



On the Origin of the Elliptic Flow and its Dependence on the Equation of State in Heavy Ion Reactions at Intermediate Energies

On print in Phys. Rev. C (2018)

by A. Le Fèvre¹, Y. Leifels¹, C. Hartnack² and J. Aichelin^{2,3}

¹GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany

²SUBATECH, UMR 6457, Ecole des Mines de Nantes - IN2P3/CNRS - Université de Nantes, France

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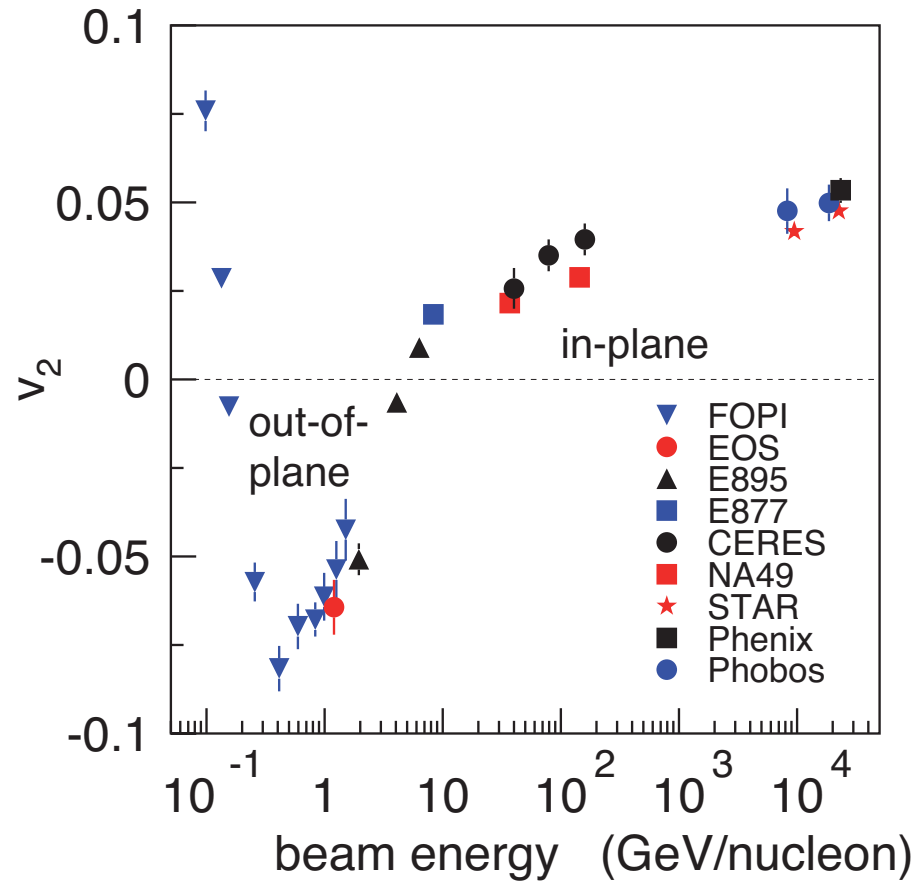
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- ▶ Introduction
- ▶ The Quantum Molecular Dynamics approach
- ▶ Elliptic flow at mid-rapidity: the strongest sensitivity to the Nuclear Equation of State
- ▶ Survey of the reaction
- ▶ Collisions versus mean field
- ▶ Incident energy dependence
- ▶ Summary



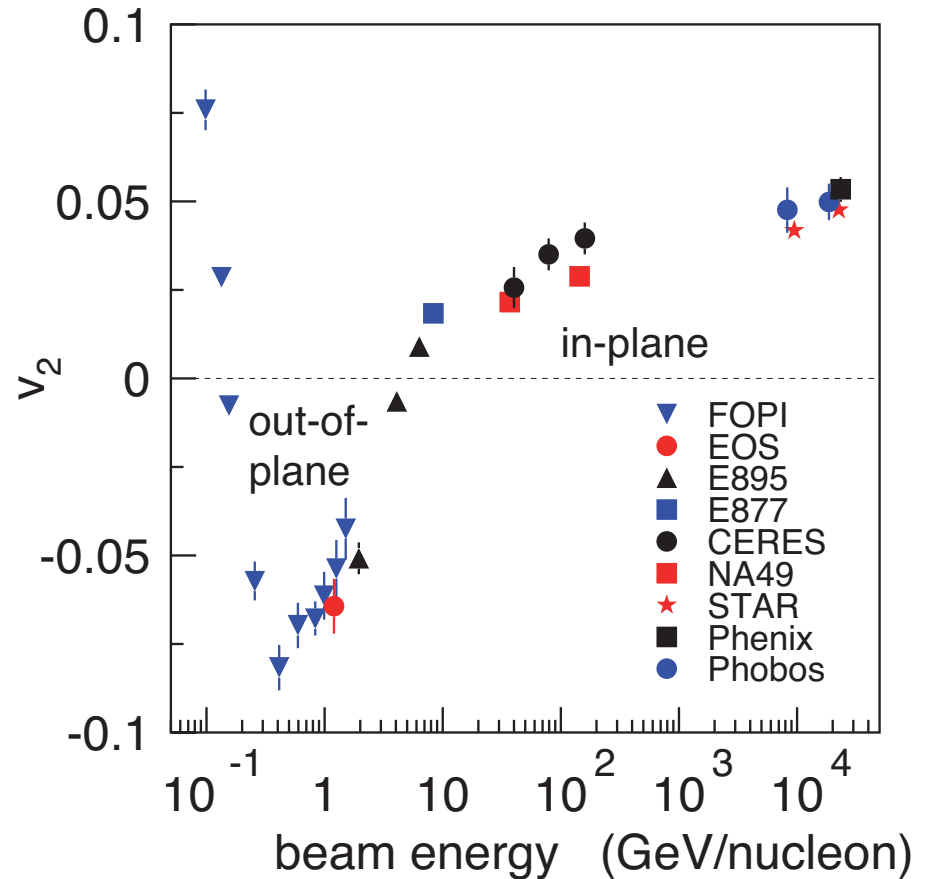
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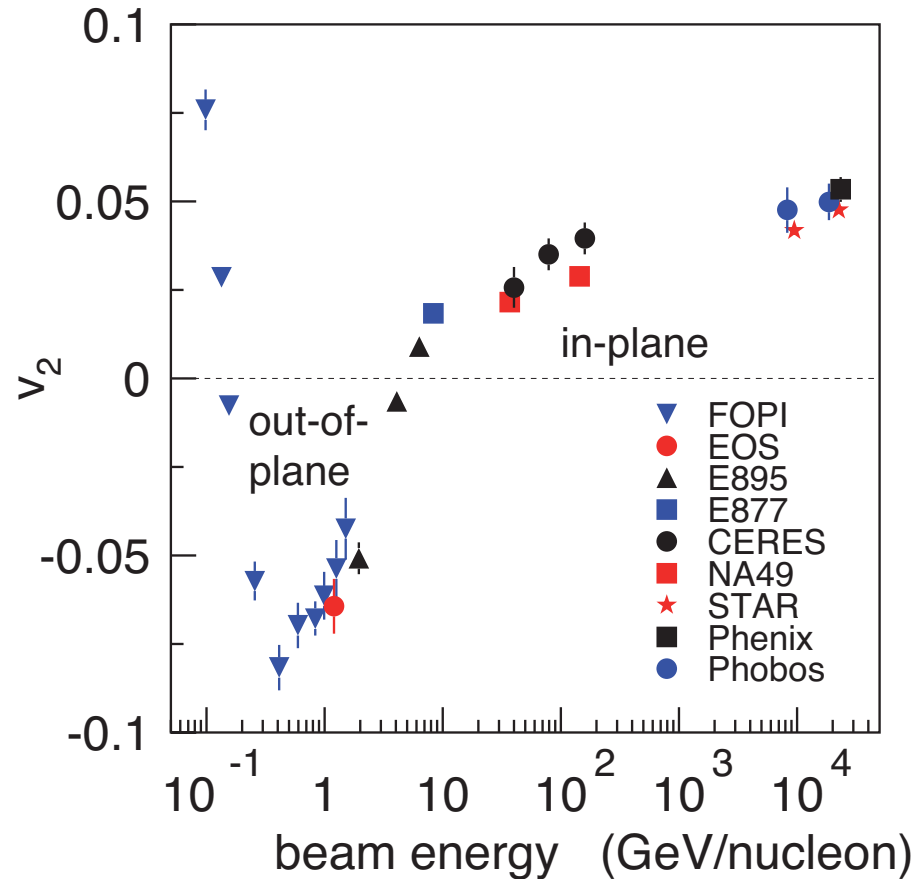




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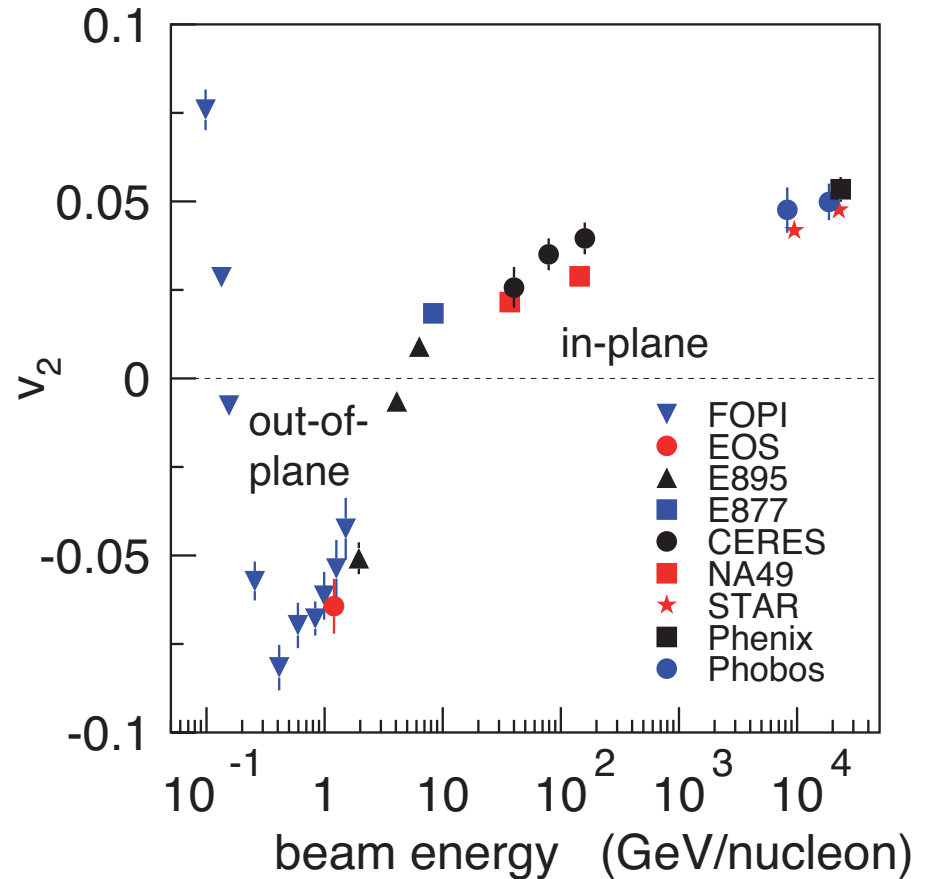


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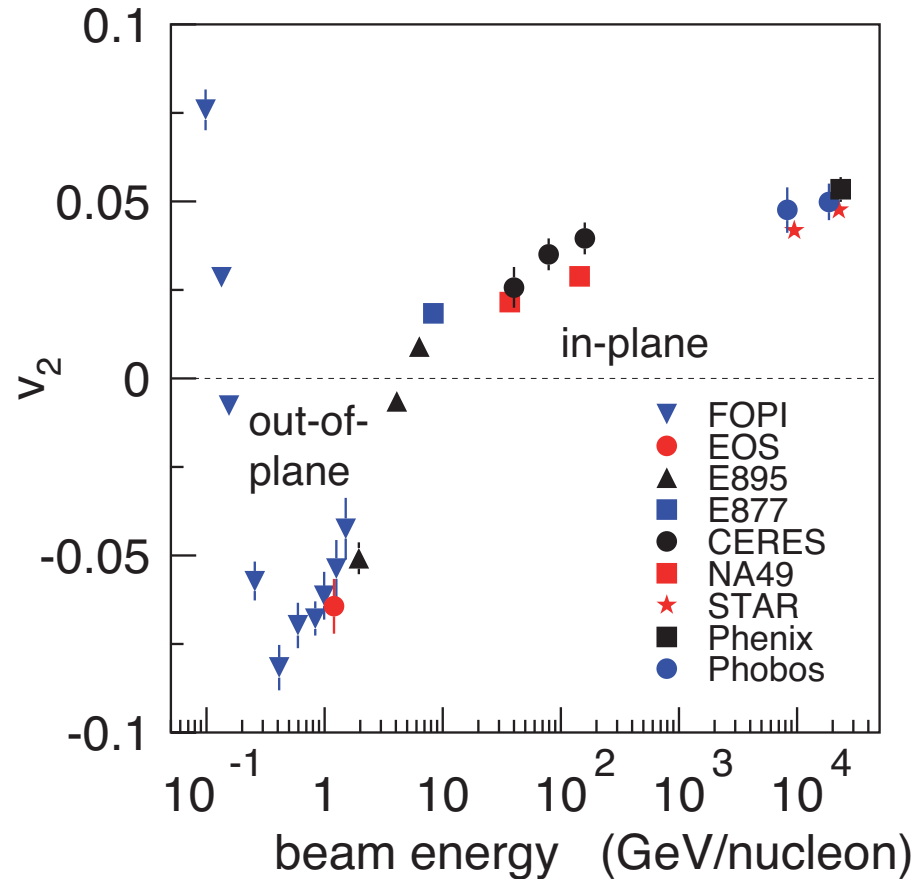
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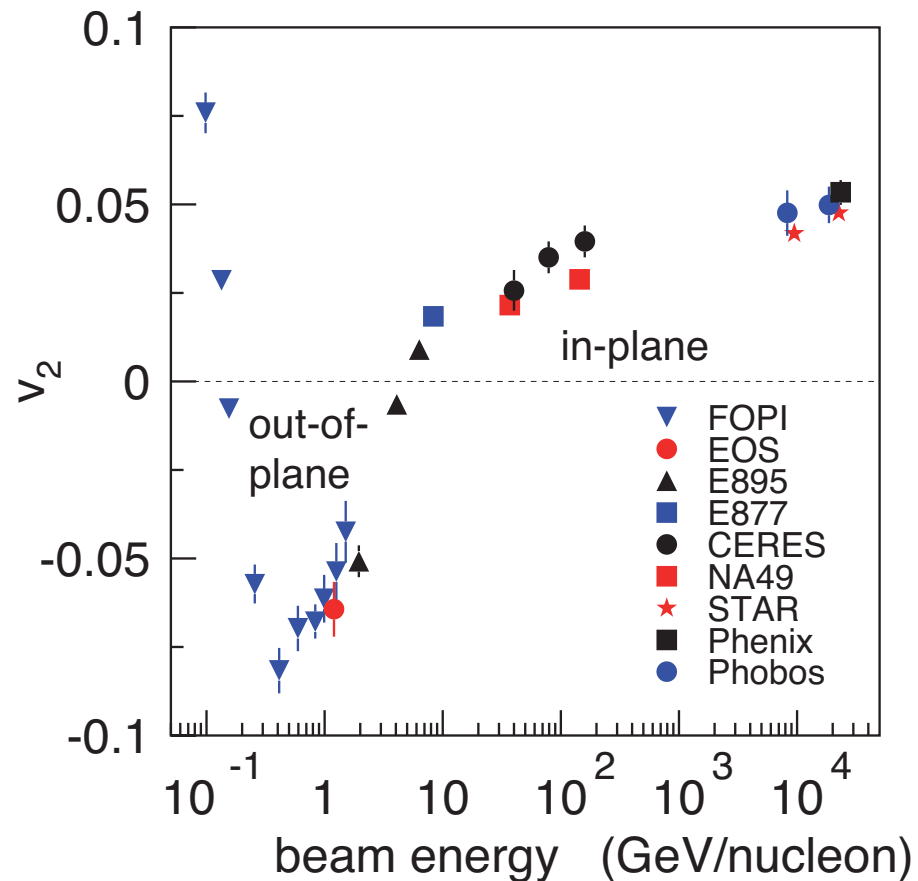
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At ultra-relativistic energies: measured v_2 and centrality dependence \Leftrightarrow expansion of initially highly compressed **almond shaped fireball** \Leftrightarrow

$v_2 > 0$ as predicted by hydrodynamics

- M. Luzum and P. Romatschke, Phys. Rev. Lett. 103 (2009) 262302





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$v_2 > 0$ as predicted by hydrodynamic models.

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0.1

Flows at high density in heavy-ion collisions

$$\frac{dN}{d(\phi - \phi_R)}(y, p_t) = \frac{N_0}{2\pi} \left(1 + 2 \sum_{n \geq 1} v_n \cos n(\phi - \phi_R) \right)^{\frac{1}{n}}$$

Y = rapidity

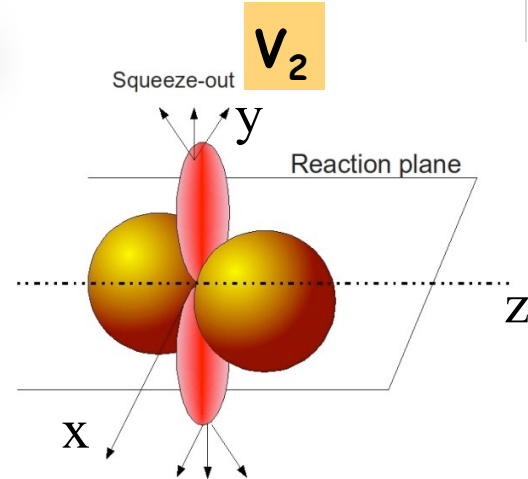
p_t = transverse momentum

Φ_R = reaction plane azimuthal angle

V_1 = 'side/directed flow', $\cos(\Phi - \Phi_R)$ mode

$$V_2(y, p_t) = \left\langle \frac{p_x^2 - p_y^2}{p_t^2} \right\rangle$$

'Elliptic flow': $\cos(2(\Phi - \Phi_R))$ mode, competition between 'in-plane' ($V_2 > 0$) and 'out-of-plane' ejection ($V_2 < 0$).



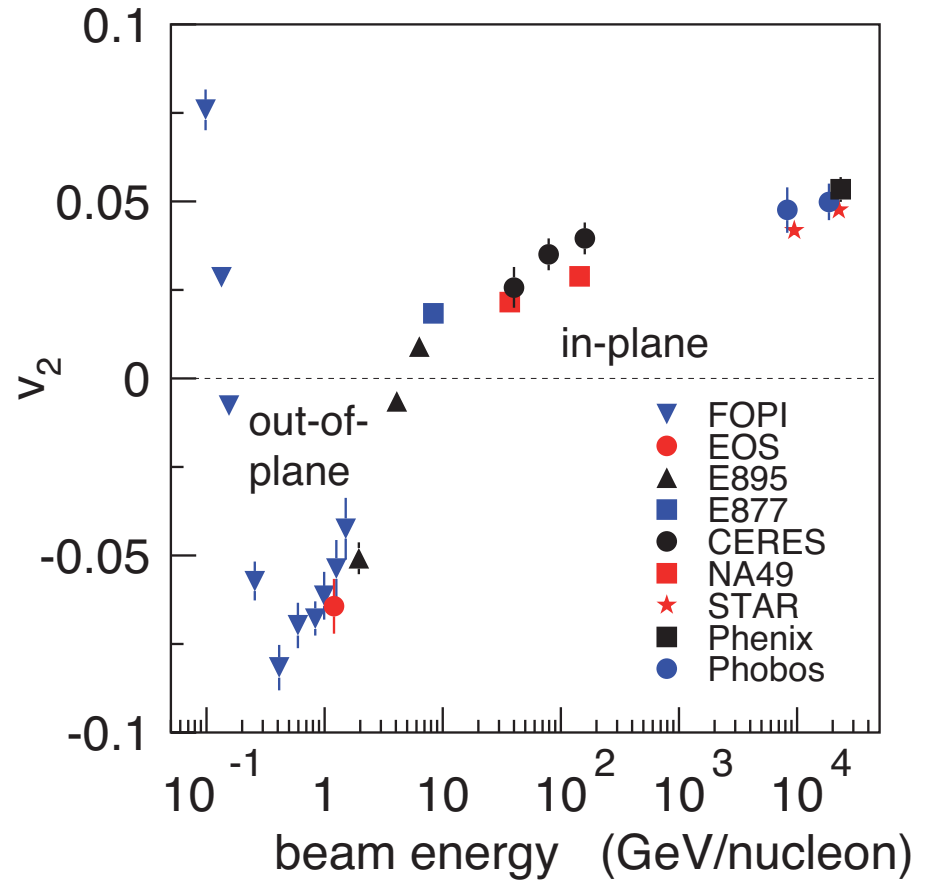
FOPI
EOS
E895
E877
CERES
NA49
STAR
Phenix
Phobos

3 4

beam energy (GeV/nucleon)



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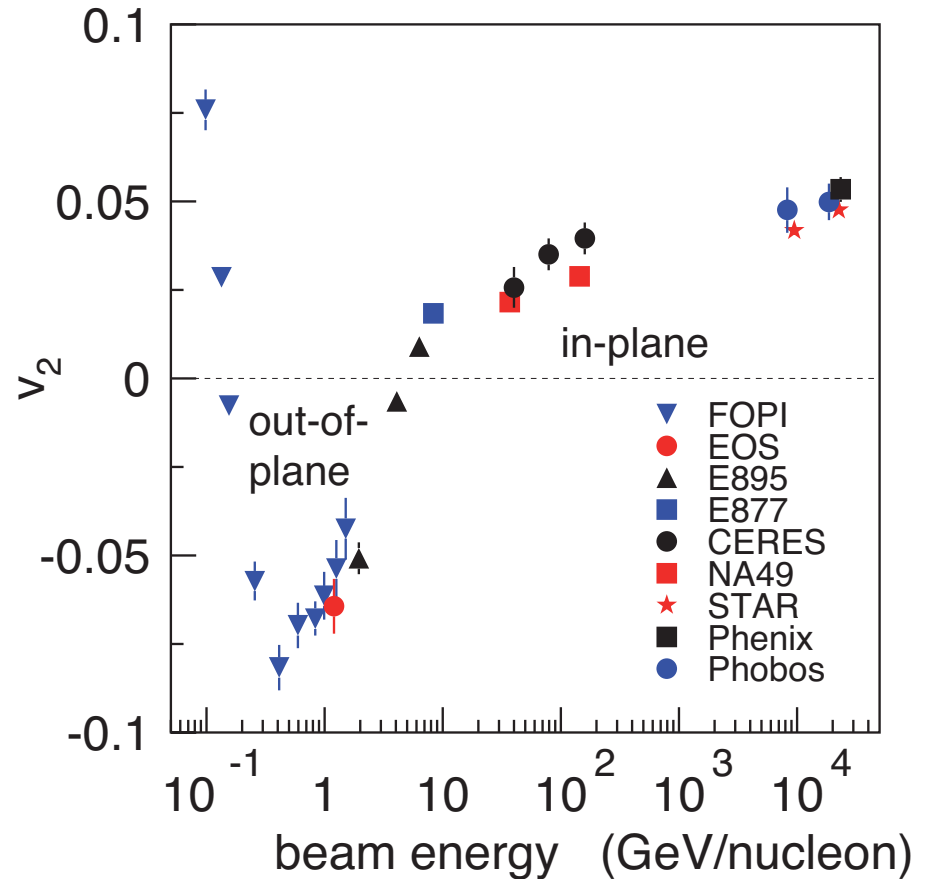




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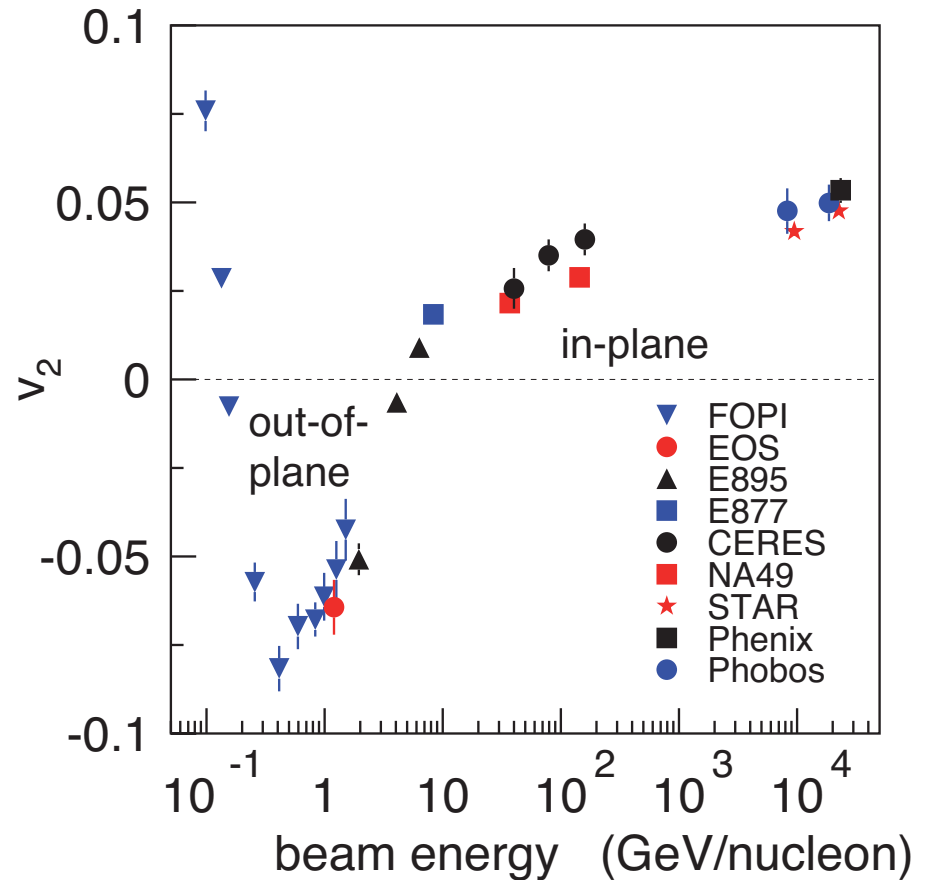
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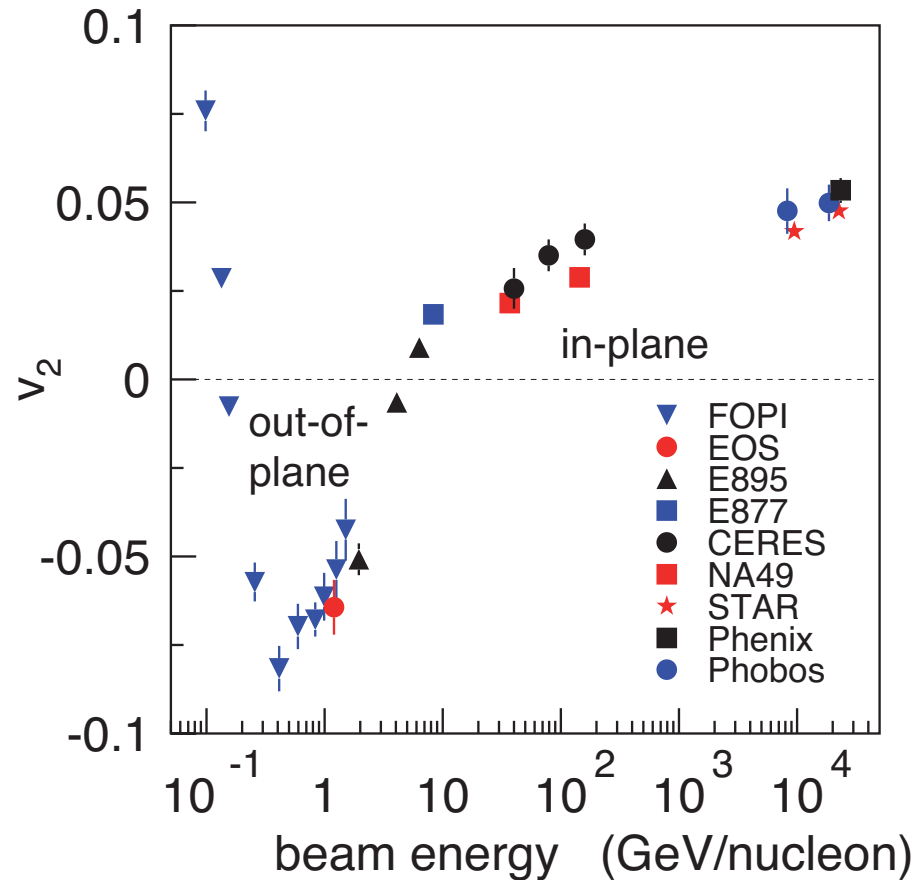
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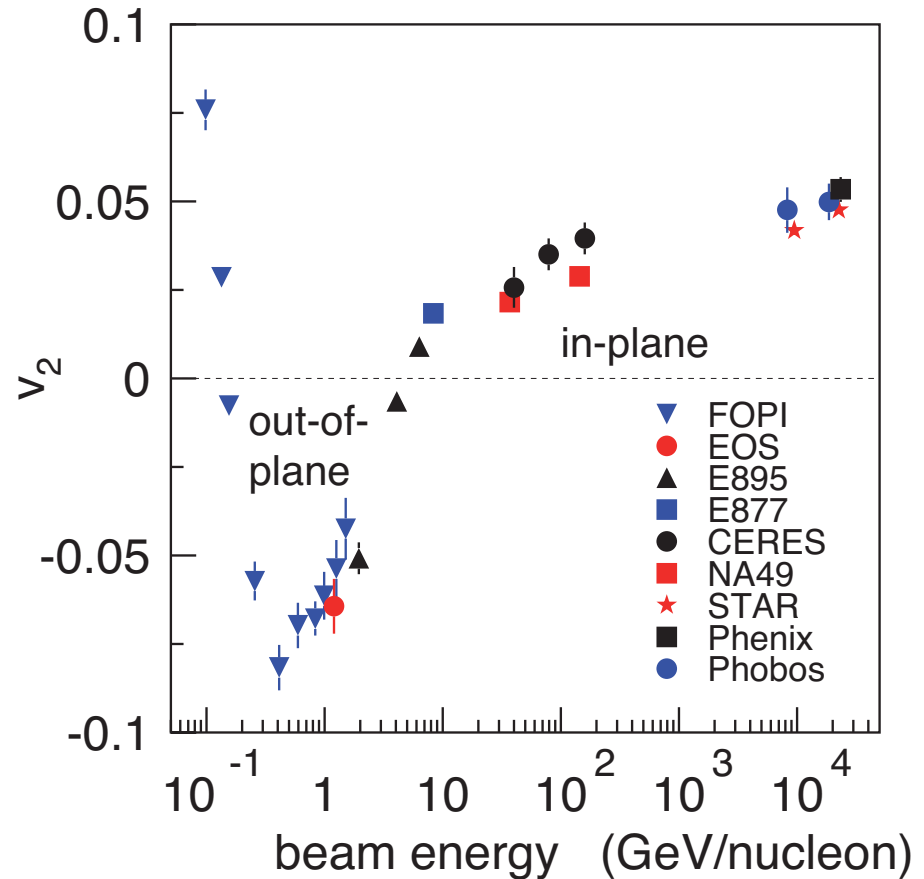
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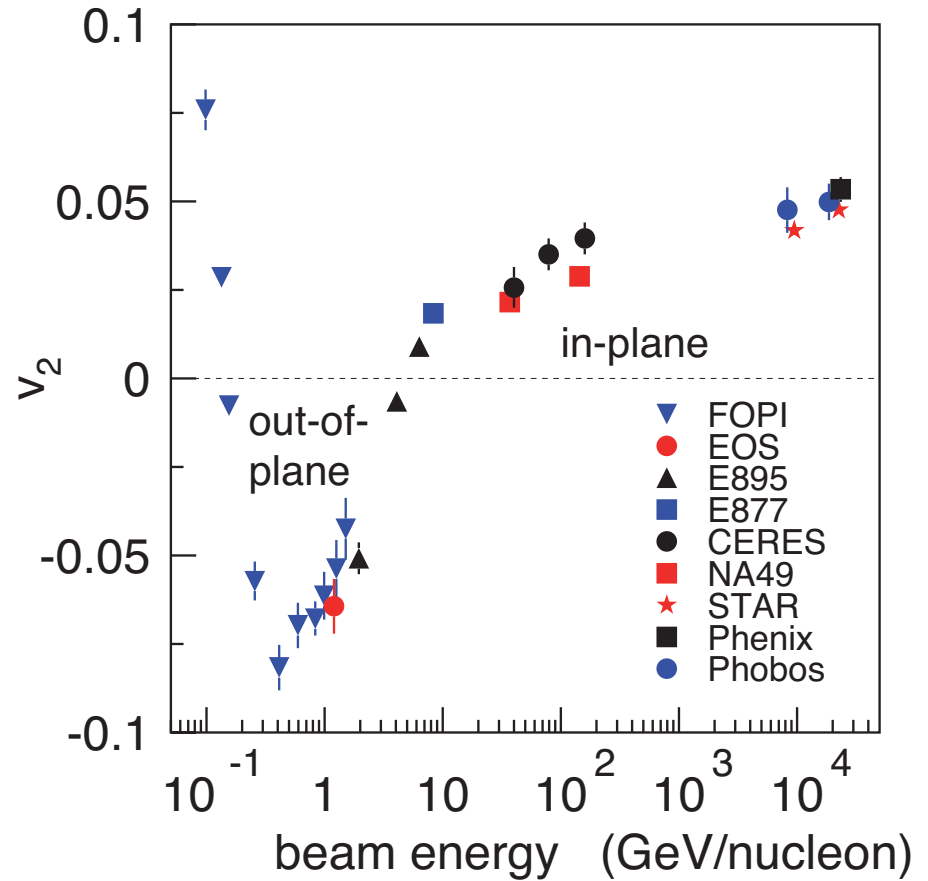
⇒ with a minimum at around 0.4-0.6 AGeV

- A. Andronic et al. [FOPI Collaboration], Phys. Lett. B 612 (2005) 173.
- G. Stoicea et al. [FOPI Collaboration], Phys. Rev. Lett. 92 (2004) 072303
- A. Andronic, J. Lukasik, W. Reisdorf and W. Trautmann [FOPI and INDRA Collaborations], Eur. Phys. J. A 30 (2006) 31.





