IWM-EC, Catania, May 22-25, 2018

Few-nucleon correlations in nuclei and nuclear matter

Gerd Röpke, Rostock

Outline

1. Quantum statistical approach to nuclear systems
2. Light elements and nuclear matter equation of state
3. Quartetting wave function and alphas in nuclei
4. Open questions
   (heavy nuclei, phase transition, transport models)

Problem: single (quasi-) particle approach
to describe the properties of nuclear systems
(mean-field approximation).
Are correlations of relevance? How to calculate?
Pauli principle: antisymmetrization of fermionic wavefunction

1. Quantum statistical approach to nuclear systems

- Nuclear systems: structure of (excited) nuclei, heavy ion collisions, compact objects in astrophysics

- Interaction? No fundamental expression, fitted to data

- Many-body system (strong interaction, quantum, Pauli principle), bound states (nuclei), Bose-Einstein condensation, phase transition

- QS approach: Green function method (numerical simulation)
  correlation functions, spectral function, self-energy, cluster decomposition

- Other fields in physics: plasma physics, semiconductor physics, Ultra-cold atoms in traps, quark-gluon plasma

- Nonequilibrium – (local) thermodynamic equilibrium

Nonequilibrium statistical operator

principle of weakening of initial correlations (Bogoliubov)

$$\rho_\epsilon(t) = \epsilon \int_{-\infty}^{t} e^{\epsilon(t_1-t)} U(t,t_1) \rho_{\rm rel}(t_1) U^\dagger(t,t_1)\, dt_1$$

time evolution operator $U(t,t_0)$

relevant statistical operator $\rho_{\rm rel}(t)$ maximum of information entropy

selection of the set of relevant observables $\{B_n\}$

self-consistency relations $\mathrm{Tr}\{\rho_{\rm rel}(t) B_n\} \equiv \langle B_n \rangle^t_{\rm rel} = \langle B_n \rangle^t$

extended von Neumann equation

$$\frac{\partial}{\partial t} \varrho_\varepsilon(t) + \frac{i}{\hbar} [H, \varrho_\varepsilon(t)] = -\varepsilon \left(\varrho_\varepsilon(t) - \varrho_{\rm rel}(t) \right)$$

$\varrho(t) = \lim_{\varepsilon \to 0} \varrho_\varepsilon(t)$ after thermodynamic limit

Many-particle theory

Equation of state
$$n_\tau^{\rm tot}(T,\mu_n,\mu_p) = \frac{1}{\Omega}\sum_{p_1,\sigma_1}\int\frac{d\omega}{2\pi}\frac{1}{e^{(\omega-\mu_\tau)/T}+1}S_\tau(1,\omega)$$

Spectral function
$$S_\tau(1,\omega;T,\mu_n,\mu_p) \qquad E(1)=\hbar^2p_1^2/2m_1$$

Green function G,
Self-energy Σ
$$S(1,\omega)=2{\rm Im}\,G(1,\omega+i0)=2{\rm Im}\frac{1}{\omega-E(1)-\Sigma(1,\omega+i0)}$$

$$S_\tau(1,\omega)=\frac{2{\rm Im}\Sigma(1,\omega-i0)}{(\omega-E(1)-{\rm Re}\Sigma(1,\omega))^2+({\rm Im}\Sigma(1,\omega-i0))^2}$$

Expansion for small damping (Im Σ)
$$S(1,\omega)\approx\frac{2\pi\delta(\omega-E^{\rm quasi}(1))}{1-\frac{d}{dz}{\rm Re}\,\Sigma(1,z)|_{z=E^{\rm quasi}(1)}}-2{\rm Im}\,\Sigma(1,\omega+i0)\frac{d}{d\omega}\frac{\mathcal{P}}{\omega-E^{\rm quasi}(1)}$$

Quasiparticle energy
$$E^{\rm quasi}(1)=E(1)+{\rm Re}\,\Sigma(1,z)|_{z=E^{\rm quasi}(1)}$$

Correlations (bound states) in Im Σ
Cluster decomposition, Bethe-Salpeter equation

Different approximations

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

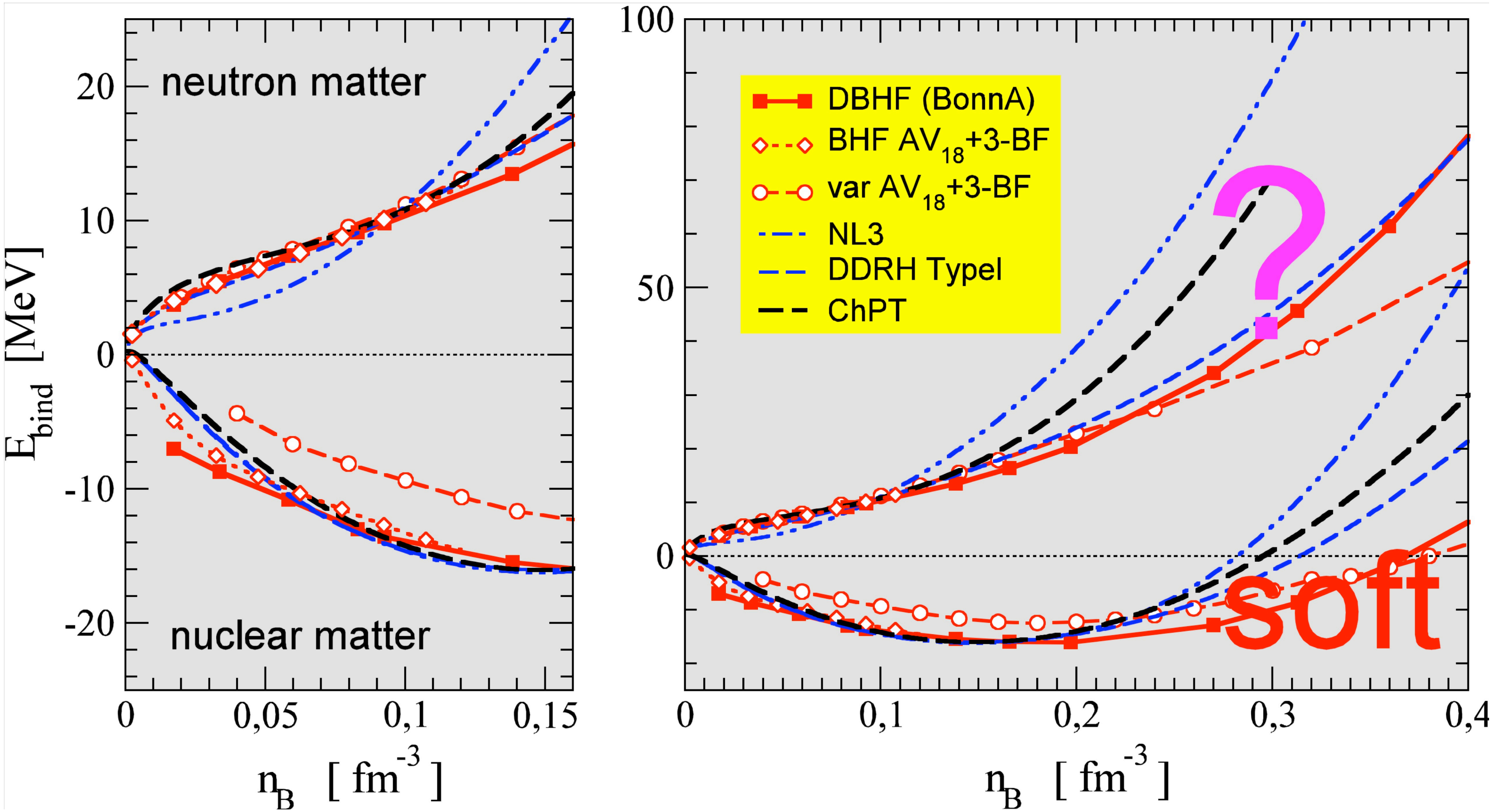
Different approximations

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

medium effects

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

Quasiparticle picture: RMF and DBHF



But: cluster formation

Incorrect low-density limit

C. Fuchs et al.;
J.Margueron et al., Phys.Rev.C **76**,034309 (2007)

Different approximations

medium effects

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

bound state formation

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

Inclusion of the light clusters (d,t,^{3}He,^{4}He)

Ideal mixture of reacting nuclides

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$
$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A,
charge Z_A,
energy $E_{A,\nu,K}$,
ν internal quantum number,
~K center of mass momentum

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

Chemical equilibrium, mass action law,
Nuclear Statistical Equilibrium (NSE)

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium with quasiparticle clusters:
self-energy and Pauli blocking

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

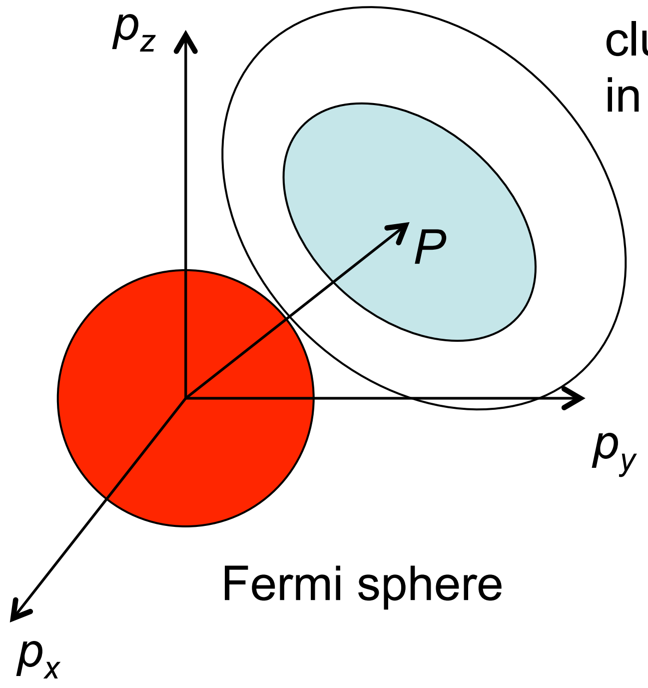
$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

Pauli blocking – phase space occupation



cluster wave function (deuteron, alpha,…)
in momentum space

P - center of mass momentum

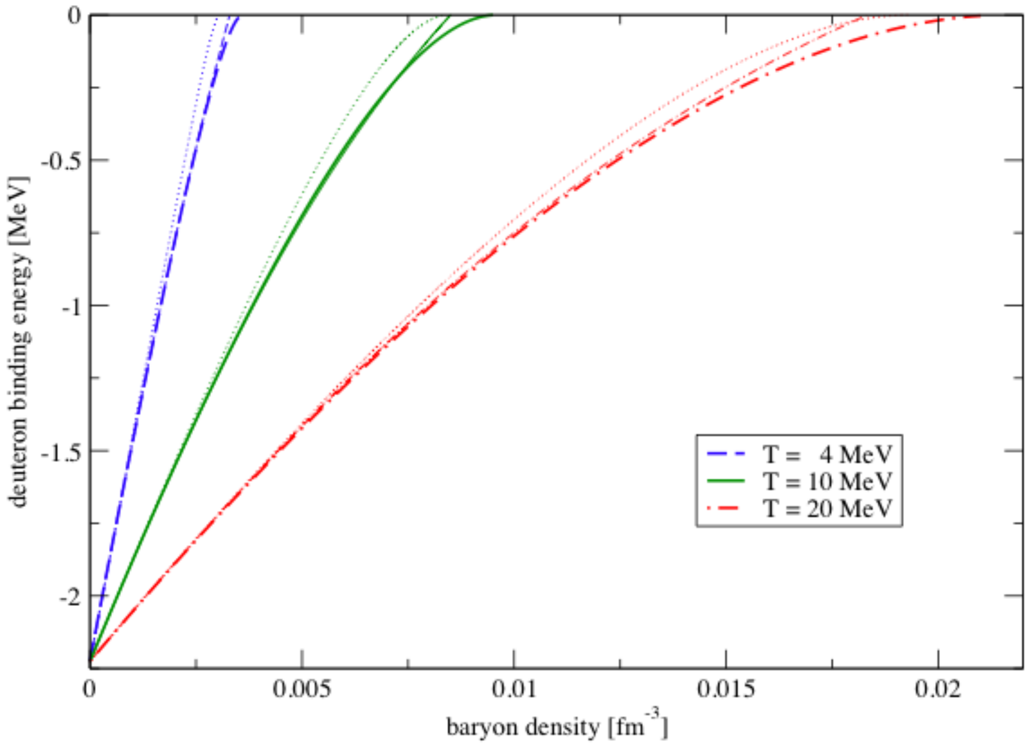
The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum *P*

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Shift of the deuteron bound state energy

Dependence on nucleon density, various temperatures,
zero center of mass momentum



thin lines:

fit formula

G.R., Nucl. Phys. A **867**, 66 (2011)

Few-particle Schrödinger equation in a dense medium

4-particle Schrödinger equation with medium effects

(self-energy shifts and Pauli blocking)

$$\left(\left[E^{HF}(p_1)+E^{HF}(p_2)+E^{HF}(p_3)+E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

$$+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4)$$

$$+\{permutations\}$$

$$=E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

Composition of dense nuclear matter

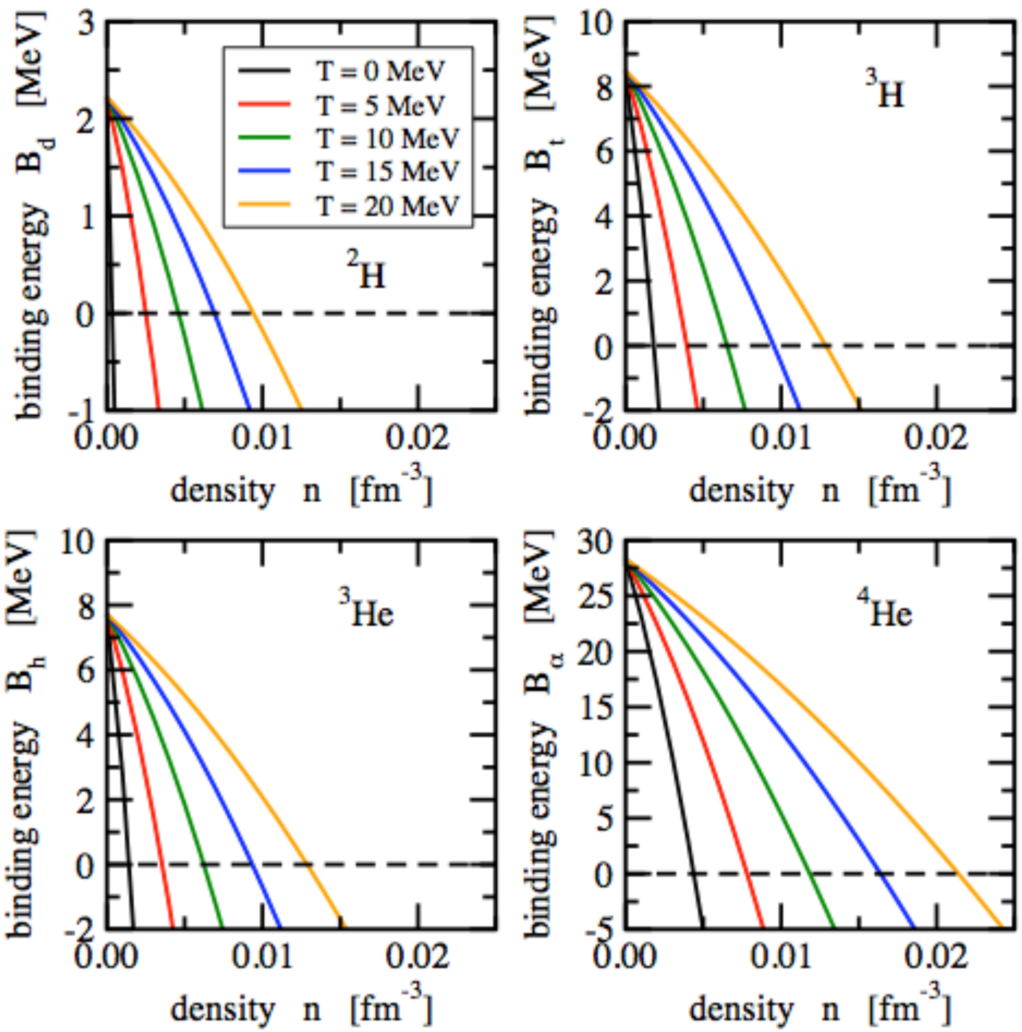
$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$
$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$
ν: internal quantum number

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

- Medium effects: correct behavior near saturation
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz, Debye)

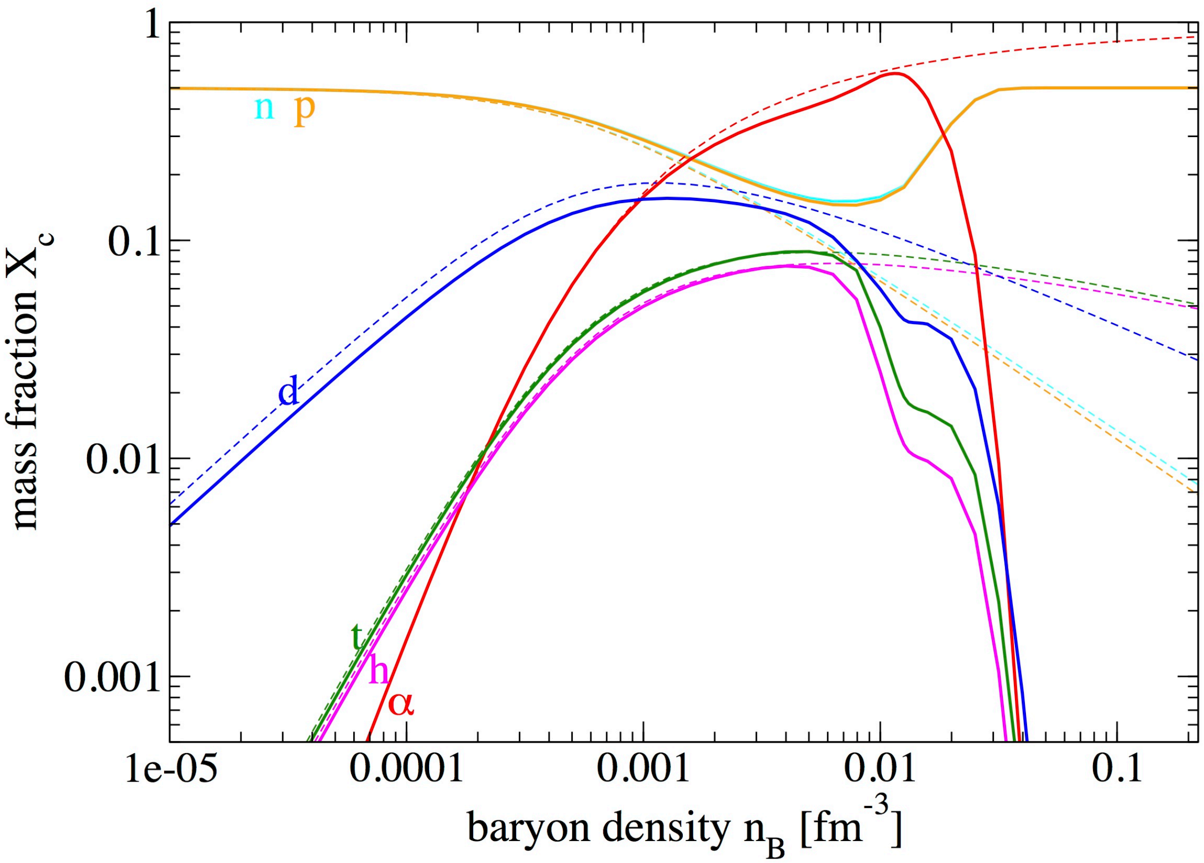
Shift of Binding Energies of Light Clusters



Symmetric matter

G.R., PRC **79**, 014002 (2009)
S. Typel et al.,
PRC **81**, 015803 (2010)

Light Cluster Abundances



Composition of symmetric matter in dependence on the baryon density n_B, T = 5 MeV.
Quantum statistical calculation (full) compared with NSE (dotted).

