Theorems from RG flows

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+ 3 minutes of memories of old times...

Outline

- FRAMEWORK: *Perturbative* QFT in Euclidean space
  - Massless ϕ^4_4 theory
  - SU(2) (pure) gauge theory

- GOAL: prove renormalizability and other properties of *renormalized* correlators in *momentum space* at *all loops*

  convergence of limits and integrals
  => *bounds on correlators*

- DIFFICULTIES: *IR divergences* (massless particles), explicit *BRST/STI breaking* from momentum cutoffs

- TOOLS: *Renormalization group flow equations* and mathematical *induction.* No Feynman diagrams around.

IR divergences in Euclidean, massless QFT

• A collection of momenta $(p_1, \cdots, p_n)$ is said to be *exceptional* if there exists a vanishing *subsum*

$$\sum_{e \in \mathbb{E}} p_e = 0 \quad \text{for some} \quad \emptyset \neq \mathbb{E} \subseteq \{1, ... n\}$$

• Correlators $\langle \phi(p_0)\phi(p_1)\cdots\phi(p_{\mathrm{N}-1})\rangle$ with mass dimension $d \leq 0$ typically *diverge* if $(p_1, \cdots, p_{\mathrm{N}-1})$ is exceptional.
($p_0 = -\sum_{e=1}^{n} p_e$ *excluded due to momentum conservation*)

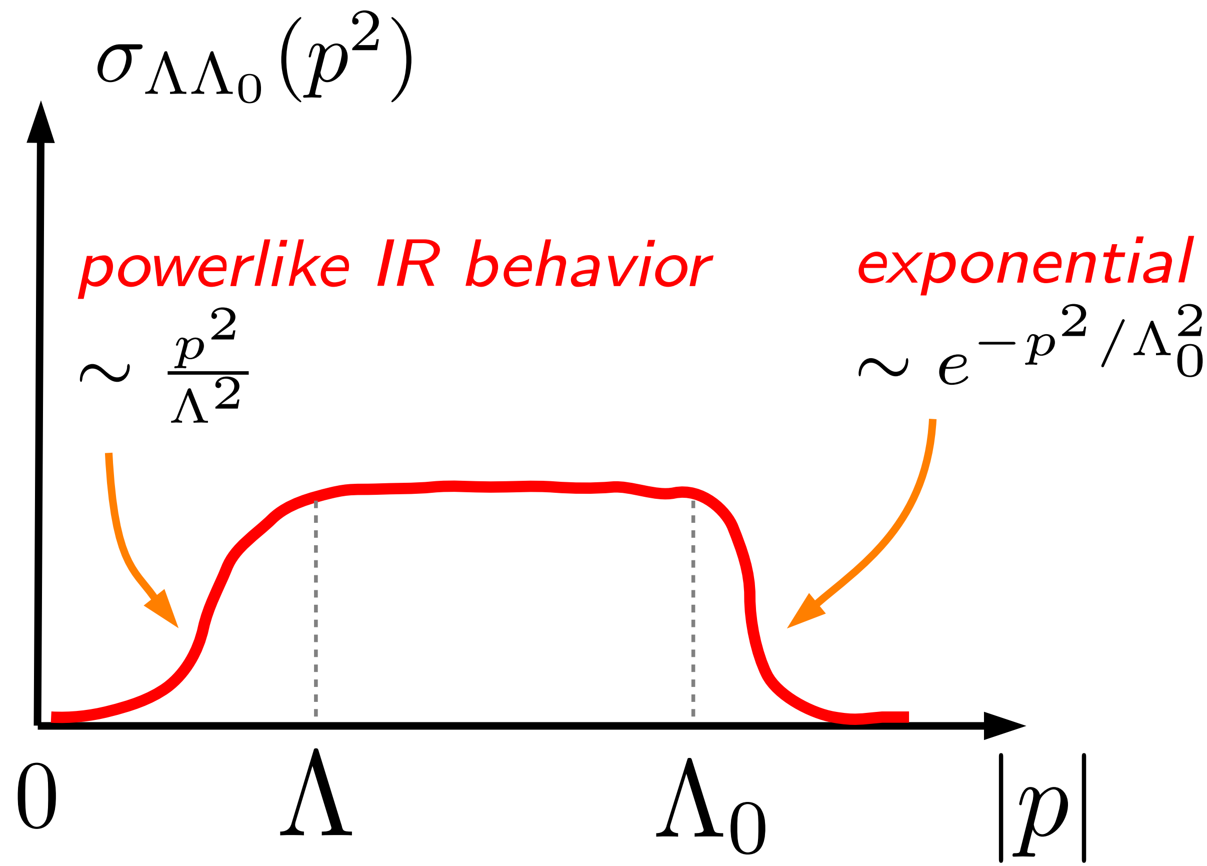
• Such IR divergences are *"physical"* and generally present in all Euclidean QFT with massless particles. (Situation even *worse* in Minkowski.)

• Mathematically: In QFT, correlators are *distributions*, not necessarily globally–defined functions.

