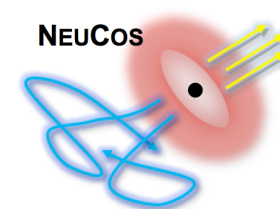


Neutrino Tomography (Generalities)

Walter Winter
DESY, Zeuthen

ISAPP summer institute: Using particle physics to
understand and image the earth

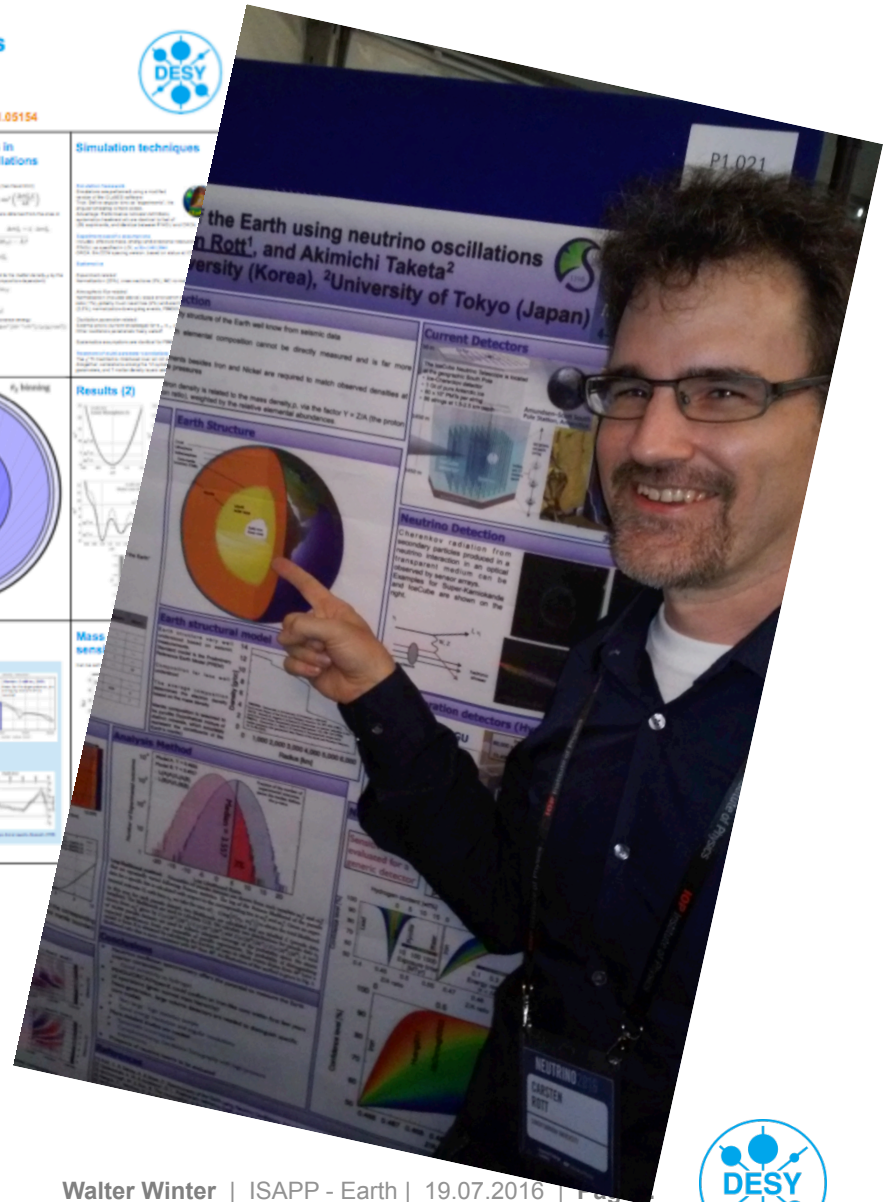
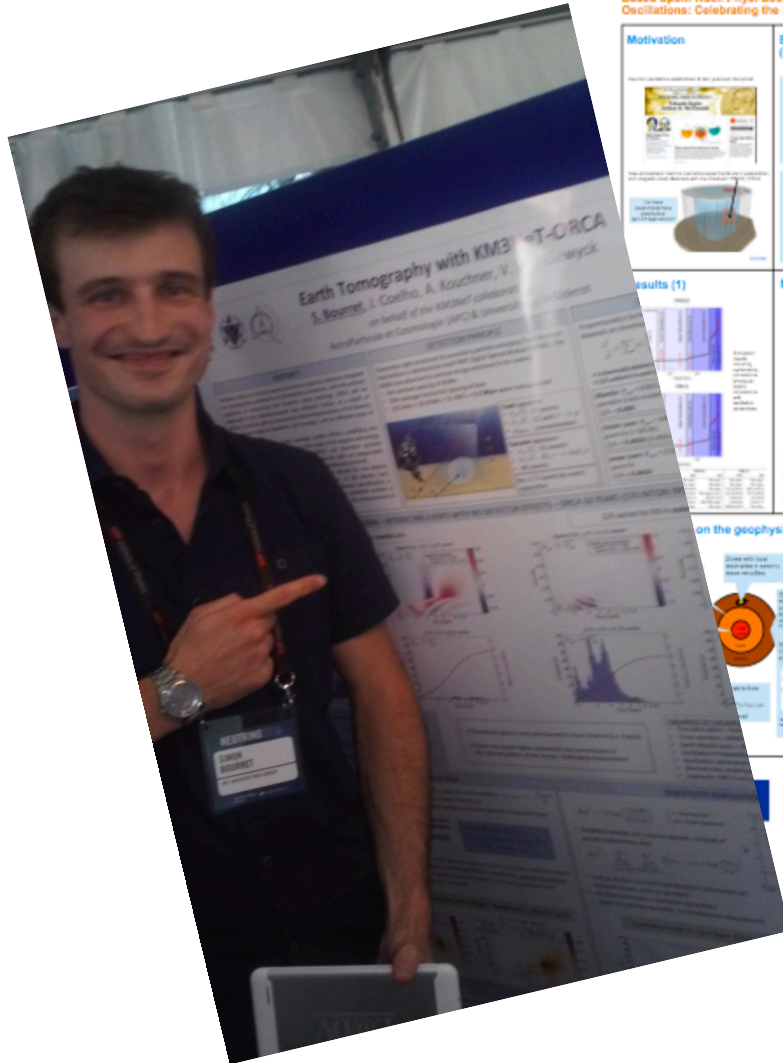
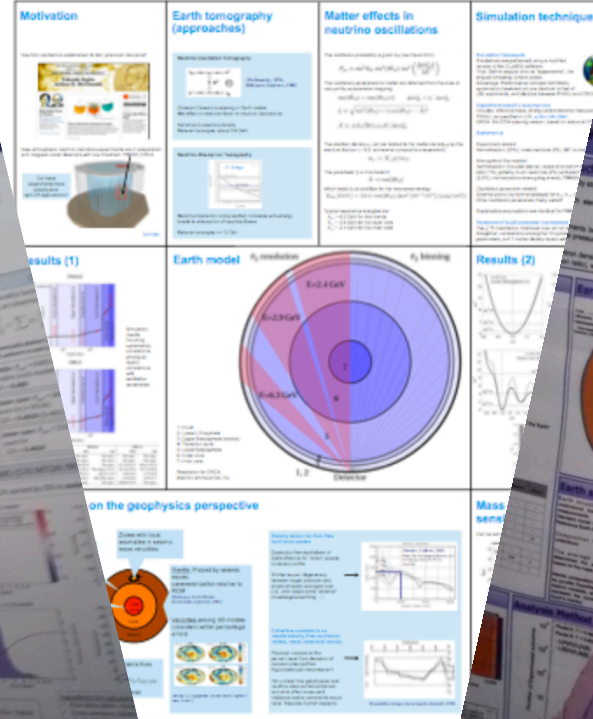
GSSI, l'Aquila, Italy
July 11-21, 2016



