PAPERS

A Bayesian Approach to Improve Estimate at Completion in Earned Value Management

Franco Caron, *Department of Management Economics and Industrial Engineering, Politecnico di Milano, Milano, Italy*
Fabrizio Ruggeri, *CNR-IMATI, Milano, Italy*
Alessandro Merli, *BvTech S.p.A., Milano, Italy*

ABSTRACT

Forecasting represents a core project management process. Estimates at completion in terms of cost and schedule provide essential data and advice to the project team in order to lead and control the project and implement suitable corrective measures. In order to improve the forecasting process, a Bayesian model has been developed within the earned value management framework aiming to calculate a confidence interval for the estimates of both cost and schedule at the completion of the project. The model is based on the integration of data records and qualitative knowledge provided by experts. The model has been tested in an oil and gas project.

KEYWORDS: project control; earned value management; estimate at completion; Bayesian approach

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INTRODUCTION

Project management is the application of tools, techniques, processes, and knowledge skills in order to manage and control a unique, temporary, and multidisciplinary task (Kliem & Ludin, 1998). Project management can be seen as a process that starts from the definition of project scope and objectives and ends with the fulfillment of project requirements. As the Project Management Institute (2008) stated, the main phases involved in project management are: initiating, planning, executing, monitoring and controlling, and closing. An important process that requires the project team to plan and control the complexity of the activities involved in the project is the calculation of estimates at completion, both in terms of cost (EAC) and time (EAC(t)). In fact, project management requires a forward control mechanism to manage the high level of uncertainty and innovation that can affect a project, and in this context estimates at completion represent a prerequisite for identifying and implementing suitable corrective measures in a timely manner.

In general, during the project life cycle, at a given time now (TN), a part of the work is completed and a part of the work still has to be done (see Figure 1).

From Figure 1, the two components of the estimate at completion may be analyzed: the actual cost (AC) of the work completed (WC) and the estimate to complete (ETC) of the work remaining (WR). It should be noted that in a typical feed-forward loop the only way to influence the overall project performance is to take actions affecting the WR. Forecasting activities are critically important in project management; therefore, the feedback drawn from the estimates at completion can increase the probability of achieving the project schedule, quality, and cost objectives, highlighting the possible need for corrective actions that can adjust the project plan (Anbari, 2003). Figure 1 also underlines the decreasing effectiveness of the corrective actions and the increasing level of information available along the project life cycle.

In order to provide an accurate estimate at completion, it should be based on two sources of information (Caron et al., 2006): (1) the data records collected during the WC, which constitute the explicit knowledge, and (2) the experts' opinions about the WR. The latter component derives from the tacit knowledge possessed by the experts, which is very valuable for forecasting purposes but difficult to formalize. Although the data records are related to the past, experts' knowledge may help to forecast the future, in particular regarding the overall WR.

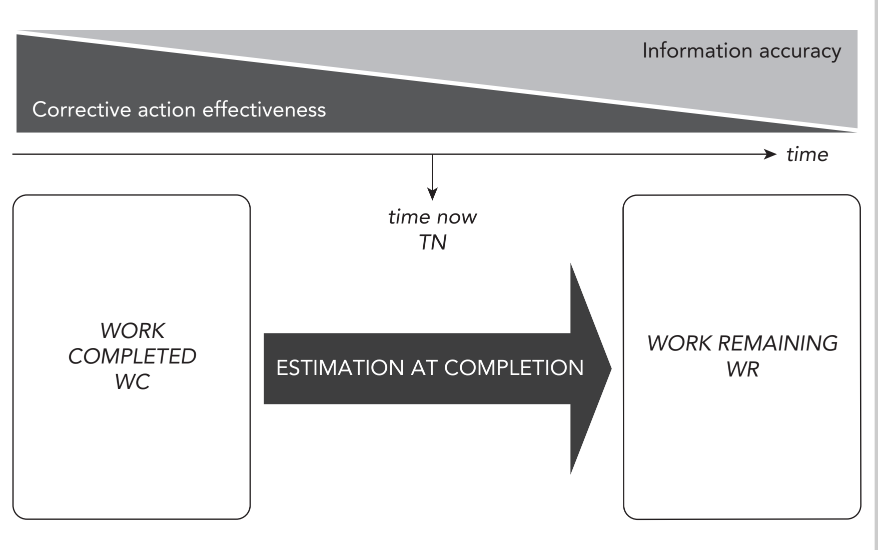


Figure 1: Feed-forward control based on estimation at completion.

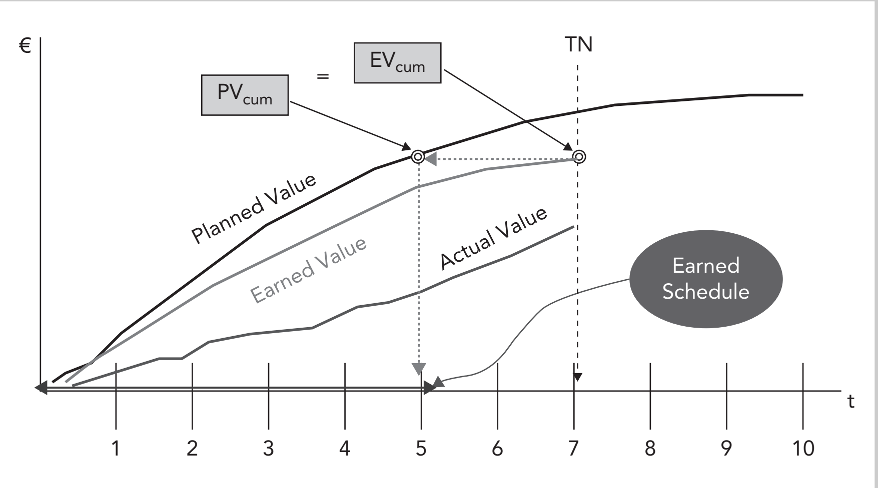


Figure 2: Earned value management.

The goal of this article is to introduce an "estimate at completion" model that utilizes the experts' tacit knowledge and uses it in combination with project data records. The integration of data records and experts' judgments can lead to an improved accuracy of estimate at completion. In particular, the main contribution that can be derived from the subjective information about the future may be related to:

1. The impact of corrective actions that management has decided to implement;
2. Typical patterns that characterize similar projects, such as the progress patterns associated with a class of similar projects (e.g., S-curves are a well-known and effective tool for describing such patterns, pointing out similar progress levels at similar milestones);
3. Anticipated future situations or events, which can arise both in terms of threats (i.e., adverse weather conditions) and opportunities (i.e., the switch to a more efficient supplier);
4. Trends in project performance, such as productivity increase/decrease (considering also nonlinear trends); and
5. Expected behavior of the stakeholders involved in the project.

The concept of estimate at completion can be applied both to the overall project and/or parts of it, by utilizing the well-known methodology represented by earned value management (EVM). EVM is an efficient and synthetic performance measurement and reporting technique for estimating cost and time at completion (PMI, 2011) (Marshall, Ruiz, & Bredillet, 2008). The basics of EVM are described by the three curves in Figure 2:

- planned value (PV), the budget cost of work scheduled at TN;
- earned value (EV), the budget cost of work completed at TN; and
- actual cost (AC), the actual cost of work completed at TN.