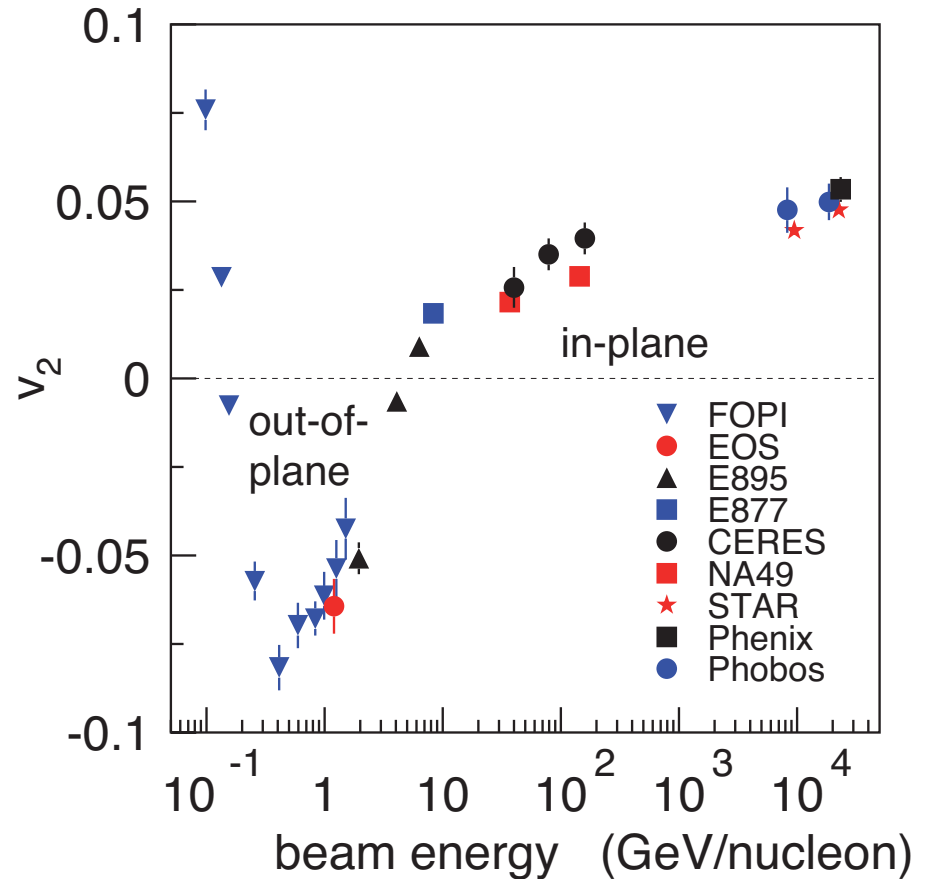
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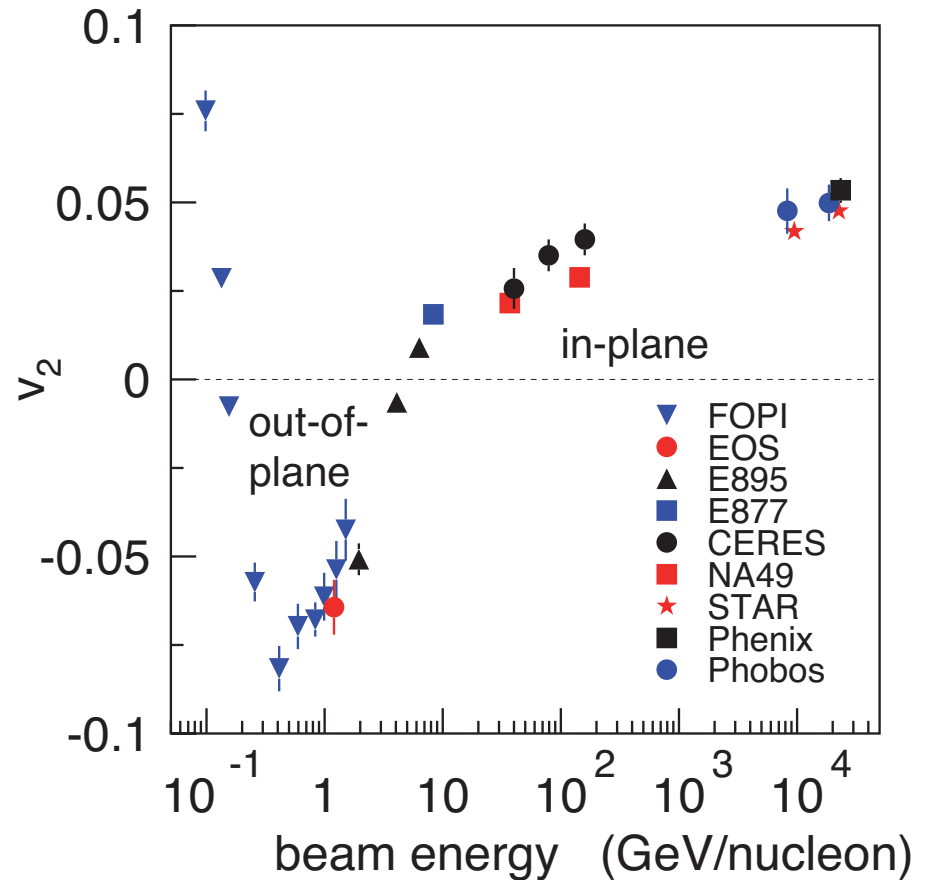


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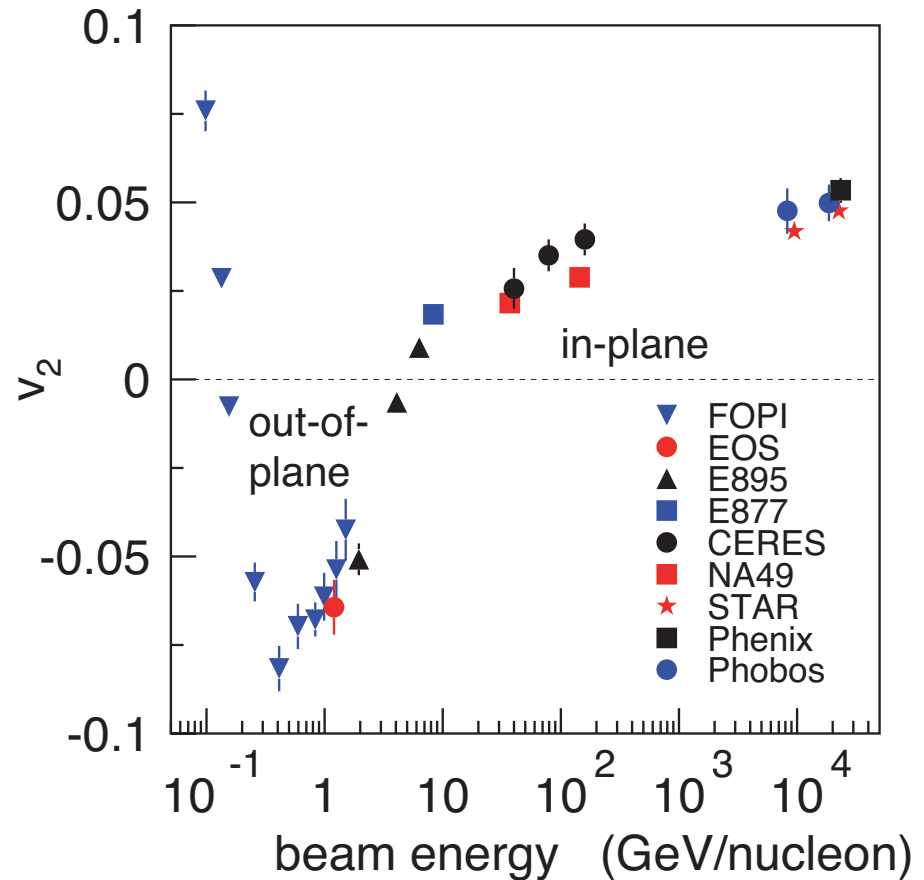
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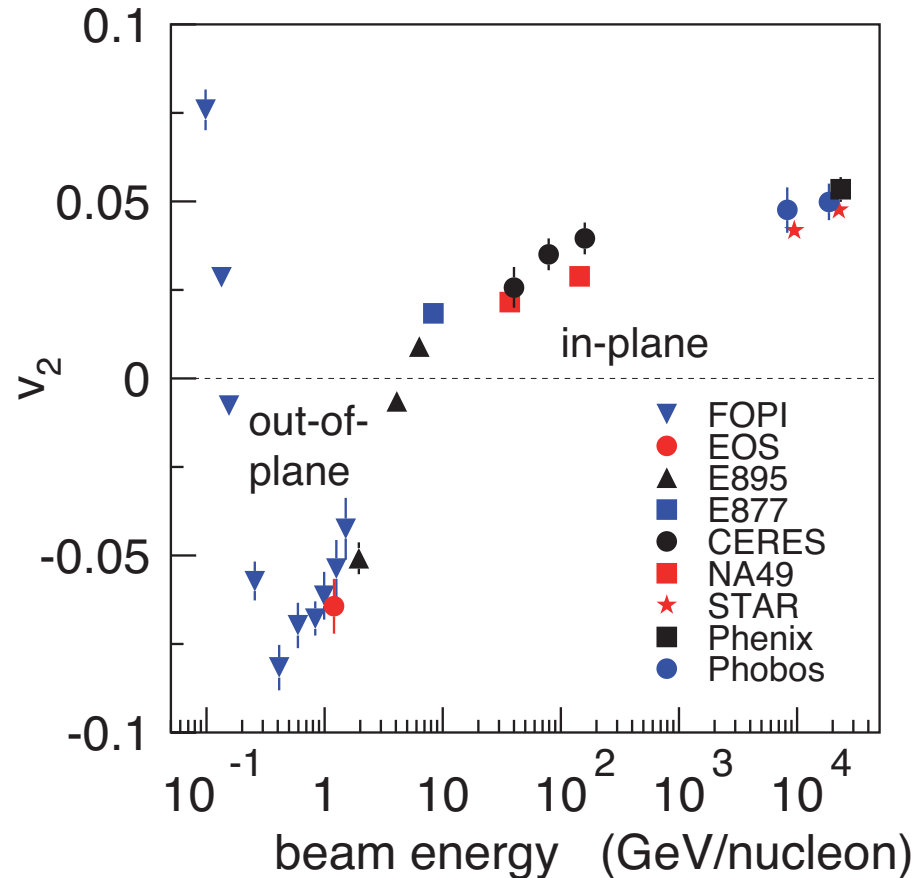
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At higher incident energies: the expansion takes place after the spectator matter has passed the compressed zone ⇒ v_2 is determined by the shape of the overlap region only ⇒ $v_2 > 0$.





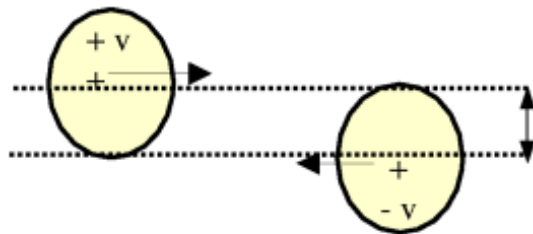
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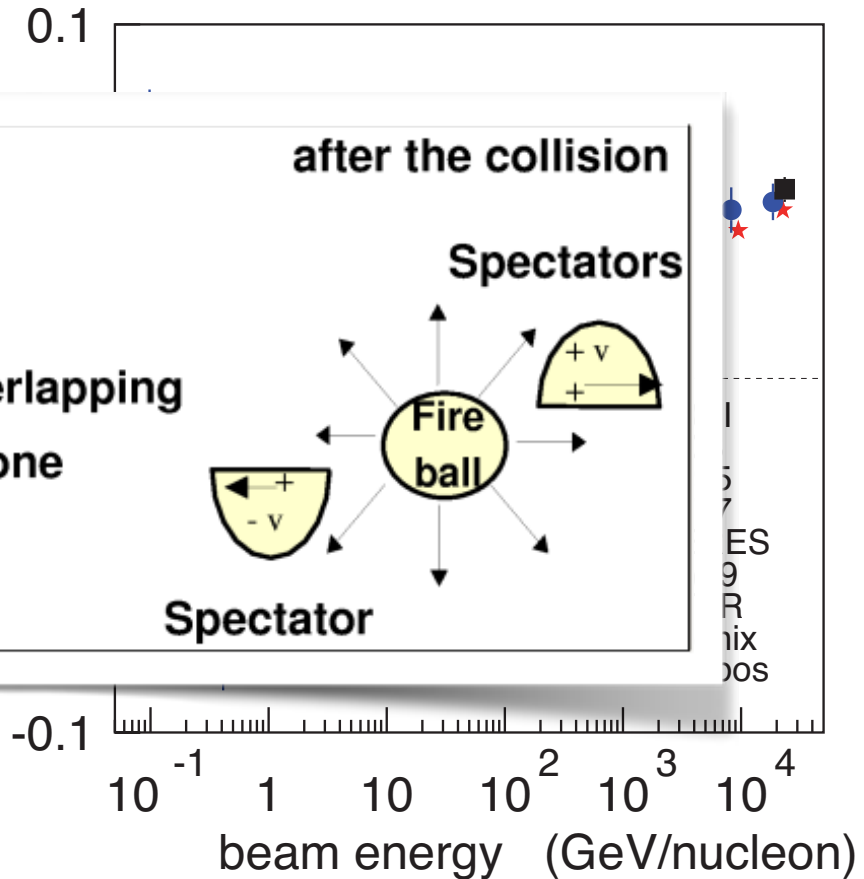
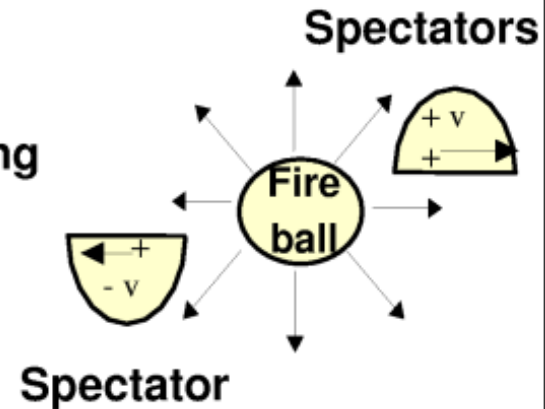
At higher in
takes place
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Before the collision



**Overlapping
zone**

after the collision



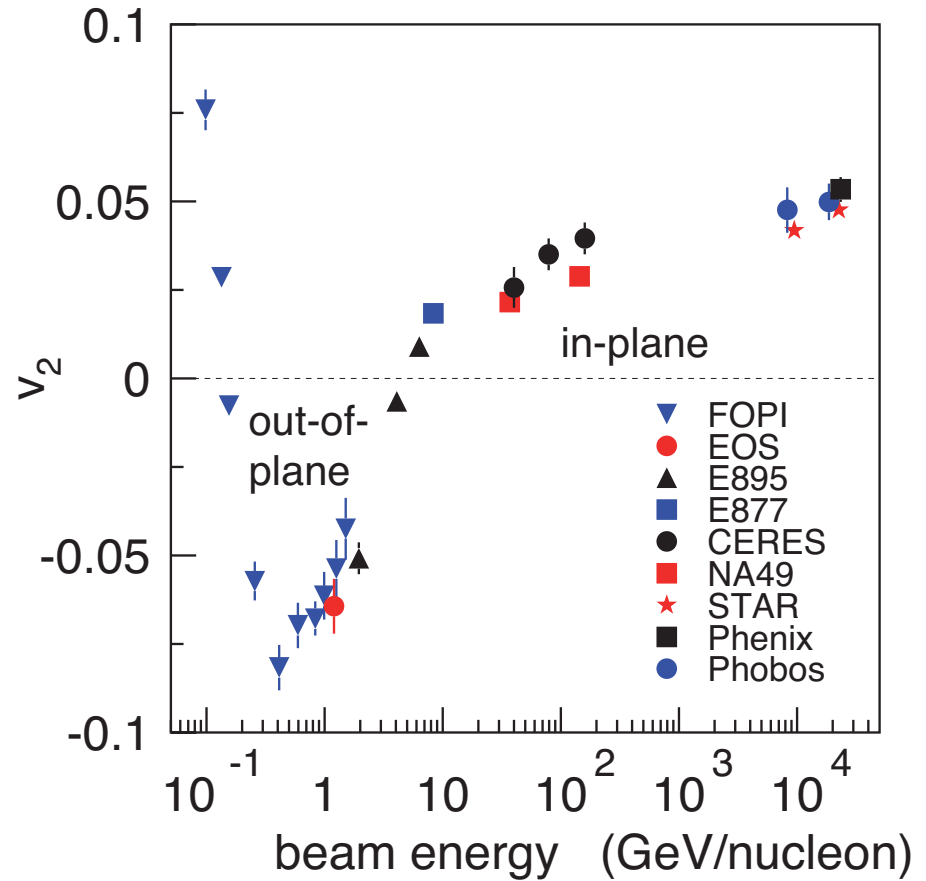
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Minimum of $v_2 \Leftrightarrow$ maximum nuclear stopping
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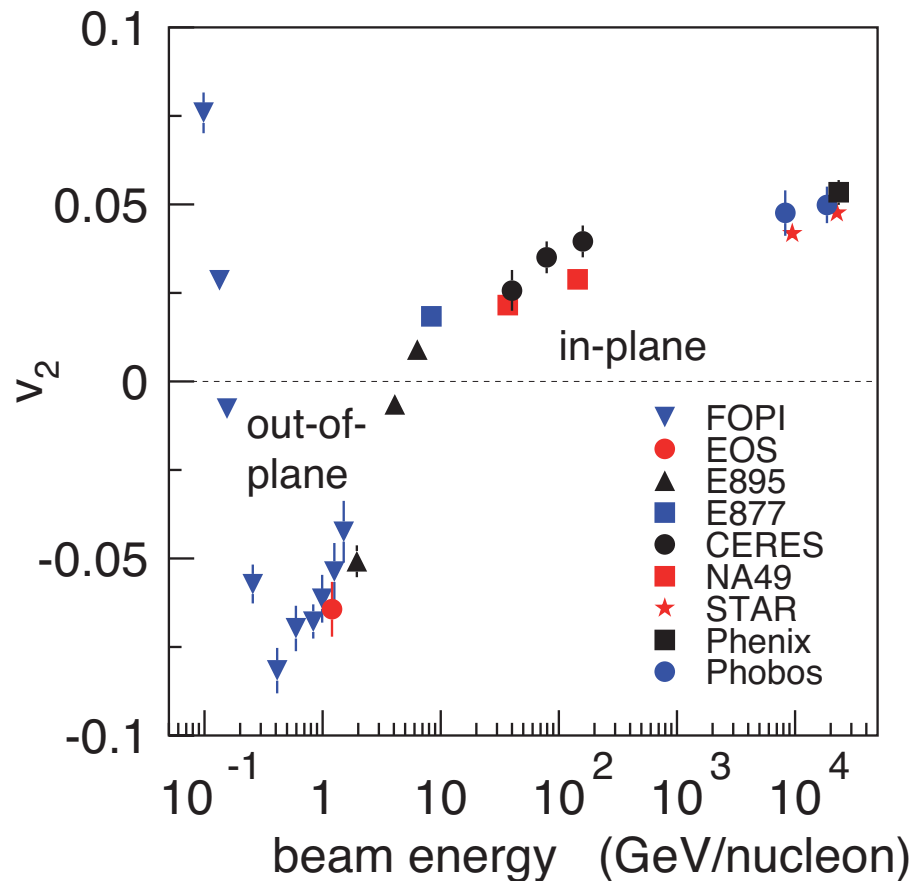
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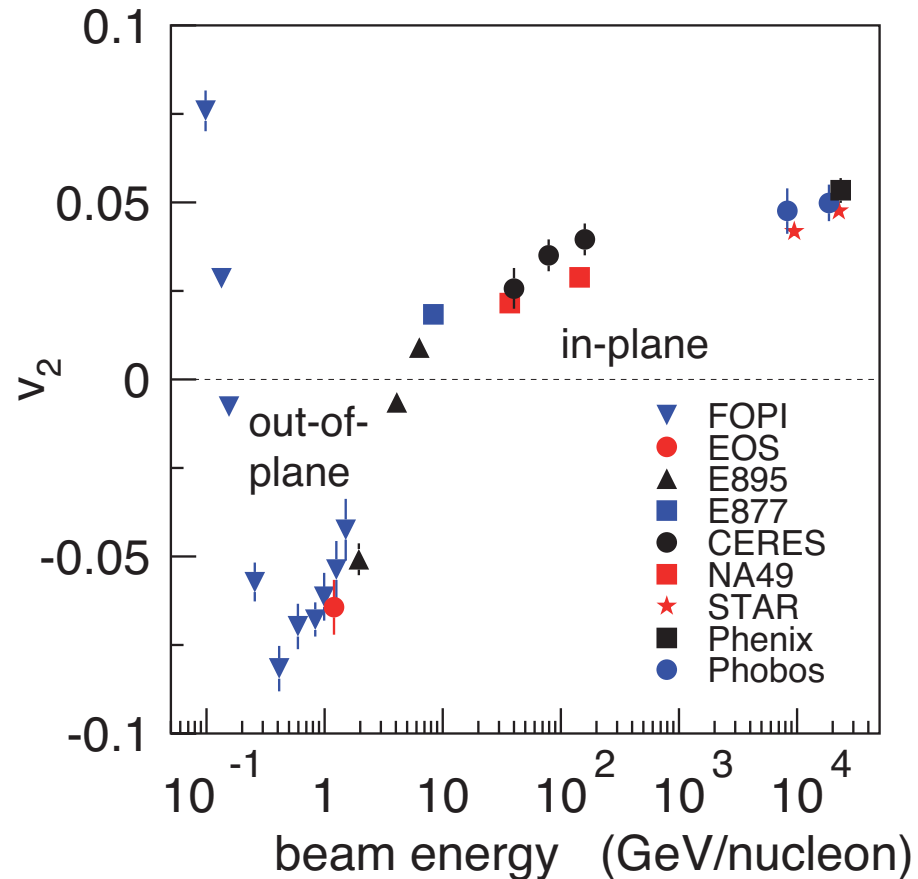
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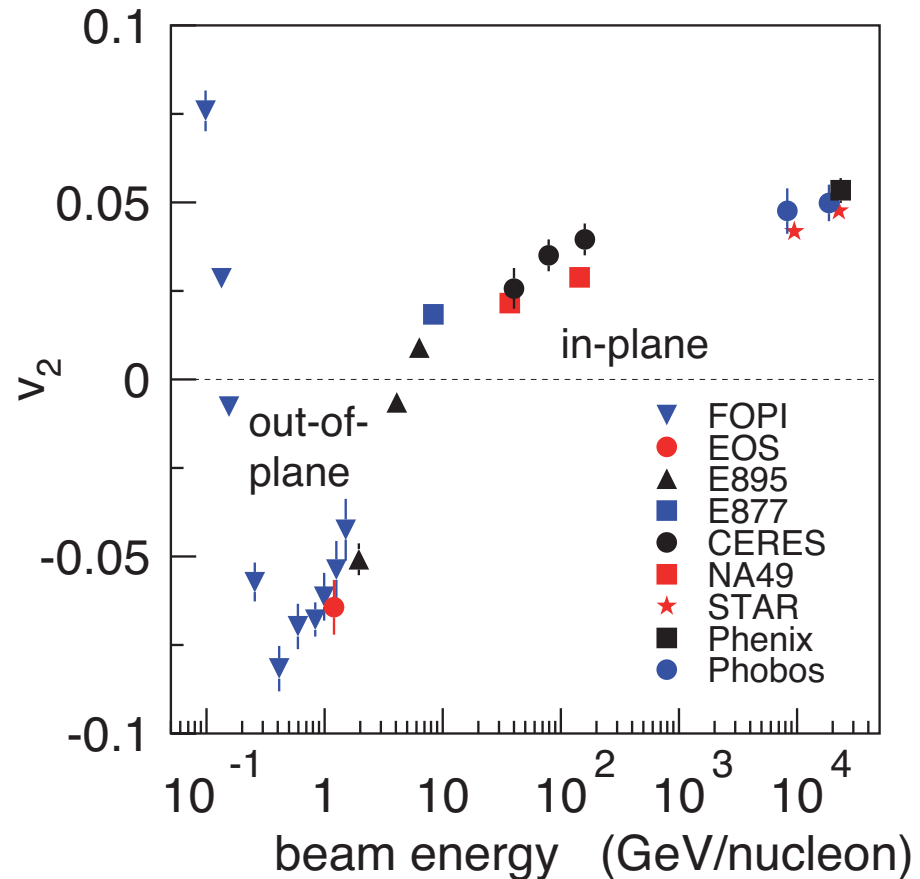
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- interactions with **spectator matter**
- **much longer collision times.**





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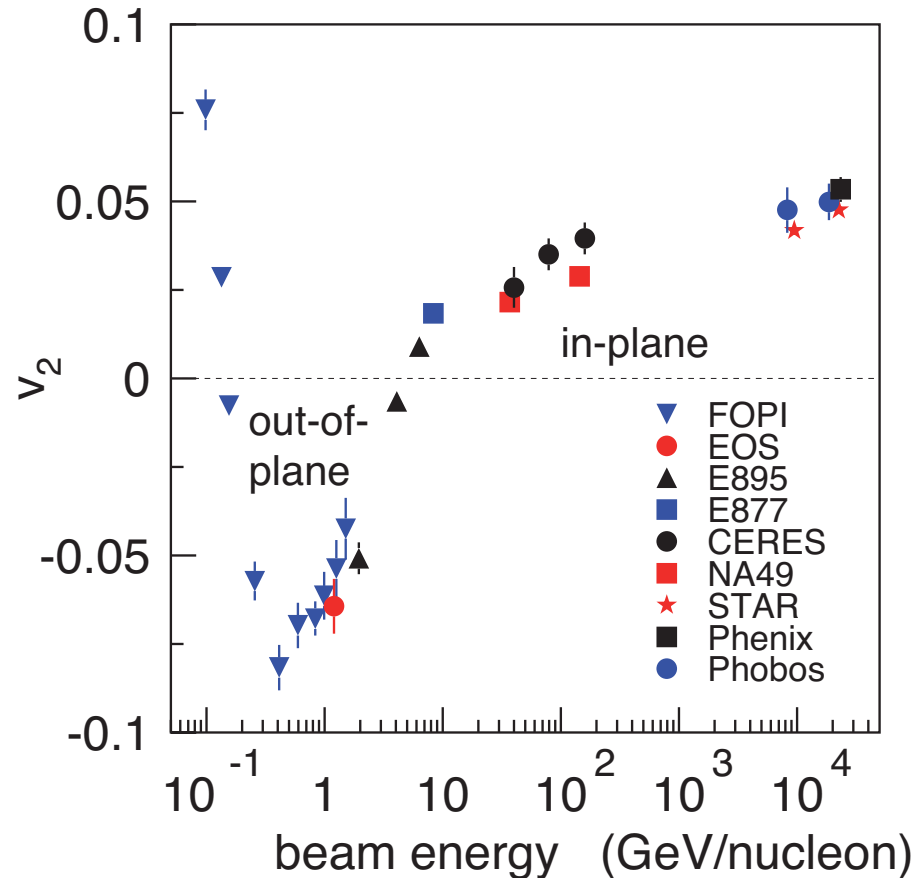
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At even lower incident energies: v_2 becomes positive again: attractive NN forces outweigh the repulsive NN collisions.

– J. Lukasik et al., Phys. Lett. B 608 (2005) 223.