G. R., PRC **92**, 054001 (2015)

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium
of quasiparticle clusters:
self-energy and Pauli blocking

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.Eq.

Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

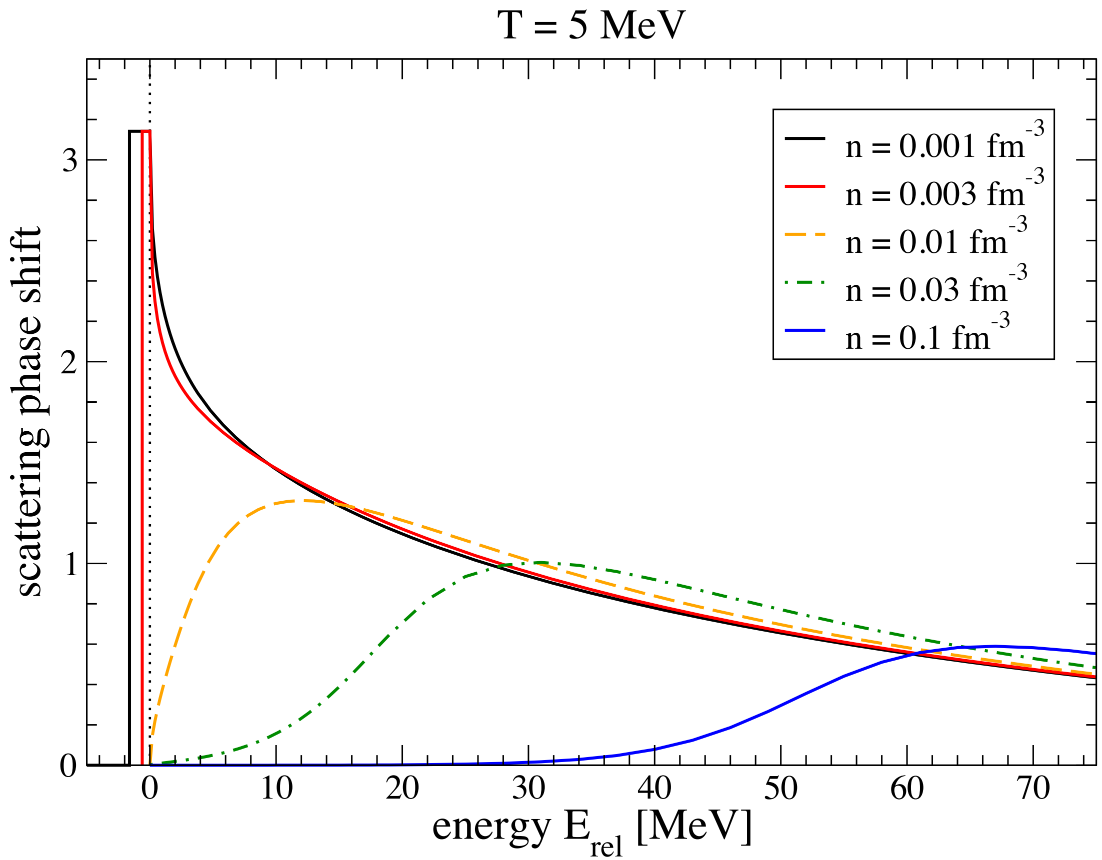
chemical & physical picture

Cluster virial approach:
all bound states (clusters)
scattering phase shifts of all pairs

Correlated medium:
phase space occupation by all bound states
in-medium correlations, quantum condensates

Deuteron-like scattering phase shifts

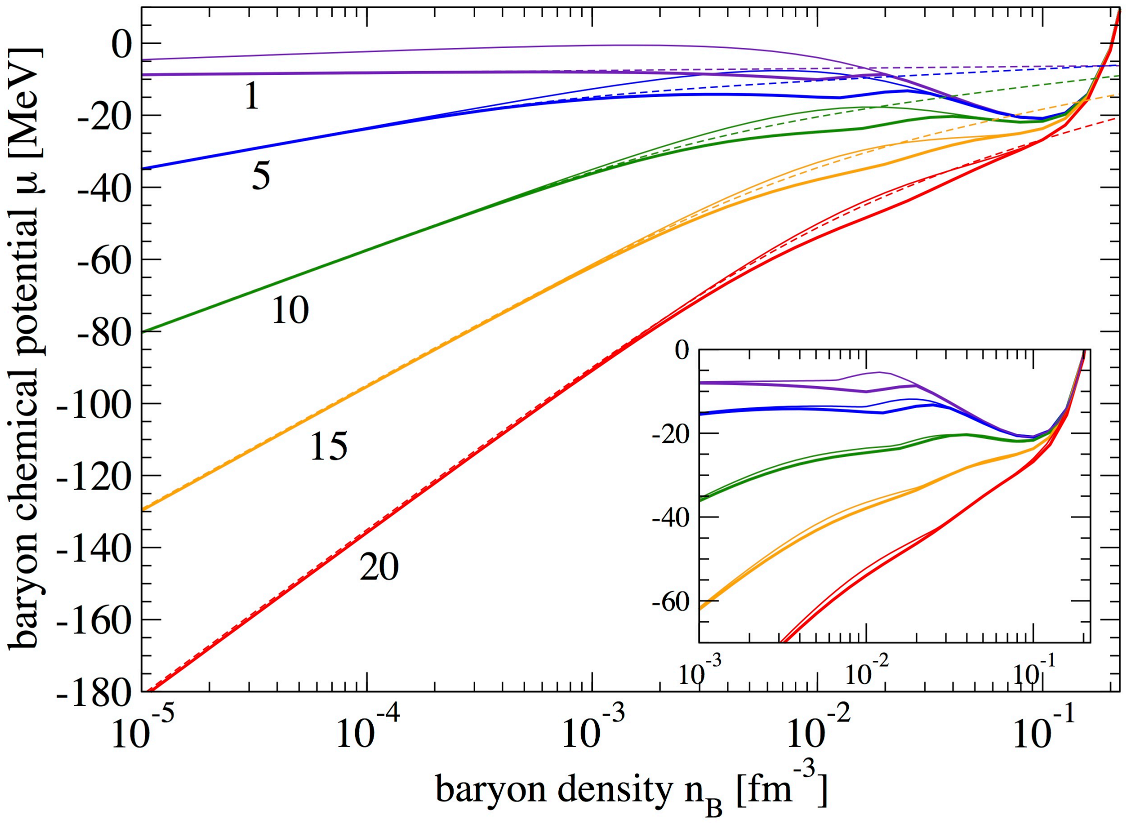
Virial coeff. $\propto e^{-E_d^0/T} - 1 + \frac{1}{\pi T}\int_0^\infty dE\; e^{-E/T}\left\{\delta_c(E) - \frac{1}{2}\sin[2\delta_c(E)]\right\}$



deuteron bound state -2.2 MeV

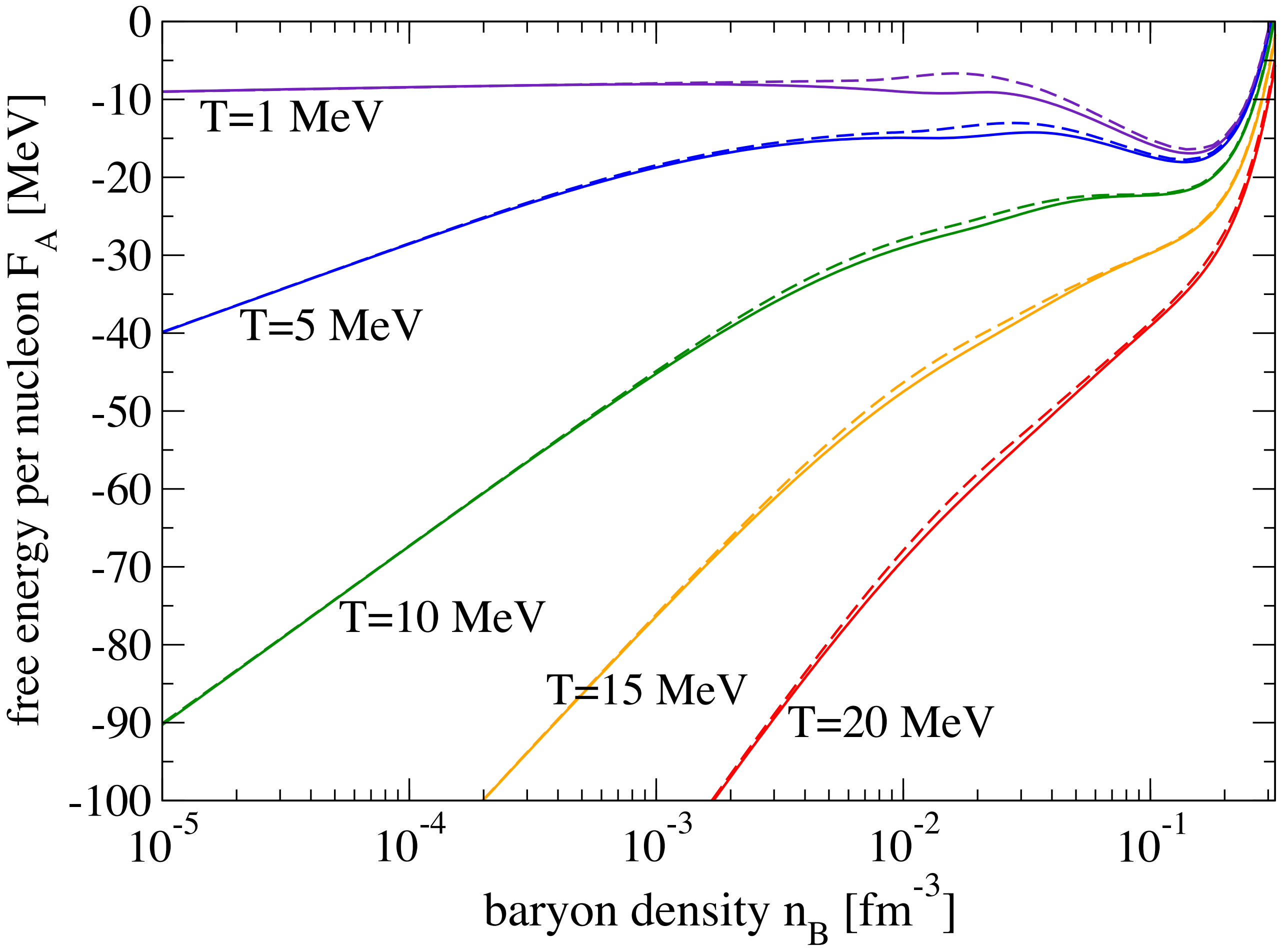
G. R., J. Phys.: Conf. Series 569, 012031 (2014).

Equation of state: chemical potential



Chemical potential for symmetric matter. T=1, 5, 10, 15, 20 MeV.
QS calculation compared with RMF (thin) and NSE (dashed).
Insert: QS calculation without continuum correlations (thin lines).

Symmetric matter: free energy per nucleon



Dashed lines: no continuum correlations

G. R., PRC **92**, 054001 (2015)

2. Heavy ion collisions

EoS at low densities from HIC

PRL **108,** 172701 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

Laboratory Tests of Low Density Astrophysical Nuclear Equations of State

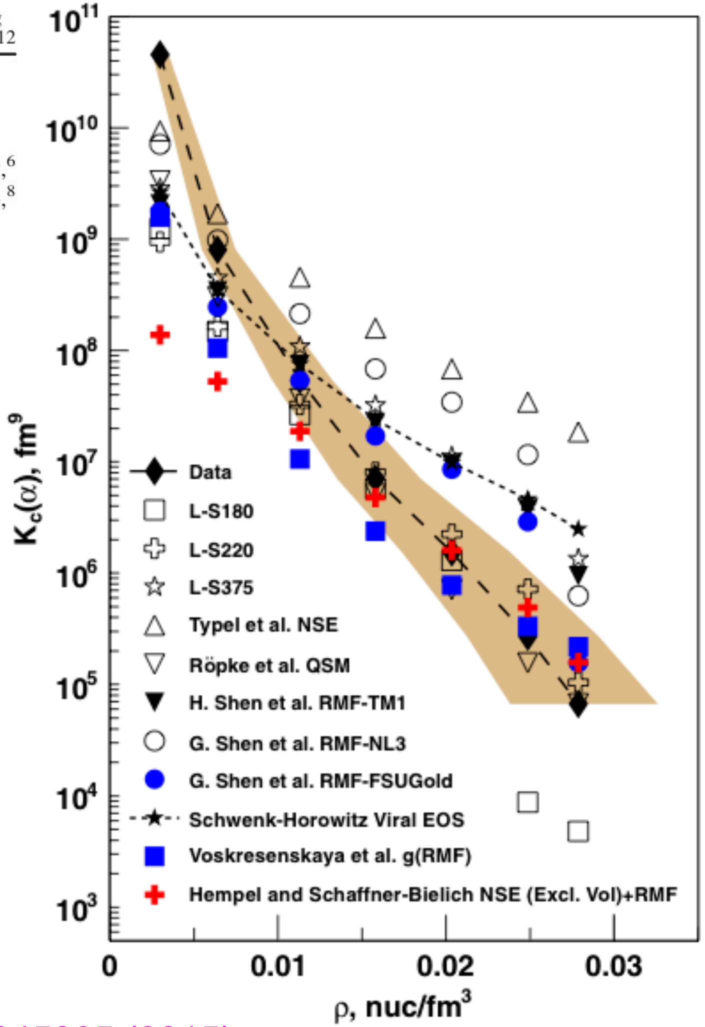
L. Qin,[1] K. Hagel,[1] R. Wada,[2,1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[6] M. Huang,[6] J. Wang,[6] H. Zheng,[1] S. Kowalski,[7] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] Z. Majka,[11] and Y. G. Ma[12]

