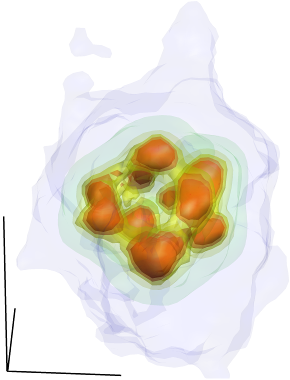
Spinodal instability growth in new stochastic approaches for the dynamics of the nuclear bulk

P. Napolitani (*IPN Orsay*), M. Colonna (*INFN-LNS*),
V. de la Mota and F. Sebille (*Subatech Nantes*)

- Spinodal multifragmentation : process driven by *mechanical instabilities*
- Since long advocated as a possible mechanism in central collisions at Fermi energy
- How to describe it quantitatively through a mean-field approach ?
- Requirements / implementation / simulations...



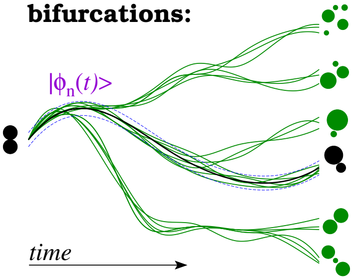
Fermionic transport with *fluctuations* in one-body dynamics

• *bulk* properties → the route we take is a *one-body* description

• But *Pure mean-field equations are not valid in regions where instabilities, bifurcations, chaos are present*

⇒ Transport description based on a *one-body H* supplemented by a contribution to introduce N-body unknown correlations

• At the ETDHF level : R.Balian, T.Alhassid, H.Reinhard, Phys.Rep131(1986)1 ; E.Suraud, P-G.Reinhard, Ann.Phys.216(1992)98



Stochastic *DYnamics of WAvelets in Nuclei*

→ a possible realization through DYWAN :

• System wave functions $|\varphi_\lambda\rangle$ projected on an orthogonal wavelet basis $|\alpha_i^\lambda\rangle$

$$\rho = n_\lambda \sum_\lambda |\varphi_\lambda\rangle\langle\varphi_\lambda| = \sum_\lambda \sum_{ij} \beta_{ij}^\lambda |\alpha_i^\lambda\rangle\langle\alpha_i^\lambda|$$

(B.Jouault, F.Sébille, V.deLaMota, NPA268(1998)119)

• *Mean field* : wavelet properties changed according to the one body density matrix evolution :

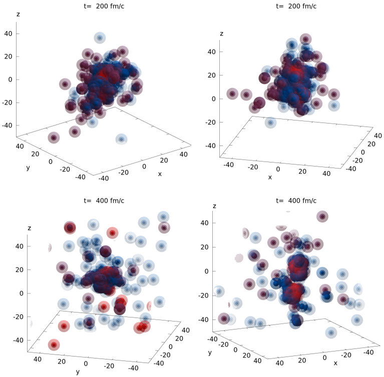
$$i\frac{d}{dt}\rho = [W(\rho), \rho]$$

• *Collisions* : Wavelet weights changed according to master equation for the occupancy rates.

• *Fluctuations* : Wavelet weights redefined by splitting the mean field in mean-field sub-ensembles.

^{136}Xe+^{124}Sn at 45A MeV, b = 2fm, two events at 200 and 400 fm/c :

variety of final configurations :



Boltzmann-Langevin Equation (*BLE*) for fermionic systems

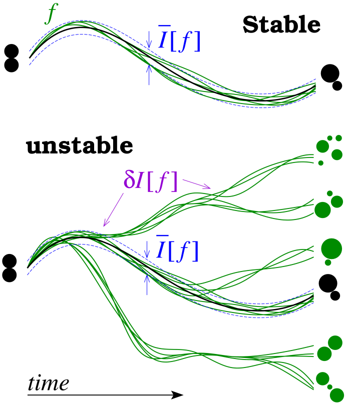
Reduction in terms of one-body distribution functions $f(\mathbf{r},\mathbf{p},t) \rightarrow$ BLE :