EVM was improved by Lipke (2003), who introduced the concept of earned schedule (ES) for measuring schedule performance in time units and overcoming the flaws associated with a schedule index defined as the rate between EV and PV. Earned schedule represents a more effective schedule index, because it avoids the problem of the convergence of the EV to PV values as the project reaches completion (Lipke, 2006a).

The above three curves and the ES value (see Figure 2) allow for the calculation of a set of indices and variances at the TN. The most important indices and variances are as follows:

- Cost Performance Index

CPI = EV/AC

- Cost Variance

CV = EV – AC

- Schedule Performance Index

SPI(t) = ES/TN

- Schedule Variance

SV(t) = ES – TN

	Variance Index	Time: SV $>$ 0; $SPI_{(t)} > 1$	Time: SV $=$ 0; $SPI_{(t)} = 1$	Time: SV $<$ 0; $SPI_{(t)} < 1$
	CV $>$ 0; CPI $>$ 1	Ahead of Schedule Under Budget	*On Schedule* Under Budget	Behind Schedule Under Budget
Cost	CV $=$ 0; CPI $=$ 1	Ahead of Schedule *On Budget*	*On Schedule* *On Budget*	Behind Schedule *On Budget*
	CV $<$ 0; CPI $<$ 1	Ahead of Schedule Over Budget	*On Schedule* Over Budget	Behind Schedule Over Budget

Table 1: Indices and variances in the earned value framework.

Case	EAC Cost	EAC(t) Time
1	AC + (BAC – EV)/CPI_f	TN + (SAC – ES)/$SPI(t)_f$
2	BAC – CV	SAC – SV(t)
3	BAC/CPI	SAC/SPI(t)
4	BAC/SCI	SAC/SCI(t)
5	BAC	SAC

Table 2: Estimation at completion formulas in earned value management.

• Schedule Cost Index
$SCI_{(t)} = CPI * SPI(t)$

In Table 1, these indices and variances are summarized together with their corresponding meaning. Many estimate at completion formulas have been proposed during almost 50 years of EVM application, but none of them has proved to be always more accurate than another one (Christensen, 1993). The indices and variances applied in cost and time estimates at completion formulas may be summarized as in Table 2 (Anbari, 2003), where BAC indicates budget at completion and SAC indicates schedule at completion. Note that EAC refers to estimate at completion in terms of cost while EAC(t) refers to estimate at completion in terms of time.

Table 2 provides a snapshot of the commonly used assumptions concerning the estimation process within the EVM framework. The table considers the following five situations:

1. Future performance significantly differs from past performance. Two new performance indices, CPI_f and $SPI(t)_f$, have been introduced concerning the WR.
2. The deviations CV and SV(t) reported at time now will not affect the rest of the project.
3. The indices CPI and SPI(t) reported at time now will remain constant until project completion.
4. The joint effect of cost and schedule performance will be considered in calculating the estimate at completion.
5. The deviations registered during the project will be absorbed by suitable corrective actions, and planned objectives will be attained.

It should be noted that only in the first formula in Table 2 (i.e., case 1) may future values be introduced for CPI_f and $SPI(t)_f$ indices, separating the two parts of the work described in Figure 1 (i.e., WC and WR) and pointing out the differences between actual past performance and estimated future performance. For instance, in case 3 (see Table 2), the same value of CPI and SPI(t) has been applied to WC and WR. Moreover, in case 3, both CPI and SPI(t) represent a cumulative value covering all the WC from the project outset to time now. In fact, relying only on past performance while developing a forecast could be misleading, because considering only past values of CPI and SPI(t) is similar to driving a car while looking just in the rearview mirror, thus making it impossible to dodge the obstacles that may lie on the route. In the following, the generic indices CPI and SPI(t) will be referred to the overall WC, while CPI_f and $SPI(t)_f$ will be referred to the overall WR.

The process of calculating an estimate at completion is usually based on a deterministic approach that is suited to projects with a low uncertainty level (Fleming, 1992). The application of a statistical approach to EVM was first proposed by Lipke (2002b), who suggested applying control charts to project performance so as to determine upper and lower limits for estimate at completion (an analogous property was investigated by Christensen [1996]) as a way to improve project planning and control processes. A further study (Lipke, 2002a) demonstrated that the monthly values CPI_m and $SPI(t)_m$ (obtained as the ratio between the difference of the value registered at the end of a month [e.g., at TN] and the value collected at the end of the previous month [e.g.,

$$CPIm = \frac{EV(TN) - EV(TN-1)}{AC(TN) - AC(TN-1)} = \frac{EVm}{ACm},$$

$$SPI(t)m = \frac{ES(TN) - ES(TN-1)}{(TN) - (TN-1)} = ESm]),$$

are log-normally distributed. This fact opened the door to establish a confidence interval for the estimate at completion (Lipke, 2006b).

According to an EVM framework, past performance can be used as a predictor of future performance (PMI, 2011). Except for the first case in Table 2 (in which, unfortunately, no guidelines are given on how to find the correct value of the future index), for forecasting purposes EVM normally considers only the data records stemming from WC. This restriction can provoke a misleading calculation of the estimate at completion, especially for long-duration, innovative, and complex projects.

This article aims to improve the choice of CPI_f and $SPI(t)_f$ values related to WR, in order to obtain more accurate estimates at completion. The proposed model is based on the use of a rigorous and formal method that takes into account available information, in terms of experts' opinions and past performance data records, in order to establish the value of the future indices CPI_f and $SPI(t)_f$. The use of experts' opinions is made possible by applying a Bayesian approach in association with expert opinions elicitation methods.

In order to implement the forecasting process, at each TN along the project life cycle, available data records correspond to the past values of CPI_m and $SPI(t)_m$ (i.e., the monthly values at times t_0 . . .,TN [where t_0 is a generic previous time period before time now, even if it generally coincides with the first period in which data are collected]); data records are combined with the opinion of the experts about WR in order to forecast the values of the indices CPI_m and $SPI(t)_m$ at time TN + 1. Such a procedure, easily implemented through a user-friendly software code, can be reiterated along the project life cycle to carry out forecasts at future times TN + 2, TN + 3, ... In the following, the estimate concerning the close future (i.e., at TN + 1) will be applied to the overall WR, because extending the forecast horizon implies a progressively decreasing accuracy of the forecast. It should be noted that instead of the overall interval, t_0 . . .,TN, only the values related to the recent past (e.g., TN – 3. . . ,TN), may be used if they allow for a more accurate description of the actual performance trend at TN.

The second section introduces the typical features of a Bayesian approach, and the third section describes the development of the Bayesian model in the EVM framework. The fourth section explains the elicitation process to obtain the experts' judgments and, consequently, the prior estimates of the CPI_f and $SPI(t)_f$ indices for WR. The fifth section describes the integration process of data records and experts' judgments in order to obtain the posterior (i.e., the updated) estimates of the CPI_f and $SPI(t)_f$ indices for WR. The sixth section develops the application of the Bayesian model to a case study concerning a project in the oil and gas industry. And the seventh section offers some conclusions about the effectiveness of the approach proposed.

