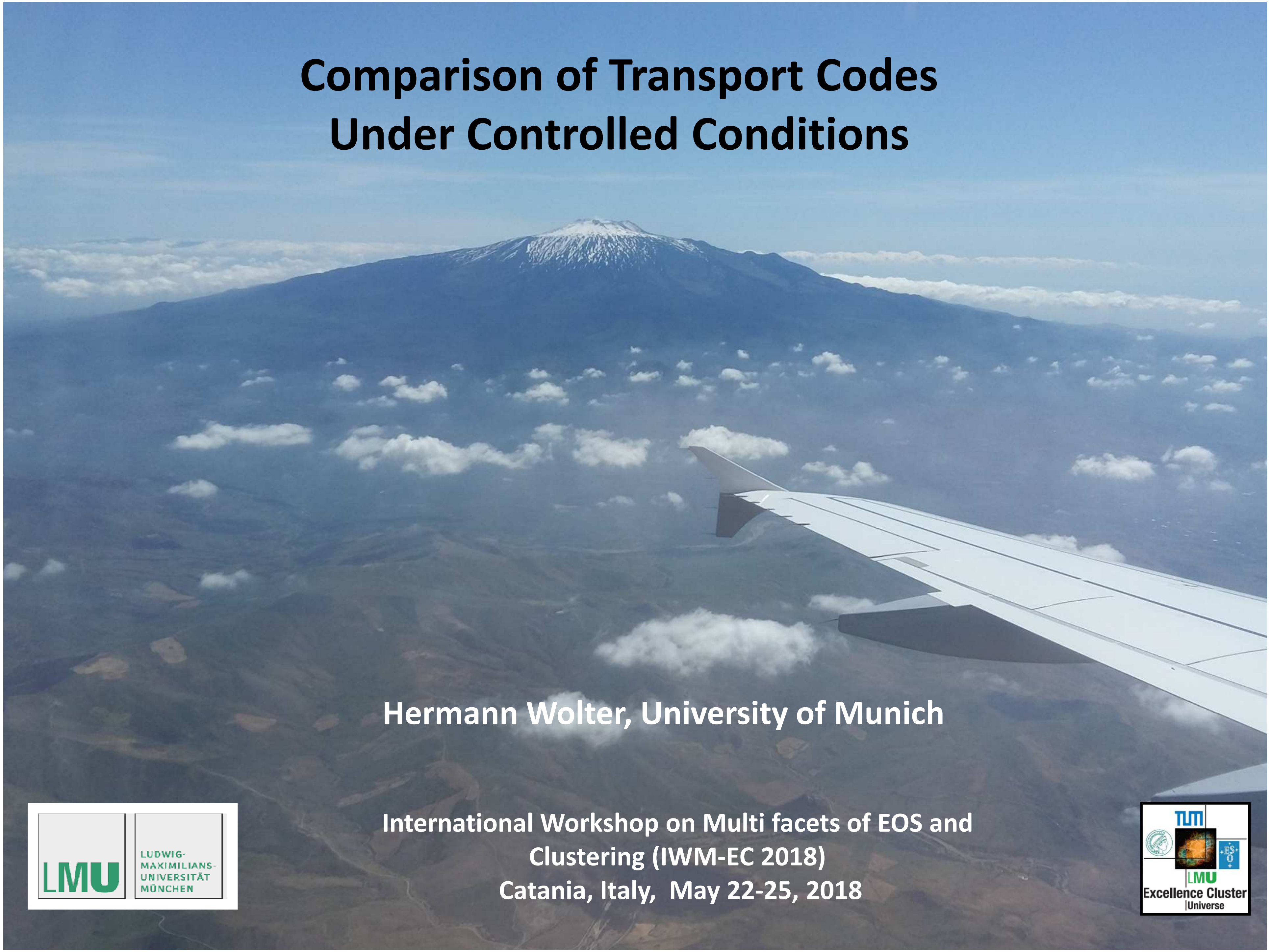
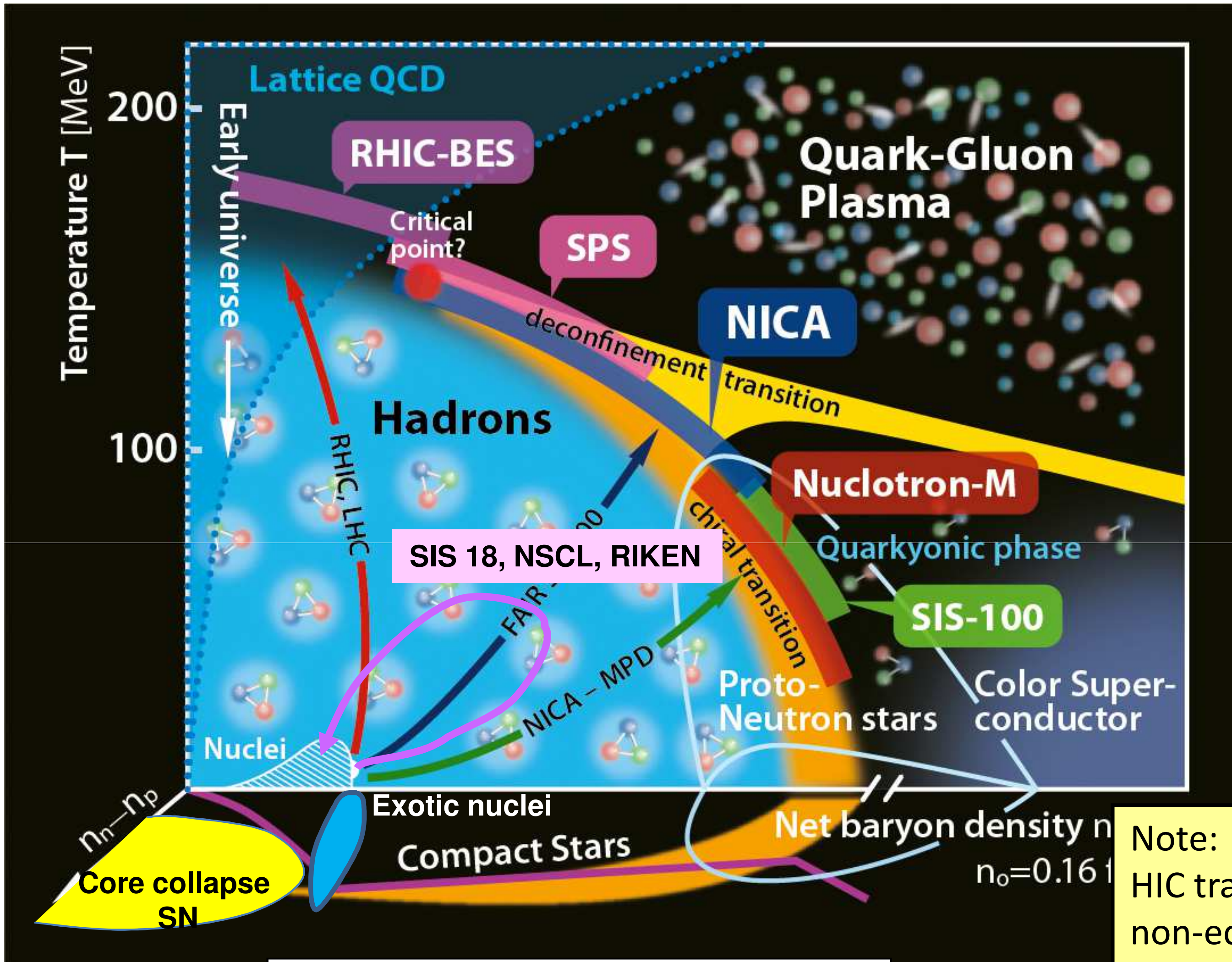
Comparison of Transport Codes Under Controlled Conditions



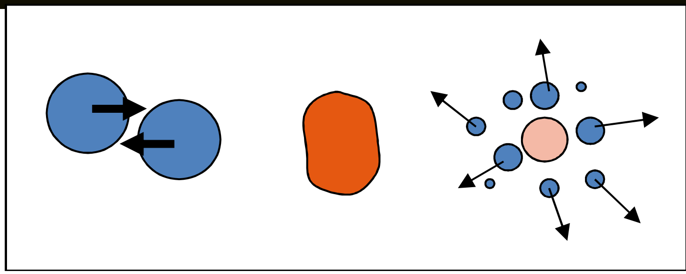
Hermann Wolter, University of Munich

International Workshop on Multi facets of EOS and Clustering (IWM-EC 2018)
Catania, Italy, May 22-25, 2018

Aim in Heavy Ion Reactions: The Phase Diagram of Strongly Interacting Matter



Asymmetry axis
--> search for symmetry energy

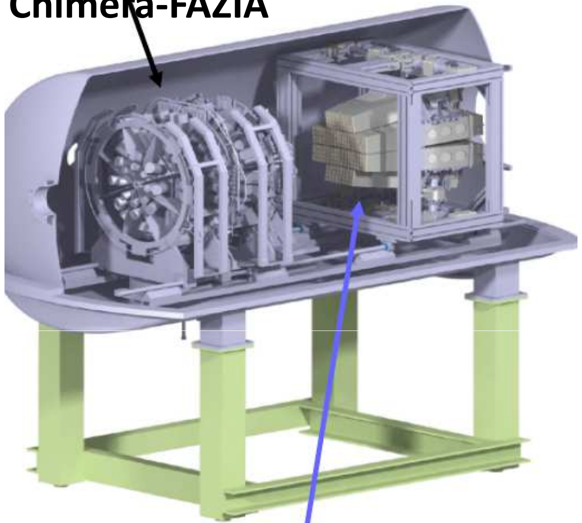


Note:
HIC trajectories are non-equilibrium processes, and are not necessarily in this diagram
→ transport theory is necessary

