Puzzles in 3D Chern–Simons–matter theories

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SUSY WILSON LOOPS

Why BPS Wilson Loops?

BPS Wilson Loops in supersymmetric gauge theories: gauge invariant non–local operators that preserve some supercharges

- They are in general non–protected operators and their expectation value can be computed exactly by using **localization techniques**.

- **Dual description** in terms of fundamental strings or M2–branes. The expectation value at strong coupling is given by the exponential of a minimal area surface ending on the WL contour. Matching with localization results provides a crucial test of the AdS/CFT correspondence.

- They are related to physical quantities like **Bremsstrahlung function** and **Cusp anomalous dimension**. Therefore, they are ultimately related to

$$\Downarrow$$

INTEGRABILITY IN AdS/CFT

Why BPS WL in 3D SCSM theories?

We will focus on

- $\mathcal{N} = 6$ ABJ(M) — Aharony, Bergman, Jafferis, Maldacena, 0806.1218; Aharony, Bergman, Jafferis, 0807.4924
- $\mathcal{N} = 4$ orbifold ABJM and more general SCSM with $\Pi_{l=1}^{r} U(N_{2l-1}) \times U(N_{2l})$ and alternating levels — Gaiotto, Witten, 0804.2907; Hosomichi, Lee, Lee, Lee, Park, 0805.3662

BPS WL in 3D SCSM theories exhibit a rich spectrum of interesting properties. Among them:

- Topological phases (**framing factors**) generally appear as overall complex phases in $\langle WL \rangle$.
- Due to dimensional reasons **scalar** and **fermions** can enter the definition of BPS WL. In general they increase the number of susy charges preserved by WL.

Prototype examples of WLs in ABJ(M)

$\mathcal{N}=6$ **susy ABJ(M) model** for $U(N_1)_k \times U(N_2)_{-k}$ CS-gauge vectors A_μ, $\hat{A}_\mu$ minimally coupled to

$$SU(4) \text{ complex scalars } C_I, \bar{C}^I \quad \text{and} \quad \text{fermions } \psi_I, \bar{\psi}^I$$

in the (anti)bifundamental representation of the gauge group with non-trivial potential. Dual to $\mathrm{AdS}_4 \times S^7/Z_k$

Bosonic BPS WL

$$W_{1/6} = \mathrm{Tr} P \exp\left[-i\int_\Gamma d\tau (A_\mu \dot{x}^\mu - \frac{2\pi i}{k}|\dot{x}| M_J^{\ I} C_I \bar{C}^J)\right]$$

Fermionic BPS WL $\quad W_{1/2} = \mathrm{Tr} P \exp\left[-i\int_\Gamma d\tau \mathcal{L}(\tau)\right]$

$$\mathcal{L}(\tau) = \begin{pmatrix} A_\mu \dot{x}^\mu - \frac{2\pi i}{k}|\dot{x}| M_J^{\ I} C_I \bar{C}^J & -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\eta_I \bar{\psi}^I \\ -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\psi_I \bar{\eta}^I & \hat{A}_\mu \dot{x}^\mu - \frac{2\pi i}{k}|\dot{x}| \hat{M}_J^{\ I} \bar{C}^J C_I \end{pmatrix}$$

How to compute $\langle WL \rangle$ in SCSM theories

$$\langle WL \rangle \sim \int D[A, \hat{A}, C, \bar{C}, \psi, \bar{\psi}]\, e^{-S}\, \mathrm{Tr} P \exp\left[-i \int_\Gamma d\tau \mathcal{L}(\tau)\right]$$

- Weak coupling $N_1/k, N_2/k \ll 1$ Perturbative evaluation
- Strong coupling $N_1/k, N_2/k \gg 1$ Holographic evaluation
- $N_1/k, N_2/k \sim 1$ Localization techniques $\langle WL \rangle \to$ Matrix Model

For ABJ(M) $\to$ non–gaussian MM Kapustin, Willett, Yaakov, JHEP 1003

$$\langle W_{1/6} \rangle = \int \prod_{a=1}^{N_1} d\lambda_a\, e^{i\pi k \lambda_a^2} \prod_{b=1}^{N_2} d\hat{\lambda}_b\, e^{-i\pi k \widehat{\lambda}_b^2} \times \left(\frac{1}{N_1} \sum_{a=1}^{N_1} e^{2\pi\lambda_a} \right) \times \frac{\prod_{a<b}^{N_1} \sinh^2(\pi(\lambda_a - \lambda_b)) \prod_{a<b}^{N_2} \sinh^2(\pi(\hat{\lambda}_a - \hat{\lambda}_b))}{\prod_{a=1}^{N_1} \prod_{b=1}^{N_2} \cosh^2(\pi(\lambda_a - \hat{\lambda}_b))}$$