Bayesian Approach

The Bayesian approach differs from others because it describes the probability in a subjective way (De Finetti, 1974) as the degree of belief that someone has in the occurrence of an event. The subjective definition of probability is justified by the dependence of probability on the state of information that someone has (D'Agostini, 2003).

In the Bayesian perspective, the parameter θ of a probability density function (PDF) of a sample of n observations, $x = (x_1 \ldots x_n)$, independently and identically distributed, is a random variable that follows a probability distribution called prior distribution (Congdon, 2003). Applying Bayes' theorem, it is possible to write the posterior density function as,

$$f(\theta|x) = \frac{f(x|\theta) \cdot f(\theta)}{\int f(x|\theta) \cdot f(\theta)\, d\theta} \quad (1)$$

In Equation 1, there are four elements:

- $f(\theta \mid x)$, posterior distribution of the parameter θ given the sample $x = (x_1 \ldots x_n)$;
- $f(x \mid \theta)$, the joint density function (conditional on θ) of x, or the likelihood $L(\theta; x)$ when considering it as a function of θ given x, defined as the product of n functions $f(x_i \mid \theta)$, PDF of x_i given the parameter θ, $I = 1, n$;
- $f(\theta)$, prior distribution of θ;
- $\int f(x \mid \theta)\, f(\theta)\, d\theta$, marginal density of x, which is the integral of the product of the likelihood function and prior distribution.

Equation 1 represents a formal method to update the prior information $f(\theta)$, which reflects experts' tacit actual knowledge, taking into account a series of n past observations x_i's, through the likelihood $L(\theta;x)$. As described later in this article, it is possible to use the posterior distribution to predict the values of the future observations. Application to non-repetitive processes like projects, updating of the probability according to the changing available information, and forecasting perspective are the main strengths of the Bayesian approach.

The Bayesian Model Applied to EVM Framework

The most delicate step in the development of a Bayesian approach is the selection of the prior distribution. The prior distribution reflects experts' knowledge about project trends, future threats and opportunities, projects' patterns, and impacts of corrective actions, corresponding to the main contribution given by experts to the forecasting process.

Drawing on Lipke's (2002a) assumption of a log-normal distribution of indices CPI_m and $SPI(t)_m$, where m stands for monthly (or periodic) values, this article aims to develop a model that combines prior opinions with the evidence provided by the data records collected during WC, in order to update the forecast values of performance indices concerning WR. The posterior distribution of CPI_f and $SPI(t)_f$ concerning WR enables us to set up a confidence interval for the estimates at completion both in terms of cost and time (i.e., EAC and EAC_t).

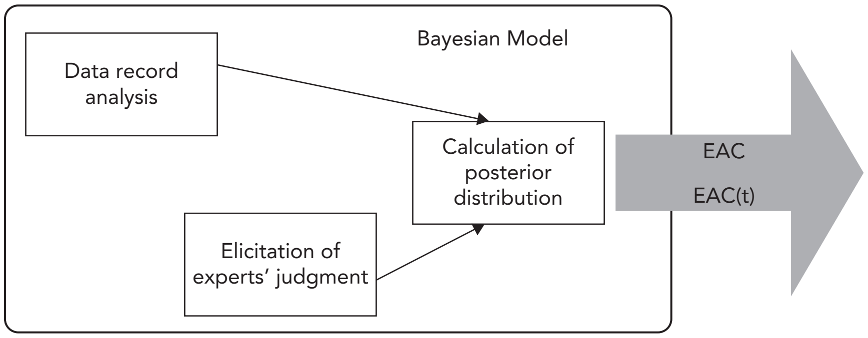


Figure 3: Bayesian model for estimate at completion.

The proposed Bayesian model is made up of three phases (see Figure 3). The main assumptions underlying the model are the log-normal distribution and the independence of the indices CPI_m and $SPI(t)_m$. The latter assumption is motivated by the consideration that the above indices are affected by a high level of volatility due to the changing conditions, month by month, of the project internal and external context.

The first phase in modeling is data analysis and logarithmic transformation of the indices' values (see Figure 3). First, the hypotheses of log-normality and independence of the indices must be verified. For this purpose, the tests developed by Anderson-Darling and Ljung-Box-Pierce are used (Brockwell & Davis, 2002). Exploratory data analysis techniques accompany the implementation of the two tests. Defining an index as x, the log-normal PDF is assumed:

$$f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}} \quad (2)$$

Next, the prior distributions for the parameters μ and σ^2 of log-normal distribution should be chosen. In order to ensure flexibility and relatively simple algebraic computations, a normal distribution (see Equation 3) and an inverse gamma distribution (see Equation 4) have been chosen, respectively.

$$f(\mu) \sim N(\theta,\xi^2) = \frac{1}{\sqrt{2\pi}\xi} e^{-\frac{(\mu-\theta)^2}{2\sigma^2}} \quad (3)$$

$$f(\sigma^2) \sim INVG(\alpha,\beta) = \frac{\beta^\alpha}{G_{(\alpha)}\sigma^{2(\alpha+1)}} e^{-\frac{\beta}{2\sigma^2}} \quad (4)$$

In fact, with these prior distributions (Equation 3 and Equation 4), it is possible to express the posterior distribution easily, apart from a constant, as:

$$f(\mu,\sigma^2|x) \propto \frac{exp\left[\frac{\sum_{i=1}^{n}(\ln(x_i)-\mu)^2)}{-2\sigma^2}\right]}{[\sigma^2]^{\frac{n}{2}}}$$

$$\times\, exp\left[\frac{(\mu-\theta)^2}{-2\xi^2}\right] \frac{exp\left[\frac{-\beta}{\sigma^2}\right]}{[\sigma^2]^{\alpha+1}} \quad (5)$$

Elicitation of Experts' Opinion

The second phase in modeling aims at transforming experts' knowledge into a prior distribution (see Figure 3). The elicitation process is focused on minimizing the biases that derive from cognitive and motivational aspects (since a person can follow personal objectives that may differ from the project ones) and affect human thinking about probability. The major biases that exercise an impact on human thinking from a cognitive point of view may be: anchoring and adjustment, availability, representativeness, range frequency compromise, and overconfidence (O'Hagan et al., 2006). From a motivational point of view, a typical example is given by excessive optimism aiming to emphasize the positive and downplay the negative in order to get the project approved. An outside view, also known as reference class forecasting, may mitigate both cognitive and motivational biases by considering data records related to a class of similar projects completed in the past (Flyvberg, 2006).

The elicitation process is very important in the construction of a probabilistic model, as it minimizes the risk that incorrect or erroneous parameters may be fed into the model. Any elicitation process consists of a set of questions that the experts respond to either directly by providing numbers or indirectly by choosing between simple alternatives or bets. A typical direct approach to elicitation requires the definition of the density function quantiles (e.g., the median value; Spetzler & Staël von Holstein, 1975). A direct approach starting from the elicitation of the median value has been applied in the case study for two reasons.

First, the median represents an easily understandable value separating the higher half of a sample from the lower one (in a more practical language, this is also known as P_{50})—for example, assuming the same probability of early or late completion for an activity. In order to let the experts understand the actual questions and refrain from confusing median with mode or sample average, the survey is presented in a straightforward form where it is clearly stated that the median corresponds to the line that divides into two halves the probability density function.