Impressions from Neutrino 2016

Atmospheric Neutrino Oscillations for Earth Tomography.

Walter Winter, DESY Zeuthen
Based upon: Nucl. Phys. 5908 (2016) 250 (in special issue "Neutrino Oscillations: Celebrating the Nobel Prize in Physics 2015"), arXiv:1511.05154



Contents

- > Introduction to neutrino oscillations
- > Neutrino tomography of Earth (1):
Approaches and ideas
- > Neutrino oscillations in matter
- > Neutrino tomography of Earth (2):
Towards realistic applications
- > Summary
- > Open issues/discussion

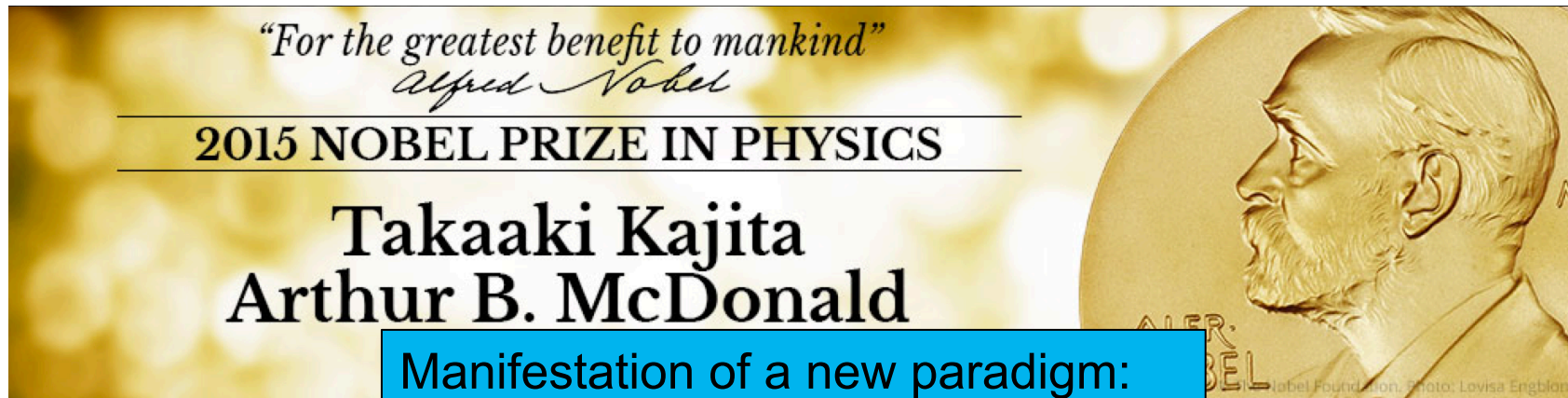
More to come on
atmospheric neutrinos
and detectors in
Veronique van
Elewyck's talk



Recap: Neutrino oscillations (a mini review)



Nobel prize 2015: Neutrino oscillations



Ill: N. Elmehed. © Nobel Media 2015

2015 Nobel Prize in Physics

The [Nobel Prize in Physics 2015](#) was awarded jointly to [Takaaki Kajita](#) and [Arthur B. McDonald](#) "for the discovery of neutrino oscillations, which shows that neutrinos have mass".

→ [Read more about the prize](#)

Manifestation of a new paradigm:
precision physics in lepton sector

Where else can this lead us?
Neutrino tomography of Earth?



Illustration: © Johan Jamesstad/The Royal Swedish Academy of Sciences

They Solved the Neutrino Puzzle

Takaaki Kajita and Arthur B. McDonald solved the neutrino puzzle and opened a new realm in particle physics. They were key scientists of two large research groups, Super-Kamiokande and Sudbury Neutrino Observatory, which discovered the neutrinos mid-flight metamorphosis.

→ [Read more](#) (pdf)