Drukker, Marino, Putrov, CMP306 (2011) 511

Plan of the talk

Puzzles typically arise in 3D SCSM theories when we try to match perturbative results with localization predictions

- Solved and unsolved puzzles
  - Framing factor in ABJ(M) ✓
  - Degeneracy of WLs in $\mathcal{N}=4$ orbifold ABJM ✓
  - Comparison with localization result for $\mathcal{N}=4$ SCSM theories
    Alert!

- Conclusions and Perspectives

M.S. Bianchi, L. Griguolo, M. Leoni, A. Mauri, D. Seminara, J-j. Zhang
PLB753, JHEP 1606, JHEP 1609, arXiv:1705.02322 + in progress

Puzzle 1: Framing factor in ABJ(M)

For the $U(N)_k$ pure Chern–Simons theory (topological theory)

$$S_{CS} = -i\frac{k}{4\pi}\int d^3x\, \varepsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho + \frac{2}{3} i A_\mu A_\nu A_\rho\right)$$

On a closed path Γ and in fundamental representation

$$\begin{aligned}\langle \mathcal{W}_{\rm CS}\rangle &= \langle \mathrm{Tr}\, P\, e^{-i\int_\Gamma dx^\mu A_\mu(x)}\rangle \\ &= \sum_{n=0}^{+\infty} \mathrm{Tr}P \int dx_1^{\mu_1}\cdots dx_n^{\mu_n}\,\langle A_{\mu_1}(x_1)\cdots A_{\mu_n}(x_n)\rangle\end{aligned}$$

1. either by using semiclassical methods in the large k limit

   Witten, CMP121 (1989) 351

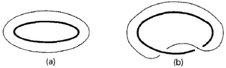
2. or perturbatively (n-pt correlation functions)

   Guadagnini, Martellini, Mintchev, NPB330 (1990) 575

Define $\langle A_{\mu_1}(x_1)\cdots A_{\mu_n}(x_n)\rangle$ at coincident points.

Using point–splitting regularization

$$\Gamma_f : \quad y^\mu(\tau) \to y^\mu(\tau) + \epsilon\, n^\mu(\tau)$$



$$\lim_{\epsilon\to 0} \oint_\Gamma dx^\mu \oint_{\Gamma_f} dy^\nu \,\langle A_\mu(x) A_\nu(y)\rangle = -i\pi\lambda\,\chi(\Gamma,\Gamma_f) \qquad \lambda = \frac{N}{k}$$

Gauss linking number

$$\chi(\Gamma,\Gamma_f) = \frac{1}{4\pi}\oint_\Gamma dx^\mu \oint_{\Gamma_f} dy^\nu \;\varepsilon_{\mu\nu\rho}\frac{(x-y)^\rho}{|x-y|^3}$$

Higher-order contributions exponentiate the one–loop result

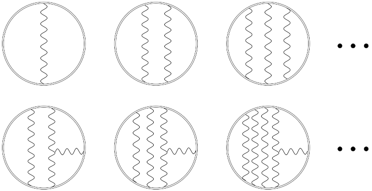
$$\langle \mathcal{W}_{\rm CS}\rangle = \underbrace{e^{-i\pi\lambda\chi(\Gamma,\Gamma_f)}}_{\text{framing factor}}\,\rho(\Gamma)$$

Exponentiation of one–loop framing term relies on the following distinguishing properties Alvarez, Labastida, NPB395 (1993) 198

1. The gauge propagator is one–loop exact. In Landau gauge

$$\langle A^a_\mu(x) A^b_\nu(y) \rangle = \delta^{ab} \frac{i}{2k} \varepsilon_{\mu\nu\rho} \frac{(x-y)^\rho}{|x-y|^3}$$

2. Only diagrams with collapsible propagators contribute to framing



3. Factorization theorem

$\mathcal{N} = 2$ susy CS theory

We are primarily interested in supersymmetric theories for which localization can be used.

$$\langle W_{\rm SCS} \rangle = \langle {\rm Tr}\, P\, e^{-i \int_\Gamma d\tau (\dot{x}^\mu A_\mu(x) - i|\dot{x}|\sigma)} \rangle$$

Localization always provides the result at framing $\chi(\Gamma, \Gamma_f) = -1$. This follows from requiring consistency between point–splitting regularization and supersymmetry used to localize: The only point-splitting compatible with susy is the one where the contour and its frame wrap two different Hopf fibers of S^3

Kapustin, Willett, Yaakov, JHEP 1003 (2010) 089

Localization is sensible to framing!

