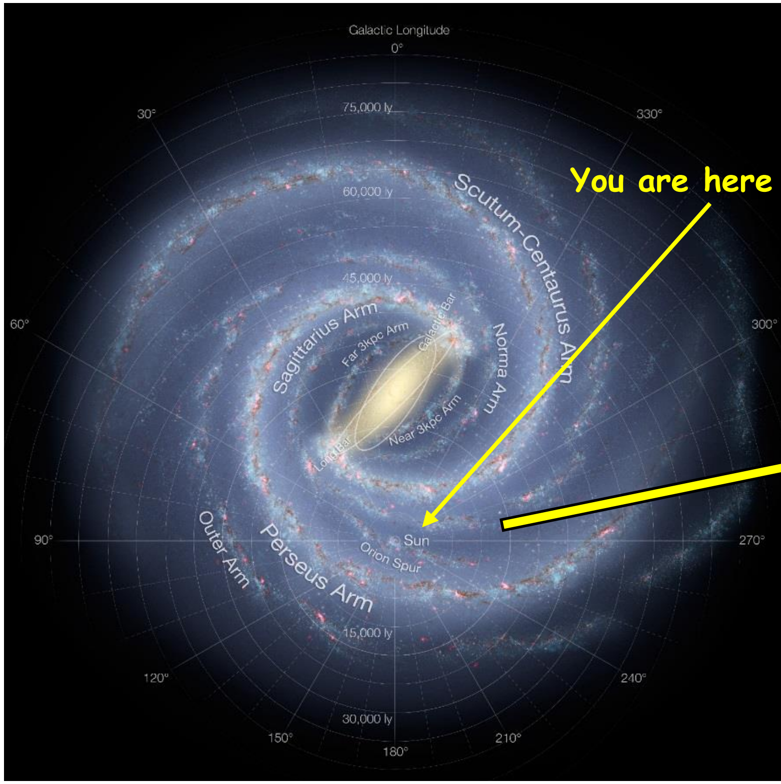
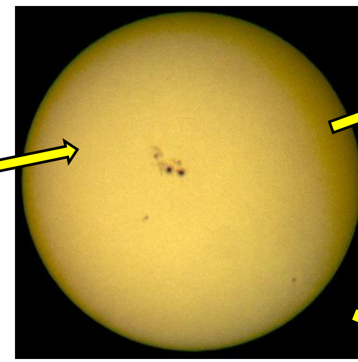
Modeling the solar interior: a not so easy task!

Sketch of the Milky Way disc



≈ 150.000 light years



(https://commons.wikimedia.org)

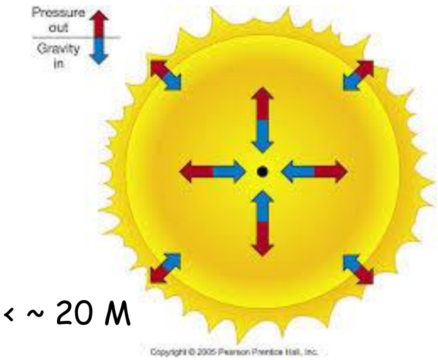
Solar telescope
(Wikipedia.org)

Gran Sasso National Laboratories
(https://www.lngs.infn.it)

Scilla Degl'Innocenti, Physics Department, Pisa University

Fest for Gianni and Dalpiaz², October 8-9, 2018

The stars in few words



- **Hydrostatic equilibrium**: stars are gaseous systems in equilibrium between the pressure (gas+radiation pressure) and the gravitational force → the equilibrium configuration is a sphere

  M ↑ T ↑ L ↑

  $$\frac{L}{L_\odot} \propto \left(\frac{M}{M_\odot}\right)^\alpha$$ $\alpha \approx 2\div4$ for $M < \sim 20$ M

- **Thermodynamic equilibrium** between matter and radiation

  ⟶ relation among luminosity (L), Radius (R) and surface (effective) temperature (T_e): $L=4\pi R^2\sigma T_e^4$

- ***An equation of state (EOS)*** is needed

- **Thermal equilibrium:** the amount of energy per unit time which exits from a given spherical region of infinitesimal thickness (shell) direct outward is equal to the amount of energy which enters in the shell plus the energy possibly produced in the shell itself

  $$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$ ε = energy production per unit mass and time = $\varepsilon_{nucl} + \varepsilon_{grav} - \varepsilon_\nu$

- **For the most of their life stars are powered by nuclear fusions**

Solar luminosity: $L_\odot \approx 3.83\times10^{33}$ erg/s   Solar mass: $M_\odot \approx 1.989\times10^{33}$ g

- Stars radiate their energy to space from the surface

- Systems with feedback: nuclear reaction efficiency exactly compensates radiation losses from the surface

- Energy transport mechanisms:

✓ Temperature gradient → radiative transport

✓ convection or electronic conduction, can be present under specific conditions

Stellar structure equations

FUNDAMENTAL STELLAR STRUCTURE EQUATIONS (FSSE) IN TIME-INDEPENDENT (STATIC) FORM

$$\frac{dP}{dr} = -G\frac{M_r \rho}{r^2}$$ HYDROSTATIC EQUILIBRIUM

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$ MASS CONSERVATION

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$$ ENERGY EQUATION

$$\left.\frac{dT}{dr}\right|_{rad} = -\frac{3}{4ac}\frac{\overline{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$ RADIATIVE TRANSPORT

$$\left.\frac{dT}{dr}\right|_{\mathrm{ad}} = -\left(1-\frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2}$$ ADIABATIC CONVECTION

These equations are solved to obtain the physical quantities and the chemical abundances point by point inside the star

opacity, $\bar{k}$: the sum of all the mechanisms of photon-matter interaction which remove energy from the outgoing flux, averaged over the photon frequency distribution.
The photon mean free path λ_γ depends on the stellar opacity and density ϱ: $\lambda_\gamma = \bar{k}\varrho$

..but there is the need to evaluate

CONSTITUTIVE RELATIONS (CR)

Equation of state
$$P = \frac{\rho k T}{\mu m_{\mathrm{H}}} + \frac{1}{3}aT^4$$

Opacity
$$\overline{\kappa} = \begin{cases} \overline{\kappa}_{bf} = \text{bound-free} \\ \overline{\kappa}_{ff} = \text{free-free} \\ \overline{\kappa}_{es} = \text{electron scattering} \end{cases}$$
FROM TABLES OR FITTED TO A FUNCTION

$$\epsilon = \varepsilon_{\mathrm{nucl}} + \varepsilon_{\mathrm{grav}} - \varepsilon_{\nu}$$ Energy generation

(Figures from https://quantumredpill.files.wordpress.com)

Stellar evolution

- Stellar evolution is driven (in the most of evolutionary phases) by the variation of the chemical composition (mainly due to the nuclear burnings)

  e.g. during the central H burning phase (main sequence) the evolution depends on the decreasing of H abundance in central regions

The chemical composition variation with time is evaluated and stellar structure equations are solved at a following time step for the corresponding new chemical composition

$\Downarrow$

Internal structure and observables quantities throughout the stellar lifetime

Main observables: electromagnetic energy flux and surface color $\Rightarrow$ luminosity and (surface) effective temperature

+ (for the Sun) neutrinos

Stellar models are the results of calculations relying on:

- **Input physics:** EOS, radiative and conductive opacities, nuclear reaction rates, neutrino emission rates, element diffusion efficiency etc.

- **Chemical composition:** initial helium, Y, and metal*, Z, fractional abundance by mass, heavy – elements mixture, etc...

Each of these ingredients is affected by not negligible **errors**

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Stellar models are still affected by significant **uncertainties**

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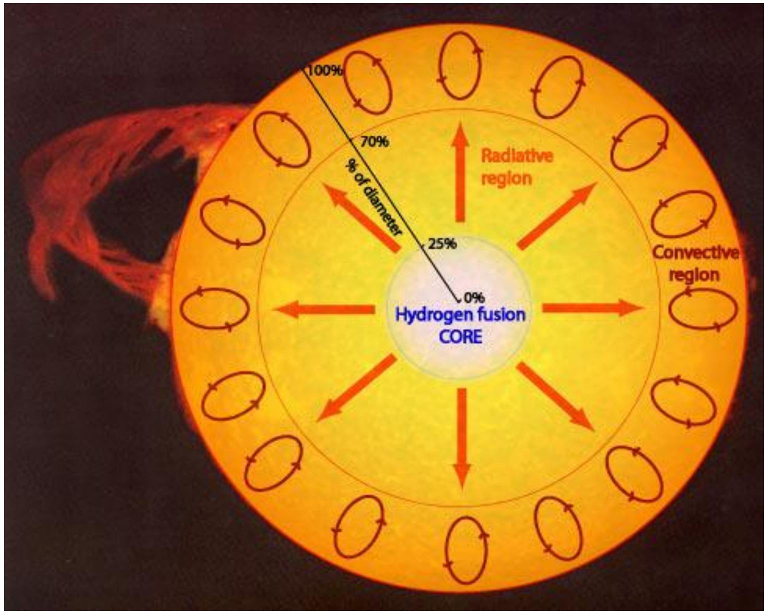
* In astrophysics all the elements heavier than helium are called "metals"

The Sun "identity card"



(Figures from http://astronomy.ohio-state.edu)

General characteristics*

Radius ~ $(6.9566 \pm 0.001)\ 10^5$ km
(~110 times Earth)

Mass ~ $(1.98855 \pm 0.00024)\ 10^{30}$ kg
(~333000 times Earth)

Surface temp. ~ 5771.8 ± 0.7 K

Age ~ (4.572 ± 0.004) Gyr

Rotation Period =
24.9 days (equator)
29.8 days (poles)

Internal structure

Nuclear fusion central region: $R \lesssim 0.2\ R_\odot$

Radiative zone: $0.2 \lesssim R/R_\odot \lesssim 0.71$

Convective zone: $R \gtrsim 0.71\ R_\odot$

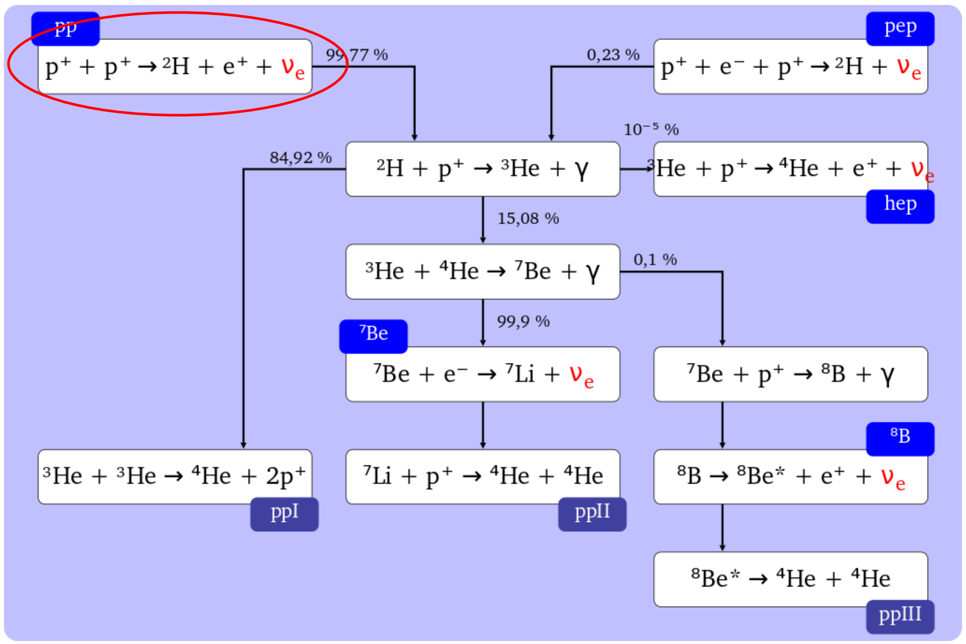
Photosphere: deepest layer of the Sun that we can directly observe

+ helioseismic observables, neutrino fluxes, surface chemical composition
..see later..

