Neutrino masses and ordering via gravitational waves, photon and neutrino detections

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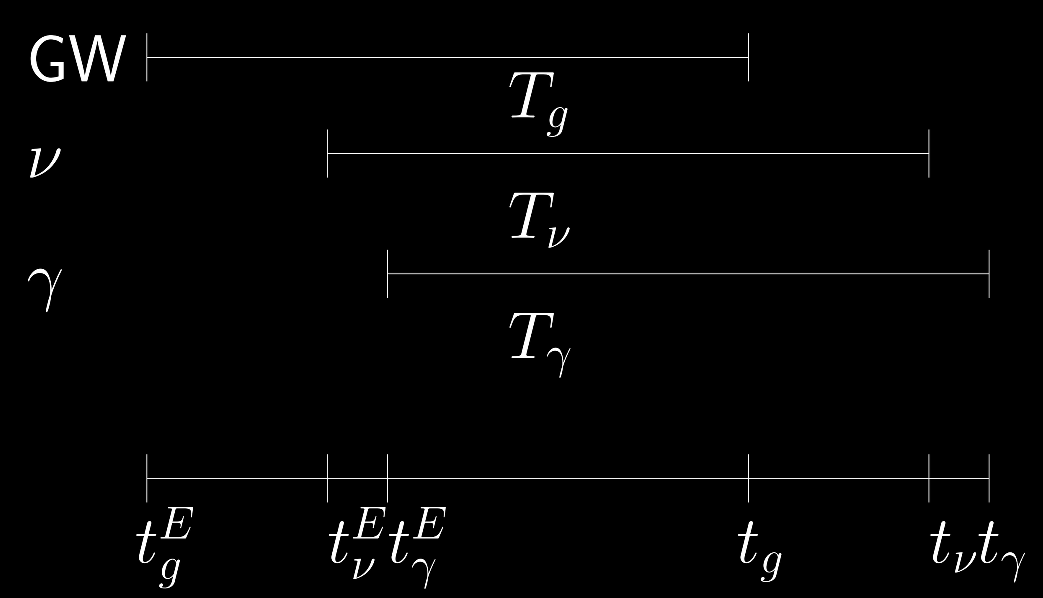
Multi-Messenger Astronomy

What can we learn from the revolutionary discovery of LIGO [1]? Let us consider a potential observation of an astrophysical catastrophe. Using the same notation of [2], we denote with $T_g \equiv L/v_g$, $T_{\nu_i} \equiv L/v_{\nu_i}$ and $T_\gamma \equiv L/v_\gamma$ respectively the time of propagation of a gravitational wave (GW), a given neutrino mass eigenstate and photons with group velocities v_g, v_{ν_i}, and v_γ. In the figure below we imagine a GW is emitted at the time t_g^E from a source at distance L and detected on Earth at t_g. Similarly, we have emission and detection times for photons and neutrinos. For instance, astrophysical catastrophes like the merging of a neutron star binary or the core bounce of a core-collapsed supernova (SN) are believed to follow this pattern.



We express the time delay between the arrival of different messengers as:

$$\Delta t_{\nu_i\nu_j} = \Delta t_i - \Delta t_j = \frac{\Delta m_{ij}^2 c^4}{2E^2} T_0 \quad \text{with} \quad T_0 = \frac{L}{c} \,,$$

with $\Delta m_{ij}^2 = m_i^2 - m_j^2$ and to leading order in m^2c^4/E^2. In this limit the time intervals don't depend on the absolute mass scale.