– M. Zheng et al., Phys. Rev. Lett. 83 (1999)

– P. K. Sahu et al., Nucl. Phys. A 672 (2000) 376





The Quantum Molecular Dynamics approach



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Here, we quote only how this approach allows for an exploration of the nuclear EoS

Nucleons are represented as Gaussian wave functions -> single-particle Wigner density:

$$f_i(\mathbf{r}, \mathbf{p}, t) = \frac{1}{\pi^3 \hbar^3} e^{-\frac{2}{L}(\mathbf{r} - \mathbf{r}_i(t))^2} e^{-\frac{L}{2\hbar^2}(\mathbf{p} - \mathbf{p}_i(t))^2}$$

The total one-body Wigner density is the sum of the Wigner densities of all nucleons



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$$\begin{aligned} V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{p}_i, \mathbf{p}_j) &= G + V_{\text{Coul}} \\ &= V_{\text{Skyrme}} + V_{\text{Yuk}} + V_{\text{mdi}} + V_{\text{sym}} + V_{\text{Coul}} \\ &= t_1 \delta(\mathbf{r}_i - \mathbf{r}_j) + t_2 \delta(\mathbf{r}_i - \mathbf{r}_j) \rho^{\gamma-1}(\mathbf{r}_i) + \\ &\quad t_3 \frac{\exp\{-|\mathbf{r}_i - \mathbf{r}_j|/\mu\}}{|\mathbf{r}_i - \mathbf{r}_j|/\mu} + \\ &\quad t_4 \ln^2(1 + t_5(\mathbf{p}_i - \mathbf{p}_j)^2) \delta(\mathbf{r}_i - \mathbf{r}_j) + \\ &\quad t_6 \frac{1}{\rho_0} T_3^i T_3^j \delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{Z_i Z_j e^2}{|\mathbf{r}_i - \mathbf{r}_j|}. \end{aligned}$$

Convolution of the distribution functions f_i and $f_j \rightarrow$ single-particle potential (« mean-field ») = $V_{\text{Skyrme}} + V_{\text{mdi}}$ (local interactions + momentum dependence)

$$U_i(\mathbf{r}_i, t) = \alpha \left(\frac{\rho_{int}}{\rho_0} \right) + \beta \left(\frac{\rho_{int}}{\rho_0} \right)^\gamma + \delta \ln^2 \left(\varepsilon (\Delta \mathbf{p})^2 + 1 \right) \left(\frac{\rho_{int}}{\rho_0} \right)$$

In nuclear matter t_1, t_2, t_4, t_5 uniquely related $\alpha, \beta, \delta,$ and ε and δ : given by fits to the optical potential extracted from elastic scattering data in pA collisions.

α, β, γ : 2 are constrained by volume energy has a minimum of $E/A(\rho_0) = -16$ MeV at ρ_0 .

	α (MeV)	β (MeV)	γ	δ (MeV)	$\varepsilon \left(\frac{e^2}{\text{GeV}^2} \right)$	K (MeV)
SM	-390	320	1.14	1.57	500	200
HM	-130	59	2.09	1.57	500	376



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The Quantum Molecular Dynamics approach

Details of the Quantum Molecular Dynamics (QMD) approach have been published in

– J. Aichelin, Phys. Rept. 202 (1991) 233.

– C. Hartnack et al., Phys. Rept. 510 (2012) 119

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Here, we quote only how this approach allows for an exploration of the nuclear EoS

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The Quantum Molecular Dynamics approach

Details of the QMD approach have

– J. Aichelin, Phys. Rev. Lett. 67 (1991) 2763

– C. Hartnack et al., Phys. Rev. Lett. 79 (1997) 4030

– C. Hartnack et al., Phys. Rev. Lett. 82 (1999) 3644

Comparisons to other approaches

in the incident energy range are published in

– W. Reisdorf et al., Phys. Rev. Lett. 87 (2012) 1

Here, we quote only the first approach since the other two are for an exploration of the nuclear EoS

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$$K = -V \frac{\partial^2 E/A(\rho)}{\partial \rho^2} \Big|_{\rho=\rho_0}$$

= compression modulus of nuclear matter

= curvature of the volume energy at $\rho = \rho_0$ (for $T=0$)

$$K = -V \frac{\partial p}{\partial V} = 9\rho^2 \frac{\partial^2 E/A(\rho)}{(\partial \rho)^2} \Big|_{\rho=\rho_0}$$

all terms:

$$E = E_{kin} + V_{mdi} + V_{sym} + V_{Coul} + V_{int}$$

$$V_{int} = \sum_{i < j} \left[\frac{f_i(\mathbf{r}_i, \mathbf{p}_i, t) f_j(\mathbf{r}_j, \mathbf{p}_j, t)}{|\mu|} + \frac{1}{2} (\mathbf{p}_i - \mathbf{p}_j)^2 \delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{Z_i Z_j e^2}{|\mathbf{r}_i - \mathbf{r}_j|} \right]$$

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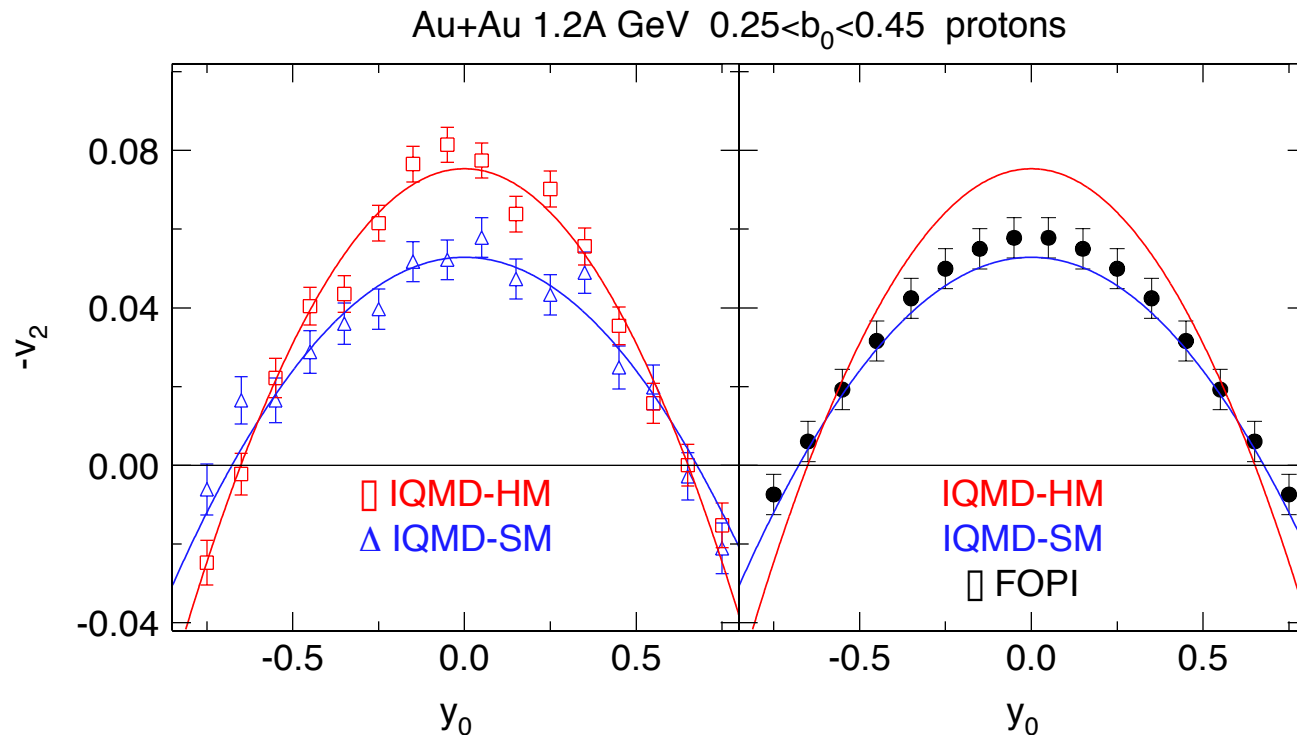
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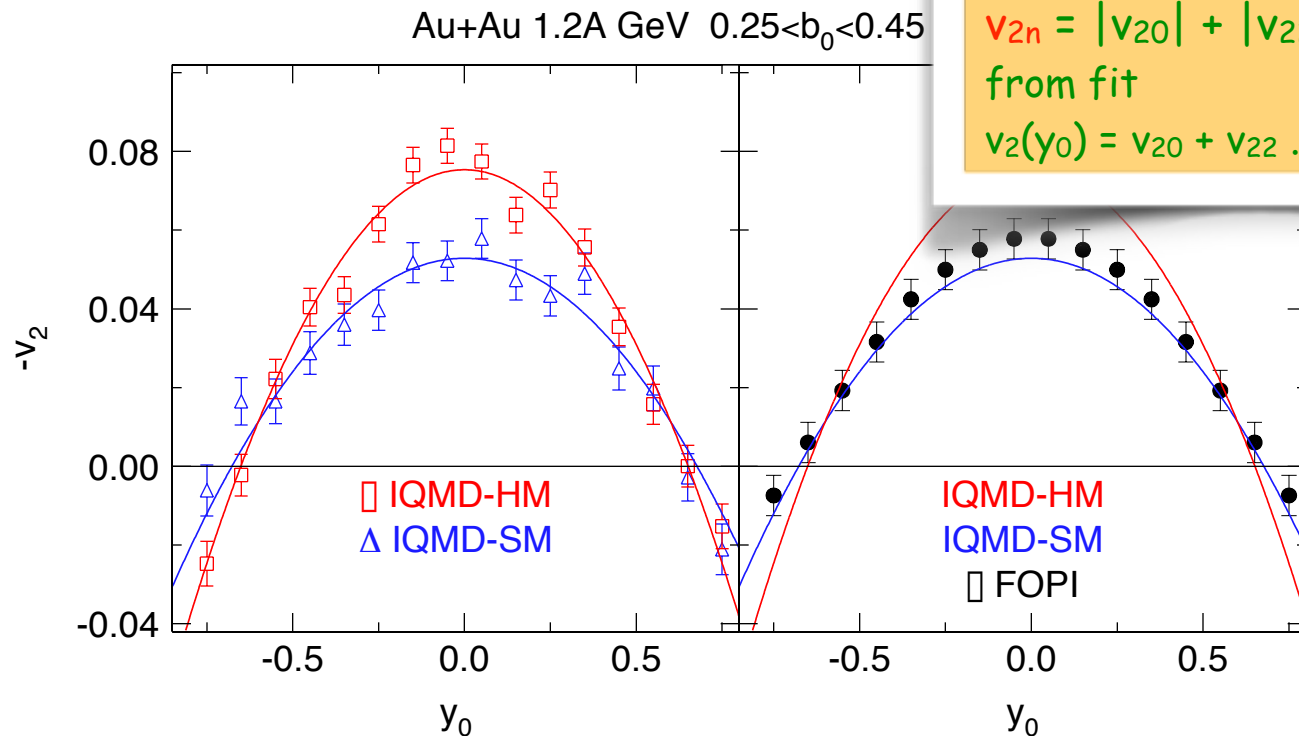
Elliptic flow at mid-rapidity: the strongest sensitivity to the Nuclear Equation of State



A. Le Fèvre et al., Nucl. Phys. A 945 (2016) 112.



Elliptic flow at mid-rapidity: the strongest sensitivity to the Nuclear Equation of State



Complete shape of $v_2(y_0)$:

a new observable:

$$v_{2n} = |v_{20}| + |v_{22}|,$$

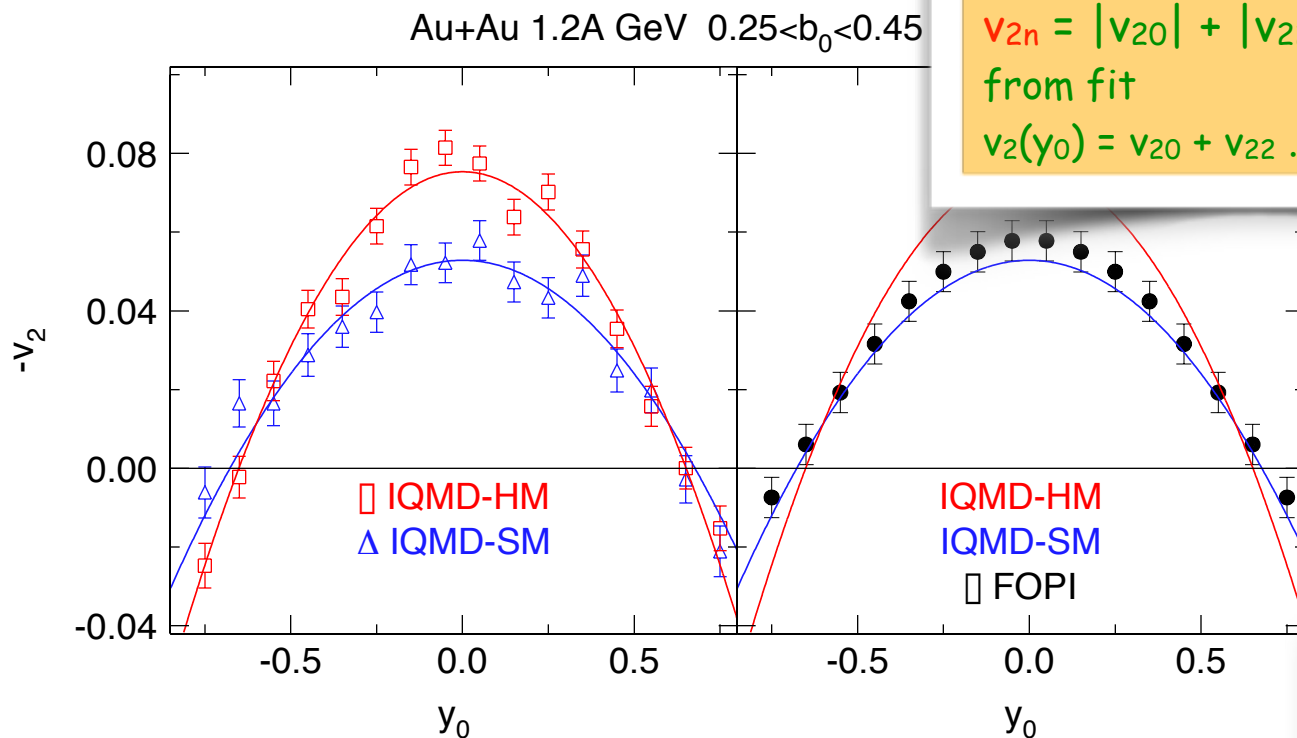
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$K =$

380 MeV ('stiff')

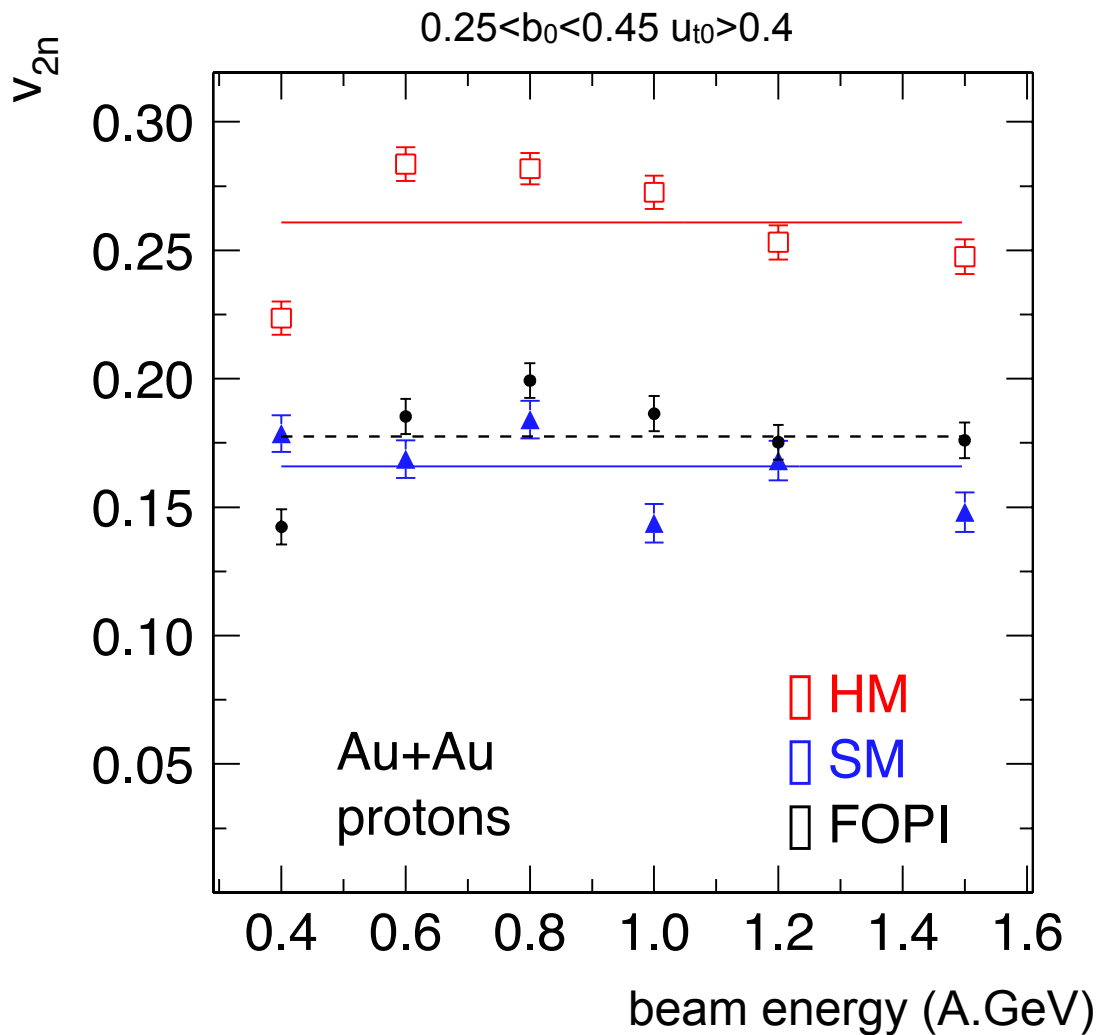
200 MeV ('soft')

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→ $v_{2n}(E_{\text{beam}})$ varies by a factor ≈ 1.6 , \gg measured uncertainty (≈ 1.1)
→ clearly favors a 'soft' EOS.

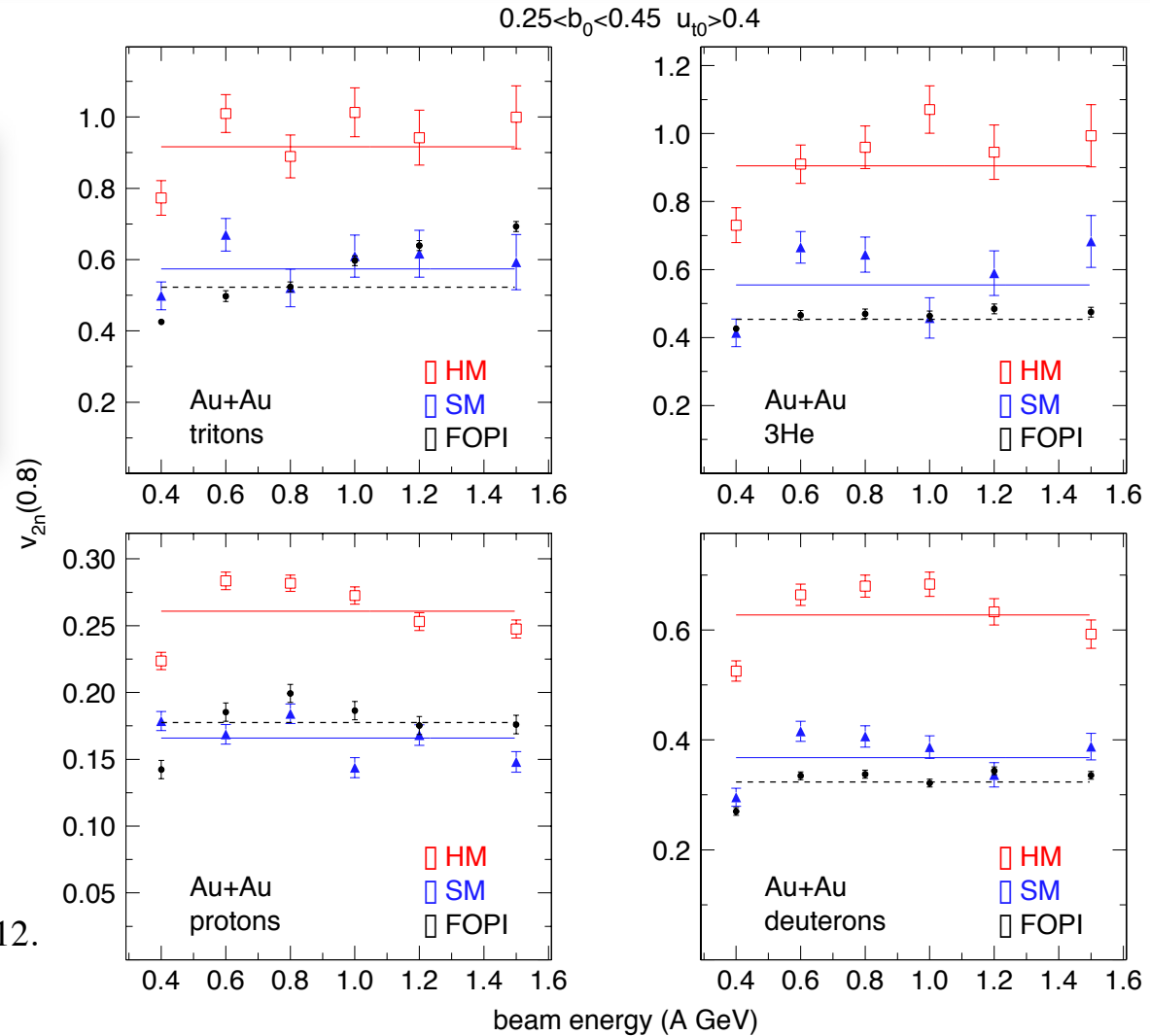


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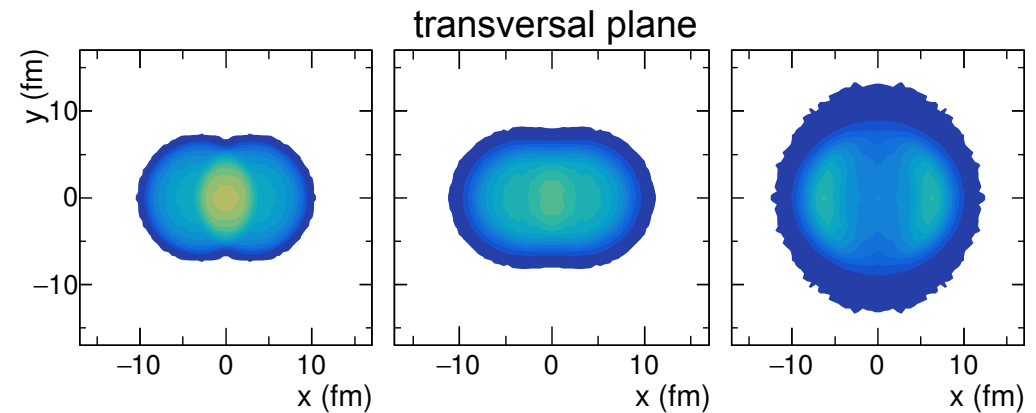
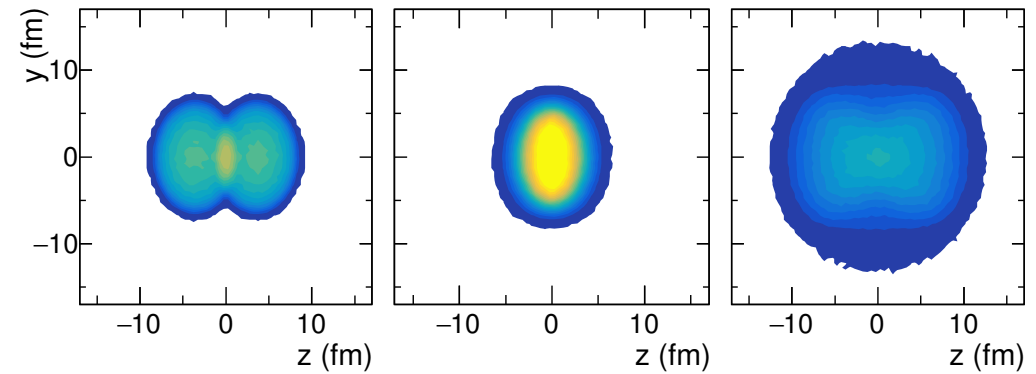
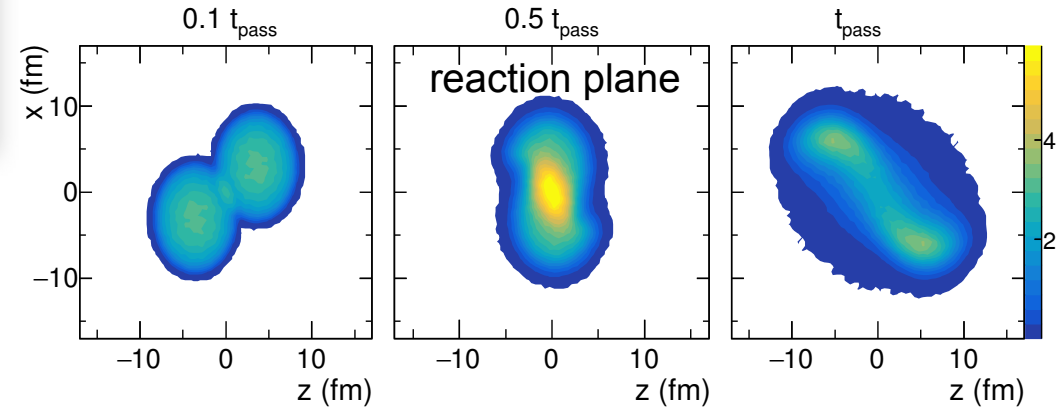
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Survey of the reaction

IQMD (SM) $^{197}\text{Au} + ^{197}\text{Au}$ at 1.5 AGeV, $b = 6$ fm **yield/event/fm²**



Only protons are considered in the following, Au+Au with $b=6$ fm as illustration

z: beam direction

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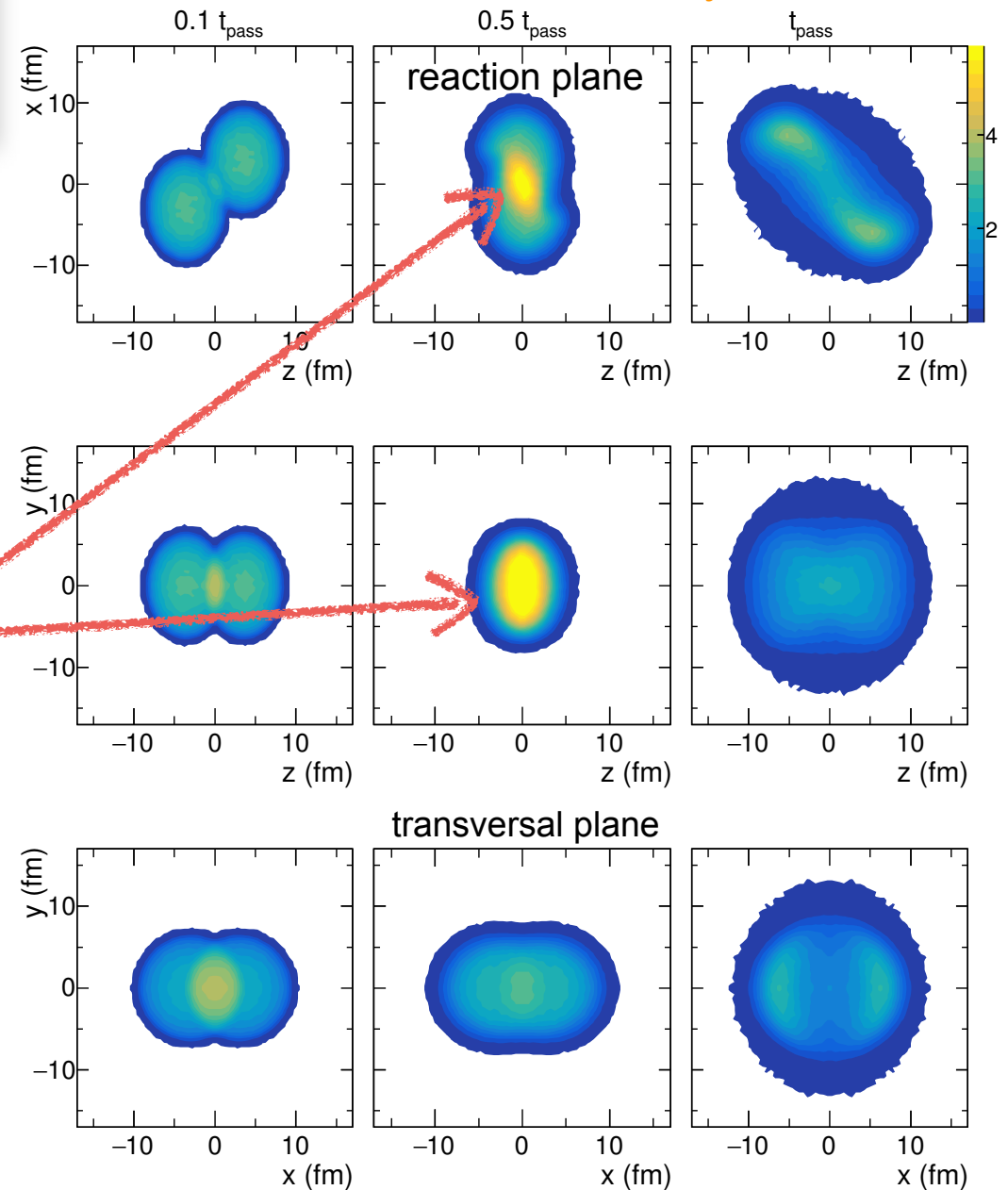
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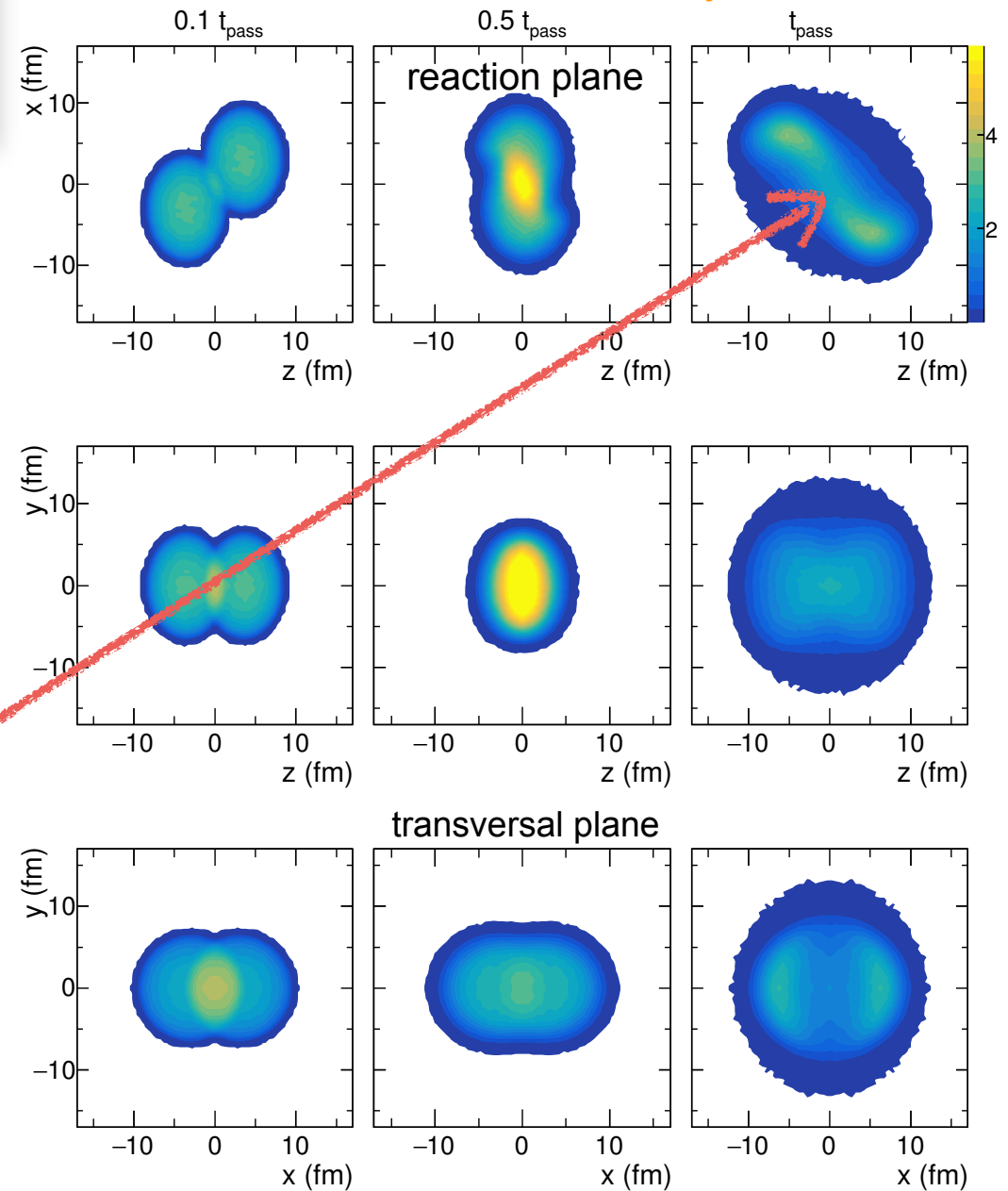
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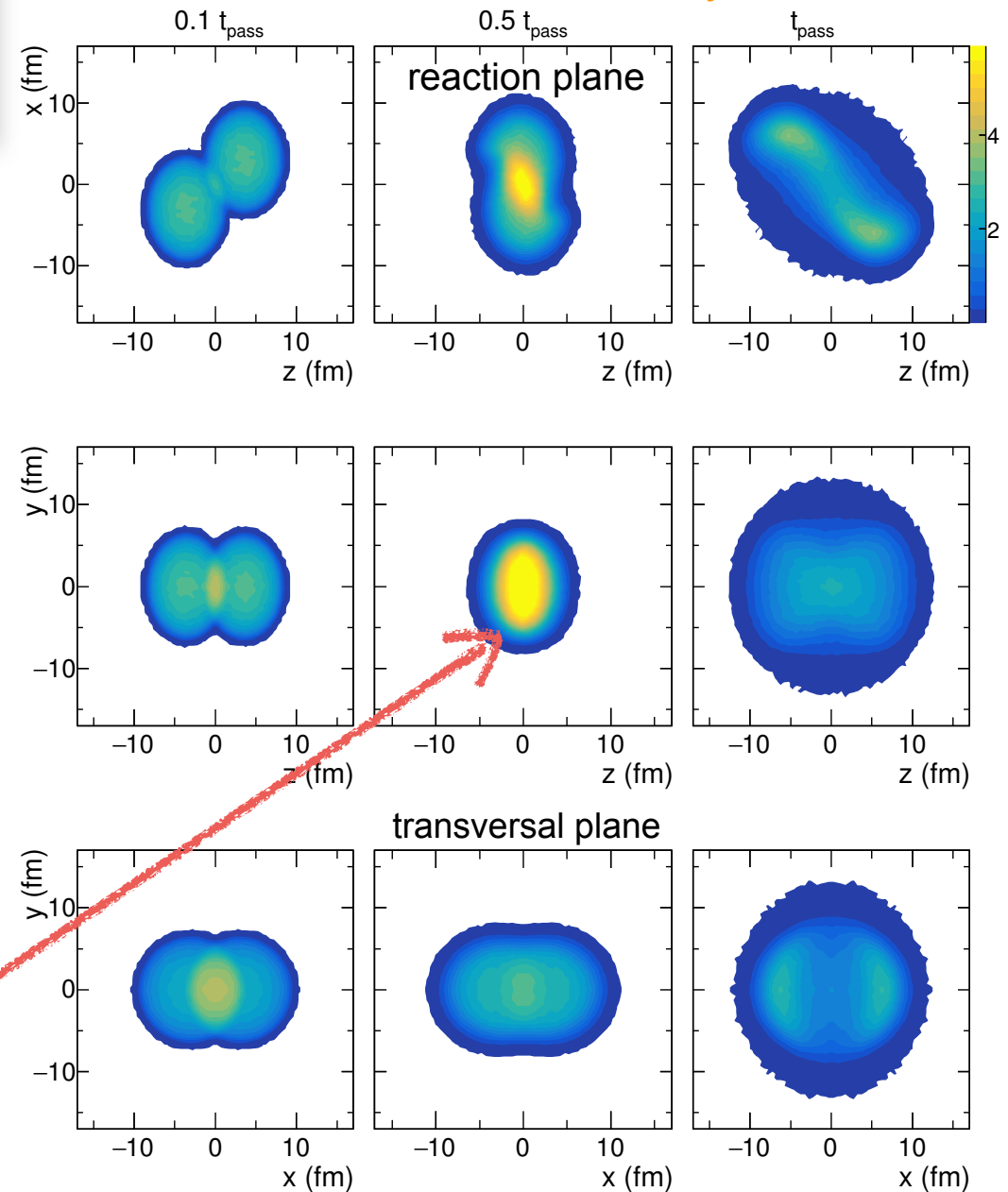
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The importance of this ridge can be seen in the zy plane at max. overlap \rightarrow the highest density at $z=0$, in the ridge.