Yields of clusters from HIC: p, n, d, t, h, α

chemical constants

$$K_c(A, Z) = \rho_{(A,Z)}/[(\rho_p)^Z(\rho_n)^N]$$

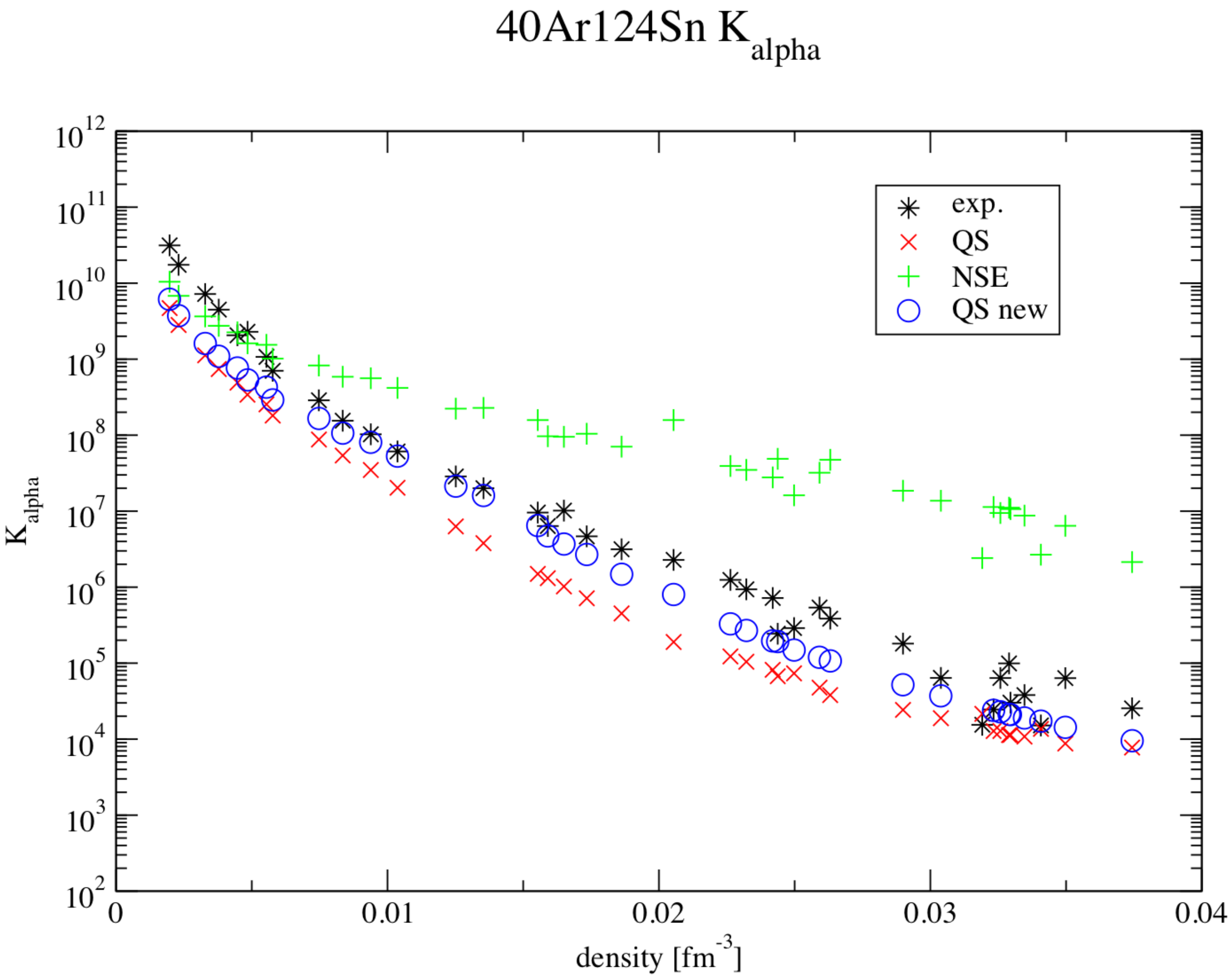
inhomogeneous,
non-equilibrium



Quantum statistics (QS), excluded volume

M. Hempel, K. Hagel, J. Natowitz, G. R., S. Typel, Phys. Rec. C 91, 045805 (2015)

QS versus NSE: comparison with data



QS new: with continuum correlations

Generalized RMF

$$\mathcal{L} = \sum_{j=n,p,d,t,h,\alpha} \mathcal{L}_j + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\omega\rho}$$

Effective Lagrangian:
quasiparticle nuclei as
new degrees of freedom

$$M_j^* = A_j m - g_{sj}\phi_0 - \left(B_j^0 + \delta B_j\right).$$

Coupling to the meson fields
depending on A

$$g_{sj} = x_{sj} A_j g_s$$

x_{sj}=0.85 for A > 1

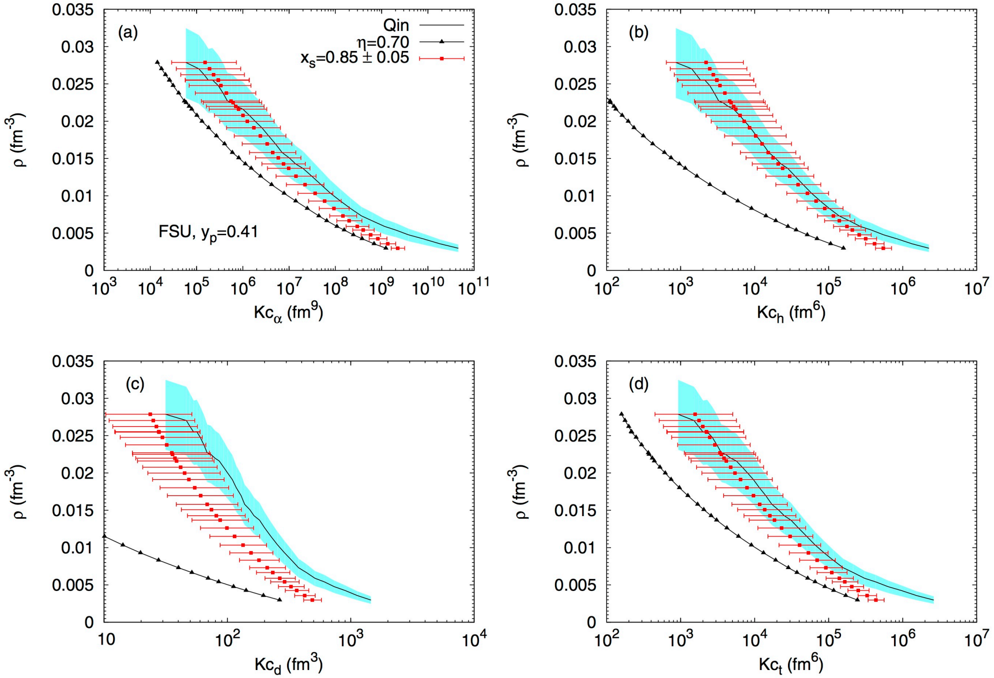
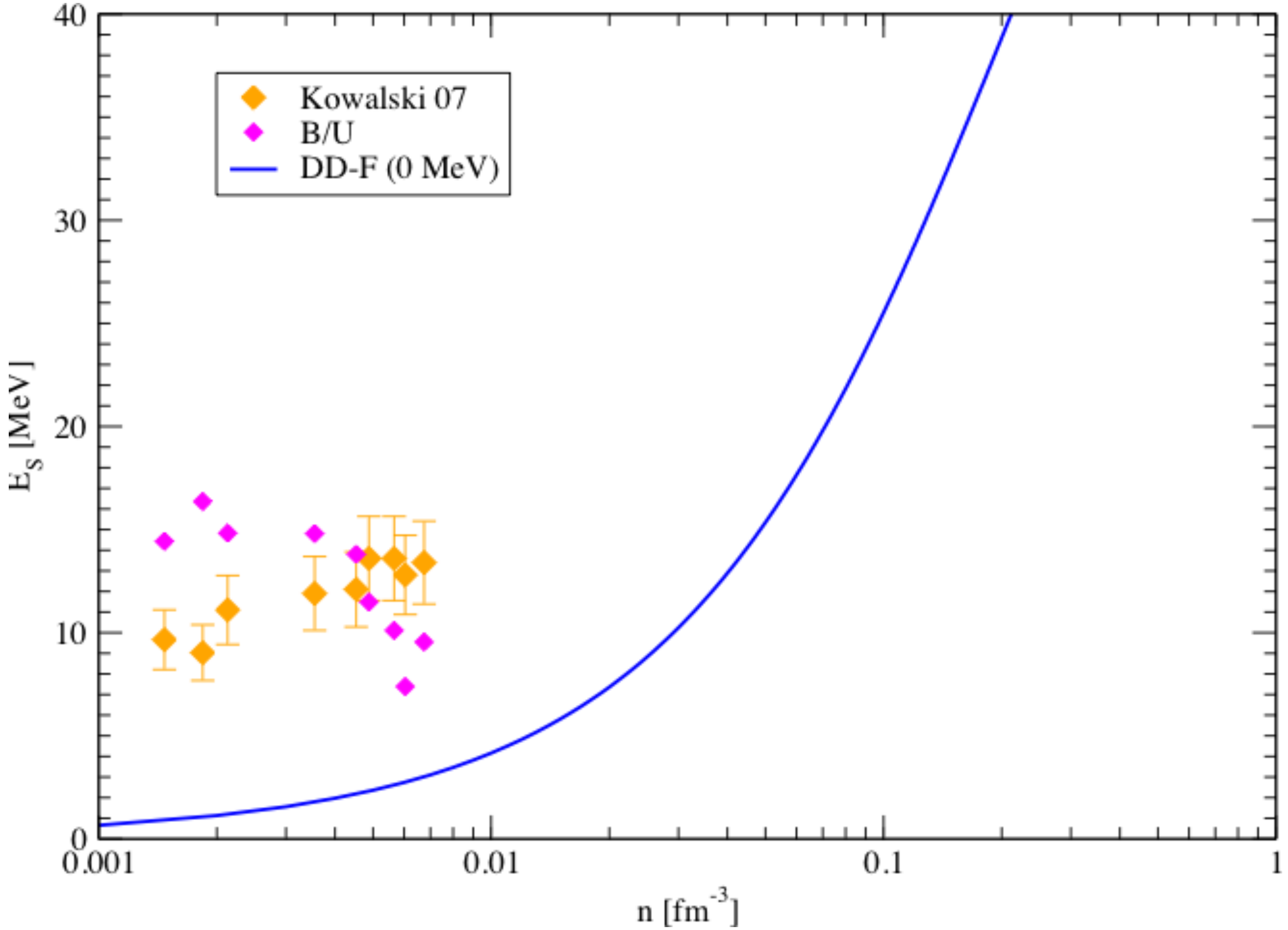


FIG. 7. Chemical equilibrium constants of α (a), helion (b), deuteron (c), and triton (d) for FSU, and $y_p = 0.41$, and for the $\eta = 0.70$ (black squares) fitting (check Ref. [17] for the complete parameter sets) and the universal g_{sj} fitting with $g_{sj} = (0.85 \pm 0.05)A_j g_s$, (red dotted lines). The experimental results of Qin *et al.* [18] (light blue region) are also shown.

H. Pais, F. Gulminelli, C. Providencia, G. R., Phys. Rev. C **97**, 045805 (2018)

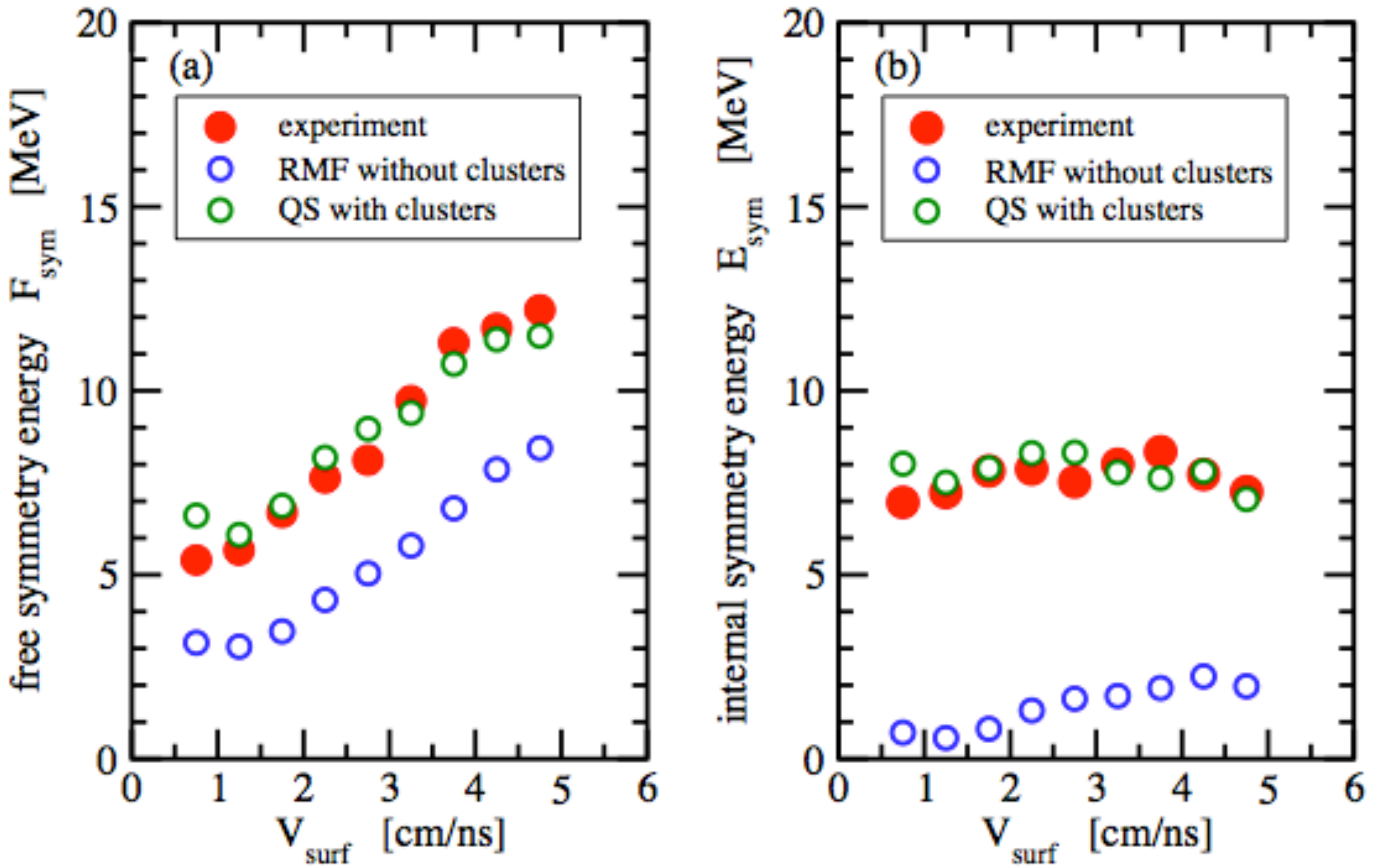
Symmetry energy

Heavy-ion collisions, spectra of emitted clusters, temperature (3 - 10 MeV), free energy



S. Kowalski et al., PRC **75**, 014601 (2007)

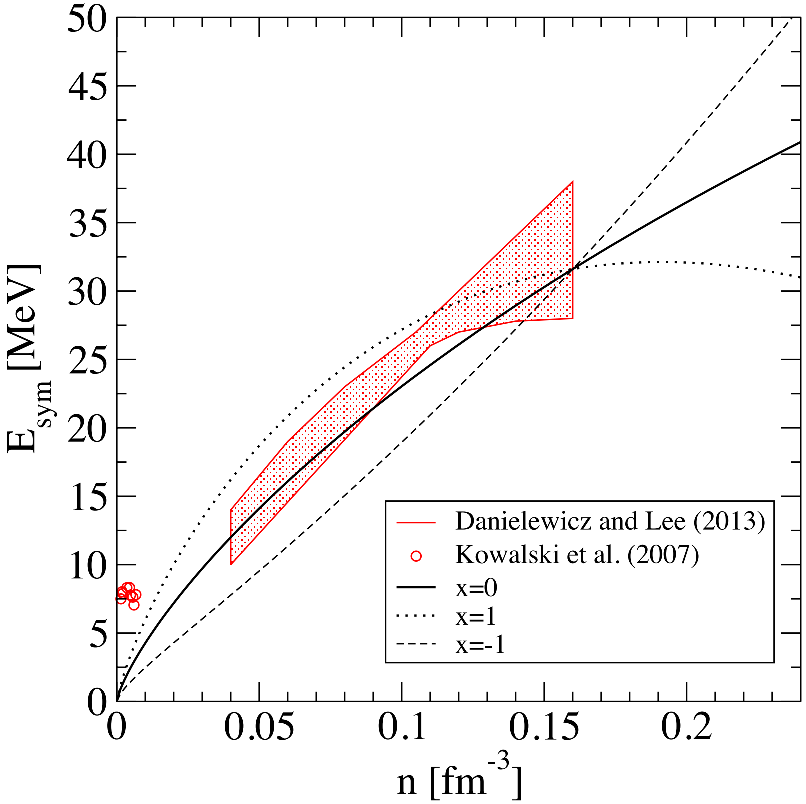
Symmetry energy, comparison experiment with theories

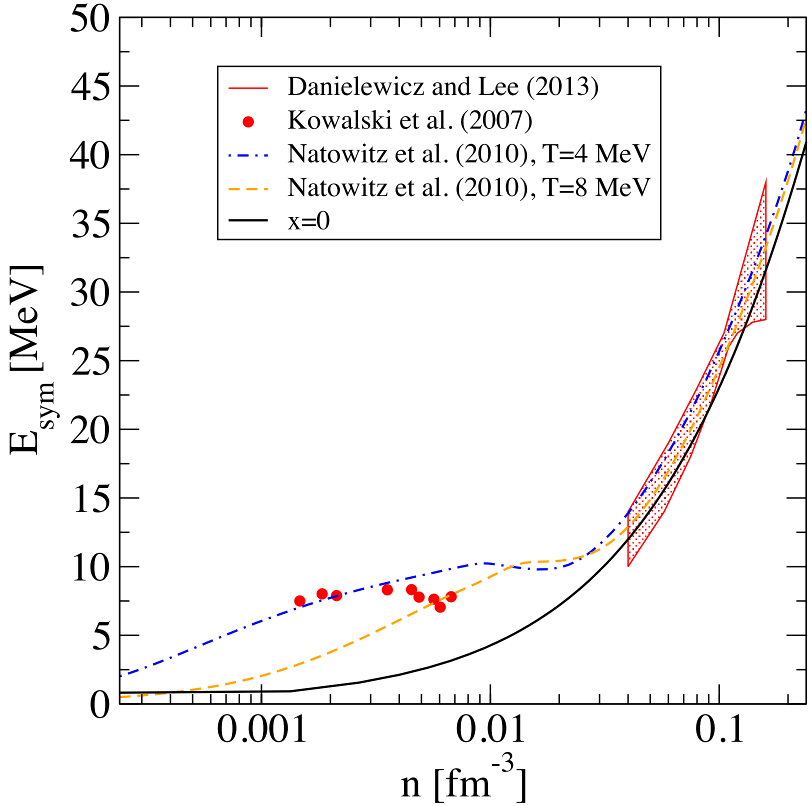


J.Natowitz et al., PRL 2010

Symmetry energy: low density limit

correlations (bound states) → larger values for the symmetry energy





K. Hagel et al., Eur. Phys. J. A (2014) 50: 39

Intermediate-mass fragment production

density value of $\rho/\rho_0 = 0.56$ from a previous analysis [26], the temperature and symmetry energy values of $T = 4.6 \pm 0.4$ MeV and $a_{\text{sym}} = 23.6 \pm 2.1$ MeV are extracted. These