Second, because a log-normal distribution (Equation 2) is used, it is simpler to ask the experts for the median value of the distribution. In fact, the median of a log-normal distribution is given by the exponential function of the μ parameter, while mean and mode depend also on the parameter σ, so the μ parameter is easy to compute and, above all, to handle with a software application

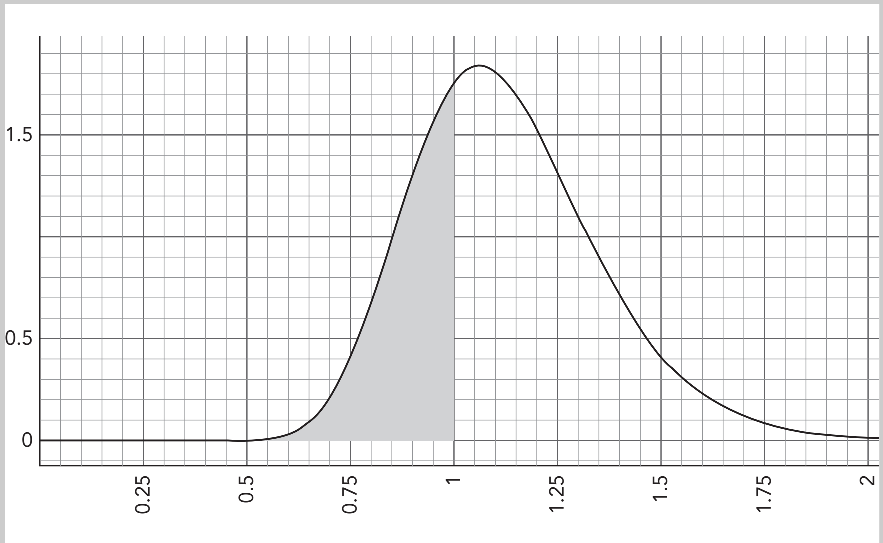


Figure 4: Probability density function of the elicited index.

since it is independent from other parameters.

Furthermore, the use of graphical software, such as Grapher, can facilitate the elicitation process, helping the experts to fully understand the implications of their choice. Any change in the experts' estimate can be verified as a change in the probability density function.

As in the example in Figure 4, based on the elicited values of median and standard deviation, the probability density function of the index has been obtained, allowing for an adjustment of the expert's initial answer. For instance, in Figure 4, the grey area, limited by the unitary value of the index (e.g., CPI_f), indicates the range of poor performance of the project. Moreover, the probability density function of the index indicates the modal value, corresponding to the most frequent value the expert should have observed in similar situations. A further check to verify the consistency of the expert's answers may regard the maximum and the minimum value of the distribution: if these values lie on the outskirts of the obtained distribution, it means the elicitation has been successfully performed; otherwise, the elicitation process has to be repeated.

After k experts are identified, they are interviewed about:

- the final median cost of the project;
- the final median completion date of the project; and
- the overall uncertainty level of the project, expressed in a qualitative way as a scale of three values corresponding to high, medium, and low.

Based on these values, the prior distribution of the index may be derived (see Figure 4). The first two values are related to the μ parameter and the third value to the σ parameter of the distribution.

It should be noted that from the final median cost, the corresponding value of CPI_f may be easily derived through the relationship $EAC = AC + BCWR/CPI_f$, where BCWR indicates the budget value of WR (see the case study section that follows). The same approach may be applied in order to estimate $SPI(t)_f$.

The experts provide their opinions about these three main project features, according to their knowledge and expertise, related not only to the current project but also to similar projects developed in the past. To reduce motivational biases, it is important to indicate that the goal of the interview is to compare experts' opinions in order to evaluate the uncertainty level affecting the project and that it is not utterly significant to recognize who provides the most accurate estimation.

With reference to the estimate at completion, it is easy to collect N experts' opinions about future median value of the indices CPI_f and $SPI(t)_f$, related to WR. Above all, the main interest is the logarithm of those values, μ_k, $k = 1, N$, which contribute to Equations 6 and 7 to determine the value of the hyper-parameters θ and ξ of μ prior distribution.

Here we consider a method of moments (see Gajoni, Dey, & Ruggeri, 2010) to estimate the hyper-parameters, considering μ_k, $k = 1, N$, the logarithm of the elicited median values, as a sample from the prior distribution, computing its sample mean and standard deviation. The logarithm of each index CPI_f and $SPI(t)_f$ follows a Gaussian distribution with mean/median μ, and we equate sample mean and standard deviation with its mean and standard deviation, as shown in Equations 6 and 7, respectively.

$$\theta = \frac{\sum_{k=1}^{N} \mu_k}{N} \qquad (6)$$

$$\xi = \sqrt{\frac{\sum_{k=1}^{N} (\mu_k - \theta)^2}{N - 1}} \qquad (7)$$

The hyper-parameters α and β of σ^2 distribution have been chosen in a different way, using a qualitative approach based on the project's uncertainty level assessed by experts. Dividing the uncertainty level U into three categories (low, medium, and high), corresponding to a range of 1 to 3, respectively, it is possible to compute an average *U level* for the project's uncertainty (the average of the values collected during the inquiry) that helps to determine the posterior value of CPI_f and $SPI(t)_f$ standard deviation.

In the case study, based on empirical considerations, hyper-parameters α and β are set according to the above-average U

level in order to obtain: $\sigma = [0; 0.1]$, corresponding to low *U level*; $\sigma = [0.1; 0.3]$, corresponding to medium *U level*; $\sigma = [0.3; 0.5]$, corresponding to the high *U level*.

As it is difficult for someone who does not have a statistical background to understand the meaning of the hyper-parameters of σ^2 for the prior distribution, an external "expert" (i.e., a statistician) who has a good knowledge of statistical curves and related properties may be necessary in order to support the setting of the inverse-gamma hyper-parameters. However, a sensitivity analysis on these hyper-parameters has to be performed to prevent any exogenous substantial influence on the posterior distribution and ensure the robustness of the model.

Posterior Distribution

The third step of the model is the calculation of the posterior distribution of CPI_f and $SPI(t)_f$ (see Figure 3). Equation 5 cannot be solved with a closed formula, so it may be broken down into two parts, each one representing the posterior conditional distribution of the parameter. After some manipulations, it is possible to obtain a normal distribution for μ (see Equation 8) and an inverse gamma distribution for σ^2 (see Equation 9).

$$f(\mu|\sigma^2,x) \sim N\left(\frac{\left[\xi^2 \cdot \sum_{i=1}^{n} \ln(x_i) + \sigma^2 \cdot \theta\right]}{\xi^2 + \sigma^2}, \left[\frac{\sigma \cdot \xi}{\sqrt{(\xi^2 + \sigma^2)}}\right]^2\right) \quad (8)$$

$$f(\sigma^2|\mu,x) \sim INVG\left(\frac{n}{2} + \alpha, \frac{1}{2} \cdot \sum_{i=1}^{n} (\ln(x_i) - \mu)^2 + \beta\right) \quad (9)$$

Both distributions are conditional with respect to data and other parameters. Therefore, Equations 8 and 9 can be used to perform the Gibbs sampling process, a Markov Chain Monte Carlo simulation technique, which obtains a sample from the posterior distribution (Gamerman & Freitas Lopes, 2006).

