

PHYSICS & GEOMETRY - REMEMBERING GIUSEPPE MORANDI

ABSTRACTS

- ❖ Giuseppe Marmo - From trajectories to vector fields and commutation relations. 45 years of collaboration with Giuseppe
- ❖ José F. Cariñena - Killing vector fields and quantisation of natural Hamiltonians
The usual canonical prescription ordinarily made for the obtention of the quantum Hamiltonian operator for a classical system leads to some ambiguities in situations beyond the simplest ones and these ambiguities arise unavoidably when the configuration space has non-zero curvature, as well as in systems in Euclidean space but with a position-dependent mass. A recently proposed method to circumvent this difficulty for natural Hamiltonians will be described. The idea is not to quantise the coordinates and their (classical) conjugate momenta (which is where the ambiguities could arise), but to work directly with Killing vector fields and associated Noether momenta in order to get in some unambiguous way the corresponding Hamiltonian operator. The examples of one-dimensional position-dependent mass systems and motions on constant curvature surfaces will be used to illustrate the method.
- ❖ A. Ibort - Geometry and spaces of classical probability states
- ❖ Giovanni Venturi - An amusing analogy between spontaneous symmetry breaking in inner and outer space
- ❖ Guido Fano - Un ricordo personale
- ❖ Giorgio Velo - Strichartz estimates for the Schroedinger equation
A family of integral over space-time estimates of the solutions of the free Schroedinger equation will be presented. Their most important applications will be briefly described.
- ❖ Arianna Montorsi - Trivial and non trivial orders induced by interaction in low dimensional quantum systems
The appearance of gapped phases induced by interaction in the low energy regime of many one dimensional quantum Hamiltonians can be discussed through a bosonization analysis of two decoupled sine-Gordon models. Each phase is characterized by a non-vanishing nonlocal order. In case the latter is of Haldane type, the phase also exhibits fractionalized charge/spin edge modes protected by particle-hole symmetry and charge/spin conservation. Such analysis is in one to one correspondence with the group cohomology classification of symmetry protected trivial and non-trivial topological phases. Generalizations to higher dimensions can be envisaged.
- ❖ Arturo Tagliacozzo - From Quantum Hall effect to Majorana Fermions in Condensed Matter
The Hall conductance of the Integer Quantum Hall effect has a fundamental topological meaning as first pinpointed by Thouless, Kohmoto, Nightingale, and den Nijs (TKNN, 1982). It is expressed as an integral that represents the Chern number of the U(1) bundle formed by the Bloch functions over the magnetic Brillouin zone. This is an invariant on the magnetic Brillouin zone and supports the high accuracy in the universal quantization of the Hall conductance. A direct connection has been established between the Hall conductance in the bulk system and that due to the edge states. Even in the case with a weak disorder, the quantization of the Hall conductance due to edge states is stable since they are chiral. The correspondence bulk-edges has been taken over in the Quantum Spin Hall 2D-systems, in the 3D-Topological Insulators which have Dirac states in the bulk gap, crossing the Fermi energy, localized at the boundaries and in Weyl semimetal. Proximity with a superconductor may turn these states into Majorana fermion zero energy excitations. A quick overview of these developments and applications will be presented, starting from the booklet on QHE by G.Morandi (Bibliopolis, 1988).
- ❖ Pierbiagio Pieri - Transition temperature of a superfluid Fermi gas throughout the BCS-BEC crossover
It has long been known that the critical temperature of a BCS weak-coupling superfluid gets considerably reduced when the Gorkov-Melik-Barkhudarov (GMB) correction is considered, whereby particle-hole excitations affect the two-fermion scattering in the medium responsible for the formation of Cooper pairs. I will discuss how the effects of the GMB correction can be included throughout the BCS-BEC crossover by inserting the Feynman diagram responsible for the GMB correction in a partially self-consistent T-matrix scheme. The resulting equations are solved numerically and yield a curve for the critical temperature which agrees very well with the Quantum Monte Carlo data available in the crossover region.
- ❖ Loris Ferrari - Vacuons and quasi-phonons. The hidden side of Bogolyubov theory
In a gas of N interacting bosons, the Hamiltonian H_C , obtained by dropping all the interaction terms between free bosons with $\hbar k = 0$, is diagonalized exactly. The resulting eigenstates $|S, k, \eta\rangle$ depend on two discrete indices $S, \eta = 0, 1, \dots$, where η numerates the quasiphonons carrying a moment $\hbar k$, responsible for transport or dissipation processes. S , in turn, numerates a ladder of 'vacua' $|S, k, 0\rangle$, with increasing equispaced energies, formed by boson pairs with opposite moment. Passing from one vacuum to another ($S \rightarrow S \pm 1$), results from creation/annihilation of new moment-less collective excitations, that we call vacuons. Exact quasiphonons originate from one of the vacua by 'creating' an asymmetry in the number of opposite moment bosons. The well known Bogoliubov collective excitations (CEs) are shown to coincide with the exact eigenstates $|0, k, \eta\rangle$, i.e. with the quasiphonons (QPs) created from the lowest-level vacuum ($S = 0$). All this is discussed, in view of existing or future experimental observations of the vacuons, a sort of bosonic Cooper pairs, which are the main factor of novelty beyond Bogoliubov theory.

❖ Piero Naldesi - Quantum simulation and many-body localization

The many-body localization transition is a dynamical quantum phase transition of a disordered quantum many-body Hamiltonian, which evolves from an ergodic to a localized phase, where laws of statistical mechanics do not hold anymore. The nature of this transition, that involves highly excited eigenstates of the system, is nowadays one of the hardest challenges for numerical simulations.