X. Liu et al., Phys. Rev. C 95, 044601 (2017)

Zhao-Wen Zhang, Lie-Wen Chen,
Phys. Rev. C 95, 064330 (2017);

J. A. Lopez, S. Terrazas Porras,
Nucl. Phys. A 957, 312 (2017)

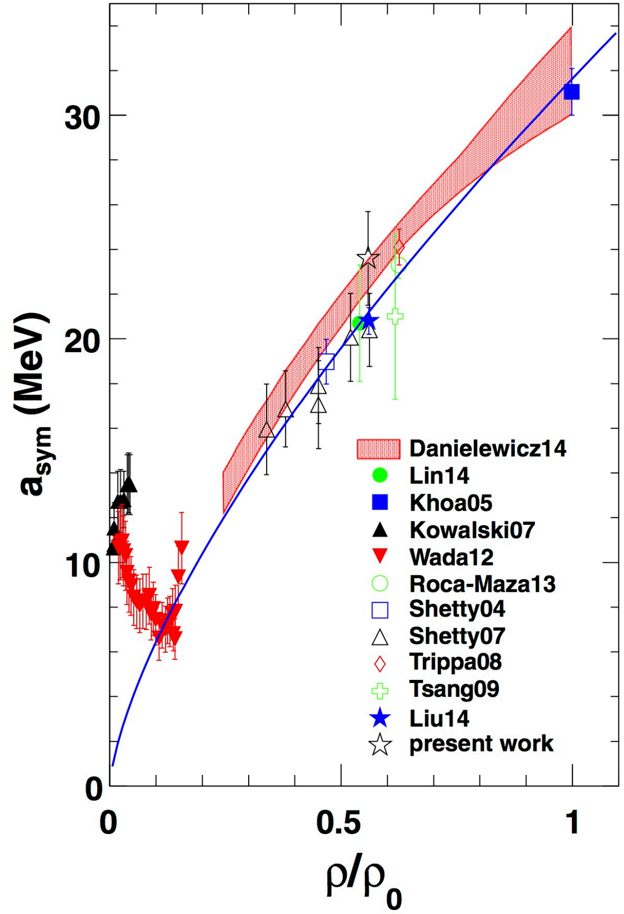


FIG. 10. Summary of the density dependent symmetry energy obtained in the present and previous studies. The line is the fit of the existing data points at $0.1 \leqslant \rho/\rho_0 \leqslant 1.0$ using Eq. (14).

Landau Fermi liquid

Strongly degenerate Fermi system: excitations near the Fermi energy, well-defined quasiparticles

Inverse of compressibility, T=0

$$K = n \frac{\partial \mu}{\partial n}\Big|_T = \frac{p_F^2}{3E_F}(1+f_0)$$

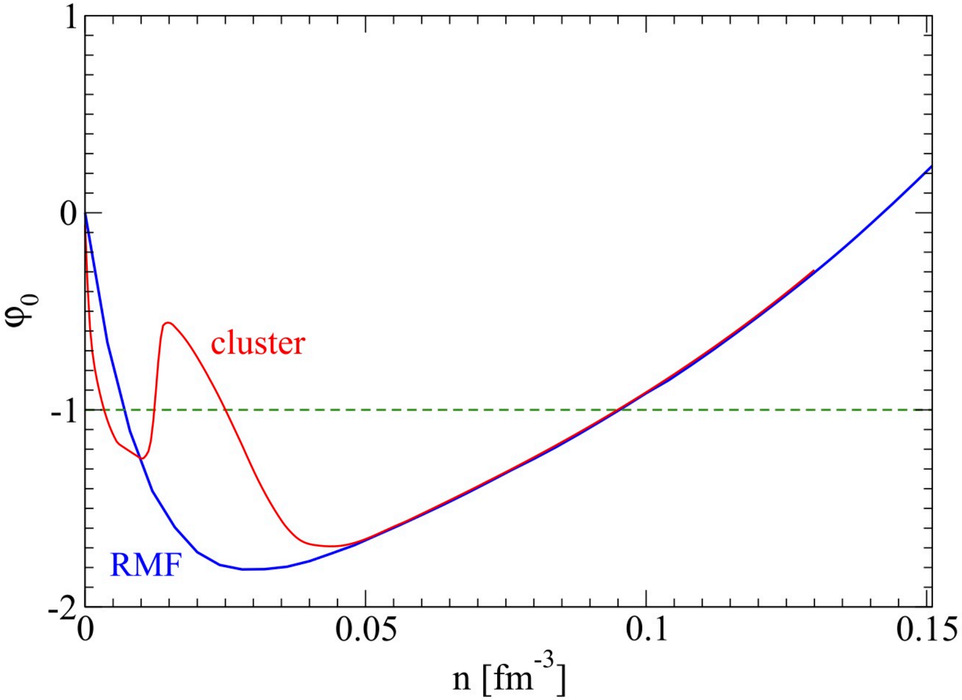
Landau-Migdal parameter f_0 (T=0)

General case, correlations, finite T

$$K(T,n) = K^{(0)}(T,n)[1+\varphi_0(T,n)]$$

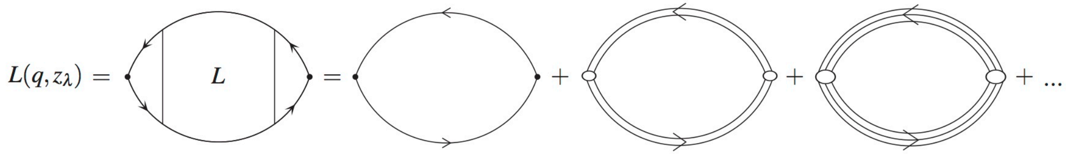
fluctuation-dissipation theorem

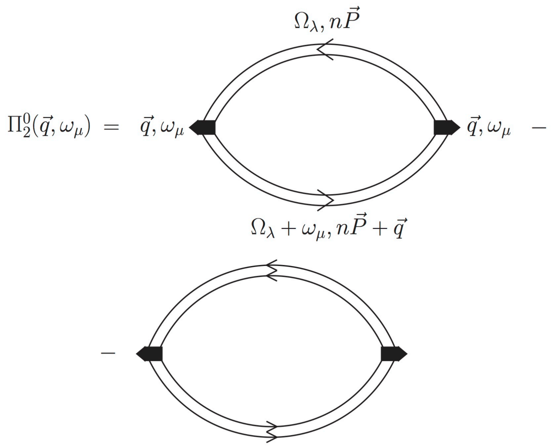
response function χ(q, ω)

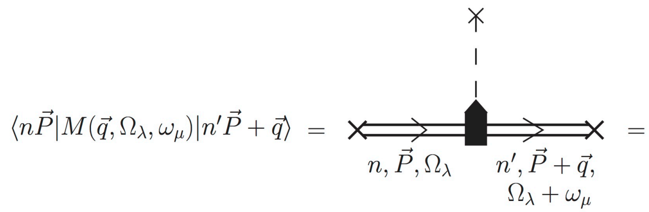


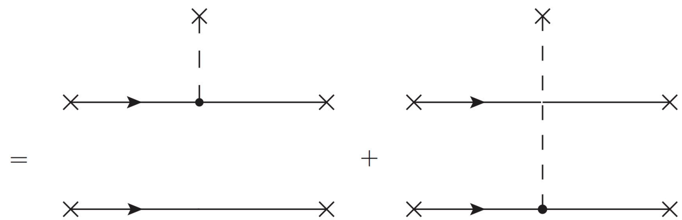
G. R., D.N. Voskresensky, I.A. Kryukov, D. Blaschke, Nucl. Phys. A 970, 224 (2018)

Cluster decomposition of the polarization function





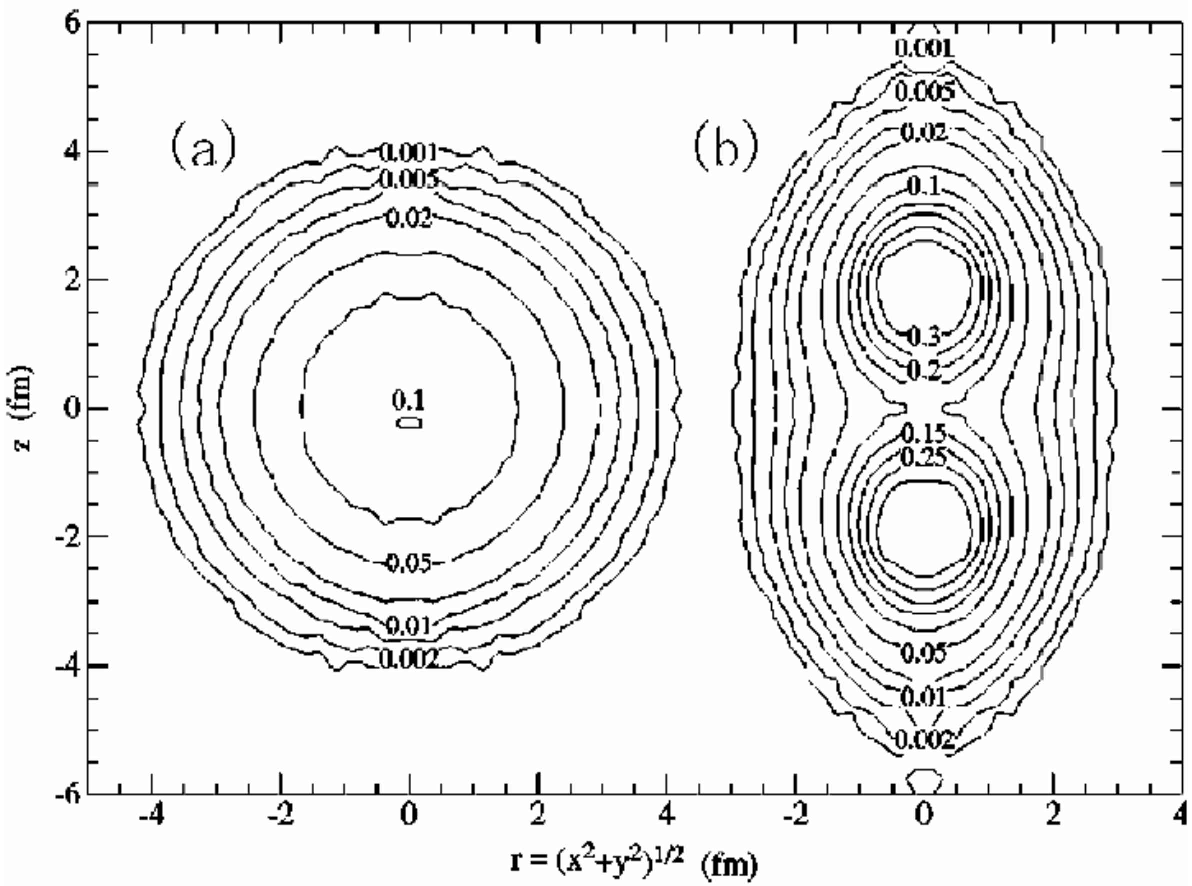




$$M_{\nu\nu'}(\mathbf{q}) = \langle \nu, \mathbf{P} | M(\mathbf{q}, z_\lambda, z_\mu) | \nu', \mathbf{P} + \mathbf{q} \rangle = \sum_{\mathbf{p}_1, \mathbf{p}_2} \psi^*_{\nu, \mathbf{P}}(p_1, p_2) [\psi_{\nu', \mathbf{P}+\mathbf{q}}(\mathbf{p}_1 + \mathbf{q}, \mathbf{p}_2) + \psi_{\nu', \mathbf{P}+\mathbf{q}}(\mathbf{p}_1, \mathbf{p}_2 + \mathbf{q})]$$

$$\kappa_{\rm iso}^{\rm (BU)}(T, \mu_n, \mu_p) = \frac{\beta}{\Omega_0 n_B^2} \left\{ \sum_{\mathbf{p}} f_p^0 (1 - f_p^0) + \sum_{\alpha, \mathbf{P}} \int_{-\infty}^{\infty} \frac{dE}{\pi} f_2 \left(E + \frac{P^2}{4m} \right) \left[1 + f_2 \left(E + \frac{P^2}{4m} \right) \right] D_{\alpha, \mathbf{P}}(E) \right\}$$

3. α cluster structures in nuclei



^{8}Be

R.B. Wiringa et al., PRC **63**, 034605 (01)

Contours of constant density, plotted in cylindrical coordinates, for ^{8}Be(0+) .
The left side is in the laboratory frame while the right side is in the intrinsic frame.

The Hoyle state in ^{12}C

^{12}C: from astrophysics: excited state predicted near the 3 α threshold energy (F. Hoyle).

a 0^+ state at 0.39 MeV above the 3 α threshold energy has been found.

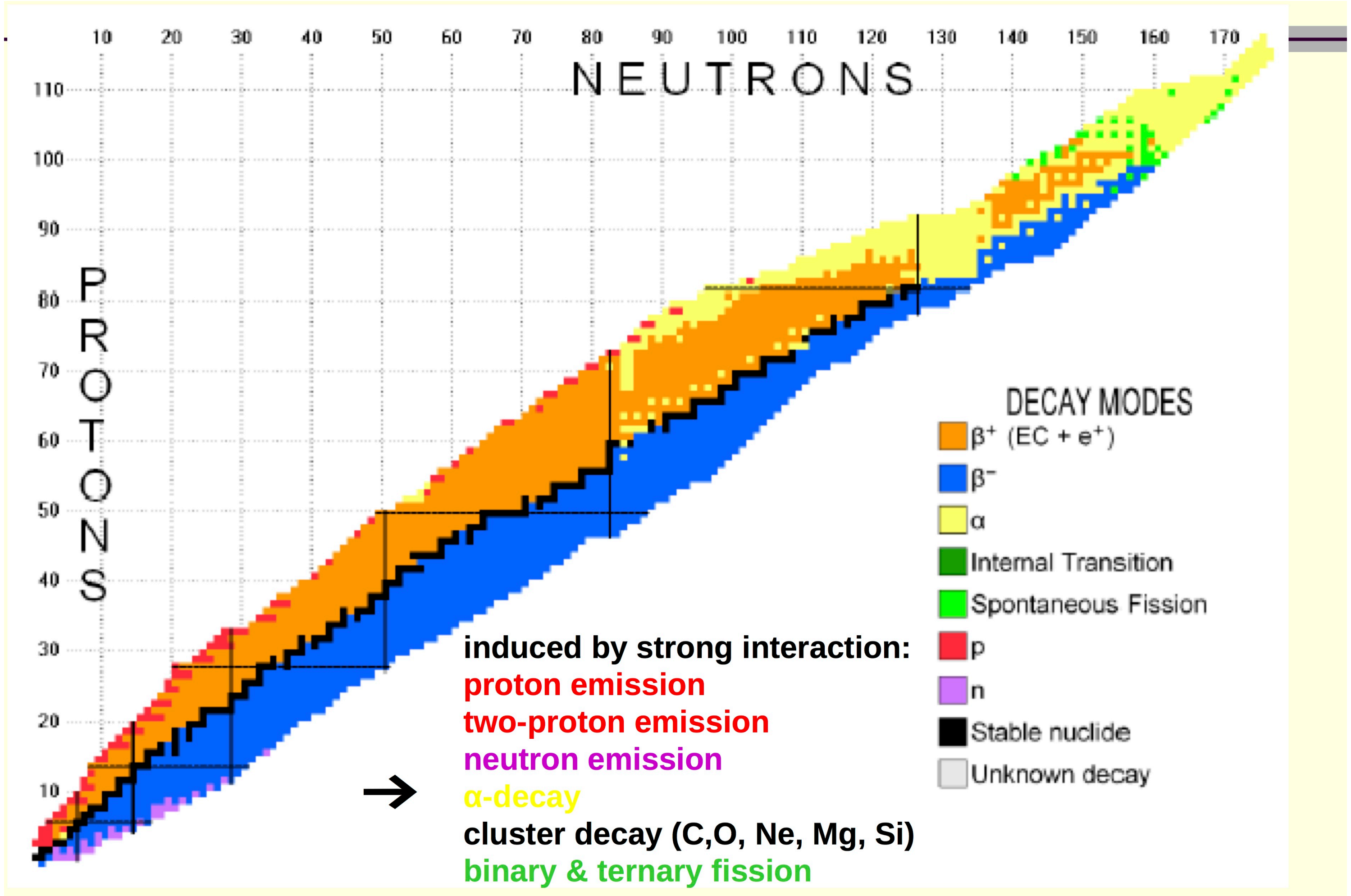
not described by shell structure calculations,
3 α cluster interact predominantly in relative S waves,
gas-like structure, THSR state

A. Tohsaki et al., PRL **87**, 192501 (2001)

α−particle condensation in low-density nuclear matter,
ρ below $\rho_{sat}/5$

nα nuclei: ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, …
cluster type structures near the n α breakup threshold energy