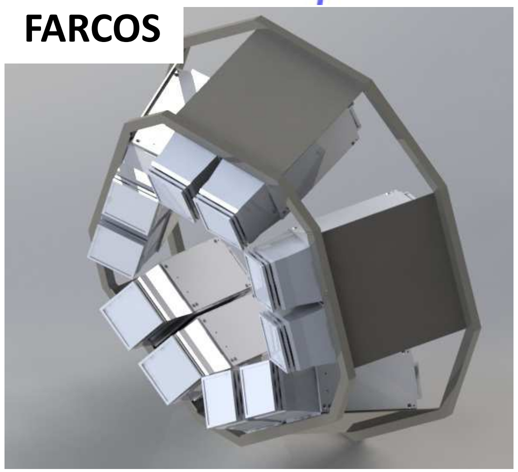
Goal: to determine the Equation-of-State of nuclear matter

Experimentalists are taking big steps to improve their tools

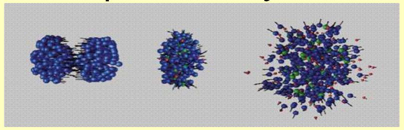
Chimera-FAZIA



FARCOS



Transport theory for HIC



$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\cdot\vec{\nabla}_r-\vec{\nabla}_r U\cdot\vec{\nabla}_p\right) f(\vec{r},\vec{p};t)=I_{\text{coll}}(\vec{r},\vec{p};t), \quad (1)$$

with the collision term

$$I_{\text{coll}}=\frac{g}{(2\pi\hbar)^3}\int d^3p_1 d\Omega\, v_{\text{rel}}\frac{d\sigma^{\text{med}}}{d\Omega}[f'f_1'(1-f)(1-f_1) - ff_1(1-f')(1-f_1')], \quad (2)$$

$$\Psi(\vec{r}_1,\ldots,\vec{r}_A;t)=\prod_{i=1}^{A}\phi_i(\vec{r}_i;t),$$

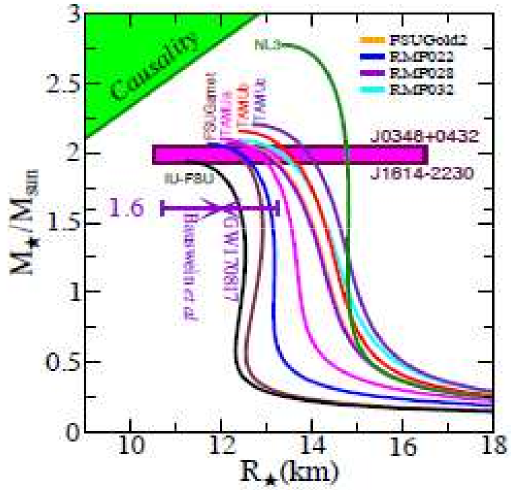
$$\phi_i(\vec{r}_i;t)=\frac{1}{[2\pi(\Delta x)^2]^{\frac{3}{4}}}\times\exp\left\{-\frac{[\vec{r}_i-\vec{R}_i(t)]^2}{4(\Delta x)^2}\right\}e^{(i/\hbar)\vec{P}_i(t)\cdot\vec{r}_i}.$$

$$f(\vec{r},\vec{p};t)=\frac{(2\pi\hbar)^3}{gN_{\text{TP}}}\sum_{i=1}^{AN_{\text{TP}}}G(\vec{r}-\vec{r}_i(t))\tilde{G}(\vec{p}-\vec{p}_i(t)).$$

$$\frac{d\vec{r}_i}{dt}=\vec{\nabla}_{p_i}H \quad\text{and}\quad \frac{d\vec{p}_i}{dt}=-\vec{\nabla}_{r_i}H\,.$$

$$\frac{dN_{\text{coll}}}{dt}=\frac{1}{2}A\rho\frac{1}{4m^4 T_B K_2^2(m/T_B)}\times\int_{2m}^{\infty}d\sqrt{s}\,s\,(s-4m^2)\,K_1(\sqrt{s}/T_B)\,\sigma^{\text{med}}$$

Increasing constraints from Neutron star observation: mass-Radius relation, NS mergers



Theory also needs to shape up their tools:
--> test and improve reliability of transport calculations

Aim of this talk:

- **discussion of transport approaches to heavy-ion collisions (HIC)**
- **not interpretation of data,**
  but accuracy of description of transport approaches
- **comparison of transport codes with identical physical input**

→ among each other for HIC
→ and in box calculations with exact limits in nuclear matter

- **highlight the role of fluctuations in the description of HIC**

On behalf of the Code Comparison Project
- of the order of 30 participants
- core group:
Maria Colonna (Catania), Akira Ono (Sendai),
Yingxun Zhang (CIAE, Beijing), Jun Xu (SINAP, Shanghai), Betty Tsang (MSU),
Pawel Danielewcz (MSU), Jongjia Wang (Houzhou), HHW (Munich)

Transport theory: based on a chain of approximations from real-time Green functions via Kadanoff-Baym eqs. to Boltzmann-Vlasov eq. (semi-classical , quasi-particle appprox.)

In practice: two families of transport approaches

Boltzmann-Vlasov-like (BUU/BL/SMF)

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\vec{\nabla}^{(r)}-\vec{\nabla}U(r)\vec{\nabla}^{(p)}\right)f(\vec{r},\vec{p};t)$$
$$=I_{coll}[\sigma^{in-med}]+\delta I_{fluc}$$

Dynamics of the 1-body phase space distribution function f with 2-body dissipation (collision term I_{coll})

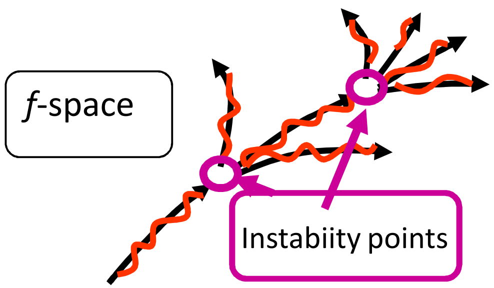
Solution with test particles, exact for $N_{TP}\rightarrow\infty$

include fluctuations around diss. solution

$$f(r,p,t)=\bar{f}(r,p,t)+\delta f(r,p,t)$$

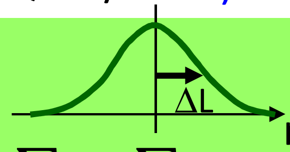
$$\frac{df}{dt}=I_{coll}+I_{fluc}$$

Boltzmann-Langevin eq.



Molecular-Dynamics-like (QMD/AMD)

$$|\Phi\rangle=A\prod_{i=1}^{A}\varphi(r;r_i,p_i)|0\rangle$$



$$\dot{r}_i=\{r_i,H\};\quad \dot{p}_i=\{p_i,H\};\quad H=\sum_i t_i+\sum_{i,j}V(r_i-r_j)$$

TD-Hartree(-Fock)

(or classical molecular dynamics with extended particles, Hamiltonian eq. of motion)

plus stochastic NN collisions

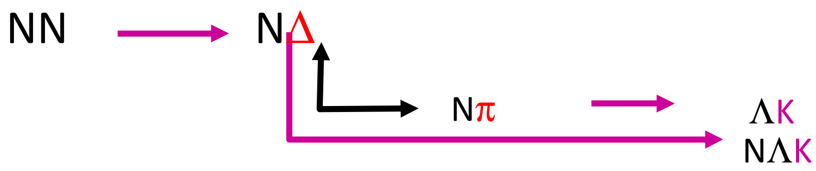
No quantum fluctuations,
but classical N-body fluctuations, damped by the smoothing.

More fluctuations in QMD than in BUU, since degrees of freedom are nucleons:

→ amount controlled by width of single particle packet ΔL

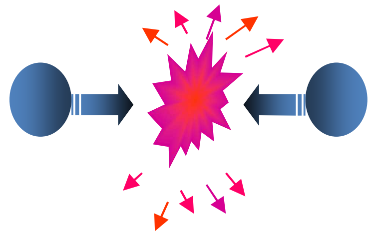
We will see, that the different amount of fluctuations accounts for much the different behaviour of BUU and QMD

Inelastic collisions: Production of particles and resonances



e.g. pion and kaon production;

coupling of Δ and strangeness channels.



$$\frac{d}{dt} f_N(x_\mu) = I_{coll}(\sigma_{NN\to NN} f_N{}^2; \sigma_{NN\to N\Delta} f_N{}^2; \ldots.)$$

$$\frac{d}{dt} f_\Delta(x_\mu) = I_{coll}(\sigma_{\Delta N\to NYK} f_N f_\Delta; \ldots.)$$

etc.

Coupled transport equations

Many new potentials, elastic and inelastic cross sections needed, π,Δ dynamics in medium

Sequence of elastic and inelastic scattering in the simulation of the collision term important

Why Code Comparison ?

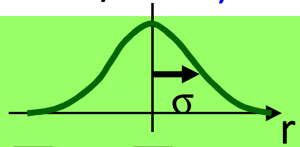
Boltzmann-Vlasov-like (BUU/BL/BLOB)

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla}^{(r)} - \vec{\nabla}U(r)\,\vec{\nabla}^{(p)}\right) f(\vec{r},\vec{p};t) = I_{coll}[\sigma^{in-med}, f_i]$$

6-dim integro-differential, non-linear eq.