$\dot{f} = \partial_t f - \{H[f], f\} = \bar{I}[f] + \delta I[f]$ $\rightarrow$ t-evolution of f in response to :

1) effective Hamiltonian $H[f]$
$\rightarrow$ l.h.s gives the Vlasov evolution for f in its own self-consist. MF

2) average Boltzmann hard two-body collision integral $\bar{I}[f]$
$\rightarrow$ mean number of transitions $d\nu_{ab\rightarrow cd}$

3) unknown N-body correlations expressed in terms of one-body f
$\rightarrow$ fluctuating term of Markovian type, defined through its correlations :



$\langle \delta I(\mathbf{r},\mathbf{p},t)\delta I(\mathbf{r}',\mathbf{p}',t')\rangle = 2D(\mathbf{r},\mathbf{p};\mathbf{r}',\mathbf{p}',t')\delta(t-t')$,
D : *diffusion coeff* related to $d\nu$
$\rightarrow$ δI acts as a dissipating force but conserves single-part. energies

P.Chomaz, M.Colonna, J.Randrup Phys.Rep.389(2004)263

BLE in *one-body* approaches: attempts, limits, recommendations

Some representative cases :

- *Stochastic initial conditions*

M.Colonna et al. PRC47 (1993) 1395, ...

- *Projection of B-L noise on a subspace* (ρ)

BOB (→ from a Brownian force Ph.Chomaz et al. PRL73 (1994) 3512, A.Guarnera et al. PLB403 (1997) 191), *SMF* (→ from Fermi gas kin. equil. fluctuations or from numerical noise M.Colonna et al. NPA642 (1998) 449) , ...

- *Fluctuations in full phase-space*

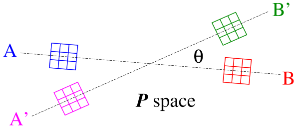
A first attempt by W.Bauer, G.F.Bertsch and S.Das Gupta PRL58 (1987) 863 :
If two test part. i, j collide successfully, the same scattering applies to two agglomerates of N_{test} test part. taken around i and j.
Transition probability divided by N_{test}.

- Criticism to Bauer-et-al method :

F.Chapelle, G.F.Burgio, Ph.Chomaz, J.Randrup NPA540 (1992) 227 : Schematic Pauli blocking implies severe effects on the development of the fluctuation amplitude in phase space
⇒ Boltzmann statistics.

- Rizzo-et-al solution in unif. matter :

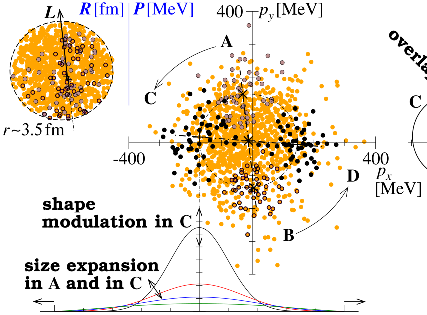
J.Rizzo, M.Colonna, Ph.Chomaz NPA806 (2008)40 : Bauer-et-al method reformulated by imposing strict Pauli blocking for the whole swarm of test particles involved in the scattering.
⇒ correct Fermi statistics (average values, variances).

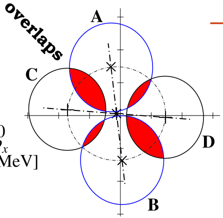


Solution of the BLE in BLOB (P.Napolitani, M.Colonna, PLB726(2013)382)

At *a given time t*,
in $(\mathbf{r}_a, \mathbf{p}_a)$,
for *elastic coll.* :

$$\dot{f}_a(\mathbf{r}_a, \mathbf{p}_a) = g \int \frac{d\mathbf{p}_b}{h^3} \int d\Omega \, W(AB \leftrightarrow CD) \, F(AB \rightarrow CD)$$