Decay modes of nuclei



induced by strong interaction:

proton emission

two-proton emission

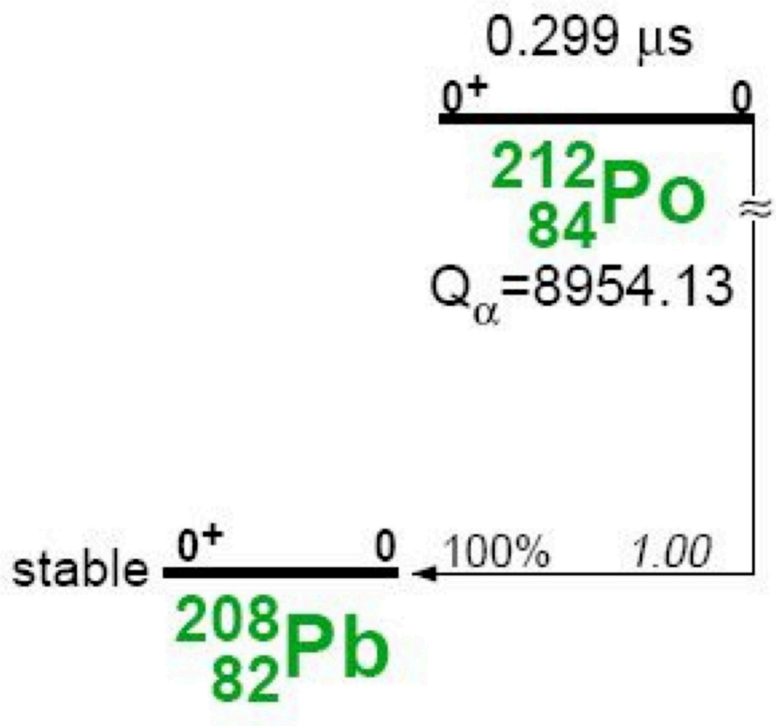
neutron emission

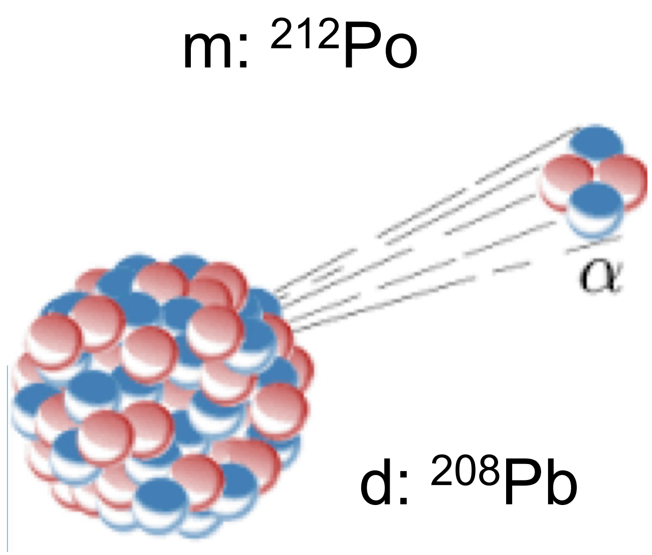
→ **α-decay**

cluster decay (C,O, Ne, Mg, Si)

binary & ternary fission

α decay of ^{212}Po





Quartetting wave-function approach

c. o. m. wave equation $$-\frac{\hbar^2}{8m}\nabla^2_{\mathbf{R}}\Phi(\mathbf{R})+W(\mathbf{R})\Phi(\mathbf{R})=E\Phi(\mathbf{R})$$

Effective c. o. m. potential

$$W(\mathbf{R})=\int d^3R'\,d^9s_j\,d^9s'_j\,\varphi_4^*(\mathbf{s},\mathbf{R})\,[T_4[\nabla_{s_j}]\delta(\mathbf{R}-\mathbf{R}')\delta(\mathbf{s}_j-\mathbf{s}'_j)+V_4(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)]\,\frac{\Phi(\mathbf{R})}{\Phi(\mathbf{R}')}\varphi_4(\mathbf{s}',\mathbf{R}')$$

$$V_4(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)=V_4^{\rm ext}(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)+V_4^{\rm intr}(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)$$

External contribution together with mean-field contribution to the effective potential

$$V_4^{\rm ext}(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)=\Big[V_{\tau_1}^{\rm mf}(\mathbf{R}+\frac{1}{2}\mathbf{s}+\frac{1}{2}\mathbf{s}_{12})+V_{\tau_2}^{\rm mf}(\mathbf{R}+\frac{1}{2}\mathbf{s}-\frac{1}{2}\mathbf{s}_{12})$$
$$+V_{\tau_3}^{\rm mf}(\mathbf{R}-\frac{1}{2}\mathbf{s}+\frac{1}{2}\mathbf{s}_{34})+V_{\tau_4}^{\rm mf}(\mathbf{R}-\frac{1}{2}\mathbf{s}-\frac{1}{2}\mathbf{s}_{34})\Big]\,\delta(\mathbf{R}-\mathbf{R}')\delta(\mathbf{s}-\mathbf{s}')\delta(\mathbf{s}_{12}-\mathbf{s}'_{12})\delta(\mathbf{s}_{34}-\mathbf{s}'_{34})$$

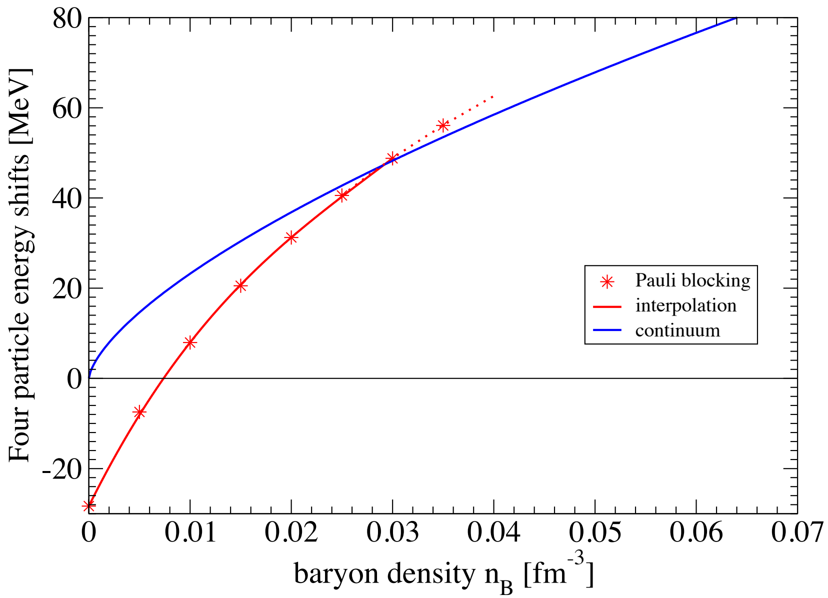
Intrinsic contribution containing Pauli blocking

$$V_4^{\rm intr}(\mathbf{r}_i;\mathbf{r}'_i)=\int d^3r''_1\,d^3r''_2\,\langle\mathbf{r}_1\mathbf{r}_2|[1-f_1(\varepsilon_{n_1})][1-f_2(\varepsilon_{n_2})]|\mathbf{r}''_1\mathbf{r}''_2\rangle\langle\mathbf{r}''_1\mathbf{r}''_2|V_{N-N}|\mathbf{r}'_1\mathbf{r}'_2\rangle\delta(\mathbf{r}'_3-\mathbf{r}_3)\delta(\mathbf{r}'_4-\mathbf{r}_4)$$
$$+\text{ five permutations}$$

Local approximation for the four nucleon effective c.o.m. potential W(R)

Thomas-Fermi model

Local density theory:
Pauli blocking is taken
from the homogeneous matter result,
energy shift of the α – like bound state
is a function of the baryon density.



At a critical density ($n_{crit} = n_{sat}/5 = 0.03$ fm^{-3})
the α – like bound state disappears.
At higher densities, the quartet consists of
four nucleons in (free) scattering states.
The quartet is added to the nucleon system
at the Fermi energy, kinetic energy = 4 E_{Fermi}

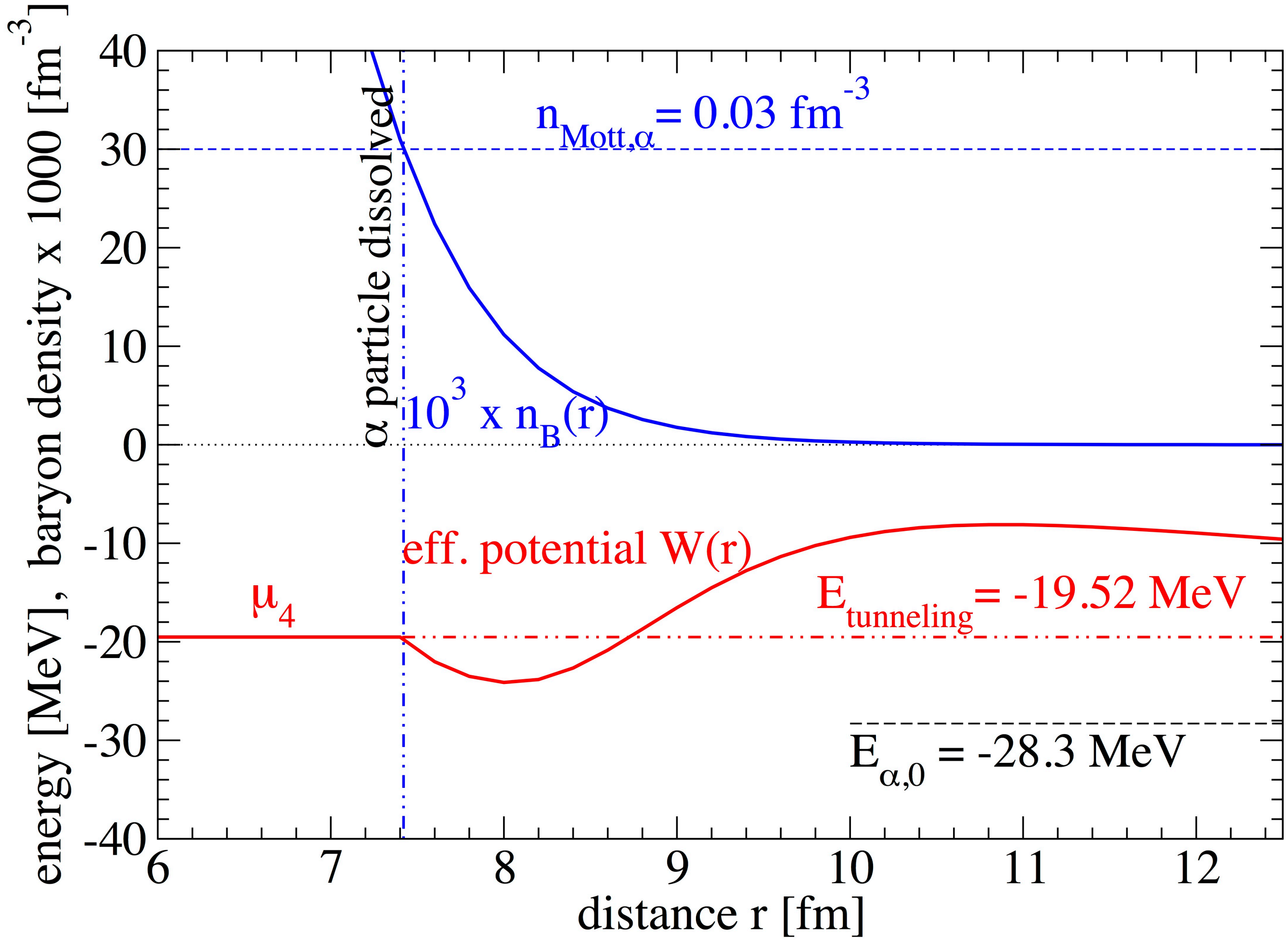
$$W^{\text{Pauli}}(n_B) \approx 4515.9\,\text{MeV fm}^3 n_B - 100935\,\text{MeV fm}^6 n_B^2 + 1202538\,\text{MeV fm}^9 n_B^3$$

Thomas-Fermi model for the core nucleus (^{208}Pb): For a given mean field $V^{mf}(r)$ all states below the Fermi energy E_{Fermi} are occupied. Inside the core, the chemical potential $\mu = V^{mf}(r)+E_{Fermi}(r)$ is not depending on r.
A quartet of 4 unbound nuclei can be added at the cluster chemical potential $\mu_4 = 4\,\mu$.

Thomas-Fermi rule: particles are taken away from the system at the chemical potential μ_4, $\mu_4=E_{tunneling}$

Double-folding M3Y potential W(R)

Thomas-Fermi approximation for the nucleons in the ^{208}Pb core: $\mu_4 = E_{tunneling}$



Results for α decay of ^{212}Po

C. Xu et al., PHYSICAL REVIEW C **93**, 011306(R) (2016) RAPID COMMUNICATIONS

Potential	c (MeV fm)	d (MeV fm)	E_{tunnel} (MeV)	Fermi energy μ_4 (MeV)
A	13866.30	4090.51	−19.346	−19.346
B	11032.08	3415.56	−19.346	−19.771

$E_{\text{tunnel}} - \mu_4$ (MeV)	Preform. factor P_α	Decay half-life $T_{1/2}$ (s)
0	0.367	2.91×10^{-8}
0.425	0.142	2.99×10^{-7}

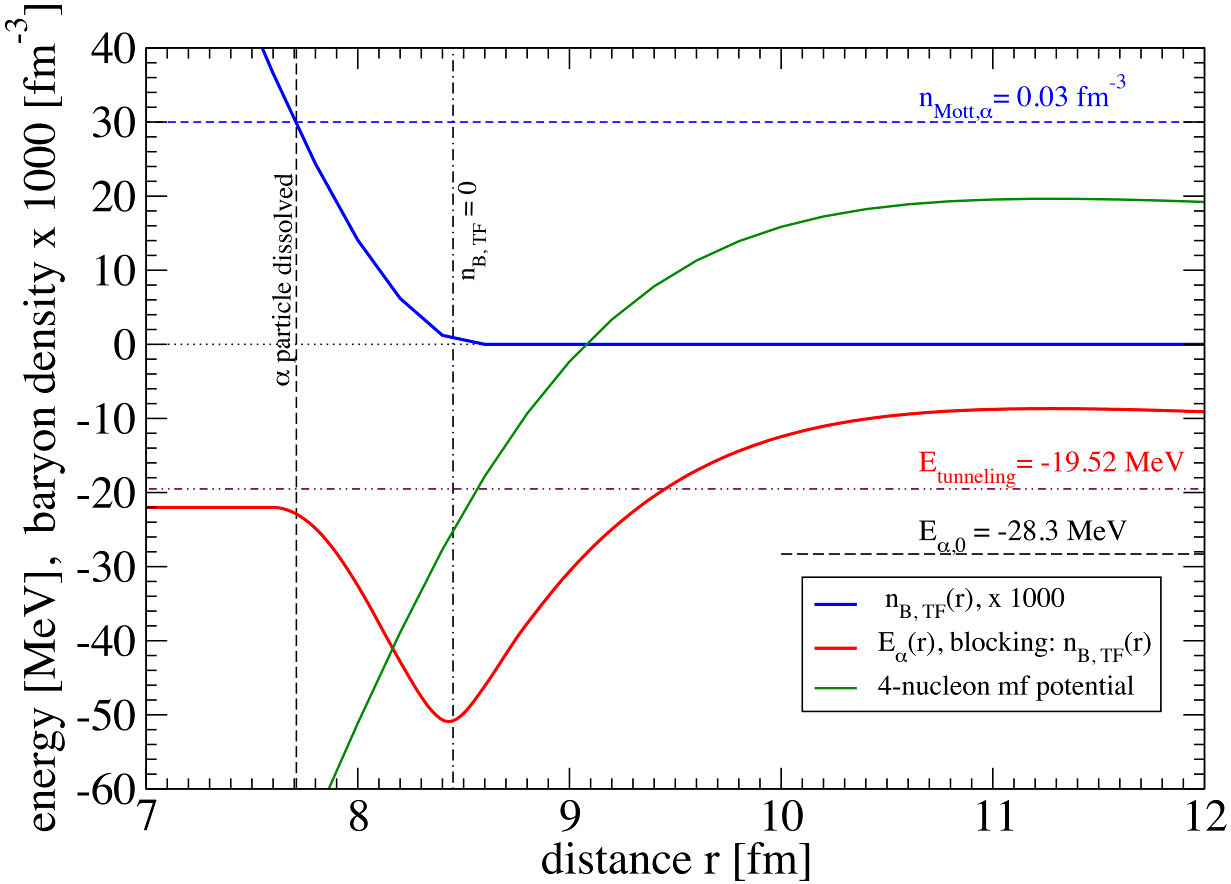
$$v(s) = c\exp(-4s)/(4s) - d\exp(-2.5s)/(2.5s)$$

^{212}Po: α on top of ^{208}Pb

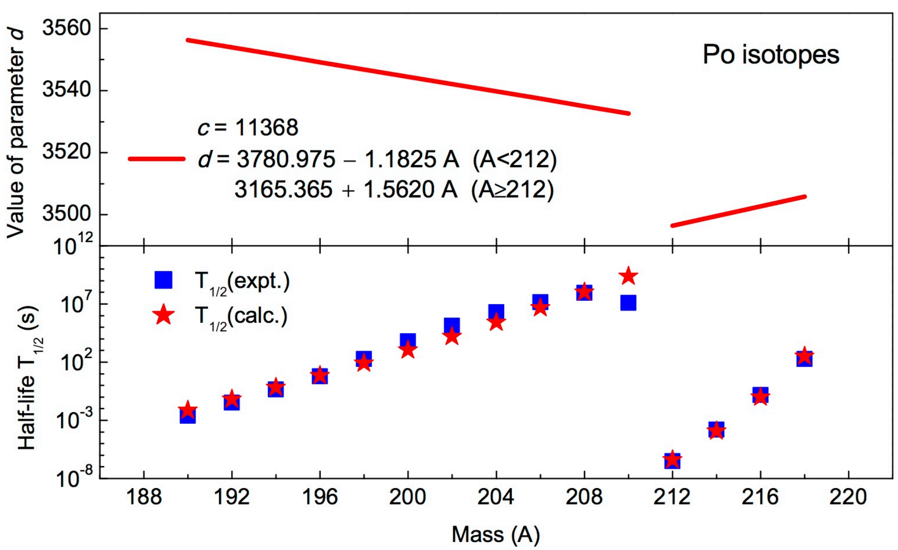
Problem:
decay half-life too short

Finite system:
discrete spectrum
of energy levels.
Relax the
Thomas-Fermi rule
$\mu_4 = E_{tunneling}$

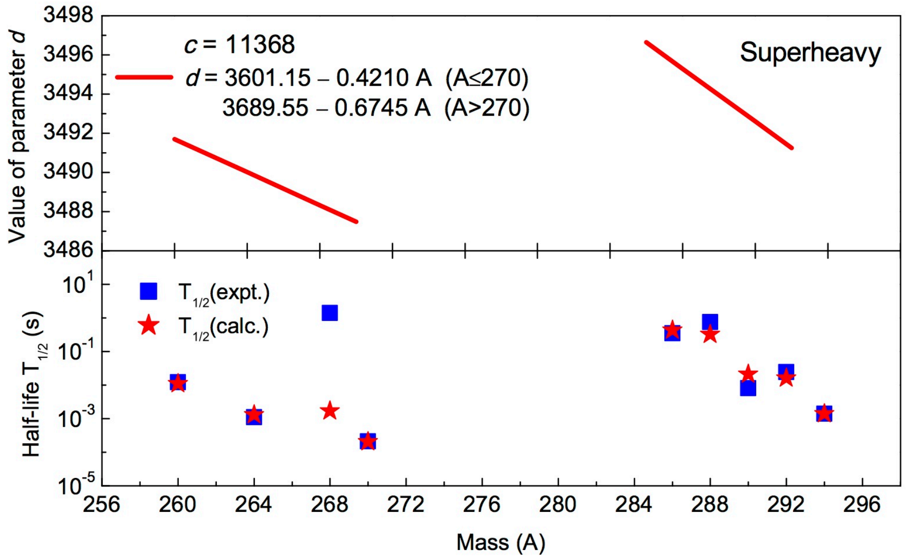
Discrete spectrum:
shell effects



G. R. et al., PRC **90**, 034304 (2014)



Comparison of experimental and calculated half-lives for the Po isotopes by using linear mass-dependent parametrization of M3Y interaction strengths.