Molecular-Dynamics-like (QMD/AMD)

$$|\Phi\rangle = \mathcal{A}\prod_{i=1}^{A}\varphi(r;r_i,p_i)|0\rangle$$



$$\dot{r}_i = \{r_i, H\}; \quad \dot{p}_i = \{p_i, H\}; \quad H = \sum_i t_i + \sum_{i,j} V(r_i - r_j)$$

6A-dim many body problem + stochastic coll.

physical input (EOS, σ_{inmed}, $\pi\Delta$ physics, ..) → **transport code** → observables

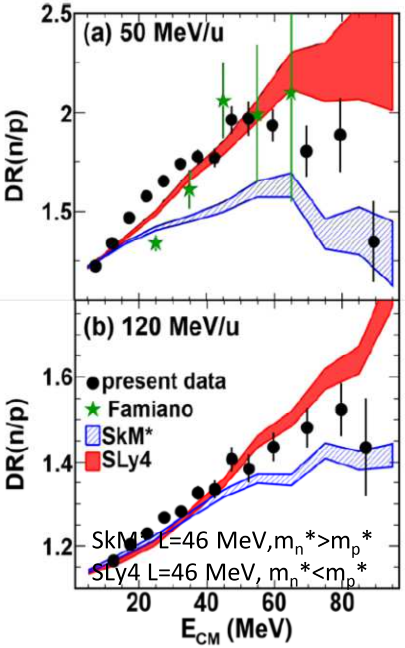
→ unique?, e.g. like a transfer reaction
→ very complex, simulation of an equation rather than a solution, introduces many technical details
→ results are sometimes not consistent
→ establish a sort of systematical theoretical error

→Transport Code Evaluation (Comparison) Project

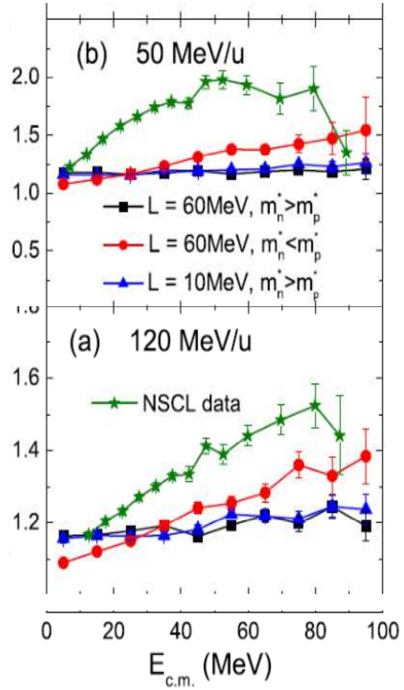
Code Comparison:
A need for more consistency in HI simulations: examples

double ratio of n/p pre-equilibrium emiss.

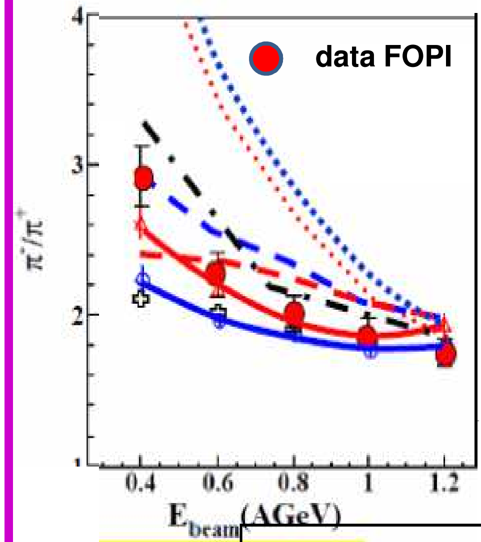
D.D.S.Coupland, et al., PRC94, 011601(R) (2016)

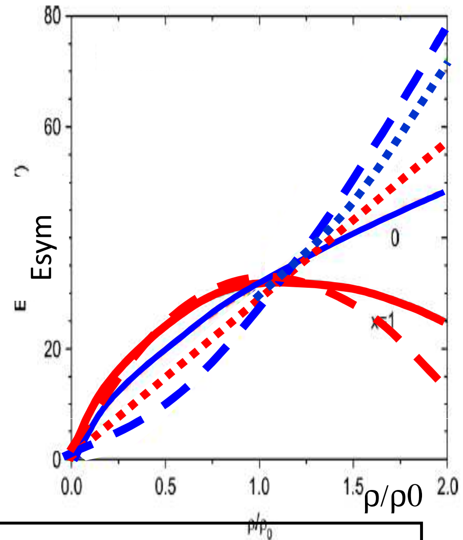


H.J.Kong, et al., PRC91,047601 (2015)



ratio of pion yields, Au+Au,0.4-1.2 GeV/A





various models
blue: stiffer symm energy
red: softer symm energy
→ no consensus, even on ordering

Reasons for differences often not clear, since calculations slightly different in the physical parameters.
→ therefore comparison of calculations with same physical input,
i.e. under controlled conditions

Code Comparison Project

History:
Workshop in Trento 2004 (1 AGeV regime, mainly particle production π,K
Workshop in Trento 2009 and Shanghai 2014 (Au+Au collisions, 100, 400 AMeV)
Workshop ICNT and NuSYM 2017, MSU 2017 (Cascade box calculations)
to be continued : Zhuhai (China, 2018) and NuSYM 2018 (Busan, Korea), Transport19 (ECT*?)

Steps in Code Comparison of Transport Simulations

1. Full heavy ion collisions (Au+Au, 100, 400 AMeV)
comparison of initialization, collision rates and observables
J. Xu et al., Phys. Rev. C 93, 064609 (2016)
-> considerable discrepancies, but difficult to disentangle

done

2. Calculations of nuclear matter (box with periodic boundary conditions)
test separately ingredients in a transport approach:
a) collision term without and with blocking (Cascade) **done**
Y.X. Zhang, et al., Phys. Rev. C 97, 034625 (2018)
b) mean field propagation (Vlasov)
c) pion, Δ production in Cascade
} **in progress**
d) instabilities , fragmentation
e) momentum dependent fields
} **planned**
……

Codes participating in the code comparison

BUU type	Code correspondents	Energy range	Reference	QMD type	Code correspondents	Energy range	Reference
BLOB	P. Napolitani, M. Colonna	0.01–0.5	[19]	AMD	A. Ono	0.01–0.3	[28]
GIBUU-RMF	J. Weil	0.05–40	[20]	IQMD-BNU	J. Su, F. S. Zhang	0.05–2	[29]
GIBUU-Skyrme	J. Weil	0.05–40	[20]	IQMD	C. Hartnack, J. Aichelin	0.05–2	[30–32]
IBL	W. J. Xie, F. S. Zhang	0.05–2	[21]	CoMD	M. Papa	0.01–0.3	[33,34]
IBUU	J. Xu, L. W. Chen, B. A. Li	0.05–2	[11,22]	ImQMD-CIAE	Y. X. Zhang, Z. X. Li	0.02–0.4	[35]
pBUU	P. Danielewicz	0.01–12	[23,24]	IQMD-IMP	Z. Q. Feng	0.01–10	[36]
RBUU	K. Kim, Y. Kim, T. Gaitanos	0.05–2	[25]	IQMD-SINAP	G. Q. Zhang	0.05–2	[37]
RVUU	T. Song, G. Q. Li, C. M. Ko	0.05–2	[26]	TuQMD	D. Cozma	0.1–2	[38]
SMF	M. Colonna, P. Napolitani	0.01–0.5	[27]	UrQMD	Y. J. Wang, Q. F. Li	0.05–200	[39,40]

→ BUU- and QMD-type, most of the commonly used codes
→ non-rel. and relativistic codes
→ antisymmetrized QMD code: AMD
→ BUU codes with explicit fluctuations: SMF, BLOB
→ many new Chinese codes: (I)QMD-XXX: much new activity in China, often originally closely related

I. Set-up of code comparison for full Heavy Ion Collisions

- typical reaction in low and intermediate energy: Au+Au, 100 and 400 AMeV, 7 fm (midcentral)
- simple physics case (not necessarily realistic)
  standard Skyrme mean field, momentum independent, equivalent RMF
  constant cross section, no inelastic collisions
- „close" initialization of colliding nuclei
  prescribed density profile, momentum in local Fermi sphere
- collision and blocking procedures as in standard use of code

- monitor: particle motion, collision numbers, energy and time,
  Pauli-blocking, observables (rapidity, flow)

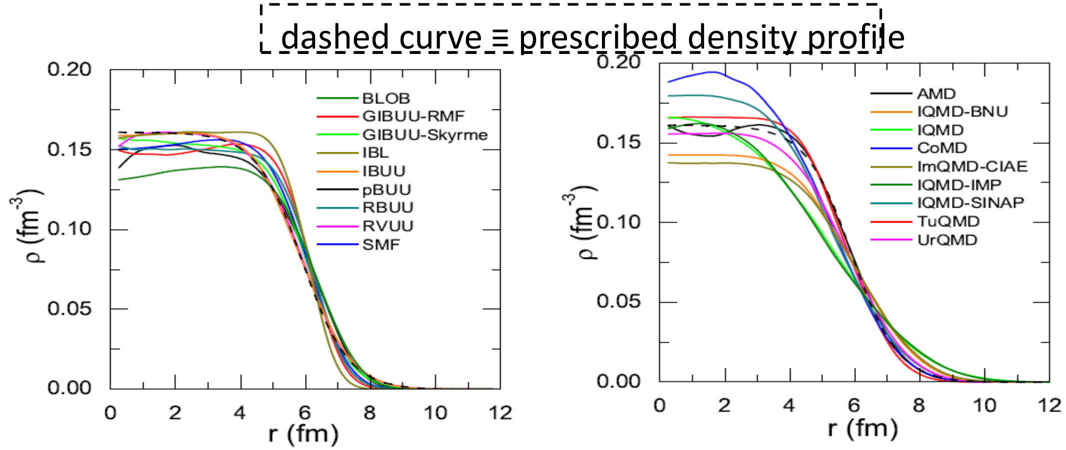
PHYSICAL REVIEW C 93, 044609 (2016)

Understanding transport simulations of heavy-ion collisions at 100A and 400A MeV: Comparison of heavy-ion transport codes under controlled conditions

core group

Jun Xu,[1,*] Lie-Wen Chen,[2,†] ManYee Betty Tsang,[3,‡] Hermann Wolter,[4,§] Ying-Xun Zhang,[5,∥] Joerg Aichelin,[6] Maria Colonna,[7] Dan Cozma,[8] Pawel Danielewicz,[3] Zhao-Qing Feng,[9] Arnaud Le Fèvre,[10] Theodoros Gaitanos,[11] Christoph Hartnack,[6] Kyungil Kim,[12] Youngman Kim,[12] Che-Ming Ko,[13] Bao-An Li,[14] Qing-Feng Li,[15] Zhu-Xia Li,[5] Paolo Napolitani,[16] Akira Ono,[17] Massimo Papa,[18] Taesoo Song,[19] Jun Su,[20] Jun-Long Tian,[21] Ning Wang,[22] Yong-Jia Wang,[15] Janus Weil,[19] Wen-Jie Xie,[23] Feng-Shou Zhang,[24] and Guo-Qiang Zhang[1]

