Modulated Vacua

Topological Solitons, Nonperturbative Gauge Dynamics and Confinement

20-21 July 2017, University of Pisa

Muneto Nitta
(Keio U.)

with Shin Sasaki (Kitasato) & Ryo Yokokura (Keio)

arXiv:1706.02938 [hep-th], arXiv:1706.05232 [hep-th]

My connection with Ken and Pisa

2006 Started collaboration on topological solitons (non-Abelian vortices and monopoles)
Visiting almost every year

Many of Ken's students and postdocs become now important collaborators and/or good friends of mine:

Giacomo Marmorini -> cond-mat, now PD @ Keio
Walter Vinci -> quantum comp
Sven Bjarke Gudnason
Yunguo Jiang
Mattia Cipriani : PhD adviser -> plasma physics

Jarah Evslin
Chandrasekhar Chatterjee -> now PD @ Keio

Minoru Eto
Toshi Fujimori
Keisuke Ohashi

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1858
CALAMVS
GLADIO
FORTIOR

Keio U.

@ Yokohama, Kanagawa

in greater Tokyo

Topological Science Project

Aimed to understand all subjects of physics in terms of Topology

5 years (2015-2020),
11 postdocs

Anyone here is very welcome to visit!!

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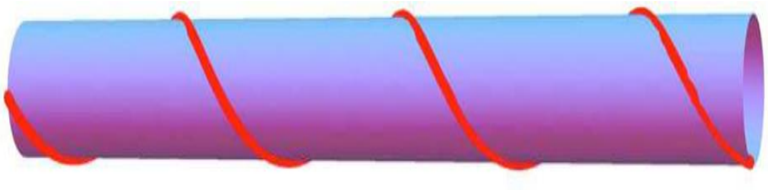
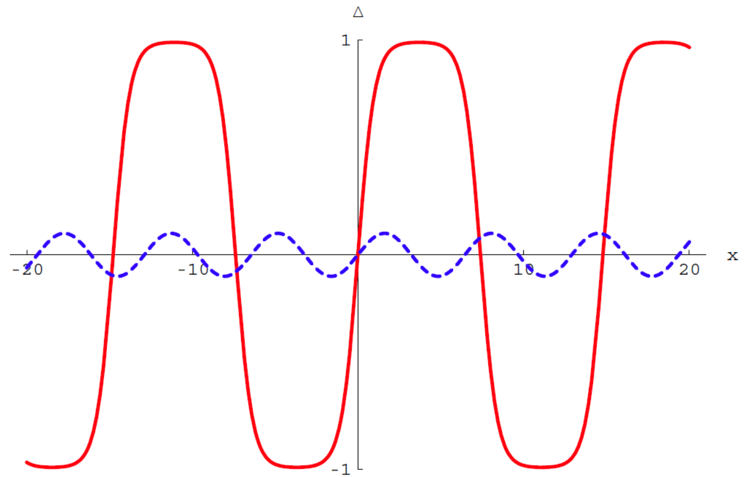
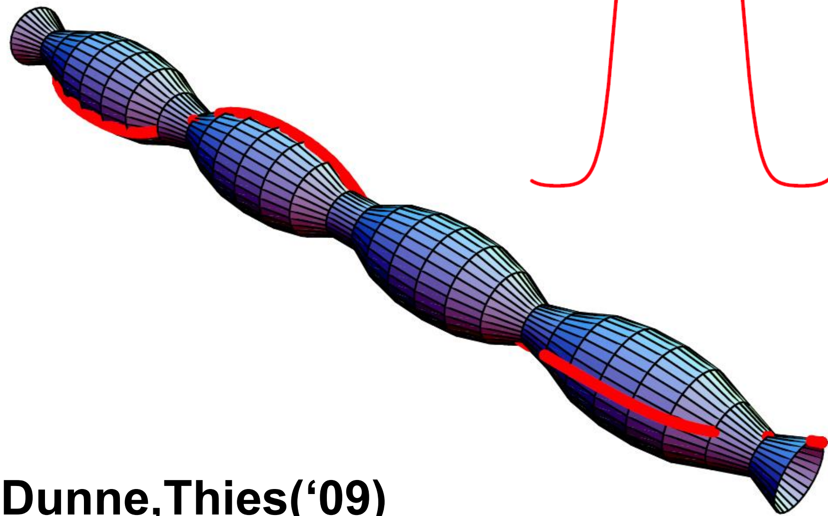
The great wave of Kanagawa

What is modulation?

A field configuration modulated in a space (periodically)



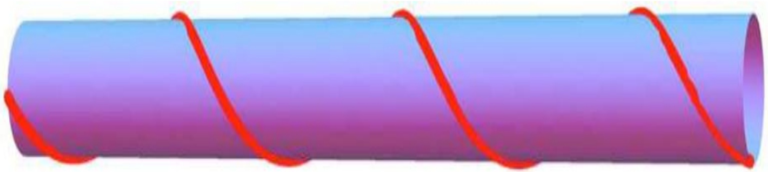




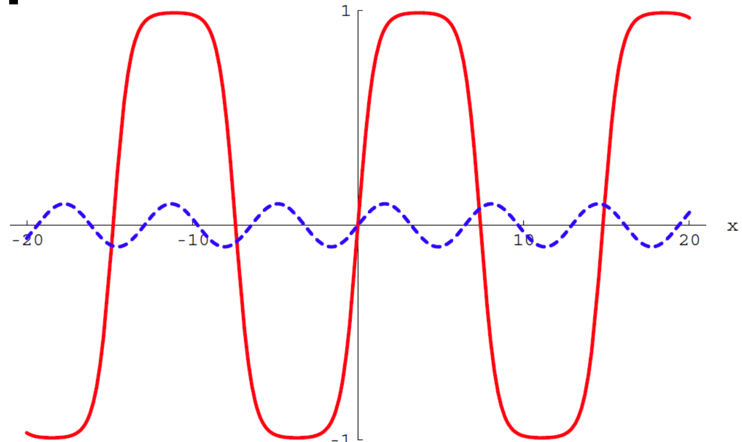
Figs from Basar,Dunne,Thies('09)