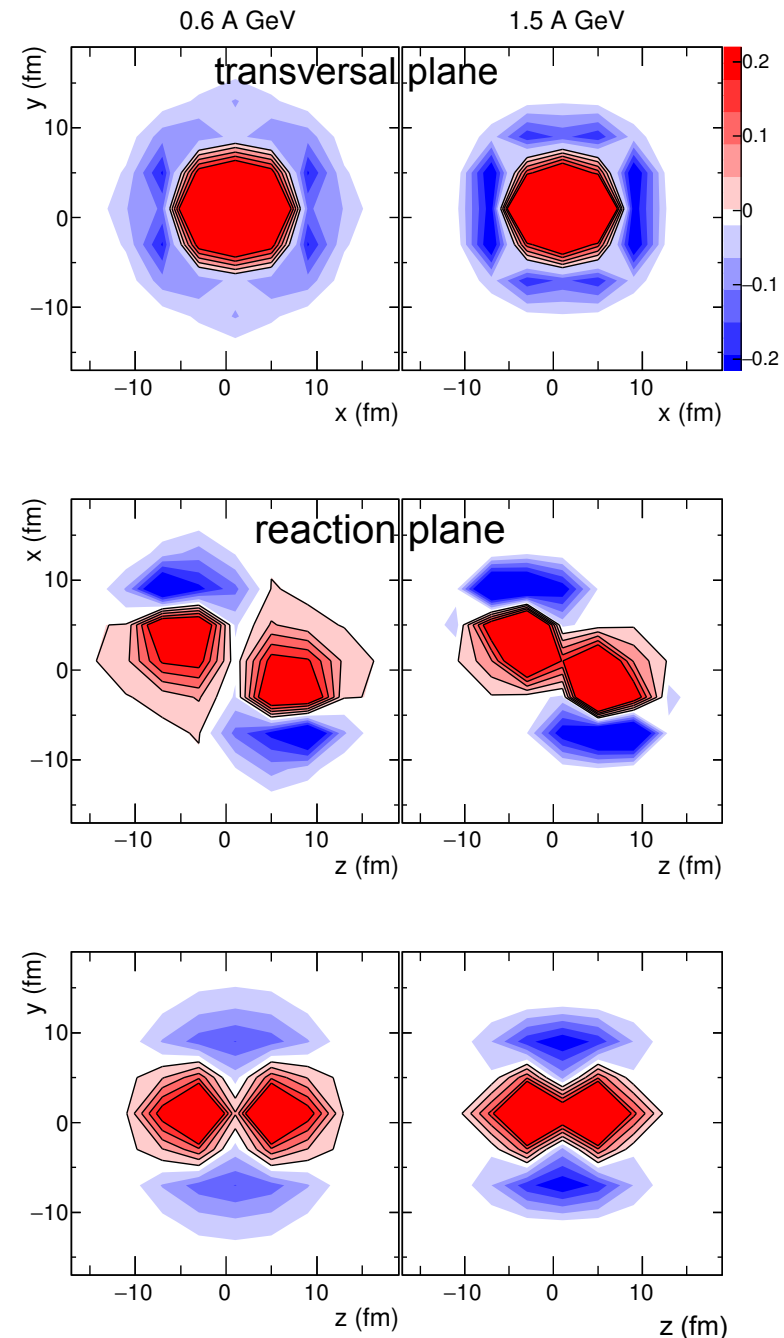


Survey of the reaction

$\Delta \rho^{\text{SM-HM}}$
at t_{pass}

The choice of the EoS influences the reaction scenario predicted by the model \Rightarrow reflected by the difference (SM-HM) of the proton densities projected onto the ij plane,

$$\Delta \rho_{ij} = \rho_{ij}^{\text{SM}} - \rho_{ij}^{\text{HM}} \text{ (event}^{-1}\text{fm}^{-2}\text{)}$$



Survey of the reaction

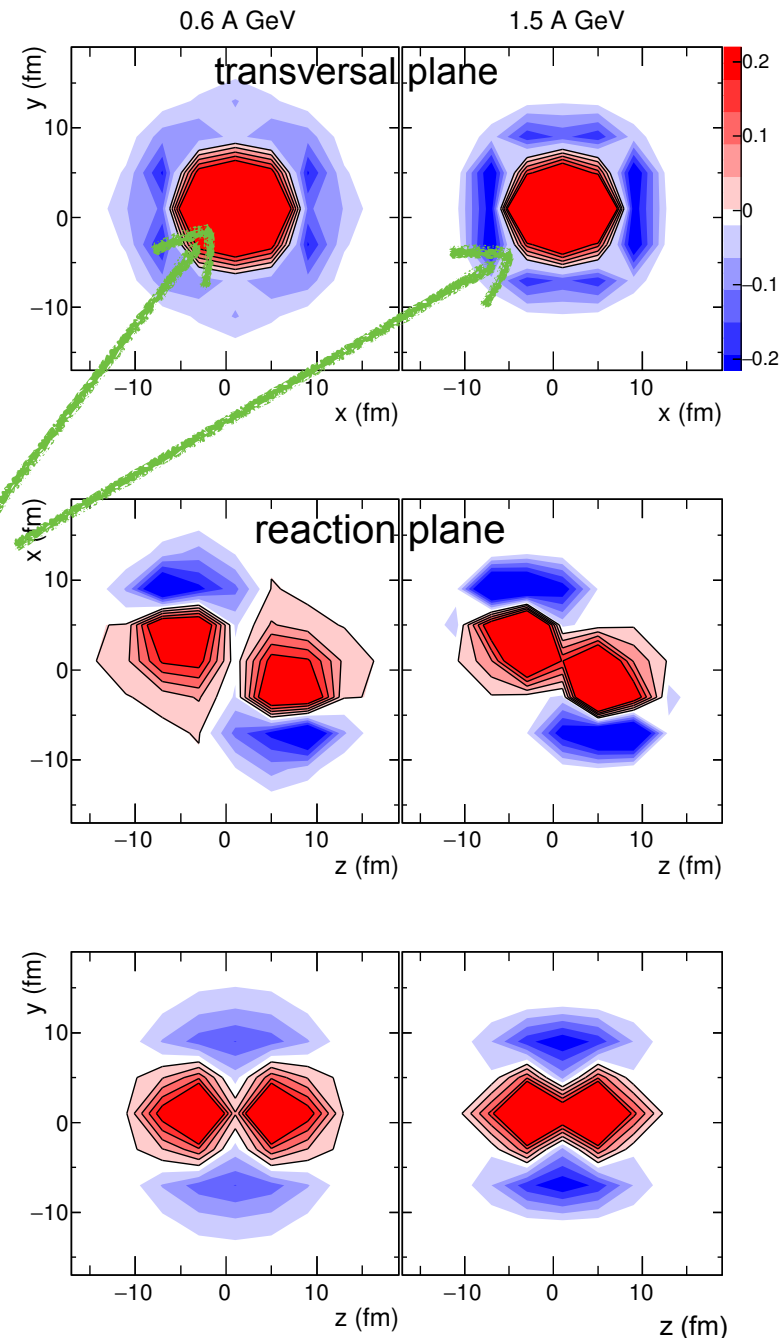
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Density of protons in the geometrical overlap region of projectile and target: higher for a soft EoS.
At larger distances from the reaction center: higher density for a hard EoS



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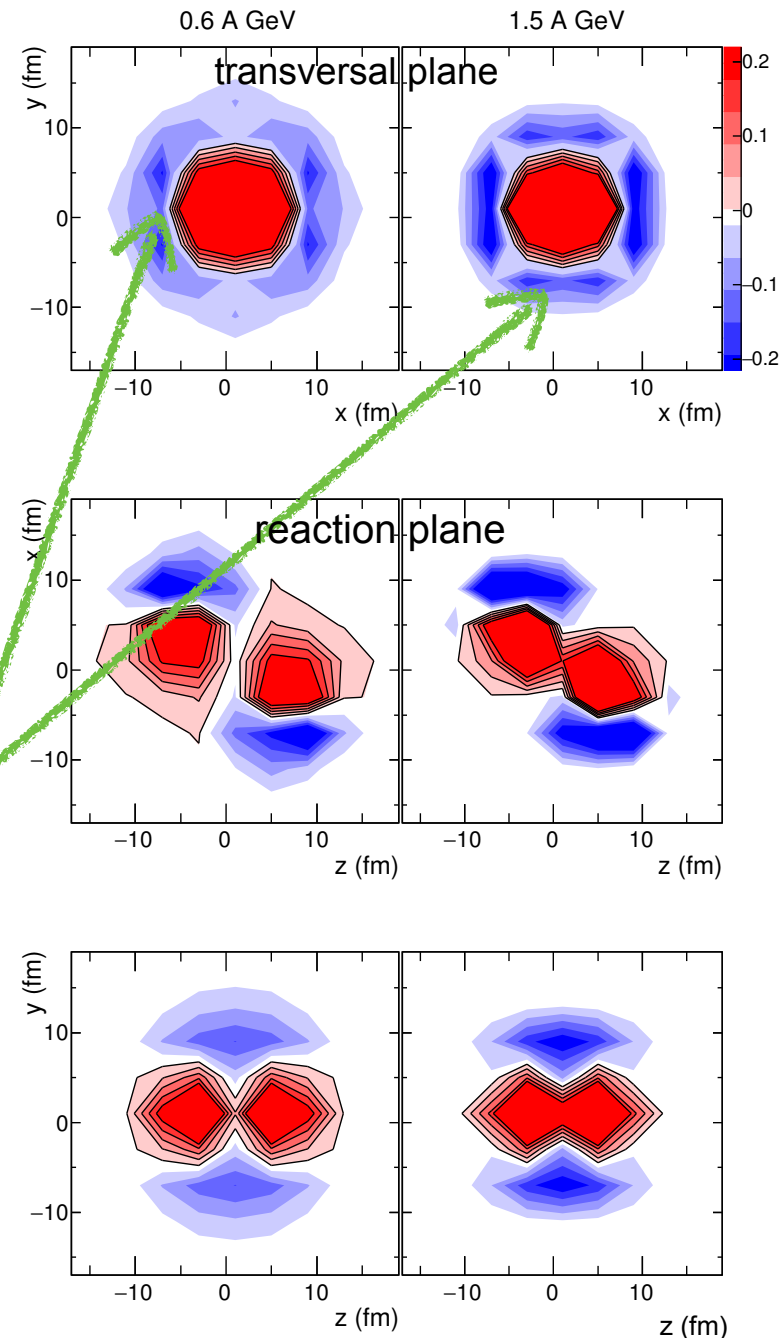
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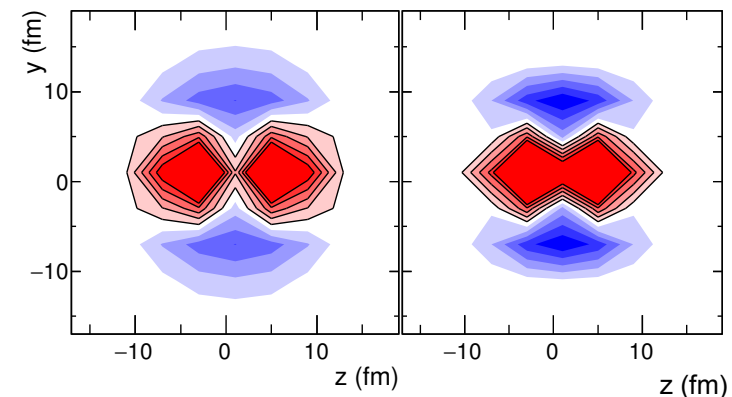
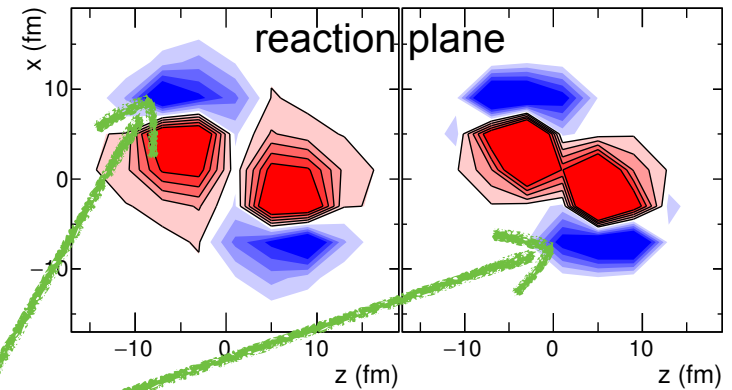
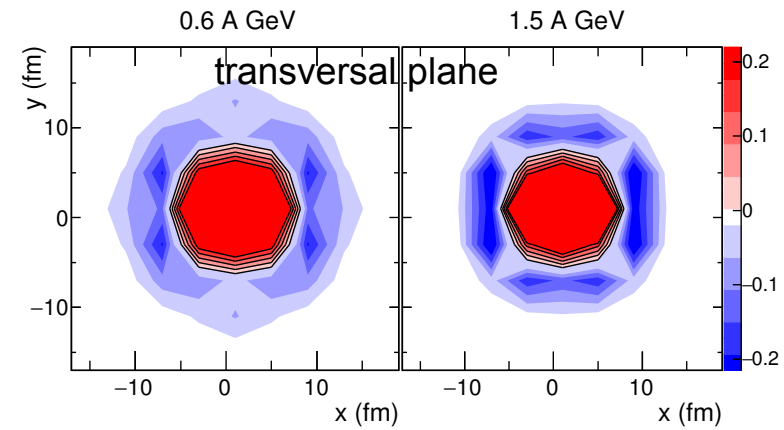
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The excess in x -direction has its origin in the in-plane flow of the spectator matter expressed by a finite directed flow (v_1):
 v_1 (hard) $\gg v_1$ (soft)



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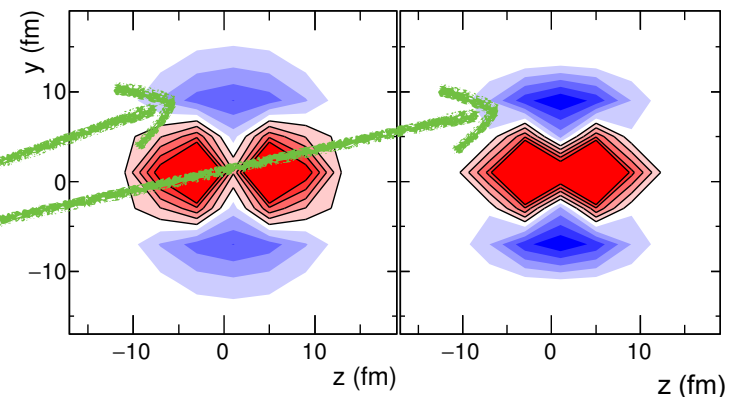
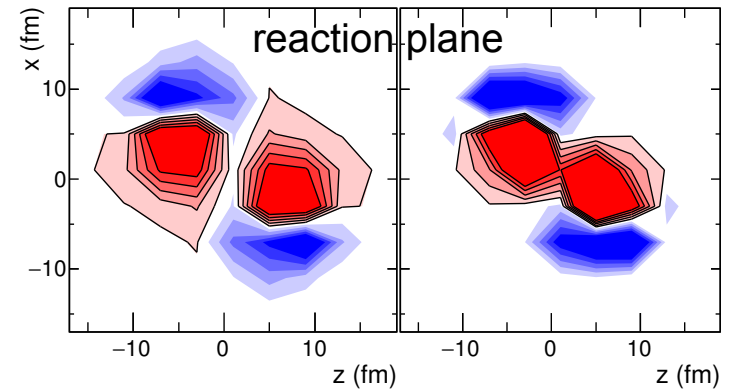
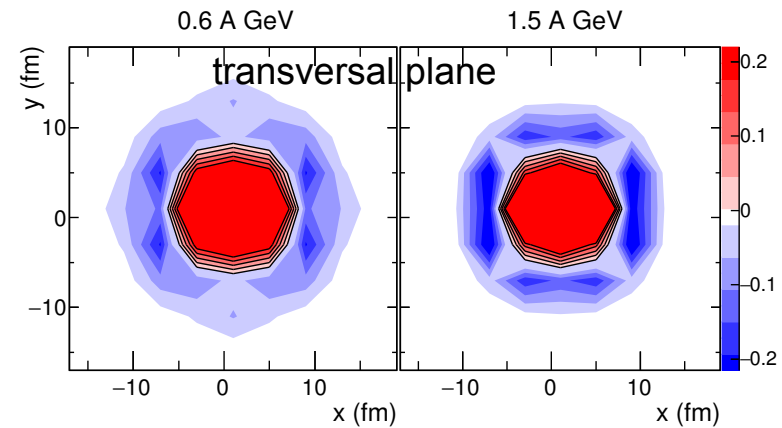
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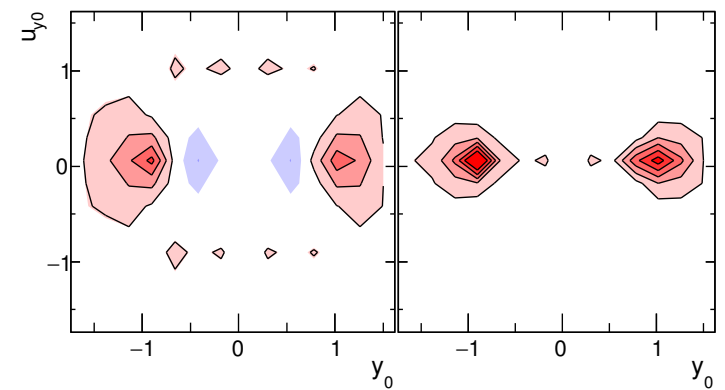
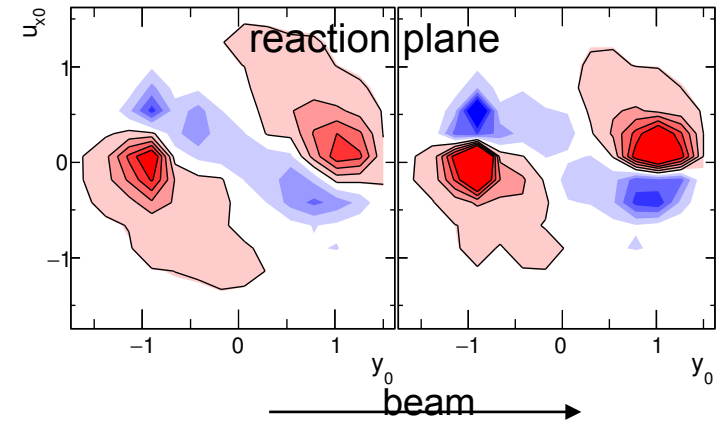
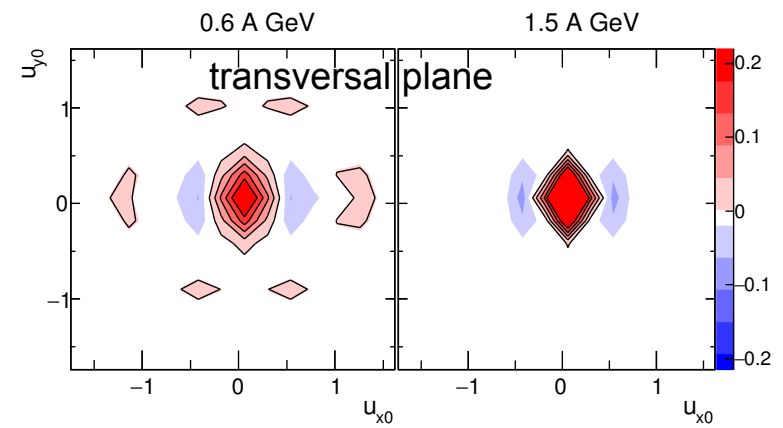
In y-direction the surplus in density of the hard EoS is concentrated at around $z=0$, being less extended but stronger at higher energies. The emission of these particles is caused by a stronger density gradient (and hence a stronger force) in y-direction for a hard (HM) EoS as compared to a soft (SM) one.



Survey of the reaction

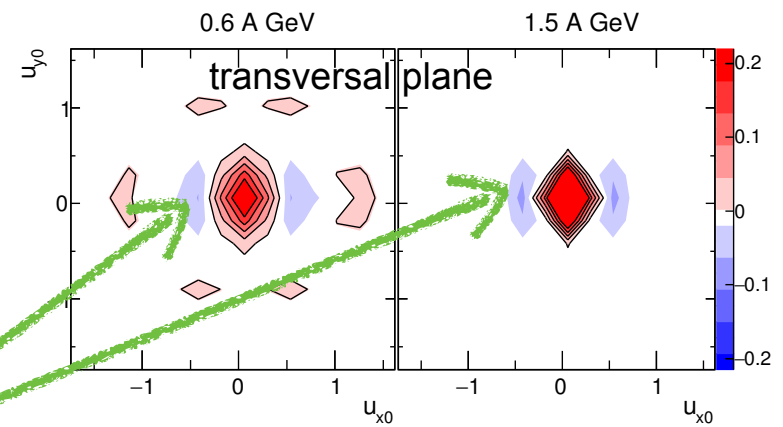
ΔQ_{SM-HM}
at t_{pass}

In velocity space we observe a complementary distribution.



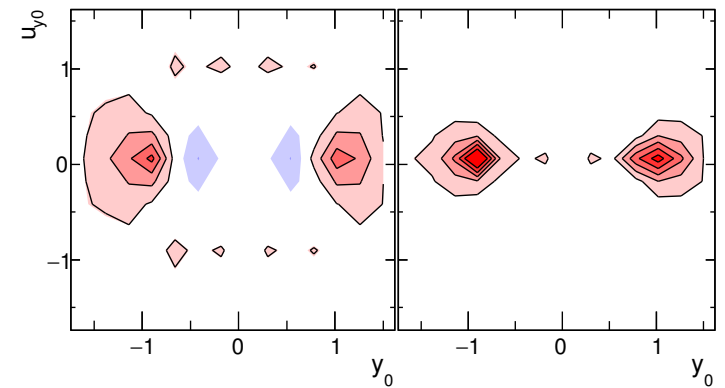
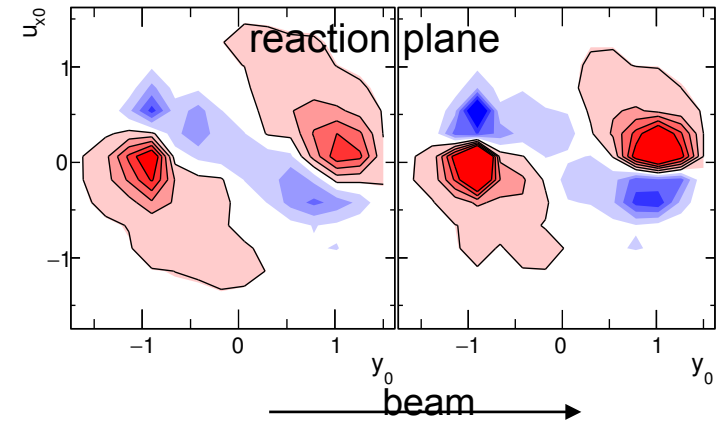
Survey of the reaction

ΔQ_{SM-HM}
at t_{pass}



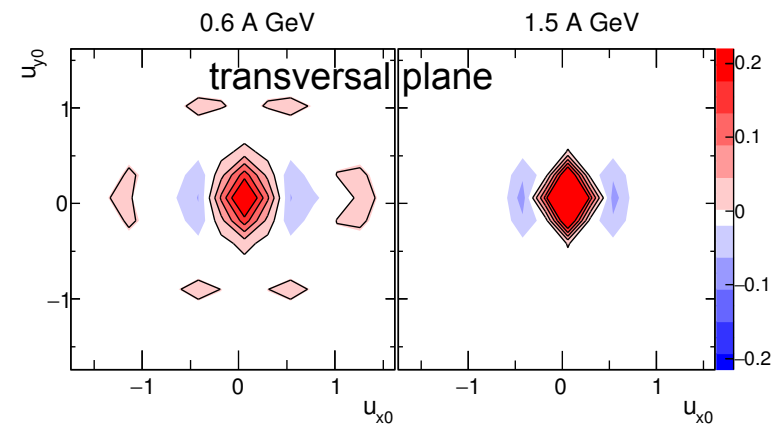
In velocity space we observe a complementary distribution.