• IR singularities *regularized* by IR cutoff $\Lambda > 0$

IR + UV regularized propagator (massless φ_4^4 case)

- Λ_0 is an *UV cutoff*; Λ is an *IR cutoff*: $0 < \Lambda \leq \Lambda_0$;
- $C^{(\Lambda,\Lambda_0)}(p) := \frac{\sigma_{\Lambda\Lambda_0}(p^2)}{p^2}$ with $\sigma_{\Lambda\Lambda_0}(p^2) := e^{-\frac{p^2}{\Lambda_0^2}} - e^{-\frac{p^2}{\Lambda^2}}$



- $\lim_{\Lambda_0\to\infty} \lim_{\Lambda\to 0^+} p^2 C^{(\Lambda,\Lambda_0)}(p) = 1$: recover usual propagator.
- In YM $\sigma_{\Lambda,\Lambda_0}(p^2) := \exp(-p^4/\Lambda_0^4) - \exp(-p^4/\Lambda^4)$

(because C^{Λ,Λ_0} is more IR singular)

Effective action L (massless φ_4^4 case)

$$e^{-\frac{1}{\hbar}L^{\Lambda,\Lambda_0}(\varphi)} := \mathcal{N}\int [\mathrm{d}\phi]\; e^{-\frac{1}{2}\int \phi\,(\hbar C^{\Lambda,\Lambda_0})^{-1}\,\phi}\; e^{-\frac{1}{\hbar}L^{\Lambda_0}(\phi+\varphi)}$$

$\mathrm{d}\mu(\phi)$: *Gaussian measure* with covariance $\hbar C^{\Lambda,\Lambda_0}$

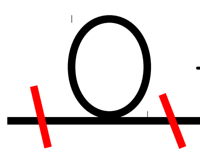
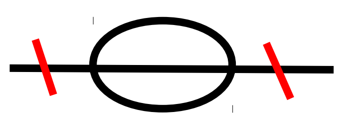
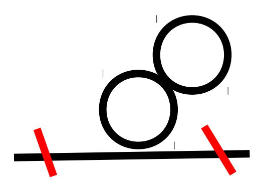
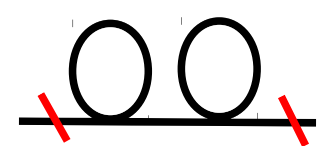
$$L^{\Lambda_0}(\phi) := \int_x \left(\frac{g}{4!}\phi^4 + \frac{\delta Z}{2}(\partial\phi)^2 + \frac{\delta m^2}{2}\phi^2 + \frac{\delta g}{4!}\phi^4 \right)$$

UV action

$\delta Z(\Lambda_0)$, $\delta m^2(\Lambda_0)$, $\delta g(\Lambda_0) = O(\hbar)$ *counterterms* fixed order-by-order by *renormalization conditions*

- $L^{\Lambda,\Lambda_0}(\varphi)$ is the generator of the *Connected Amputated Schwinger (CAS)* functions $\mathcal{L}^{\Lambda,\Lambda_0}_{\mathtt{N},\mathtt{L}}$ = sum of *Connected* & *C^{Λ,Λ_0}-Amputated* graphs $\propto \hbar^{\mathtt{L}}$.

$$L^{\Lambda,\Lambda_0}(\varphi) = \sum_{\mathtt{N}=1}^{\infty}\frac{1}{\mathtt{N}!}\sum_{\mathtt{L}=0}^{\infty}\hbar^{\mathtt{L}}\int_{p_0,\cdots p_{\mathtt{N}-1}} (2\pi)^4\delta\Big(\sum_{e=0}^{\mathtt{N}-1}p_e\Big)\,\mathcal{L}^{\Lambda,\Lambda_0}_{\mathtt{N},\mathtt{L}}(p_1,\cdots,p_{\mathtt{N}-1})\prod_{e=0}^{\mathtt{N}-1}\varphi(-p_e)$$

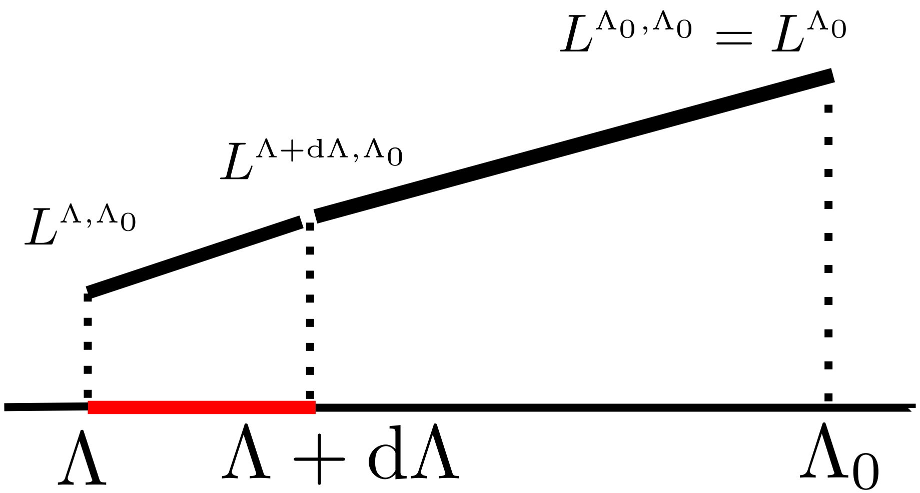
$\mathcal{L}^{\Lambda,\Lambda_0}_2(p) = 0 +$  $+$  $+$  $+$  $+\mathrm{ct} + \ldots$