Initialization and Stability

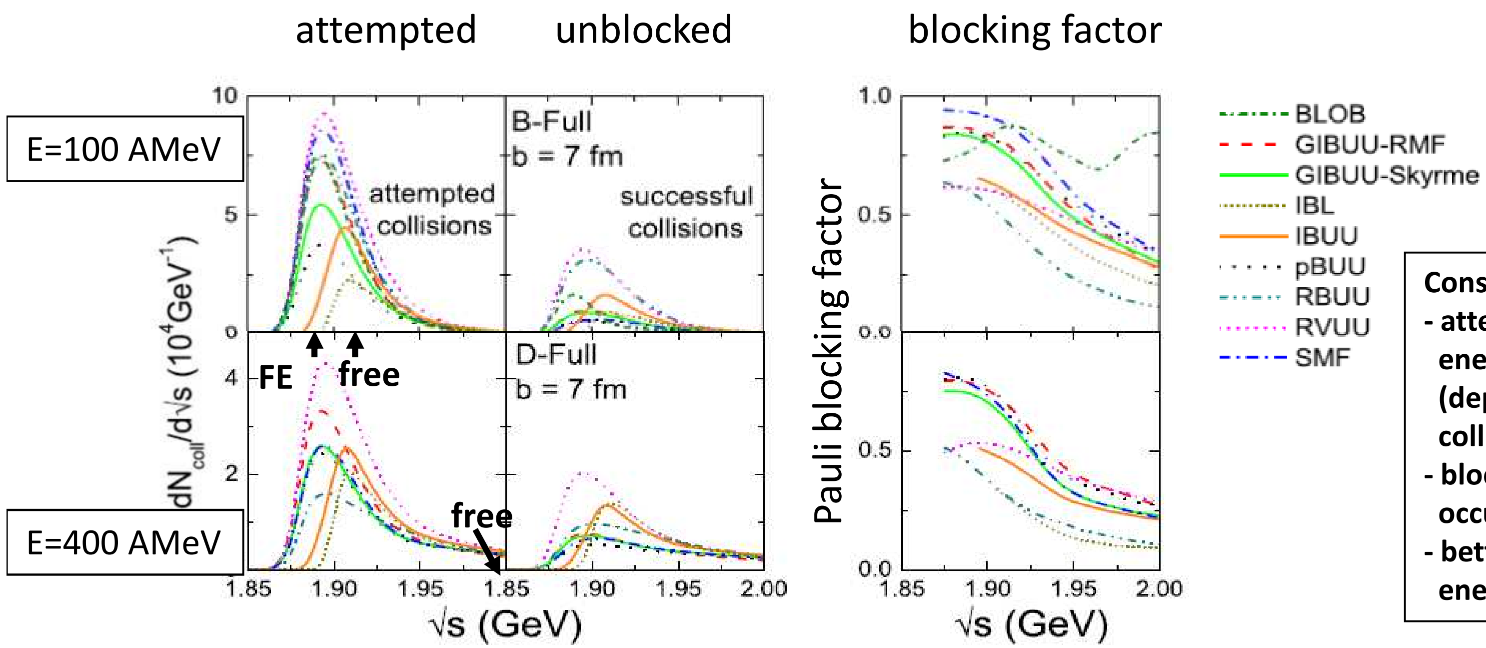


„identical“ initialization difficult, since it depends also on repesentation of (test) particles

- prescribed density profile is not neccessarily ground state and may be non-stationary

- diff. initializations affect evolution also in case of a collision

NN Collision rates per energy bin

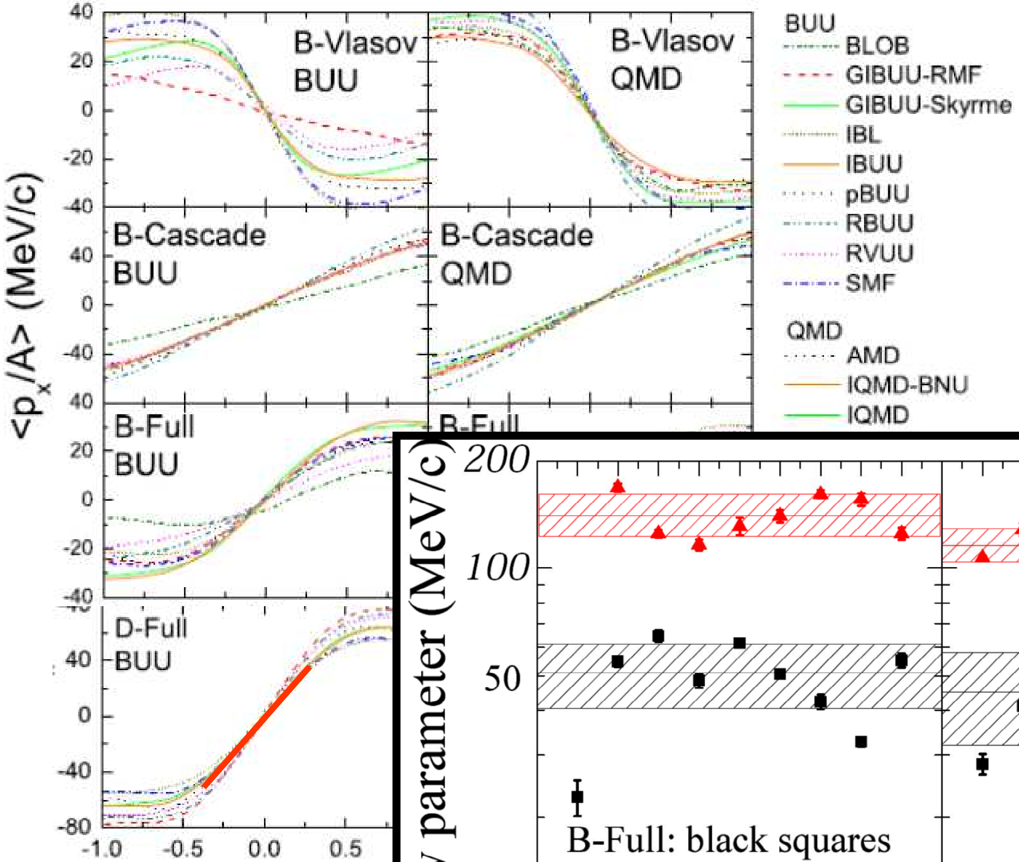


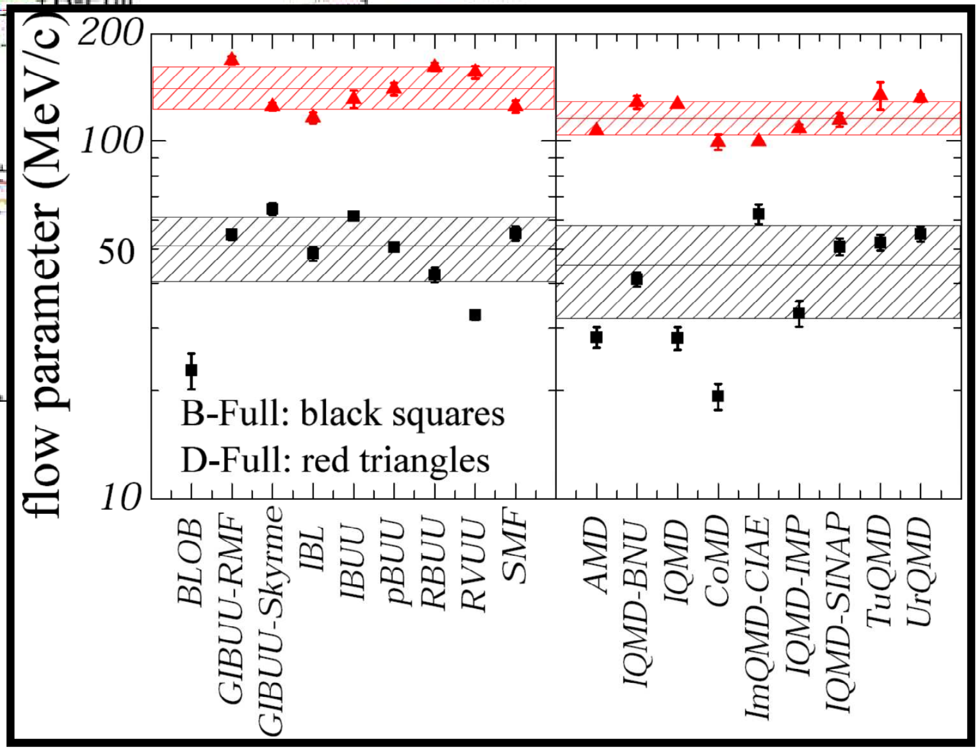
Considerable difference both for :
- **attempted collisions, mostly low energy(!) (depends on strategy for finding collision pairs)**
- **blocking factor (depends on occupation of final state)**
- **better consistency for higher energy**