Table 1: *Benchmark time lapses for ν_1, ν_2 and ν_3 respectively. We consider a distance of 1 (10) Mpc and a neutrino energy of $E = 5$ MeV.*

m_{min} [eV]	Δt_{ν_i} [s]	
	NO	IO
0	0	$4.91 \cdot 10^{-3}(10^{-2})$
	$1.54 \cdot 10^{-4}(10^{-3})$	$5.06 \cdot 10^{-3}(10^{-2})$
	$5.06 \cdot 10^{-3}(10^{-2})$	0
0.01	$2.06 \cdot 10^{-4}(10^{-3})$	$5.11 \cdot 10^{-3}(10^{-2})$
	$3.60 \cdot 10^{-4}(10^{-3})$	$5.27 \cdot 10^{-3}(10^{-2})$
	$5.27 \cdot 10^{-3}(10^{-2})$	$2.06 \cdot 10^{-4}(10^{-3})$

Neutrino Ordering

The neutrino ordering is unknown: it could be normal ($m_1 < m_2 < m_3$) or inverted ($m_3 < m_1 < m_2$). From a multi-messenger scenario like the one that we study here, in addition to the time information, also the ratio between the amplitudes of the different neutrinos reaching the detector can be measured. Since the distances considered here are very large, neutrinos will reach the detector incoherently such that the probability reads:

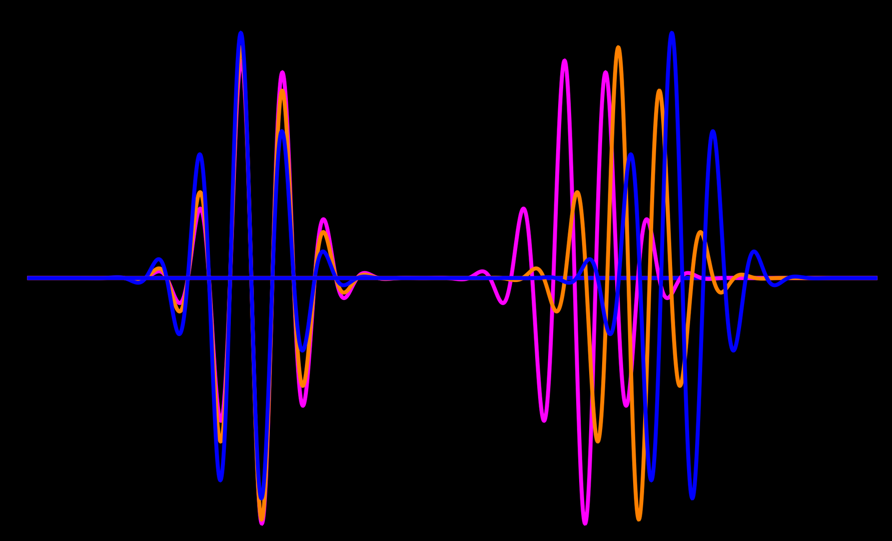
$$P\left(\nu_\alpha \to \nu_\beta\right) = \sum_i |U_{\alpha i}|^2 \, |U_{\beta i}|^2 \,,$$

where α and β are flavour eigenstates.

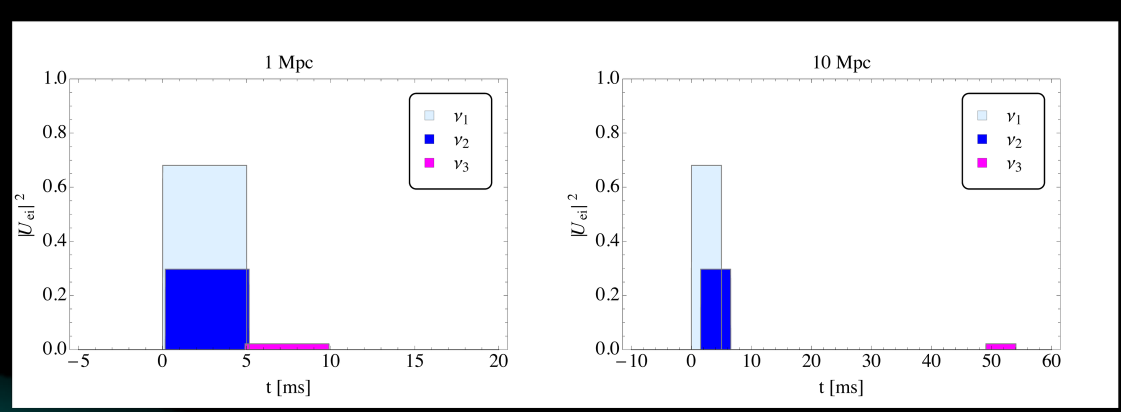
In fact, this expression holds true whenever the time arrival differences among the three mass eigenstates is smaller than the detector time resolution. However, when $\Delta t_{\nu_i\nu_j}$ is larger than the detector resolution, then each mass eigenstates ν_i can be detected independently and will interact with the detector with probability

$$P\left(\nu_\alpha \to \nu_\beta\right)_i = |U_{\alpha i}|^2 \, |U_{\beta i}|^2 \,.$$

