Topological Solitons, Nonperturbative Gauge Dynamics and Confinement

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Vortex line formation in He II

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written in NORDITA 35 years ago

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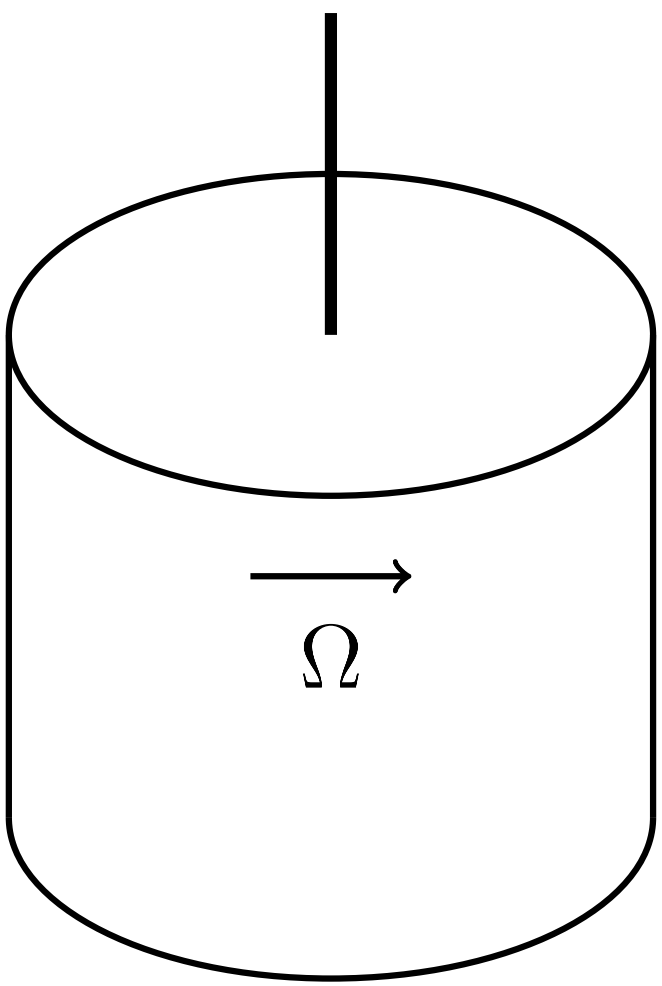
For $T_0 \simeq 2.18$ K, the behaviour of helium He^4 is similar to the behaviour of a two-components liquid in which

- one component, which has velocity $\boldsymbol{v}_s$ and mass density ρ_s, corresponds to the so-called superfluid motion; this fluid component has no viscosity and carries zero entropy;
- the second component, with velocity $\boldsymbol{v}_n$ and mass density ρ_n, corresponds to the normal motion and behaves as a normal viscous fluid.

This quantum liquid can be described (Landau) by means of:

- a gas of quasi-particles (the localized energy fluctuations of the system above its ground state)
- additional degrees of freedom which are related with the global (zero entropy) motion of the ground state wave function = global motion of the condensate

Mass density : $\rho = \rho_s + \rho_n$
Momentum density : $\boldsymbol{P}/V = \rho_s \boldsymbol{v}_s + \rho_n \boldsymbol{v}_n$



Rotating container

For small Ω, the viscous component is rotating, whereas the condensate is at rest

$$\boldsymbol{v}_n = \boldsymbol{\Omega} \wedge \boldsymbol{r} \quad , \quad \boldsymbol{v}_s = 0$$

Landau argument: in the rotating system the boundary conditions for the viscous fluid coincide with the static boundary conditions. So the Gibbs factor $= e^{-E'/kT}$, where $E' = \epsilon - \boldsymbol{\Omega}(\boldsymbol{r} \wedge \boldsymbol{p})$.
As Ω increases, the value Ω_0 is reached in which the condensate also starts moving.
For one vortex $|\boldsymbol{v}_s| = v_s(r_\perp) = \frac{\hbar}{m\, r_\perp}$

$\Omega_0 = ??$

- minimize $U'_{vor} = U_{vor} - \boldsymbol{\Omega} \boldsymbol{M}_{vor}$

Then $\overline{\Omega}_0 = \frac{\hbar}{mR^2} \ln\left(\frac{R}{a}\right)$

When $\boldsymbol{v} = \boldsymbol{v}_n - \boldsymbol{v}_s \neq 0$, the energy spectrum of a single quasi-particle (which belongs to this part of the liquid) with momentum $\boldsymbol{p}$ is given by

$$E_v(\boldsymbol{p}) = \varepsilon(p) - \boldsymbol{v}\boldsymbol{p} = \varepsilon(p) - (\boldsymbol{v}_n - \boldsymbol{v}_s)\boldsymbol{p}$$