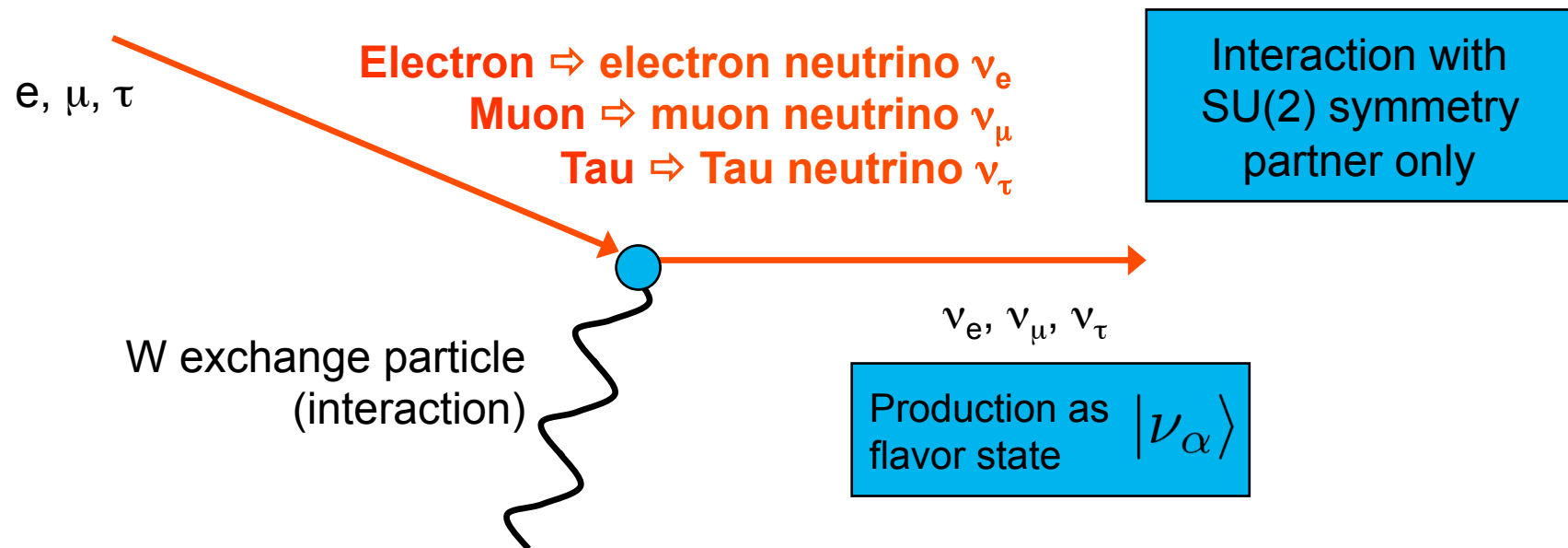
"I Gave My Wife a Hug!"

"It's ironic, in order to observe the sun you have to go kilometers under ground. That's not what you would expect." An interview with Arthur B. McDonald, awarded the 2015 Nobel Prize in Physics.



Neutrino production/detection

- > Neutrinos are only produced and detected by the weak interaction:



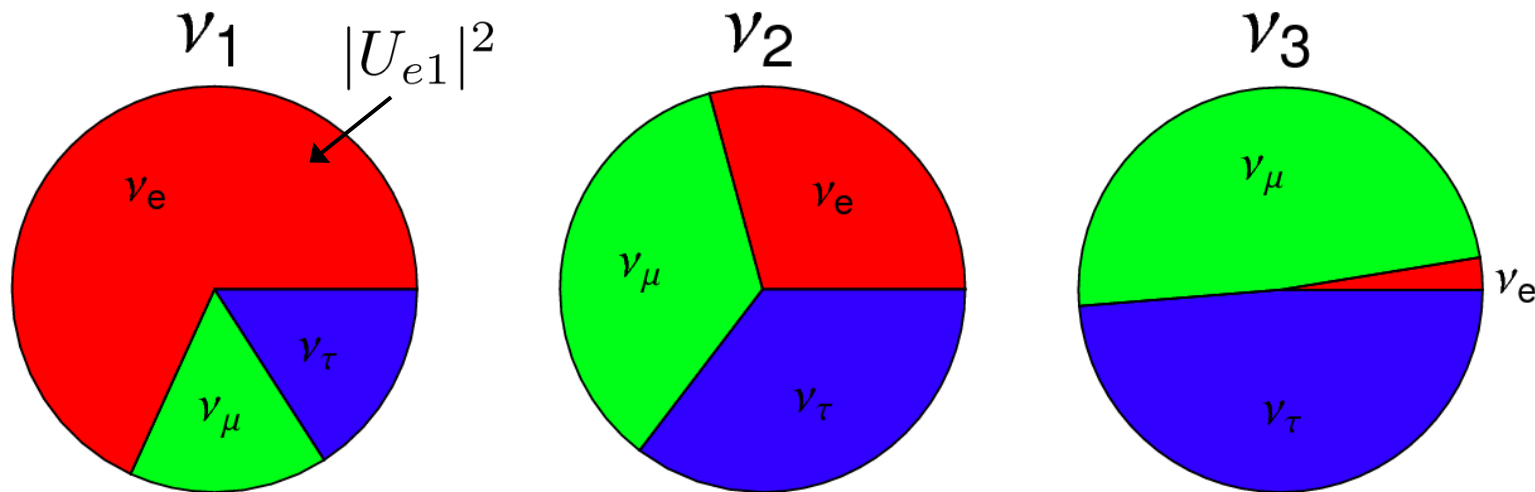
- > The dilemma: One cannot assign a mass to the flavor states ν_e, ν_μ, ν_τ !

Which mass do the neutrinos have?

> There is a set of neutrinos ν_1, ν_2, ν_3 , for which a mass can be assigned. $|\nu_i\rangle$

> **Mixture** of flavor states:

$$|\nu_\alpha\rangle = \sum_{k=1}^3 U_{\alpha k}^* |\nu_k\rangle$$



$\sin^2 2\theta_{13} = 0.1, \delta = \pi/2$

- > Not unusual, know from the Standard Model for quarks
- > However, the mixings of the neutrinos are much larger!



Neutrino oscillations (two flavor limit)

> Only two parameters:

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

Lower limit for neutrino mass!

$$\Delta m^2 = \Delta m_{21}^2 \equiv m_2^2 - m_1^2$$

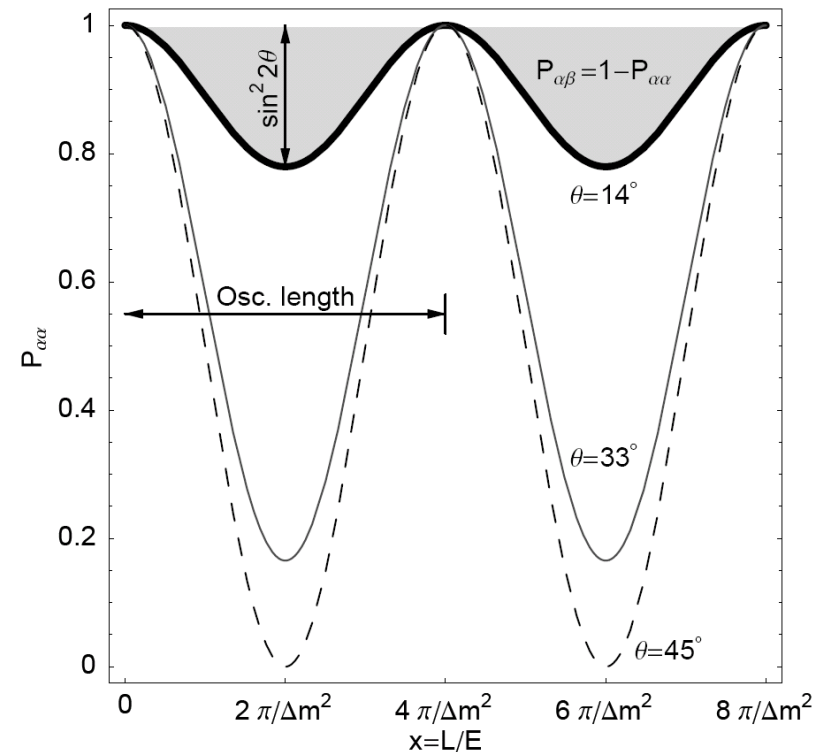
> **Disappearance** or **survival** probability

