In this work, we investigate the spontaneous emission process and its detrimental effects on coherent free-electron laser (FEL) emission. In our model, the electron dynamics are described by a discrete Wigner distribution coupled to Maxwell’s equations. For a FEL operating in the quantum regime of single photon recoil, insights on the variation of momentum distribution, bunching factor, and radiation power are presented. We also show a simple differential equation that describes the evolution of the radiated power in the linear regime. It is shown that the essential results of this work agree with those predicted by a density matrix approach.

1. The classical “conventional” FEL

- High-gain FEL mechanism involves two processes:
  - Radiation fields $\mathbf{E}$, $\mathbf{B}$ bunch electrons by Lorentz Force
  - Bunched electrons drive radiation (Maxwell’s wave Eq. $F = -\varepsilon_0 \mathbf{E} 	imes \partial \mathbf{B}/\partial t$)

  \[ F = -\varepsilon_0 \mathbf{E} \times \partial \mathbf{B}/\partial t \]

- High-gain FEL is described by [1]:
  \[ \frac{d\theta}{dz} = (\kappa_w + k) z - \omega t \]
  Pandemonietic phase
  \[ \frac{d\beta}{dz} = -\left(A \eta - \omega t \right) \]
  Scaled energy change
  \[ |\Delta |^2 \text{ or } \eta \text{ for } \eta_{\text{max}} \text{ or } \eta_{\text{min}} \]
  Scaled EM field intensity
  \[ z = z_{\text{fwhm}} \text{ or } \eta_{\text{fwhm}} \]
  Scaled position in wigglar
  \[ \rho = 1 \left( \omega_{\text{L}} / \omega_{\text{BE}} \right) (\omega_{\text{L}} / \omega_{\text{BE}})^2 \]
  Classical FEL parameter

2. The quantum regime of FEL: What and how to reach?

3. Discrete Wigner model for quantum FEL

\[ \beta \text{ is assumed to be a periodic variable in } (0,2\pi). \] This hypothesis assures that the conjugate momentum variable $p$ is discrete [4].

- The electron wavefunction $\Psi(z,\theta) = \frac{1}{\sqrt{2\pi}} \sum_{n=1}^{\infty} c_n(z) |n\rangle$.

- Discrete changes of momentum $n=1\ldots$ Eigen-value equation: $|n\rangle = \exp(i\hat{n}) |n\rangle$.

- The electron is described by a Schrödinger-like equation:

\[ \frac{\partial \Psi(z,\theta)}{\partial z} = H \Psi(z,\theta) \quad H = \frac{\beta^2}{2} - (\beta \eta + c.c.) \]

- The coupled equations that describes the FEL including the spontaneous emission:

\[ \frac{d\beta}{dz} = \frac{\beta}{\beta_0} \left( A \eta - \omega t \right) \quad \beta_0 \text{ initial momentum} \]

- From the above equations, we get the coupled equations in the form of:

\[ w_s(\theta,\phi) = \frac{1}{2\pi} \sum_{m=0}^{\infty} w_m \tilde{w}_m e^{i m \phi} \]

- A distribution function $Q$ is defined as [5]:

\[ Q(\theta,\phi) = \sum_{m=0}^{\infty} \left[ w_m(\theta,\phi) + w_{m+1}(\theta,\phi) \right] \]

4. Results 1 (Field dynamics)

- Wigner function is periodic in $\theta$ and is expressed by:

\[ w_s(\theta,\phi) = \frac{1}{2\pi} \sum_{m=0}^{\infty} w_m \tilde{w}_m e^{i m \phi} \]

- From above equations, we get the coupled equations in the form of:

\[ \frac{d\beta}{dz} = -\left(A \beta + c.c.\right) - D \phi \]

\[ P_0 = \frac{w_0}{2\pi} \]

\[ P_1 = \frac{w_1}{2\pi} + D \phi \]

5. Conclusion

We have presented a discrete Wigner model for the quantum FEL, including spontaneous emission. This model describes the momentum as a discrete variable, as it should be assumed spatial periodic boundary conditions. We have shown that, in the quantum regime, the equations reduce to these for two-momentum states coupled to the coherent radiation field. Spontaneous emission is there interpreted as responsible for the loss of coherence (i.e. bunching) and the transfer of electrons into and out of the two momentum states via rate equation terms.

6. References