Charmed Pseudoscalar and Vector Mesons: a Comprehensive QCD Sum-Rule View on Their Decay Constants

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QCD Sum-Rule Applications in a Nutshell

QCD sum rules are relations between features of hadrons (the bound states governed by the strong interactions) and the parameters of their underlying quantum field theory, QCD. Such relations may be established by analyzing vacuum expectation values of non-local products of interpolating operators (in particular, appropriate quark currents) at both QCD and hadron levels; upon application of Wilson's operator product expansion (OPE) for casting at QCD level the non-local products in the shape of series of local operators, contributions of both perturbative and non-perturbative (NP) origin enter: the former are usually represented by dispersion integrals of certain spectral densities while the latter (also called the "power" contributions) involve the vacuum expectation values of the local OPE operators, crucial quantities in this context going under the notion of "vacuum condensates." Performing a Borel transformation (from one's momentum variable to a new variable, the Borel parameter τ) lessens the relevance of hadronic excited and continuum states for such Borelized sum rules and removes subtraction terms. Our lack of knowledge about higher states is dealt with by postulating quark–hadron duality: the contributions of excited and continuum hadronic states roughly cancel against those of perturbative QCD above effective thresholds $s_{\rm eff}(\tau)$.

Decay Constants of Charmed Mesons $D_{(s)}^{(*)}$

To predict the decay constants $f_{\rm P,V}$ of charmed pseudoscalar (P) and vector (V) mesons of mass $M_{\rm P,V}$, regarded as bound states of a charmed quark c of mass m_c and a light quark $q = d, s$ of mass m_q, we use two-point correlators of adequate currents to arrive at QCD sum rules involving spectral densities $\rho^{(\rm P,V)}(s,\mu)$ and non-perturbative terms $\Pi^{(\rm P,V)}_{\rm NP}(\tau,\mu)$ at the renormalization scale of relevance, μ. Pseudoscalar currents yield for pseudoscalar mesons P

$$f_{\rm P}^2\,M_{\rm P}^4\exp\!\left(-M_{\rm P}^2\,\tau\right)=\int\limits_{(m_c+m_q)^2}^{s_{\rm eff}(\tau)}{\rm d}s\,{\rm e}^{-s\,\tau}\rho^{(\rm P)}(s,\mu)+\Pi^{(\rm P)}_{\rm NP}(\tau,\mu)\equiv\widetilde\Pi_{\rm P}(\tau,s_{\rm eff}(\tau))\ .$$

The dual correlator $\widetilde\Pi_{\rm P}(\tau,s_{\rm eff}(\tau))$ defines dual masses and decay constants:

$$M^2_{\rm dual}(\tau)\equiv-\frac{\rm d}{{\rm d}\tau}\log\widetilde\Pi_{\rm P}(\tau,s_{\rm eff}(\tau))\ ,\qquad f^2_{\rm dual}(\tau)\equiv\frac{{\rm e}^{M_{\rm P}^2\,\tau}}{M_{\rm P}^4}\,\widetilde\Pi_{\rm P}(\tau,s_{\rm eff}(\tau))\ .$$

Starting from vector currents leads to similar relations for vector mesons V:

$$f_{\rm V}^2\,M_{\rm V}^2\exp\!\left(-M_{\rm V}^2\,\tau\right)=\int\limits_{(m_c+m_q)^2}^{s_{\rm eff}(\tau)}{\rm d}s\,{\rm e}^{-s\,\tau}\rho^{(\rm V)}(s,\mu)+\Pi^{(\rm V)}_{\rm NP}(\tau,\mu)\equiv\widetilde\Pi_{\rm V}(\tau,s_{\rm eff}(\tau))\ ,$$

$$M^2_{\rm dual}(\tau)\equiv-\frac{\rm d}{{\rm d}\tau}\log\widetilde\Pi_{\rm V}(\tau,s_{\rm eff}(\tau))\ ,\qquad f^2_{\rm dual}(\tau)\equiv\frac{{\rm e}^{M_{\rm V}^2\,\tau}}{M_{\rm V}^2}\,\widetilde\Pi_{\rm V}(\tau,s_{\rm eff}(\tau))\ .$$

Numerical parameter values adopted as input to the charmed-meson OPEs:

Quantity	Numerical input value
$\overline{m}_d(2\ {\rm GeV})$	(3.42 ± 0.09) MeV
$\overline{m}_s(2\ {\rm GeV})$	(93.8 ± 2.4) MeV
$\overline{m}_c(\overline{m}_c)$	(1275 ± 25) MeV
$\alpha_{\rm s}(M_Z)$	0.1184 ± 0.0020
$\langle\bar qq\rangle(2\ {\rm GeV})$	$-[(267\pm17)\ {\rm MeV}]^3$
$\langle\bar ss\rangle(2\ {\rm GeV})$	$(0.8\pm0.3)\times\langle\bar qq\rangle(2\ {\rm GeV})$
$\left\langle\frac{\alpha_{\rm s}}{\pi}GG\right\rangle$	$(0.024\pm0.012)\ {\rm GeV}^4$

Advanced Extraction of Hadron Features

The accuracy of QCD sum-rule predictions for meson observables extracted by the traditional technique may be greatly improved by dropping both the requirement of Borel stability [1], reflecting one's hope that the value of such observable at an extremum in τ is a good approximation to its actual value, and the perhaps too naïve belief that the QCD-level effective threshold does not know about τ [2]: earlier analyses [1] (backed up by quantum mechanics, where exact solutions can be derived by just solving Schrödinger equations) forced us to conclude that predictions relying on Borel stability may emerge rather far from the truth and that effective thresholds do depend on τ,[1] and culminated in a simple prescription [2] for the extraction of hadron features:

- The admissible τ range is determined by requiring, at its lower end, the ground-state contribution to be sufficiently large and, at the upper end, the contribution of nonperturbative corrections to be reasonably small. In the case of the charmed pseudoscalar [3] and vector [4] mesons, these demands can be satisfied if choosing for the Borel windows the intervals

$$\begin{aligned} 0.1\ \text{GeV}^{-2} < \tau < 0.5\ \text{GeV}^{-2} \qquad & \text{for} \quad D,\ D^*,\ D_s^*\ , \\ 0.1\ \text{GeV}^{-2} < \tau < 0.6\ \text{GeV}^{-2} \qquad & \text{for} \quad D_s\ . \end{aligned}$$

- The threshold function $s_{\text{eff}}(\tau)$ is found by adopting a power-law Ansatz

$$s_{\text{eff}}^{(n)}(\tau) = \sum_{j=0}^{n} s_j\, \tau^j\ ,$$

  with expansion coefficients s_j determined by minimizing the expression

$$\chi^2 \equiv \frac{1}{N} \sum_{i=1}^{N} \left[M_{\text{dual}}^2(\tau_i) - M_{\text{P,V}}^2\right]^2$$

  over a set of N equidistant discrete points τ_i in the allowable range of τ.

- The spread of results for $n = 1, 2, 3$ yields their intrinsic sum-rule error.

This systematic error is subject to at least two effects demanding attention: optimal perturbative behaviour and fake impact of renormalization scale μ.

[1] Rebutting a suspicion expressed some years ago [5], we do not assume the effective continuum threshold in QCD sum rules to depend also on the cosmological constant.

Issue: Maximal Perturbative Convergence

Perturbatively, the coefficients of the local operators in the OPE are derived as series in powers of the strong coupling $\alpha_s(\mu)$; the one of the unit operator ends up in the spectral density, presently determined to three-loop order [6]:

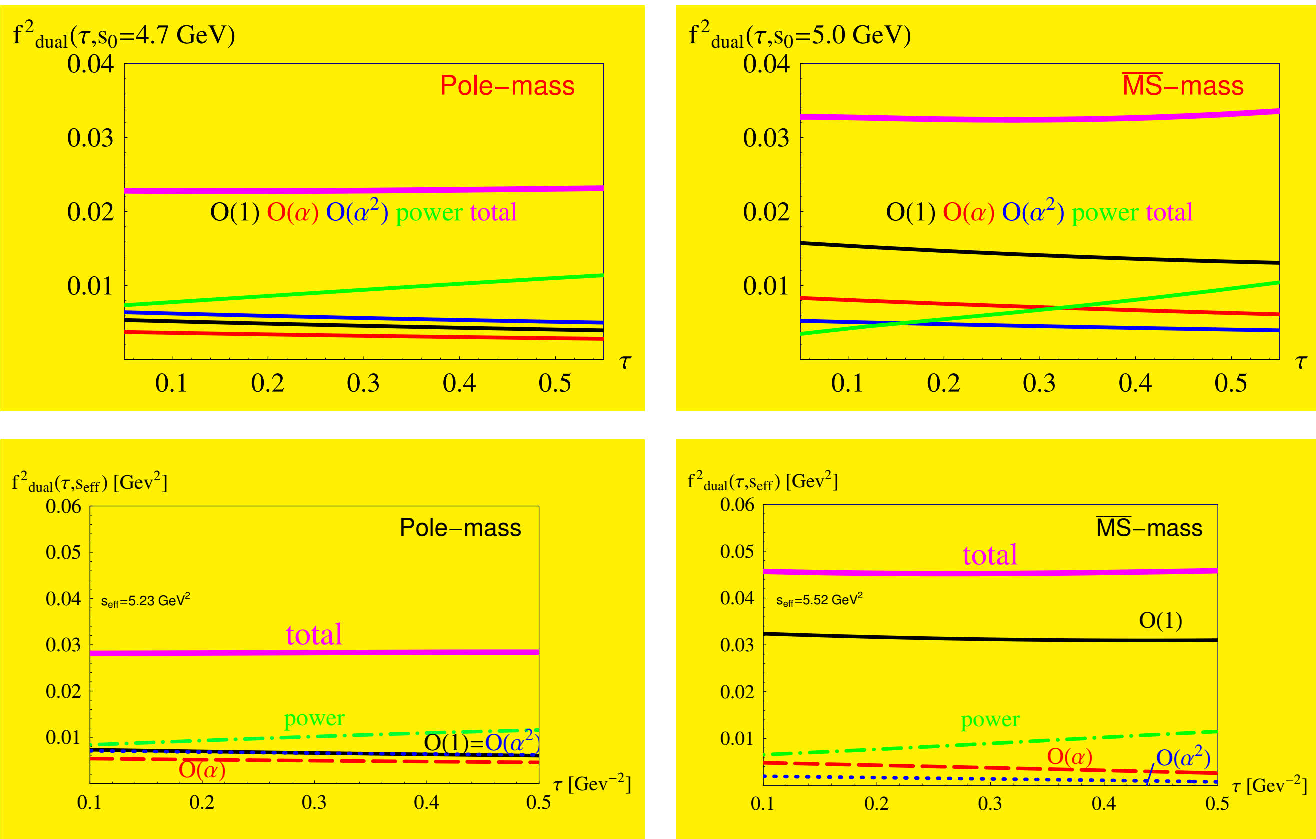
$$\rho(s, m_c, \mu) = \rho_0(s, m_c) + \frac{\alpha_s(\mu)}{\pi}\,\rho_1(s, m_c) + \frac{\alpha_s^2(\mu)}{\pi^2}\,\rho_2(s, m_c, \mu) + \cdots \ .$$

The rate of convergence of this expansion is sensitive to the renormalization scheme defining the c-quark mass. In this respect, adopting the $\overline{\mathrm{MS}}$ running mass, $m_c = \overline{m}_c(\overline{m}_c) = (1275 \pm 25)$ MeV, is superior to using the pole mass, $m_c = \mathring{m}_c = 1699$ MeV, both related in terms of given [7] expressions $r_{1,2}$ by

$$\overline{m}_c(\mu) = \mathring{m}_c \left(1 + \frac{\alpha_s(\mu)}{\pi}\,r_1 + \frac{\alpha_s^2(\mu)}{\pi^2}\,r_2 + \cdots\right).$$

The gain in perturbative credibility is evident, and larger for vector mesons.

Hierarchy of the OPE contributions to the dual decay constants $f_{\rm dual}(\tau)$ for both charmed pseudoscalar meson D (top row) and charmed vector meson D^* (bottom row), obtained in the pole-mass (left column) and the $\overline{\mathrm{MS}}$-mass (right column) renormalization scheme (for fixed threshold s_0 or $s_{\rm eff}$, resp.):



Issue: Renormalization-Scale Dependence

Needless to say, correlation functions do not depend on any renormalization scale(s) μ. However, due to practically inevitable truncations to finite-order perturbative expansions or to finite-dimensional vacuum condensates, both spectral densities and power contributions, and thus the predicted hadronic features, do. Defining the average values $\overline{\mu}$ of the renormalization scale μ by $f_{\rm dual}(\overline{\mu}) = \langle f_{\rm dual}(\mu)\rangle$, such unphysical decay-constant sensitivity to μ turns out to be more pronounced for the vector than for the pseudoscalar mesons:

$$f_D(\mu) = 208.3\ \text{MeV}\left(1 + 0.06\log\frac{\mu}{\overline{\mu}} - 0.11\log^2\frac{\mu}{\overline{\mu}} + 0.08\log^3\frac{\mu}{\overline{\mu}}\right),$$

$$f_{D_s}(\mu) = 246.0\ \text{MeV}\left(1 + 0.01\log\frac{\mu}{\overline{\mu}} - 0.03\log^2\frac{\mu}{\overline{\mu}} + 0.04\log^3\frac{\mu}{\overline{\mu}}\right),$$