L: Baseline (distance source-detector)
E: Neutrino energy

$$P_{\alpha\alpha} = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

Appearance probability

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$



Three flavors: Mixings

- > Use same parameterization as for CKM matrix (quark sector)

Potential CP violation $\sim \theta_{13}$

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$(s_{ij} = \sin \theta_{ij} \quad c_{ij} = \cos \theta_{ij})$

$$= \left(\begin{array}{c} \text{[Image of neutrino oscillation experiment]} \end{array} \right) \times \left(\begin{array}{c} \text{[Image of nuclear reactor]} \end{array} \right) \times \left(\begin{array}{c} \text{[Image of the Sun]} \end{array} \right)$$

Pontecorvo-Maki-Nakagawa-Sakata matrix

- > Neutrinos \Leftrightarrow Anti-neutrinos: $\mathbf{U} \Leftrightarrow \mathbf{U}^*$ (neutrino oscillations)
- > If neutrinos are their own anti-particles (Majorana neutrinos):
 $\mathbf{U} \Leftrightarrow \mathbf{U} \text{diag}(1, e^{i\alpha}, e^{i\beta})$ - do enter $0\nu\beta\beta$, but not neutrino oscillations

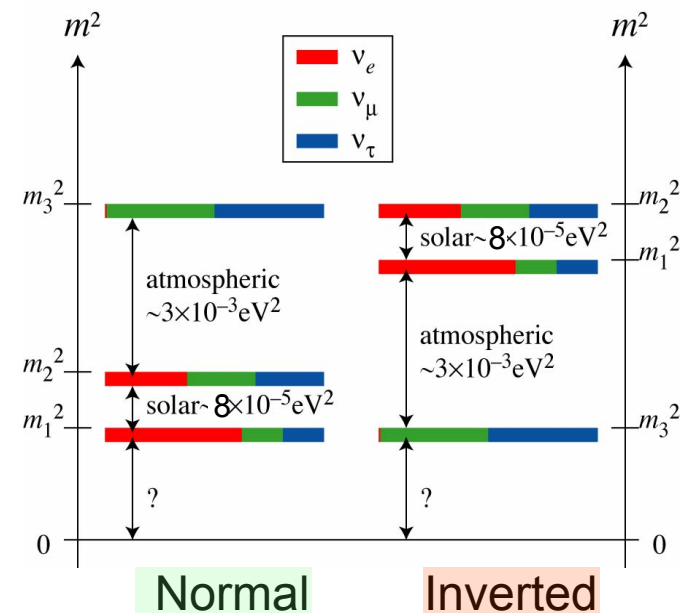


Three active flavors: Masses

- > Two independent mass squared splittings, typically Δm_{21}^2 (solar) Δm_{31}^2 (atmospheric)

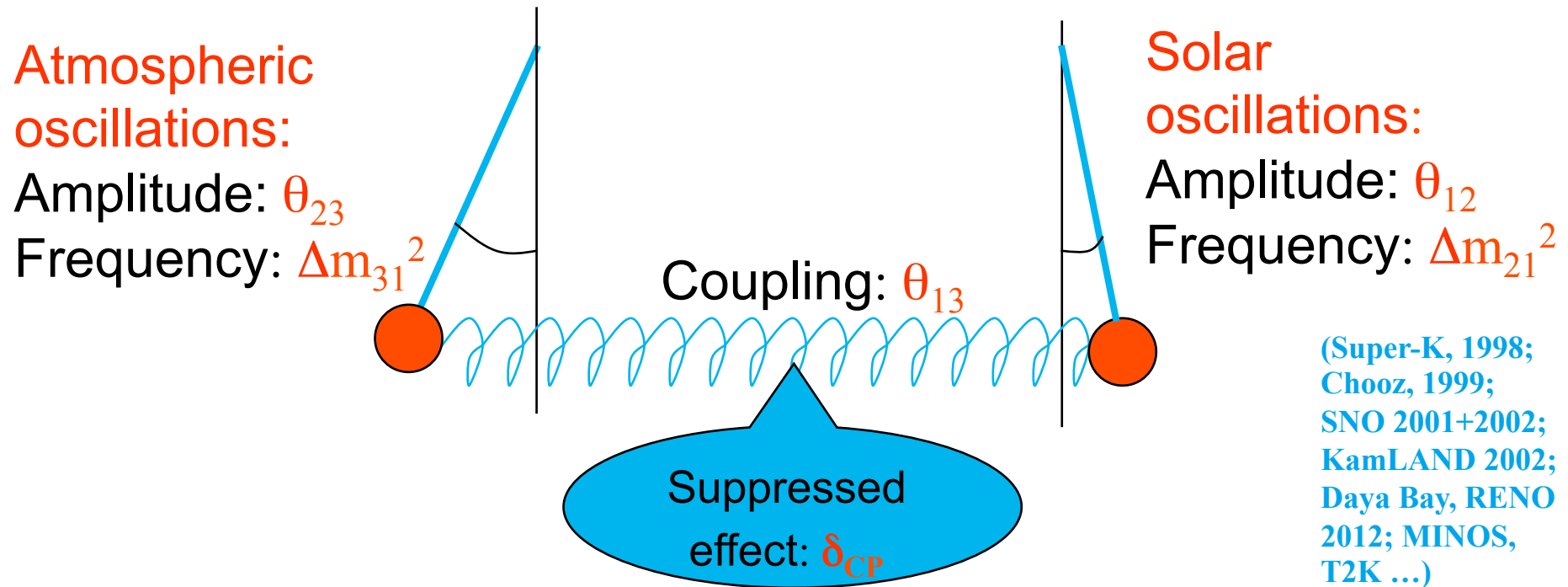
Will be relevant for neutrino oscillations!