The application of the Gibbs sampling process is based on an iterative algorithm comprising three main steps:

1. Choose initial values for μ and σ^2. The initial values do not affect Gibbs sampling because the results of the initial iterations are usually removed (burn-in process).
2. At each iteration *j*, generate a random value μ_j from the normal distribution in Equation 8.
3. Generate a random value σ_j^2 from the inverse gamma distribution in Equation 9. Then go back to step 2 as long as conditional distributions convergence is achieved (the usual adequate tests have been employed to ensure it).

Once some iterations are run (repeating steps 2 and 3 about 1,000 times), it is possible to obtain the posterior predictive density of the indexes CPI_f and $SPI(t)_f$ by inserting $I - m$ values of μ_j, σ_j^2, obtained through Gibbs sampling, into the log-normal PDF (see Equation 10):

$$f(x_k)_{TN+1} \cong \frac{\sum_{m+1}^{I} LogN(x_k|\mu_j,\sigma_j^2)}{I - m} \quad (10)$$

A step function is obtained that approximates the posterior predictive density, for a grid of values x_k, ranging from 0 to infinity (in practice, to a large number). The terms introduced in Equation 10 stand for:

- *I*, the number of iterations; usually a value of 6,000 is sufficient;
- *m*, the number of values removed as a result of the burn-in process, typically 1,000;
- *TN + 1*, the next period considered;
- x_k, the values of the CPI_f and $SPI(t)_f$ indices, from 0 to ∞ theoretically, even if the range considered is actually stopped beforehand, when the $f(x_k)$ value becomes negligible.

The posterior predictive PDF should integrate to 1; therefore, the values x_k should be chosen very carefully so that the step function approximating the density function would be given by:

$$\sum_{k=1}^{K} f(x_k)_{TN+1} \cdot (x_{k+1} - x_k) \cong 1 \quad (11)$$

Eventually, the step function is normalized to exactly reach the unitary value in Equation 11 and is used to compute the central value corresponding to the discrete sample average, and the upper and lower limits, by removing a cumulative area of 2.5% from the right and left tail of the curve, respectively. Moreover, the sample standard deviation value of the normalized step function is an indicator of the correct setting of α and β hyper-parameters as stated in the "Elicitation of Experts' Opinion" section.

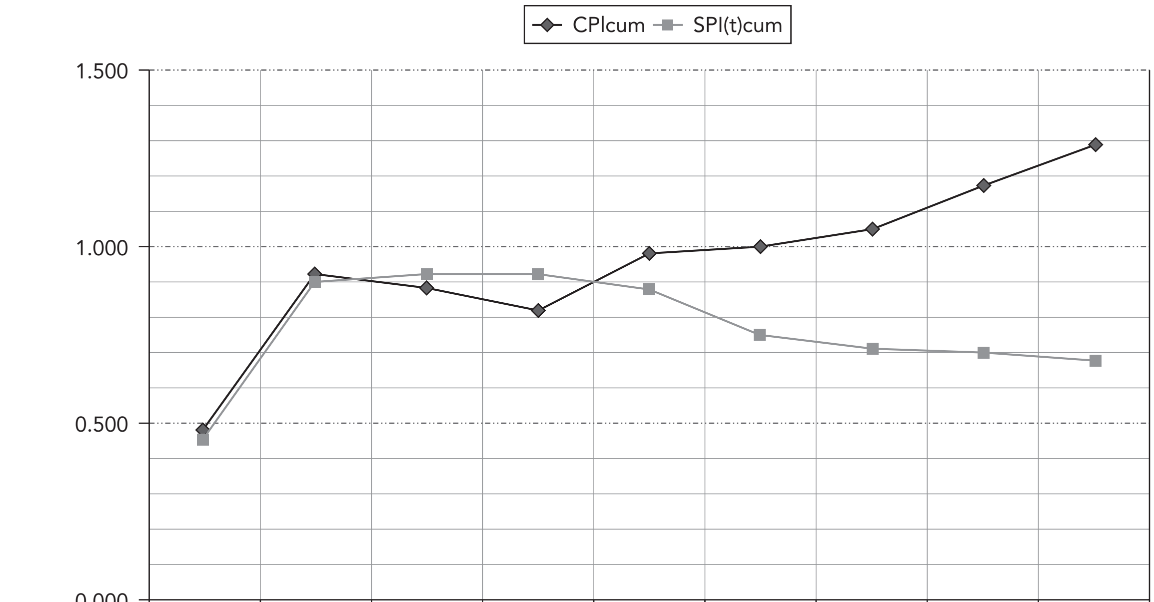
It should be noted that in the model the contribution deriving from the data records to the estimate at completion is considered accurate just for the near future (i.e., at time t + 1). If a more extended horizon were considered, the reliability of the estimation based just on data records would rapidly decrease, so the contribution deriving from the subjective information expressed through experts' opinions would become predominant.

Case Study

The Bayesian model has been applied to the construction of two pipelines within a development project for a sweetening and stabilization plant in the oil and gas industry, in order to compare its accuracy with the traditional EVM formulas. First, the CPI and SPI(t) trends are shown in Figure 5, where the cumulative indices denote good efficiency but an increasing delay in the project.

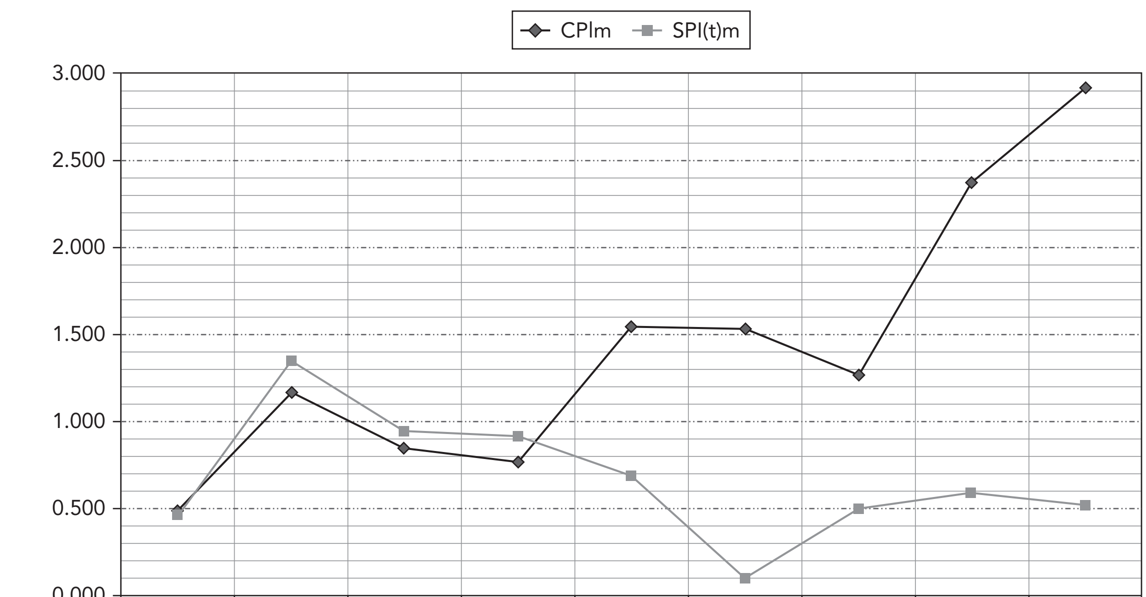
The same situation (a little bit amplified because monthly indices are more sensitive to contingent projects' situations) can be seen in Figure 6, where the CPI_m and the $SPI(t)_m$ values registered along the project duration are shown.