❖ Jesús Clemente-Gallardo - Geometric description of open quantum systems

We review the tensorial description of geometric quantum mechanics and use it to build a geometric description of the space of states of a finite-dimensional quantum system and of the Markovian evolution associated with the Kossakowski-Lindblad operator. From that construction, we are able to encode the Markovian evolution as dynamical system defined on the space of Poisson and Jordan tensor fields on the set of states. We also discuss within this framework some approaches used in the Literature to describe open quantum systems.

❖ Alessandro Zampini - Examples of Hodge - de Rham Dirac operators on non commutative algebras

❖ Davide Vodola - Qubit Losses in Topological Color Codes

Quantum information is the generalisation of the classical information theory to quantum systems. If for a classical computer the bits are the fundamental units, quantum computers are built on an analogous concept, the quantum bits that are currently implemented in different physical systems (atoms, ions, molecules) due to recent experimental advances in the control of cold and ultra-cold gases. However, these experimental setups are so delicate that qubits can be corrupted by environmental noise and even get completely lost from the apparatus resulting in the partial or total loss of the information memorised. In this talk, we will consider how losses can affect a particular class of quantum code (the so-called color codes). We introduce a protocol for dealing with losses and we show that checking whether the logical information is still recoverable is equivalent to a generalized percolation process.

❖ Manuel Asorey - Bulk-edge dualities in topological matter

The conducting properties of topological materials are characterized by topological invariants. In some cases there are dual characterizations of these properties, either in terms of bulk or edge topological invariants. We clarify in some cases the relations between both characterizations.

❖ Davide Pastorello - Geometry of quantum mechanics in complex projective spaces

This talk is focused on the geometric Hamiltonian formulation of finite-dimensional quantum mechanics where the projective Hilbert space (as a Kähler manifold) plays the role of phase space. Within such a framework quantum observables are represented by phase space functions, quantum states are described by Liouville densities (phase space probability densities), and Schrödinger dynamics is induced by the flow of a Hamiltonian vector field w.r.t. a natural symplectic structure. Then I will discuss how this viewpoint leads to a new approach to quantum control theory based on the Riemannian structure of the projective space.

❖ Mauro Spera - Remarks on Landau levels, braid groups and Laughlin wave functions

In this talk, after presenting the approach of [1] to the holomorphic geometric quantization of a charged particle on a plane subject to a constant magnetic field perpendicular to the latter (see also [2] and the original source [3]), we shall discuss the geometric approach to unitary Riemann surface braid group representations via stable holomorphic bundles on Jacobians developed in [5] and the ensuing construction of generalized Laughlin wave functions.

Physical aspects will be emphasised throughout. The lucid and enlightening monograph [4] provided constant inspiration in carrying out this research programme.

[1] A. Galasso and M. Spera: "Remarks on the geometric quantization of Landau levels", *Int.J.Geom.Meth.Mod.Phys.* 13 (10) (2016), 1650122 (19 pages).

[2] J. Klauder and E. Onofri: Landau levels and geometric quantization, *Int.J.Modern Phys. A* 4 (1989) 3939–3949.

[3] L. D. Landau: "Diamagnetismus der Metalle", *Z. Phys.* 64 (1930) 629.

[4] G. Morandi: "Quantum Hall Effect", Bibliopolis, Naples, 1988.

[5] M. Spera: "On the geometry of some unitary Riemann surface braid groups representations and Laughlin-type wave functions", *J.Geom.Phys.* 94 (2015), 120-140.

❖ Annalisa Marzuoli - Projective spin networks and their symmetries

The 1968 paper 'Semiclassical limit of Racah coefficients' by Giorgio Ponzano and Tullio Regge stands at the basis of a lot of developments and applications in quantum gravity, low-dimensional geometric topology, integrable quantum many-body systems and quantum computing. I will highlight a few algebraic features of the Ponzano_Regge model with an emphasis on the role of Regge symmetries of the $6j$ symbol framed within an improved interpretation in terms of Desargues configuration.

❖ Valter Moretti - Why does relativistic quantum mechanics need complex Hilbert spaces?

Quantum Theories, in view of Piron-Solér's theorem can be formulated in real, complex or quaternionic Hilbert spaces. No evidence of either real or quaternionic Hilbert spaces exists however. We prove how, for elementary relativistic systems, Poincaré symmetry (viewed as a continuous symmetry of the lattice of elementary propositions) as well as some characterization result on irreducible (real/complex/quaternionic) von Neumann algebras and continuous unitary (real/complex/quaternionic) representation theory of Lie groups force the theory to be formulated in a complex setup, ruling out the real and quaternionic case.

[The talk is based on two joint works with M. Oppio: *Rev. Math. Phys.* 29 (2017) arXiv:1611.09029 and arXiv:1709.09246]

❖ Paolo Facchi - Boundaries without boundaries

Starting with a quantum particle on a closed manifold without boundary, we consider the process of generating boundaries by modding out by a group action with fixed points, and we study the emergent quantum dynamics on the quotient manifold.

As an illustrative example, we consider a free nonrelativistic quantum particle on the circle and generate the interval via parity reduction. A free particle with Neumann and Dirichlet boundary conditions on the interval is obtained, and, by changing the metric near the boundary, Robin boundary conditions can also be accommodated.

We also indicate a possible method of generating non-local boundary conditions. Then, we explore an alternative generation mechanism which makes use of a folding procedure and is applicable to a generic Hamiltonian through the emergence of an ancillary spin degree of freedom.

[Joint work with: Giancarlo Garnero, Giuseppe Marmo, Joseph Samuel, and Supurna Sinha]