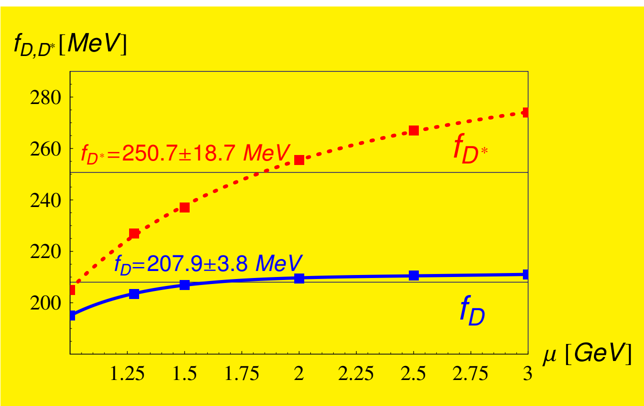
$$f_{D^*}(\mu) = 252.2\ \text{MeV}\left(1 + 0.233\log\frac{\mu}{\overline{\mu}} - 0.096\log^2\frac{\mu}{\overline{\mu}} + 0.17\log^3\frac{\mu}{\overline{\mu}}\right),$$

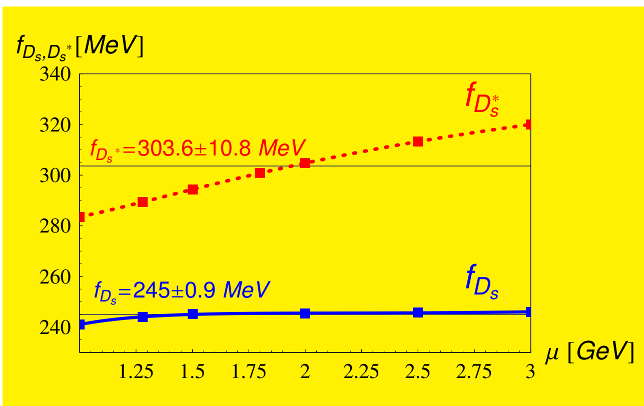
$$f_{D_s^*}(\mu) = 305.5\ \text{MeV}\left(1 + 0.124\log\frac{\mu}{\overline{\mu}} + 0.014\log^2\frac{\mu}{\overline{\mu}} - 0.034\log^3\frac{\mu}{\overline{\mu}}\right);$$

the averages are a bit larger for the vector than for the pseudoscalar mesons:

Meson	D	D_s	D^*	D_s^*
$\overline{\mu}$ (GeV)	1.62	1.52	1.84	1.94

Dependences on the renormalization scale, μ, of our QCD sum-rule findings for the dual decay constants of the charmed, non-strange mesons D and D^* ($f_{D^{(*)}}$, left), as well as the charmed, strange mesons D_s and D_s^* ($f_{D_s^{(*)}}$, right)





Observations, Outcomes, and Conclusions

The simultaneous scrutiny of QCD sum-rule predictions for charmed vector and pseudoscalar mesons [3,4] discloses similarities as well as dissimilarities:

- With respect to the perturbative features of the extraction procedures, both types of mesons certainly prefer the use of the $\overline{\text{MS}}$-mass definition; the advantage of this is clearer for vector than for pseudoscalar mesons.
- For both kinds of mesons, the decay-constant predictions relying on the $\overline{\text{MS}}$ mass are significantly higher than those arising from the pole mass.
- While the pseudoscalar mesons do not seem to care too much about the renormalization scale μ, its impact on the vector mesons dominates the OPE-related errors of their decay constants, as our final findings reveal:

$$
\begin{aligned}
f_D &= (206.2 \pm 7.3_{\text{OPE}} \pm 5.1_{\text{syst}})\ \text{MeV}\ ,\\
f_{D_s} &= (245.3 \pm 15.7_{\text{OPE}} \pm 4.5_{\text{syst}})\ \text{MeV}\ ,\\
f_{D^*} &= (252.2 \pm 22.3_{\text{OPE}} \pm 4_{\text{syst}})\ \text{MeV}\ ,\\
f_{D_s^*} &= (305.5 \pm 26.8_{\text{OPE}} \pm 5_{\text{syst}})\ \text{MeV}\ .
\end{aligned}
$$

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[5] Sorry, no names, only flat space-time coordinates: Martina Franca, 22 June 2010. (Non-scientific arguments call for non-scientific replies.)

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