- > The third is given by $\Delta m_{32}^2 = \Delta m_{31}^2 - \Delta m_{21}^2$
- > The (atmospheric) mass **ordering** (hierarchy) is unknown (normal or inverted)
- > The absolute neutrino mass scale is unknown ($< eV$)



Three flavors: Summary

- > Three flavors: 6 params (3 angles, one phase; 2 x Δm^2)

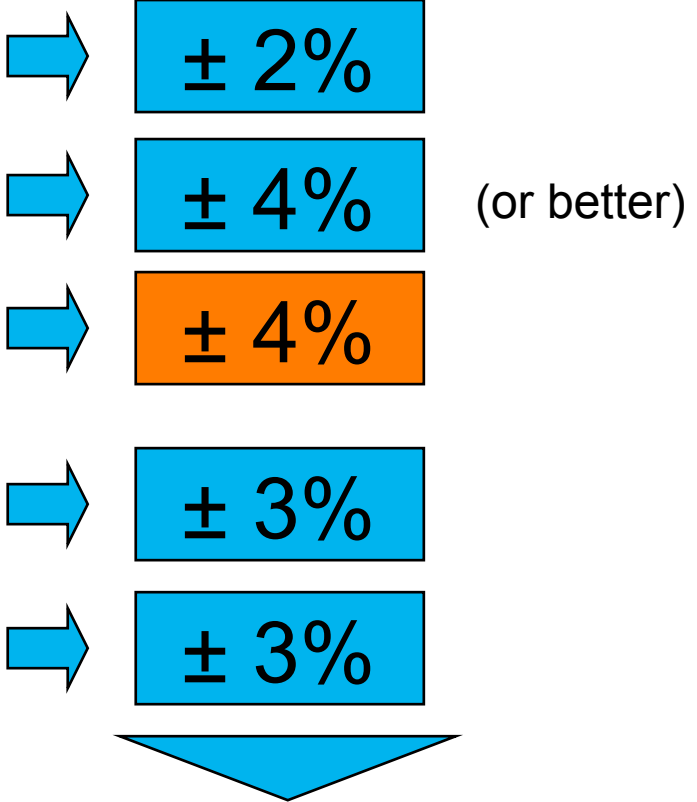


- > Describes solar and atmospheric neutrino anomalies, as well as reactor antineutrino disappearance!

Precision of parameters?

	bf μ $\pm 1\sigma$	3σ range
$\sin^2 \theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.271 \rightarrow 0.346$
$\theta_{12}/^\circ$	$33.57^{+0.77}_{-0.75}$	$31.38 \rightarrow 36.01$
$\sin^2 \theta_{23}$	$0.446^{+0.007}_{-0.007} \oplus 0.587^{+0.032}_{-0.037}$	$0.366 \rightarrow 0.663$
$\theta_{23}/^\circ$	$41.9^{+0.4}_{-0.4} \oplus 50.0^{+1.9}_{-2.2}$	$37.2 \rightarrow 54.5$
$\sin^2 \theta_{13}$	$0.0229^{+0.0020}_{-0.0019}$	$0.0170 \rightarrow 0.0288$
$\theta_{13}/^\circ$	$8.71^{+0.37}_{-0.38}$	$7.50 \rightarrow 9.78$
$\delta_{CP}/^\circ$	265^{+56}_{-61}	$0 \rightarrow 360$
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.45^{+0.19}_{-0.16}$	$6.98 \rightarrow 8.05$
$\frac{\Delta m_{31}^2}{10^{-3} \text{ eV}^2}$ (N)	$+2.417^{+0.013}_{-0.013}$	$+2.247 \rightarrow +2.623$
$\frac{\Delta m_{32}^2}{10^{-3} \text{ eV}^2}$ (I)	$-2.410^{+0.062}_{-0.062}$	$-2.602 \rightarrow -2.226$

NuFIT 1.2 (2013)



Open issues:

- Degeneracies (mass ordering, octant)
- CP phase

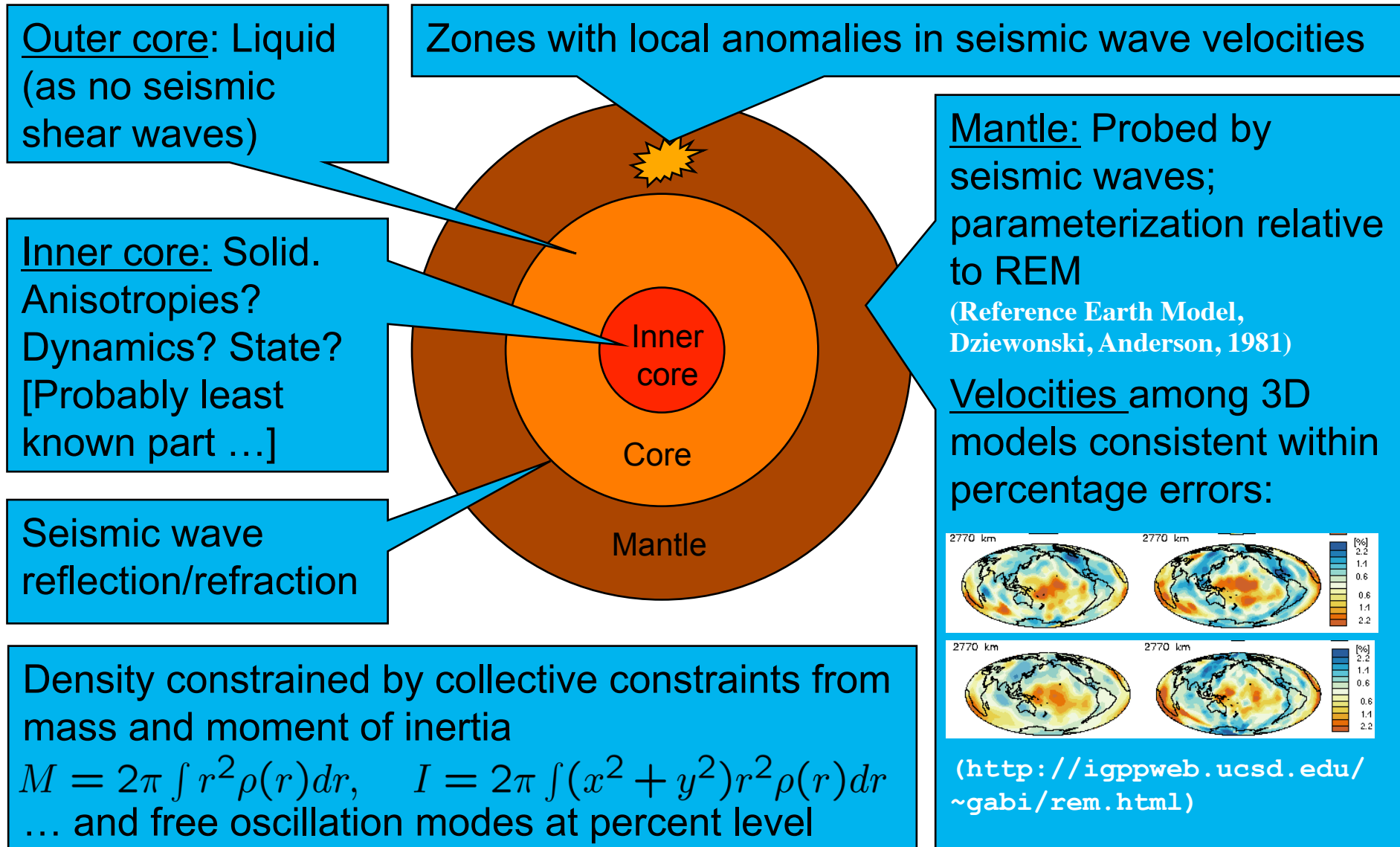
Require new, dedicated experiments!
(some are useful for tomography ...)