A potential data problem could be represented by the sixth $SPI(t)_m$ observation, corresponding to January 2010,



	aug 09 1	sep 09 2	oct 09 3	nov 09 4	dec 09 5	jan 10 6	feb 10 7	mar 10 8	apr 10 9
CPIcum	0.486	0.924	0.886	0.822	0.979	1.000	1.046	1.175	1.292
SPI(t)cum	0.456	0.903	0.918	0.919	0.874	0.746	0.712	0.697	0.678

Figure 5: Trend of cumulative CPI and SPI(t).



	aug 09 1	sep 09 2	oct 09 3	nov 09 4	dec 09 5	jan 10 6	feb 10 7	mar 10 8	apr 10 9
CPIm	0.486	1.167	0.856	0.770	1.552	1.534	1.268	2.369	2.921
SPI(t)m	0.456	1.350	0.949	0.922	0.693	0.109	0.506	0.588	0.525

Figure 6: Trend of monthly CPI_m and $SPI(t)_m$.

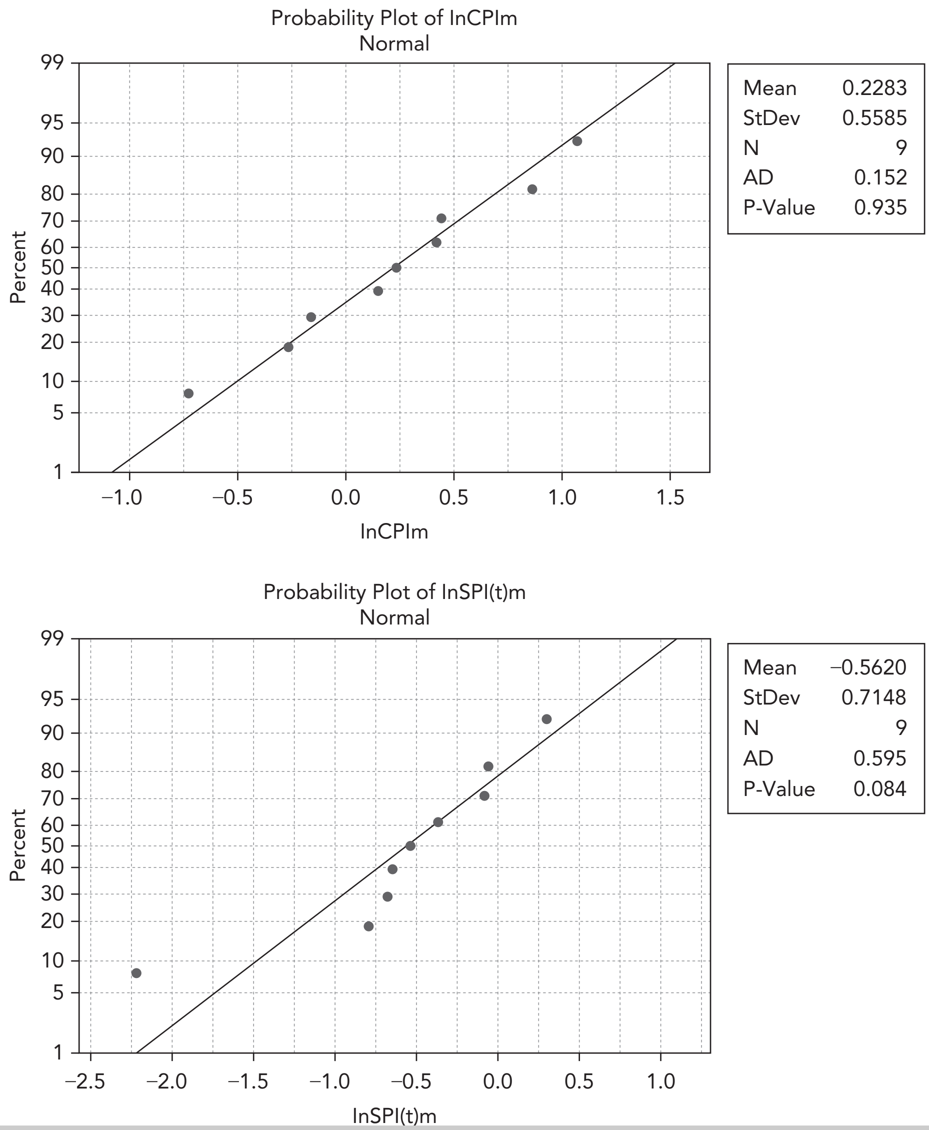


Figure 7: Anderson-Darling test.

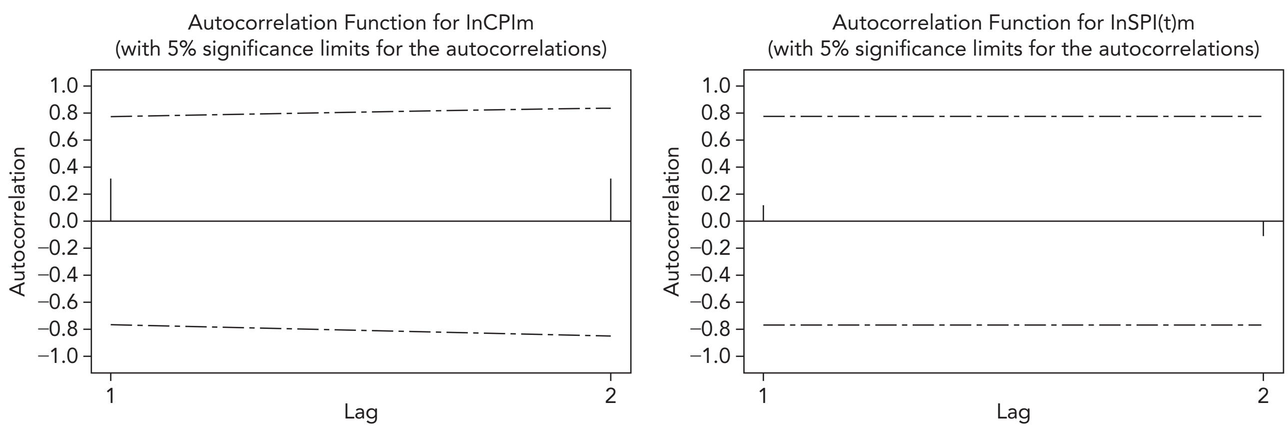


Figure 8: Ljung-Box-Pierce test.

Expert	CPI_f	$SPI(t)_f$	$\ln CPI_f$	$\ln SPI(t)_f$	Uncertainty Level
Pipelines superintendent	***	0.966	***	–0.034	3
Pipelines manager	1.150	0.983	0.140	–0.017	2
Project control manager	1.107	1.000	0.102	0.000	1
Contract engineer	1.118	***	0.111	***	2
Cost engineer	1.087	***	0.084	***	1
Project management (headquarters)	1.139	0.950	0.130	–0.051	1
Project control manager (headquarters)	0.917	0.950	–0.086	–0.051	2

Table 3: Experts' answers.