- *(When it exists)* $\lim_{\Lambda_0\to\infty}\lim_{\Lambda\to 0^+}\mathcal{L}^{\Lambda,\Lambda_0}_{\mathtt{N},\mathtt{L}}$ gives the *renormalized* CAS

Flow equation for L (φ_4^4 massless)

[Wilson]
[Wegner+Hougton]
[Polchinski]

Notation: $\dot{X} := \partial_\Lambda X$, $\langle f, g\rangle := \int_x f(x)g(x)$, $p_{[n]} := (p_1, \ldots, p_n)$



- The *RG flow equation* rules the evolution of the effective action L^{Λ,Λ_0} in terms of Λ, from $\Lambda = \Lambda_0$ to $\Lambda \to 0$

$$\dot{L}^{\Lambda,\Lambda_0} = \frac{\hbar}{2}\left\langle \frac{\delta}{\delta\varphi}, \dot{C}^{\Lambda,\Lambda_0}\frac{\delta}{\delta\varphi}\right\rangle L^{\Lambda,\Lambda_0} - \frac{1}{2}\left\langle \frac{\delta L^{\Lambda,\Lambda_0}}{\delta\varphi}, \dot{C}^{\Lambda,\Lambda_0}\frac{\delta L^{\Lambda,\Lambda_0}}{\delta\varphi}\right\rangle$$

- *Mixed UV+IR boundary conditions* $q_{[3]}, q$ *ren. points,* $|q_{[3]}|, |q| = O(M)$

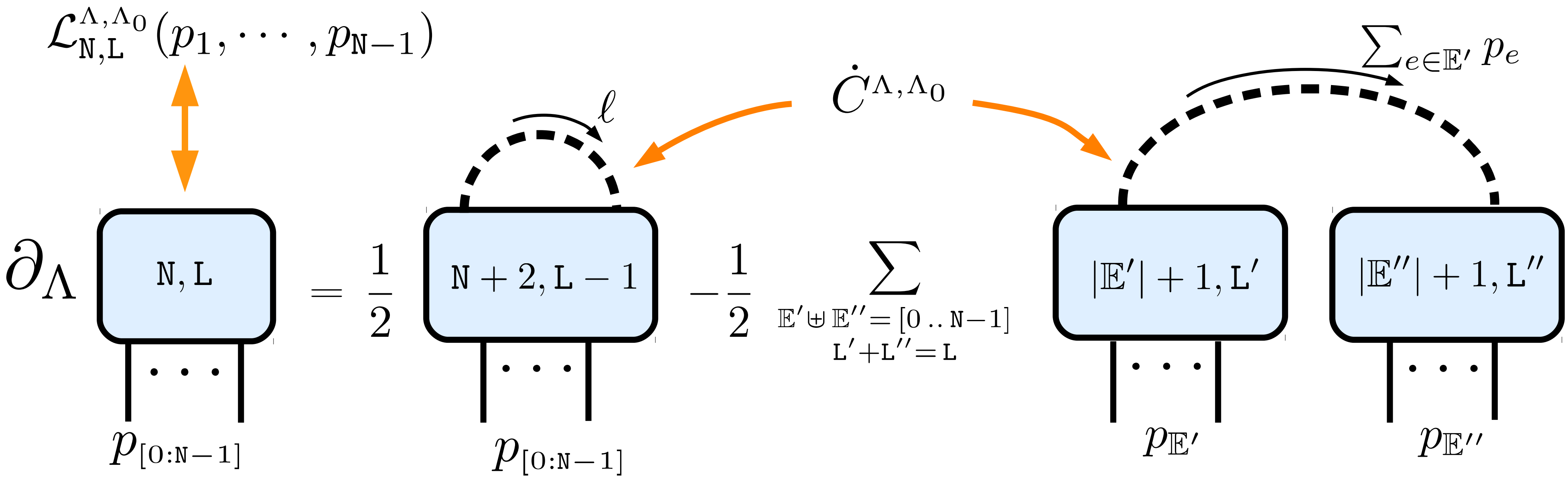
UV: $\Lambda = \Lambda_0 : L^{\Lambda_0,\Lambda_0} = L^{\Lambda_0}$ (UV action) $\Rightarrow \mathcal{L}^{\Lambda_0,\Lambda_0}_{\mathtt{N},\mathtt{L}} = 0 \quad \forall\, \mathtt{N} > 4, \mathtt{L}$

IR: $\Lambda \to 0 : \mathcal{L}^{0,\Lambda_0}_{4,\mathtt{L}}(q_{[3]}) = g_{\mathtt{L}}; \quad \partial_{p^2}\mathcal{L}^{0,\Lambda_0}_{2,\mathtt{L}}(q^2) = z_{\mathtt{L}}; \quad \mathcal{L}^{0,\Lambda_0}_{2,\mathtt{L}}(0) = 0;$

[Wilson]
[Wegner+Hougton]
[Polchinski]