Observables: average in-plane flow

at 100 AMeV
Vlasov and Cascade
opposite slope:
~ balance energy,
  sensitive region,
 →large discrepancies

at 400 AMeV
more consistent





quantify spread of simulations by value of „flow“=slope at midrapidity

BUU and QMD approx. consistent

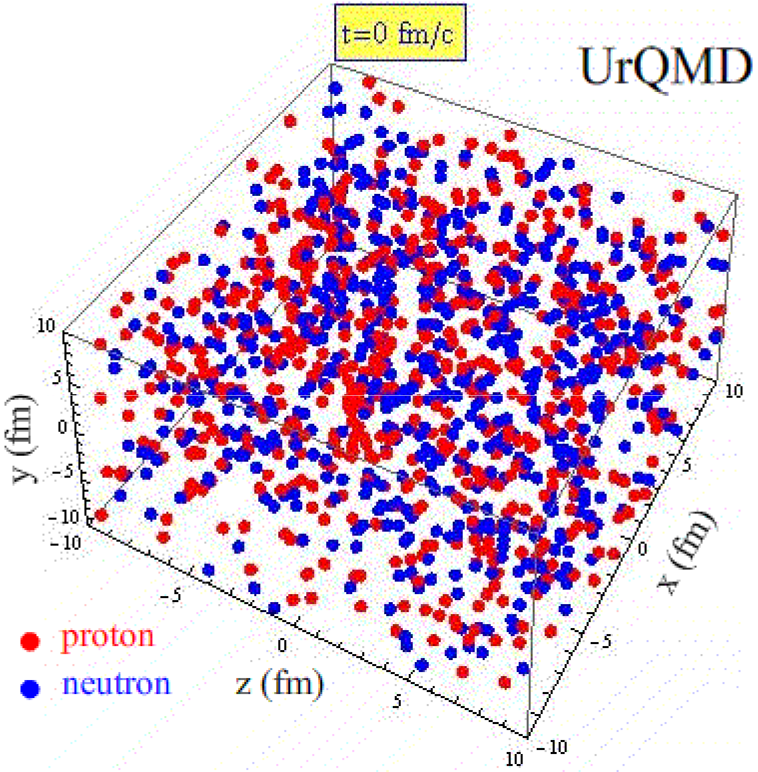
uncertainity

100 AMeV: ~30%
400 AMeV: ~13%

Difficult to disentangle origin of discrepancies

2. Box calculation comparison

simulation of the static system of infinite nuclear matter,
→ solve transport equation in a periodic box



Useful for many reasons:

- check consistency of calculation
e.g. thermodynamical consistency

- check consistency of simulation:
collision numbers, blocking
(exact limits from kinetic theory)

- check aspects of simulation separately
Cascade: only collisions
without/with blocking
Vlasov: only mean field propagation

- check ingredients of particle production
e.g. pion production

PHYSICAL REVIEW C **97**, 034625 (2018)

Comparison of heavy-ion transport simulations: Collision integral in a box

Ying-Xun Zhang,[1,2,*] Yong-Jia Wang,[3,†] Maria Colonna,[4,‡] Pawel Danielewicz,[5,§] Akira Ono,[6,∥] Manyee Betty Tsang,[5,¶] Hermann Wolter,[7,#] Jun Xu,[8,**] Lie-Wen Chen,[9] Dan Cozma,[10] Zhao-Qing Feng,[11] Subal Das Gupta,[12] Natsumi Ikeno,[13] Che-Ming Ko,[14] Bao-An Li,[15] Qing-Feng Li,[3,11] Zhu-Xia Li,[1] Swagata Mallik,[16] Yasushi Nara,[17] Tatsuhiko Ogawa,[18] Akira Ohnishi,[19] Dmytro Oliinychenko,[20] Massimo Papa,[4] Hannah Petersen,[20,21,22] Jun Su,[23] Taesoo Song,[20,21] Janus Weil,[20] Ning Wang,[24] Feng-Shou Zhang,[25,26] and Zhen Zhang[14]

Collision term in box calculations

collision probability

no blocking

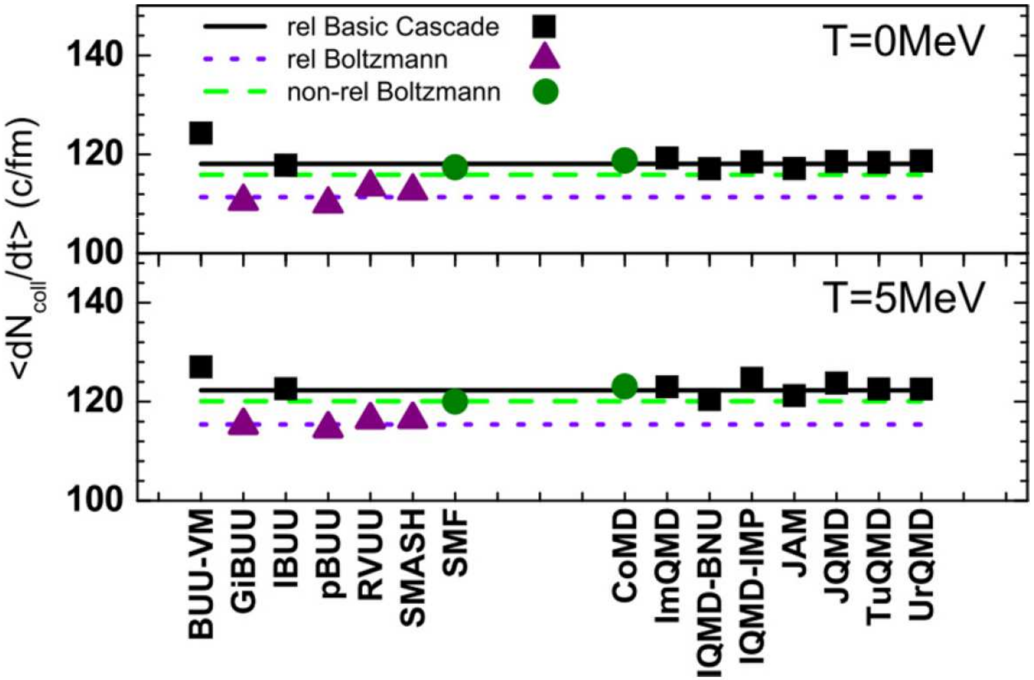
$$I_{coll} = \int d\vec{p}_2 d\vec{p}_{1'} d\vec{p}_{2'}\, v_{21}\, \sigma_{12}^{in-med}(\Omega)(2\pi)^3 \delta(p_1 + p_2 - p_{1'} - p_{2'}) \left[f_{1'} f_{2'} (1-f_1)(1-f_2) - f_1 f_2 (1-f_{1'})(1-f_{2'}) \right]$$

Collision rates in a cascade box calculation (w/o mean field, T=0 and 5 MeV)

without blocking
Comparison to exact limit

$$\frac{dN_{\rm coll}}{dt} = \frac{A}{2\rho} g^2 \int \frac{d^3p\, d^3p_1}{(2\pi\hbar)^6} v_{\rm rel}\, \sigma^{\rm med} f(p) f(p_1) = \frac{1}{2} A \rho \langle v_{\rm rel}\, \sigma^{\rm med} \rangle .$$

(v_{rel} and average depend on treatment of relativity)



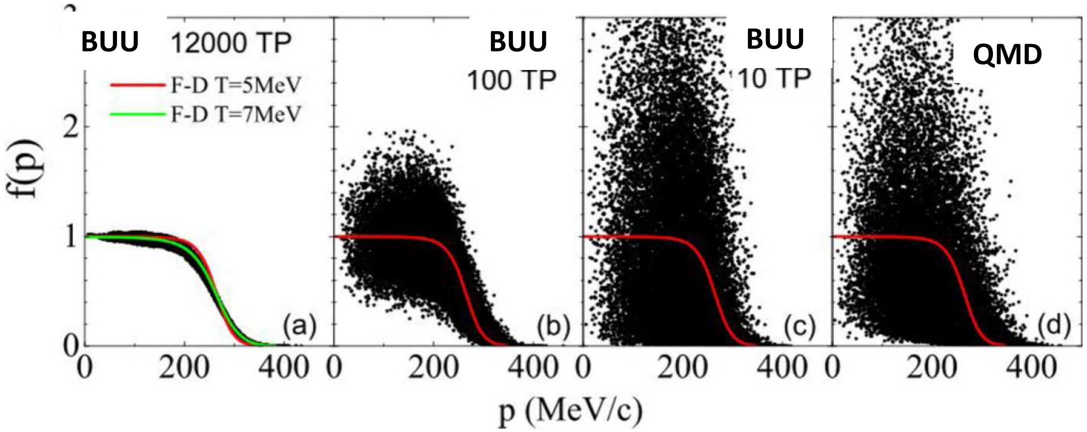
good agreement with corresponding exact result
collision probability ok

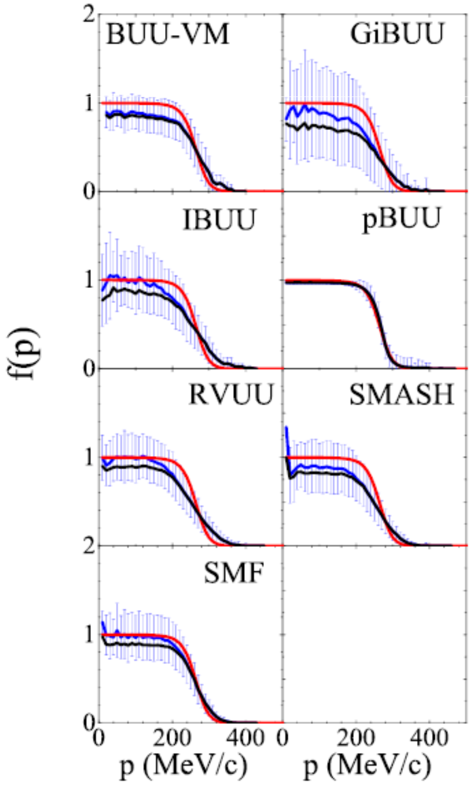
$$I_{coll} = \int d\vec{p}_2 d\vec{p}_{1'} d\vec{p}_{2'} \, v_{21} \, \sigma_{12}^{in-med}(\Omega) \, (2\pi)^3 \delta(p_1 + p_2 - p_{1'} - p_{2'}) \left[f_{1'} f_{2'} (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_{1'})(1 - f_{2'}) \right]$$

