Compactification of dualities with decoupled operators and 3d mirror symmetry

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ICTP

Pisa, July 21 2017

Based on: S. Benvenuti and S.G. arXiv:1706.02225 [hep-th],
S. Benvenuti and S.G. arXiv:1706.04949 [hep-th],
S. Benvenuti and S.G. arXiv:1707.05113 [hep-th].

Infrared dualities and compactification

Infrared dualities: two different field theories have the same long distance dynamics. They are ubiquitous in supersymmetric theories and are often essential in providing insights about nonperturbative effects.

In 4d $SU(N)$ SQCD with $N_f < \frac{3N}{2} = SU(N_f - N)$ with N_f flavors.

Once we have a duality at hand, we can obtain dualities in lower dimensions by compactifying the theory.
Subtleties due to nonperturbative effects (such as monopole superpotentials): Aharony, Razamat, Seiberg, Willett '13.

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3d supersymmetric theories and mirror symmetry

- Vectormultiplet: (A_μ, σ, Φ).
- Hypermultiplet: $(\Phi_1,\ \Phi_2)$.
- Monopole operators: $d\gamma = *dA$, $\mathfrak{M}^{\pm} = e^{\sigma \pm i\gamma}$

Coulomb branch: space of vacua where only vectormultiplet scalars and monopoles have a vev. It is modified by quantum corrections.

Higgs Branch: space of vacua where only hypermultiplet scalars have a vev. Unaffected by quantum corrections.

Mirror Symmetry (K. Intriligator, N. Seiberg '96)

Duality between $\mathcal{N} = 4$ theories exchanging Coulomb and Higgs branches.

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Unitarity bound and decoupled operators

In supersymmetric theories chiral operators satisfy the relation $\Delta(\mathcal{O}) \propto R(\mathcal{O})$.

Sometimes a chiral operator violates the unitarity bound and decouples in the infrared.

The IR fixed point is then described by a free sector of decoupled fields and an interacting sector. Kutasov, Parnachev, Sahakyan '03.

If the interacting sector describes IR fixed point of another theory we have a duality.

Since unitarity bounds are different in different dimensions:

$$R(\mathcal{O}) \geq \frac{2}{3}\,(d=4); \quad R(\mathcal{O}) \geq \frac{1}{2}\,(d=3)$$

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Modifying Kutasov's prescription and compactifications

Old Proposal: If $\mathcal{O}$ violates the unitarity bound we decouple it.

- The IR a central charge changes as follows: $a_{new} = a_{old} - a_{\mathcal{O}}$
- The partition functions change as follows: $Z_{new} = Z_{old}/\Gamma(\mathcal{O})$
- Unclear how chiral ring relations are affected.

Our Proposal: we introduce by hand a chiral multiplet β and turn on the superpotential $\delta\mathcal{W} = \beta\mathcal{O}$ (i.e. we flip $\mathcal{O}$).

- In the modified theory $a_{new} = a_{old} + a_{\beta}$ and $Z_{new} = Z_{old}\Gamma(\beta)$ (same as before since $a_{\beta} + a_{\mathcal{O}} = 0$ and $\Gamma(\beta)\Gamma(\mathcal{O}) = 1$).
- We have an ordinary lagrangian description of the interacting sector alone, so we can study the chiral ring of the theory.
- F-term for β sets $\mathcal{O} = 0$ in the chiral ring in **every** dimension.

Duality survives in lower dimension only for the modified theory!

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Argyres-Douglas theories and their lagrangian

Argyres-Douglas (AD) theories were found at singular points on the Coulom Branch of $\mathcal{N} = 2$ gauge theories.

- 4d $\mathcal{N} = 2$ SCFT's describing vectormultiplets interacting with electrons and monopoles.
- Labelled by ADE groups and the number of d.o.f. scales like N at large N.
- Their Coulomb Branch coordinates have fractional dimension, so they do not have a lagrangian description.
- The class $\mathcal{S}$ realization suggests that the dimensional reduction of AD theories is equivalent to abelian gauge theories. Nanopoulos, Xie '10.

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AD theories from SQCD

Let's consider $\mathcal{N}=2$ $SU(N)$ SQCD with $2N$ flavors and add to it a "dual meson" A^{ij} with superpotential

$$\mathcal{W}=\sum_i \tilde{q}_i \Phi q_i + A^{ij}\tilde{q}_i q_j$$

If we now give maximal nilpotent vev to A^{ij} we find SQCD with one flavor and

Maruyoshi, Song '16.

$$\mathcal{W}=\tilde{q}\Phi^{2N}q+\sum_{r\geq 0}\alpha_r \tilde{q}\Phi^r q$$

All the Casimirs $\mathrm{Tr}\,\Phi^k$ and some α_j's violate unitarity and decouple. The a and c central charges in the IR match those of AD A_{2N-1} theories. α_j $(j\leq N-2)$ identified with CB operators.