Uncertainty Level	**1.71**
μ Prior Distribution for Cost:	
θ: ***theta***	0.08013
ξ: ***csi***	0.08389
μ Prior Distribution for Schedule:	
θ: ***theta***	–0.03089
ξ: ***csi***	0.02226

Table 4: Results of expert opinions elicitation.

but there are no non-statistical grounds for rejecting or discarding this observation, so it is included in the analysis. A radical change in project performance occurred in the last three months (i.e., between February and April 2010; see monthly values in Figure 5), due to changes in health and safety policies determining a positive impact on project performance—a positive impact that only through subjective knowledge could have been foreseen and estimated, so confirming the validity of the approach adopted in the article.

The tests of normality (see Figure 7) and independency (see Figure 8) of the logarithmically transformed data have been carried out using the software Minitab.

The two tests confirm that the hypotheses underlying the model are verified; therefore, the next step is the elicitation of experts' opinions. Table 3 provides the answers from each expert, and Table 4 provides the results of the application of Equations 6 and 7.

Table 3 shows the experts' answers concerning the future median values of CPI_f and $SPI(t)_f$ indices. Also, the corresponding logarithmic values are indicated. These values may be derived from the experts' estimate of the final median cost and the median completion date of the project. For instance, see Equations 12 and 13, where the value of the index at completion CPI_{AC} (or $SPI(t)_{AC}$) has been derived as the ratio between planned value BAC (or SAC) and the estimate at completion EAC (or EAC(t)).

$$CPI_{ACi} = \frac{BAC}{EACi} \tag{12}$$

$$SPI(t)_{ACi} = \frac{SAC}{EAC(t)i} \tag{13}$$

In the same way, the values in Table 3 arise from the inversion of the first formula in Table 2 (see Equations 14 and 15), from the estimation of the final median cost and the median completion date of the project given by the k experts.

$$CPI_{f(k)} = \frac{BAC - EV}{EACk - AC} \tag{14}$$

$$SPI(t)_{f(k)} = \frac{SAC - ES}{EAC(t)k - TN} \tag{15}$$

Regardless of the actual values of CPI = 1.292 and SPI(t) = 0.678 at time now (see April 2010 in Figure 4), the experts do not agree that the past trend that emerges from WC data should be extended to WR, and think that the estimate for WR will be more similar to the planned values (experts' mean estimate is 1.086 for CPI_f and 0.971 for $SPI(t)_f$). In this case, subjective knowledge exercises a significant influence in the forecasting process by changing the trend deriving from data records.

In Table 4, in addition to the results of Equations 6 and 7 corresponding to the mean and standard deviation of the Gaussian distribution used to model the logarithm of the indices CPI_f and $SPI(t)_f$, it can be seen how the uncertainty affecting the project is estimated to be at the medium/low level, based on widespread opinion corresponding to the different points of view of the experts involved in different organizational roles and specializations.

In order to test the validity of the elicitation process, the experts were asked to give an optimistic and a pessimistic value for cost and schedule performance. Because the most optimistic and pessimistic values, indicated by the experts, lie on the outskirts of the corresponding prior distributions, it can be stated that the obtained results are satisfactory.

Simulation Output			σ^2 Prior Distribution
CPI_f	Upper bound	2.35	$\alpha = 75$
	Mean value	1.589	
	Lower bound	1.05	$\beta = 1$
	Standard deviation	0.316	
SPI_f	Upper bound	1.525	$\alpha = 75$
	Mean value	0.957	
	Lower bound	0.55	$\beta = 1.5$
	Standard deviation	0.237	

Table 5: Output of the Bayesian model.

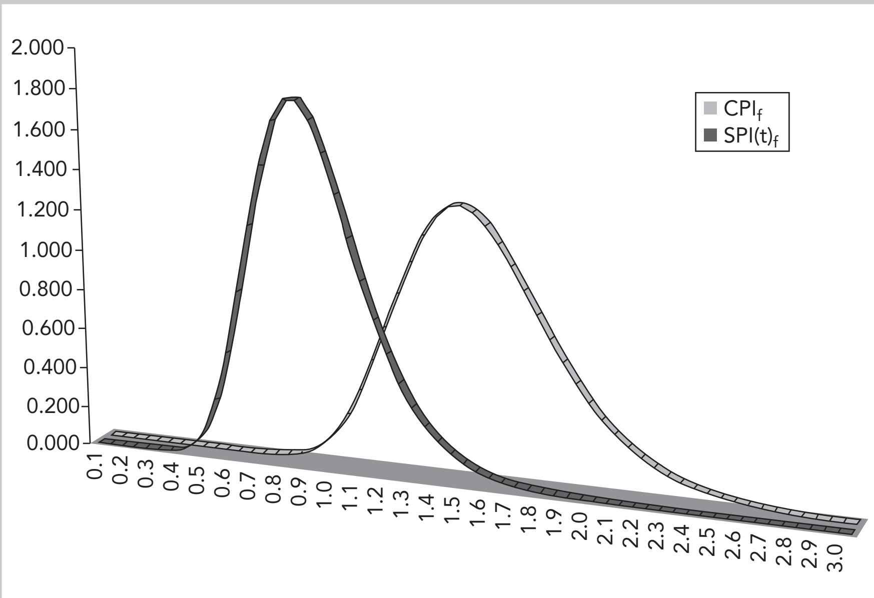


Figure 9: Posterior distribution of future performance indices.

The last step of the model is the application of the Gibbs sampling process, whose implementation is supported by the software @Risk. This software allows for random number generation and assessment of conditional distributions convergence. In Table 5, the output of the simulation is represented, in which a bilateral confidence interval of 95% has been used for the indices CPI_f and $SPI(t)_f$, corresponding to lower and upper values indicated in Table 5. Table 5 also reports the values set for the hyperparameters of σ^2 prior distribution.

Means and standard deviation values are computed with the classical formulas used for a discrete distribution while upper and lower values determine a right and left area of 2.5%, respectively (in other words, the value closer to 97.5% and 2.5% of the discrete distribution respectively; see Equation 11).

Figure 9 shows the posterior distributions for both cost and schedule future performance indices.

The posterior distribution for CPI_f is wider than that of $SPI(t)_f$. In general, in the oil and gas sector, the schedule performance is the most critical, because delays in the first oil date (the day on which the plant starts the production) can cause a huge financial loss for the owner. For this reason, the project team focuses its efforts on completing the project as soon as possible, in spite of the cost performance trend that is expected to worsen due to the incentives for the subcontractors in order to accelerate project progress.

The sensitivity analysis of the simulation results shows that there are no significant variations of the output by changing the input values for the hyperparameters α and β of σ^2 prior distribution (see Table 6, in which the results of six simulation runs from A to F are reported, where the A case is the one represented in Table 5 and Figure 9).