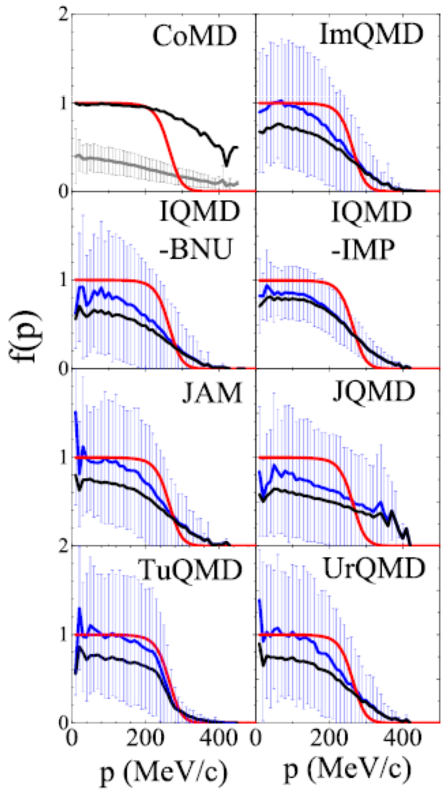
with blocking

Sampling of occupation prob. in comp. to prescribed FD distribution (red)

- **fluctuation in BUU controlled by TP number, can be made arbitrarily small**
- **fluctuation in QMD given by width of wave packet**



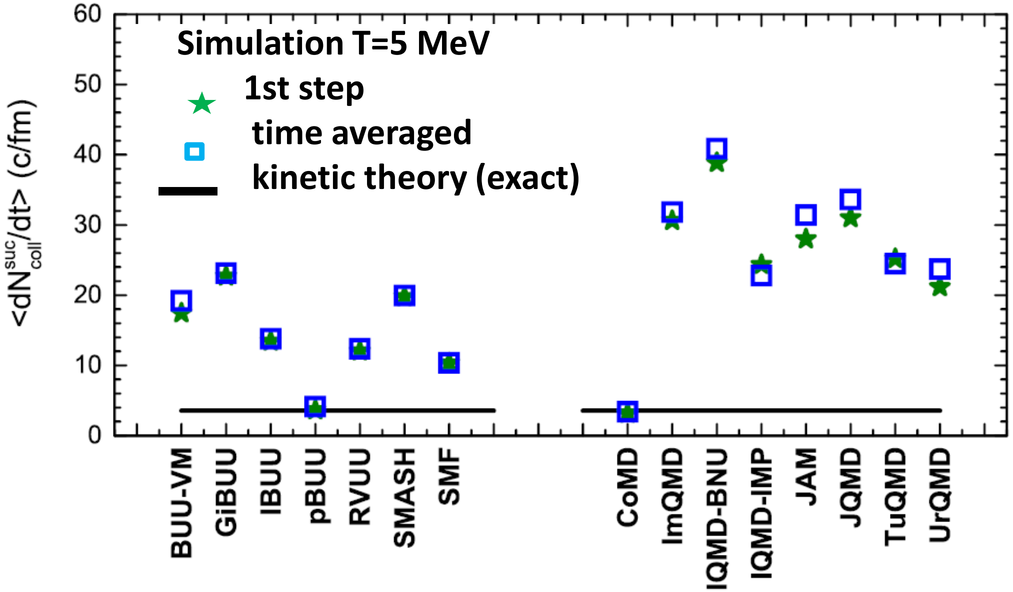




width and averages of calculated occupation numbers in different codes

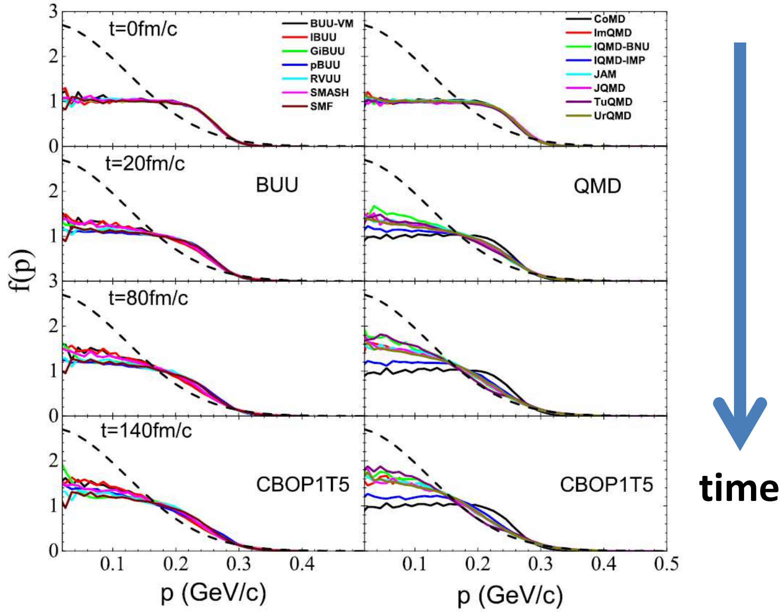
- (red line) **prescribed occupation**
- (blue line) **average calculated occupation**
- (black line) **average of f<1 occupation (used for the blocking)**

Collision rates with blocking



- almost all codes have too little blocking, i.e. allow too many collisions,
- QMD codes more, because of larger fluctuations

Evolution of momentum distributions



- the momentum distribution moves away from the stable Fermi-Dirac distribution towards the classical Maxwell-Boltzmann distribution (dotted line),
- depending on collision rates

Fluctuations influence dynamics of transport calculations.
However the proper treatment of fluctuations in transport is under debate.

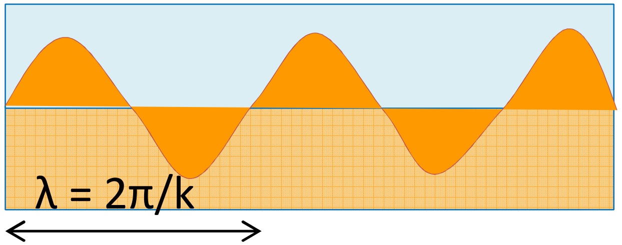
Box simulations: test of m.f. dynamics
(in progress! preliminary)

Maria Colonna

-- Symmetric matter --
- Only mean-field potential
- No surface terms
- Compressibility K=240 and 500 MeV

- Study the time evolution of $\rho(z)$

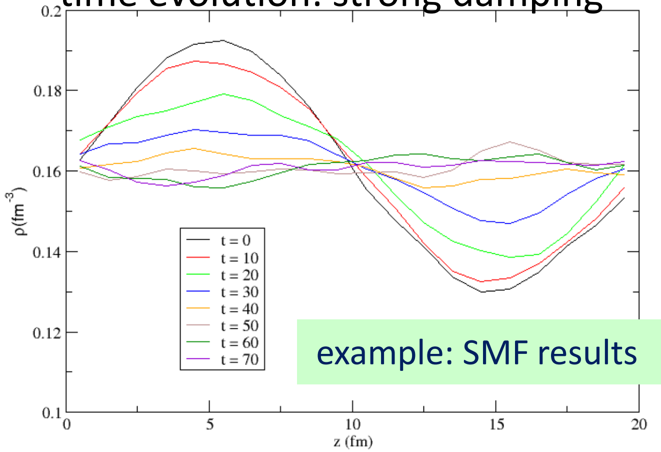
L = 20 fm



$\rho(z,t=t_0) = \rho_0 + a_\rho \sin(k_i z)$

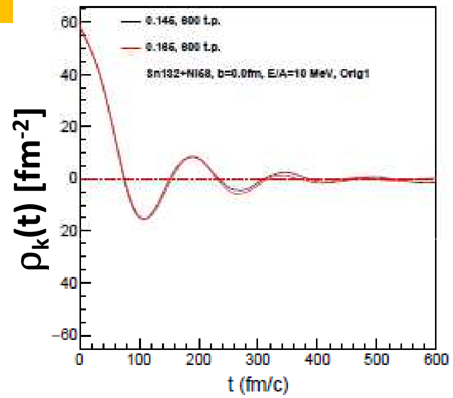
$k_i = n_i\, 2\pi/L, \quad a_\rho = 0.2\,\rho$

time evolution: strong damping



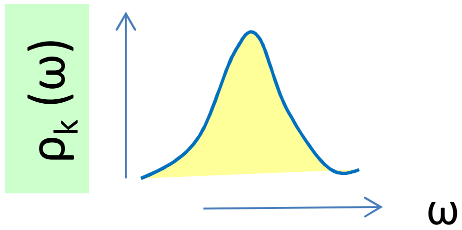
1. Extract the Fourier transform in space

$\rho_k(t) = \int dz\, \sin(kz)\, \rho(z,t)$

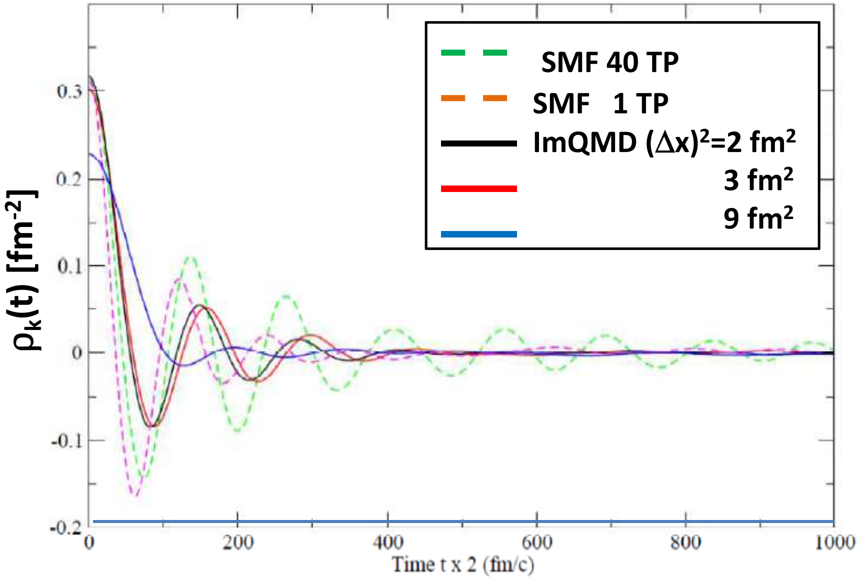


2. Fourier transform in time:
extract the oscillation frequency

$\rho_k(\omega) = \int dt\, \cos(\omega t)\, \rho_k(t)$



Time evolution of Fourier transform ρ_k (K=500 MeV)