- *"nucleon wave packets"* → → *phase-space agglomerates* of N_{test} test-particles of equal isospin $(a \in A, b \in B \ldots)$
- at each Δt : all phase space is scanned for collisions and all agglomerates are redefined

trans. rate : $W(AB \leftrightarrow CD) = \left\langle |v_a - v_b| \frac{d\sigma}{d\Omega} \right\rangle_{AB \rightarrow CD} = \left\langle W(ab \leftrightarrow cd) \right\rangle_{AB \rightarrow CD}$

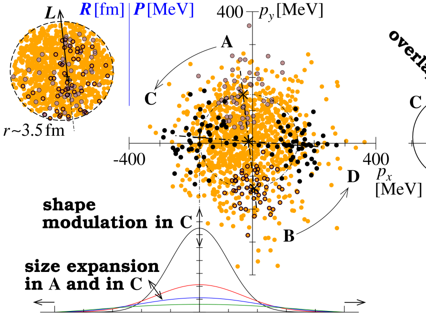
occupancy : $F(AB \rightarrow CD) = \bar{f}_A \bar{f}_B f_C f_D - f_A f_B \bar{f}_C \bar{f}_D = \left\langle F(ab \rightarrow cd) \right\rangle_{AB \rightarrow CD}$

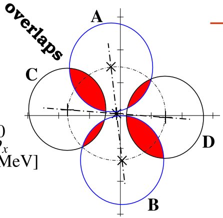
⇒ The above scheme introduces *N-N correlations*

Solution of the BLE in BLOB (P.Napolitani, M.Colonna, PLB726(2013)382)

At *a given time t*, in $(\mathbf{r}_a, \mathbf{p}_a)$, for *elastic coll.* :

$$\dot{f}_a(\mathbf{r}_a, \mathbf{p}_a) = g \int \frac{\mathrm{d}\mathbf{p}_b}{h^3} \int \mathrm{d}\Omega \; W(\mathrm{AB} \leftrightarrow \mathrm{CD}) \, F(\mathrm{AB} \rightarrow \mathrm{CD})$$





- *"nucleon wave packets"* → → *phase-space agglomerates* of N_{test} test-particles of equal isospin $(a \in A, b \in B \ldots)$
- at each Δt : all phase space is scanned for collisions and all agglomerates are redefined

Then, to introduce *phase-space fluctuations* (P.N, M.C, EPJ Web of Conf.31,00027) :

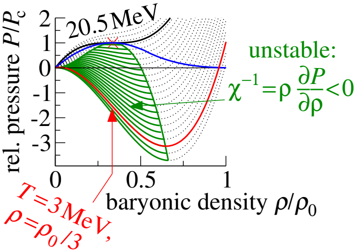
- Random sorting of the effective collision probability $W \times F$ *accounting for the extension of initial distributions and overlap geometry*
- modulation functions adapt final states to the vacancy profile
- the most compact shape chosen according to energy conservation.

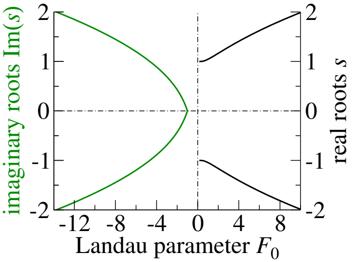
Propagation of density waves in Fermi liquids

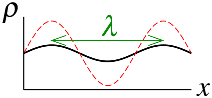
Aim : transport model for HIC which develops fluctuations spontaneously and with the correct dispersion-relation characteristics.

"Laboratory" : the mechanically unstable region of the EOS (e.g. $T' \approx 3MeV, \rho' \approx \rho/3$)
$\rightarrow$ negative incompressibility χ^{-1}

$\Rightarrow$ we get imaginary solutions of the *dispersion relation*, (T=0 form) :
$\frac{s}{2} ln\left(\frac{s+1}{s-1}\right) = 1 + \frac{1}{F_0}$,
with the *Landau parameter*
$F_0(\mathbf{k}) = \partial_\rho U(\mathbf{k}) \frac{3\rho}{2\epsilon_F}$ $\rightarrow$ amplification of ρ disturbance (i.e. undulation of wavelength λ and wave number k)
[M.Colonna,Ph.Chomaz,PRC49(1994)1908]