Framing identified as imaginary contributions

$$\langle W_{\rm SCS} \rangle = e^{i\pi\lambda} \rho(\Gamma)$$

Adding matter → ABJ(M) case

1/6-BPS Wilson loop Drukker, Plefka, Young, JHEP 0811 (2008) 019

Chen, Wu, NPB 825 (2010) 38, Rey, Suyama, Yamaguchi, JHEP 0903 (2009)

$$\langle W_{1/6} \rangle = \langle \mathrm{Tr} P \exp \left[-i \int_\Gamma d\tau (A_\mu \dot{x}^\mu - \frac{2\pi i}{k} |\dot{x}| M_J^{\ I} C_I \bar{C}^J) \right] \rangle$$

$$M_I^{\ J} = \mathrm{diag}(+1, +1, -1, -1)$$

- **Localization result**. $\langle WL \rangle$ → non-gaussian MM computed exactly

  Drukker, Marino, Putrov, (2011); Klemm, Marino, Schiereck, Soroush, (2013)

$$\lambda_1 = N_1/k, \ \lambda_2 = N_2/k \ \ll 1$$

$$\langle W_{1/6} \rangle = \underbrace{e^{i\pi\lambda_1}} \left(1 - \frac{\pi^2}{6}(\lambda_1^2 - 6\lambda_1\lambda_2) \underbrace{-i\frac{\pi^3}{2}\lambda_1\lambda_2^2} + \mathcal{O}(\lambda^4) \right)$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

pure CS framing (-1) factor   extra imaginary term   ???

- **Perturbation theory** (framing $= 0$) $\rightarrow$ no contributions at odd orders

Rey, Suyama, Yamaguchi, JHEP 0903 (2009)

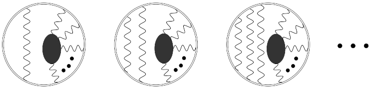
Conjecture: Matter contributes to framing

PROOF: **perturbative 3–loop calculation at framing (-1)**

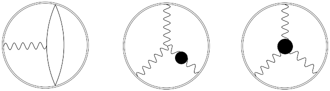
Matter contributes to framing in two different ways:

1. Matter gives non-trivial corrections to the the gauge propagator (FINITE at two loops). Collapsible propagators

$$\langle A_\mu(x) A_\nu(y) \rangle \to \frac{i}{2k} \left[1 - \frac{\pi^2}{2} \left(\underbrace{\lambda_2^2} + \underbrace{\lambda_1 \lambda_2 \left(\frac{1}{4} + \frac{2}{\pi^2} \right)} \right) \right] \varepsilon_{\mu\nu\rho} \frac{(x-y)^\rho}{|x-y|^3}$$



2. Matter vertex-like diagrams cancel lower-transcendentality terms



Exponentiation still works, so we can write

$$\langle W_{1/6}\rangle_1 = \underbrace{e^{i\pi\left(\lambda_1 - \frac{\pi^2}{2}\lambda_1\lambda_2^2 + \mathcal{O}(\lambda^5)\right)}}\left(1 - \frac{\pi^2}{6}(\lambda_1^2 - 6\lambda_1\lambda_2) + \mathcal{O}(\lambda^4)\right)$$

$$\Downarrow$$

perturbative framing function $\qquad f(\lambda_1,\lambda_2) = \lambda_1 - \frac{\pi^2}{2}\lambda_1\lambda_2^2 + \mathcal{O}(\lambda^5)$

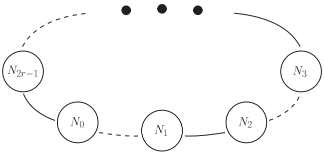
$$\langle W_{1/6}\rangle_0 = \left|\langle W_{1/6}\rangle_1\right|$$

Puzzle solved ✓

Puzzle 2: WL degeneracy in $\mathcal{N}=4$ SCSM theories

Gaiotto, Witten, JHEP 06 (2010) 097; Hosomichi, Lee[3], Park, JHEP 07 (2008) 091

$\Pi_{l=1}^{r} U(N_{2l-1}) \times U(N_{2l})$ quiver gauge theories with alternating $\pm k$ levels