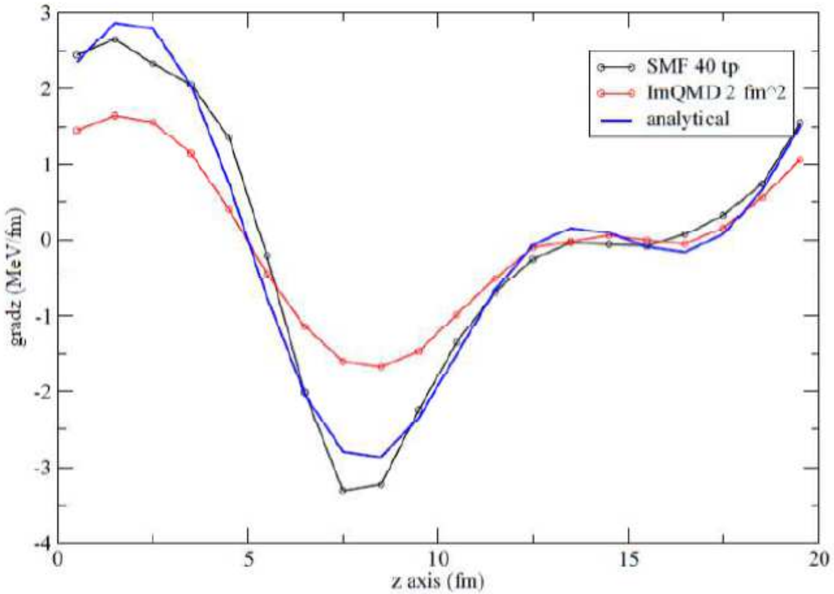
Generally: strong damping

- SMF (BUU-like, dashed curves)
smaller no of TP: more damping, larger frequency

- ImQMD (solid curves)
increasing width Δx of wave packet:
larger fluctuations in QMD $\rightarrow$ stronger damping
smaller effective forces in QMD $\rightarrow$ larger frequencies

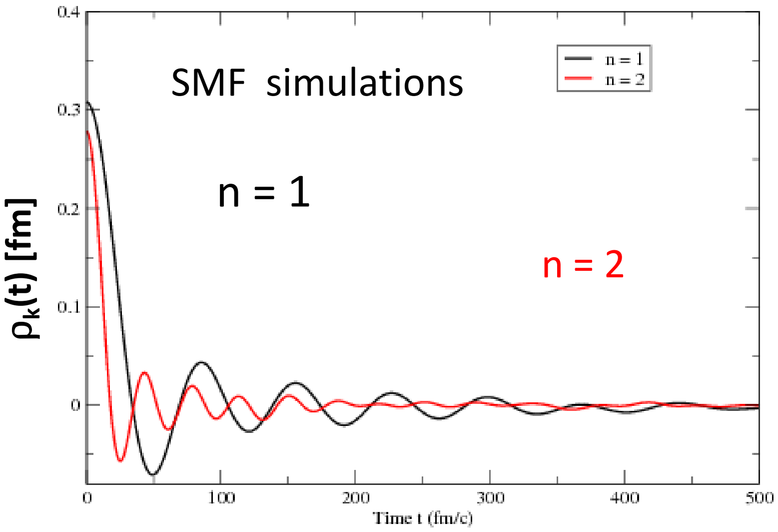
Gradient along z-axis

- SMF with 40 TP (1 event) good
- QMD too low,
effect of an approximation (which can be improved)



$$\rho_k(t) = \int dz \sin(kz)\, \rho(z,t)$$

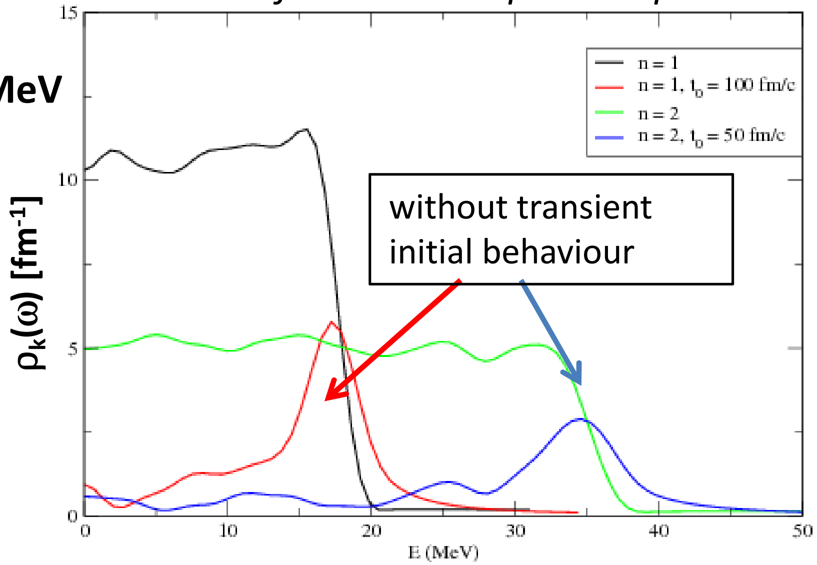
Fourier transform with respect to space



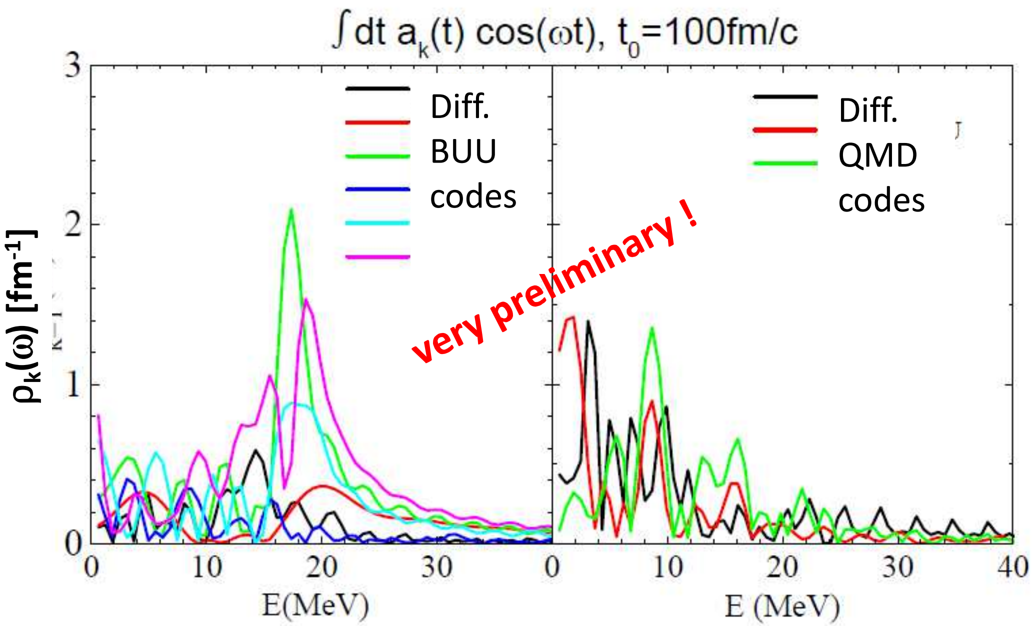
$$\rho_k(\omega) = \int dt \cos(\omega t)\, \rho_k(t)$$

Fourier transform with respect to space and time

K=240 MeV



$\omega \,/\, (k\, v_F) \sim 1$ n = 1, E ~ 18 MeV



- QMD-like models:
appear structureless, large damping
- BUU-like models:
differences in frequency and damping

Fluctuations strongly influence propagation of collective modes

π,Δ production in box cascade calculation:
(in progress, preliminary!)

Akira Ono and Jun Xu

N,Δ, no pions

— kinetic solution (rate eqs.)

one- way only

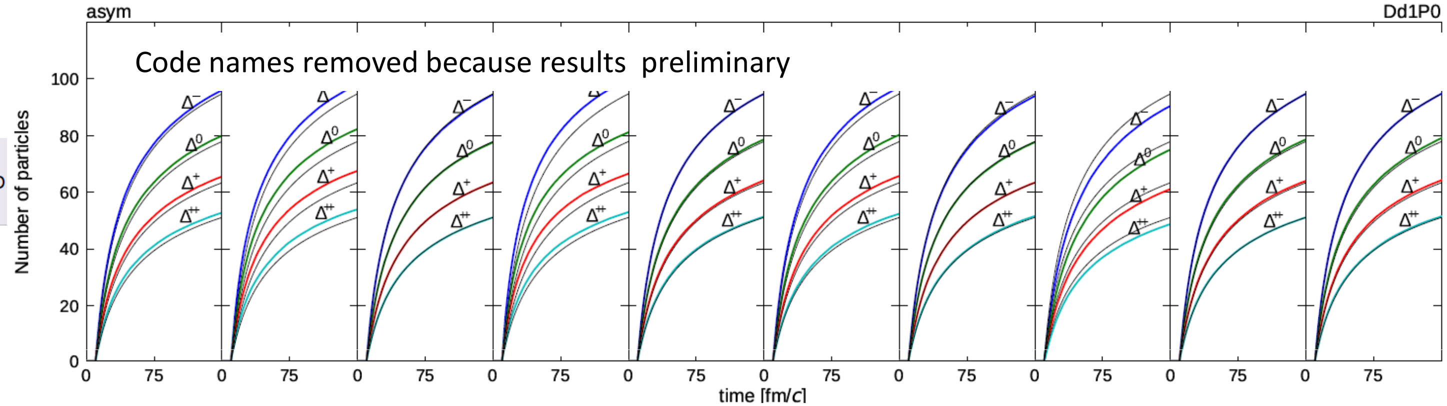
$$NN \rightarrow N\Delta$$

energy dep cross sect.