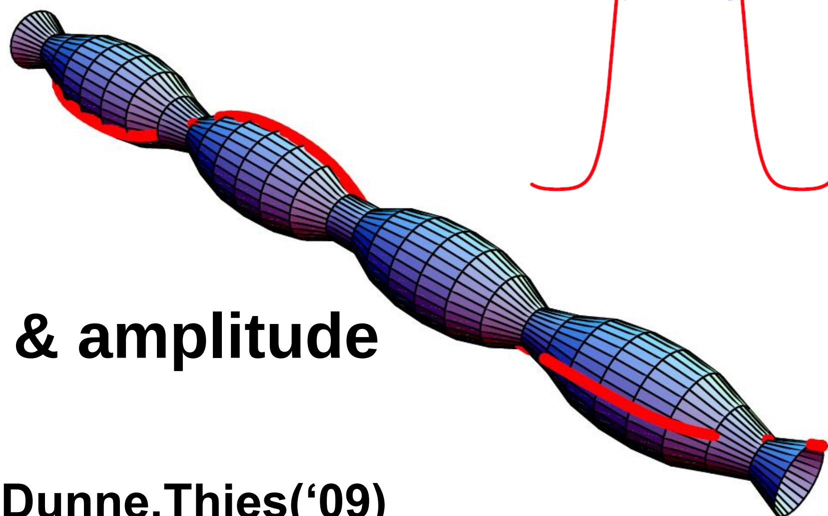
History In a superconductor

Fulde & Ferrell (FF) (‘64)
Phase modulation

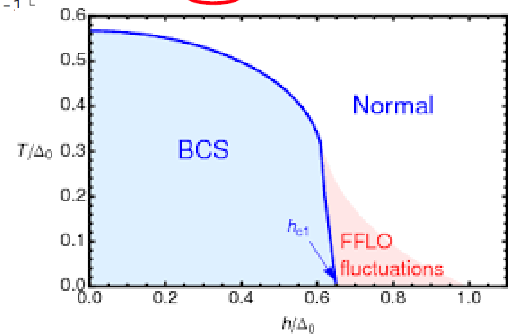


Larkin & Ovchinnikov(LO) (‘64)
Amplitude modulation





FFLO:
both phase & amplitude



Figs from Basar,Dunne,Thies(‘09)

History

In a superconductor,

--- experimentally, some evidences but not conclusive

Other systems:

ultracold atomic fermion gases

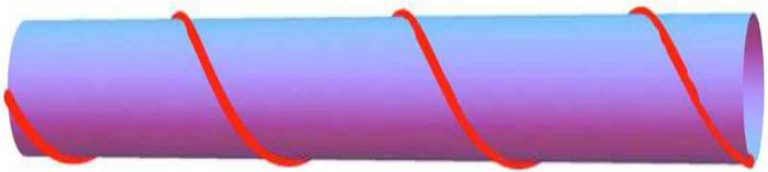
--- experimentally, some evidences but not conclusive

History In chiral Gross-Neveu model in d=1+1

Exact self-consistent solution by Basar & Dunne ('08)