Density of free energy for the quasi-particles gas

$$[F/V]_{q.p.} = kT \int d\tau \, \ln\left(1 - e^{-(\varepsilon - \boldsymbol{v}\boldsymbol{p})/kT}\right)$$

Condensate contribution $[F/V]_c = [U/V]_s = \frac{1}{2}\rho v_s^2$

Density of free energy for Helium II :

$$F/V = F_0/V + \tfrac{1}{2}\,\rho\, v_s^2 - \tfrac{1}{2}\rho_n(\boldsymbol{v}_n - \boldsymbol{v}_s)^2$$

where

$$F_0/V = kT \int d\tau\, \ln\left(1 - e^{-\varepsilon/kT}\right)$$

and

$$\rho_n = \int d\tau\, (p^2/3)\, \left[-\frac{\partial n(\varepsilon)}{\partial \varepsilon}\right]$$

For fixed Ω (fixed $\boldsymbol{v}_n$) consider the free energy

$$F = \int d^3r \left\{ F_0/V + \tfrac{1}{2}\,\rho\, v_s^2 - \tfrac{1}{2}\rho_n(\boldsymbol{v}_n - \boldsymbol{v}_s)^2 \right\}$$

one has $F = \widetilde{F} + F_I + F_{II}$ where

$$\widetilde{F} = F_0 - \tfrac{1}{2} \int d^3r\, \rho_n |\boldsymbol{v}_n|^2$$

and

$$F_I = \int d^3r\, \rho_n\, \boldsymbol{v}_n \boldsymbol{v}_s$$

$$F_{II} = \tfrac{1}{2} \int d^3r\, \rho_s\, |\boldsymbol{v}_s|^2$$

When $F_I + F_{II} < 0$ one has a vortex formation.

$$F_I = \pm\,\rho_n \frac{\pi L R^2 \hbar}{m}\,\Omega$$

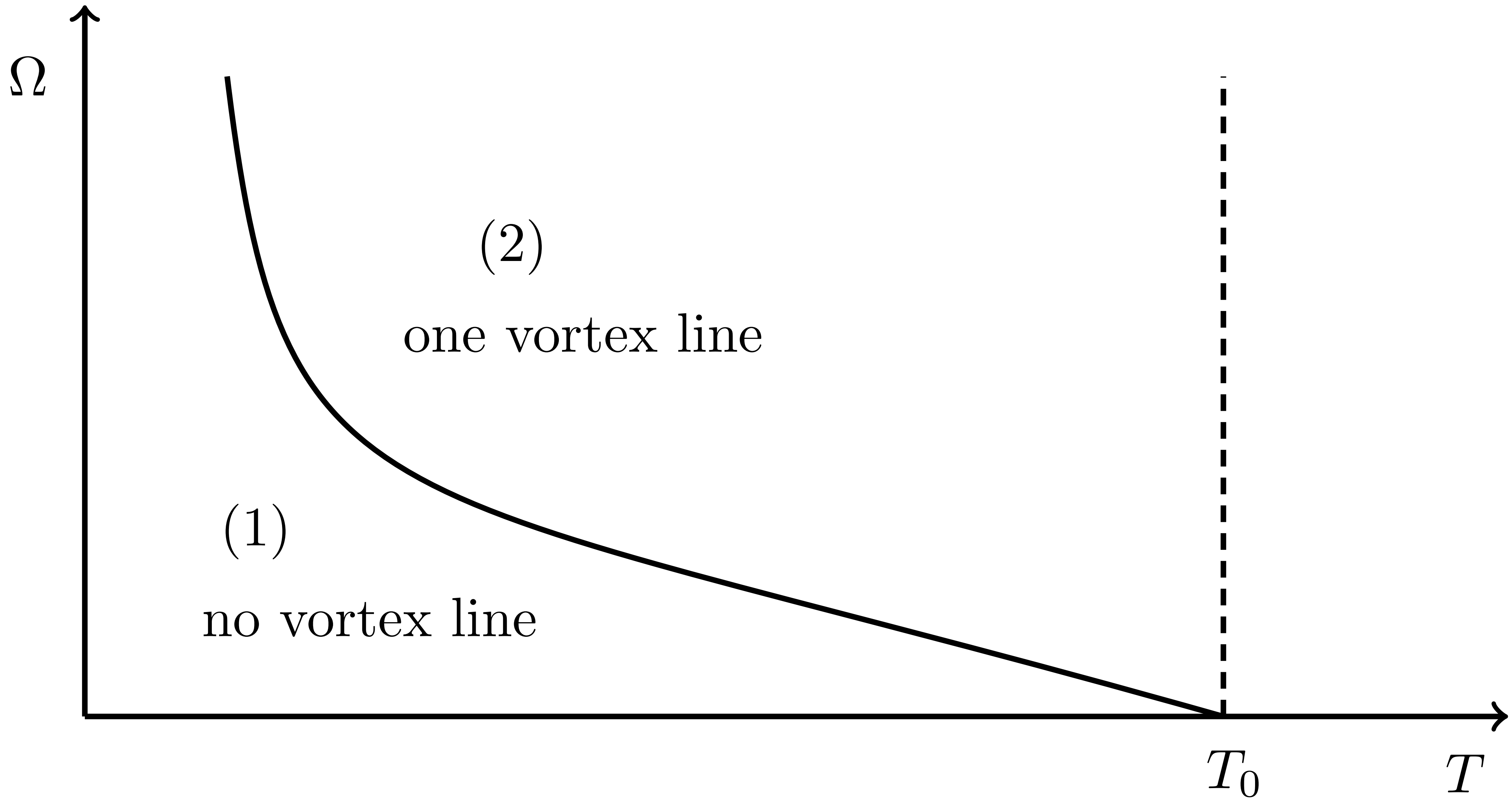
$$F_{II} = \rho_s \frac{\pi L \hbar^2}{m^2}\,\ln\left(\frac{R}{a}\right)$$