Flow equation for CAS

$$\dot{L}^{\Lambda,\Lambda_0} = \frac{\hbar}{2}\left\langle \frac{\delta}{\delta\varphi}, \dot{C}^{\Lambda,\Lambda_0}\frac{\delta}{\delta\varphi}\right\rangle L^{\Lambda,\Lambda_0} - \frac{1}{2}\left\langle \frac{\delta L^{\Lambda,\Lambda_0}}{\delta\varphi}, \dot{C}^{\Lambda,\Lambda_0}\frac{\delta L^{\Lambda,\Lambda_0}}{\delta\varphi}\right\rangle$$



$(\mathtt{H},\mathtt{L})$ $\quad$ $(\mathtt{H},\mathtt{L}-1) \prec (\mathtt{H},\mathtt{L})$ $\quad$ $(\mathtt{H}',\mathtt{L}') \prec (\mathtt{H},\mathtt{L})$

Infinite hierarchy of equations which can be organized recursively i.e. LHS is expressed in terms of known RHS

1) $\mathtt{N}' := |\mathbb{E}'|+1 \leq \mathtt{N}+1$ and $\mathtt{L}' \leq \mathtt{L}$
2) $\mathtt{H}' = \mathtt{N}' + 2\mathtt{L}' \leq \mathtt{N}+1+2\mathtt{L} = \mathtt{H}+1$
3) $\mathtt{H}' = \mathtt{H}+1$ *forbidden* by $\mathcal{L}^{\Lambda,\Lambda_0}_{1,0} = 0$
4) $\mathtt{H}' = \mathtt{H}$ *forbidden* by $\mathcal{L}^{\Lambda,\Lambda_0}_{2,0} = 0$

... so $\mathtt{H}' < \mathtt{H}$

```
iterate over H:=N+2L=2,4,6,···
 iterate over L=0, 1,...,(H-2)/2
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$(\mathtt{H},\mathtt{L})_{\mathtt{RHS}} \prec (\mathtt{H},\mathtt{L})_{\mathtt{LHS}}$

Inductive proofs: reconstruction

WARNING:
integrals must converge
$\Rightarrow$ bounds are needed!

```
iterate over H=N+2L=2,4,6,···
    iterate over L = 0, 1, ..., (H − 2)/2
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N,L

$\|w\| = w_{max}$
w multi-index counting momentum derivatives

irrelevant terms (i.e. $d < 0$)

$$\partial_p^w \mathcal{L}_{\mathtt{N},\mathtt{L}}^{\Lambda,\Lambda_0}(p_{[\mathtt{N}-1]}) := \underbrace{0}_{\text{UV-BC}} + \int_{\Lambda_0}^{\Lambda} \mathrm{d}\lambda\, \underbrace{\partial_p^w \dot{\mathcal{L}}_{\mathtt{N},\mathtt{L}}^{\lambda,\Lambda_0}(p_{[\mathtt{N}-1]})}_{\text{flow}}$$

decrease $\|w\|$
(i.e. increase $d = 4 - \mathtt{N} - \|w\|$)

relevant terms (i.e. $d \geq 0$) $q_{[3]}$ *ren. point*, $|q_{[3]}| = O(M)$

$$\mathcal{L}_{4,\mathtt{L}}^{\Lambda,\Lambda_0}(p_{[3]}) := g_{\mathtt{L}} + \int_0^{\Lambda} \underbrace{\mathrm{d}\lambda\, \dot{\mathcal{L}}_{4,\mathtt{L}}^{\lambda,\Lambda_0}(q_{[3]})}_{\text{flow}} + \underbrace{\int_0^1 \mathrm{d}t\, \frac{\mathrm{d}Q_e^\alpha}{\mathrm{d}t}(t)\, \partial_{p_e^\alpha} \mathcal{L}_{4,\mathtt{L}}^{\Lambda,\Lambda_0}(Q(t))}_{\text{interpolation from } q_{[3]} \text{ to } p_{[3]}},$$

IR-BC = ren. cond.

All order uniform bounds for massless φ_4^4

[RG+Kopper, arXiv:1103.5692 (sketch)]
[RG+ Kopper: Full paper still pending] *(shame on the speaker!)*

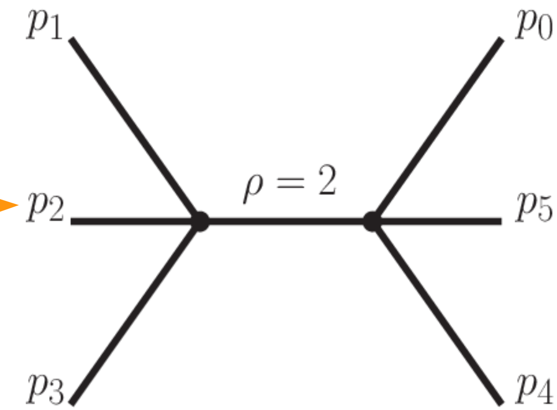
- $\forall \mathtt{N}, \forall \mathtt{L}$ we prove accurate bounds for $\mathcal{L}_{\mathtt{N},\mathtt{L}}^{\Lambda,\Lambda_0}(p_1,\ldots,p_{\mathtt{N}-1})$ that
  1) hold *uniformly* for all p, Λ, Λ_0
  2) encode *IR divergences* at *exceptional momenta*
  3) exibit a physical *power-like falloff* (up to *logarithms*) for large non-exceptional momenta.