Matter in (anti)bifundamental representation of adjacent gauge groups and in $(2,1)$ and $(1,2)$ of $SU(2) \times \hat{SU}(2)$ R–symmetry $\qquad \phi^I \qquad \phi^{\hat{I}}$

Dual to M–theory on $\mathrm{AdS}_4 \times S^7/(Z_r \otimes Z_r)/Z_k$

Orbifold ABJM: $\quad N_0 = N_1 = \cdots = N_{2r-1}$

Dual to M–theory on $\mathrm{AdS}_4 \times S^7/(Z_r \otimes Z_{rk})$

BPS WL defined locally for quiver nodes $(2l-1, 2l) \to W^{(l)}$

or globally $W = \sum_{l=1}^{r} W^{(l)}$

Higgsing procedure allows to construct two classes of 1/2 BPS WLs

Exiting heavy **particle** dof $\to$ class $\mathcal{C}$

Exiting heavy **anti–particle** dof $\to$ class $\hat{\mathcal{C}}$

For ABJ(M) models, representatives of different classes preserve different sets of supercharges only partially overlapping.

In $\mathcal{N} = 4$ SCSM theories, for each W_{ψ_1} representative in $\mathcal{C}$ we can find a representative W_{ψ_2} in $\hat{\mathcal{C}}$ that preserves the **same set of supercharges**.

Crooke, Drukker, Trancanelli, JHEP 10 (2015); Lietti, Mauri, Zhang, SP, 1705.03322

Puzzle ??

We have proved that (embedding S^7 in $\mathrm{R}^8 \cong \mathrm{C}^4 \to z_{1,2,3,4}$)

$W_{\psi_1} \to$ **M2–brane** wrapped on $|z_1| = 1$ and localized at $z_{2,3,4} = 0$

$W_{\psi_2} \to$ **M2–antibrane** wrapped on $|z_2| = 1$ and at $z_{1,3,4} = 0$

The two brane configurations preserve the **same set of supercharges**.

Puzzle solved ✓

$$\psi_i = \frac{1}{N_1 + N_2} \operatorname{Tr} P \exp\left(-i \int_\Gamma d\tau \mathcal{L}_{\psi_i}(\tau) \right)$$

where

$$\mathcal{L}_{\psi_1} = \begin{pmatrix} \mathcal{A}_{(1)} & \bar{c}_\alpha \psi^\alpha_{(1)\hat{1}} \\ c^\alpha \bar{\psi}^{\hat{1}}_{(1)\alpha} & \mathcal{A}_{(2)} \end{pmatrix}$$

$$\mathcal{A}_{(1)} = \dot{x}^\mu A_{(1)\mu} - \frac{i}{k} \left(q^I_{(1)} \delta_I{}^J \bar{q}_{(1)J} + \bar{q}_{(0)\hat{I}} (\sigma_3)^{\hat{I}}{}_{\hat{J}}\, q^{\hat{J}}_{(0)} \right) |\dot{x}|$$

$$\mathcal{A}_{(2)} = \dot{x}^\mu A_{(2)\mu} - \frac{i}{k} \left(\bar{q}_{(1)I} \delta^I{}_J\, q^J_{(1)} + q^{\hat{I}}_{(2)} (\sigma_3)_{\hat{I}}{}^{\hat{J}} \bar{q}_{(2)\,\hat{J}} \right) |\dot{x}|$$

$$\mathcal{L}_{\psi_2} = \begin{pmatrix} \mathcal{B}_{(1)} & \bar{d}_\alpha \psi^\alpha_{(1)\hat{2}} \\ d^\alpha \bar{\psi}^{\hat{2}}_{(1)\alpha} & \mathcal{B}_{(2)} \end{pmatrix}$$

$$\mathcal{B}_{(1)} = \dot{x}^\mu A_{(1)\mu} - \frac{i}{k} \left(-q^I_{(1)} \delta_I{}^J \bar{q}_{(1)J} + \bar{q}_{(0)\hat{I}} (\sigma_3)^{\hat{I}}{}_{\hat{J}}\, q^{\hat{J}}_{(0)} \right) |\dot{x}|$$

$$\mathcal{B}_{(2)} = \dot{x}^\mu A_{(2)\mu} - \frac{i}{k} \left(-\bar{q}_{(1)I} \delta^I{}_J\, q^J_{(1)} + q^{\hat{I}}_{(2)} (\sigma_3)_{\hat{I}}{}^{\hat{J}} \bar{q}_{(2)\,\hat{J}} \right) |\dot{x}|$$

Puzzle 3: Localization vs. perturbation

At quantum level?