*Data from the 2015 IAU resolutions B2 and B3

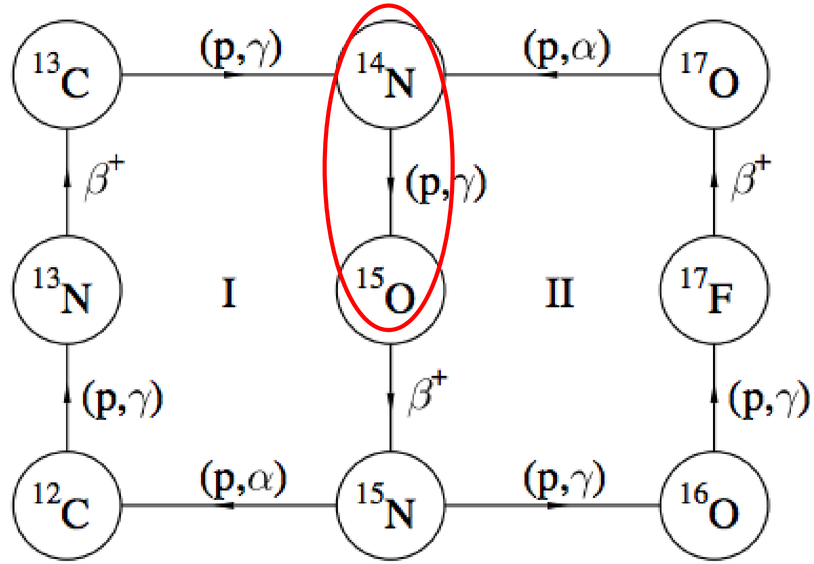
Hydrogen burning reactions

Proton proton chain



Dorottya Szam https://commons.wikimedia.org

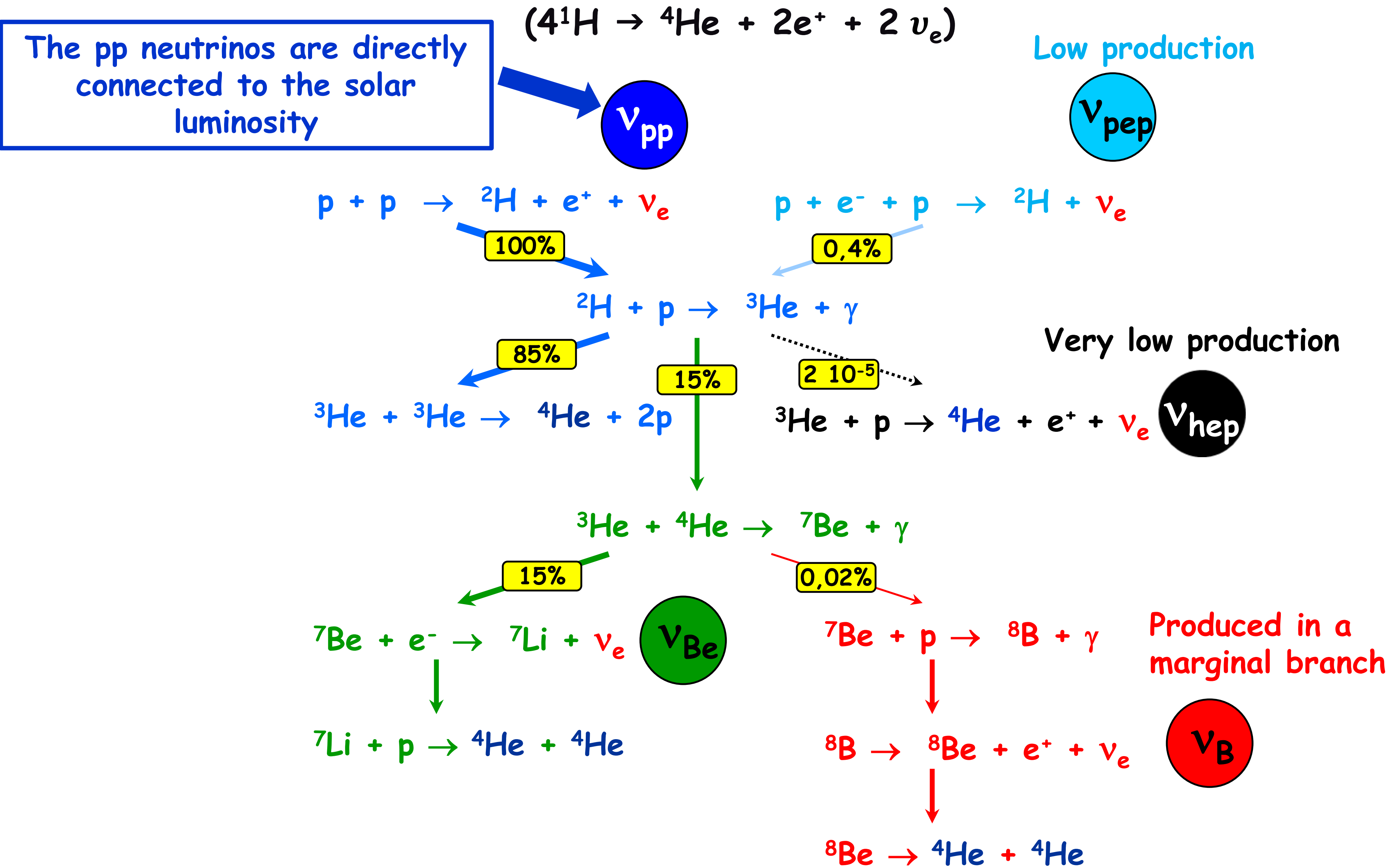
CN-NO bicycle



In the Sun H burning through the proton-proton chain highly dominates

In each case: $4\,{}^1H \to {}^4He + 2e^+ + 2\upsilon_e$ ($Q \approx 26.7$ MeV)

Solar neutrino production: the proton proton chain
(highly dominant ~ 98.5% of the energy production)



(Adapted from D. Vignaud at Neutrinos at the forefront of particle physics and astrophysics, 2012)

CN-NO by-cycle

(disfavoured ~ 1.5% of the solar energy production)

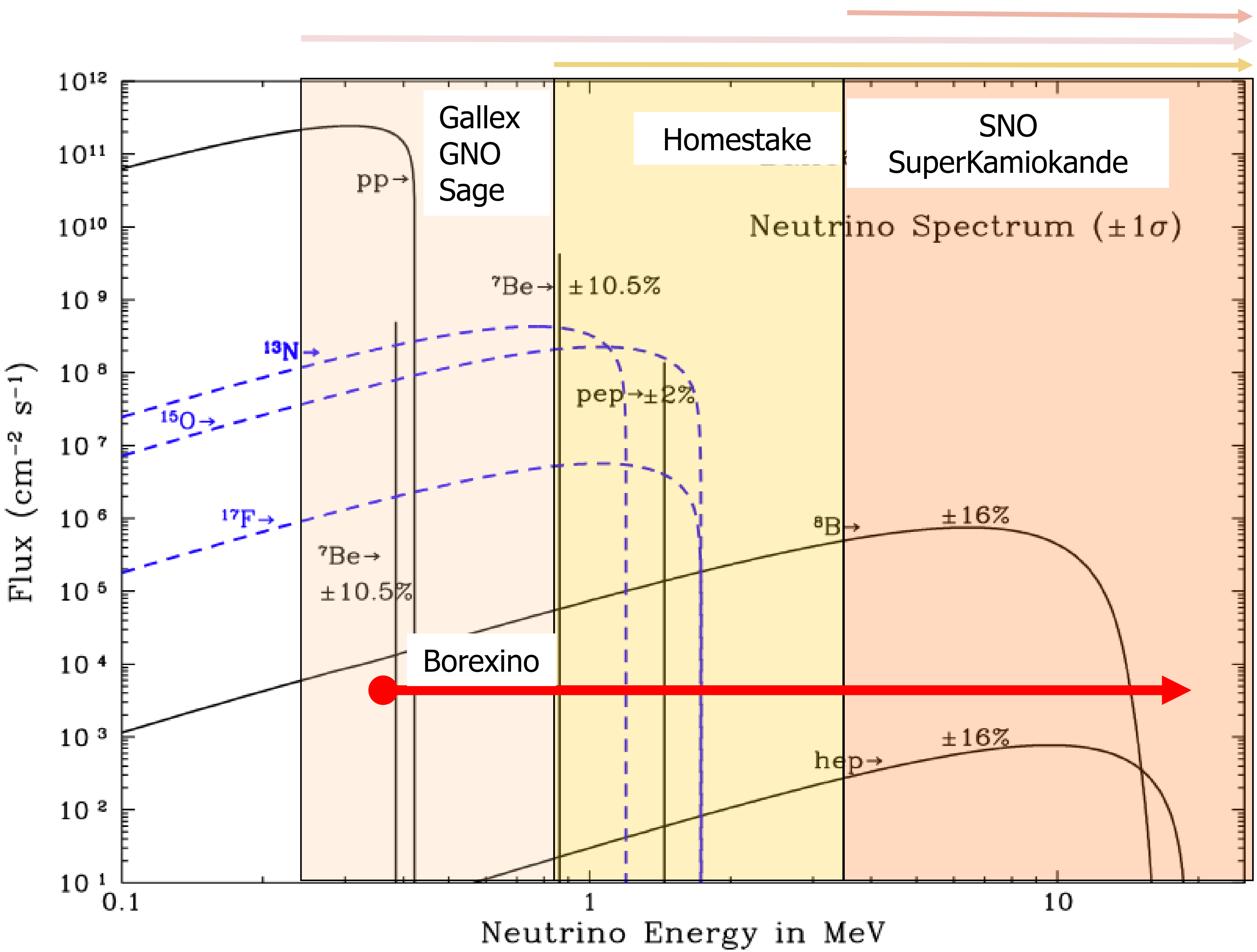
CN-cycle
~ 1.49%

$$\begin{cases} {}^{12}\mathrm{C} + \mathrm{p} \rightarrow {}^{13}\mathrm{N} + \gamma \\ {}^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + \mathrm{e}^{+} + \nu_e + \gamma \quad (\nu_N) \\ {}^{13}\mathrm{C} + \mathrm{p} \rightarrow {}^{14}\mathrm{N} + \gamma \\ {}^{14}\mathrm{N} + \mathrm{p} \rightarrow {}^{15}\mathrm{O} + \gamma \\ {}^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu_e + \gamma \quad (\nu_O) \\ {}^{15}\mathrm{N} + \mathrm{p} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He} + \gamma \end{cases}$$