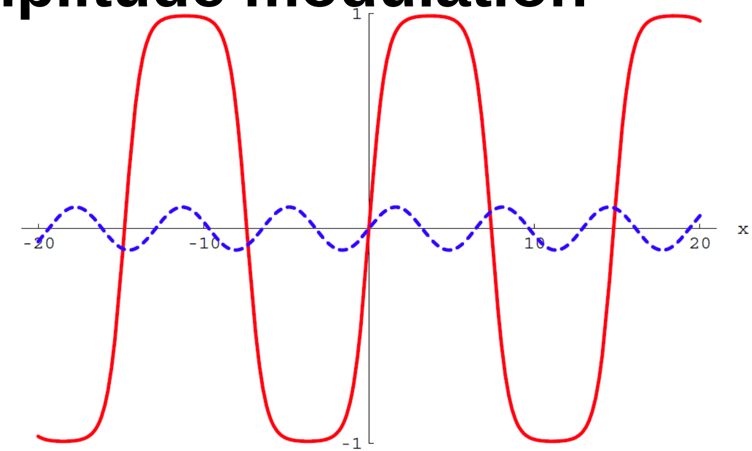
Chiral spiral

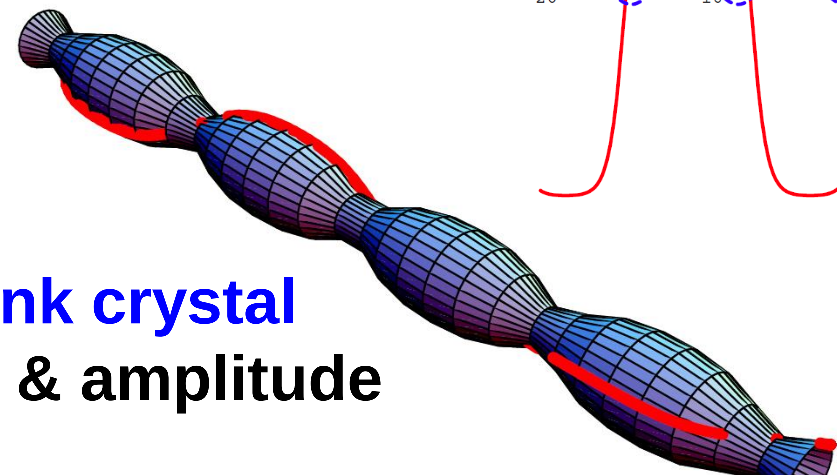
Phase modulation



Real kink crystal

Amplitude modulation



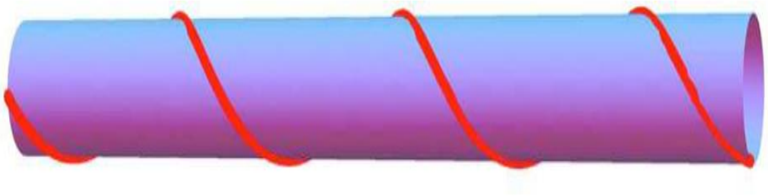


Complex kink crystal

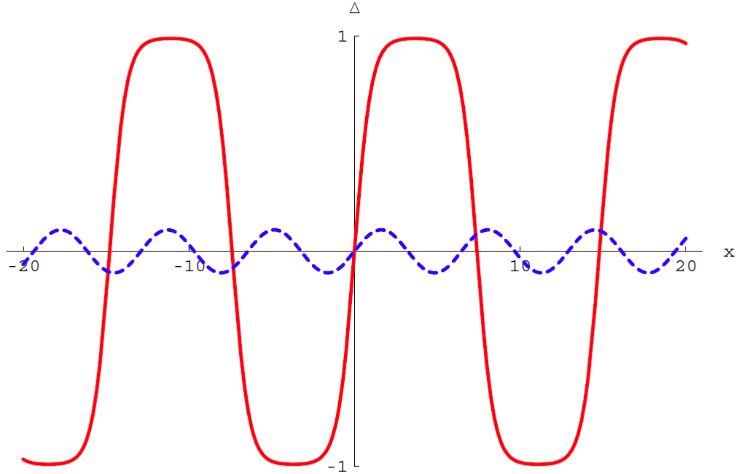
both phase & amplitude

History In Nambu-Jona-Lasino in d=3+1

Nakano & Tatsumi ('04)
Chiral spiral
Phase modulation



Nickel ('09) Real kink crystal
Amplitude modulation



In terms of Ginzburg-Landau effective theory

Nickel ('09)

$$\langle \overline{\psi}\psi \rangle \approx \phi(\mathbf{x})$$

$$\Omega_{GL}(T,\mu;\phi(\mathbf{x}))$$
$$= c_2(T,\mu)\phi(\mathbf{x})^2 + c_{4,a}(T,\mu)\phi(\mathbf{x})^4 + c_{4,b}(T,\mu)(\nabla\phi(\mathbf{x}))^2$$
$$+c_{6,a}(T,\mu)\phi(\mathbf{x})^6 + c_{6,b}(T,\mu)(\nabla\phi(\mathbf{x}))^2\phi(\mathbf{x})^2 + c_{6,c}(T,\mu)\boxed{(\Delta\phi(\mathbf{x}))^2}$$

When $c_{4,b} < 0 \quad c_{6,b},\ c_{6,c} > 0$, **modulation can be favored**

Higher derivative terms are needed

Analogous to frustrated magnet (Sutcliffe's talk)

Purpose of our work

Modulations considered so far in cond-mat., QCD are ground states @ finite temperature/ density/ magnetic field, and so in **non-relativistic theories**.

(1) Modulation in relativistic field theory

arXiv:1706.02938 [hep-th]

Unusual Nambu-Goldstone boson

Cf)No-go theorem by Son for relativistic fermion condensates

(2) Modulation in supersymmetric field theory

arXiv:1706.05232 [hep-th]

Goldstino

New mechanism of SUSY breaking

$$\delta\psi_\alpha = i\sqrt{2}(\sigma^m)_{\alpha\dot\alpha}\bar\xi^{\dot\alpha}\underset{\neq 0}{\boxed{\partial_m\varphi}} + \sqrt{2}\xi_\alpha \underset{= 0}{\boxed{F(\varphi,\bar\varphi)}}$$

Plan of talk

§ 1 Intoduction: What is modulation?

§ 2 Modulated vacua in bosonic theory

§ 3 Modulated vacua in SUSY

§ 4 Summary & Discussion

Plan of talk

Remember

$$\Omega_{GL}(T,\mu;\phi(\mathbf{x}))$$
$$= c_2(T,\mu)\phi(\mathbf{x})^2 + c_{4,a}(T,\mu)\phi(\mathbf{x})^4 + c_{4,b}(T,\mu)(\nabla\phi(\mathbf{x}))^2$$
$$+c_{6,a}(T,\mu)\phi(\mathbf{x})^6 + c_{6,b}(T,\mu)(\nabla\phi(\mathbf{x}))^2\phi(\mathbf{x})^2 + c_{6,c}(T,\mu)\boxed{(\Delta\phi(\mathbf{x}))^2}$$

Naively replace

non-relativistic **relativistic**

Ostrogradski instability

$$\nabla \longrightarrow \partial_m \qquad m = 0,1,2,3$$

$$\Delta = \nabla^2 \longrightarrow \Box \cancel{= \partial_m \partial^m = \partial_t^2} - \nabla^2 \longrightarrow \textbf{Ghost}$$

$$L = f\left(\partial_m \varphi\, \partial^m \varphi, \varphi\right)$$

Ex) Nambu-Goto

$$L = \sqrt{1 - \partial_m \varphi\, \partial^m \varphi}$$

Ostrogradski's ghost instability

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{2M^2}(\Box\phi)^2$$

eliminating χ

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\left(\chi^2 + \frac{2\chi}{M}\Box\phi\right)$$

$$= -\frac{1}{2}(\partial\phi)^2 + \frac{1}{M}\partial\chi\partial\phi - \frac{1}{2}\chi^2$$

$$= -\frac{1}{2}(\partial\tilde{\phi})^2 + \frac{1}{2}(\partial\tilde{\chi})^2 - \frac{M^2}{2}\tilde{\chi}^2$$

$$\tilde{\chi} \equiv \frac{1}{M}\chi + \varphi$$

ghost

(wrong sign)

Let's consider a complex scalar field φ

Global stability condition

The highest order n of $|\partial\varphi|^2$ must be odd

$$L = \mp|\partial\varphi|^{2n} + \cdots = \mp\left(-|\dot{\varphi}|^2 + |\nabla\varphi|^2\right)^n + \cdots$$

$$\pi = \frac{\delta L}{\delta\dot{\varphi}} = \mp(-1)^n n\dot{\varphi}^* |\dot{\varphi}|^{2n-2} + \cdots$$

highest $\dot{\varphi}$ **highest $\nabla\varphi$**

$$H = \pi\dot{\varphi} + \pi^*\dot{\varphi}^* - L = \mp(-1)^n(2n-1)|\dot{\varphi}|^{2n} \pm |\nabla\varphi|^{2n}$$

odd n for temporal stability

upper sign for spatial stability

The simplest is $n = 3$

A model

$k, \lambda, \alpha > 0 \quad m = 0,1,2,3$

$$\mathcal{L} = -k\partial_m\varphi\partial^m\bar{\varphi} + (\lambda - \alpha\partial_m\varphi\partial^m\bar{\varphi})(\partial_n\varphi\partial^n\varphi)(\partial_p\bar{\varphi}\partial^p\bar{\varphi})$$