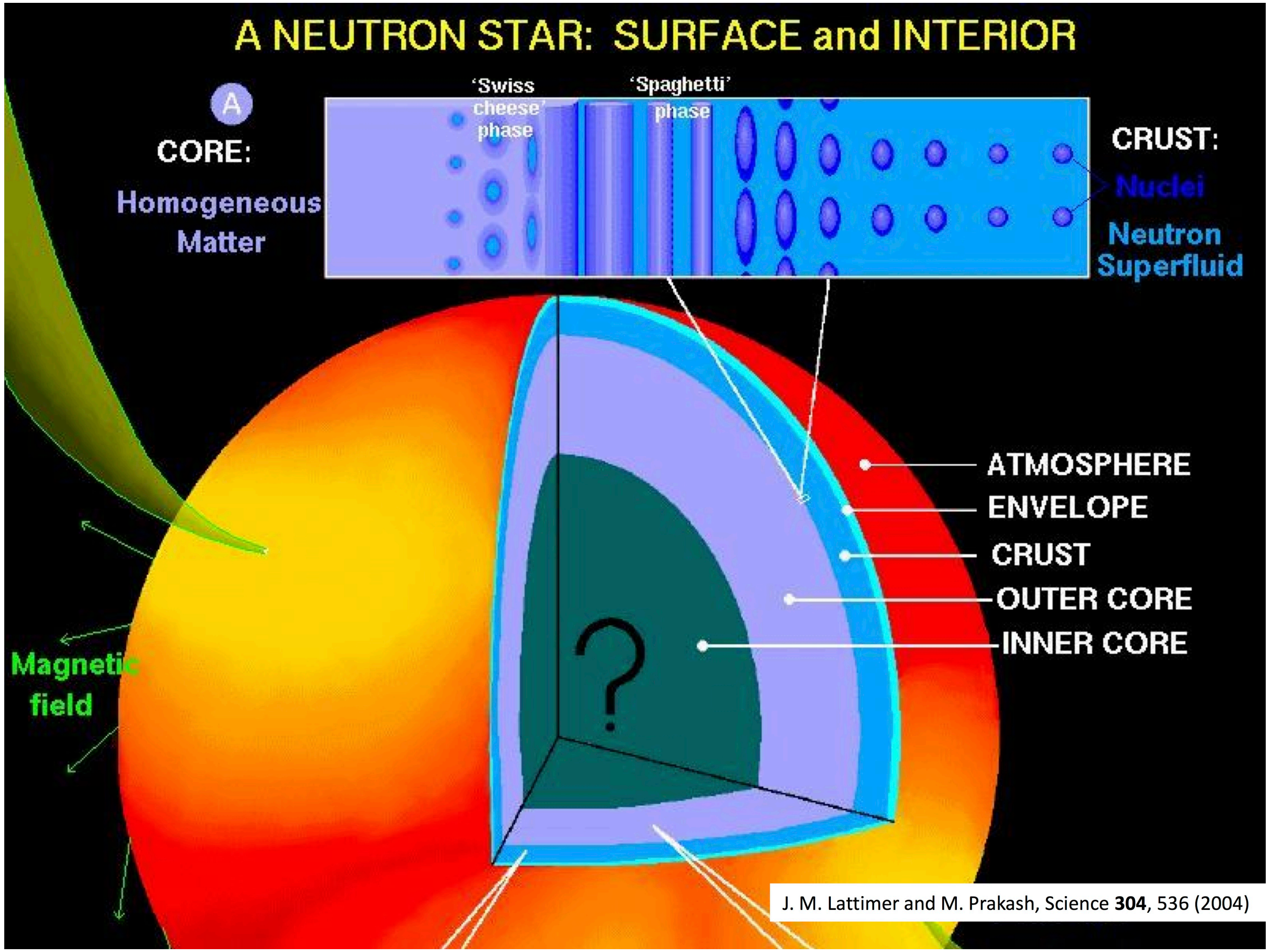


Comparison of experimental and calculated half- lives for the superheavy nuclei by using linear mass-dependent parametrization of M3Y interaction strengths.

TABLE I. The α-cluster formation probabilities of even-even superheavy nuclei by the quartetting wave function approach. Strong deviations indicating a possible proton shell closure are highlighted in bold face.

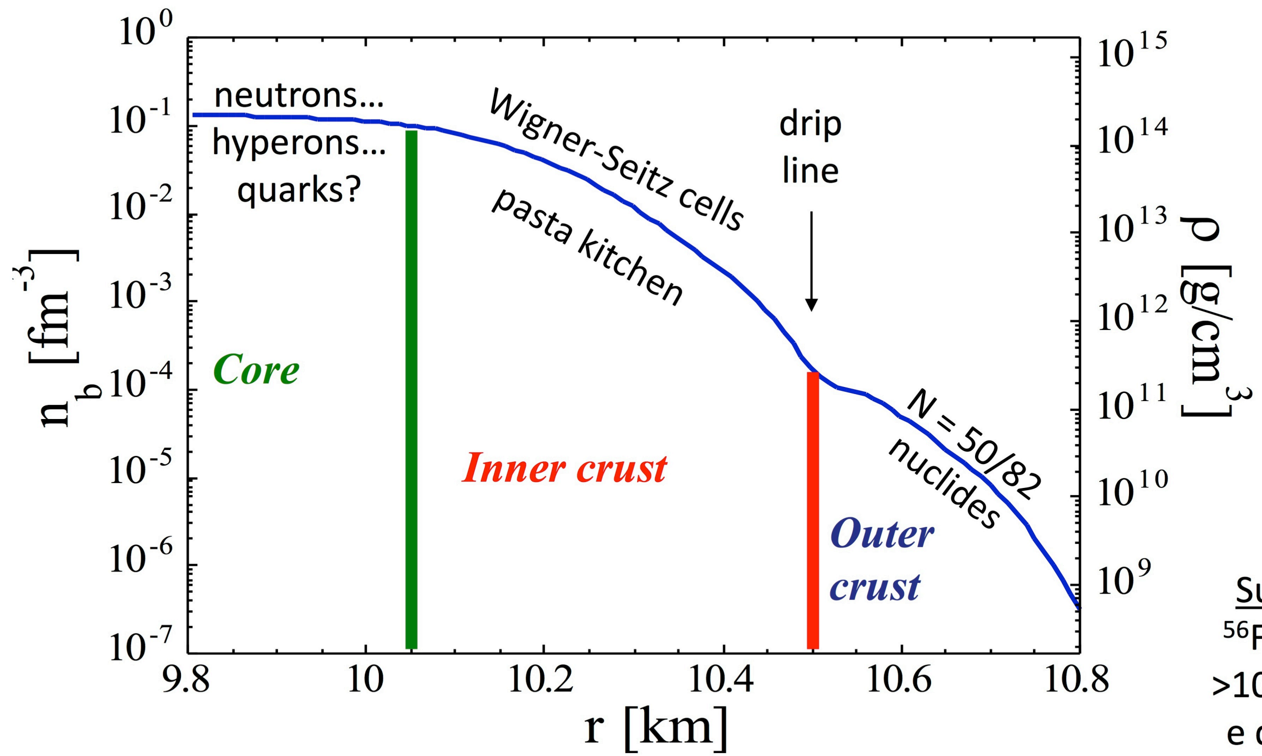
Mass	Z	N	Q_α MeV	Half-life $T_{1/2}$ (s)	c (MeV fm)	d (MeV fm)	Fermi energy μ_4 (MeV)	E_{tunnel} (MeV)	$E_{\mathrm{tunnel}} - \mu_4$ (MeV)	P_α
294	118	176	11.810	1.4×10^{-3}	17066.70	4847.61	−16.889	−16.490	0.399	0.110
292	116	176	10.774	2.4×10^{-2}	19237.20	5365.62	−17.772	−17.526	0.246	0.197
290	116	174	10.990	8.0×10^{-3}	19027.50	5315.41	−17.568	−17.310	0.258	0.191
288	114	174	10.072	7.5×10^{-1}	18743.70	5251.07	−18.549	−18.228	0.320	0.156
286	114	172	10.370	3.5×10^{-1}	17237.40	4892.79	−18.349	−17.930	0.419	0.104
270	**110**	**160**	11.117	2.1×10^{-4}	17079.10	4847.45	−17.547	−17.183	**0.364**	**0.144**
268	**108**	**160**	9.623	1.4×10^{0}	15653.10	4516.39	−19.171	−18.677	**0.494**	**0.077**
264	108	156	10.591	1.1×10^{-3}	17054.60	4843.76	−18.088	−17.709	0.379	0.140
260	106	154	9.901	1.2×10^{-2}	17488.80	4948.93	−18.759	−18.399	0.360	0.152

4. Astrophysics



Neutron stars, inner crust: pasta structures

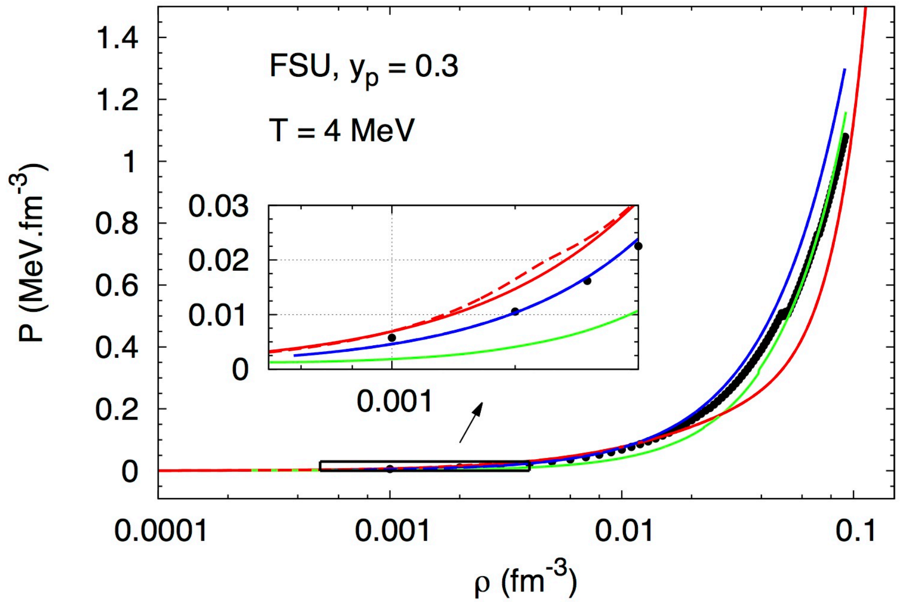
Density of neutron star crust

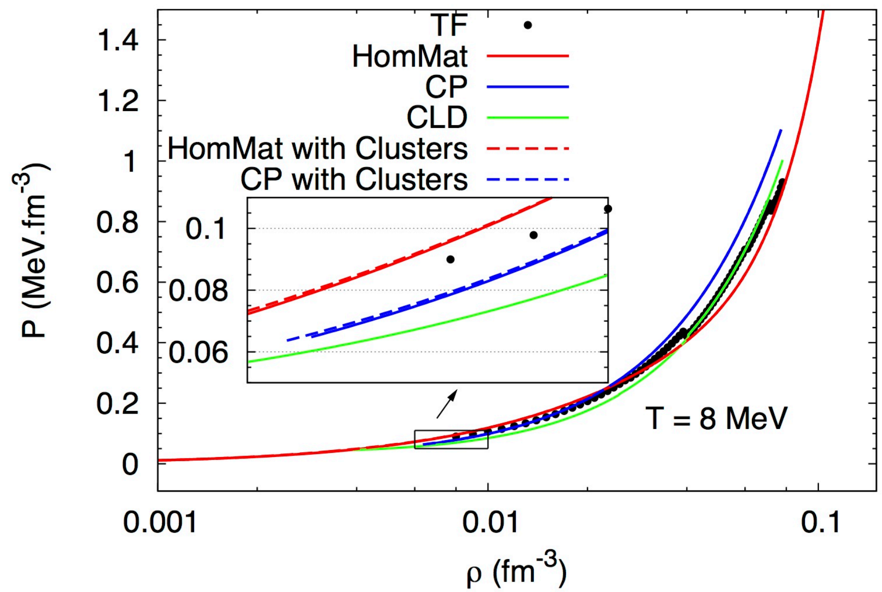


Surface:
^{56}Fe; ^{62}Ni;
$>10^7$ g/cm^3
e degen.:
p + e → n

J. M. Pearson *et al.*, PRC **83**, 065810 (2011)

Light clusters and pasta phases in core-collapse supernova matter





Pressure as function of density, Yp=0.3, T=4 MeV / 8 MeV.
With and without pasta, including or not clusters. TF - Thomas-Fermi,
CP – coexisting-phases method, CLD – compressible liquid drop

H. Pais, S. Chiacchiera, C. Providencia, PRC **91**, 055801 (2015)

Light Clusters and Pasta Phases in Warm and Dense Nuclear Matter

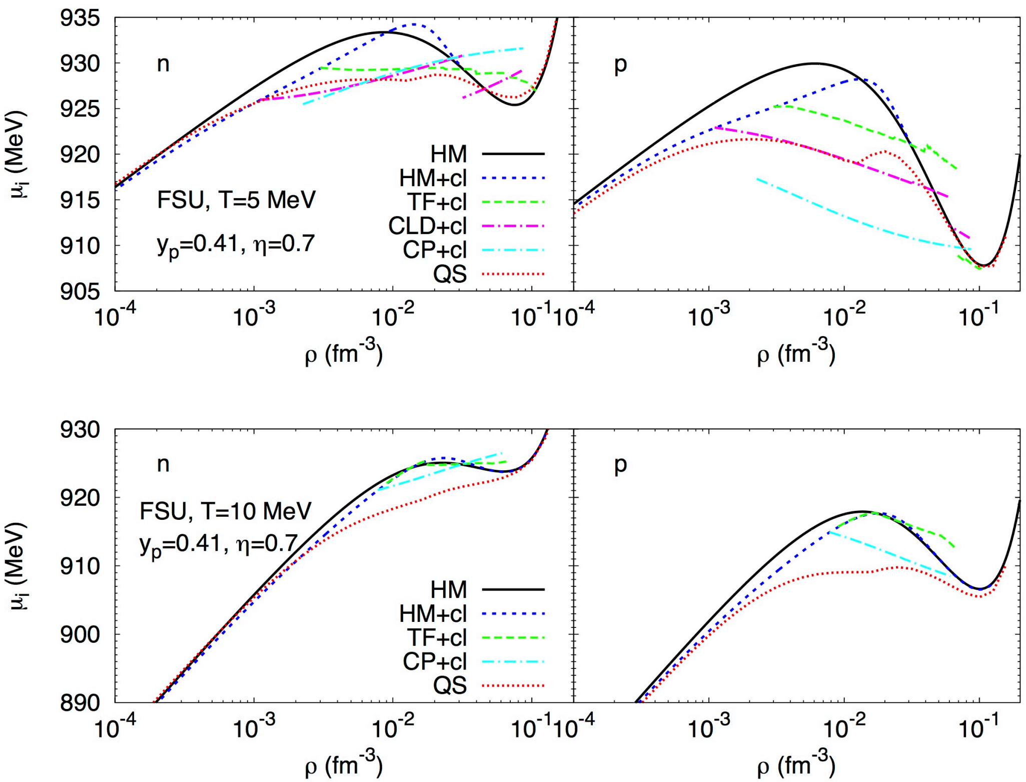


FIG. 8. Neutron (left panels) and proton (right panels) chemical potentials with $\eta = 0.7$ and $Y_p = 0.41$ as a function of density at $T = 5$ MeV (top) and $T = 10$ MeV (bottom), for homogeneous nuclear matter (HM) (solid), nuclear matter with light clusters (blue short-dashed), and mean-field pasta calculations with clusters [TF (green, dashed), CLD (pink, dash-dotted), CP (cyan, dash-dotted)]. QS results (red, dotted) are also shown.