NO-cycle
~ 0.01%

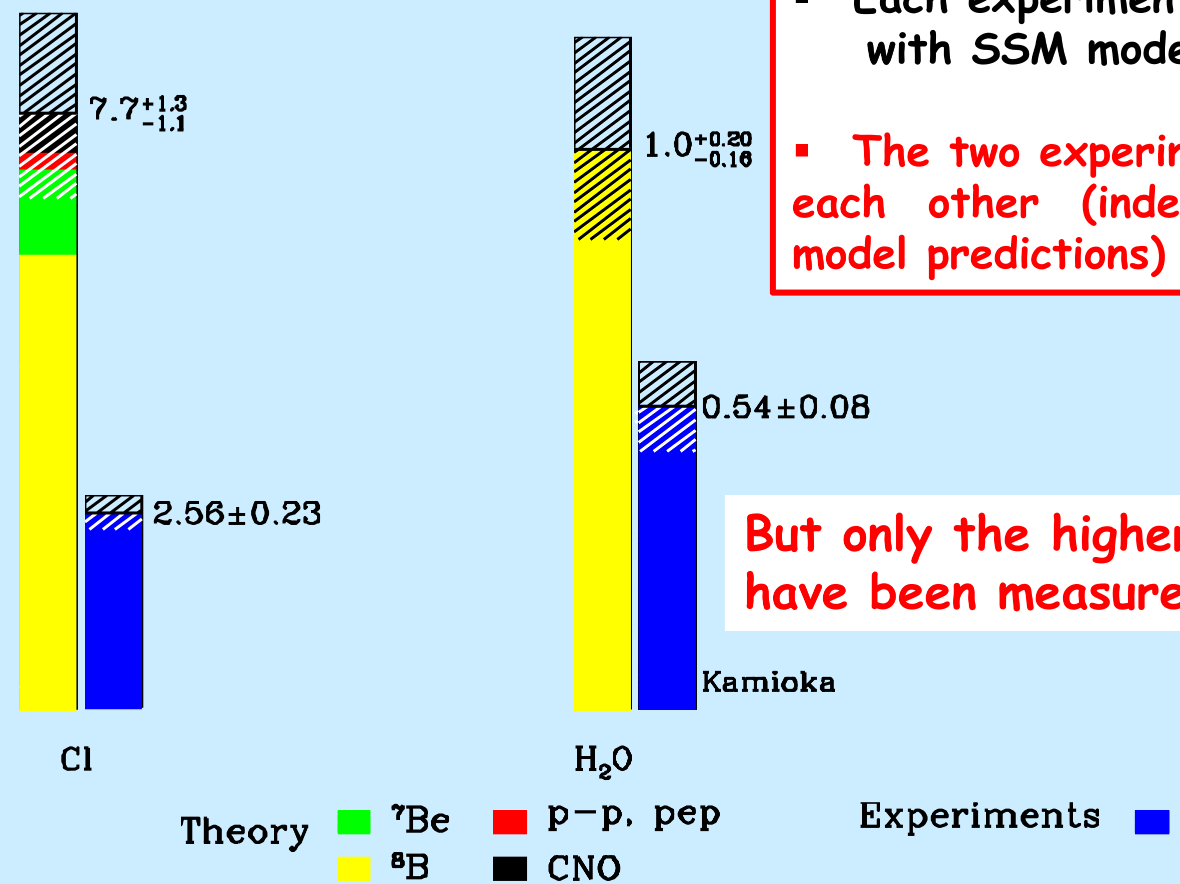
$$\begin{cases} {}^{15}\mathrm{N} + \mathrm{p} \rightarrow {}^{16}\mathrm{O} + \gamma \\ {}^{16}\mathrm{O} + \mathrm{p} \rightarrow {}^{17}\mathrm{F} + \gamma \\ {}^{17}\mathrm{F} \rightarrow {}^{17}\mathrm{O} + \mathrm{e}^{+} + \nu_e + \gamma \quad (\nu_F) \\ {}^{17}\mathrm{O} + \mathrm{p} \rightarrow {}^{14}\mathrm{N} + {}^{4}\mathrm{He} + \gamma \end{cases}$$

Energy spectrum of solar neutrinos



(Adapted from Serenelli et al. 2011)

Results of the Kamiokande and Chlorine experiments compared with Standard Solar Model predictions



Two "solar neutrino problems"

- Each experiment is in disagreement with SSM models predictions
- The two experiments do not agree each other (independently of solar model predictions)

But only the higher energy neutrinos have been measured

(SSM and comparison from Bahcall & Pinsonneault 2000)

*The Chlorine signal is in Solar Neutrino Unit (SNU): 10^{-36} captures per target atom per second
The Kamioka signal is normalized to the SSM prediction

Two kinds of proposed solutions

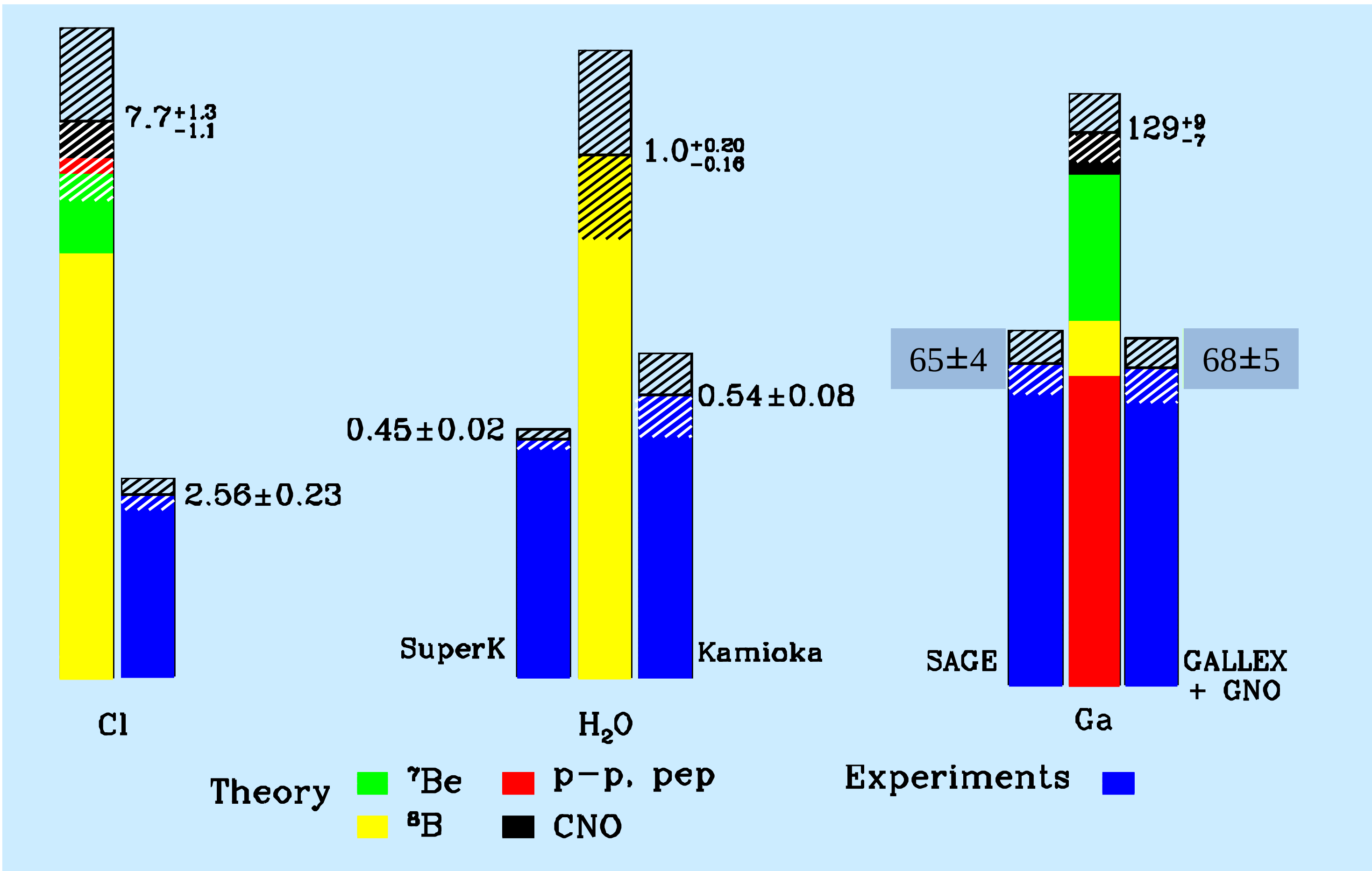
- There is something wrong in the standard solar models → but the results of the two experiments cannot be reconciled

- Neutrinos have "not standard properties" → neutrino oscillations


What about a collaboration on solar models and solar neutrinos ?
HOTEL CARLTON
FERRARA
Vittorio Castellani

Results of Chlorine, Super-Kamiokande and Gallium experiments compared with SSM predictions