Dispersion relation

Ideal hydrodynamical behaviour

The more matter is to be relocated, the longer it takes (i.e. the larger is the *characteristic time* Γ)
$\Rightarrow$ *Growth rate* $\hbar/\Gamma \to 0$ for $k \to 0$ (i.e. for large λ)

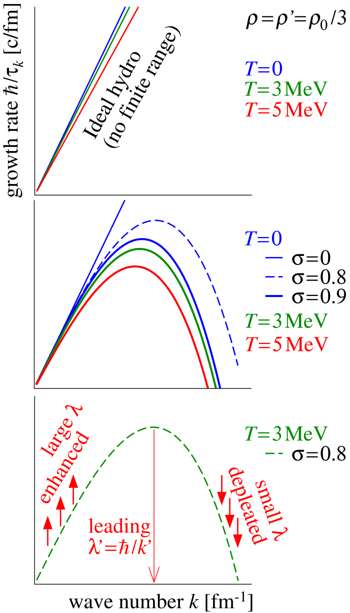
Finite range, all k modes decoupled

Finite range ($F_0 \to g(k)F_0$) excludes short λ
$\Rightarrow \hbar/\Gamma \to 0$ for $k \to k_{max}(\gamma, T, \rho)$

Finite range, coupled k modes

Small λ compose into large λ
$\Rightarrow$ large k depleted, small k enhanced

Ph.Chomaz,M.Colonna,J.Randrup Phys.Rep389,263

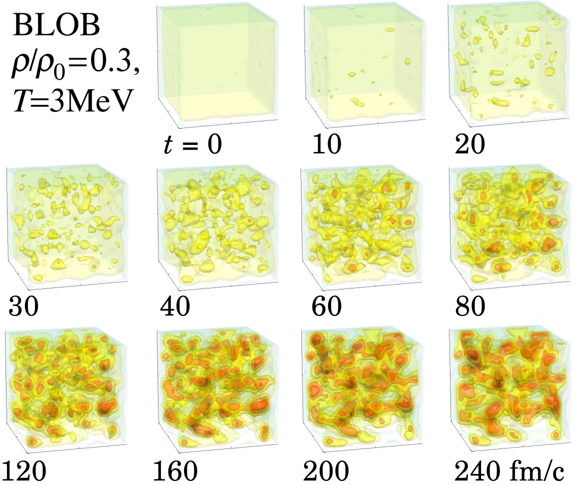


BLOB in nuclear matter

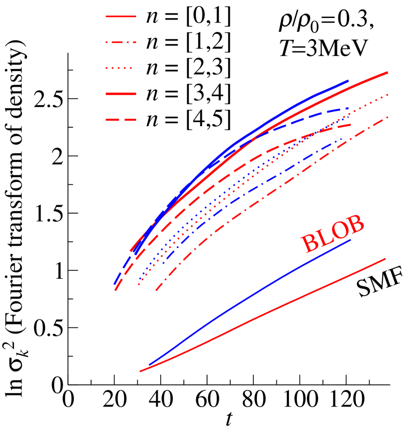
Calculation in a periodic box of 39fm :

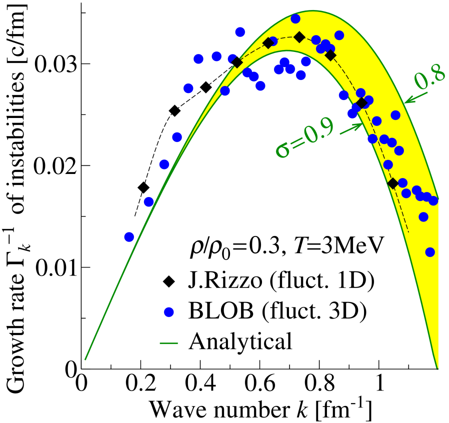
- 1584 n and 1584 p
- 40 test part. per nucleon

- tested in a unif. syst. at T, ρ', corresp. to the leading wave k'
→ BL term agitates ρ profile over several k waves spontaneously