- Bounds for $\partial_\Lambda^h \partial_{\Lambda_0}^k \mathcal{L}_{\mathtt{N},\mathtt{L}}^{\Lambda,\Lambda_0}(p_1,\ldots,p_{\mathtt{N}-1})$ are also established $(h,k \in \{0,1\})$

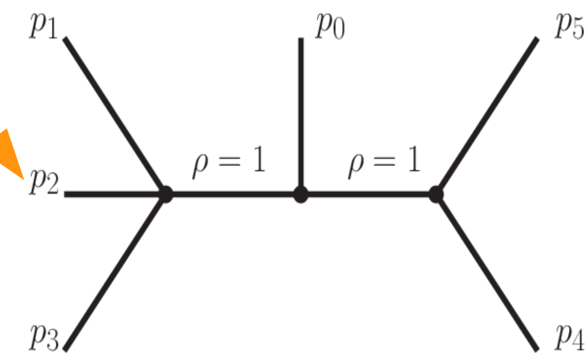
- *Renormalizability:* the existence of *pointwise UV+IR limit* for non–exceptional momenta follows from the bounds.

Bounds: *baby* version

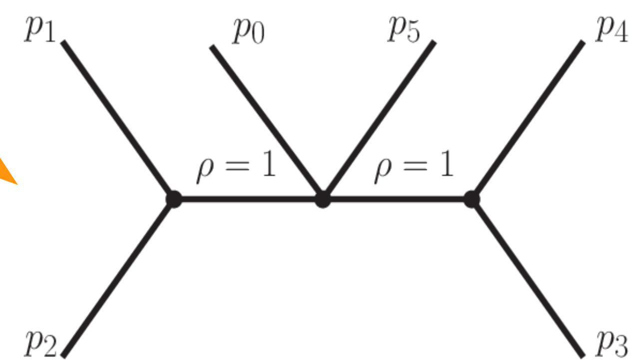
$$|\mathcal{L}^{\Lambda,\Lambda_0}_{6,\mathtt{L}}(p_{[5]})| \leq \Big(\quad |p_1+p_2+p_3|_\Lambda^{-2}$$



$$+\,|p_1+p_2+p_3|_\Lambda^{-1}\,|p_4+p_5|_\Lambda^{-1}$$



$$+\,|p_1+p_2|_\Lambda^{-1}\,|p_3+p_4|_\Lambda^{-1}$$



$$+ \text{ perms.} \Big)$$

$$\Big\{ \mathcal{P}_{\mathtt{L}}\left(\log_+ \frac{|p_{[5]}|_M}{\kappa(\Lambda, p_{[5]}, M)}, \log_+ \frac{\Lambda}{M}\right)$$

$\deg \mathcal{P}_{\mathtt{L}} = \mathtt{L}$

large eg when $\Lambda = 0$ and $p_{[5]}$ exceptional

large when $\Lambda = \Lambda_0 \gg M$

- $\log_+(x) := \log(\max(1, x))$
- M *renormalization scale*
- $p_{[n]} := (p_1, \cdots, p_n)$;
- $|p_{[n]}|_\Lambda := \max(\Lambda, \sqrt{\sum p_e^2})$

  IR regulated Euclidean norm
- $\kappa := \min\Big(|p_i|_\Lambda,\ |p_i+p_j|_\Lambda,\ \cdots,\ |p_1+\cdots+p_5|_\Lambda,\ |M|_\Lambda\Big)$ *dynamical IR cutoff*

weighted trees !!!

Bounds: *teenager* version

$$\left|\partial_p^w \mathcal{L}^{\Lambda,\Lambda_0}_{\mathtt{N}\geq 4,\mathtt{L}}\left(p_{[\mathtt{N}-1]}\right)\right| \leq \left(\sum_{T\in\mathcal{T}_{\mathtt{N},w}} \prod_{i\in\mathcal{I}(T)} |k_i|_\Lambda^{-\theta(i)}\right) \quad \mathcal{P}_{\mathtt{L}}(\text{logs})$$

$w \in \mathbb{N}_0^{4(\mathtt{N}-1)}$ is a *multi–index*

$\mathcal{T}_{\mathtt{N},w}$ is a set of *weighted trees*

$\mathcal{I}(T)$ is the set of *internal lines* of the weighted tree T

k_i is the *momentum* flowing through the internal line i.

$\theta(i) = \rho(i) + \ldots$ is the *total weight* associated to i

sum rule $\sum_i \theta(i) = \mathtt{N} + \|w\| - 4 = -d$

Renormalization of SU(2) pure gauge theory

[Efremov+RG+Kopper, arXiv:1704.06799] [Efremov, PhD Thesis 2017]

- Explicit renormalization conditions at *"physical points"'*
  $\Lambda = 0^+$, p *non-exceptional*.