Ordinary kinetic

Derivative corrections

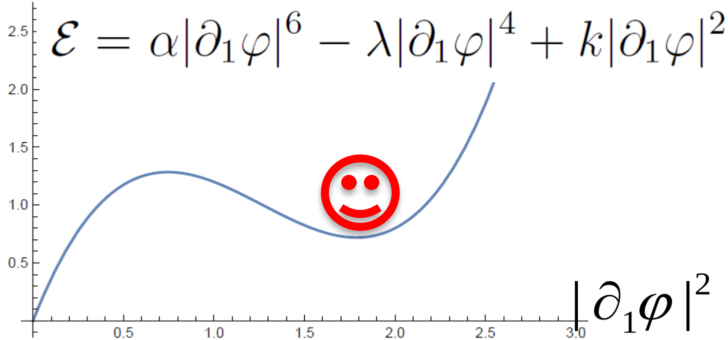
wrong sign for 4th & correct sign for 6th

$$\boxed{\langle 0 | \partial_m \varphi | 0 \rangle \neq 0} \xrightarrow[SO(3,1)]{} \langle 0 | \partial_1 \varphi | 0 \rangle \neq 0$$

Assume spatial

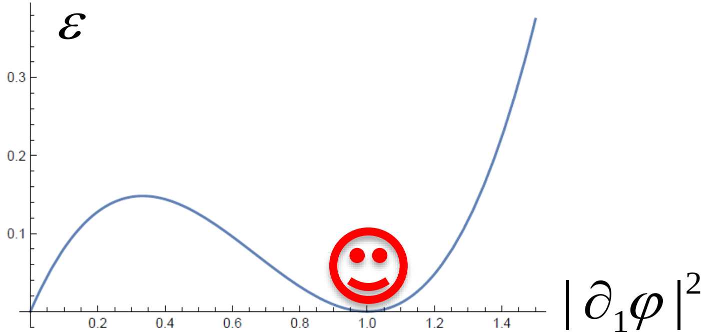
$$\varphi = \varphi(x^1) \Longrightarrow \mathcal{E} = \alpha|\partial_1\varphi|^6 - \lambda|\partial_1\varphi|^4 + k|\partial_1\varphi|^2$$

$$\begin{aligned}
\mathcal{E} &= \pi_\varphi\dot{\varphi} + \pi_{\bar{\varphi}}\dot{\bar{\varphi}} - \mathcal{L} \\
&= k(|\dot{\varphi}|^2 + |\partial_i\varphi|^2) + \{\lambda - \alpha(-|\dot{\varphi}|^2 + |\partial_i\varphi|^2)\}\{3|\dot{\varphi}|^4 - \dot{\varphi}^2(\partial_i\bar{\varphi})^2 - \dot{\bar{\varphi}}^2(\partial_i\varphi)^2 - (\partial_i\varphi)^2(\partial_j\bar{\varphi})^2\} \\
&+ 2\alpha|\dot{\varphi}|^2\{|\dot{\varphi}|^4 - \dot{\varphi}^2(\partial_i\bar{\varphi})^2 - \dot{\bar{\varphi}}^2(\partial_i\varphi)^2 + (\partial_i\varphi)^2(\partial_j\bar{\varphi})^2\},
\end{aligned}$$



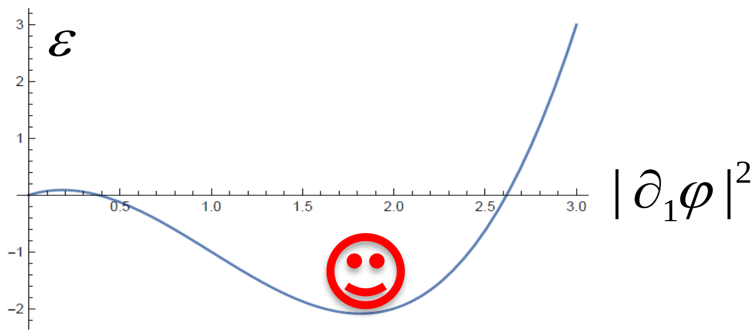
Metastable (local minimum)

$$\lambda^2 - 4\alpha k < 0$$



Stable (degenerate)

$$\lambda^2 - 4\alpha k = 0$$



Stable (global minimum)

$$\lambda^2 - 4\alpha k > 0$$

All cases

$$\lambda^2 - 3\alpha k > 0$$

Local min @

$$|\partial_1\varphi|^2 = \frac{\lambda + \sqrt{\lambda^2 - 3\alpha k}}{3\alpha}$$

Modulate vacua

$$\varphi(x^1) = \varphi_0 e^{ipx^1}$$

$$p^2\varphi_0^2 = \frac{\lambda + \sqrt{\lambda^2 - 3\alpha k}}{3\alpha}$$

satisfying EOM

Spontaneous symmetry breaking

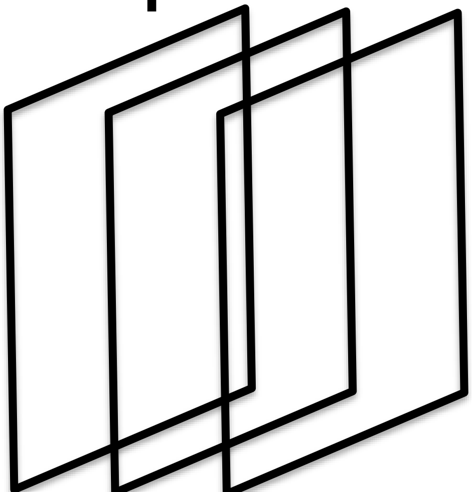
$$G = U(1)_{\varphi} \times ISO(3,1) = U(1) \times (P_{x,y,z} \times P_t) \ltimes SO(3,1)$$

$$H = \boxed{U(1)_{x+\varphi}} \ltimes ISO(2,1) = (U(1)_{x+\varphi} \times P_{y,z} \times P_t) \ltimes SO(2,1)$$

translation + phase

Order Parameter Space (OPS)

$$\frac{G}{H} = U(1)_{x-\varphi} \ltimes \frac{ISO(3,1)}{ISO(2,1)}$$



Inverse map of φ

How many Nambu-Goldstone (NG)?