The IR fixed point of this theory is AD theory plus free fields!

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Chiral ring stability

1. Consider SQED with 2 flavors and $\mathcal{W} = A\tilde{q}q + B\tilde{p}p + A^2\tilde{p}p$
   Its mirror dual is the same model with Aharony et al. '97.

   $$\mathcal{W} = S_1\tilde{q}q + S_2\tilde{p}p + AS_1 + BS_2 + A^2S_2 \to 0$$

   Its mirror is then $\mathcal{W} = A\tilde{q}q + B\tilde{p}p$.
2. Consider SQED with 1 flavor and $\mathcal{W} = \alpha\mathfrak{M}^+\mathfrak{M}^-$
   Its mirror is $\mathcal{W} = XYZ + \alpha YZ = (X + \alpha)YZ$

In (1) F-term for B sets $\tilde{p}p = 0$. In (2) there is the chiral ring relation $\mathfrak{M}^+\mathfrak{M}^- = 0$.

Chiral ring stability

If in the theory obtained removing a superpotential term $\int d^2\theta\mathcal{O}$ we have the relation $\mathcal{O} = 0$; that term should be dropped.

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The lagrangian for Argyres-Douglas

Start from Maruyoshi-Song lagrangian for A_3 AD theory: $SU(2)$ SQCD with

$$\mathcal{W} = \tilde{q}\Phi^4 q + \alpha_0 \tilde{q} q + \alpha_1 \tilde{q}\Phi q + \alpha_2 \tilde{q}\Phi^2 q$$

$\operatorname{Tr}\Phi^2$, α_1 and α_2 decouple, so we consider

$$\mathcal{W} = \tilde{q}\Phi^4 q + \alpha_0 \tilde{q} q + \beta \operatorname{Tr}\Phi^2$$

Since $\Phi^2 = -\frac{1}{2}\operatorname{Tr}\Phi^2 I_2$, $\tilde{q}\Phi^4 q \propto (\operatorname{Tr}\Phi^2)^2 \tilde{q} q$ and by chiral ring stability, we can drop the first term.

The resulting model flows in the IR to AD theory without extra free sectors and there are no unitarity bound violations!

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4d vs 3d and abelianization

Keeping the term $\tilde{q}\Phi^4 q$, one would impose $2R(q) + 4R(\Phi) = 2$. In 4d cancellation of $U(1)_R$ anomaly imposes the same constraint! In 3d there are no anomalies, so we gain a $U(1)$ symmetry by discarding $\tilde{q}\Phi^4 q$.

In 3d $U(1)_R$ is determined by minimizing the S^3 partition function:

Jafferis '10.

$$\mathcal{Z}_{S^3} = \int d\mu_{Haar} \prod_{\phi_i} s(1 - R_{\phi_i}, \mu)$$

We find $R(q) = 1/2$ and $R(\Phi) = 0$. The contribution from Φ cancels against the Haar measure and the partition function neatly reduces to that of SQED with two flavors!

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Susy enhancement and Maruyoshi-Song model in 3d

We found a duality between $SU(2)$ SQCD with one flavor

$$\mathcal{W} = \alpha_0 \tilde{q} q + \beta \,\mathrm{Tr}\,\Phi^2$$

and $\mathcal{N} = 4$ SQED with two flavors ($\mathcal{W} = \tilde{q}_i \phi q^i$)

In SQED we have the relation $\mathfrak{M}^+\mathfrak{M}^- = \phi^2$. Borokhov, Kapustin, Wu '02.

In the $SU(2)$ theory: $B = \epsilon^{ab} q_a (\Phi q)_b$, $\tilde{B} = \epsilon_{ab} (\tilde{q}\Phi)^a \tilde{q}^b$

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only α_2 and $\mathrm{Tr}\,\Phi^2$ decouple in 3d, but not α_1!

This theory is equivalent to $\mathcal{N} = 2$ SQED with $\mathcal{W} = 0$.

There is no supersymmetry enhancement and the duality with AD is lost going to three dimensions!

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Outlook

We understood compactification of dualities with unitarity bound violations and clarified why AD theories are related to abelian gauge theories in 3d. In the process we found several interesting phenomena which deserve further study:

- We can understand unitarity bound violations by introducing singlets and suitable superpotential couplings,
- Sometimes the lagrangian can be "simplified" thanks to chiral ring stability,
- Nonabelian gauge theories in 3d can abelianize and are actually equivalent to abelian theories.

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