In the xy plane, the shift of protons in x direction is smaller for a soft (SM) than for a hard (HM) EoS



Survey of the reaction

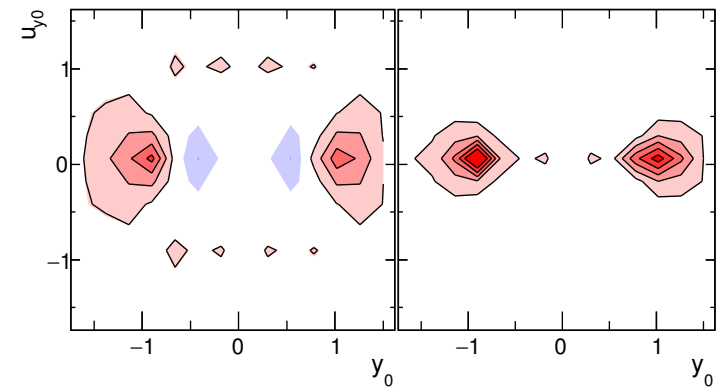
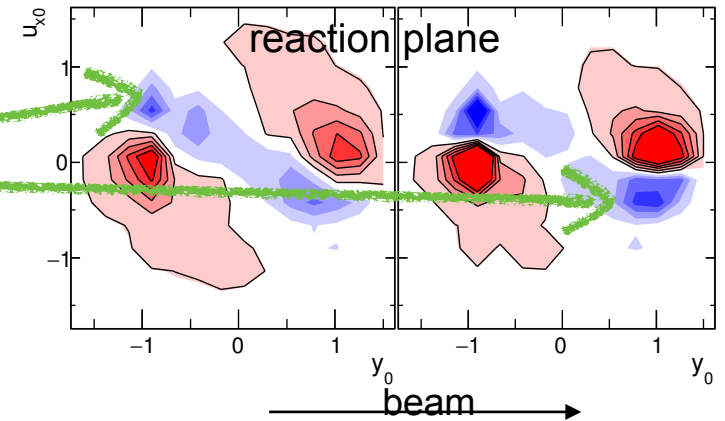
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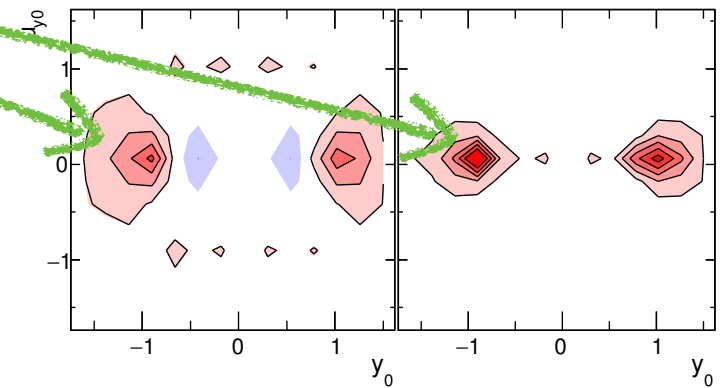
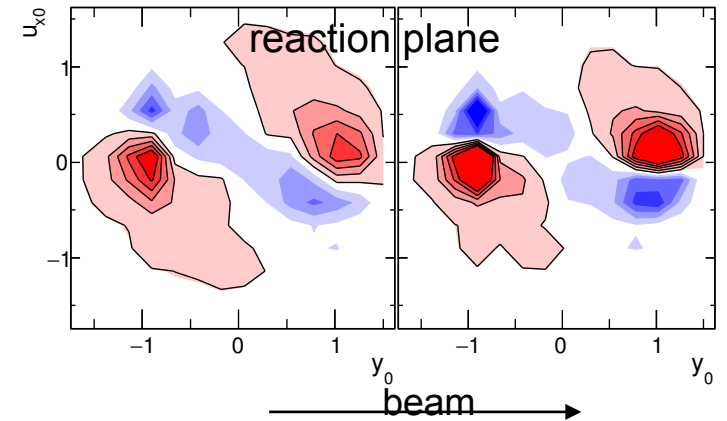
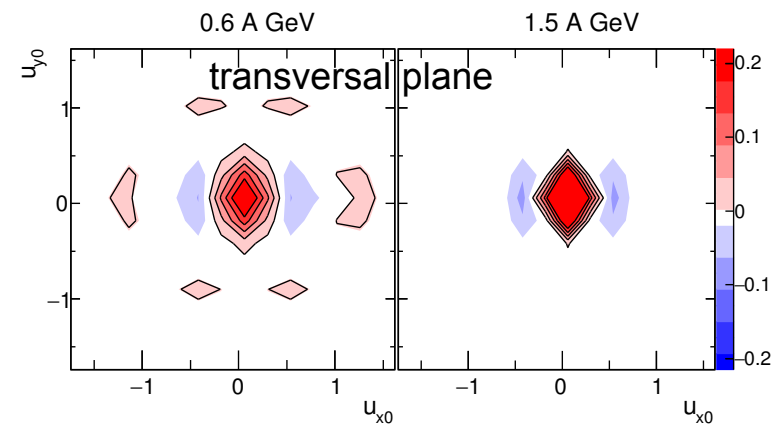
In the xy plane, the shift of protons in x direction is smaller for a soft (SM) than for a hard (HM) EoS

This is due to a smaller acceleration yielding a weaker in-plane flow and hence a smaller velocity in x-direction



Survey of the reaction

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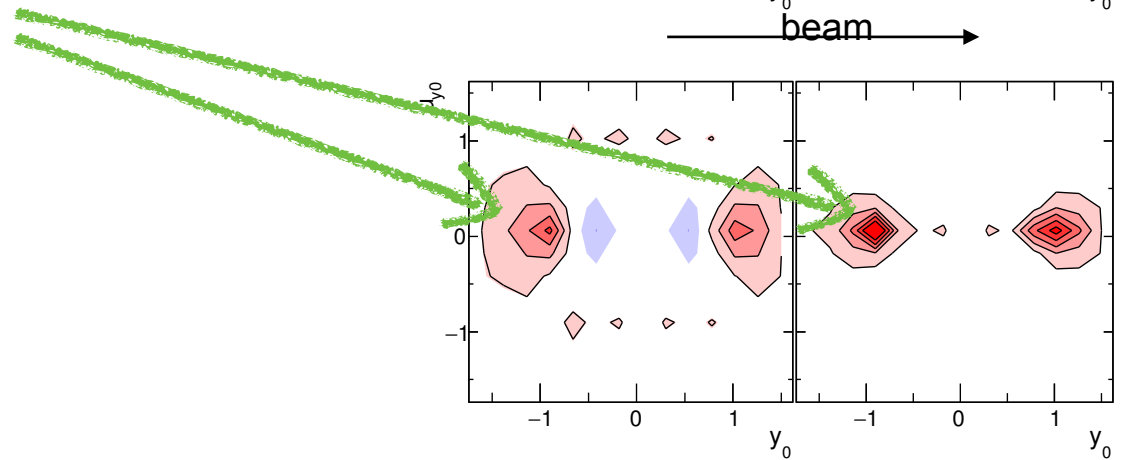


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In the xy plane, the shift of protons in x direction is smaller for a soft (SM) than for a hard (HM) EoS

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The soft EoS leads also to less stopping

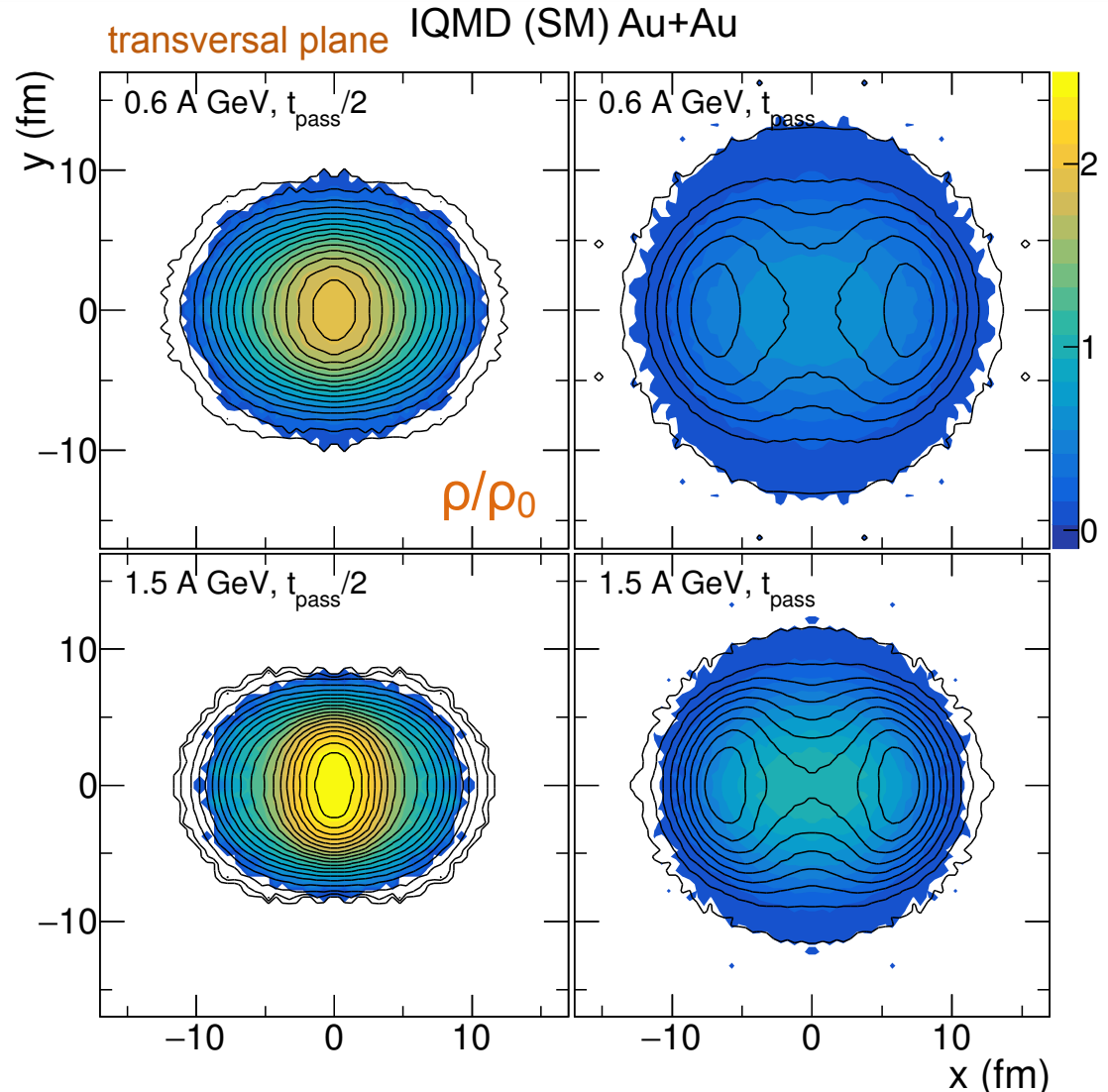


Survey of the reaction



We select now fast moving particles in the transverse direction at mid-rapidity:

$|y_0| < 0.2$, $u_{t0} > 0.4$ (used by the FOPI collaboration for the v_2 investigation) in color. Compared to all (black contours).



Survey of the reaction



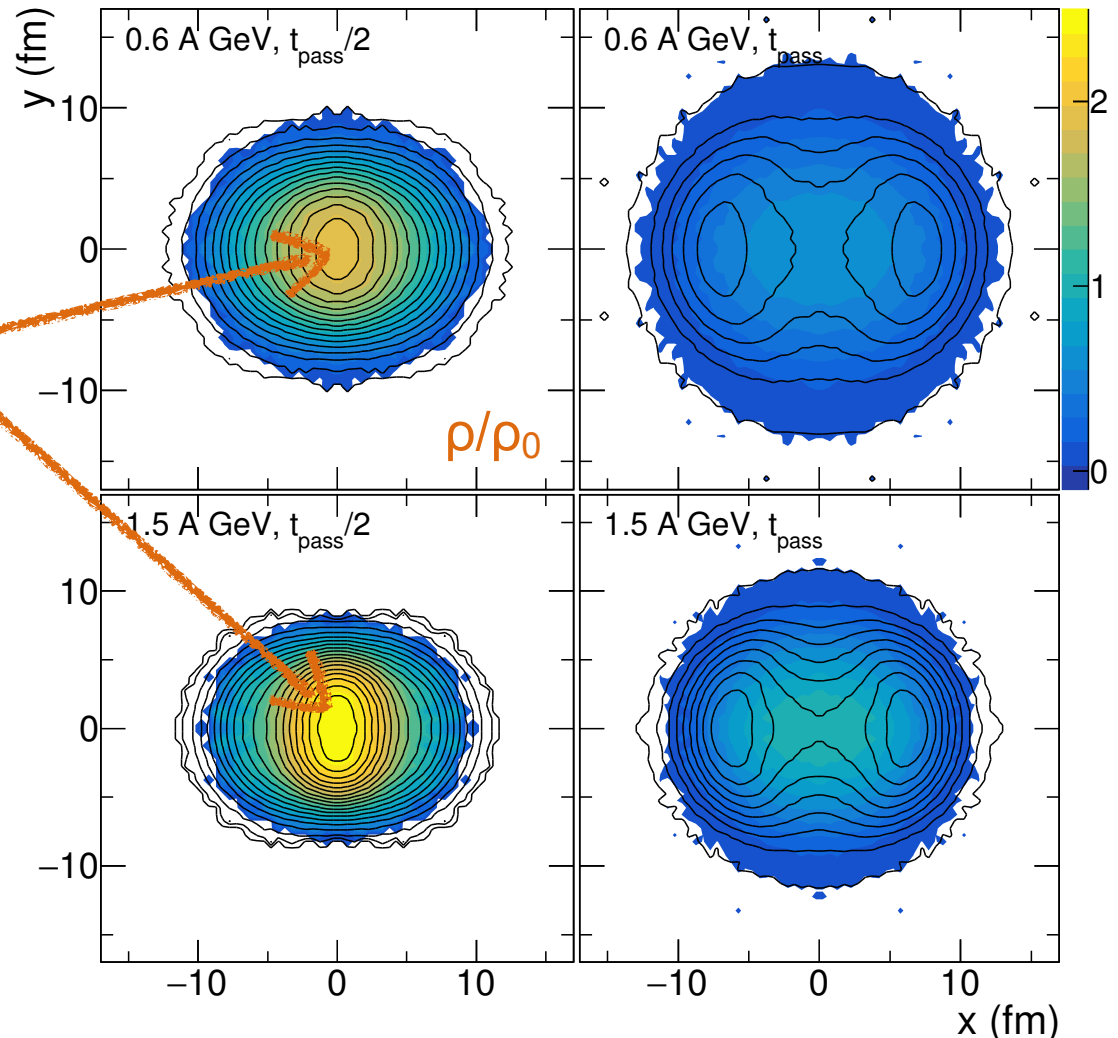
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At full overlap:

- the innermost participants = a dense almond shaped core, out-of-plane elongated, compression is highest.

transversal plane IQMD (SM) Au+Au



Survey of the reaction

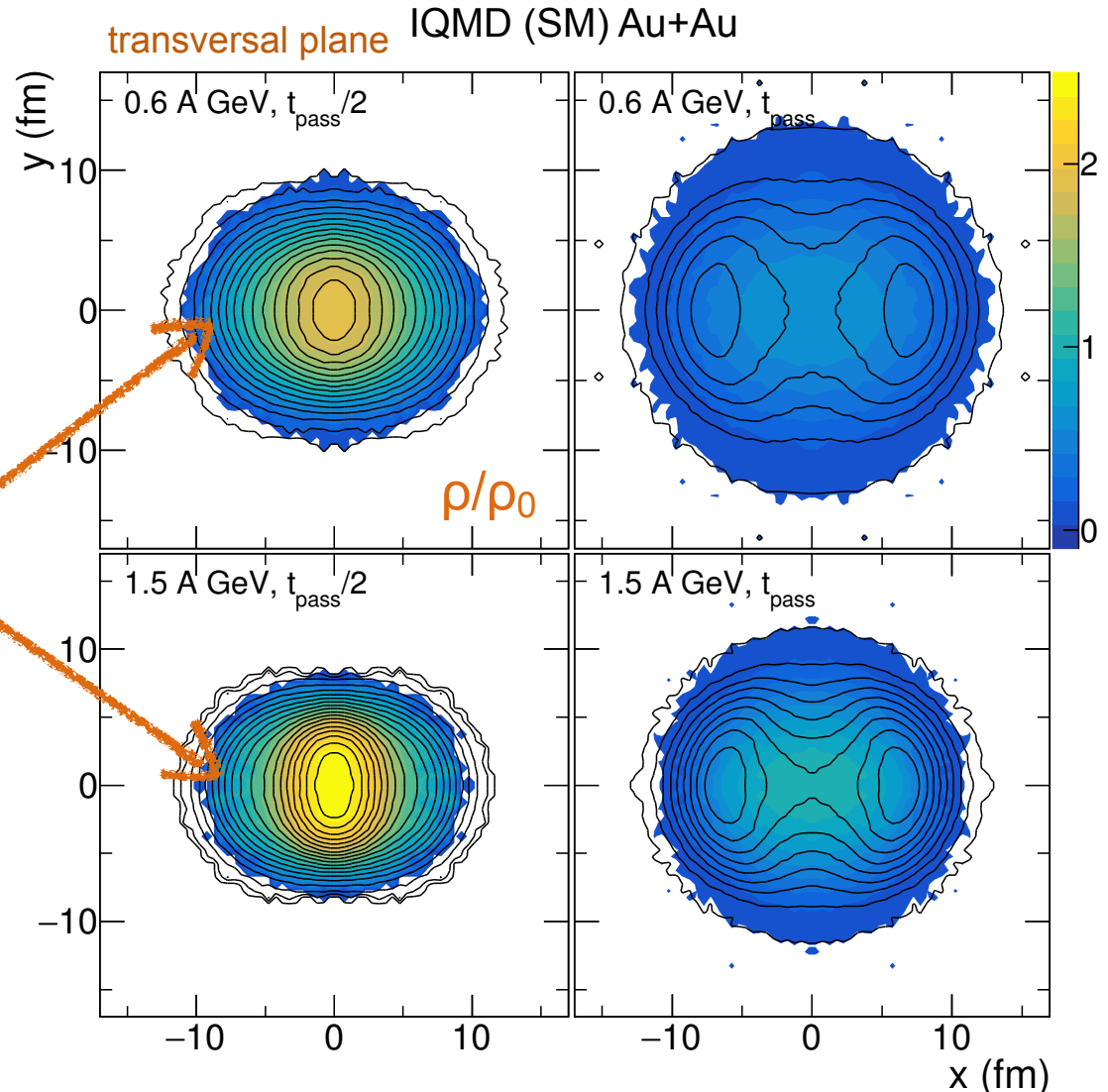


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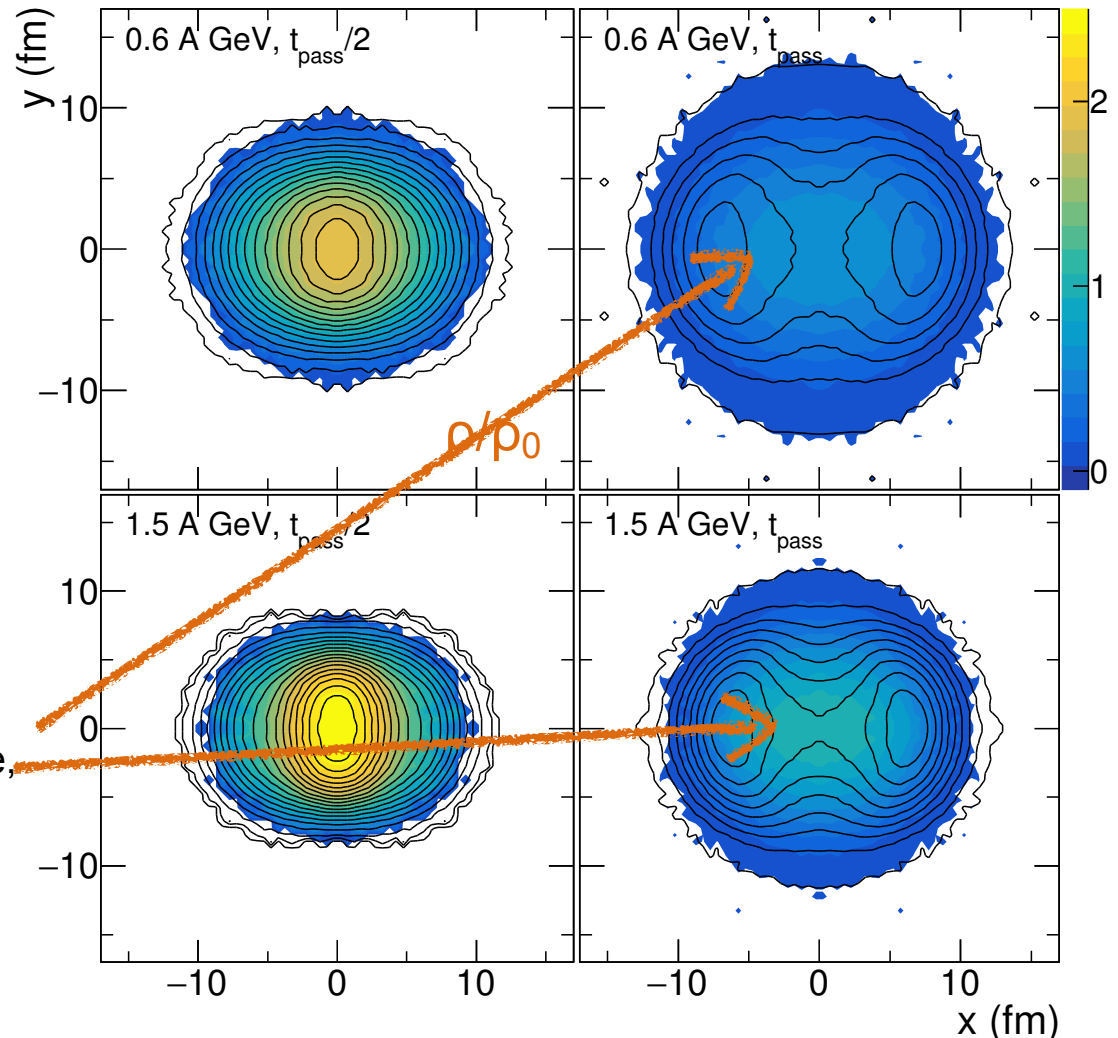
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At passing time, the innermost (compressed) participants expand in-plane, but **not with enough pressure** to produce a positive elliptic flow v_2 (seen later), in contrast to higher bombarding energies

transversal plane IQMD (SM) Au+Au

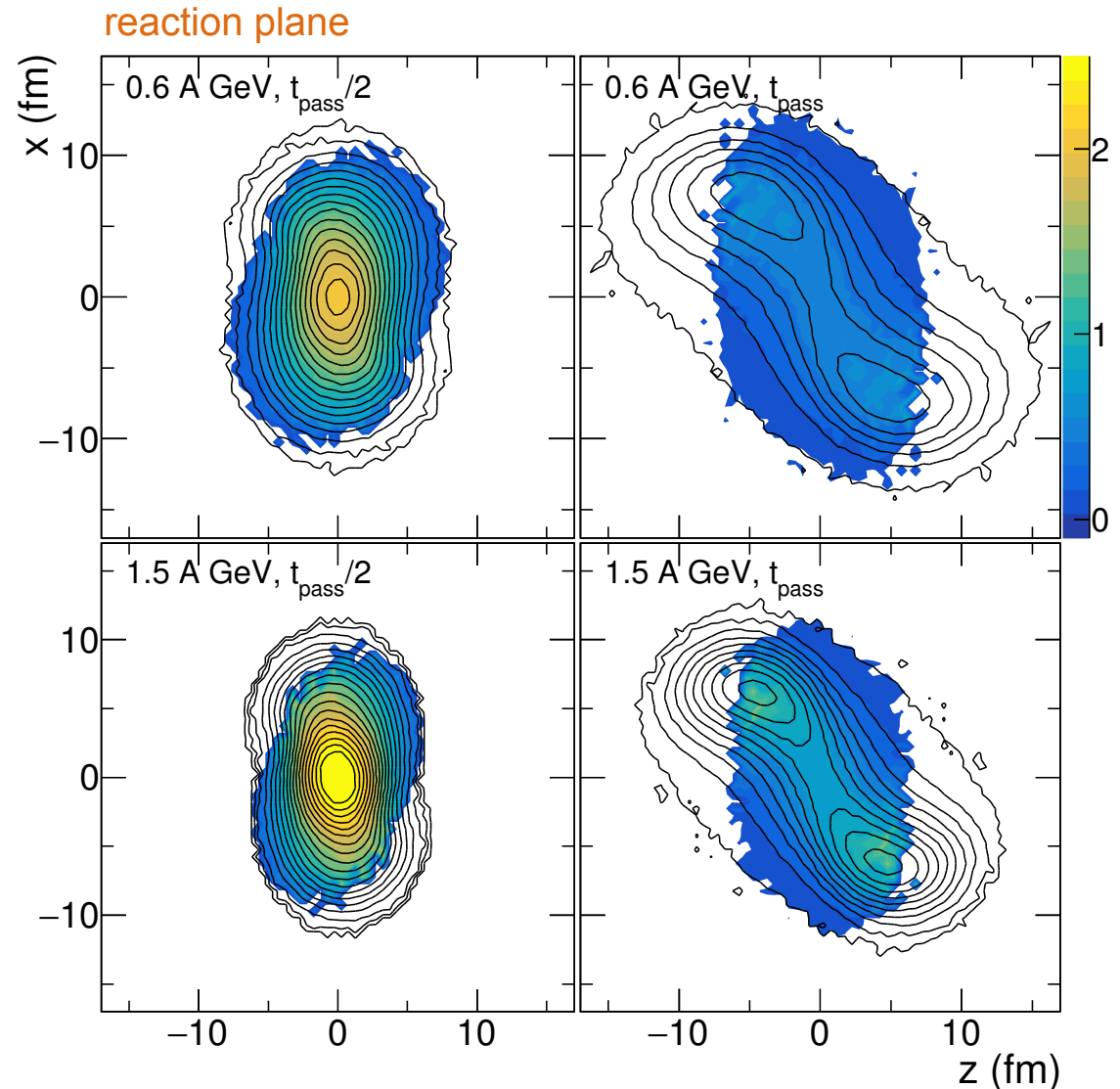




Survey of the reaction

Formation of an in-plane ridge
between the bulk of the spectators.

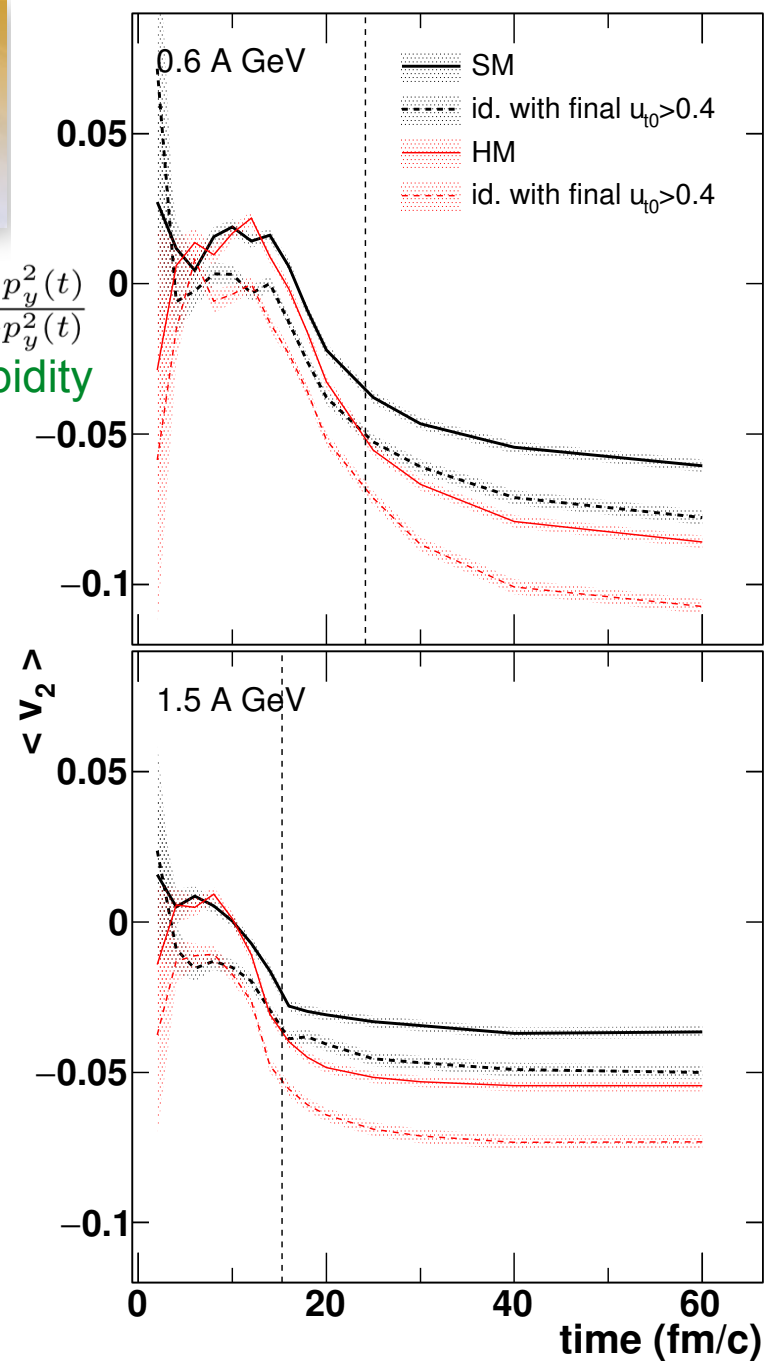
Incident energy \nearrow \Rightarrow ridge & initial
almond core density \nearrow