Mean field response in presence of instabilities





characteristic wavelength $\lambda=\hbar/k$

disper

Compares well with analytical relation in linear response regime :

$\tau_k^{-1} = f(k, \chi, \rho', T, \sigma)$ σ : smearing, (*all k decoupled*)

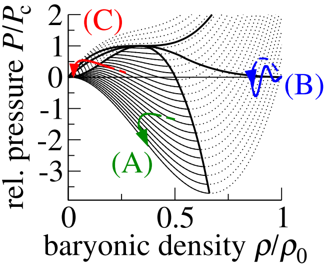
- Difficulty : (*small λ combine into large λ*)

One-body fragment phenomenology in a finite system

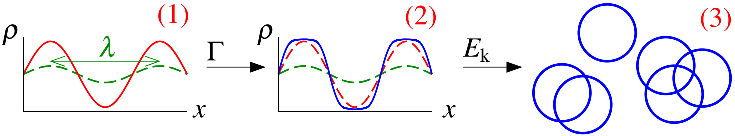
A) *Inside* of the spinodal ($T' \approx 3 MeV$, $\rho' \approx \rho/3$) :

1. Fluctuation seeds initiate the process : the *leading disturbance* λ' is the wavelength with the *largest growth rate* :
2. blobs of size $A' \approx \rho' \lambda'^3$ develop.

3a. They may separate into fragments of size $A' \approx$ Neon, Oxygen, if radial expansion is large enough.



3b. Otherwise they coalesce giving fragments of size $> A'$.

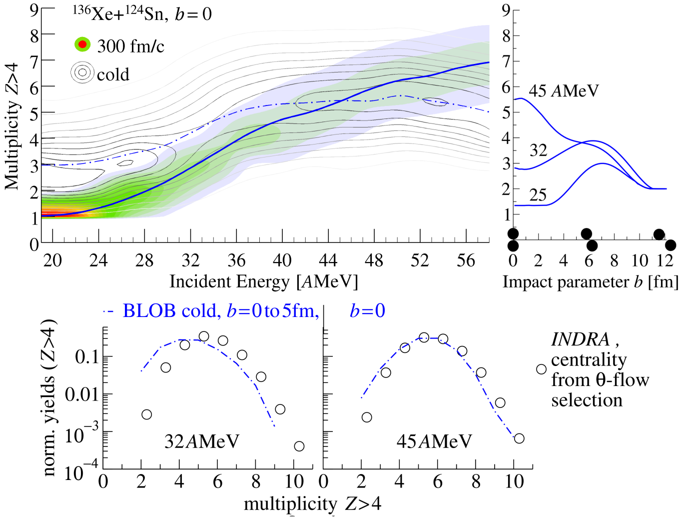


B,C) *Outside* of the spinodal region : the system is either

- either not enough excited and diluted → *damped dynamics* → *C.N.*
- or very hot and very diluted → *vaporisation* → clusters beyond MF

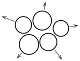
BLOB in nuclei

Competition between instability growth and MF resilience determines the fragmentation thresholds :



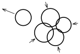
Data from INDRA : PRC86(2012)044617, arXiv :1310.5000(2013)

Instability growth versus mean-field resilience



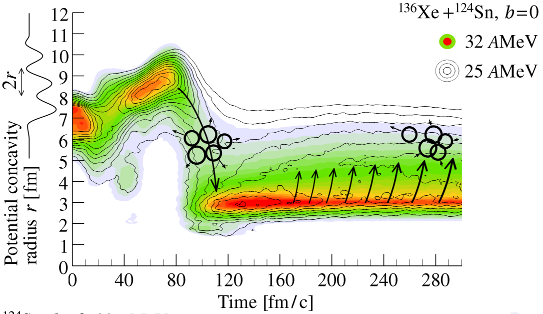
instability growth :