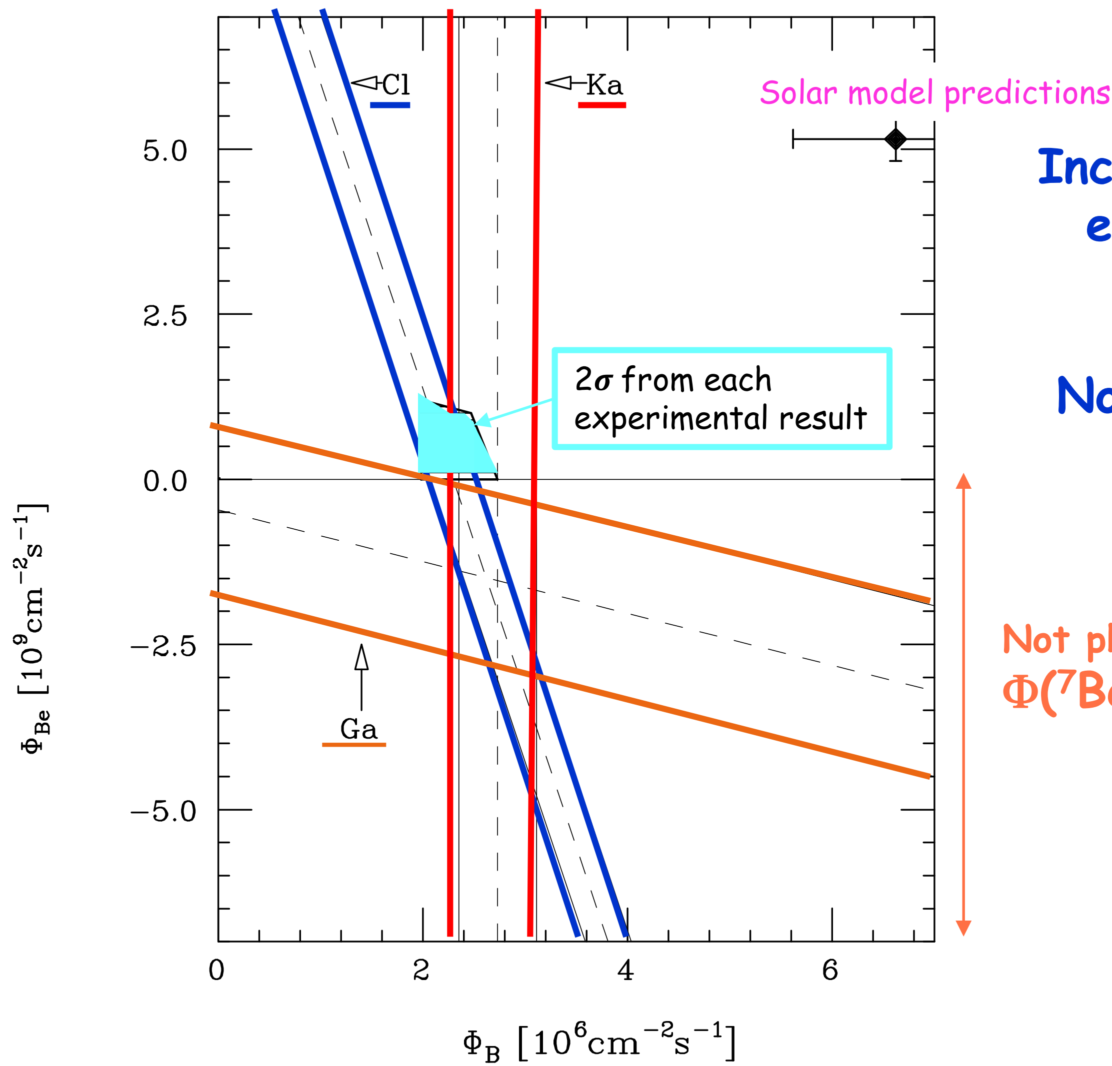
The signal in Gallium experiments is (more or less) the one due only to pp neutrinos → no room for neutrinos from other branches → inconsistency among experiments



(SSM and comparison from Bahcall & Pinsonneault 2000)

*The Chlorine and Gallium signals are in Solar Neutrino Unit (SNU): 10^{-36} captures per target atom per second. The Kamioka signal is normalized to the SSM prediction.

Information on the ^{7}Be and ^{8}B neutrino fluxes derived from the Chlorine (Cl), Kamiokande (Ka) and Gallex (Ga) experiments and the solar luminosity



(Castellani et al. 1997)

Inconsistency among the experimental results

⬇

Non standard neutrino properties

Not physical region:
$\Phi(^7\mathrm{Be}) < 0$

Standard Solar Models

Bahcall (1995): "A SSM is one which reproduces, within uncertainties, the observed properties of the Sun, by adopting a set of physical and chemical inputs chosen within the range of their uncertainties".

Recipe: evolve a model with 1 $M_\odot$, with good microphysics and with assumed original helium and metal abundances, starting from a chemically homogeneous star (in the Pre-Main Sequence phase) to present solar age.

Fixed quantities		
Solar mass	$M_\odot$=1.989×10^{33}g 0.01%	Kepler's 3rd law
Solar age	$t_\odot$=4.57 ×10^9yrs 0.1%	Meteorites

Quantities to match		
Solar luminosity	$L_\odot$=3.827 ×10^{33}erg s^{-1} 0.04%	Solar constant
Solar radius	$R_\odot$=6.9566 ×10^{10} cm 0.01%	Angular diameter
Solar photospheric metals/hydrogen ratio	$(Z/X)_\odot$= 0.0183* ~10%	Photosphere and meteorites

*The precise value for the solar chemical composition is still under debate

Data from the 2015 IAU resolutions B2 and B3

Standard Solar Models

3 (more or less) free parameters:

- The initial Helium abundance Y^*_{in}
- The initial total metals abundance Z_{in}
- External convection efficiency → semi-empirical treatment with one free parameter, α_{MLT} , which determines the convection efficiency

Construct a $1M_\odot$ initial model with Y_{in}, Z_{in} and α_{MLT}, evolve it during $t_\odot$ and match $(Z/X)_\odot$, $L_\odot$ and $R_\odot$ to better than one part in 10^{-5}

*In astrophysics the fractional mass abundance of H, He and elements heavier than helium (metals) are indicated, respectively, as: X, Y and Z, thus X+Y+Z=1

"MLT " means "mixing length theory" the approximate semi-empirical theory generally adopted to treat convection in the stellar envelopes

Microscopic diffusion

Microscopic diffusion (origin in pressure, temperature and concentration gradients). Very slow process $t_{diff} \approx$ few 10^{10} yr

Dominant effect in stars: **gravitational settling: H↑ – He & metals ↓**

⬇

- **He and heavy elements sink relative to hydrogen in the radiative interior of the Sun →** lower surface helium abundance

- **Surface metal abundance should decrease too** but it's fixed to the observed value → higher original metal abundance

- **Higher external opacity →** deeper convective envelope

Standard Solar Models

Effects of the three free parameters:

- Y_{in} mainly influences the model luminosity

increasing Y_{in} the Sun is brighter and a given luminosity is reached in a shorter time

- Z_{in} essentially determines the present photospheric metallicity, that is the surface $(Z/X)_\odot$ predicted value ⟵ influenced by diffusion efficiency

- α_{MLT} only affects the model radius

to reproduce $R_\odot$ one adjusts the efficiency of external convection. If $\alpha\uparrow$, convection is more efficient, $dT/dr\downarrow$ and $T_{sur}\uparrow$ thus, since $L_\odot$ is fixed , the radius decreases

Three free parameters to reproduce three observables ⟶ not a so big achievement!

Confidence in the SSM is also based on the successes of stellar evolution theory to reproduce the characteristics of stars similar to the Sun observed in the Galaxy in different phases of their life .

Moreover ….

Helioseismology

- In the 1960s, it was discovered that the solar structure oscillates around its equilibrium configuration supporting at least 10^5 eigenmodes (Leighton et al. 1962)
- Solar oscillations may be regarded as a superposition of many standing waves. Frequencies of order mHz (5-min oscillations)
- Solar oscillations are acoustic waves (p-modes, pressure is the restoring force) stochastically excited by convective motions which reflect off the photosphere
- The oscillation frequencies are linked to the sound speed profile inside the Sun
- Doppler observations of spectral lines: velocities of a few cm/s are measured (relative accuracy in frequencies ~ 10^{-5})

Analysis of the oscillation frequencies tell us about the inside of the Sun

Helioseismic "observables"

By comparing the measured frequencies with the calculated ones (based on a SSM) one can determine:

- **The sound speed profile**
- **Depth of the convective envelope**

(transition between radiative and convective zone)

$R_{cz}/R_{\odot}$ =0.713 ± 0.001

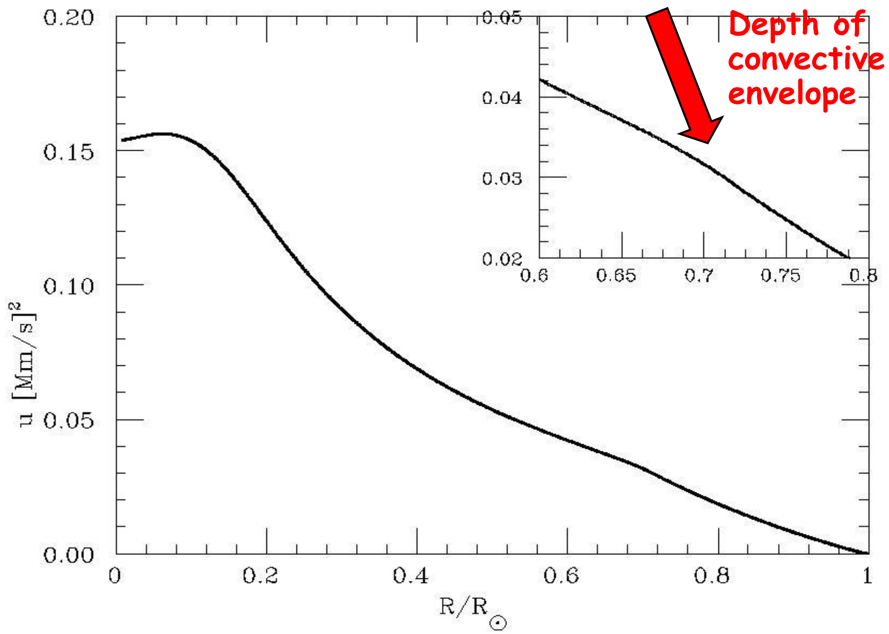
(Basu & Antia 1997)

- **The present surface He abundance**

Y_s= 0.2485 ± 0.0034

(Basu & Antia 2004)

Profile of the inferred sound speed inside the Sun



Ricci B. (2002)

$$(c^2 = \Gamma_1 \frac{P}{\rho}) \text{ with } \Gamma_1 = \frac{dlogP}{dlog\rho}|_{adiabatic}$$

Three other observables are added, the models are now constrained !!

→ **Helioseismic models**

Predictions of standard solar models

- **Physical quantities** as a function of radius: T, P, density, luminosity, mass …

- **Chemical profiles** $X(r)$, $Y(r)$, $Z_i(r) \rightarrow$ electron density profiles

- **Helioseismic quantities**: surface helium abundance, depth of the convective envelope, sound speed

- **Eight neutrino fluxes**: production profiles and integrated values

Main uncertainty sources for solar models

Chemical composition determination

Adopted input physics

(nuclear reaction rates, opacity, EOS..)