This probability takes into account that the three mass eigenstates becomes decoherent as shown in the picture below.



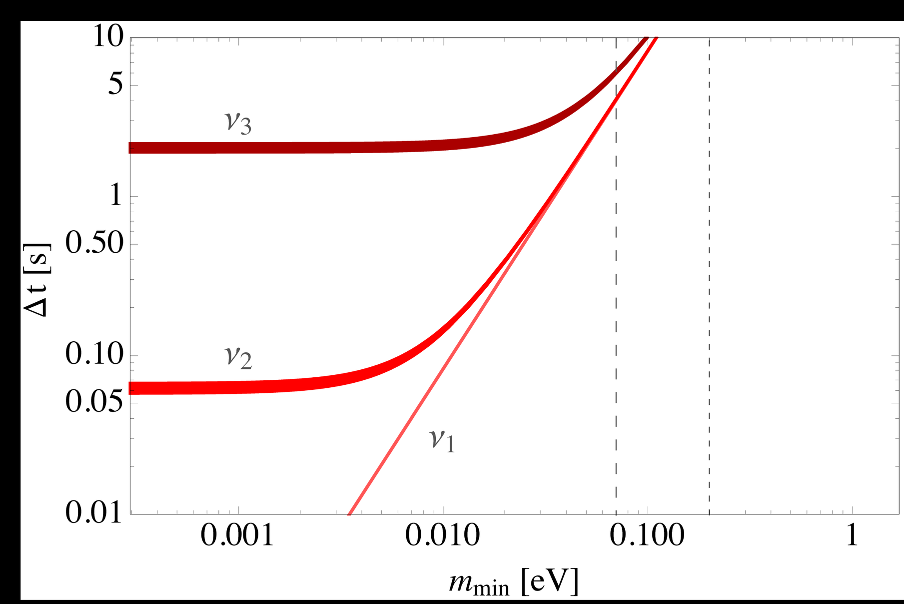
Below we illustrate a possible pattern of neutrino detection. Depending on the source, its distance and the experimental time sensitivity the figure shows that, at least in principle, one can observe interesting time-patterns reflecting the neutrino ordering and mixing.



Schematic representation of the square root of the probability given above of detecting flavor state ν_e if the source emits a short burst of ν_e as a function of time. The top panel is for 1 Mpc and the lower is for 10 Mpc at an energy of 5 MeV. For definiteness we assume normal ordering and each bin corresponds to a fiducial collective time of 5 ms.

Absolute Neutrino mass

For signals coming from sources very far away, the time delay between the arrival of the gravitational wave and the fastest neutrino, will potentially be large enough to be disentangled from uncertainties in emission and detection. Such a measurement, will give constraints on the absolute mass of the neutrinos. In the figure below, we transform the known square mass differences of the neutrinos into the time delays that would be measured from a distance of the 400 Mpc. This is the distance of the event measured by LIGO (GW150914).



The time lapses for normal ordering between the three neutrino mass eigenstates. We use the distance of the source of GW150914, $L = 400$ Mpc, a neutrino energy $E_\nu = 5$ MeV and the 3σ uncertainty in the oscillation parameters.