Ivanov & Ogievetsky ('75)

"Inverse Higgs mechanism"

Low & Manohar ('02)

Only translation gives NG

Comment

More general solution

$$\varphi(x^1) = \sqrt{x_+}\int_c^{x_1} ds\ e^{iF(s)}$$

$$\varphi(x^1) = \varphi_0 e^{ipx^1}$$ preserves the highest symmetry

A vacuum alignment problem.

(Quantum correction may pick up a state with the highest symmetry.)

Liner Analysis

$$\varphi \longrightarrow \underbrace{\langle\varphi\rangle}_{\text{VEV}} + \underbrace{\tilde{\varphi}}_{\text{fluctuation}} \qquad \langle\varphi\rangle = \varphi_0 e^{ipx^1}$$

$$\mathcal{E}_{\mathrm{quad.}} = \frac{1}{2}\vec{\varphi}^\dagger \mathbf{M}\vec{\varphi}.$$

$$\vec{\varphi} = \begin{pmatrix} \partial^{\hat{m}}\tilde{\varphi} \\ \partial^{\hat{m}}\tilde{\varphi}^\dagger \\ \partial_1\tilde{\varphi} \\ \partial_1\tilde{\varphi}^\dagger \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} a \\ a \end{matrix}}\right\} \text{transverse} \\ \left.\vphantom{\begin{matrix} a \\ a \end{matrix}}\right\} \text{modulation} \end{matrix}$$

$\hat{m} = 0,2,3$

$$\mathbf{M} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

$$M_1 = \begin{pmatrix} k + \alpha x_+^2 & 2(\lambda - \alpha x_+)p^2\varphi_0^2 e^{2ipx^1} \\ 2(\lambda - \alpha x_+)p^2\bar{\varphi}_0^2 e^{-2ipx^1} & k + \alpha x_+^2 \end{pmatrix} \quad M_2 = \begin{pmatrix} 9\alpha x_+^2 - 4\lambda x_+ + k & 2(\lambda - 3\alpha x_+)p^2\varphi_0^2 e^{2ipx^1} \\ 2(\lambda - 3\alpha x_+)p^2\bar{\varphi}_0^2 e^{-2ipx^1} & 9\alpha x_+^2 - 4\lambda x_+ + k \end{pmatrix}$$

Diagonalize M_1 M_2

M_1

$$s_1 > 0 \;:\; \mathbf{e}_1 = \frac{1}{\sqrt{2}|\varphi_0|}\begin{pmatrix} \langle\varphi\rangle \\ \langle\bar{\varphi}\rangle \\ 0 \\ 0 \end{pmatrix}, \qquad s_2 = 0 \;:\; \mathbf{e}_2 = \frac{1}{\sqrt{2}|\varphi_0|}\begin{pmatrix} \langle\varphi\rangle \\ -\langle\bar{\varphi}\rangle \\ 0 \\ 0 \end{pmatrix}$$

M_2

$$t_1 > 0 \;:\; \mathbf{e}_3 = \frac{1}{\sqrt{2}|\varphi_0|}\begin{pmatrix} 0 \\ 0 \\ -\langle\varphi\rangle \\ \langle\bar{\varphi}\rangle \end{pmatrix}, \qquad t_2 = 0 \;:\; \mathbf{e}_4 = \frac{1}{\sqrt{2}|\varphi_0|}\begin{pmatrix} 0 \\ 0 \\ \langle\varphi\rangle \\ \langle\bar{\varphi}\rangle \end{pmatrix}$$

$$U_1 = \frac{1}{\sqrt{2}|\varphi_0|}\begin{pmatrix} \langle\bar{\varphi}\rangle & \langle\varphi\rangle \\ -\langle\bar{\varphi}\rangle & \langle\varphi\rangle \end{pmatrix}, \qquad U_2 = \frac{1}{\sqrt{2}|\varphi_0|}\begin{pmatrix} -\langle\bar{\varphi}\rangle & \langle\varphi\rangle \\ \langle\bar{\varphi}\rangle & \langle\varphi\rangle \end{pmatrix}$$

Diagonalize M_2 in the modulated direction

$$\tilde{\varphi}_{\mathrm{NG}} = \frac{1}{\sqrt{2}|\varphi_0|}\left(\langle\bar{\varphi}\rangle\partial_1\tilde{\varphi} + \langle\varphi\rangle\partial_1\tilde{\varphi}^\dagger\right)$$

NG mode **No quadratic kinetic term**

$$\tilde{\varphi}_{\mathrm{H}} = \frac{1}{\sqrt{2}|\varphi_0|}\left(-\langle\bar{\varphi}\rangle\partial_1\tilde{\varphi} + \langle\varphi\rangle\partial_1\tilde{\varphi}^\dagger\right)$$

Higgs mode **gapless**

$$\tilde{\varphi}_{\mathrm{NG}} = \partial_1 A - ipB, \qquad \tilde{\varphi}_{\mathrm{H}} = \partial_1 B - ipA.$$

$$\mathcal{L}_{\mathrm{quad.}} = -\frac{1}{2}s_1\partial_{\hat{m}}A\partial^{\hat{m}}A - \frac{1}{2}t_1|\partial_1 B - ipA|^2$$

higher $$\mathcal{L}_{\mathrm{quart.}} = \frac{1}{4}(\lambda - 6\alpha x_+)\tilde{\varphi}_{\mathrm{NG}}^4 + \cdots,$$

Plan of talk

§ 1 Intoduction: What is modulation?