Nuclear fusions

Energetic | Neutrino production | Element nucleosynthesis

The fusion reactions happen among thermalized charged nuclei of the stellar plasma screened by the plasma electrons through tunnel effect

$$\sigma(E) = \frac{1}{E}\boxed{S(E)}\boxed{\exp\left(-2\pi\frac{Z_1 Z_2 e^2}{\hbar v}\right)}$$

Penetration probability through the Coulomb barrier
(strong energy dependence)

The "astrophysical factor", a function smoothly varying with E, which includes the nuclear information and the normalization of the cross section; E=$\frac{1}{2}\mu v^2$

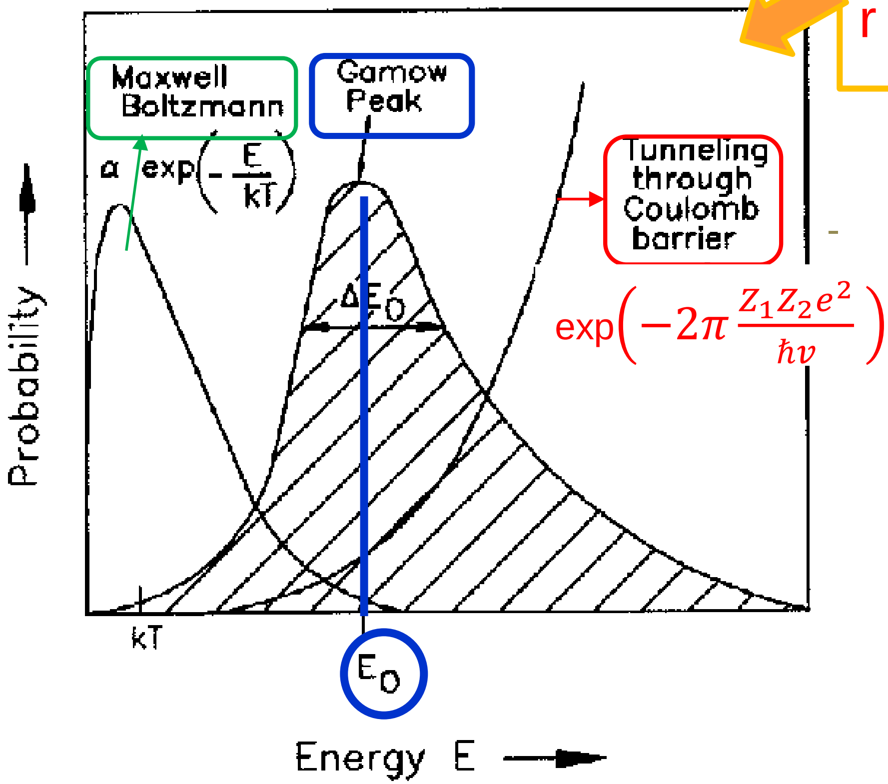
Reaction rate

$$\mathbf{r} = \mathbf{f}\,\frac{1}{1+\delta_{12}}\,n_1 n_2\left(\frac{8}{\pi\mu}\right)^{1/2}\frac{1}{(KT)^{3/2}}\int_0^{\infty}\boxed{\sigma(E)}\,E\,exp\left(-\frac{E}{KT}\right)dE$$

f=the plasma electron screening factor; $\mu = \frac{m_1 m_2}{m_1+m_2}$, n=number density

The fusion rate is significant only in a restricted range of energy (Gamow peak) around the energy E_O, corresponding to the reactivity peak.

$$r \propto \int_0^{\infty} \exp\left(-2\pi \frac{Z_1 Z_2 e^2}{\hbar v}\right) \exp\left(-\frac{E}{KT}\right) dE$$



(Figure adapted from Rolf & Rodney «Cauldrons in the cosmos», 1988)

Where E_G = Gamow energy = b^2

With $b = (2\mu)^{1/2} \pi e^2 Z_1 Z_2/\hbar$

$$E_O = \left(\frac{bkT}{2}\right)^{2/3} = 1.22 \left(Z_1^2 Z_2^2\ \mu\ T_6^2\right)^{\frac{1}{3}} \text{keV}$$

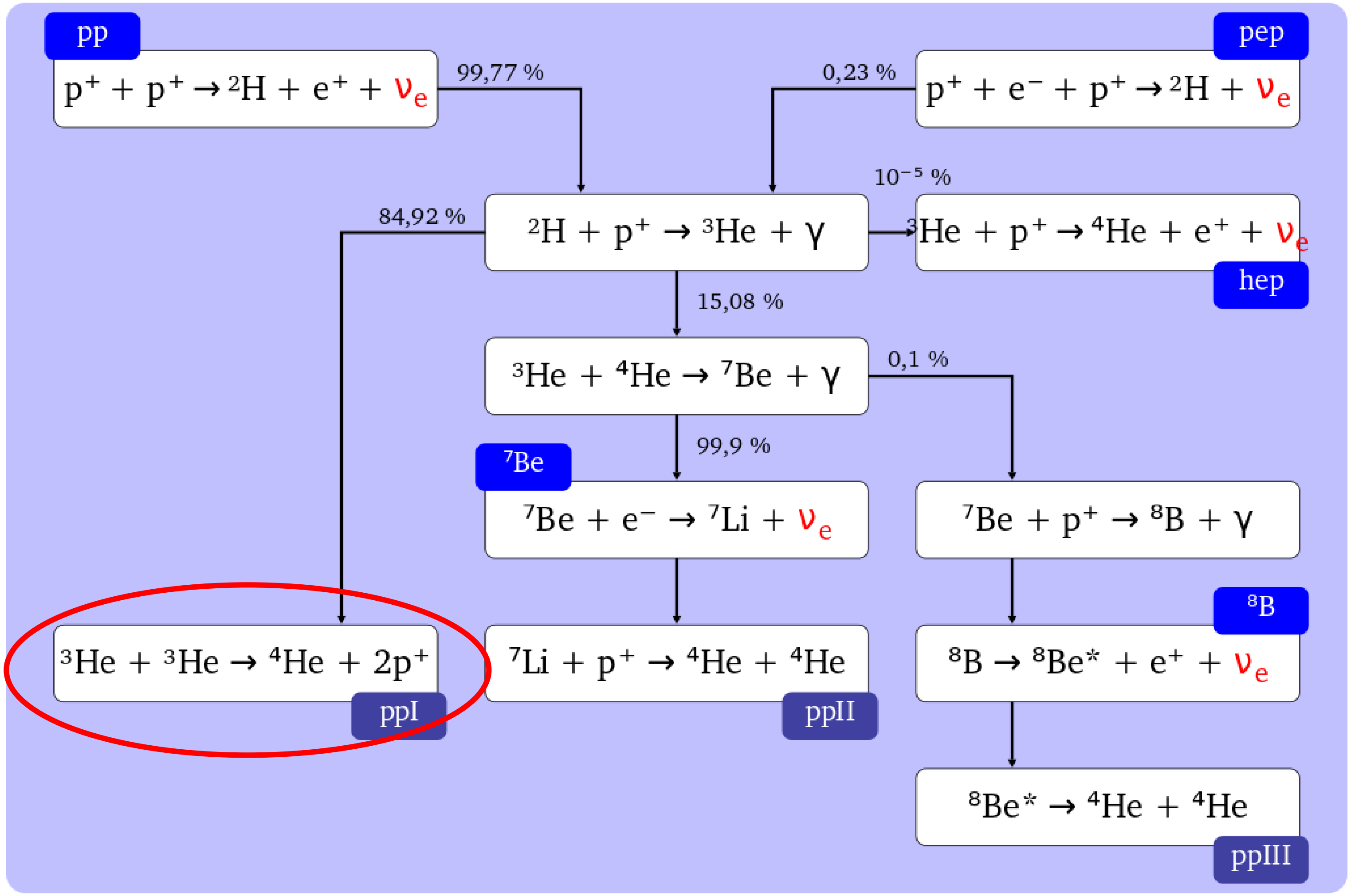
with $T_6 = \dfrac{T}{10^6}$

Typical values for H burning in the Sun:
$E_O \approx$ 10-20 keV

$$r \propto f \frac{n_1 n_2}{1+\delta_{12}} \frac{E_o^{1/2}}{KT} S(E_o) e^{-\frac{3E_O}{KT}}$$

E_O depends on the stellar temperature and on the charge of the reacting nuclei, the higher is their charge the higher is the Coulomb barrier and more difficult is the fusion $\longrightarrow$ higher temperatures are needed

.



The $^3He+^3He$ reaction was not measured (until 1999) at the Gamow peak energies -> resonance at low energies?

"Some people are so crazy that they actually venture into deep mines to observe the stars in the sky"

De origine animalium – Aristotele

Nuclei in the Cosmos I / 1990 / Baden/Vienna, Austria

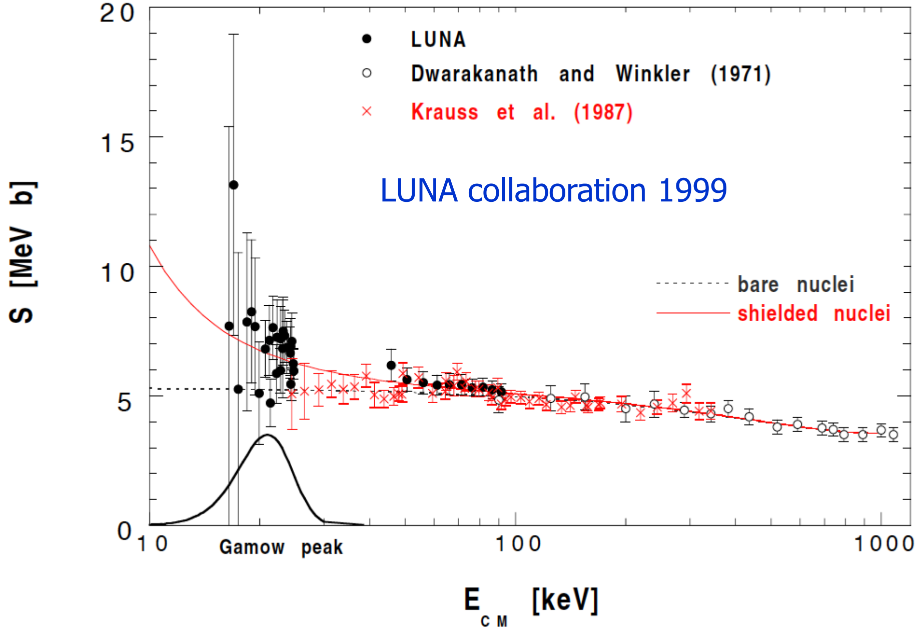
Why are we not going into the Gran Sasso?