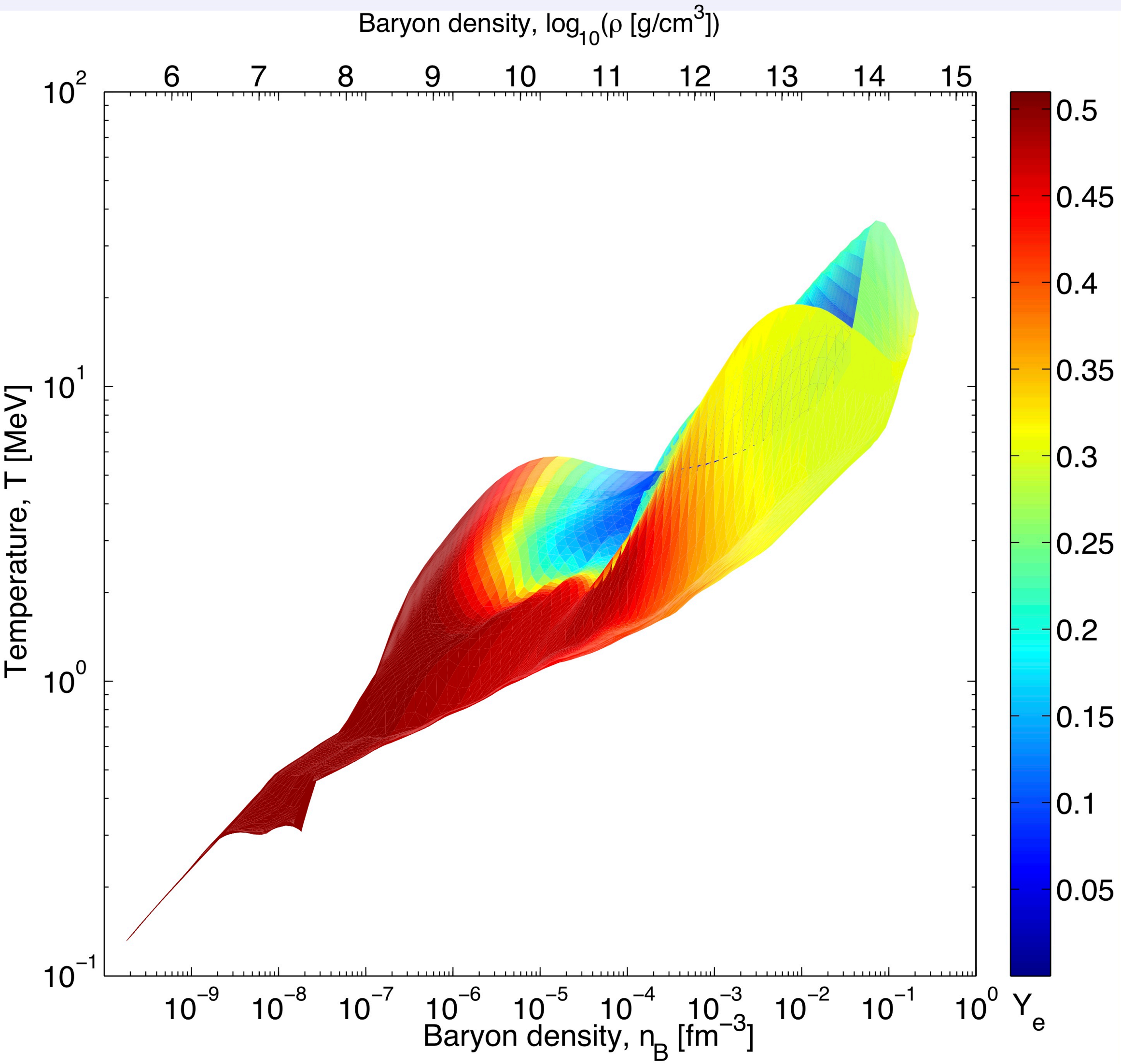
S. S. Avancini et al., Phys. Rev. C **95**, 045804 (2017)

Nuclear matter phase diagram

Core collapse supernovae

Relevant Parameters:

- **density**:
  $10^{-9} \lesssim \varrho/\varrho_{\text{sat}} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\text{sat}} \approx 2.5 \cdot 10^{14}$ g/cm^3
  ($n_{\text{sat}} = \varrho_{\text{sat}}/m_n \approx 0.15$ fm^{-3})
- **temperature**:
  0 MeV $\leq k_B T \lesssim 50$ MeV
  ($\hat{=}\ 5.8 \cdot 10^{11}$ K)
- **electron fraction**:
  $0 \leq Y_e \lesssim 0.6$



T. Fischer, GSI Darmstadt

Summary

- A quantum statistical approach can be given to describe correlations and antisymmetrization in nuclear systems

- Many-particle theory: Equation of state
  QCD? Effective interactions, Green functions, spectral functions

- Low-density limit: cluster formation
  Mass action law, nuclear statistical equilibrium, virial expansion

- Near saturation: medium effects
  mean-field and quasiparticles, dissolution of bound states

- Quantum condensates:
  transition from BEC to BCS, Hoyle states, pairing and quartetting

- Correlations of nucleons and formation of “pasta” structures are of importance in the crust of neutron stars.

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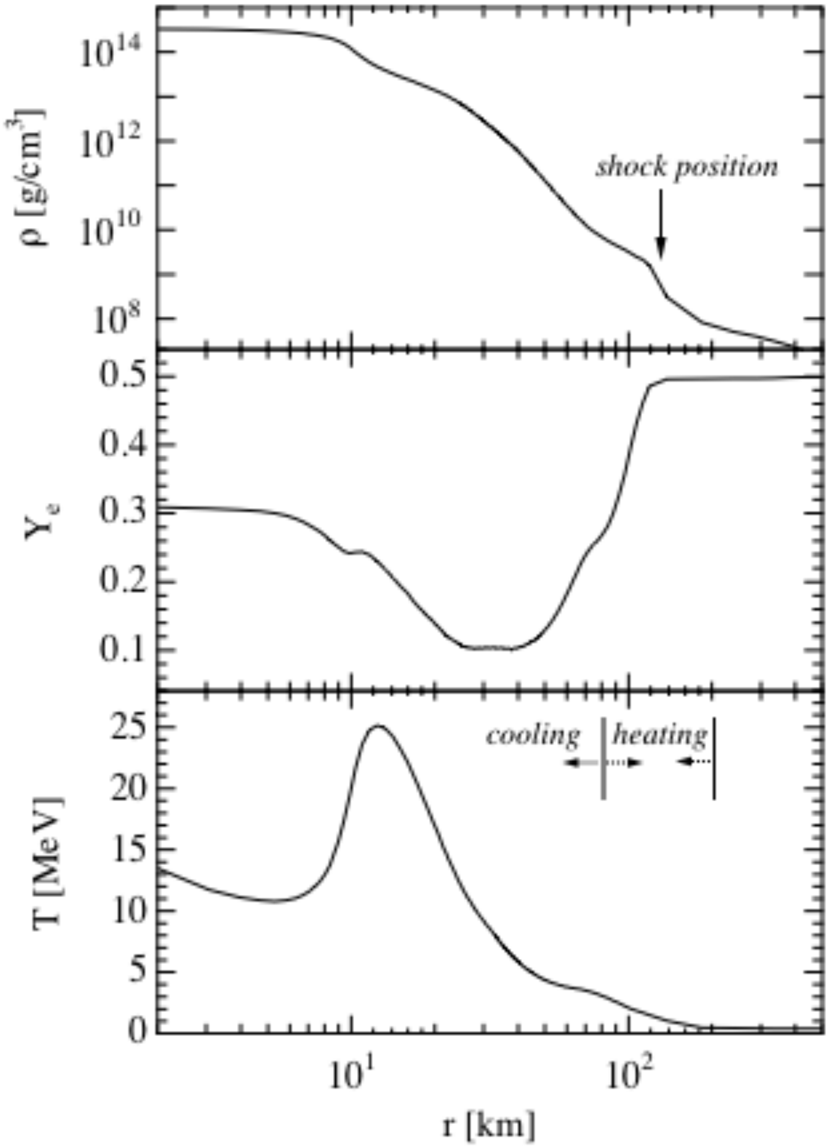
A. Tohsaki, S. Typel, H. Wolter, C. Xu, T. Yamada, B. Zhou
for collaboration

to you

for attention

D.G.

Core-collapse supernovae



Density.

electron fraction, and

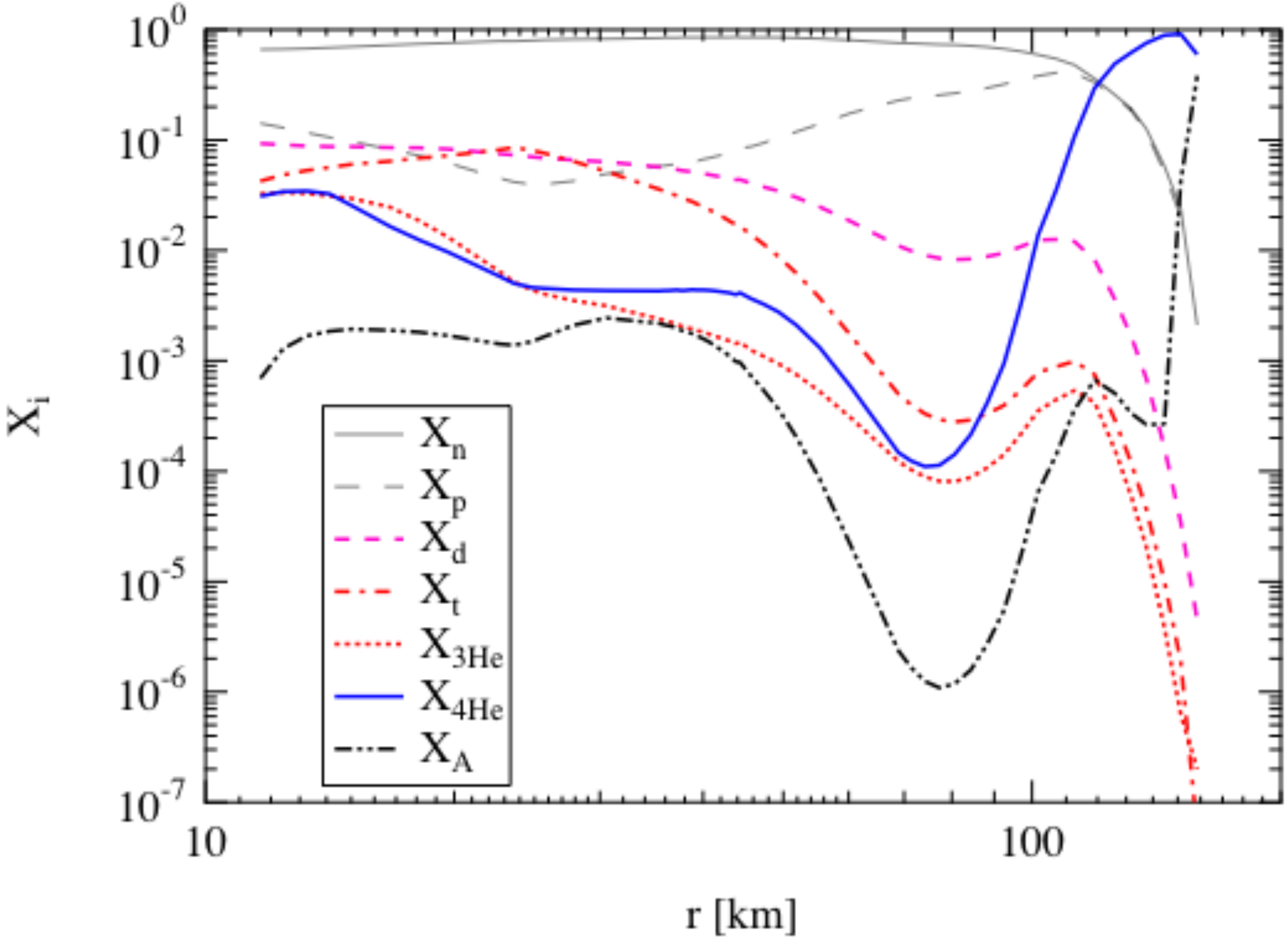
temperature profile

of a 15 solar mass supernova at 150 ms after core bounce as function of the radius.

Influence of cluster formation on neutrino emission in the cooling region and on neutrino absorption in the heating region ?

K.Sumiyoshi et al., Astrophys.J. **629**, 922 (2005)

Composition of supernova core



Mass fraction X of light clusters for a post-bounce supernova core

K.Sumiyoshi, G. R., PRC **77**, 055804 (2008)

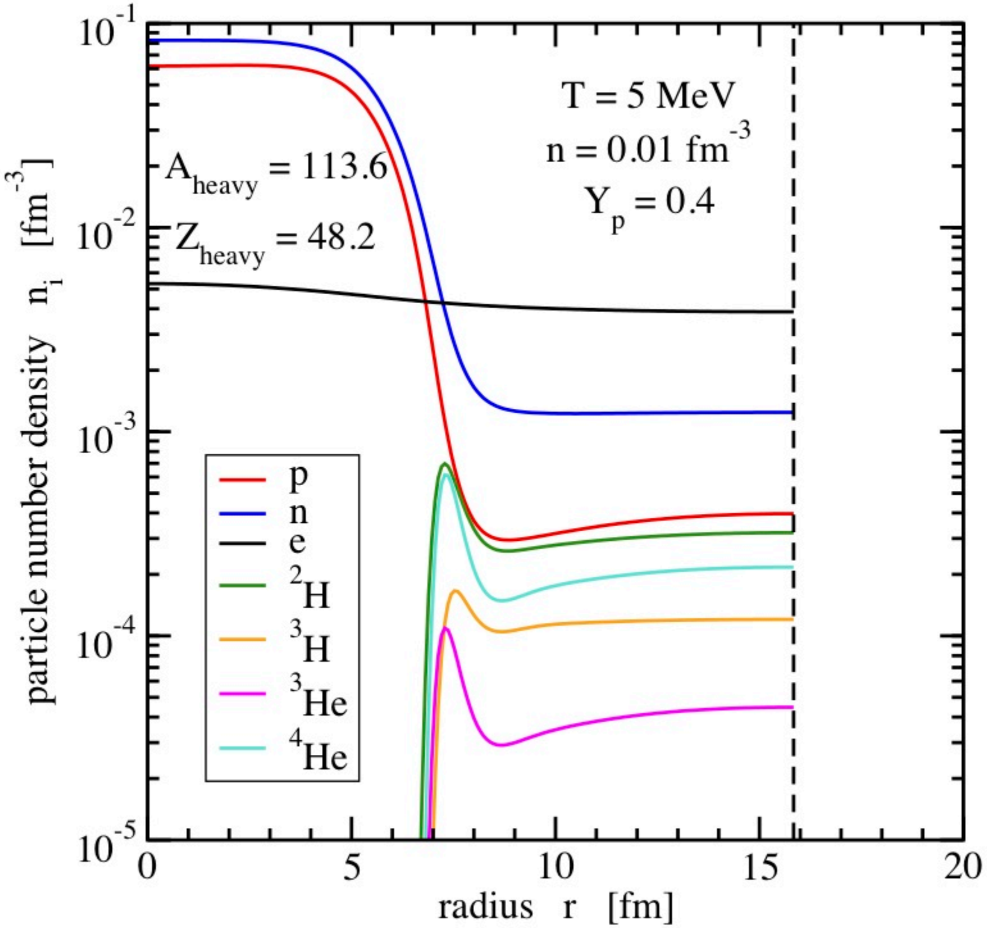
α cluster in astrophysics

Crust of neutron stars

Protons in droplets (heavy nuclei)

α−cluster outside, at the surface, condensate?