- *Uniform & all-order bounds in momentum space* for *OPI functions* (and their Λ_0-derivatives) with an arbitrary number of insertions of the composite operators ψ^{Λ_0}, Ω^{Λ_0} generating the *renormalized BRST* transformation

- *Renormalizability: pointwise IR+UV convergence* of such OPI functions at non-exceptional momenta

- Slavnov–Taylor identities (STI) for the OPI functions [Zinn-Justin,1975], [Kluberg-Stern+Zuber,1975] are *explicitly broken* by cutoffs.

- We prove *uniform bounds* on the *"STI violations"* from which follows that they *vanish* at non-exceptional momenta in the IR+UV limit:

$$\left|\text{violations}^{\Lambda\Lambda_0}(\vec{p})\right| \leq \frac{\Lambda + |\vec{p}| + M}{\Lambda_0} \times \text{trees} \times \mathcal{P}_r(\text{logs}) \times \mathcal{Q}_s\left(\frac{|\vec{p}|}{|M| + \Lambda}\right) \times \left(1 + \frac{|\vec{p}|}{\Lambda_0}\right)^4$$

with degrees r, s linear in L

SU(2) YM: tree level Lagrangian, no cutoffs

- *Lorentz* gauge fixing with auxiliary field B and *Faddeev-Popov* ghosts $c, \bar{c}$

- The *Total, 0-loop* Lagrangian density in the limit $\Lambda \to 0$, $\Lambda_0 \to \infty$ is

$$\mathscr{L}^{0,\infty}_{\mathrm{tot},0} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{\xi}{2} B^2 - iB\partial_\mu A_\mu - \partial_\mu \bar{c} D_\mu c$$

- $\mathscr{L}^{0,\infty}_{\mathrm{tot},0}$ is *invariant* under *BRST* transformation

$$\delta\Phi = \epsilon\, s\Phi \qquad s\, A = Dc, \qquad s\, c = \frac{i}{2} g\{c, c\}, \qquad s\, \bar{c} = iB, \qquad s\, B = 0,$$

ϵ is a *Grassmann parameter*
$\{c, c\}^d = i\epsilon_{abd} c^a c^b$

- *the s operator is nilpotent:* $s^2 = 0$

Dependence on B field is trivial

• CHOICE: *Vanishing renormalization conditions* for all relevant (i.e. $d \geq 0$) terms involving B

$$\partial^w \mathsf{\Gamma}_{\mathrm{L}}^{0\Lambda_0;B\vec{\phi}}(\vec{q}) = 0\,,$$

• LEMMA: for all $\Lambda > 0$

$$\mathsf{\Gamma}_{\mathrm{L}}^{\Lambda\Lambda_0;B\vec{\phi}}(\vec{p}) = 0$$

• COROLLARY: *no counterterms involving B*

• COROLLARY: *trivial (tree-level) dependence* on B:

$$\Gamma^{\Lambda\Lambda_0}(A,B,c,\bar{c}) = \frac{1}{2\xi}\int d^4x\,(\xi B - i\partial A)^2 + \tilde{\Gamma}^{\Lambda\Lambda_0}(A,c,\bar{c})$$

SU(2) YM: all-order Lagrangian + cutoffs

- *All-order, total, regularized Lagrangian*, with $\Phi := (A, B, c, \bar{c})$:

$$\mathscr{L}_{\rm tot}^{\Lambda,\Lambda_0} = \frac{1}{2}\Phi\,\mathbf{C}_{\Lambda\Lambda_0}^{-1}\,\Phi + \mathscr{L}_{\rm int,0} + \mathscr{L}_{\rm ct}^{\Lambda_0}$$

$$\mathscr{L}_{\rm int,0} = g\epsilon_{abc}(\partial_\mu A_\nu^a)A_\mu^b A_\nu^c + \frac{g^2}{4}\epsilon_{cab}\epsilon_{cds}A_\mu^a A_\nu^b A_\mu^d A_\nu^s - g\epsilon_{abc}(\partial_\mu \bar{c}^a)A_\mu^b c^c$$

$$\begin{aligned}\mathscr{L}_{\rm ct}^{\Lambda_0} = {} & r^{\bar{c}c\bar{c}c}\bar{c}^b c^b \bar{c}^a c^a + r_1^{\bar{c}cAA}\bar{c}^b c^b A_\mu^a A_\mu^a + r_2^{\bar{c}cAA}\bar{c}^a c^b A_\mu^a A_\mu^b \\ & + r_1^{A^4} A_\mu^b A_\nu^b A_\mu^a A_\nu^a + r_2^{A^3} A_\nu^b A_\nu^b A_\mu^a A_\mu^a + 2\epsilon_{abc} r^{A^3}(\partial_\mu A_\nu^a)A_\mu^b A_\nu^c \\ & - r_1^{A\bar{c}c}\epsilon_{abd}(\partial_\mu \bar{c}^a)A_\mu^b c^d - r_2^{A\bar{c}c}\epsilon_{abd}\bar{c}^a A_\mu^b \partial_\mu c^d + \Sigma^{\bar{c}c}\bar{c}^a \partial^2 c^a \\ & - \frac{1}{2}\Sigma_T^{AA} A_\mu^a(\partial^2\delta_{\mu\nu} - \partial_\mu\partial_\nu)A_\nu^a + \frac{1}{2\xi}\Sigma_L^{AA}(\partial_\mu A_\mu^a)^2 \\ & + \delta m_{AA}^2 A_\mu^a A_\mu^a + \delta m_{\bar{c}c}^2 \bar{c}^a c^a\end{aligned}$$

- We include *all* possible counterterms compatible with the global symmetries: 11 marginal (i.e. $d = 0$) counterterms and 2 strictly relevant (i.e. $d > 0$) counterterms . NB: No counterterms with B in our case.