Neutrino tomography of Earth: Approaches and ideas



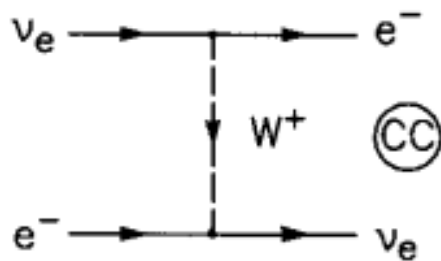
Earth's interior: What we know (served with apologies to geophysicists ...)



Neutrino tomography: Principle approaches

Matter effects in neutrino oscillations

- > Coherent forward scattering in matter leads to phase shift
- > Net effect on electron flavor:

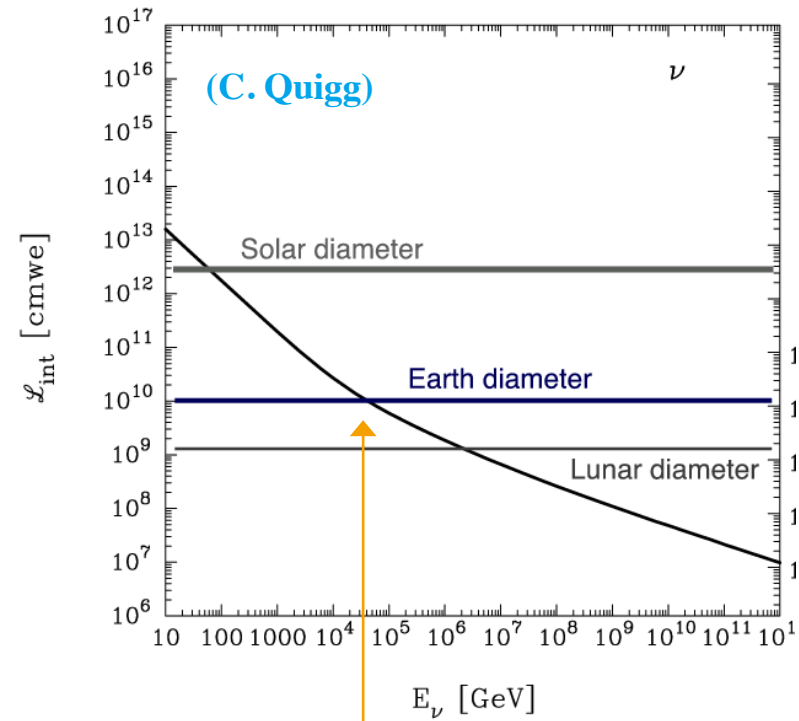


(Wolfenstein, 1978;
Mikheyev,
Smirnov, 1985)

(Earth matter does not contain muons and taus!)

- > Evidence: Neutrino conversion in the Sun, solar day-night-effect; more to come (NO ν A etc)
- > Relevant energy \sim 3-6 GeV (later)

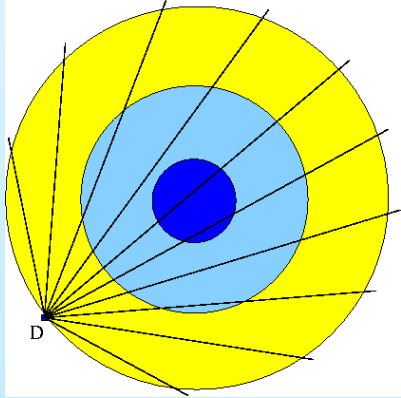
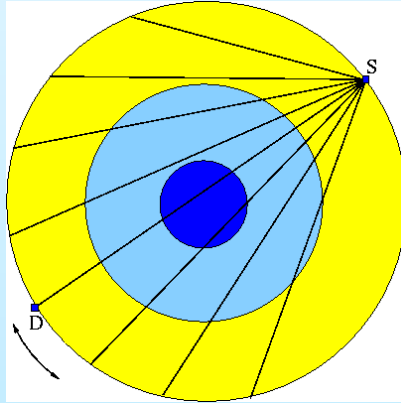
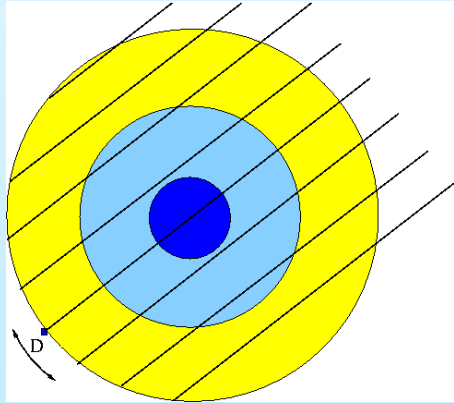
Neutrino absorption of energetic neutrinos