The elliptic flow time evolution

$$v_2(t) = \frac{p_x^2(t) - p_y^2(t)}{p_x^2(t) + p_y^2(t)}$$

at mid-rapidity

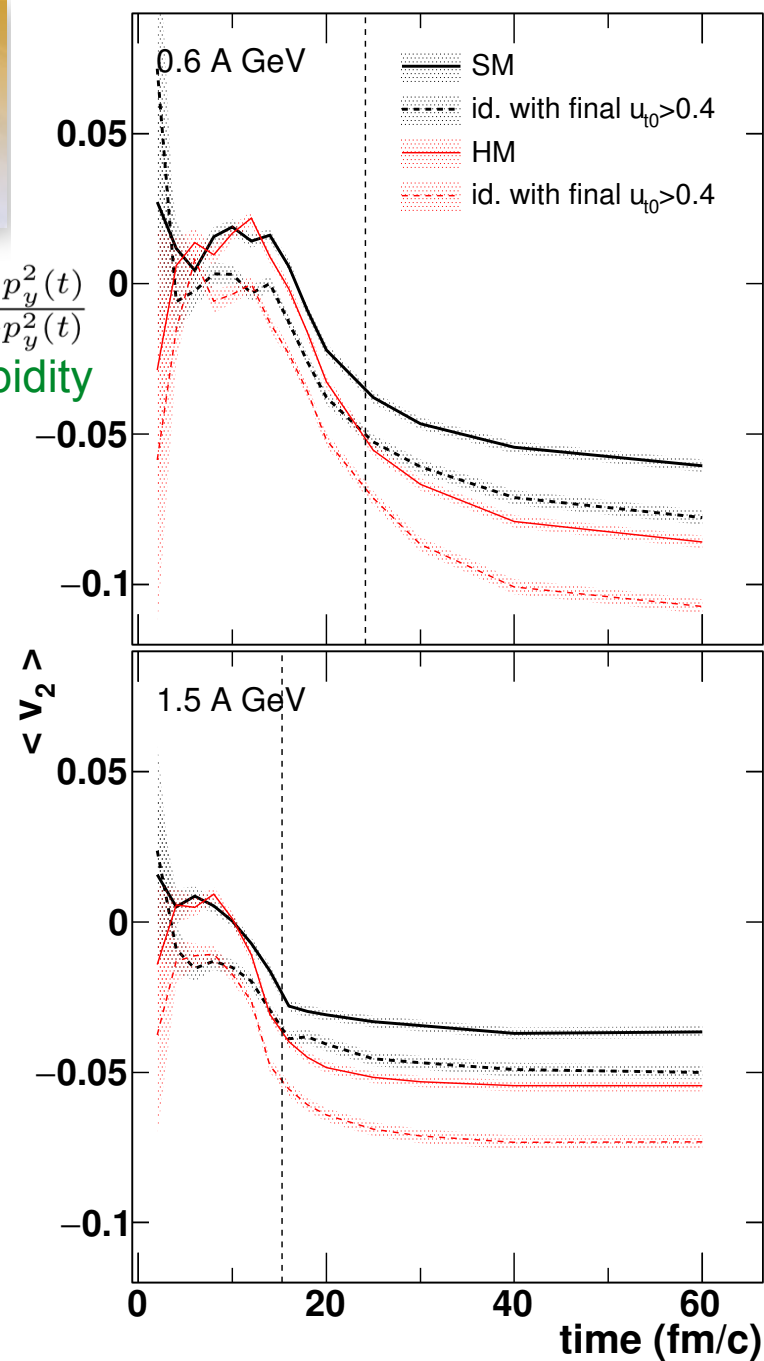


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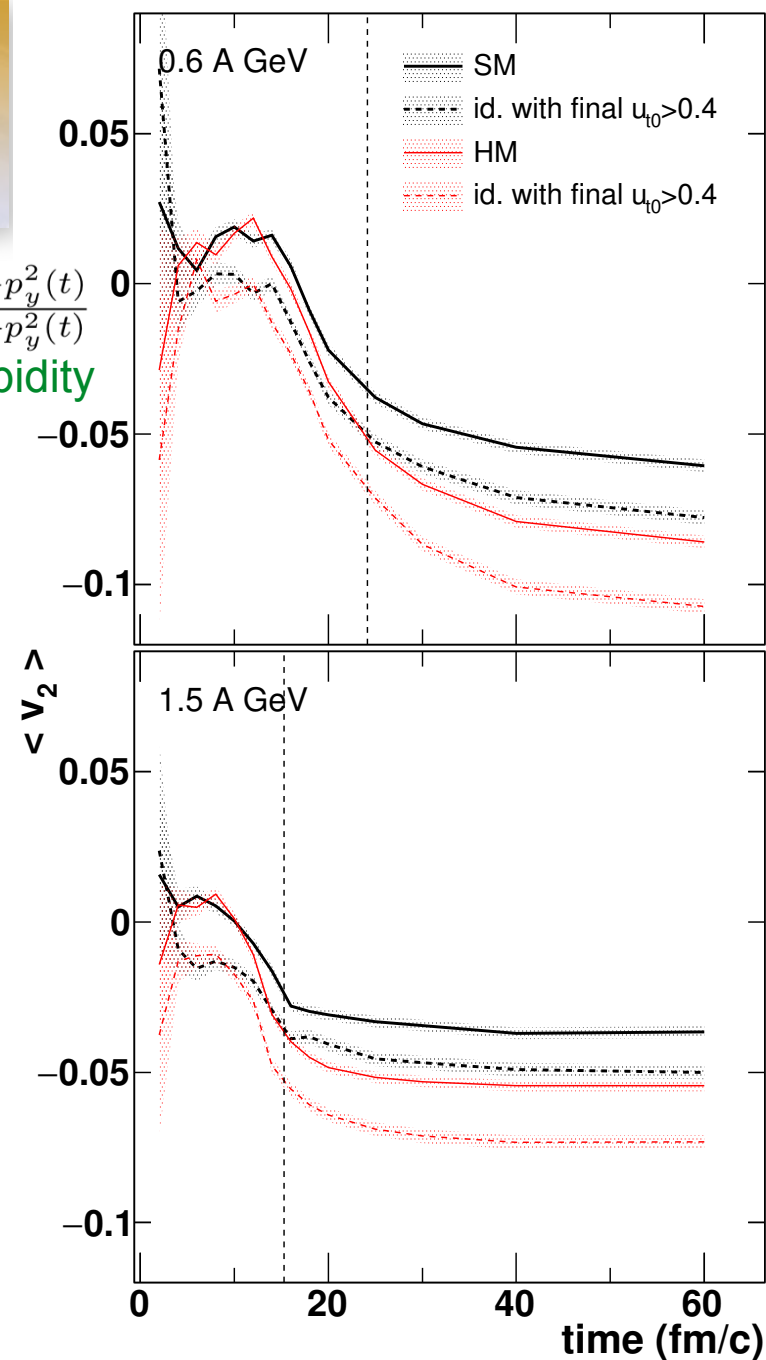


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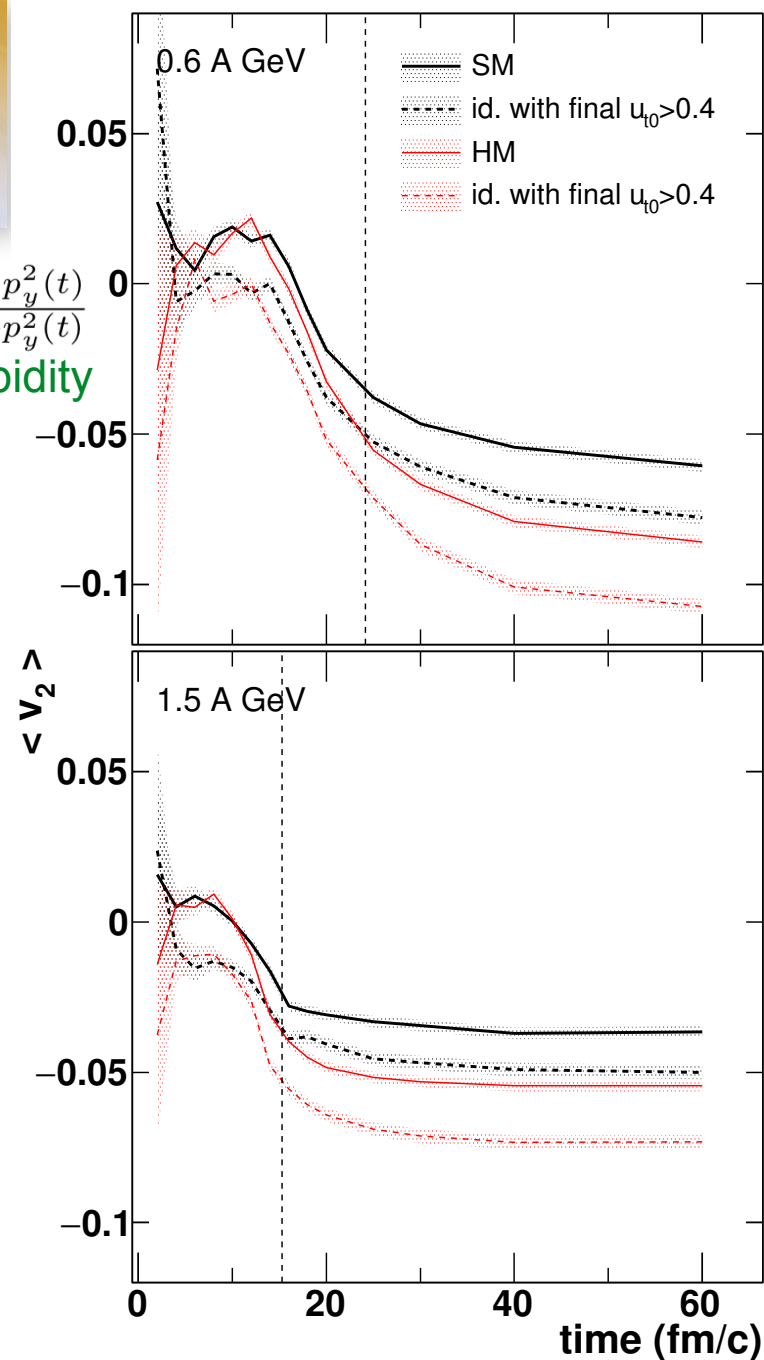


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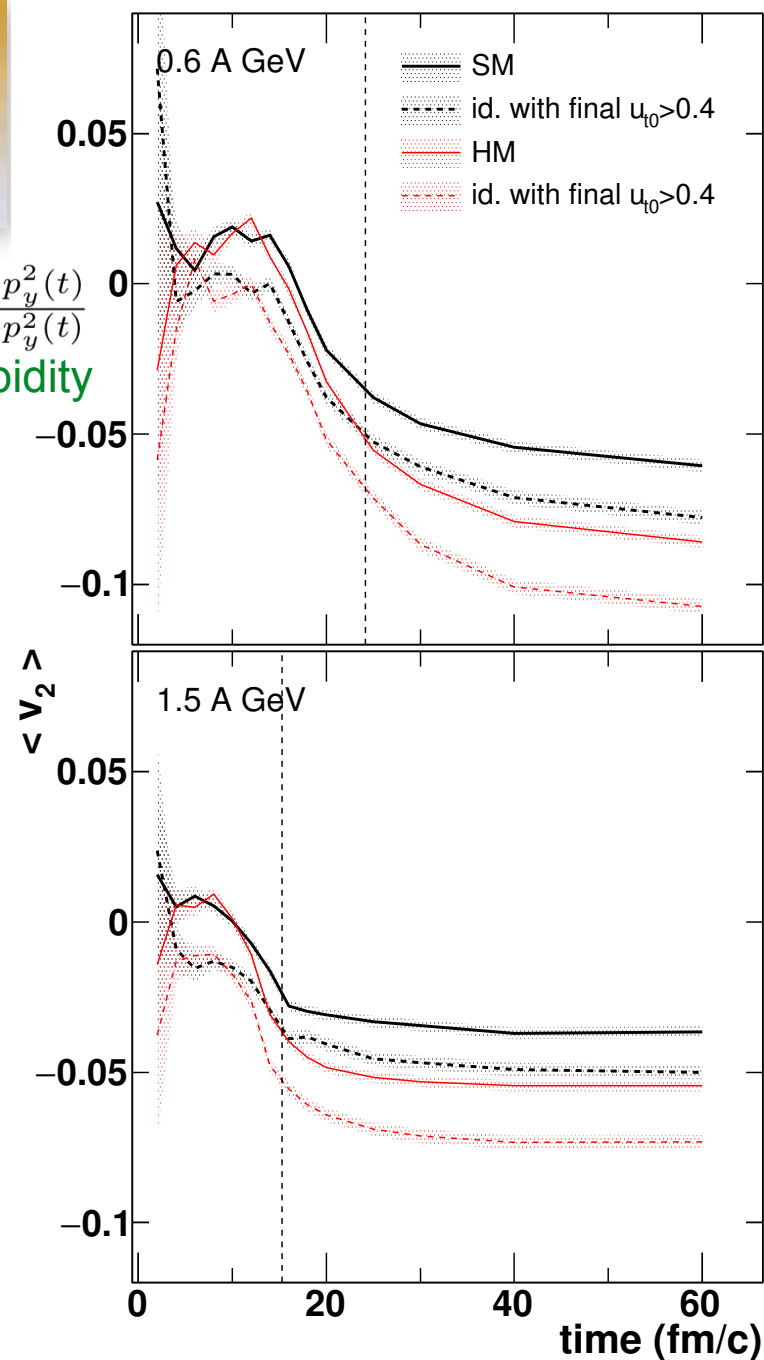


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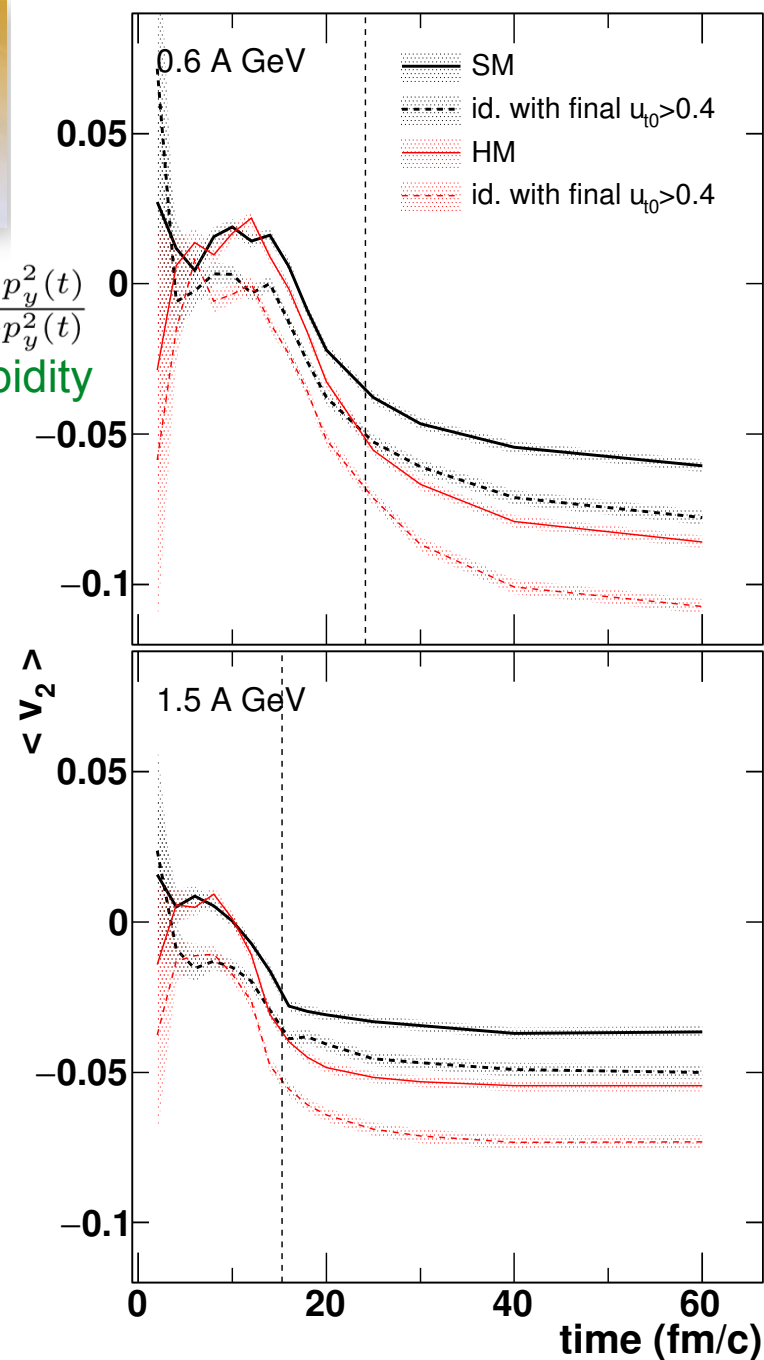


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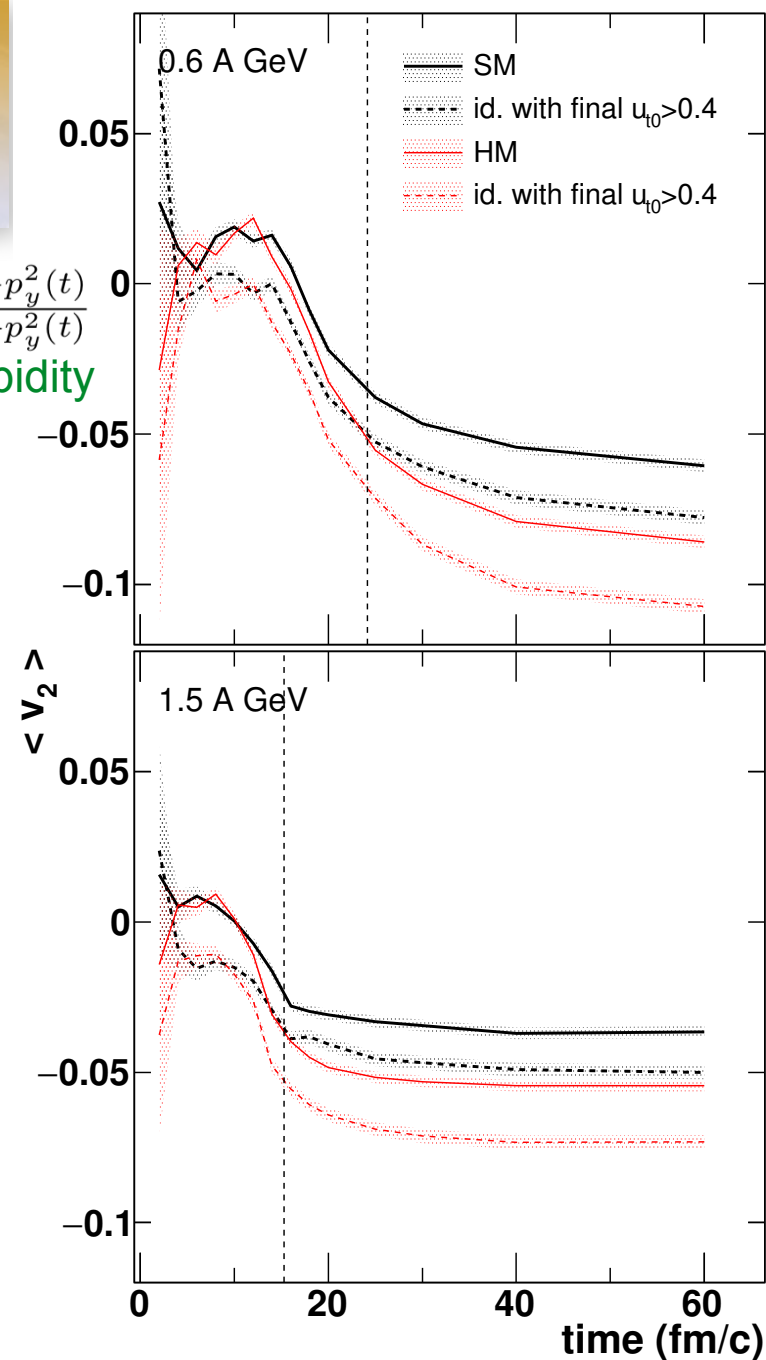


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- SM vs HM: v_2 at mid-rapidity depends strongly on the EoS; effect enhanced for fastest protons.





The elliptic flow: collisions versus mean field

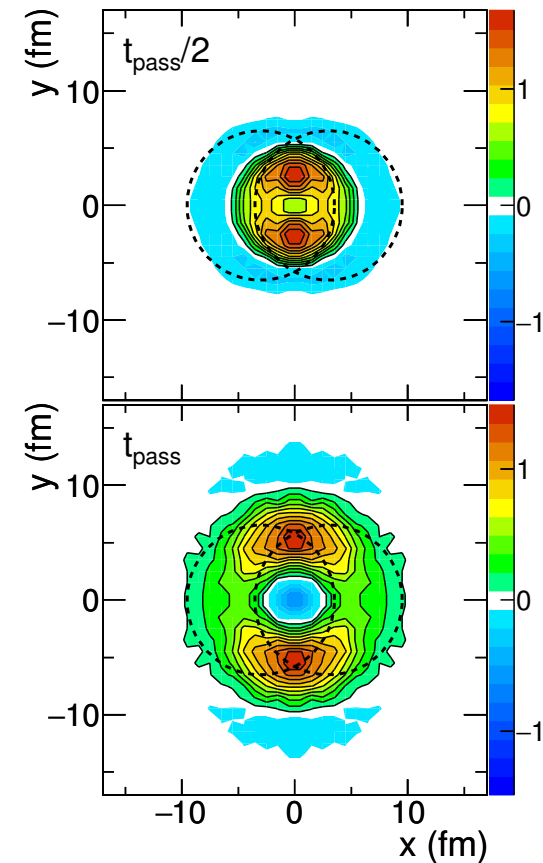
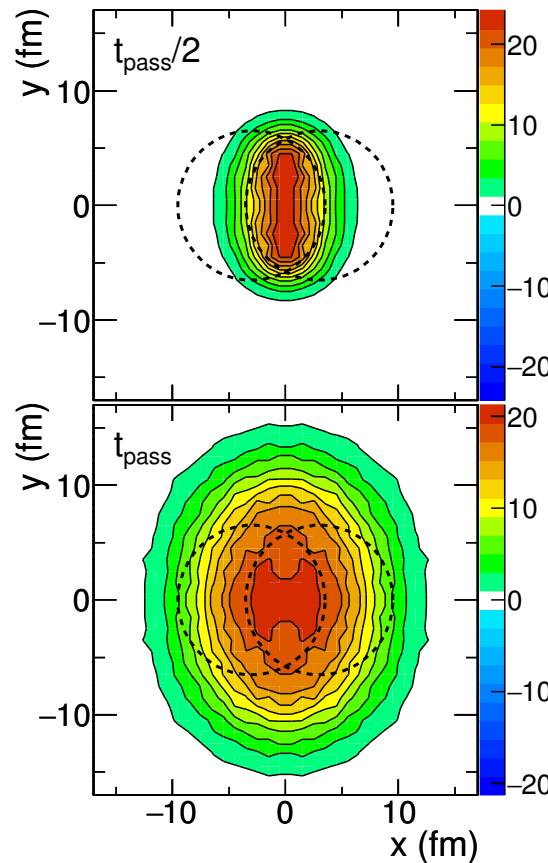
An observable to quantify their respective contribution to it: transverse momentum modification induced projected on the direction of the final momentum:

$$\langle \Delta P_t^o(t) \rangle = \langle \Delta \mathbf{P}_t(t) \cdot \frac{\mathbf{p}_{final}}{|\mathbf{p}_{final}|} \rangle$$

reaction plane
collisions

0.6 A.GeV, mid-rapidity, $u_{t0} > 0.4$

mean field





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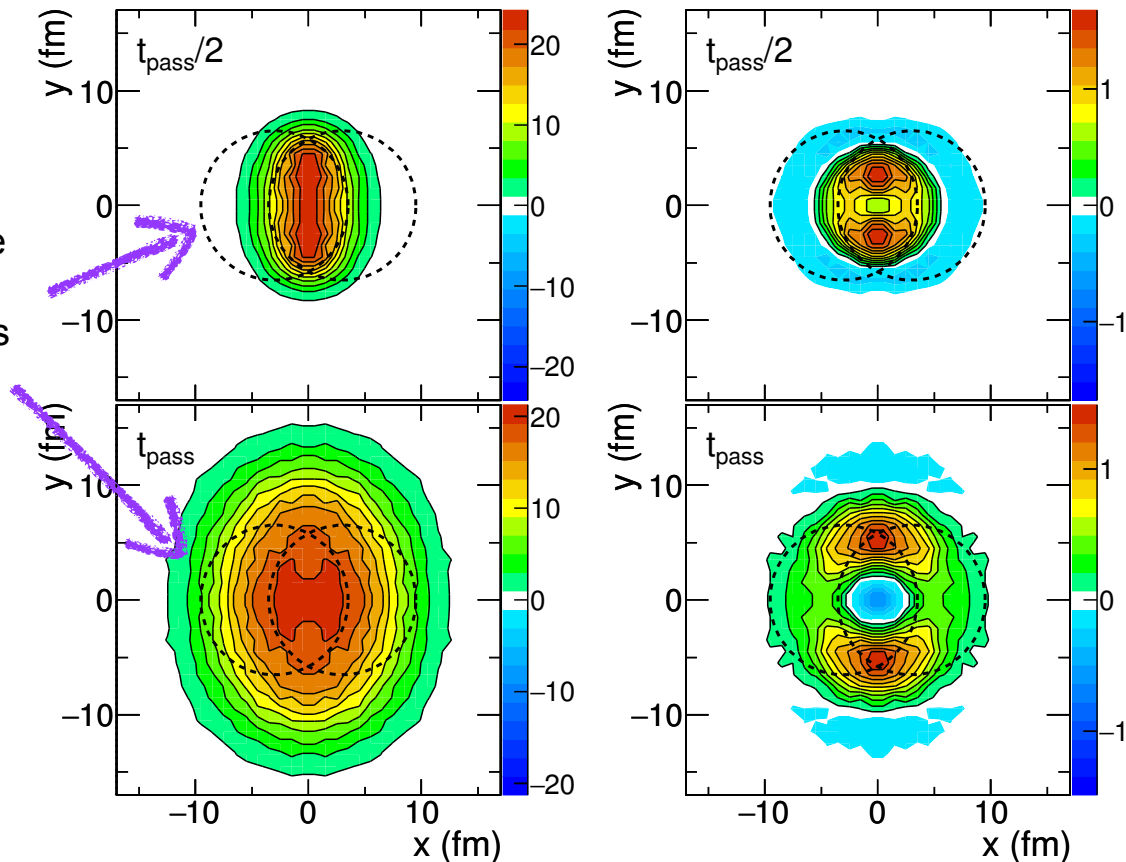
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From collisions: about an order of magnitude larger than from mean field, set fast in the overlap zone \Rightarrow this zone of violent collisions expands rapidly keeping its almond shape.

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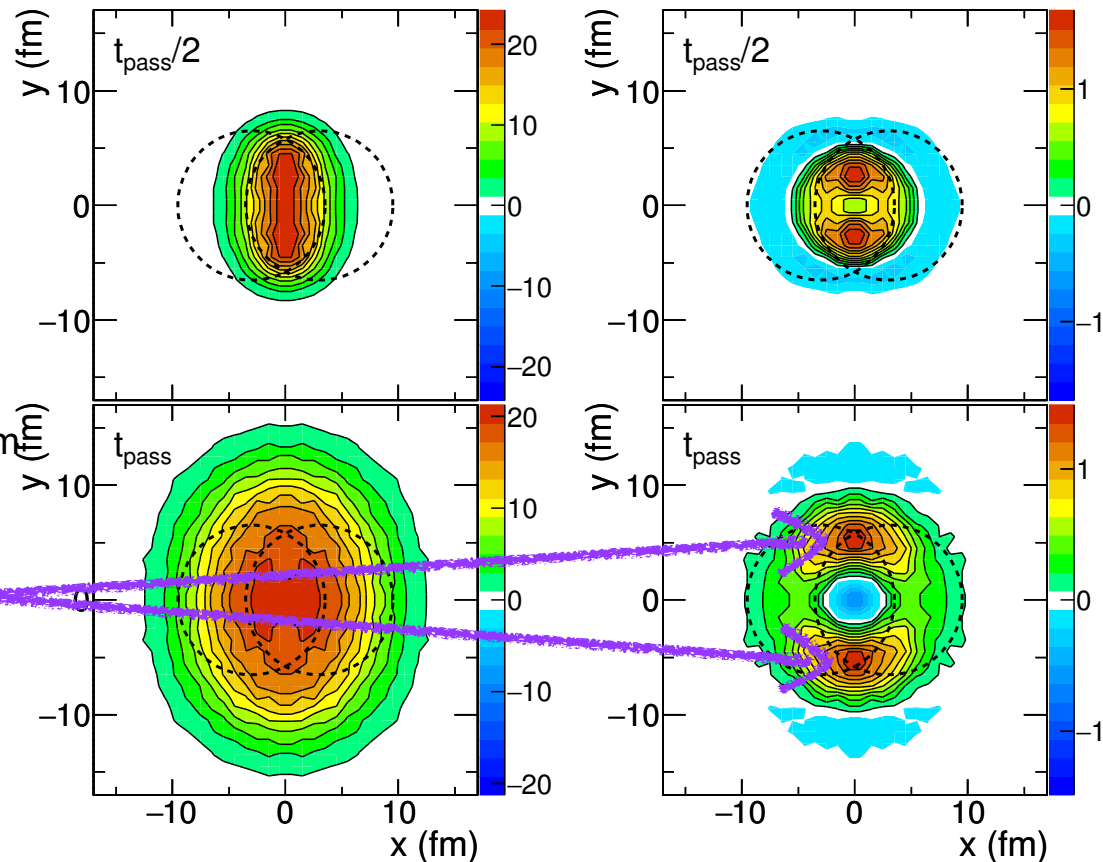
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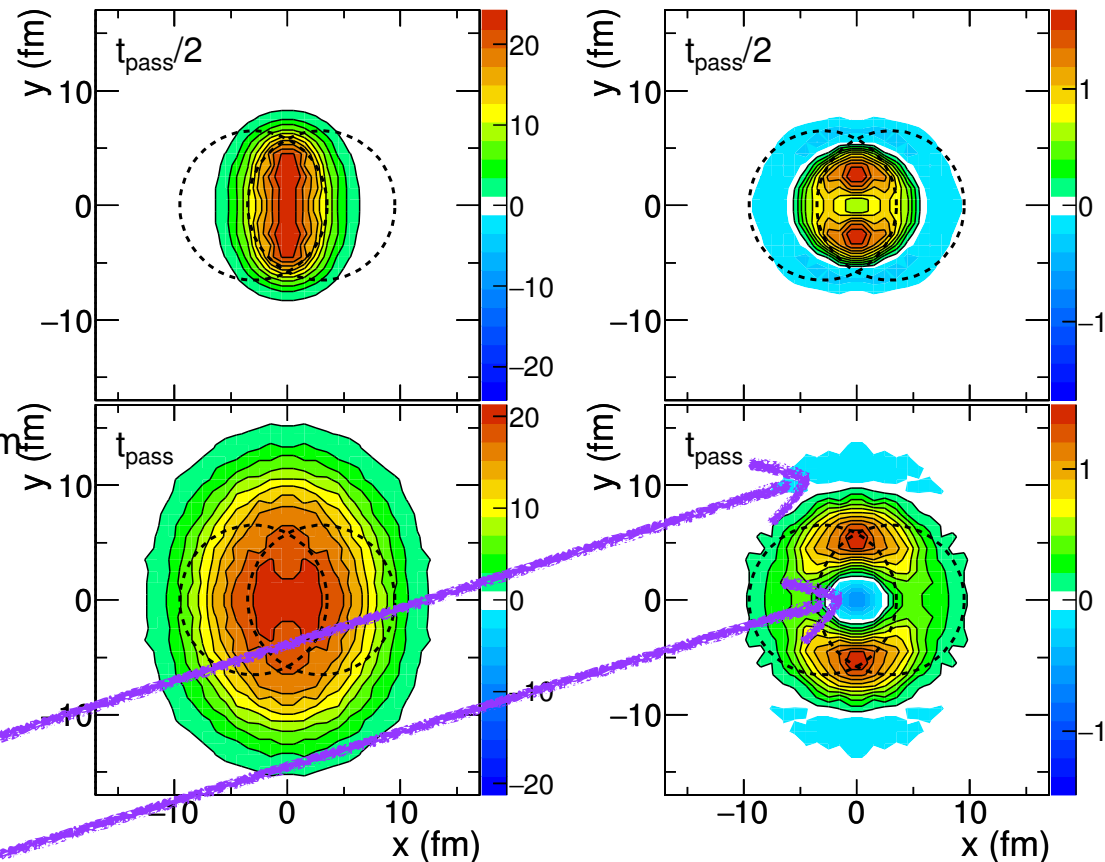
Outer blue areas \Leftarrow attractive potential of the remnant, deceleration.

