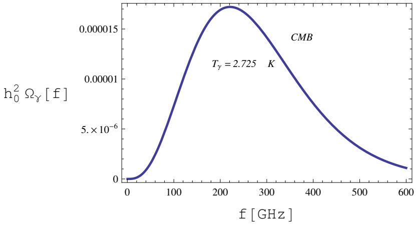
$$\rho'_{\gamma\gamma} = -\frac{\Gamma_\gamma}{Ha}\left[\frac{m^2_{g\gamma}}{m^2_\lambda}(\rho_{\gamma\gamma}-\rho_{gg}) + (\rho_{\gamma\gamma}-\rho^{eq}_{\gamma\gamma})\right]$$

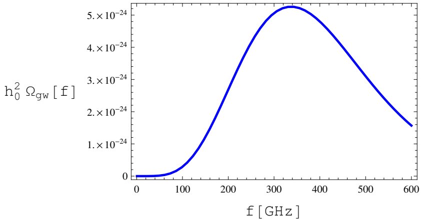
$$\rho'_{gg} = \frac{\Gamma_\gamma}{Ha}\left[\frac{m^2_{g\gamma}}{m^2_\lambda}(\rho_{\gamma\gamma}-\rho_{gg})\right].$$

- everything is calculated numerically!

CMB photons into gravitons, DE'13

- CMB photons make non resonant transformations into gravitons because of their low energy
- there is no temperature anisotropy (contrary to as found in **P. Chen' 94**) but its interesting from the GW formation
- the detection of such GW background is a future prospective but very challenging!





Eq. of motion for photon-pseudoscalar mixing

- The Lagrangian density of photons+pseudoscalars and their interaction is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha^2}{90m_e^4}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(\tilde{F}_{\mu\nu}F^{\mu\nu})^2\right] + \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m_\phi^2\phi^2\right) - \frac{g_{\phi\gamma}}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}\phi,$$

- similarly to the photon-graviton case, equation of motions in WKB regime are ($\lambda_p \ll \lambda_B$)

$$\left[(\omega + i\partial_{\mathbf{x}})\mathbf{I} + \begin{bmatrix} m_+ & 0 & 0 \\ 0 & m_\times & m_{\phi\gamma} \\ 0 & m_{\phi\gamma} & m_a \end{bmatrix}\right]\begin{bmatrix} A_+ \\ A_\times \\ \phi \end{bmatrix} = 0\,,$$

where $m_+ = \omega(n-1)_+$, $m_\times = \omega(n-1)_\times$, $m_{\phi\gamma} = g_{\phi\gamma}B_T/2$, $m_a = -m_\phi^2/2\omega$

Eq. of motion for photon-pseudoscalar mixing

- An important difference in comparison with photon-graviton mixing is that ALPs interact with medium!
- equations of density operator are more complicated!

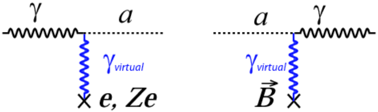
$$\begin{aligned}
\rho'_\gamma &= \frac{2m_{\phi\gamma} I + \Gamma_\gamma (\rho_\gamma - \rho^\gamma_{\rm eq})}{HT}, \\
\rho'_\phi &= \frac{-2m_{\phi\gamma} I + \Gamma_\phi (\rho_\phi - \rho^\phi_{\rm eq})}{HT}, \\
R' &= \frac{-(m_\times - m_a) I + (\Gamma_\gamma + \Gamma_\phi) R/2}{HT}, \\
I' &= \frac{(m_\times - m_a) R + (\Gamma_\gamma + \Gamma_\phi) I/2 + m_{\phi\gamma}(\rho_\phi - \rho_\gamma)}{HT},
\end{aligned}$$

- the system of equations is highly stiff! very difficult to solve numerically!

Coherence breaking terms for photon-pseudoscalar mixing

- another complications is that m_ϕ and $g_{\phi\gamma}$ are independent
- significant conversion of photons into pseudo-scalars can take place before recombination
- we focus at the post BBN epoch
- apart the Compton scattering there are two other scattering terms which do not conserve the photon number
- one of them is Primakoff effect

$$\Gamma_{\gamma\to a} = \frac{g_{\phi\gamma}^2 T k_c^2}{32\pi}\left[\left(1+\frac{k_c^2}{4\omega^2}\right)\log\left(1+\frac{4\omega^2}{k_c^2}\right)-1\right],$$



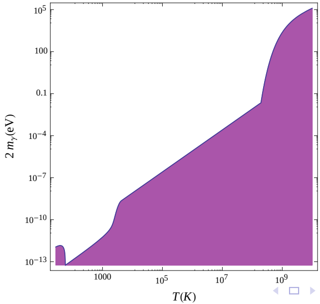
Coherence breaking terms for photon-pseudoscalar mixing