The results of the Bayesian model are compared (see Table 7) with traditional EVM formulas listed in Table 2. All the cases listed in Table 7 address the problem of estimating at time now the final cost and duration of the project. While EVM case 1 and the Bayesian model (see the upper part of Table 7) are directly comparable because they introduce a distinction between WC and WR, EVM cases 2, 3, 4, and 5 (see the lower part of Table 7) do not consider such a distinction; nevertheless, they contribute to give a general view of typical formulas used in the EVM context.

In Table 7, the estimated values of CPI_{AC} and $SPI(t)_{AC,}$ (i.e., the values of the indexes at completion, are reported for the five cases considered in Table 2, plus the Bayesian model (see column "case"). The first column indicates the underlying assumptions for each case. As for the first case, in the last column of Table 7, m indicates the monthly value and 3 and 6 indicate the three-month and six-month moving average values, respectively. In the last column of Table 7, where the index is not pointed out, it means the use of cumulative CPI and SPI(t) for estimating EAC and EAC(t), respectively. In the third case,

n°	Simulation Output						σ^2 Prior Distribution	
		CPI_f			$SPI(t)_f$		CPI_f	$SPI(t)_f$
Sim—A	2.35	1.589	1.05	1.525	0.957	0.55	$\alpha = 75$	$\alpha = 75$
							$\beta = 1$	$\beta = 1.5$
Sim—B	2.35	1.612	1.05	1.65	0.976	0.525	$\alpha = 50$	$\alpha = 50$
							$\beta = 0.1$	$\beta = 1$
Sim—C	2.4	1.649	1.1	1.6	0.968	0.525	$\alpha = 60$	$\alpha = 60$
							$\beta = 0.2$	$\beta = 2$
Sim—D	2.4	1.645	1.1	1.525	0.957	0.55	$\alpha = 70$	$\alpha = 70$
							$\beta = 0.5$	$\beta = 1.2$
Sim—E	2.35	1.610	1.075	1.55	0.952	0.55	$\alpha = 70$	$\alpha = 70$
							$\beta = 0.7$	$\beta = 1.5$
Sim—F	2.4	1.644	1.1	1.6	0.966	0.525	$\alpha = 55$	$\alpha = 55$
							$\beta = 0.1$	$\beta = 0.8$

Table 6: Sensiivity analysis results.

Assumptions	Case	CPI_{AC}	$SPI(t)_{AC}$	Index Type/Time Interval
Distinction between WC and WR. The index is applied only to WR.	**1**	1.995	0.581	m
	1	1.649	0.591	3
	1	1.369	0.603	6
	1	1.435	0.996	SCI_m
	1	1.136	0.854	SCI_3
	1	0.922	0.738	SCI_6
Distinction between WC and WR. The index is applied only to WR.	**1—Bayesian model ETC**	1.806	0.993	lower bound
	1—Bayesian model ETC	1.465	0.813	mean
	1—Bayesian model ETC	1.128	0.598	upper bound
No distinction between WR and WC. Index or variance is applied to the overall work.	**2**	1.091	0.831	(variance)
	3—Lipke	2.834	0.863	lower bound
	3—Lipke	1.292	0.678	cum
	3—Lipke	0.589	0.532	upper bound
	4	0.712	0.876	SCI_{cum}
The project will be completed as planned.	**5**	1.000	1.000	(as planned)
Actual performance computing at 96% of physical progress.		1.46	0.92	progress at 96%

Table 7: Comparison of different estimate at completion formulas.

the approach proposed by Lipke (2002b, 2006b) has been applied, allowing for the calculation of a lower and upper bound of the index value.

In comparison with the predicted values of the indices listed in Table 7, the actual performance of the project, related to 96% cumulative physical progress, shows a value of 1.46 for CPI_{AC} and a value of 0.92 for $SPI(t)_{AC}$. Actual project performances are included in the range provided by the Bayesian model and demonstrate its validity and accuracy.

Among the different indices reported in Table 7, the most accurate seems to be SCI_m, hardly ever used in estimate at completion because of its pessimistic bias. SCI_3 and SCI_{cum} appear to be good estimators of schedule performance but do not perform well in cost performance. In general, SCI_m, SCI_3, SCI_6, and SCI_{cum} seem to work quite well in this case because there is a clear trade-off between time and cost performances. This trade-off also has been recognized by subjective judgments, which provide accurate estimates at completion for both cost and schedule, as actual performances are within the model range and, moreover, close to its mean.

The other indices listed in the lower part of Table 7 fail, as they cannot provide an accurate estimate at completion for both cost and schedule. In particular, the Bayesian model provides confidence limits (5% of significance) narrower than the estimation proposed by Lipke's formula for the cost performance, while for the schedule performance the range is very similar. The range reduction is about 70% for cost performance thanks to the improved information obtained from the subjective judgments. Moreover, the range contains many values of CPI_{AC} and $SPI(t)_{AC}$ deriving from traditional formulas. The model proposed by Lipke seems to be accurate only for CPI_{AC}, even if this result can be explained by considering that the range calculated is so large that it is unlikely that a project performance value may lie outside it, but it does not contain the actual value of $SPI(t)_{AC}$.

Conclusion

The use of a Bayesian approach, based on expert opinion elicitation, permits the exploitation of subjective judgments in a rigorous and formal way, leading to an improvement in the accuracy of estimates at completion within the EVM framework. The advantage of the proposed model is the integration of experts' knowledge with the project data records in order to create a future-oriented and more valuable support tool allowing for the improvement of the forecasting process, which in turn determines a corresponding improvement of the decision-making process concerning project control.

The strength of the proposed model is its robustness in every project phase, in particular during the early project phase when data records are few or scarcely reliable. Moreover, it allows for the determination of a confidence interval describing the future scenario the project is going to face.

The application of the Bayesian approach to a project in the oil and gas industry demonstrates its applicability and effectiveness. Furthermore, the research results also suggest that the same approach could be applied to other industries in order to improve the project control process.

The Bayesian model has been translated into a software package allowing for a more user-friendly management of input and output data. Based on the software package, a large-scale plan of testing will be implemented in order to evaluate the accuracy and general applicability of the model. ■

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Franco Caron is a professor with the Management, Economics and Industrial Engineering Department at Politecnico di Milano, and administers the course Management of Large Engineering Projects both in the systems engineering and industrial engineering programs. At MIP-Politecnico, he heads the course on project risk analysis and management, both in the Master in Project Management (MPM) and the Master in Strategic Project Management, developed jointly by MIP-Politecnico di Milano, Heriot Watt University Edinburgh, and UMEA University (Sweden). He is a member of PMI's Northern Italy Chapter.

Fabrizio Ruggeri, who holds a BSc from Milan University, an MSc from Carnegie Mellon, and a PhD from Duke University, is research director at the Italian National Research Council and adjunct faculty at New York University. He is a Fellow of the American Statistical Association, past ENBIS and ISBA president. He is editor-in-chief of *Applied Stochastic Models in Business and Industry* and Wiley's *Encyclopedia of Statistics in Quality and Reliability,* and author of more than 100 papers and several books.

Alessandro Merli holds a master's degree in management and finance from Politecnico di Milano, where he specialized in large engineering projects. His thesis concerned the use of subjective judgments in estimates at completion. He is currently working as a project management consultant at SIA S.p.A. He is currently managing major projects related to the implementation of financial services in the payment card industry.