LUNA
Laboratory for Underground Nuclear Astrophysics

Courtesy of Paolo Prati

The first experiment on $^3He(^3He,2p)^4He$ at solar energies

LUNA-I has measured the $^3He+^3He \rightarrow ^4He + 2p$ at the solar Gamow peak, by using a 50 kV underground accelerator at the Gran Sasso National Laboratories

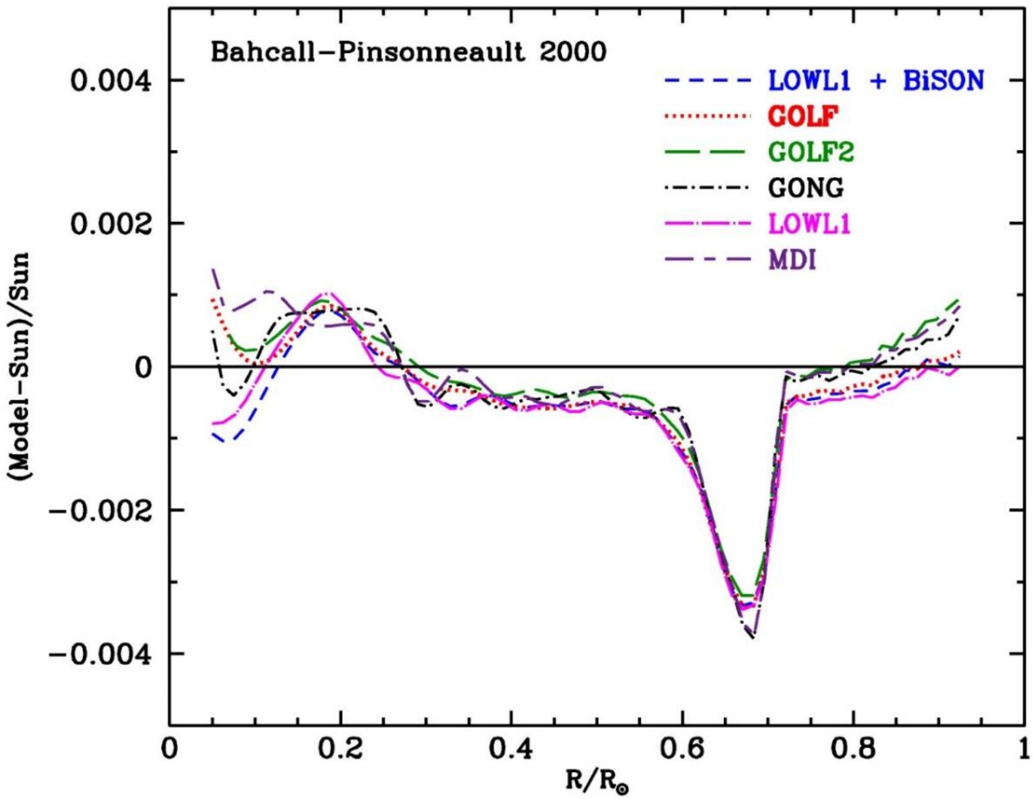


Courtesy of Paolo Prati

The "idyllic" situation for the Sun until 2004:

Several SSMs (Guzik et al. 2001, Bahcall Pinsonneault & Basu 2001, Couvidat et al. 2003, Castellani et al. 1999, Christensen-Dalsgaard 2002, Di Mauro & Paternò 2003) **reproduced the sound speed profile determined from seismic inversion to within 0.4% as well as the seismically-inferred convection zone depth and the surface helium abundance**

Relative difference between the predicted and the helioseismologically inferred (with various frequency sets) solar sound speed profile



Bahcall, Pinsonneault & Basu (2001)

But…

Solar Composition changes

Element	"Old" GS98	"New" AGSS09met
C	8.52 ± 0.06	8.43 ± 0.05
N	7.92 ± 0.06	7.83 ± 0.05
O	8.83 ± 0.06	8.69 ± 0.05
Ne	8.08 ± 0.06	7.93 ± 0.10
Mg	7.58 ± 0.01	7.53 ± 0.01
Si	7.56 ± 0.01	7.51 ± 0.01
S	7.20 ± 0.06	7.15 ± 0.02
Ar	6.40 ± 0.06	6.40 ± 0.13
Fe	7.50 ± 0.01	7.45 ± 0.01
$(Z/X)_\odot$	0.02292	0.01780

"Old" (Gresse & Sauval 1998) and "new" (Asplund et al. 2009)* solar abundances expressed as: $\log \epsilon_i \equiv \log N_i/N_H + 12.$ where N_i is the numerical abundance of the i-th element.

Reduction of O/Ne abundance by about 40%

Reduction of C/N abundance by about 20%

Reduction of refractory elements by about 10%

Reduction of the total metallicity by about 25%

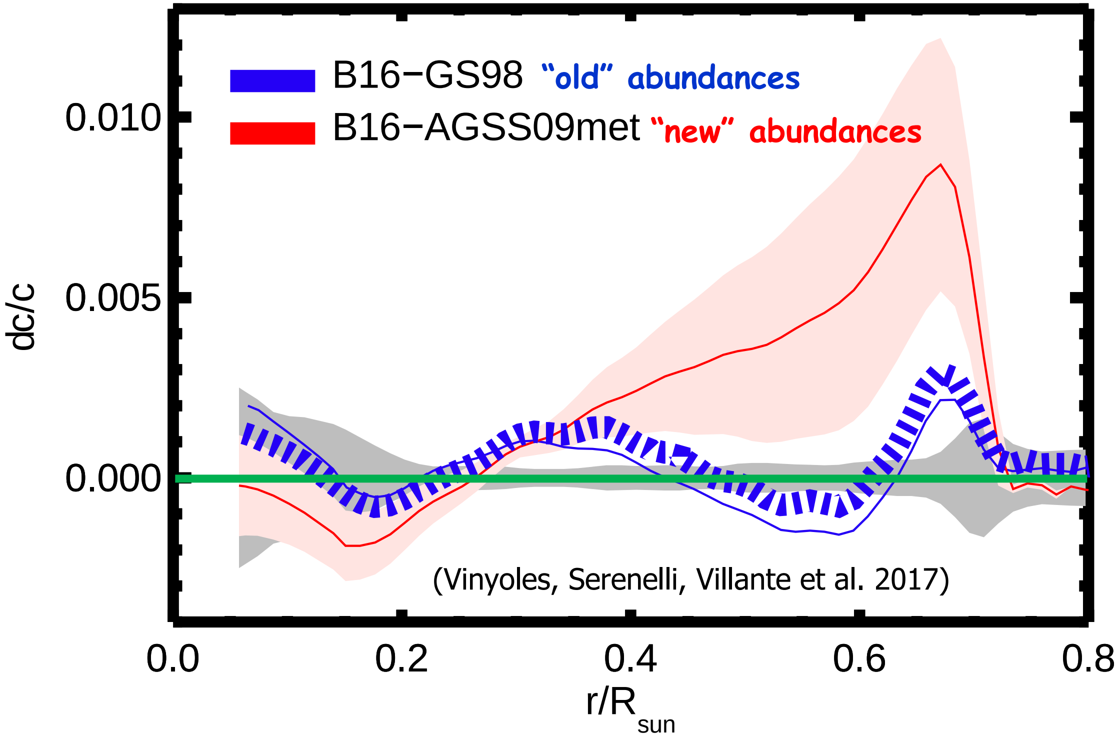
(In astrophysics the metallicity, Z, is the fractional abundance in mass of elements heavier than helium)

*See also Asplund et al. 2005, Scott et al. 2015a,b. The estimates by these authors are substantially confirmed by Melendez 2004, Socas-Navarro & Norton 2007 but somehow questioned by Ayres et al. 2006, Caffau et al. 2008, 2009, 2011.

The comeback to the reality

With the revision of the solar abundances the situation got worse

Relative difference between the predicted and the helioseismologically inferred solar sound speed profile



With respect to helioseismic results

$R_{cz}/R_\odot$ =0.713 ± 0.001

Y_s= 0.2485 ± 0.0034

- lower surface helium abundance
- shallower convective envelope

Worse agreement with the solar quantities inferred by helioseismology

Low original solar helium abundance: Y~0.255÷0.265 (see e.g. Bahcall et al. 2005, Guzik et al. 2005, Turch-Chièze et al. 2004, Yang & Bi 2007, Pisa SSM 2018, Christensen-Dalsgaard et al. 2009, Basu 2010, Vinyoles, Serenelli, Villante et al. 2017) **with respect to the present estimates for the primordial helium abundance (Y_P=0.2446$\pm$0.0029)** (Peimbert et al. 2016, see also Cyburt et al. 2016, Planck satellite data, Aver et al. 2015, Izotov et al. 2014, Coc et al. 2013)

...But the Sun is a "common star" ➡ low helium enrichment for the stars (i.e. the interstellar medium) during the history of the Galaxy?

The estimates of helium to metal enrichment for galactic stars, $\frac{\Delta Y}{\Delta Z}$, vary from 0.5 to 5 (at least), currently preferred values are $\frac{\Delta Y}{\Delta Z}$ = 2 ± 1

(see e.g. Pagel & Portinari 1998, Lebreton et al. 1999, Jimenez et al. 2003, Balser 2006, Casagrande et al. 2007, Bertelli et al. 2008, Portinari et al. 2010, Gennaro et al. 2010, Serenelli & Basu 2010, Lebreton et al. 2014, etc..)

Several works searched for the solution of solar composition controversy

(see e.g. Song, Gonzalez-Garcia, Villante et al. 2018, Basu & Antia 2013, 2008, 2007, Serenelli et al. 2011, Basu 2010, Guzik 2008, Montalban et al. 2004, Bahcall et al. 2005, 2006)

➡ **The problem is still open and constitutes a "warning" for stellar evolution theories**

* In astrophysics the fractional mass abundance of H, He and elements heavier than helium (metals) are indicated, respectively, as: X, Y and Z, thus X+Y+Z=1

Chemical composition determination

Main uncertainty sources for solar models

Adopted input physics

(nuclear reaction rates, opacity, EOS..)

Opacity

(photon-matter interaction processes)

Opacity affects the temperature gradient and thus the extension of the region of energy production

- Opacity is a complex function of temperature, density and chemical composition of the solar plasma. → It's very difficult to calculate its uncertainty

- **The accuracy of opacity calculations is supposed to get worse moving from the stellar center to the base of the convective envelope**

- **In solar interior all theoretical opacities** (see e.g. Badnell et al. 2005,OP opacities, Blanchard et al. 2012, Mondet et al. 2015, OPAS, Colgan et al. 2016, OPLIB, Krief et al. 2016, STAR) **agree with each other within 5% and the most widely adopted ones** (OPAL and OP opacities) **agree with 2.5%** (see also Haxton & Serenelli 2008, 2013, Serenelli et al. 2013, Villante et al. 2014, Vinyoles, Serenelli, Villante et al. 2017, Song, Gonzalez-Garcia, Villante et al. 2018)

- **At the bottom of the convective envelope opacity uncertainty is assumed not lower than 7%**

- **The profile of the opacity error inside the Sun is unknown**

The $p(p, e^+\nu_e)^2H$ cross section

Recently Marcucci et al. 2013 revised the pp cross section calculations and Acharya et al. 2016 made a careful quantitative analysis of the theoretical uncertainty for the proton proton reaction rate

Marcucci et al. 2013, MSV13* : $$S(0) = (4.030 \pm 0.006) X \; 10^{-25} \; \mathrm{MeV \; b}$$

Acharya et al. 2016, ACEFP16 : $$S(0) = (4.047^{+0.024}_{-0.032}) X 10^{-25} MeV \; b$$

Conservative estimate of the uncertainty on the pp cross section of the order of 1%

The very small error on the $p(p, e^+\nu_e)^2H$ rate determination has a neglible effect on standard solar models calculations

* The Marcucci et al. $S_{pp}(E)$ shows differencies not only in S(0) with respect to previous estimates (Adelberger et al. 2011) but also in the higher orders in the Taylor expansion of $S_{pp}(E)$. For the Sun the "new" pp rate is about 1.3% lower than the one by Adelberger et al. 2011. See Tognelli et al. 2015 for a discussion of the effects on solar models.