§ 2 Modulated vacua in bosonic theory

§ 3 Modulated vacua in SUSY

§ 4 Summary & Discussion

New mechanism of SUSY breaking

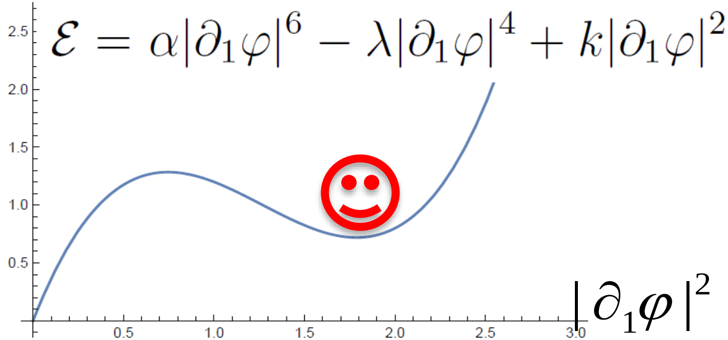
SUSY transformation

$$\delta\psi_\alpha = i\sqrt{2}(\sigma^m)_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}\underbrace{\boxed{\partial_m\varphi}}_{\neq 0} + \sqrt{2}\xi_\alpha\underbrace{\boxed{F(\varphi,\bar{\varphi})}}_{= 0}$$

$$\delta\psi \neq 0 \quad \partial\varphi \neq 0 \Longrightarrow \text{Goldstino}$$

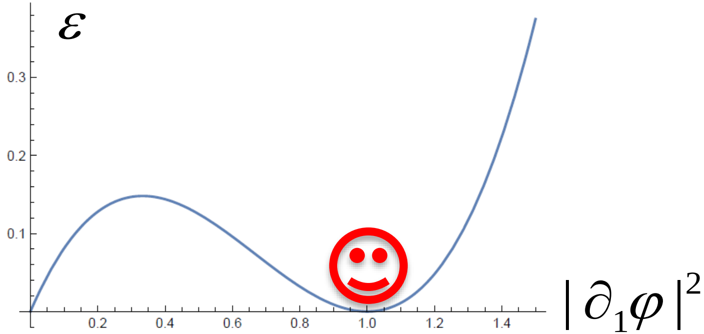
Cf) BPS solitons (*not* vacuum)

BPS domain wall $\partial_1\varphi \sim F \neq 0$



Metastable (local minimum)

$$\lambda^2 - 4\alpha k < 0$$



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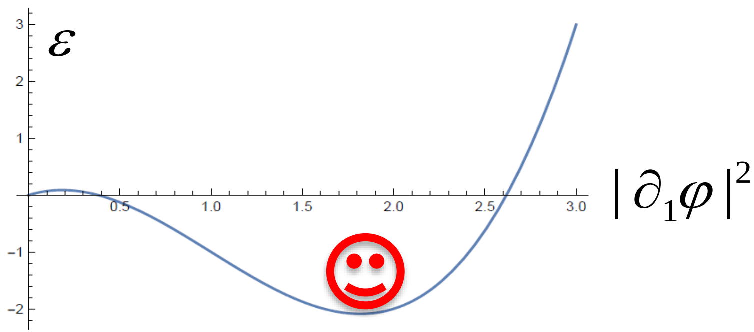

Stable (global minimum)

$$\lambda^2 - 4\alpha k > 0$$

All cases

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Local min @ ☺

$$|\partial_1\varphi|^2 = \frac{\lambda + \sqrt{\lambda^2 - 3\alpha k}}{3\alpha}$$

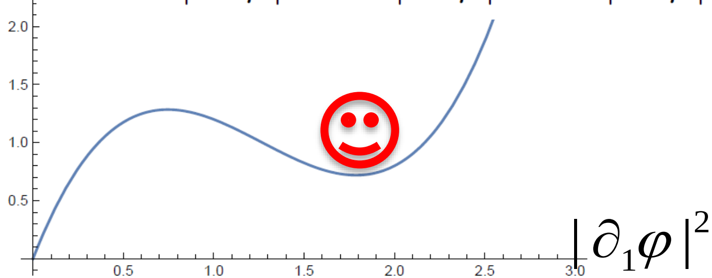
Modulate vacua

$$\varphi(x^1) = \varphi_0 e^{ipx^1}$$

$$p^2\varphi_0^2 = \frac{\lambda + \sqrt{\lambda^2 - 3\alpha k}}{3\alpha}$$

satisfying EOM

$$\mathcal{E} = \alpha|\partial_1\varphi|^6 - \lambda|\partial_1\varphi|^4 + k|\partial_1\varphi|^2$$



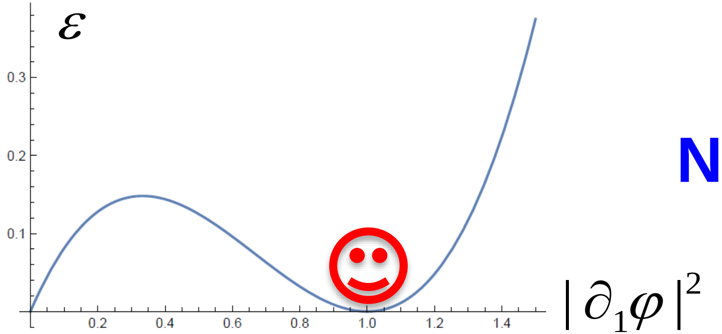
Metastable

Ordinary Goldstino

$E > 0$

SUSY algebra

$$\boxed{\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2(\sigma^m)_{\alpha\dot{\alpha}} P_m}$$

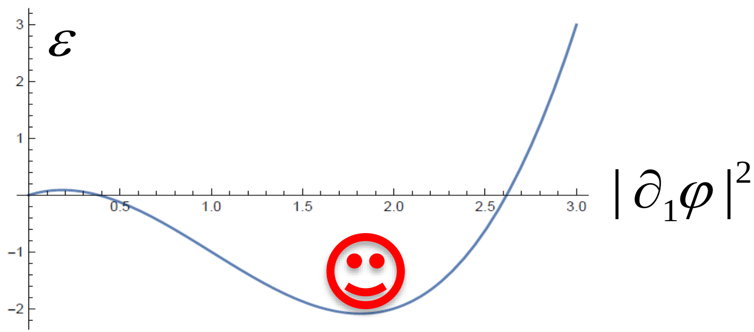


Stable

No Goldstino

$E = 0$

$$E = \langle \Psi | P^0 | \Psi \rangle \sim \sum_{\alpha=1,2} \left| Q_\alpha | \Psi \rangle \right|^2 + \sum_{\dot{\alpha}=1,2} \left| \overline{Q}_{\dot{\alpha}} | \Psi \rangle \right|^2$$