- In presence of cutoffs $0 < \Lambda < \Lambda_0 < \infty$ the BRST invariance of the total Lagrangian is *explicitly broken* already at tree level

violated STI for Z

- *renormalized BRST* $\delta_\epsilon \Phi$:

$$\delta_\epsilon A = \epsilon\, \sigma_{0\Lambda_0} \psi^{\Lambda_0}, \quad \delta_\epsilon c = -\epsilon\, \sigma_{0\Lambda_0} \Omega^{\Lambda_0}, \quad \delta_\epsilon \bar{c} = \epsilon\, \sigma_{0\Lambda_0} iB \quad \delta_\epsilon B = 0$$

$$\psi^{\Lambda_0} = R_1^{\Lambda_0} \partial c - ig R_2^{\Lambda_0} [A, c], \qquad \Omega^{\Lambda_0} = \frac{1}{2i} g R_3^{\Lambda_0} \{c, c\}$$

- The *3* functions $R_i^{\Lambda_0} = 1 + O(\hbar)$ encode operators' counterterms
- Define $Z^{0\Lambda_0}(K, \chi)$ functional of the sources $K := (j, b, \bar{\eta}, \eta)$, $\chi := (\gamma, \omega)$:

$$Z^{0\Lambda_0}(K, \chi) := \int [\mathrm{d}\Phi] \exp\Big(-\frac{1}{\hbar} \int_x \Big(\mathscr{L}_{\mathrm{tot}}^{0,\Lambda_0} + \underbrace{\gamma \psi^{\Lambda_0} + \omega \Omega^{\Lambda_0}} - K\Phi \Big)\Big)$$

No contact terms $\gamma^2, \omega^2, \omega\gamma$ *by global symmetries*

- The change of variable $\Phi' = \Phi + \delta_\epsilon \Phi$ gives the *violated* STI:

$$\left\langle \int_x \left(\delta_\epsilon \Phi \frac{\mathscr{L}_{\mathrm{tot},0}^{0,\Lambda_0}}{\delta \Phi} + \gamma \delta_\epsilon \psi^{\Lambda_0} + \omega \delta_\epsilon \Omega^{\Lambda_0} \right) \right\rangle_{K,\chi} = \left\langle \int_x K \delta_\epsilon \Phi \right\rangle_{K,\chi}$$

$$\langle X(\Phi) \rangle_{K,\chi} := \int [\mathrm{d}\Phi] \exp\Big(-\frac{1}{\hbar} \int_x \Big(\mathscr{L}_{\mathrm{tot}}^{0,\Lambda_0} + \gamma \psi^{\Lambda_0} + \omega \Omega^{\Lambda_0} - K\Phi \Big)\Big) X(\Phi)$$

violated STI for Γ

$$\int_x \left(\Gamma^{0\Lambda_0}_{\rho(x)} + (iB + \frac{1}{\xi}\partial A)\, \Gamma^{0\Lambda_0}_{\beta(x)} \right) = \frac{1}{2}\mathcal{S}\, \underline{\Gamma}^{0\Lambda_0}$$

- ρ and β are sources for the 2 operators encoding the STI violations
- Gammology: three Gamma's differing by quadratic terms

standard Γ $\Rightarrow$ $\Gamma^{\Lambda\Lambda_0}(A,B,c,\bar{c}) = \frac{1}{2\xi}\int d^4x\,(\xi B - i\partial A)^2 + \tilde{\Gamma}^{\Lambda\Lambda_0}(A,c,\bar{c})$

reduced Γ *(flow equation)* $\Rightarrow$ $\Gamma^{\Lambda\Lambda_0} = \tilde{\Gamma}^{\Lambda\Lambda_0} - \frac{1}{2}\langle A, \sigma^{-1}_{\Lambda\Lambda_0} C^{-1} A\rangle - \langle \bar{c}, \sigma^{-1}_{\Lambda\Lambda_0}\partial^2 c\rangle$

auxiliary $\underline{\Gamma}$ *(defines* $\mathcal{S}$*)* $\Rightarrow$ $\underline{\Gamma}^{0\Lambda_0} := \tilde{\Gamma}^{0\Lambda_0} + i\langle B, \bar{\omega}\rangle + \frac{1}{2\xi}\langle A, \partial\partial\, A\rangle$