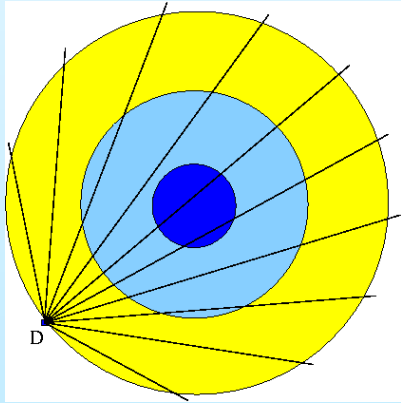
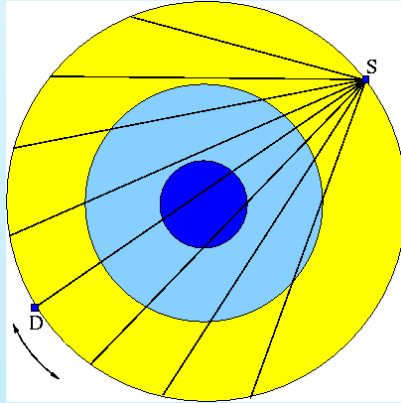
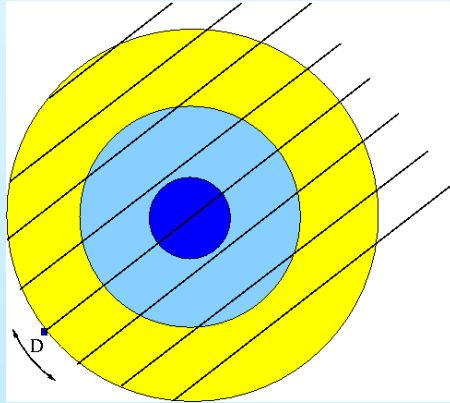
Relevant for $E \gg 10$ TeV
Example: Neutrino telescopes!



Ideas using *absorption* tomography

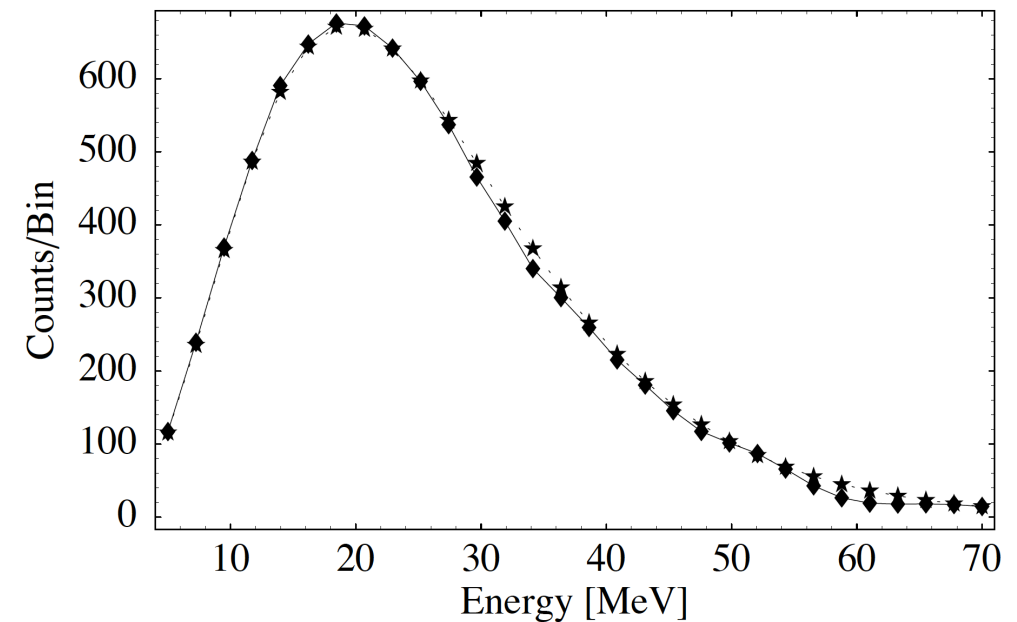
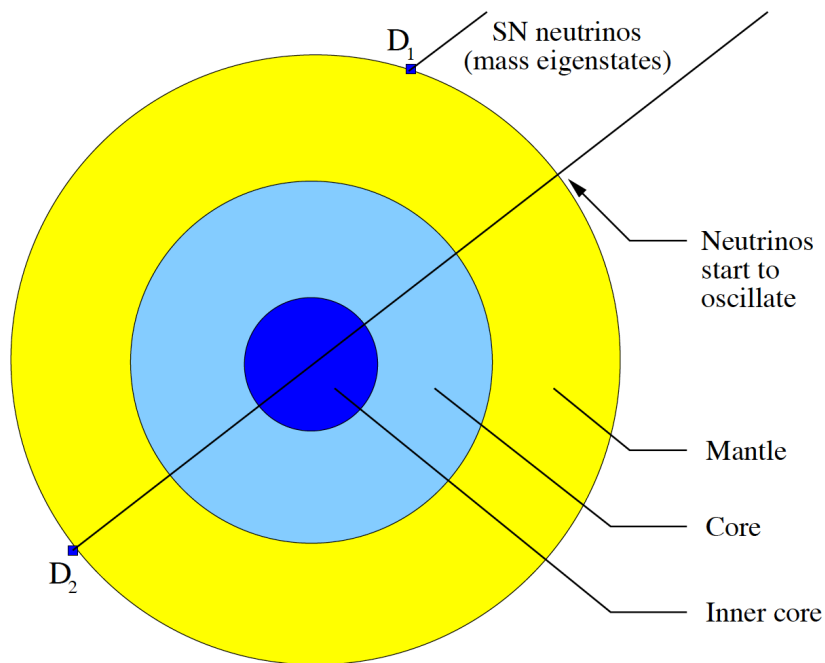
	<p>Isotropic flux (cosmic diffuse, atmospheric)</p> 	<p>TeV beam</p> 	<p>Astro point source</p> 
+	Sources available	Potentially high precision	Earth rotation → different baselines
-	<p><u>Atmospheric neutrinos</u>: low statistics at $E > 10$ TeV</p> <p><u>Diffuse cosmic flux</u>: low statistics, unknown flux normalization</p>	Build and safely operate a TeV neutrino beam (need FCC-scale accelerator); moving decay tunnel+ detector?	No sources resolved yet; most probably low statistics
	<p>Jain, Ralston, Frichter, 1999; Reynoso, Sampayo, 2004; Gonzales-Garcia, Halzen, Maltoni, 2005; ...</p>	<p>De Rujula, Glashow, Wilson, Charpak, 1983; Askar`yan, 1984; Borisov, Dolgoshein, Kalinovskii, 1986; ...</p>	<p>Wilson, 1984; Kuo, Crawford, Jeanloz, Romanowicz, Shapiro, Stevenson, 1994; ...</p>

Ideas using *oscillation tomography*

	<p>Isotropic flux (atmospheric, diffuse cosmic)</p> 	<p>Neutrino beam</p> 	<p>Astro point source (supernova, Sun)</p> 
+	Sources available, atmospheric ν just right	Potentially high precision	Earth rotation → different baselines
-	<u>Diffuse cosmic flux</u> : too high neutrino energies	Moving decay tunnel+ detector? Or: new dedicated experiment?	<u>Supernovae</u> in neutrinos are rare events <u>Solar neutrinos</u> have somewhat too low E
	Rott, Taketa, Bose, 2015; Winter, 2016 + some earlier ideas; ...	Ohlsson, Winter, 2002; Winter, 2005; Gandhi, Winter, 2007; Arguelles, Bustamante, Gago, 2015; ...	Lindner, Ohlsson, Tomas, Winter, 2003; Akhmedov, Tortola, Valle, 2005; ...