One finds

- $\Omega > \Omega_0$, with

$$\Omega_0 = \left(\frac{\rho_s}{\rho_n}\right)\,\frac{\hbar}{mR^2}\,\ln\left(\frac{R}{a}\right)$$

- the condensate starts moving in the opposite direction of the viscous normal component of the fluid (*i.e.* $\boldsymbol{v}_n \boldsymbol{v}_s = -|\boldsymbol{v}_n|\,|\boldsymbol{v}_s| < 0$).



Critical curve $\Omega_0(T)$ in the (Ω, T)-plane.

Along the critical curve, one has $F_{(1)} = F_{(2)}$. Since $dF = -SdT - Jd\Omega$, where J corresponds to the vertical component of the angular momentum of the quasi-particles gas, from

$$-S_{(1)}dT - J_{(1)}d\Omega_0 = -S_{(2)}dT - J_{(2)}d\Omega_0$$

one gets

$$\frac{d\Omega_0}{dT} = -\frac{S_{(2)} - S_{(1)}}{J_{(2)} - J_{(1)}} = -\frac{\lambda}{T(J_{(2)} - J_{(1)})}$$

where $\lambda = T(S_{(2)} - S_{(1)})$ denotes the latent heat for the vortex formation.

The discountinuous change of J, which is due to the formation of a vortex line, is given by

$$\Delta J = J_{(2)} - J_{(1)} = -\widehat{\boldsymbol{z}} \left(\int d^3 r \, \rho_n \, \boldsymbol{r} \wedge \boldsymbol{v}_s \right) = \rho_n \frac{\pi L R^2 \hbar}{m}$$

whereas the total angular momentum of helium II decreases

$$\Delta \left(\boldsymbol{J}_z + \boldsymbol{M}_z \right) = -\rho_s \frac{\pi L R^2 \hbar}{m}$$

$$\lambda = T(S_{(2)} - S_{(1)}) = (2\rho_n^* - \rho_n) \left(\frac{\rho}{\rho_n} \right) \frac{\pi L \hbar^2}{m^2} \, \ln \left(\frac{R}{a} \right)$$

where

$$\rho_n^* = \frac{5}{2} \, \rho_{n,ph} + \rho_{n,r} \left[\frac{\Delta}{2kT} + \frac{1}{4} \right]$$

$$\Omega_0 = \left(\frac{\rho_s}{\rho_n}\right) \frac{\hbar}{mR^2} \ln\left(\frac{R}{a}\right) = D \frac{\hbar}{mR^2} \ln\left(\frac{R}{a}\right)$$

D		2.34×10^4	1.03×10^3	1.31×10^2	33
ρ_n/ρ		4.27×10^{-5}	9.66×10^{-4}	7.52×10^{-3}	2.92×10^{-2}
T (K)		0.6	0.8	1	1.2
D	12	4.88	2.12	0.78	0.35
ρ_n/ρ	7.54×10^{-2}	0.17	0.32	0.56	0.74
T (K)	1.4	1.6	1.8	2.0	2.1