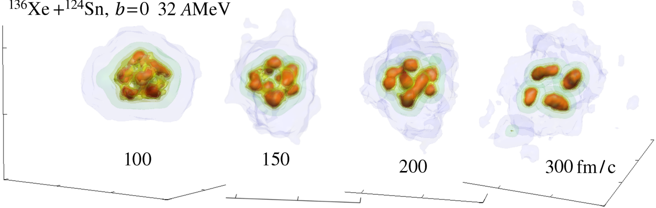
- → many ρ blobs of comparable size $A_{frag} \approx \rho(\lambda')^3$ ($\approx$ region of O, Ne)



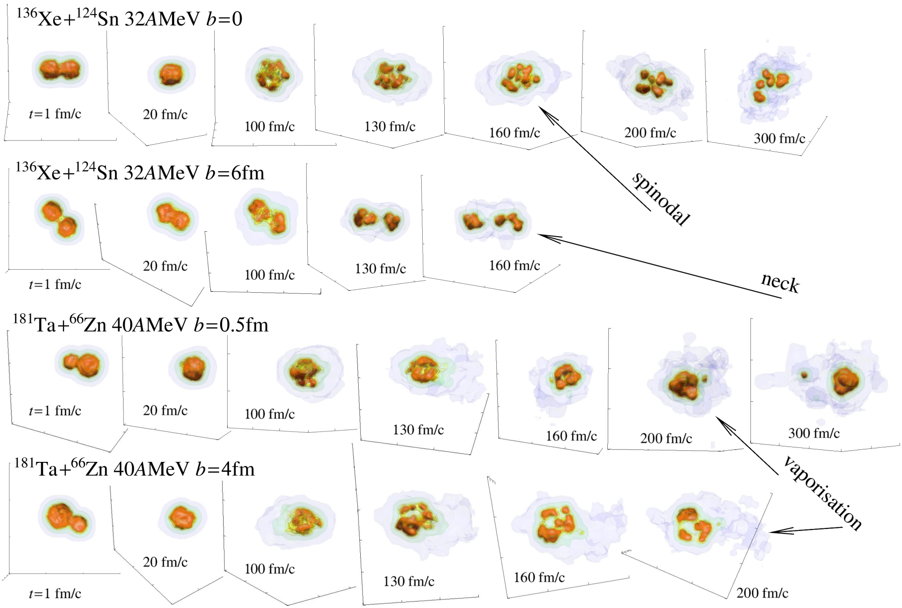
partial coalescence :

- → few fragments of various size asymmetries



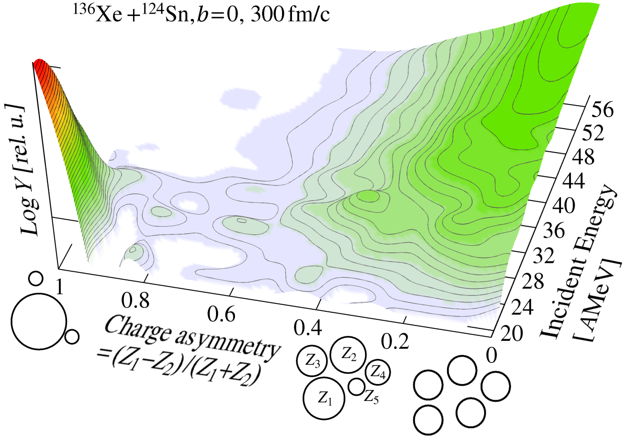


Some examples of fragment production in BLOB



Bimodality

Same macroscopic initial conditions → E fluctuations
→ oscillation between two energetically favoured configurations
→ 1^{st}o. phase trans. features found as a result of fragm. dynamics



→ *bimodality* in fragm. observables at b = 0 at rather small energy
(so far not investigated experimentally in these conditions)

Conclusions

Fluctuations :

• Lead to a variety of channels in dissipative collisions → bifurcations

• feed spinodal instability growing → important for fragment production at $\rho\ 1/3\rho_0$

Models :

• *BLE* → correct description of dispersion relation, fluctuations originate spontaneously, 3D.

• *ETDHF* → Fully stochastic mean-field approach on a wavelet basis