Inner blue area: inner density decreases and attraction by the moving spectators \Rightarrow transverse velocity decreases

reaction plane collisions

0.6 A.GeV, mid-rapidity, $u_{t0} > 0.4$

mean field

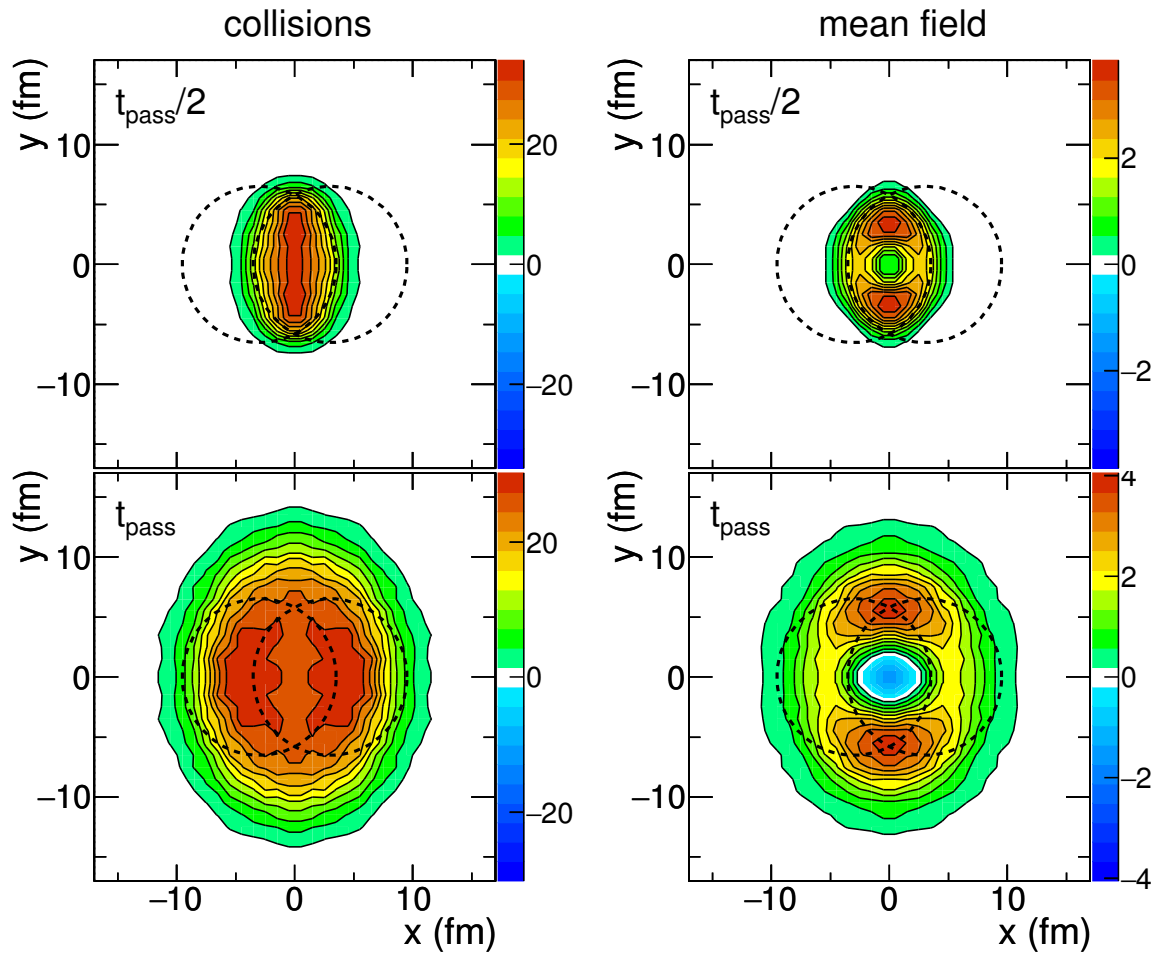




The elliptic flow: collisions versus mean field

Little difference between 0.6 AGeV and at 1.5 AGeV.

1.5 A.GeV, mid-rapidity, $u_{t0} > 0.4$



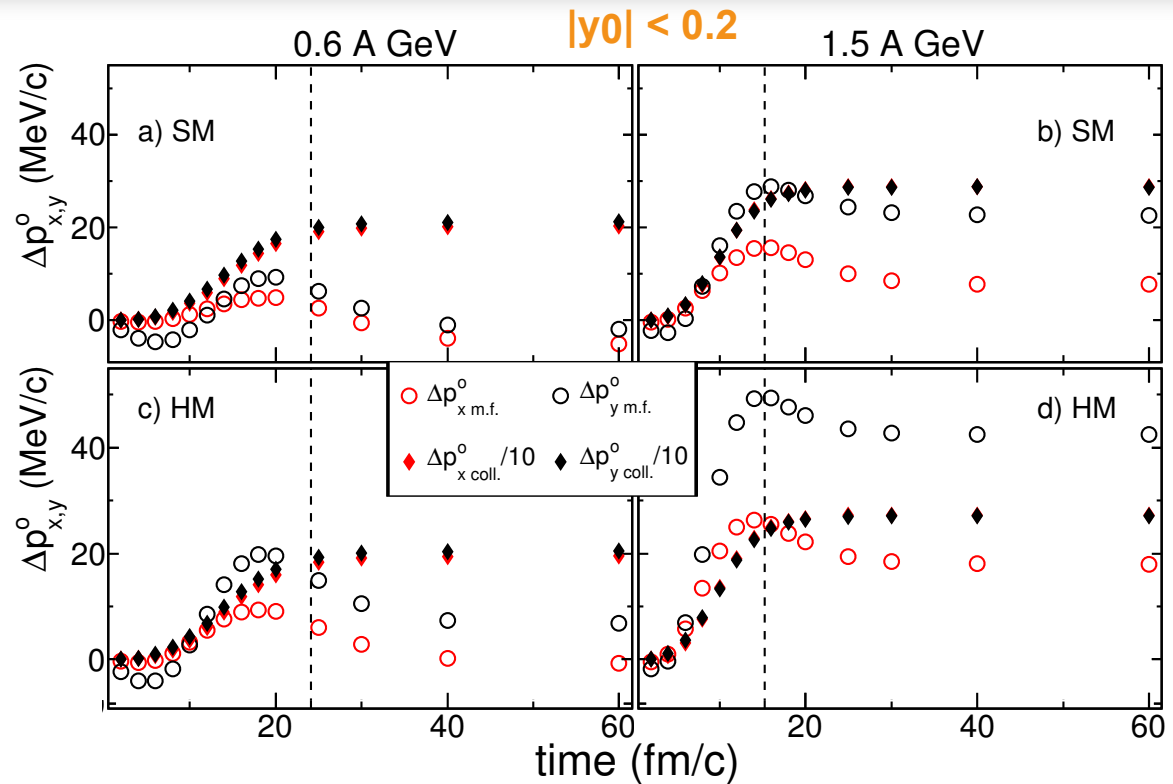


The elliptic flow: collisions versus mean field

v_2 directly related to its anisotropy in x and y .

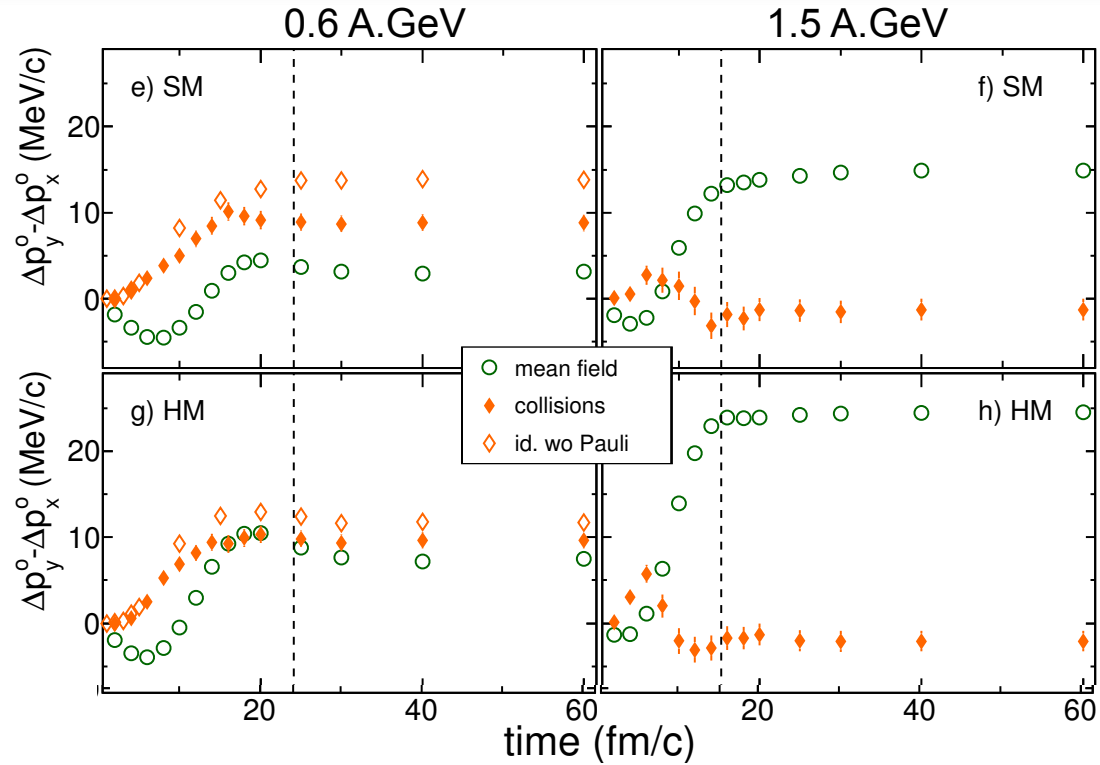
$$\langle \Delta P_i^o(t) \rangle = \langle \Delta P_i(t) \cdot \frac{p_{i,final}}{|p_{i,final}|} \rangle$$

Collision contribution: always much larger than that of mean field.





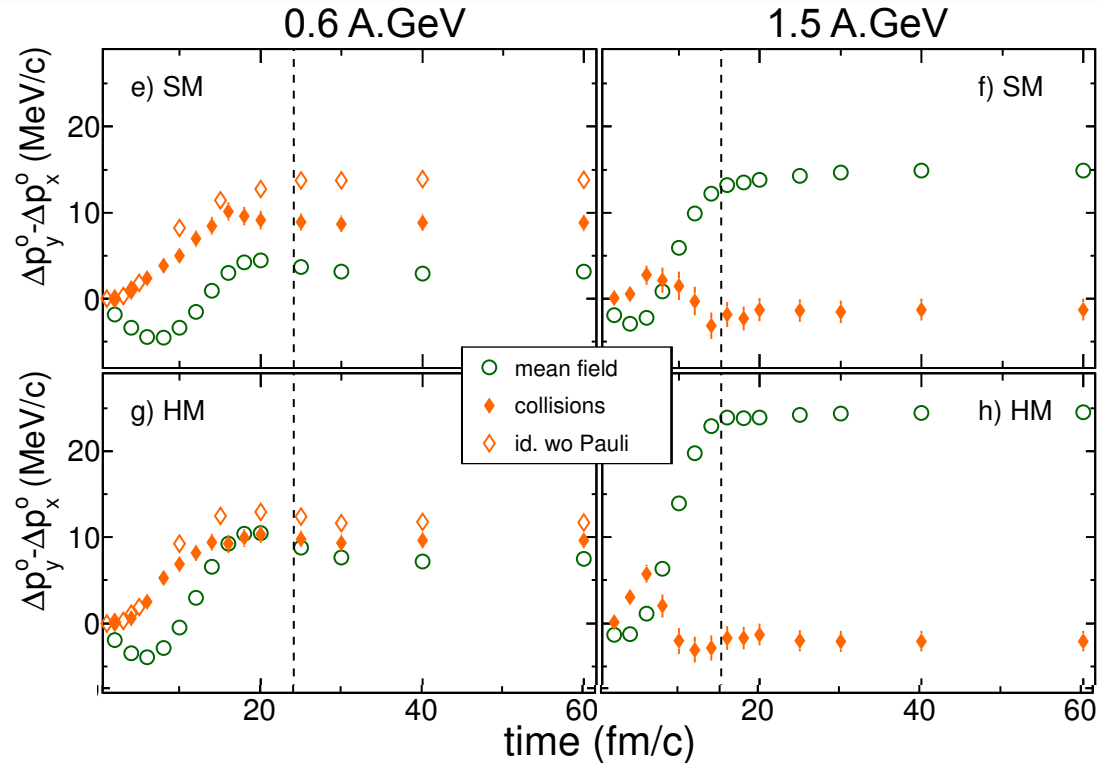
The elliptic flow: collisions versus mean field





The elliptic flow: collisions versus mean field

Excess in the y- direction: clearly visible for the mean field AND the collisions. For the collisions: becomes smaller with higher projectile velocity until it vanishes at 1.5 AGeV incident energy.

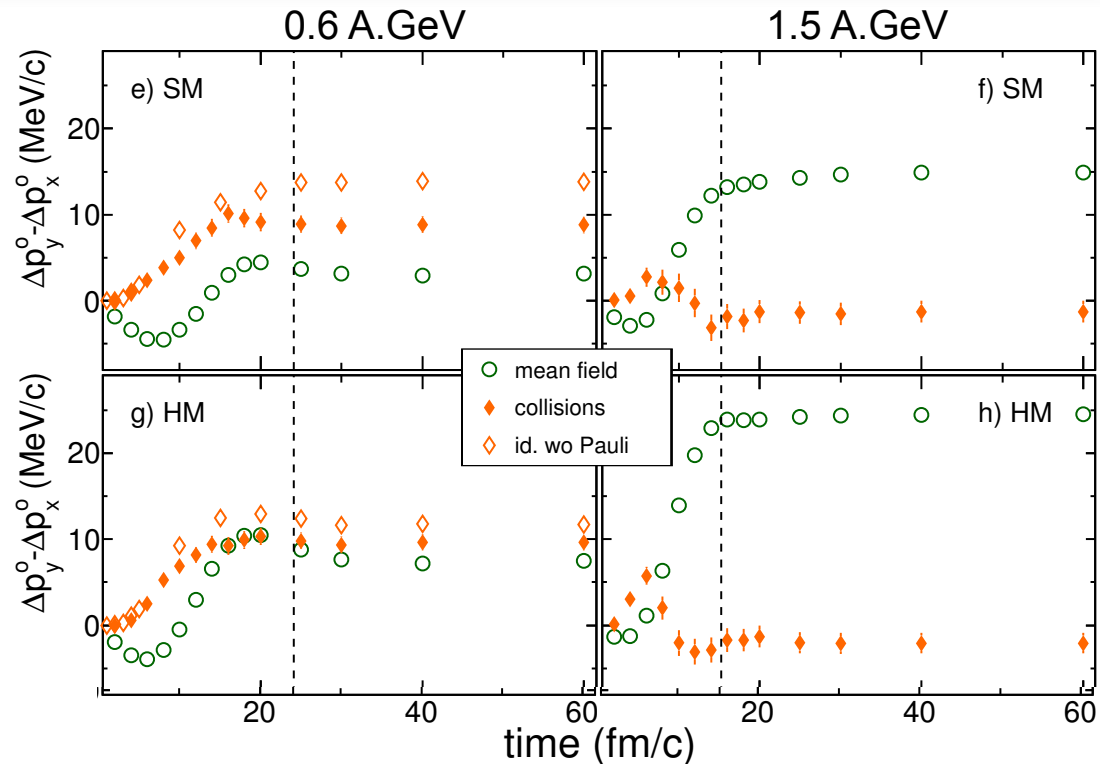




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K_0 has no visible influence on the amplitude of the **collisional** out-of-plane momentum excess because the number of collisions is almost unchanged by the choice of the EoS.



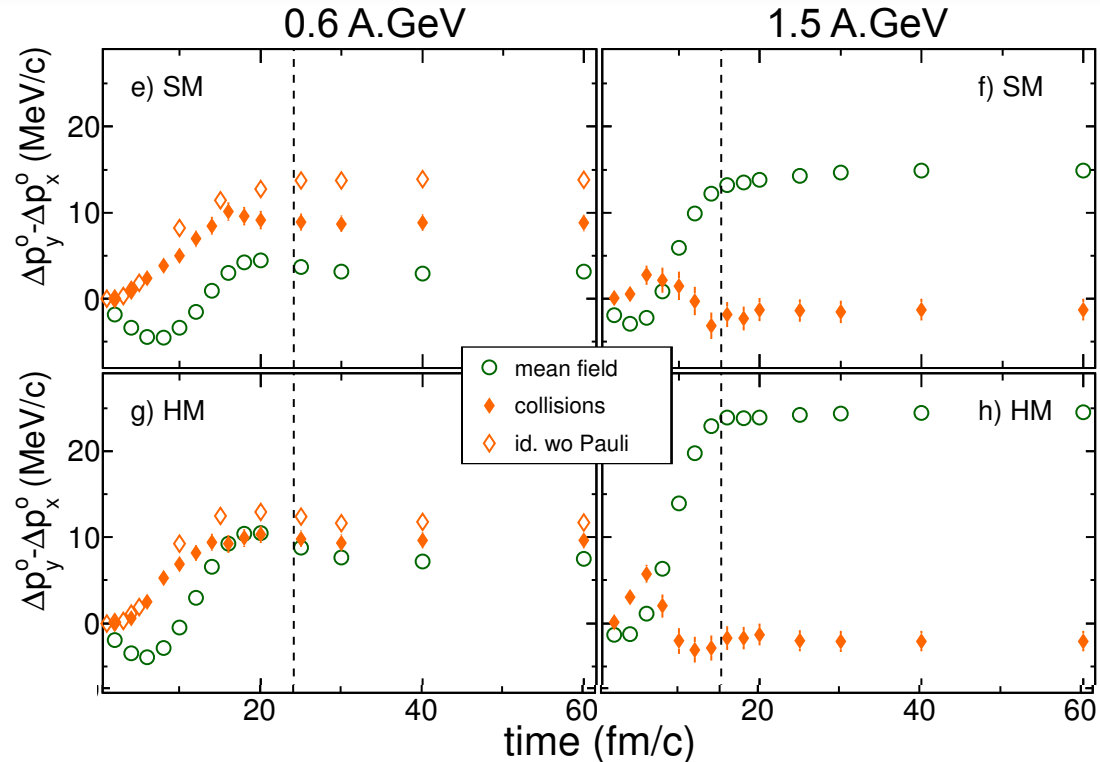


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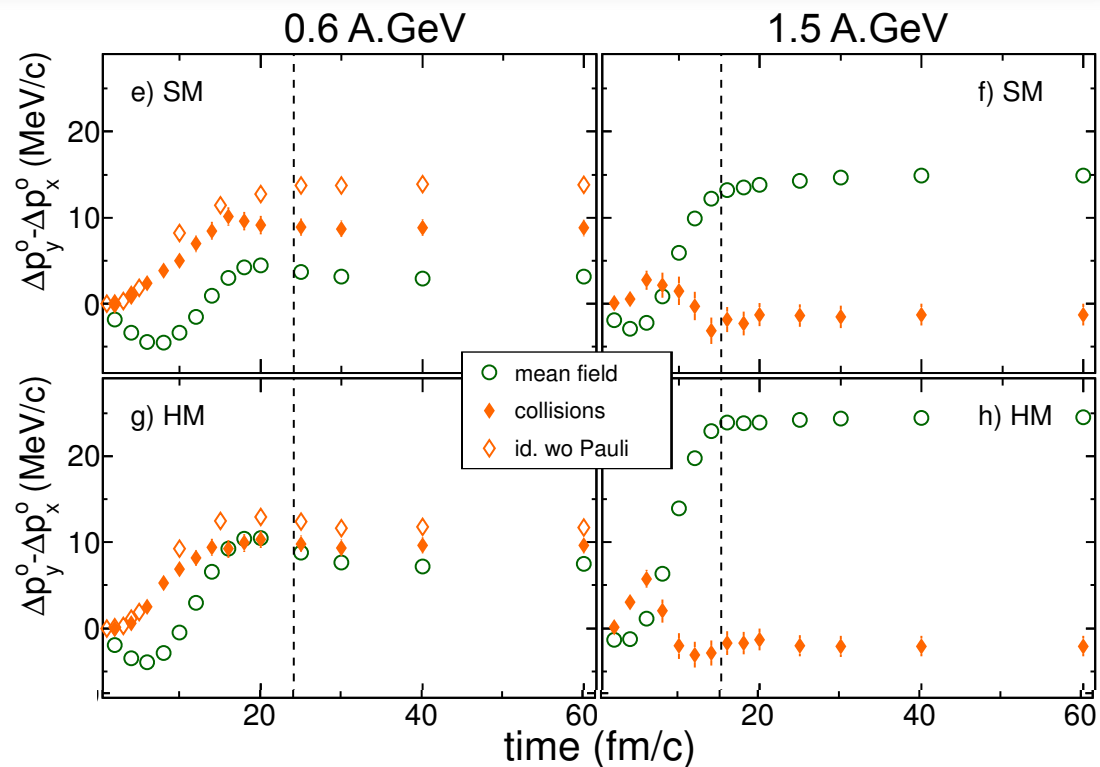
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⇒ Without Pauli blocking, there would be a collisional contribution to the EoS dependence of v_2 .





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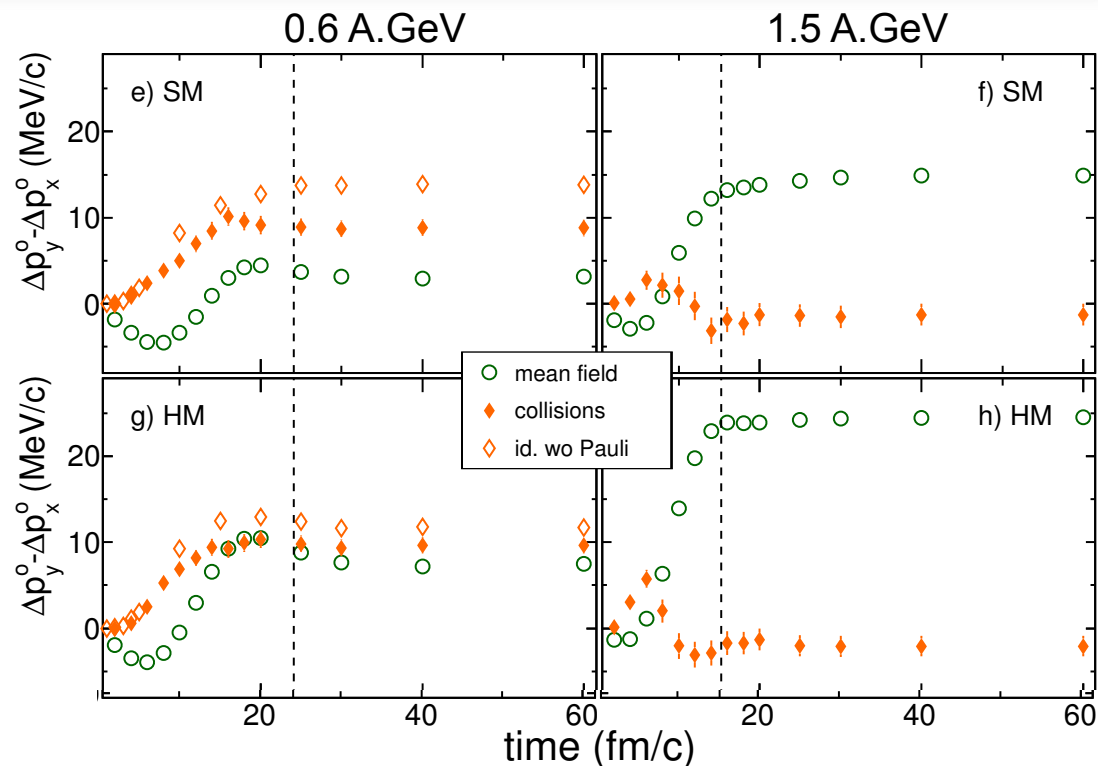
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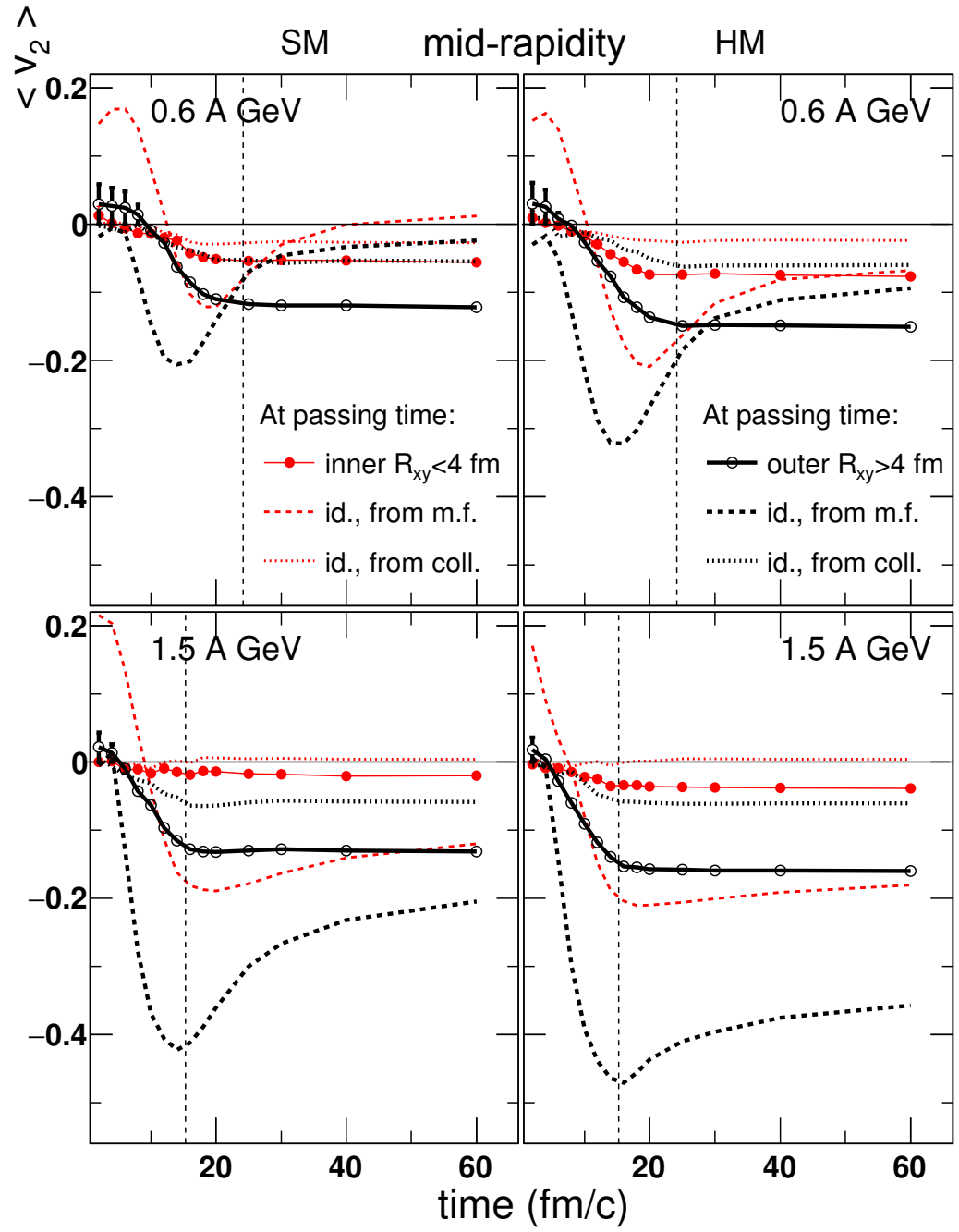
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Mean field contribution to $v_2 < 0$: dependent on incident energy and K_0 : moderate at 0.6 AGeV with the soft EoS, contributing to only 30% of the total $\Delta P_y^0 - \Delta P_x^0$, very strong and dominating at 1.5 AGeV with the stiffer EoS.

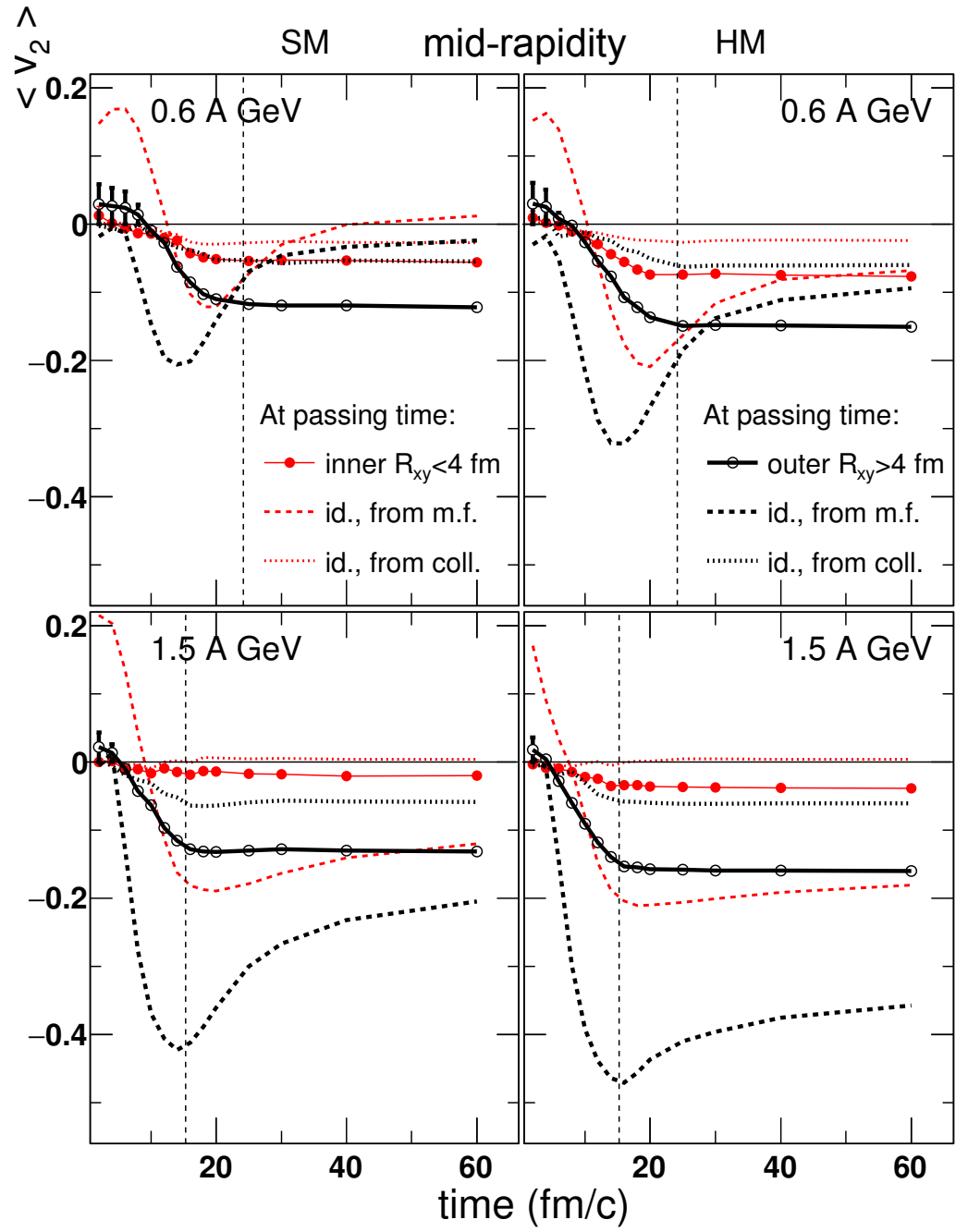


The elliptic flow: collisions vs mean field



The elliptic flow: collisions vs mean field

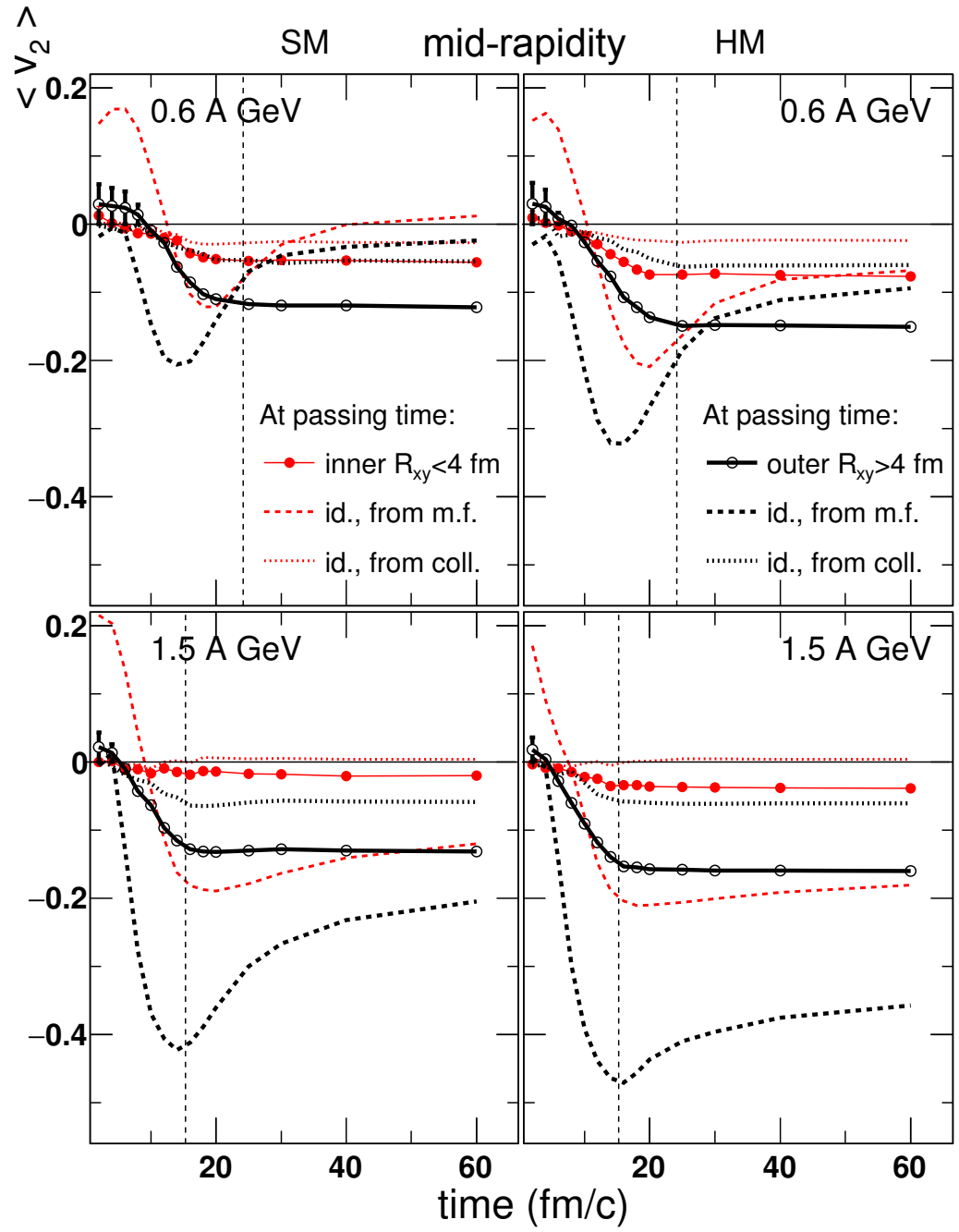
Outermost nucleons ($R_{xy} > 4$ fm) = the main source of the overall negative v_2 :