- with $\frac{\delta}{\delta\tilde{c}} := \frac{\delta}{\delta\bar{c}} - \partial\frac{\delta}{\delta\gamma}$ we define:

$$\mathcal{S} := \sum_{(\phi,\phi^\star)\in\{(A,\gamma),(-c,\omega),(\tilde{c},\bar{\omega})\}} \Big\langle \frac{\delta\underline{\Gamma}^{0\Lambda_0}}{\delta\phi}, \sigma_{0\Lambda_0}\frac{\delta}{\delta\phi^\star}\Big\rangle + \Big\langle \frac{\delta\underline{\Gamma}^{0\Lambda_0}}{\delta\phi^\star}, \sigma_{0\Lambda_0}\frac{\delta}{\delta\phi}\Big\rangle$$

- *Restricted nilpotency* of $\mathcal{S}$ implies a vital *consistency condition*:

$$\mathcal{S}^2\,\underline{\Gamma}^{0\Lambda_0} = 0 \Rightarrow \mathcal{S}\int_x \left(\Gamma^{0\Lambda_0}_{\rho(x)} + (iB + \frac{1}{\xi}\partial A)\Gamma^{0\Lambda_0}_{\beta(x)} \right) = 0$$

[Becchi, arXiv:hep-th/9607188]

Free and constrained renormalization constants

- $\partial^w \Gamma^{0\Lambda_0;\vec{\phi}}_{\vec{\varkappa}}(0) = 0$, $\varkappa_i \in \{\gamma, \omega\}$, for all *strictly relevant* terms

- $11 + 3 = 14$ marginal counterterms $\Leftrightarrow$ 14 *marginal renormalization constants*

- $\Gamma^{M\Lambda_0;c\bar{c}c\bar{c}}(0) = 0$, $\Gamma^{M\Lambda_0;c\bar{c}A^2}(0) = 0$, $\partial_A \Gamma^{M\Lambda_0;c\bar{c}A}(0) = 0 \Leftarrow 4$ *RC*
  LEMMA: the 4 counterterms $r^{\bar{c}c\bar{c}c}$, $r_1^{\bar{c}cA^2}$, $r_2^{\bar{c}cA^2}$, $r_2^{A\bar{c}c}$ vanish

- The *3* renormalization constants corresponding to CTs r^{A^3} , Σ_T^{AA} , $\Sigma^{\bar{c}c}$ are *free*

- The *7* remaining renormalization constants must satisfy *7* additional relations in order to make the marginal violation terms $\int_x \Gamma_\rho^{\vec{\phi};w}$ and $\Gamma_\beta^{\vec{\phi};w}$ at the renormalization point *comply* with the bounds (i.e. *be small in UV limit*)

- We prove the *existence* of a solution for this system of relations that does not depend on the UV cutoff.

- Agreement with [Bonini+D'Attanasio+Marchesini, arXiv:hep-th/9602156]

(selected) results using the FE framework

• All-order bounds for correlation functions of gauge-invariant operators in Yang-Mills theory [Fröb+Holland+Holland, arXiv:1511.09425] (P conserved $\Rightarrow$ triviality of BRST cohomology)

• φ_4^4 massive: explicit bounds on the constants in the polynomials and proof of local existence of Borel transform [Kopper 2010, CMP295]

• OPE convergence at fixed L: for φ_4^4 massive [Hollands+Kopper, arXiv:1105.3375] $n > 2$ [Holland+Hollands, arXiv:1205.4904], φ_4^4 massless [Holland+Hollands+Kopper, arXiv:1411.1785]

•OPE: formulas for derivatives wrt coupling of Wilson coefficients; [Holland+Hollands, arXiv:1401.3144]

• OPE: extension to YM [Fröb+Hollands, arXiv:1603.08012]

• Renormalization Proof for Massive φ_4^4 Theory on Riemannian Manifolds [Kopper+Müller, arXiv:math-ph/0609089]

• Renormalization of Finite Temperature massive φ_4^4 [Kopper+Müller, arXiv:hep-th/0003254]

• Minkowski space: renormalization, analiticity of $\mathcal{L}_2(p^2)$ near mass shell and continuity of $\mathcal{L}_4(p_{[3]})$ on $\mathbb{R}^{12}$ [Keller+Kopper+Schophaus, arXiv:hep-th/9605137], [Kopper, arXiv:math-ph/0701071]

Memories….

The beginning...

- 1989, Genoa University, Italy: I knocked at Ken's office door asking for a graduation (*Laurea*) subject ...
- (As usual) Ken was immediately hospitable and gave me a problem

  ... *in spite of the fact that I did not follow his QM course*

  (two QM courses alternated yearly at that time)
- Ken gave me also a drop of his wisdom (?Japanese) :

  'I can lead a horse to water but I can't make it drink'

  ... *upsetting but motivating!*
- This was the beginning of *a long and fruitful scientific collaboration,* that continued with my PhD and after, till beginning 1996 when I started my post-doc in Saclay
- Long days of hard work ...

  ... *and short weeks because at that time Ken arrived at Genoa U on Tuesday and quit for Pisa on Thursday!*

...a close friendship started as well !

- *It has been (and it is) a* ***pleasure*** *to interact with Ken!*

- ***Everybody*** *meeting Ken is enriched by his human qualities:*

... long list omitted ...

- *Ken acted with me more as a* ***second father*** *than as a PhD director!*

... so, ***sincerely****:*

Thanks for all, KEN!

小西さん
お誕生日
おめでとうございます

Happy birthday
KONISHI-SAN !!!