- for γ also the free-free absorption $\gamma + p + e \to p + e$

$$\Gamma_{free-free} = (32\pi^3/3)^{1/2}(Z^2e^2)(m_e/T)^{1/2}(n/\omega^3)\left(1 - e^{-\omega/T}\right)\Gamma_C$$

- for ALPs the two photon decay $\phi \to 2\gamma$ (if $m_\phi > 2m_\gamma$)

$$\Gamma_{\phi\to 2\gamma} = g^2_{\phi\gamma} m^3_\phi/(64\pi)$$



Steady state approximation

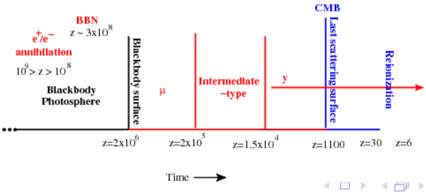
- Even in the case of photon-pseudoscalar mixing $l_{\rm osc} \ll H^{-1}$, $I' \simeq R' \simeq 0$ where $I' \simeq 0$ and $R' \simeq 0$
- a closed system of diff. equation is obtained here

$$\rho'_\gamma = \frac{1}{HT}\left[\Gamma_\gamma(\rho_\gamma - \rho_\gamma^{eq}) + \frac{4\Gamma m_{\phi\gamma}^2}{4\Delta m^2 + \Gamma^2}(\rho_\gamma - \rho_\phi)\right],$$
$$\rho'_\phi = \frac{1}{HT}\left[\Gamma_\phi(\rho_\phi - \rho_\phi^{eq}) - \frac{4\Gamma m_{\phi\gamma}^2}{4\Delta m^2 + \Gamma^2}(\rho_\gamma - \rho_\phi)\right].$$

where $\Delta m = m_\times - m_a$

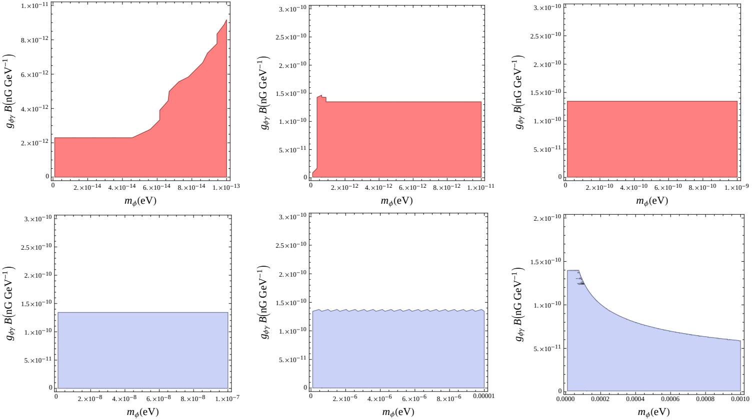
- the initial conditions are $\rho_\phi(T_i) = 0$ and $\rho_\gamma(T_i) = 1/2$ where $T_i = 10^{10}$ K.

- Compton scattering when efficient can restore the BE distribution of distorted CMB with $|\mu| \neq 0$
- inelastic processes lead to a decrease of $|\mu|$ and restore complete equilibrium for $z \geq 2 \times 10^6$ **Khatri and Sunyaev 2013**
- for $2 \times 10^5 \lesssim z \lesssim 2 \times 10^6$, energy release or absorption would eventually create the BE distribution of CMB with $|\mu| \neq 0$, $|\mu| < 9 \times 10^{-5}$ at **Fixsen 1996**
- for $z \lesssim 1.5 \times 10^4$, the Compton scattering cannot establish a thermal equilibrium $\rightarrow$ y-distortions where $y < 1.5 \times 10^{-5}$ **Fixsen 1996**



limits on $m_\phi - g_\phi B$ parameter space

- more stringent limits for higher values of B. For 10^{-25} eV $< m_\phi < 10^{-15}$ eV, $g_\phi B \sim 2 \times 10^{-12}$ nG GeV^{-1}



Conclusions

- CMB turn out again to be one of the most important means which we have to test fundamental physics
- It can couple to large scale magnetic field and oscillate into low mass bosons (gravitons and pseudo-scalars)
- the oscillation probability depends essentially on $G = 1/m_{Pl}$ for gravitons and $g_{\phi\gamma}, m_\phi$ for pseudo-scalars
- The produced background of gravitons is tough to detect at present but is a good opportunity to test quantization of gravity in case of detection
- Axions and ALPs are extremely important for the SM, its extension and string theory. However, we don't know $g_{\phi\gamma}, m_\phi$
- CMB temperature anisotropy gives stringent bounds on $g_{\phi\gamma}, m_\phi$ for light pseudo-scalars 10^{-25} eV$\lesssim m_\phi \lesssim 10^{-4}$ eV