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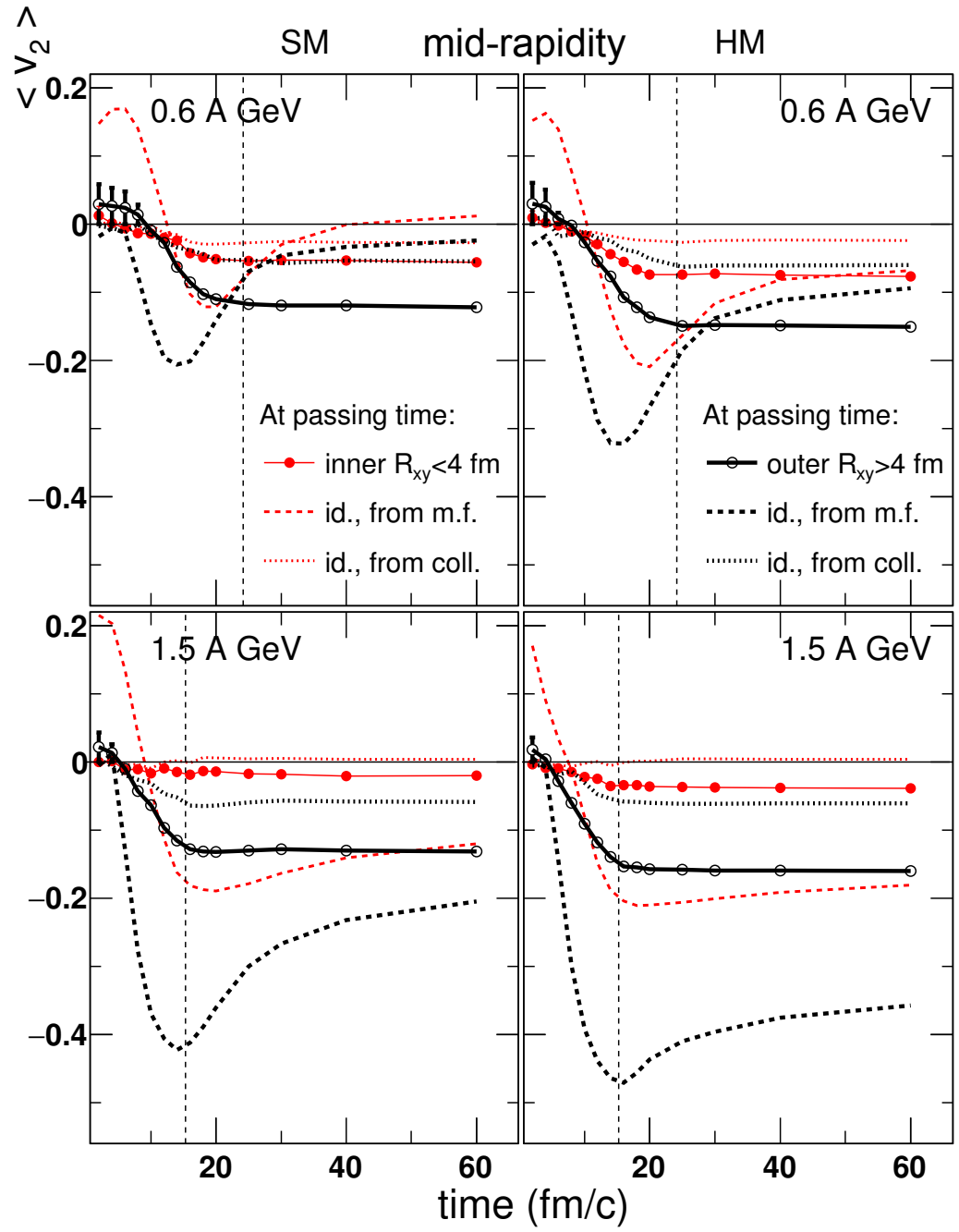
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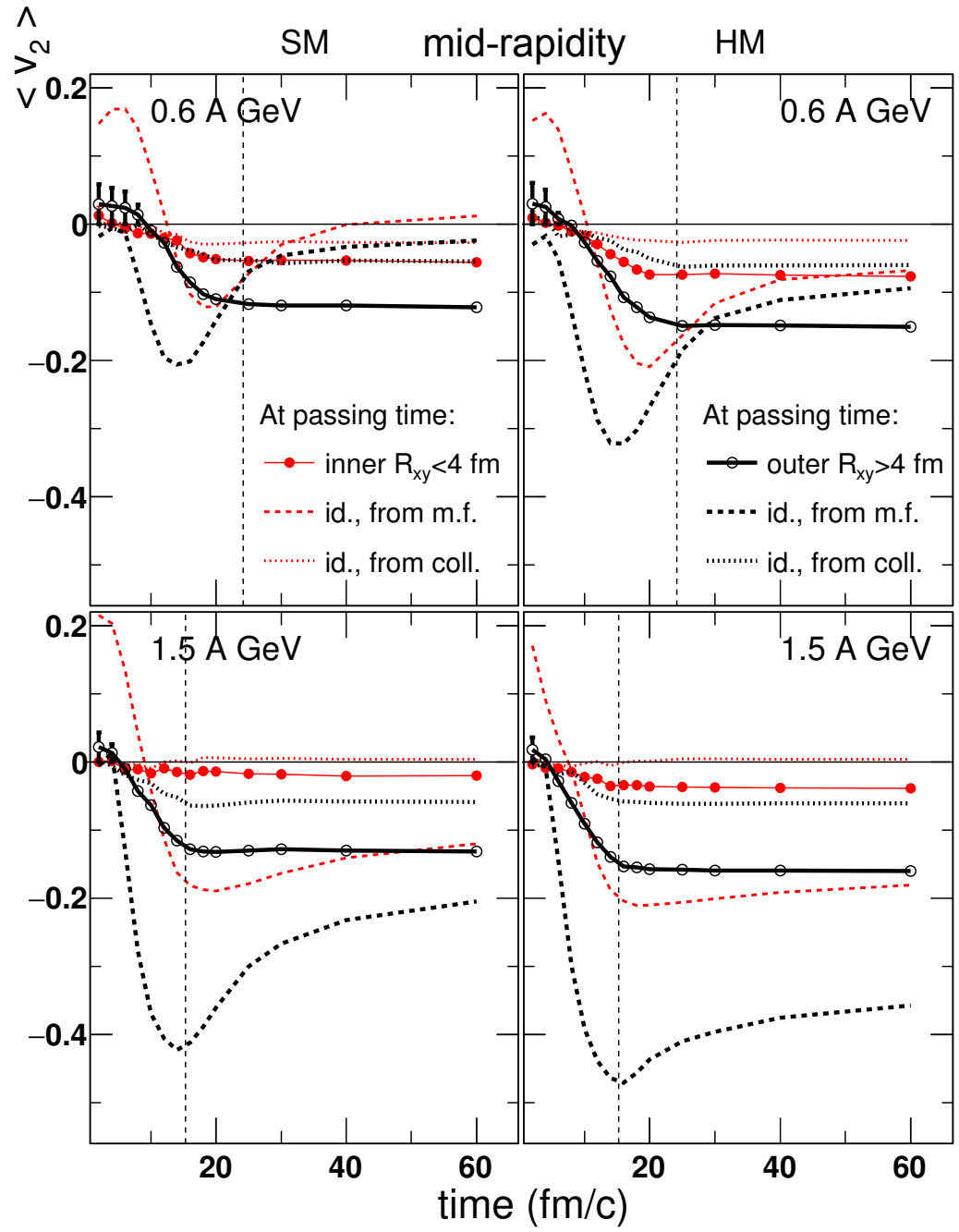
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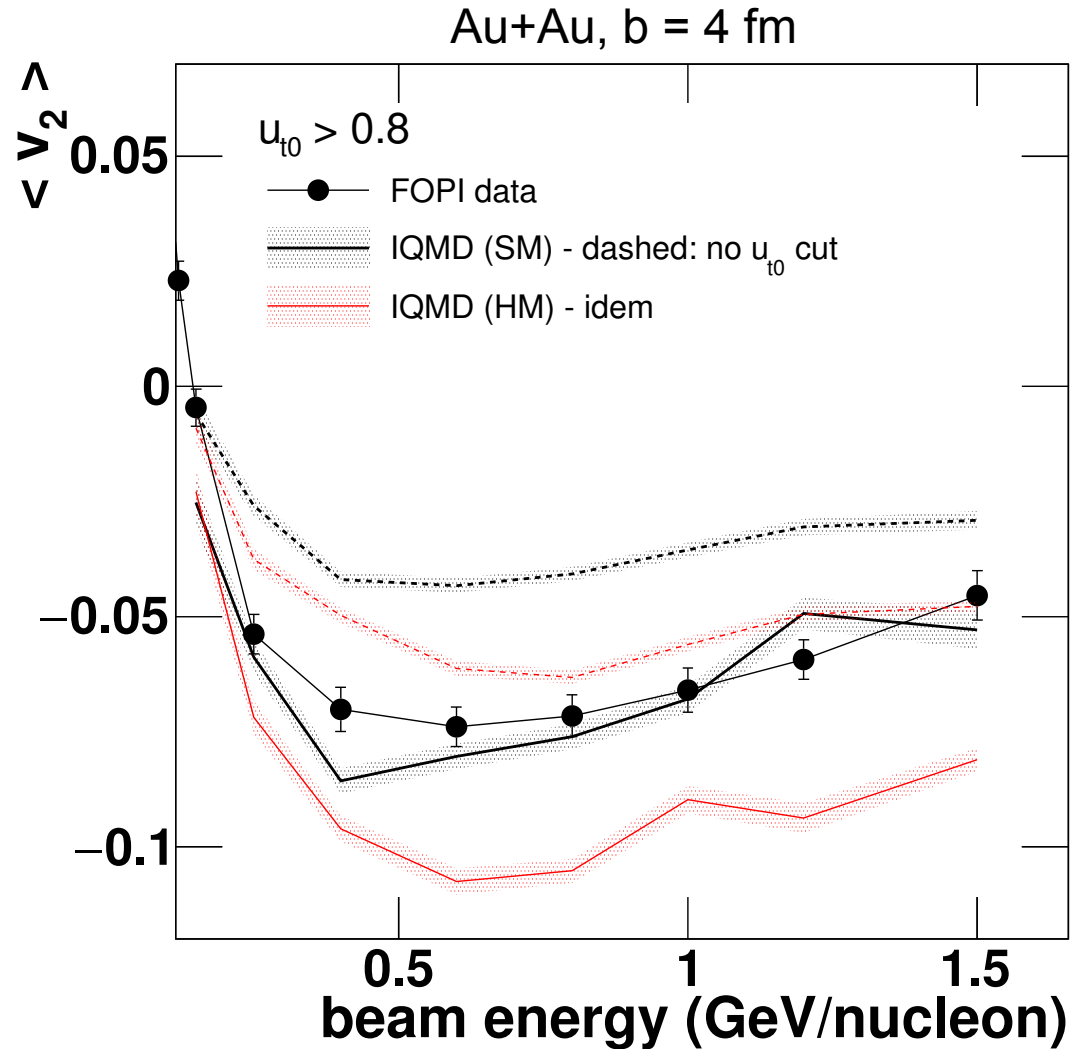
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* Asymptotically, the mean field = the main origin of the overall out-of-plane v_2 , apart from reactions at energies below 1 AGeV where the collisions contribute equally when the nuclear matter EoS is soft, i.e. the number of collisions is large.





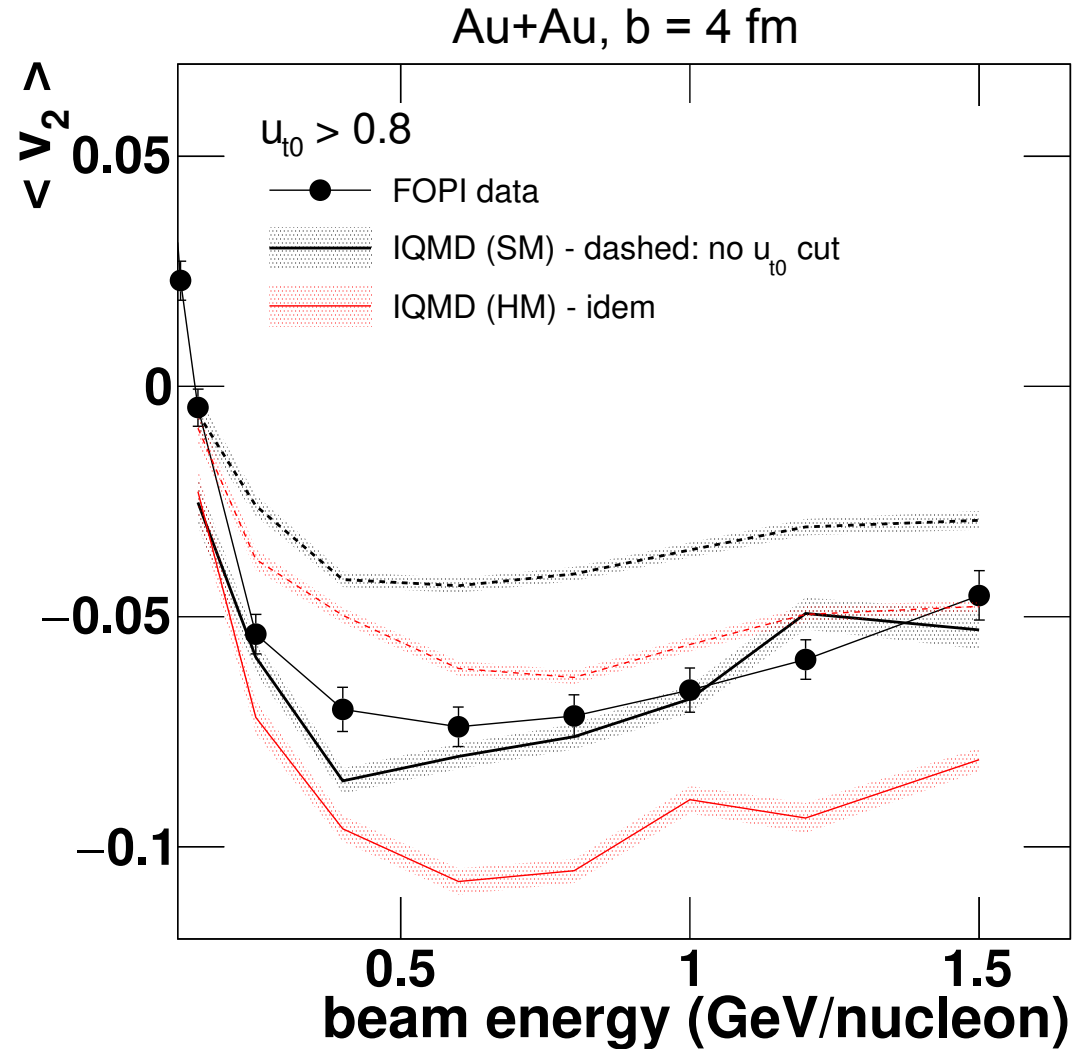
The elliptic flow: incident energy dependance





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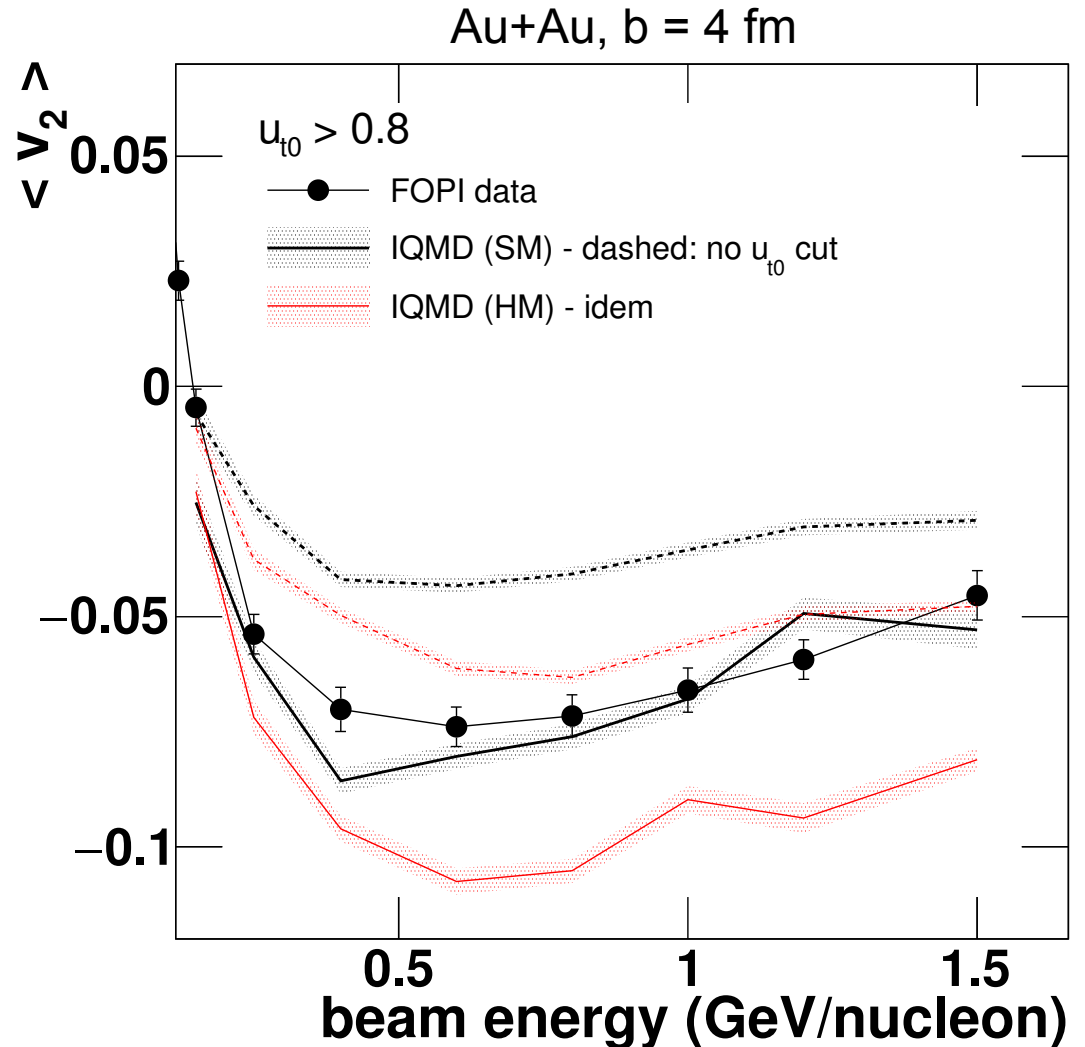
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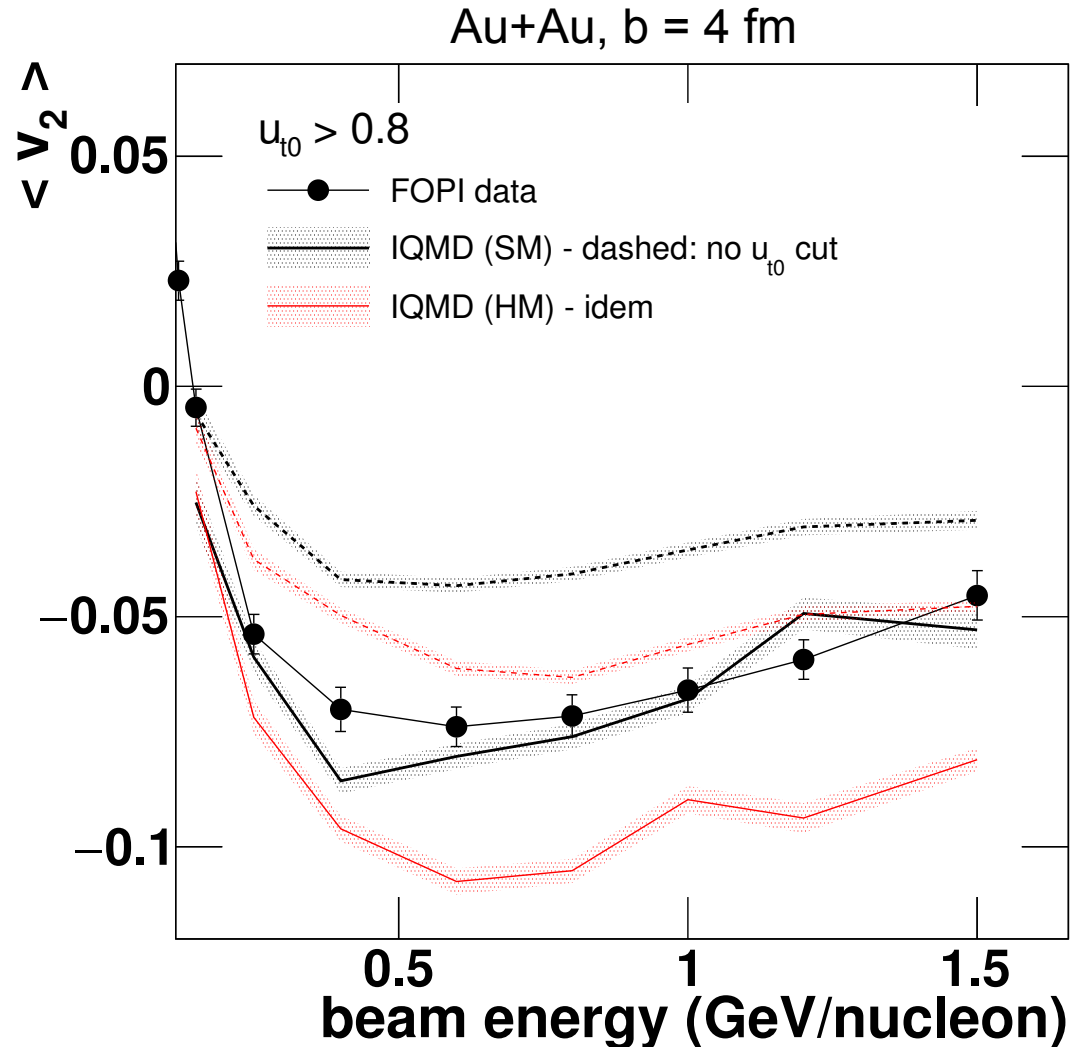
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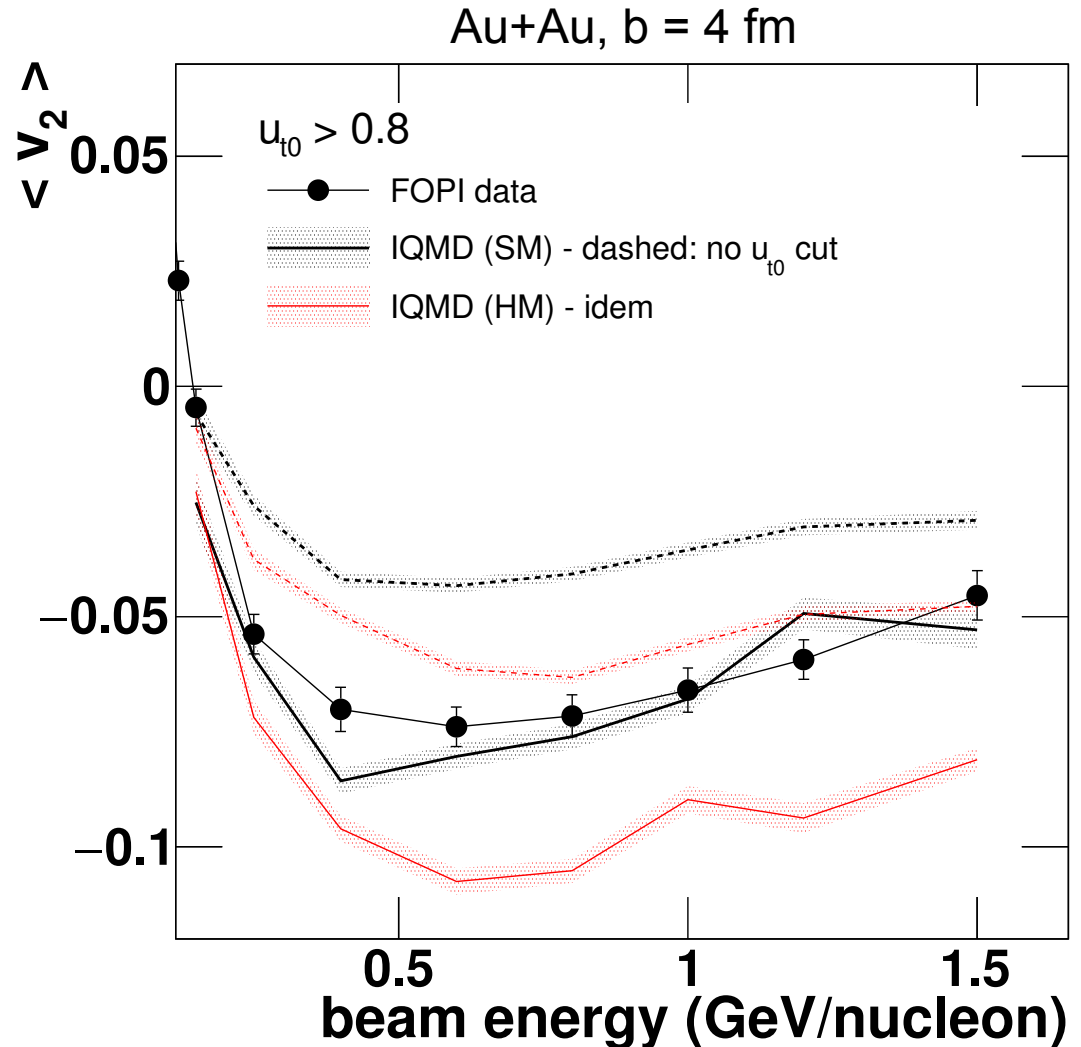
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The elliptic flow: incident energy dependance

- Strong beam energy dependence for $E_{inc} > 0.4$ AGeV
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- Strength enhanced with protons with a large transverse velocity.
- Comparison with FOPI observations (protons with $u_{t0} > 0.8$, same impact parameter) \Rightarrow good agreement (amplitude and evolution) using the soft (SM) EoS.





Summary



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- ❖ The elliptic flow observed in the reactions around $E_{\text{kin}} \approx 1 \text{ AGeV}$ for protons at mid-rapidity ($|y_0| < 0.2$) has two origins:



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- ❖ This effect is amplified if one selects particles with a high transverse velocity.
- ❖ The calculations with a soft EoS (SM) are in better agreement with the experimental data than that with a hard equation of state (HM).



Thank you for your attention!

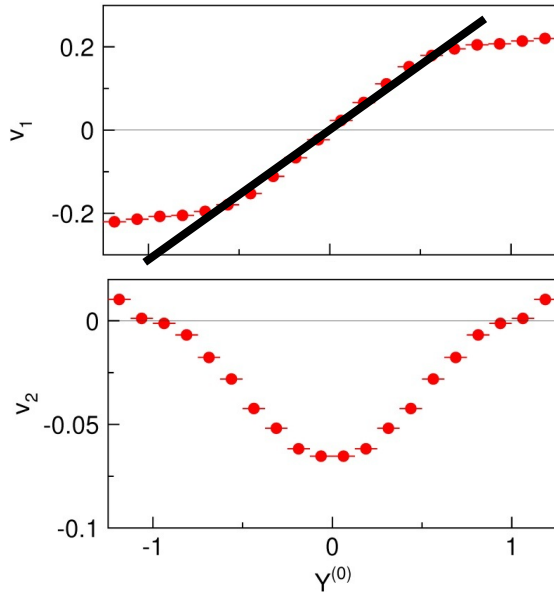


Introduction

- ▶ **Alternative method:** in earth laboratories, heavy ion collisions over a wide range of incident energies, system sizes and compositions.
 - ▶ limited to $E_{\text{beam}} < 10 \text{ A.GeV}$ ← some kind of a clock is available (sound velocity versus participant-spectator interaction).
 - ▶ KaoS (1990's), C+C, Au+Au, K^+ yields → 'soft' EOS. **But:**
 - ▶ kaons rare at $E_{\text{beam}} = 0.8 \text{ A.GeV}$ (max. sensitivity to the EOS).
 - ▶ all 'bulk' observables (multiplicities, clusterisation, stopping, flow) under control in the transport model ?
 - ▶ EoS (1996), Au+Au @ 0.25 to 1.15 A.GeV, radial & sideward flow, squeeze-out versus QMD → no strong sensitivity on the nuclear incompressibility K_0 .
 - ▶ FOPI (2005), Au+Au @ 0.09-1.5 A.GeV, $Z=1$ elliptic flow, versus 4 different transport codes → 'no strong constraint on the EOS can be derived at this stage'.
 - ▶ BEVALAC & AGS accelerators, proton flows versus transport theories → $K_0 = 167\text{-}200 \text{ MeV}$ (soft) from V_1 , $K_0 = 300 \text{ MeV}$ (semi-stiff) from V_2 → contradictions.



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The elliptic flow