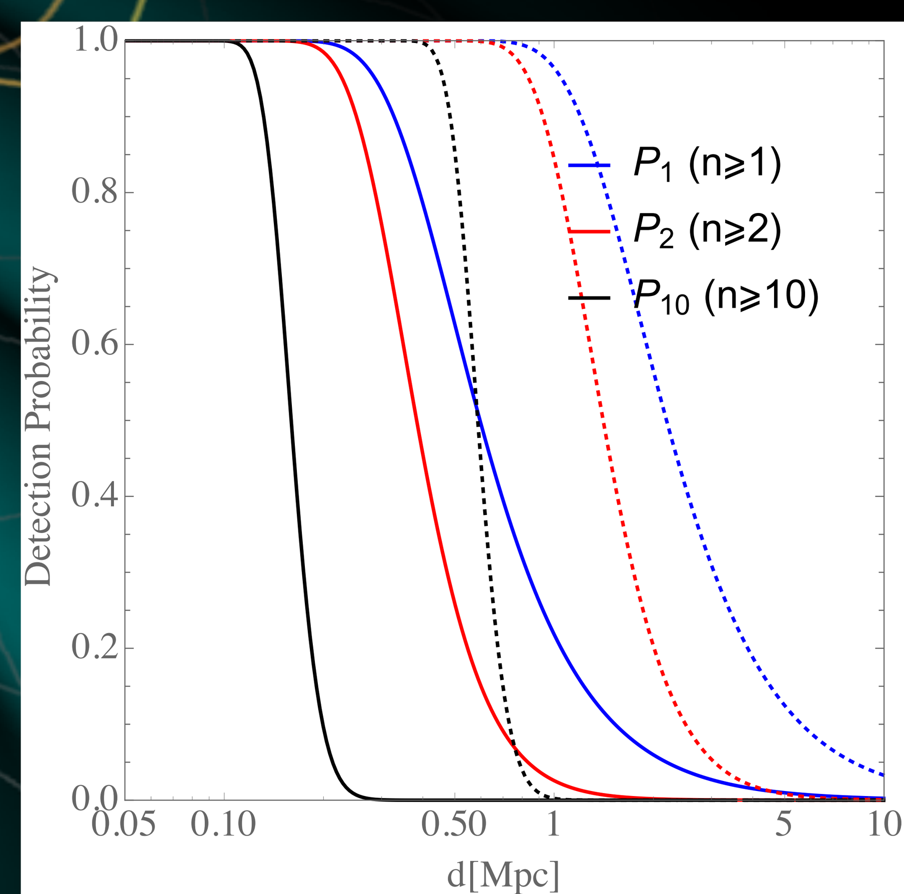
Probability of detection

Three parameters are vital to increase the time lapse between mass eigenstates: the distance from the source L, the energy of the emitted neutrino, E_ν, and the absolute neutrino mass m_{min}. Conversely, the larger the distance is, the smaller is the rate. The neutrino detection rate over the relevant time interval reads:

$$\frac{dN}{dt} = n_p \int_{E_e^{th}} dE_e \int_{E_\nu^{th}} dE_\nu \, \mathcal{F}(E_\nu, t) \, \sigma'(E_e, E_\nu) \, \epsilon,$$

where n_p is the number of protons in the target, $E_{\nu,e}$ are respectively the (anti)neutrino and the (electron) positron energy of the event, $\mathcal{F}(E_\nu, t)$ is the flux per unit time, area and energy and ϵ is the detector efficiency. Finally $\sigma'(E_e, E_\nu) = d\sigma/dE_e$ is the differential cross section of the process under study. We will assume the efficiency of the detector to be 100% for energies larger than the energy threshold of the detector, $E_\nu > E_\nu^{th}$.

For low rates it is useful to estimate the actual detection probability as function of the distance from the source. To assess this, we use the Poisson probability to detect n events as $P_n = \lambda^n e^{-\lambda}/n!$ where λ is the expected number of events by IBD for example.



Detection probability of neutrinos versus distance from the source to Hyper-Kamiokande (solid lines) and to a hypothetical future 5 Mton experiment (dotted lines) using a $7-30$ MeV energy range. Blue, red and black curves represent the detection probability resulting in requiring observation of at least one, two, and ten events per burst, respectively.

[1] B. P. Abbott *et al.* [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. **116** (2016) 6, 061102.

[2] A. Nishizawa and T. Nakamura, Phys. Rev. D **90** (2014) 4, 044048.