S. Typel, GSI



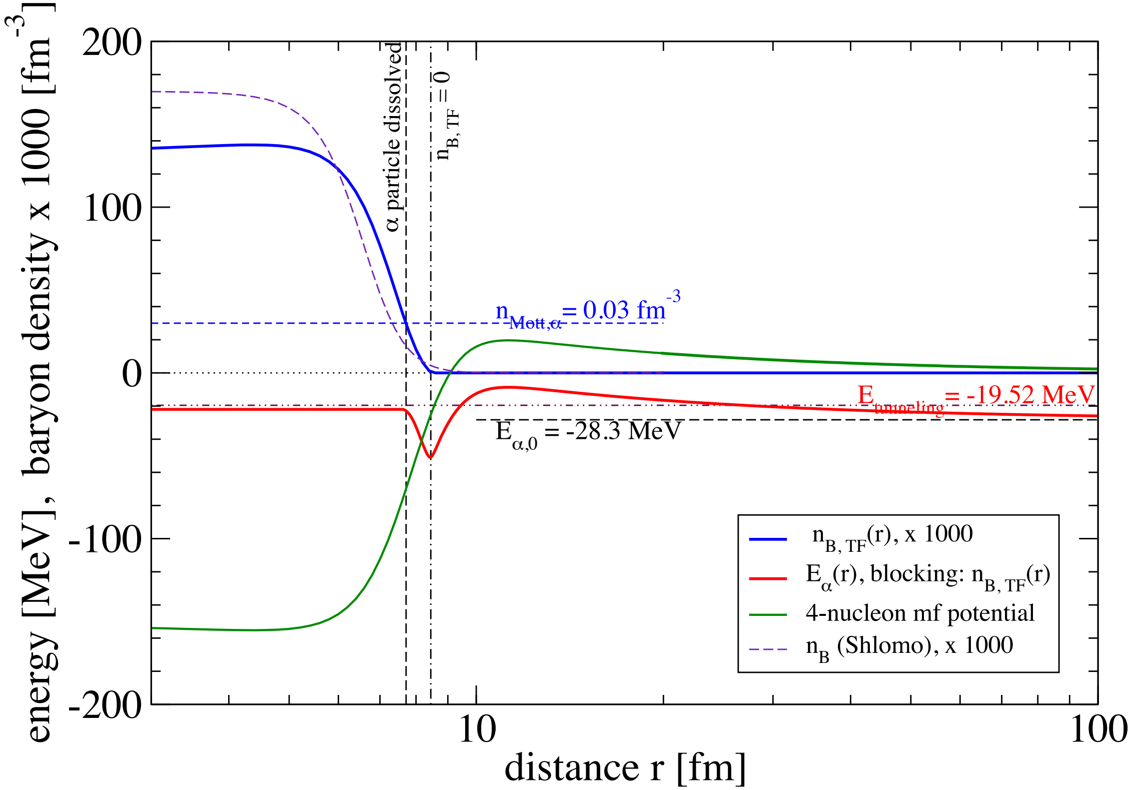
^{212}Po: α on top of ^{208}Pb

Local effective potential W(**R**) of the quartet (2 neutrons and 2 protons) with respect to the ^{208}Pb core.

Mean field:
Double-folding potential
M3Y (Bertsch et al.), nucleon-nucleon interaction with two parameter c and d

Pauli blocking:
Thomas-Fermi model

Pocket formation



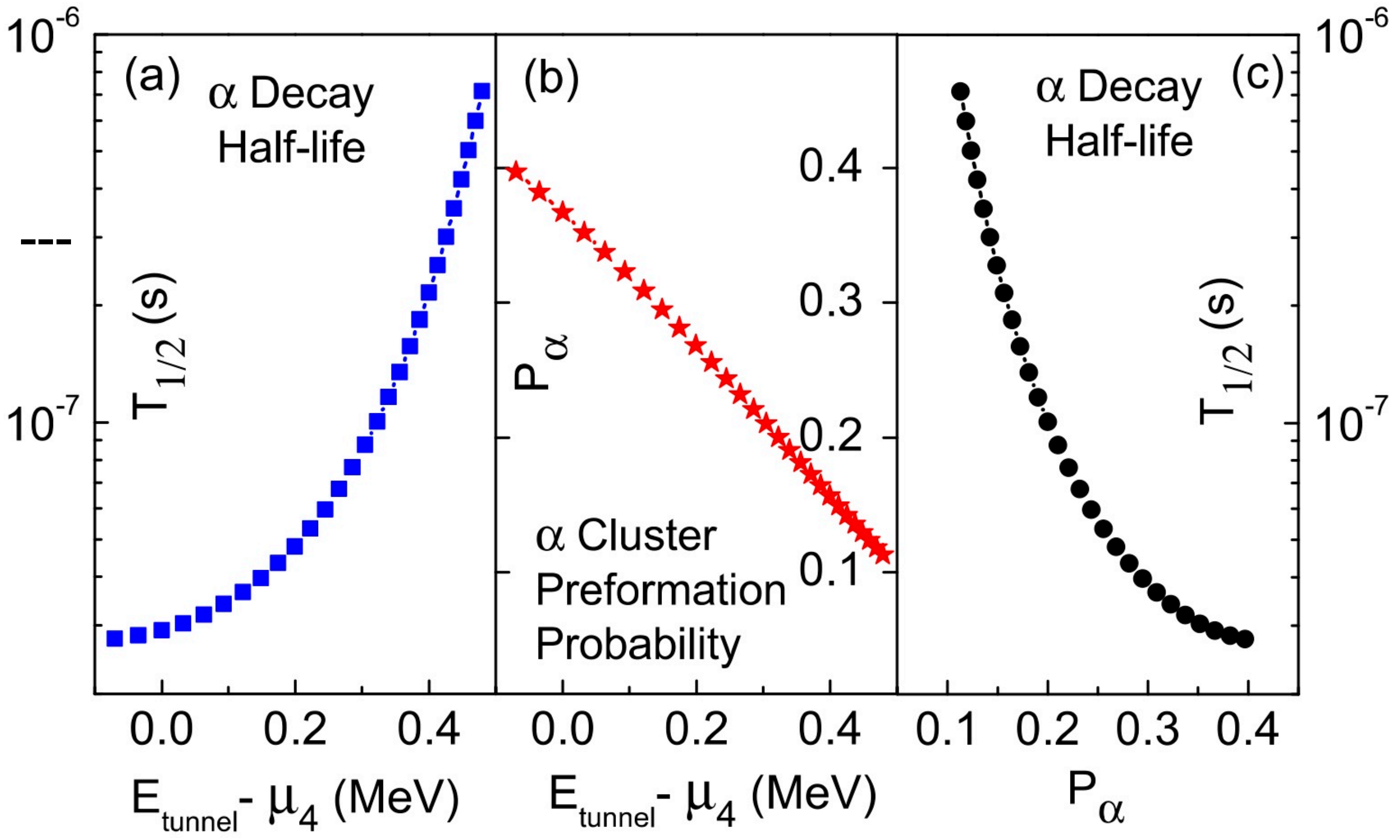
$$W(\mathbf{R}) = W^{\text{ext}}(\mathbf{R}) - B_\alpha + W^{\text{Pauli}}[n_B(\mathbf{R})]$$

C. Xu et al., PRC **93**, 011306(R) (2016)

Modification of the mean-field potential

experiment:
$T_{1/2} = 2.99 \times 10^{-7}$ s

Relax the Thomas-Fermi rule
$\mu_4 = E_{tunneling}$



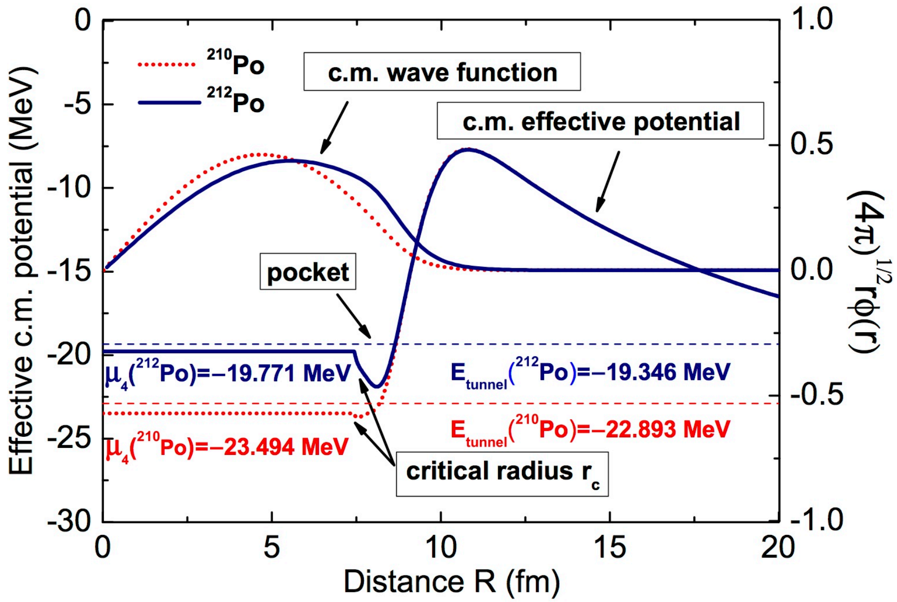
parameterized M3Y–type nucleon–nucleon effective interaction with c, d fitted to data

$$v(s) = c\exp(-4s)/(4s) - d\exp(-2.5s)/(2.5s)$$

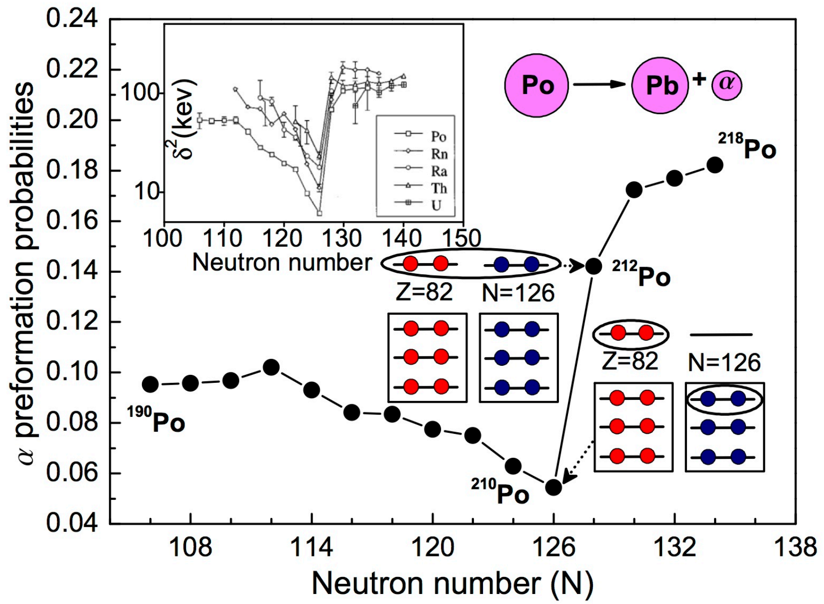
α-cluster formation and decay

parameterized M3Y–type nucleon–nucleon effective interaction

$$v(s) = c\exp(-4s)/(4s) - d\exp(-2.5s)/(2.5s)$$



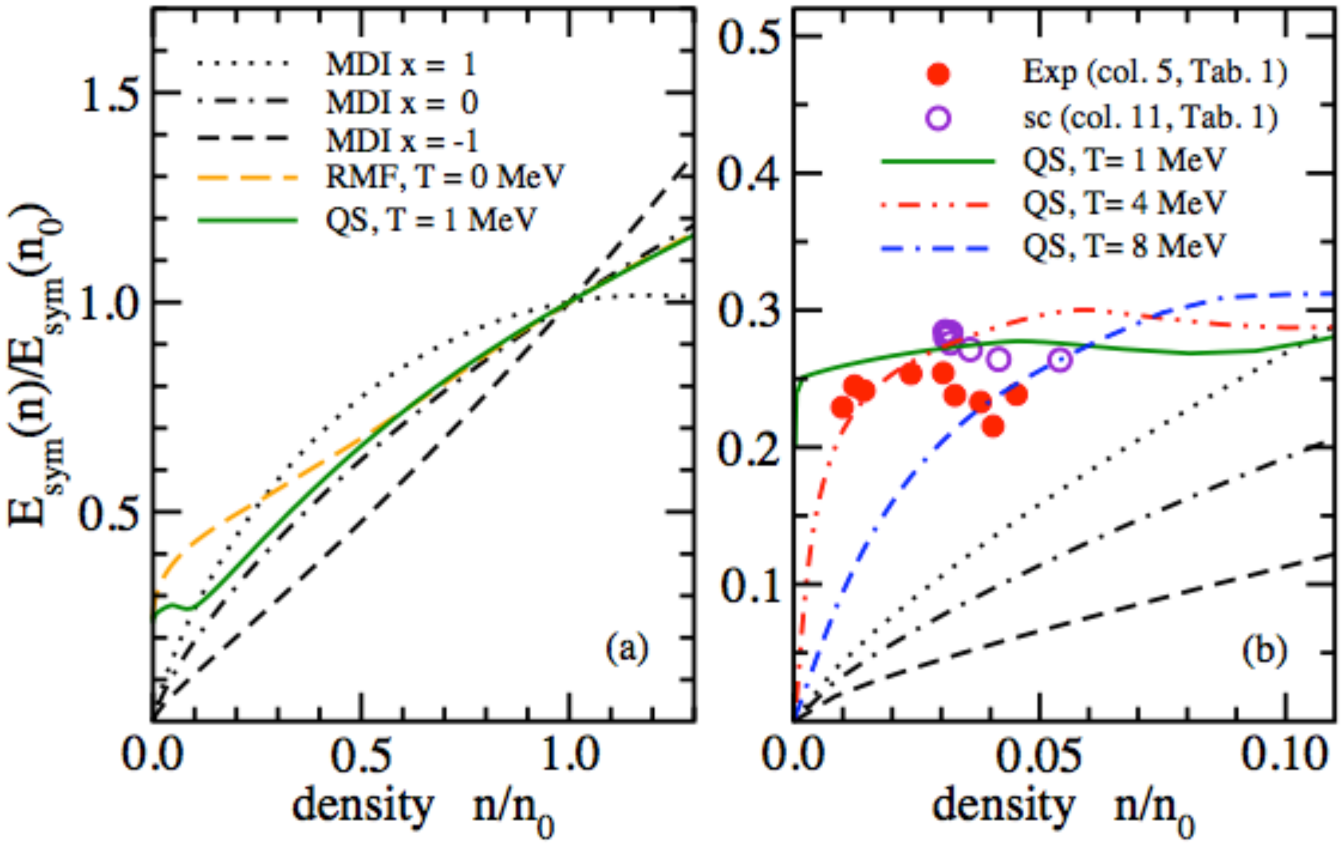
Comparison of the c.m. effective potentials, the c.m. wave functions, and the Fermi energies for two neighboring α emitters ^{210}Po and ^{212}Po.



α-cluster preformation probability P of even-even Po α isotopes by the quartetting wave function approach.

Chang Xu et al., Phys. Rev. C **95**, 061306(R) (2017)

Symmetry Energy

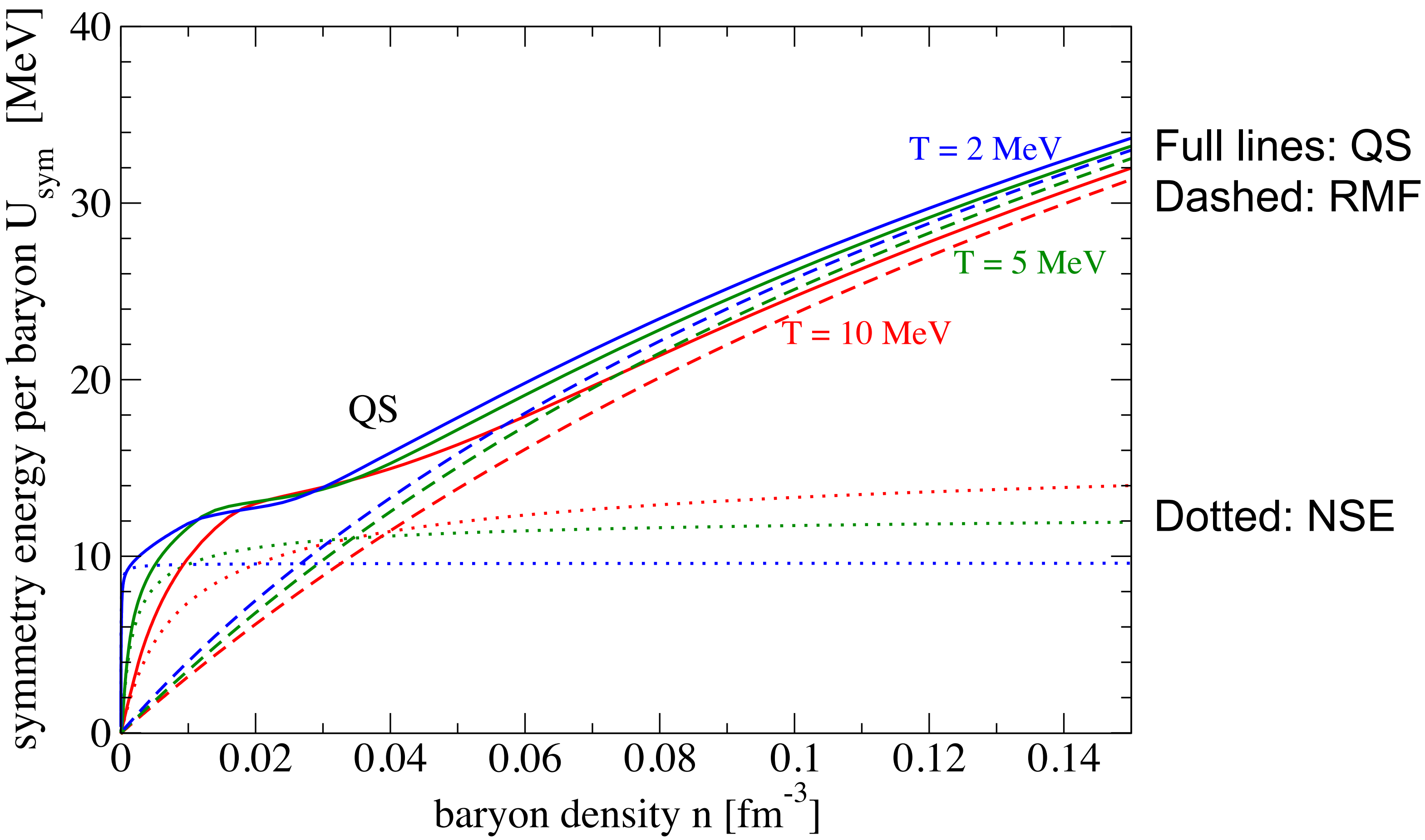


Scaled internal symmetry energy as a function of the scaled total density.
MDI: Chen et al., QS: quantum statistical, Exp: experiment at TAMU

J.Natowitz et al. PRL, May 2010

Light clusters and symmetry energy

dependent on T



K. Hagel et al.Eur. Phys. J. A (2014) 50: 39