Uncertainty in diffusion efficiency

Element diffusion in stars (Eddington 1926, Aller & Chapman, 1960) includes different processes:
• gravitational settling • thermal diffusion • diffusion driven by composition gradients
• radiative acceleration of individual ions (Michaud 1970) . Often a turbulent diffusion term (Schatzman 1969) is included

•He and Z settling is a long term process (~ Gyr)

• Diffusion is certainly active in the Sun

(see e.g. Demarque & Guenter 1988, Cox et al. 1989, Bahcall & Loeb 1990, Bahcall & Pinsonneault 1992, 1995, Guzik & Cox 1992,1993, Proffit (1994), Gough et al. 1996, Christensen-Dalsgaard et al. 1993, Lydon et al. 1993, Basu et al. 2000…)

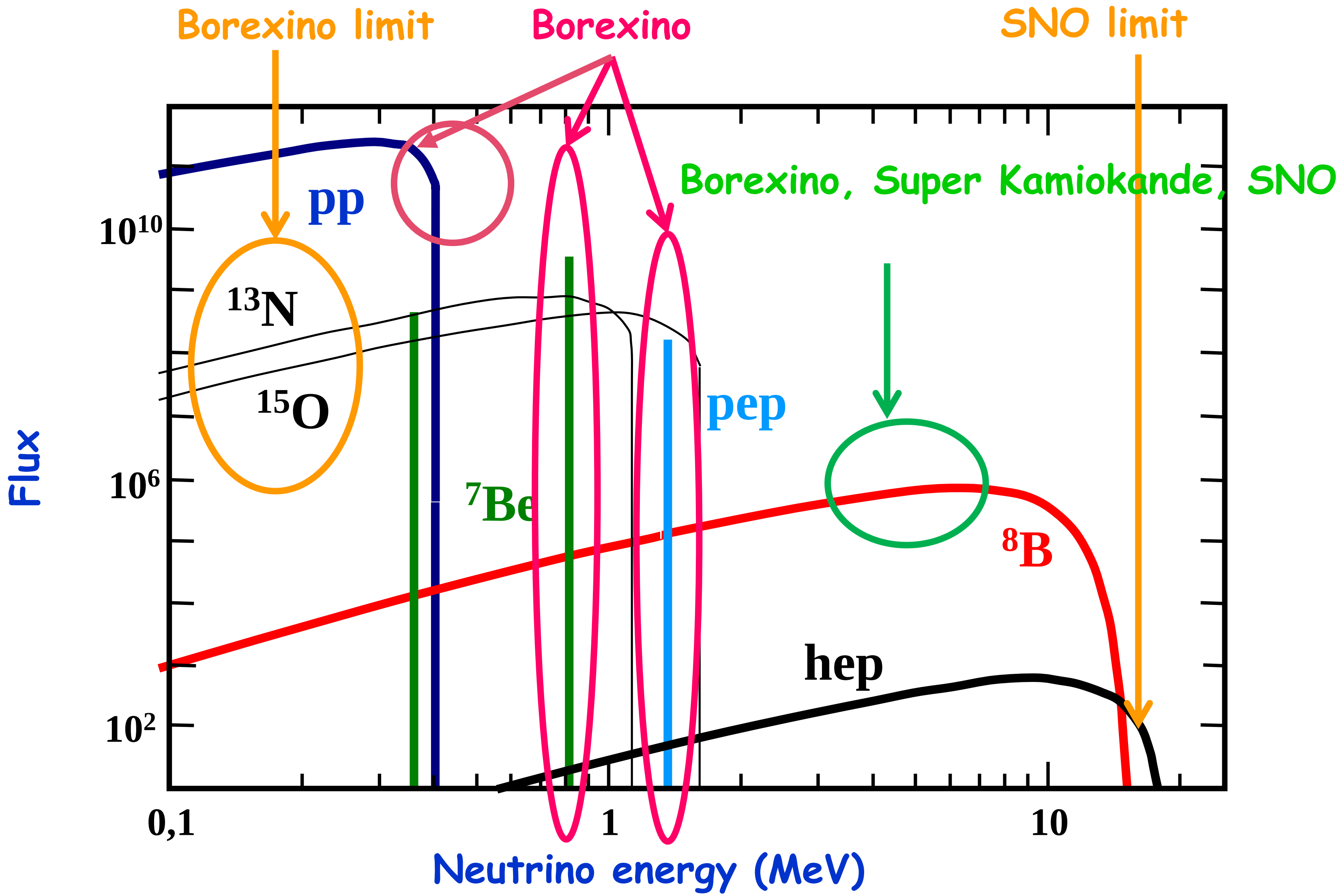
• Estimated uncertainty of diffusion (for the Sun) ~ 10%-15%

(Thoul et al. 1994, see also discussions in e.g. Bahcall & Pinsonneault 1995, Fiorentini et al. 1998, Vinyoles et al. 2017, etc.. but also discussions in Roussel-Dupré 1982, Turcotte et al. 1998, Schlattl 2002, Schlattl & Salaris 2003, Montalbán et al. 2006, Bahcall et al. 2006,Thoul & Montalbán 2007, etc..)

• For the Sun an extra-mixing at the bottom of the convective envelope seems to be present

(see e.g. Richard et al. 1996, Antia & Chitre 1998, Brun et al. 1999, 2011, Schattl & Basu 2009, Christensen-Dalsgaard & Di Mauro 2007, Christensen-Dalsgaard et al. 2011, 2018, etc..)

Could solar neutrino measurements help to solve the "solar abundance problem"?



Experimental results for solar neutrinos

Solar neutrinos already individually detected

pp [Borexino, radiochemical Ga experiments]

^{7}Be [Borexino, radiochemical Cl experiment]

pep [Borexino]

^{8}B [SNO, Super-Kamiokande, Borexino]

Solar neutrinos still to be individually detected

CNO Borexino upper limit < $7.9\ 10^8\ cm^{-2}s^{-1}$

hep SNO upper limit < $1.9 \times 10^4\ cm^{-2}\ s^{-1}$
SK upper limit < $1.5\ 10^5\ cm^{-2}s^{-1}$

For experimental results see e.g. : Aharmin B. et al. 2013, Bellerive et al. 2016, Mastbaum 2016 (SNO collaboration), Cravens et al. 2008, Abe et al. 2001, 2016 (Super Kamiokande collaboration), Bellini et al. 2011, 2012, 2014 Bellini et al., 2014, Nature, arXiv:1709.00756v1 [hep-ex] 2017, arXiv:1707.09279v2 [hep-ex] 2017 (Borexino Collaboration)

Solar neutrino fluxes inferred from global fits to solar neutrino data

($cm^{-2}s^{-1}$ units)

Flux	Solar value	Error (~)
pp	5.97×10^{10}	0.6%
pep	1.45×10^8	0.9%
hep	19×10^3	+63% -47%
^{7}Be	4.82×10^9	5%
^{8}B	5.16×10^6	2%
^{13}N	$\leq 13.7 \times 10^8$	
^{15}O	$\leq 2.8 \times 10^8$	
^{17}F	$\leq 85 \times 10^6$	

(Bergstrom et al. 2016, see also Vinyoles et al. 2017, Capozzi et al. 2018)

These results have been obtained from the neutrino signal in the various experiments, in the neutrino flavour oscillation framework, with the constraint that the sum of the thermal energy generation rates associated with each of the solar neutrino fluxes coincides with the solar luminosity. This last constraint strongly bounds pp and pep fluxes.

Dependence of solar neutrino fluxes on central temperature

- **The dependence of the solar neutrino fluxes on central temperature variation can be approximated as a power law*** (Bahcall 1989)**:**

$$\phi \propto T_c^{\alpha}$$

Flux	α
pp	-0.7
pep	-1.2
^{7}Be	10
^{8}B	20
NO	20
^{17}F	23

- Be neutrinos strongly depends on T_c, due to Gamow factor in ^{3}He+^{4}He
- B neutrinos has a stronger dependence due both to ^{3}He+^{4}He and (mainly) to ^{7}Be+p
- NO strongly depends on T_c, due to Gamow factor in ^{14}N+p
- For the conservation of total flux, pp neutrinos decrease with increasing T_c
- The pep rate goes approximatly as $R_{pp}\ T^{-0.5}$

Update of Castellani et al. 1997 for solar models with diffusion and Grevesse& Sauval 1998 chemical composition

The central temperature depends on the adopted physical and chemical inputs

α values are weakly dependent on which parameter is varied to obtain the change in T_c

*See e.g. Bahcall & Ulrich 1988, Bahcall & Ulmer 1996, Castellani et al. 1993, Castellani et al. 1997, Degl'Innocenti et al. 1998, Antia & Chitre 1999, Fiorentini & Ricci 2002, Serenelli et al. 2013, Vinyoles, Serenelli Villante et al. 2017

Solar neutrino fluxes

Environmental inputs (Lum.,opacity, age, Z/X…) which affect physical conditions of the production region, mainly the temperature

Nuclear inputs (cross sections for the pp chain and CNO cycle reactions)

Dependence of solar neutrino fluxes on physical and chemical inputs

The sensitivity of the ν fluxes (and of the solar central temperature) to (small) changes of physics and chemical inputs can be expressed in terms of power laws.