$$\sigma(NN \rightarrow N\Delta) = \frac{(\sqrt{s} - 2M_N - M_\pi)^2}{(0.015\ \text{GeV}^2) + (\sqrt{s} - 2M_N - M_\pi)^2} \times 20\ \text{mb}$$

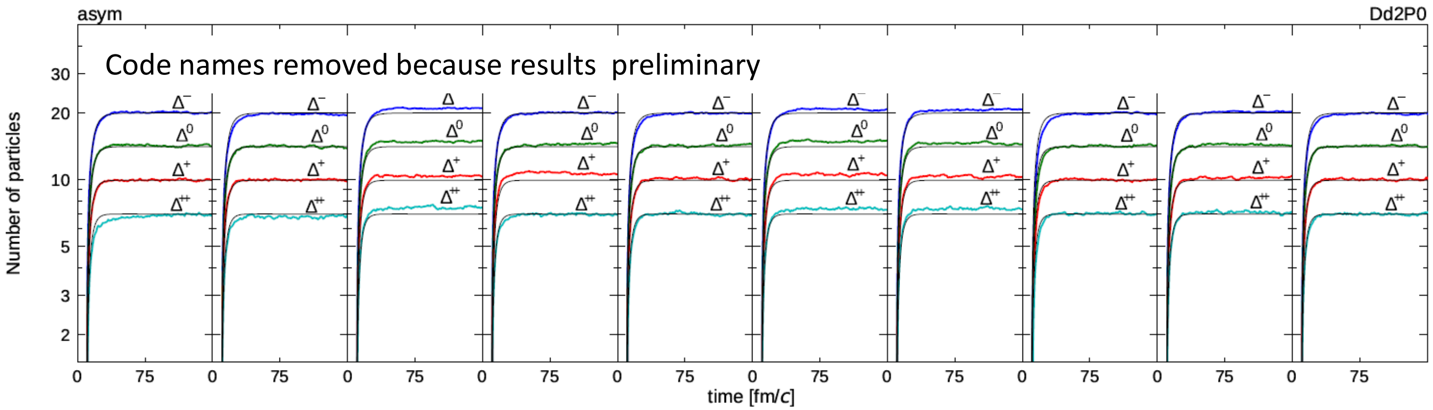
Δ spectral function

$$A(m) = \frac{4M_\Delta^{0\,2}\Gamma_\Delta}{(m^2 - M_\Delta^{0\,2})^2 + M_\Delta^{0\,2}\Gamma_\Delta^2}$$



two- ways

$$NN \leftrightarrow N\Delta$$



Looks reasonably ok! Now switch on pions

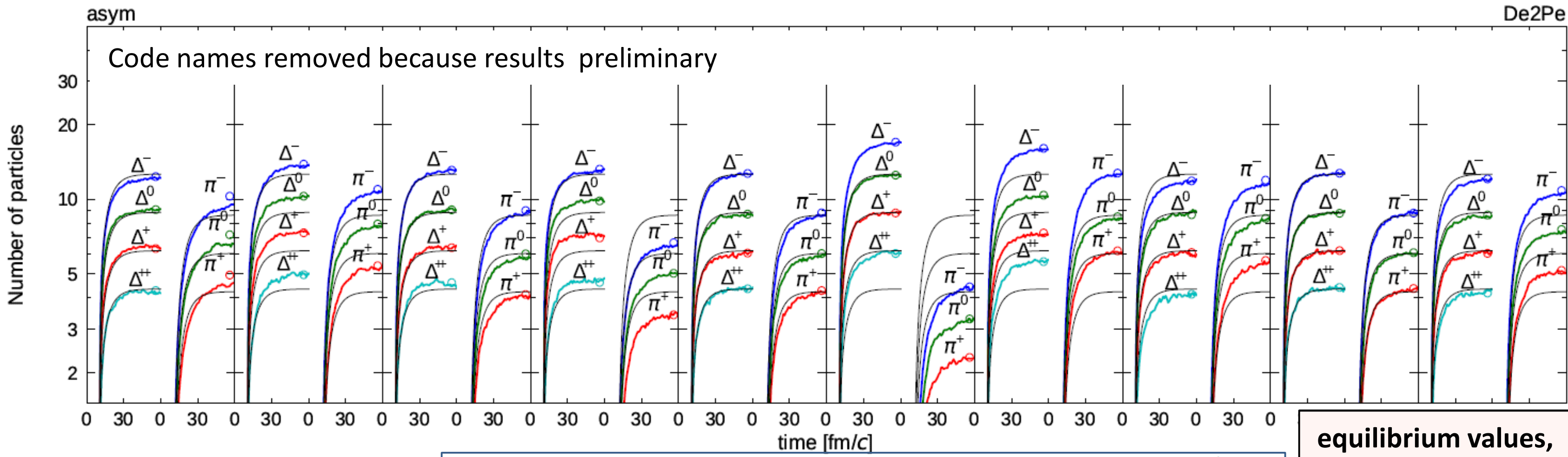
π,Δ production in box cascade calculation: (in progress, preliminary!)

now including pions

$NN \leftrightarrow N\Delta, \quad \Delta \leftrightarrow N\pi$

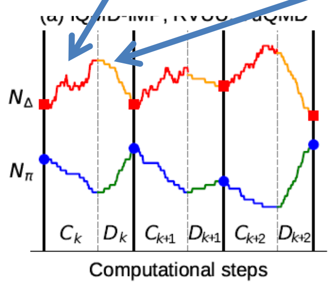
kinetic solution (rate eqs.)

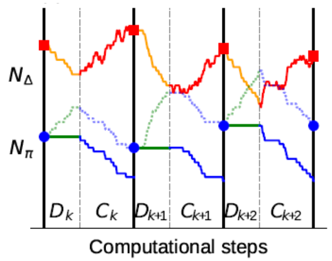
large differences between models and exact result



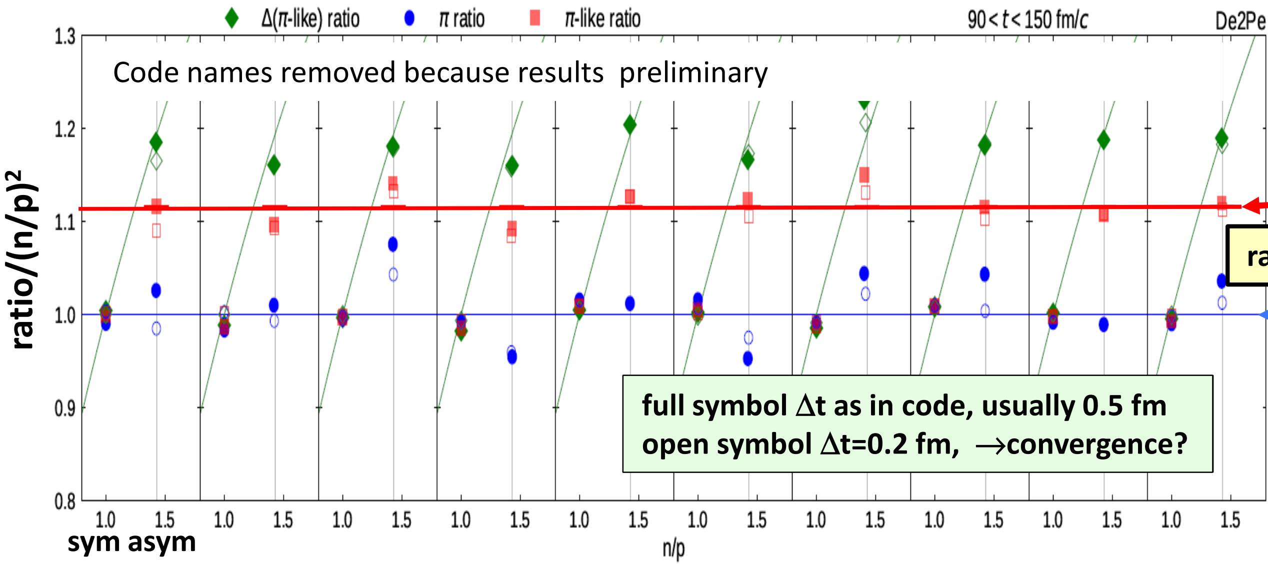
equilibrium values, not quite as good for small times

(partly) due to sequence of handling collisions (C_k) and decays (D_k)





ratios $\pi \text{ ratio} = \pi^- / \pi^+$ $\quad \pi\text{-like ratio} = \dfrac{\pi^- + \Delta^- + \frac{1}{3}\Delta^0}{\pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+}$



rate eq.

full symbol Δt as in code, usually 0.5 fm
open symbol Δt=0.2 fm, $\rightarrow$convergence?

$\rightarrow$ towards a better understanding of the pion ratios

Summary

-Transport approaches are an important method to extract physics information from complex non-equilibrium processes, as e.g. heavy ion collisions.

However, there are open problems in the application of transport theories:

- **physical (which degrees of freedom, esp. for phase transitions, fluctuations, correlations, short range)**
- **questions of implementation: simulation, rather than solution of the transport equations**
- **involves strategies not strictly given by the equations, such as**
  representation of the phase space, coarse graining, criteria for collisions and Pauli blocking

- **these may affect the deduction on physical properties from collisions and lead to a kind of systematical theoretical error**
- **here attempt to understand, quantify and hopefully reduce these uncertainities in a Transport Code Comparison under Controlled Conditions**

Results:

- **Comparison of full HIC makes evident the discrepancies (initializations, collision term), but difficult to disentangle**
- **Box calculations to study the different ingredients of transport**
  (collisions, blocking, mf evolution, particle production)
- **Important influence of fluctuations on the simulations**
- **Fluctuations (and correlations) go beyond the one-body description. Implementions differ in BUU (explicit fluctuation term) and QMD (classical correlations + smoothing by wave packet)**
- **particle production and decay: sequence of treatment in collision term important**
- **continue in the future, e.g. in fragmentation in instable regime, pion production in full HIC, …**

Thank you for your attention