Cohomological equivalence

$$W_{\psi_1} = W_{1/4} + QV_1 \qquad W_{\psi_2} = W_{1/4} + QV_2$$

- Localization (framing–one) $\langle W_{\psi_1} \rangle_1 = \langle W_{\psi_2} \rangle_1 = \langle W_{1/4} \rangle_1$

  Ouyang, Wu, Zhang, Chin.Phys. C40 (2016)

  We expect $\langle W_{\psi_1} \rangle_0 = \langle W_{\psi_2} \rangle_0 = |\langle W_{1/4} \rangle_1|$ (Proved at 3 loops)

- Perturbation theory (framing–zero): For planar contour

$$\langle W_{\psi_1} \rangle_0^{(L)} = (-1)^L \langle W_{\psi_2} \rangle_0^{(L)}$$

Bianchi, Griguolo, Leoni, Mauri, SP, Seminara, JHEP 1609 (2016) 009

Consistency requires

$$\langle W_{\psi_1} \rangle_0^{(odd)} = \langle W_{\psi_2} \rangle_0^{(odd)} = 0$$

Is it true?

• From localization $|\langle W_{1/4}\rangle_1|$ vanishes at odd orders (checked up to three loops).

• From a perturbative calculation: One loop result vanishes. We need a **3–loop** calculation

- Orbifold ABJM: Too many diagrams to compute. Still open question
- $\mathcal{N}=4$ SCSM theories: The number of diagrams can be drastically reduced by restricting to the range–3 color sectors $N_{l-1}N_lN_{l+1}$

  We have found $(l=1)$

  Bianchi, Griguolo, Leoni, Mauri, SP, Seminara, JHEP1609 (2016)

$$\langle W_{\psi_1}\rangle^{(3L)} = -\langle W_{\psi_2}\rangle^{(3L)} = \frac{5}{8\pi}\,\frac{N_0N_1^2N_2 + N_1N_2^2N_3}{(N_1+N_2)}\,\frac{1}{k^3}$$

Alerting puzzle!

Possible explanation?

- Neither $\langle W_{\psi_1} \rangle^{(3L)}$ nor $\langle W_{\psi_2} \rangle^{(3L)}$ match the localization result.
- It is a matter of fact that $\langle \frac{W_{\psi_1}+W_{\psi_2}}{2} \rangle^{(odd)} = 0$ and matches the localization result.
- It is hard to believe that two non–BPS operators give rise to a BPS operator when linearly combined.
- If the dual description works as in the orbifold case, it points towards the fact that both W_{ψ_1} and W_{ψ_2} should be BPS at quantum level. But we don't know ...

Only possibility: W_{ψ_1} and W_{ψ_2} are BPS, but the cohomological equivalence is broken by quantum effects

$$\langle W_{\psi_1} \rangle = \langle W_{1/4} \rangle + \mathcal{A} \qquad \langle W_{\psi_2} \rangle = \langle W_{1/4} \rangle - \mathcal{A}$$

such that $\frac{1}{2}(W_{\psi_1} + W_{\psi_2})$ is BPS and Q–equivalent to $W_{1/4}$.

A direct check requires computing $\langle W_{\psi_1} \rangle_1$ and $\langle W_{\psi_2} \rangle_1$ at framing one in perturbation theory.

Puzzle unsolved X

Conclusions

- We have understood the framing mechanism in CS theories with matter. This is important for its relation with the Bremsstrahlung function

$$B_{1/2} = \frac{1}{8\pi} \tan \Phi_{1/6}$$

Bianchi, Griguolo, Leoni, Mauri, SP, Seminara, JHEP 1406 (2014)

But

- Better understand contributions from vertex-like diagrams.
- Framing from matter in fermionic WLs: understand framing from fermionic diagrams
- What happens at higher orders? Divergences?
- Framing in new classes of less supersymmetric fermionic WLs in ABJ(M) and $\mathcal{N} = 4$ SCSM theories

- Cohomological equivalence in $\mathcal{N} = 4$ SCSM theories is still an open problem

- Framing at strong coupling?

KEN !!!