Unstable

Stable

Ghost Goldstino

$E < 0$

The unique term free from the auxiliary field problem

$$\int d^4\theta\, \Lambda_{ik\bar{j}\bar{l}}(\Phi,\Phi^\dagger) D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\dagger\,\bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger\,\bar{l}}$$

$\Lambda_{ik\bar{j}\bar{l}}$ **(2,2) tensor**

4 derivative term

I.L. Buchbinder, S. Kuzenko, and Z. Yarevskaya, NPB411, 665 (1994) **LEEA of susy**

A. T. Banin, I. L. Buchbinder, and N. G. Pletnev, PRD74, 045010 (2006) **WZW**

J. Khoury, J.-L. Lehners, and B. Ovrut, PRD83,125031 (2011) **Ghost condensate**

J. Khoury, J.-L. Lehners, and B. Ovrut, PRD84,043521 (2011) **Galileon**

M. Koehn, J.-L. Lehners, and B. A. Ovrut, PRD86, 085019 (2012) **SUGRA**

C. Adam, J.M. Queiruga, J. Sanchez-Guillen, and A.Wereszczynski, JHEP05 (2013)108 **SUSY baby Skyrmion**

C. Adam, J.M. Queiruga, J. Sanchez-Guillen, and A.Wereszczynski, Phys. Rev. D 86, 105009 (2012) **SUSY k-field theory**

S. Sasaki, M. Yamaguchi, and D. Yokoyama, PLB718, 1(2012) **suerpotential**

Expansion to quadratic order

$$x_+ = \frac{\lambda + \sqrt{\lambda^2 - 3\alpha k}}{3\alpha}$$

$$\mathcal{L}_{\mathrm{quad.}\psi} = iC\bar{\psi}\bar{\sigma}^{\hat{m}}\partial_{\hat{m}}\psi + iC_{\mathrm{mod}}\bar{\psi}\bar{\sigma}^1\partial_1\psi$$
$$+ px_+^2\left\{\alpha - (\lambda - \alpha x_+)p\varphi_0 - 2pe^{-2ipx^1}\right\}\psi\sigma^1\bar{\psi}.$$

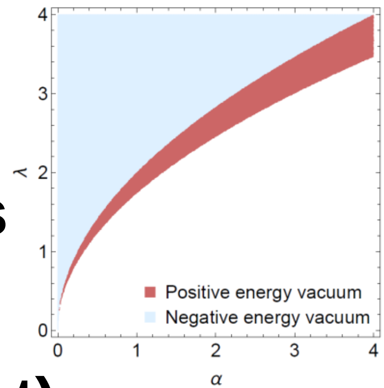
$$C \equiv -k + x_+(\lambda - \alpha x_+) = -k + \frac{1}{3\alpha}(\lambda + \sqrt{\lambda^2 - 3\alpha k})\left(\lambda - \frac{1}{3}(\lambda + \sqrt{\lambda^2 - 3\alpha k}\ )\right)$$

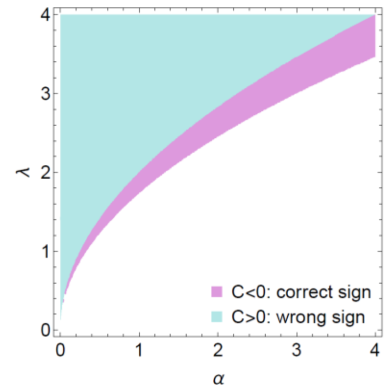
$$C_{\mathrm{mod}} \equiv C - 2\alpha x_+^2(1 - \cos(2px^1)) < C$$

$E > 0 \iff C < 0$ **Correct sign**

$E = 0 \iff C = 0$ **No dynamics**

$E < 0 \iff C > 0$ **Wrong sign (fermionic ghost)**





Plan of talk

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§ 2 Modulated vacua in bosonic theory

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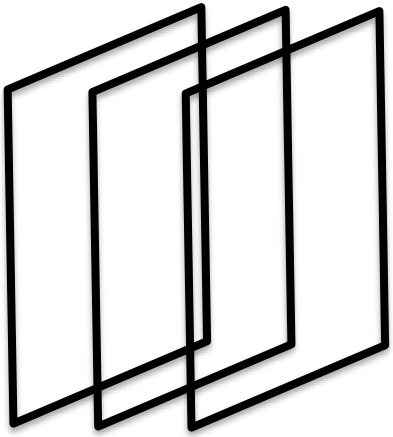
Summary

1. Relativistic modulation
   NG boson without quadratic term, gapless Higgs
2. Supersymmetric modulation
   (usual, non-dynamical, ghost) Goldstino E(>0,=0,<0)

Discussion

1. Generalized NG theorem
2. Gauging U(1) , generalized Higgs mechanism
3. More general models: higher order Skyrme model
4. Higher co-dimensional/ temporal modulation
5. Topological solitons in modulations