Y \ X	S_{pp}	S_{33}	S_{34}	S_{17}	$S_{1,14}$	L	Z/X	opa	age	dif
pp	0.114	0.029	-0.062	0	-0.019	0.73	-0.076	-0.12	-0.088	-0.02
Be	-1.03	-0.45	0.87	0	-0.027	3.5	0.60	1.18	0.78	0.17
B	-2.73	-0.43	0.84	1	-0.02	7.2	1.36	2.64	1.41	0.34
N	-2.59	0.019	-0.047	0	0.83	5.3	1.09	1.82	1.15	0.25
O	-3.06	0.013	-0.038	0	0.99	6.3	2.12	2.17	1.41	0.34
T_c	**-0.14**	**-0.0024**	**0.0045**	**0**	**0.0033**	**0.34**	**0.078**	**0.14**	**0.083**	**0.016**

(Ricci. B., 2002, see also Fiorentini & Ricci 2002)

$$\frac{Y}{Y_{SSM}} \propto \left(\frac{X}{X_{SSM}}\right)^{\alpha}$$

The α coefficients obtained numerically are physically understood

Values of $d\ln Y / d\ln X$ computed by using models including element diffusion and Grevesse & Sauval 1998 chemical composition

(See also Bahcall & Ulrich 1988, Bahcall 1989, Castellani et al. 1993, Bahcall & Ulmer 1996, Castellani et al. 1997, Haxton & Serenelli 2008, Villante & Ricci 2010, Serenelli et al. 2013)

Main uncertainty sources for solar characteristics and neutrino fluxes

Quant.	Dominant theoretical error sources in %			
$\Phi(pp)$	$L_\odot$: 0.3	S_{34}: 0.3	κ: 0.2	Diff: 0.2
$\Phi(pep)$	κ: 0.5	$L_\odot$: 0.4	S_{34}: 0.4	S_{11}: 0.2
$\Phi(hep)$	S_{hep}: 30.2	S_{33}: 2.4	κ: 1.1	Diff: 0.5
$\Phi(^7Be)$	S_{34}: 4.1	κ: 3.8	S_{33}: 2.3	Diff: 1.9
$\Phi(^8B)$	κ: 7.3	S_{17}: 4.8	Diff: 4.0	S_{34}: 3.9
$\Phi(^{13}N)$	C: 10.0	S_{114}: 5.4	Diff: 4.8	κ: 3.9
$\Phi(^{15}O)$	C: 9.4	S_{114}: 7.9	Diff: 5.6	κ: 5.5
$\Phi(^{17}F)$	O: 12.6	S_{116}: 8.8	κ: 6.0	Diff: 6.0
α_{MLT}	O: 1.3	Diff: 1.2	κ: 0.7	Ne: 0.7
Y_{ini}	κ: 1.9	Ne: 0.5	Diff: 0.4	Ar: 0.3
Z_{ini}	O: 4.7	C: 2.0	Ne: 1.7	Diff: 1.6
Y_S	κ: 2.2	Diff: 1.1	Ne: 0.6	O: 0.3
Z_S	O: 4.8	C: 2.0	Ne: 1.8	κ: 0.7
R_{CZ}	κ: 0.6	O: 0.3	Diff: 0.3	Ne: 0.2

(Vinyoles, Serenelli, Villante et al. 2017)

Now the global uncertainty is calculated also by means of Monte Carlo simulations of solar models in which the values of the input quantities are chosen randomly from their respective distribution (see e.g. Bahcall et al. 2006, Vinyoles, Serenelli, Villante et al. 2017, Song, Gonzalez-Garcia, Villante et al. 2018)

Relevant errors on nuclear cross sections

$^{7}Be(p,\gamma)^{8}B$

$S(0)_{1,7}$ = 21.3 eV-b $\pm$ 4.7%

(Zhang et al. et al. 2015, see also Adelnerger et al. 2011 and the discussion in Vinyoles, Serenelli, Villante et al. 2017)

$^{14}N(p,\gamma)^{15}O$

$S(0)_{1,14}$ = 1.59 keV-b $\pm$ 7.5%

(LUNA Collaboration, Marta et al. 2011, see also Marta et al. 2008, Imbriani et al. 2005)

$^{3}He(^{4}He,\gamma)^{7}Be$

$S(0)_{3,4}$ = 0.56 keV-b $\pm$ 5.2%

(Adelberger et al. 2011, see also deBoer et al. 2014 and Iliadis et al. 2016 which give values which bracket the quoted one)

$^{3}He(^{3}He,2p)^{4}He$

$S(0)_{3,3}$ = 5.21 MeV-b $\pm$ 5.2%

The still present uncertainty on the $^{7}Be(p,\gamma)^{8}B$ and $^{14}N(p,\gamma)^{15}O$ cross sections leads to an indetermination on ^{8}B and ^{13}N, ^{15}O neutrino fluxes of the order of 5÷7.5%

SSM predictions for neutrino fluxes

Flux ($cm^{-2}\ s^{-1}$)	GS98 (Old composition)	AGS09 (New composition)	Error (~)	Effect of composition change
pp (10^{10})	5.98	6.03	0.6%	
pep (10^{8})	1.44	1.46	1.1%	
hep (10^{3})	7.98	8.25	30%	
^{7}Be (10^{9})	4.93	4.50	6%	~ 10%
^{8}B (10^{6})	5.46	4.50	14%	~ 20%
^{13}N (10^{8})	2.78	2.04	14%	~ 30%
^{15}O (10^{8})	2.05	1.44	16%	~ 30%
^{17}F (10^{6})	5.29	3.26	19%	

(Solar models from Vinyoles, Serenelli, Villante 2018)

Differences of the central temperature between models with the two compositions are of the order of 1%

Comparison between theoretical predictions and experimental results for solar neutrino fluxes

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^{13}N	$\leq 13.7 \times 10^{8}$	
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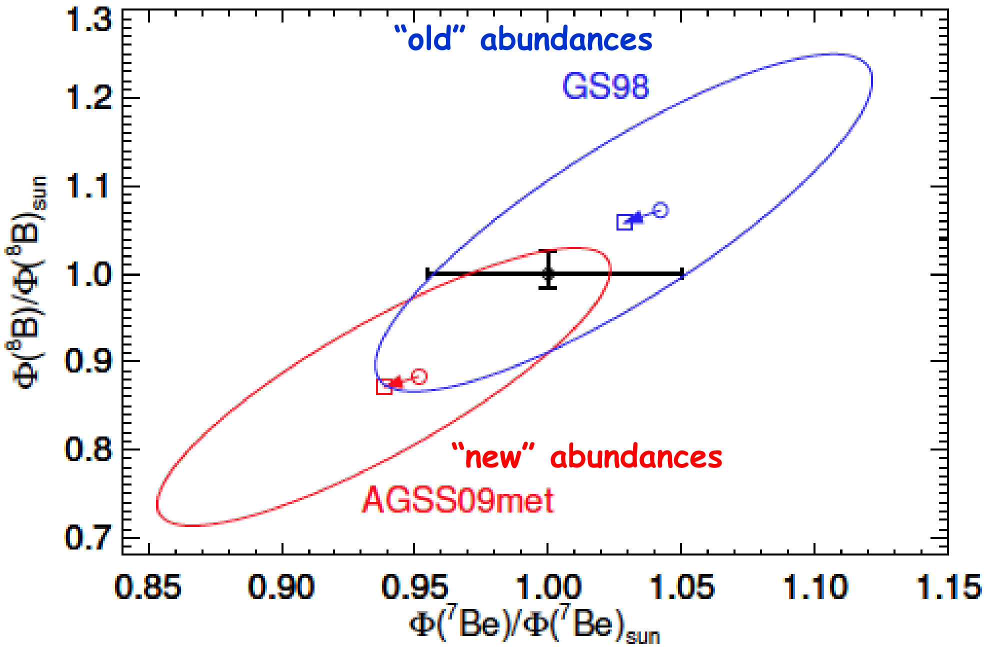
^{17}F	$\leq 85 \times 10^{6}$	

(Bergstrom et al. 2016)

Experimental results are in agreement with solar models predictions in the neutrino flavor oscillation framework and within theoretical and experimental uncertainties (independently of the assumed chemical composition)

Comparison between experimental and predicted ^{7}Be and ^{8}B neutrino fluxes



(Vinyoles, Serenelli, Villante et al. 2017)

Figure 1. $\Phi(^{8}\mathrm{B})$ *and* $\Phi(^{7}\mathrm{Be})$ *fluxes normalized to solar values [43]. Black circle and error bars: solar values. Squares and circles: results for B16 (current) and (older) generation of SSMs respectively. Ellipses denote theoretical* 1σ *C.L. for 2 dof.*

Experimental errors on ^{7}Be and ^{8}B neutrino fluxes are smaller than the theoretical ones !

The solar composition only impacts ^{7}Be and ^{8}B fluxes indirectly by altering the solar core temperature

Both solar compositions lead to standard solar models which are consistent at 1σ with ^{7}Be and ^{8}B "observed" neutrino fluxes

Could solar neutrino measurement help to solve the "solar abundance problem"?

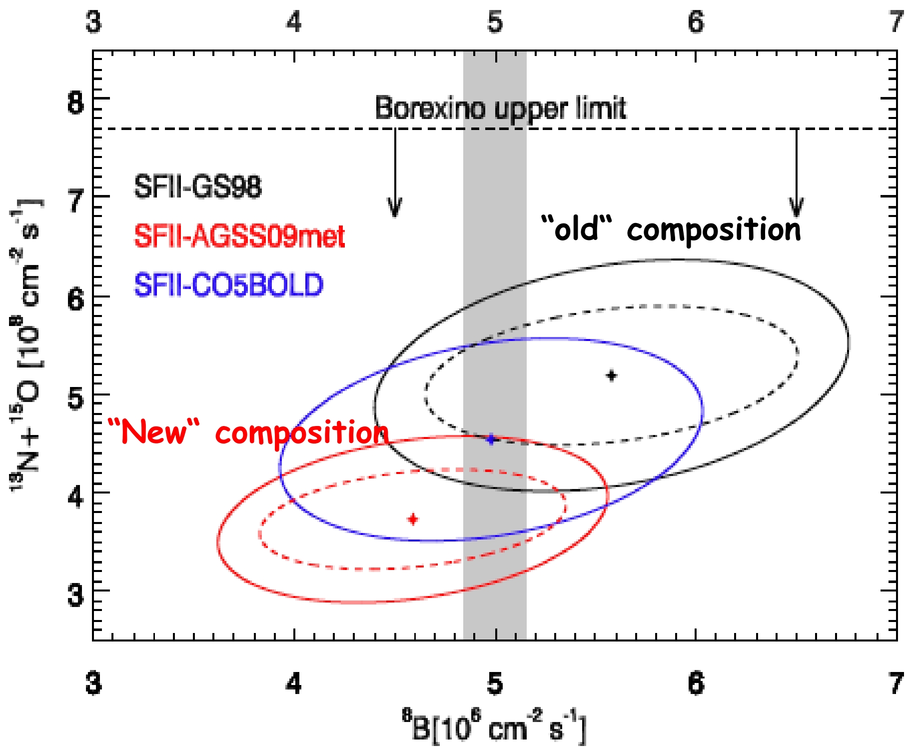
- CNO neutrinos depend on "environmental variables" in the solar core which affects solar temperature and linearly on CNO abundances

- ^{8}B neutrinos have (more or less) the same dependence as CNO ones on the environmental factors

Yes! One could discriminate between low and high metallicity solar models

The upper limit from Borexino is still higher than SSM results regardless of the adopted solar composition

The problem is still open…



(Serenelli, A. 2016)


Thanks for the attention!
Gianni and friends 2003