Topological solitons in modulation



Order Parameter Space (OPS)

$$\frac{G}{H} = U(1)_{x-\varphi} \ltimes \frac{ISO(d,1)}{ISO(d-1,1)}$$

$$\pi_1(G/H) = \mathbf{Z} \qquad \pi_{d-1}(G/H) = \mathbf{Z}$$

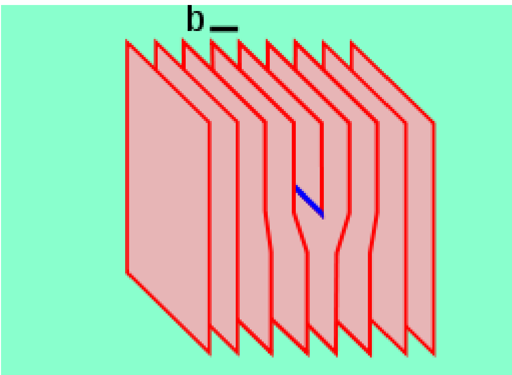
$\cup$

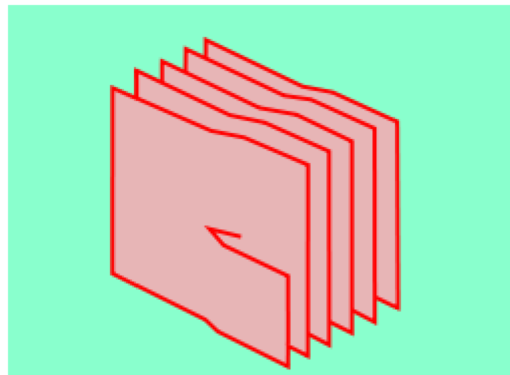
S^{d-1}

vortices

vortex⊥ modulation

vortex // modulation





Crystal

Edge dislocation

Screw dislocation

小西さん、
古稀(Koki=70 years old)、おめでとうございます。
これまで、ありがとうございます。
これからもよろしくお願いいたします。

Thank you for your attention

Notorious Problem: Auxiliary Field Problem

Chiral superfield $y^m = x^m + i\bar{\theta}\sigma^m\theta$

Auxliary field

$$\Phi(y,\theta) = \varphi(y) + \theta\psi(y) + \theta\theta F(y)$$

H.D terms→ ∂_m **acts on**

In general, F cannot be eliminated algebraically.
F may become dynamical DOF.
Is SUSY OK? Introduce of further fermion partner?

Is it always the case?
If not, how can we construct higher derivative term without this problem?

$$
\begin{aligned}
\frac{1}{16}(D\Phi)^2(\bar{D}\Phi^\dagger)^2 = \theta^2\bar{\theta}^2\Big[\ & (\partial_m\varphi)^2(\partial_n\bar{\varphi})^2 - 2\bar{F}F\partial_m\varphi\partial^m\bar{\varphi} + \bar{F}^2F^2 \\
& - \frac{i}{2}(\psi\sigma^m\bar{\sigma}^n\sigma^p\partial_p\bar{\psi})\partial_m\varphi\partial_n\bar{\varphi} + \frac{i}{2}(\partial_p\psi\sigma^p\bar{\sigma}^m\sigma^n\bar{\psi})\partial_m\varphi\partial_n\bar{\varphi} \\
& + i\psi\sigma^m\partial^n\bar{\psi}\partial_m\varphi\partial_n\bar{\varphi} - i\partial^m\sigma^n\bar{\psi}\partial_m\varphi\partial_n\bar{\varphi} + \frac{i}{2}\psi\sigma^m\bar{\psi}\left(\partial_m\bar{\varphi}\Box\varphi - \partial_m\varphi\Box\bar{\varphi}\right) \\
& + \frac{1}{2}\left(F\Box\varphi - \partial_m F\partial^m\varphi\right)\bar{\psi}^2 + \frac{1}{2}(\bar{F}\Box\bar{\varphi} - \partial_m\bar{F}\partial^m\bar{\varphi})\psi^2 \\
& + \frac{1}{2}F\partial_m\varphi(\bar{\psi}\bar{\sigma}^m\sigma^n\partial_n\bar{\psi} - \partial_n\bar{\psi}\bar{\sigma}^m\sigma^n\bar{\psi}) + \frac{1}{2}\bar{F}\partial_m\bar{\varphi}\left(\partial_n\sigma^n\bar{\sigma}^m\psi - \psi\sigma^n\bar{\sigma}^m\partial_n\psi\right) \\
& + \frac{3}{2}i\bar{F}F(\partial_m\psi\sigma^m\bar{\psi} - \psi\sigma^m\partial_m\bar{\psi}) + \frac{i}{2}\psi\sigma^m\bar{\psi}(F\partial_m\bar{F} - \bar{F}\partial_m F)\ \Big] \\
& + \sqrt{2}i\bar{\theta}^2(\partial_m\varphi)^2(\theta\sigma^n\bar{\psi})\partial_n\bar{\varphi} - \sqrt{2}i\theta^2(\partial_m\bar{\varphi})^2(\psi\sigma^n\bar{\theta})\partial_n\varphi \\
& + \sqrt{2}\theta^2F\partial_m\bar{\varphi}\left(i\bar{F}(\psi\sigma^m\bar{\theta}) + (\bar{\theta}\bar{\sigma}^m\sigma^n\bar{\psi})\partial_m\varphi\right) \\
& + \sqrt{2}\bar{\theta}^2\bar{F}\partial_m\varphi\left(-iF(\theta\sigma^m\bar{\psi}) + (\psi\sigma^m\bar{\sigma}^n\theta)\partial_n\bar{\varphi}\right) \\
& - \frac{1}{2}\bar{\theta}^2(\partial_m\varphi)^2\bar{\psi}\bar{\psi} - \frac{1}{2}\theta^2(\partial_m\bar{\varphi})^2\psi\psi + 2(\psi\sigma^m\bar{\theta})(\theta\sigma^n\bar{\psi})\partial_m\varphi\partial_n\bar{\varphi} \\
& + 2\bar{F}F(\theta\psi)(\bar{\theta}\bar{\psi}) + i(\theta\sigma^m\bar{\theta})(F\partial_m\varphi\bar{\psi}\bar{\psi} - \bar{F}\partial_m\bar{\varphi}\psi\psi) + \frac{1}{2}\theta^2F^2\bar{\psi}\bar{\psi} + \frac{1}{2}\bar{\theta}^2\bar{F}^2\psi\psi \\
& + \sqrt{2}\bar{F}F(\bar{F}(\theta\psi) + F(\bar{\theta}\bar{\psi})) + i(\psi\sigma^m\bar{\psi})(F\partial_m\bar{\varphi} - \bar{F}\partial_m\varphi). \qquad (39)
\end{aligned}
$$