Example: Supernova neutrinos

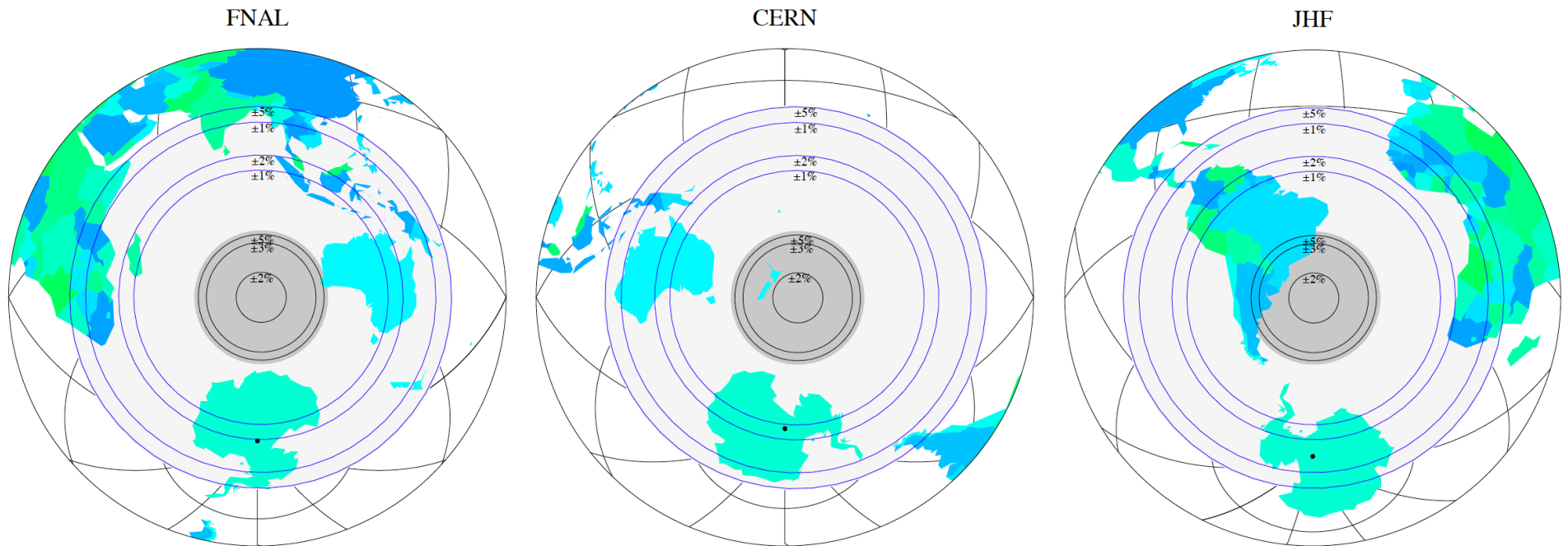
- Supernova neutrinos detected by two different detectors (with and without Earth in between)
- Spectral distortions from Earth matter effects expected at $E \gg 20$ MeV
- Hyper-K-like detectors: Expected precision (outer core density) at per cent level, inner core contrast can be seen



Example: Dedicated ν beam

- > Neutrino factory with (near) vertical baseline
- > Potential detector locations from certain laboratories:

	$L = 2 \cdot R_E$		$L = 12510 \text{ km}$	
	% error on $\bar{\rho}_{IC}$		% error on $\bar{\rho}_C$	
$\sin^2 2\theta_{13}$	1σ	3σ	1σ	3σ
Combination with $L = 3000 \text{ km}$:				
0.1	-0.5/+0.5	-1.4/+1.4	-0.2/+0.2	-0.6/+0.6
0.01	-1.8/+1.7	-5.5/+5.0	-0.6/+0.6	-1.8/+1.7
0.001	-8.3/+6.9	-27/+21	-1.8/+2.2	-4.9/+7.0
Core crossing baseline alone:				
0.1	-0.5/+0.5	-1.4/+1.4	-0.3/+0.2	-0.8/+0.6
0.01	-2.1/+5.8	-7.2/+9.2	-0.9/+0.9	-2.4/+2.7
0.001	-9.9/+19	-40/+35	-2.3/+2.5	-14/+10



(Winter, Phys.Rev. D72 (2005) 037302) $\sin^2 2\theta_{13} = 0.01$



How does it work?

Neutrino oscillations in matter

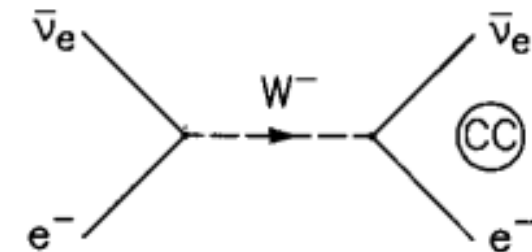
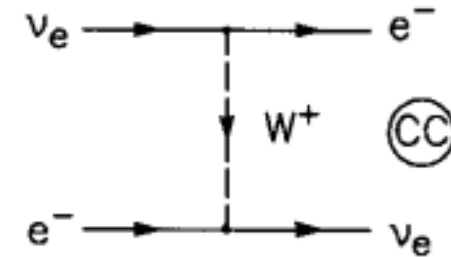
(Neutrino oscillation tomography)



Matter effect (MSW effect)

- > Ordinary matter: electrons, but no μ , τ
- > Coherent forward scattering in matter: Net effect on electron flavor
- > Hamiltonian in matter (matrix form, flavor space):

(Wolfenstein, 1978; Mikheyev, Smirnov, 1985)



$$\mathcal{H}(n_e) = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m_{21}^2}{2E} & 0 \\ 0 & 0 & \frac{\Delta m_{31}^2}{2E} \end{pmatrix} U^\dagger + \begin{pmatrix} V(n_e) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V_\nu = +\sqrt{2}G_F n_e, \quad V_{\bar{\nu}} = -\sqrt{2}G_F n_e, \quad n_e = Y \rho_j / m_N$$

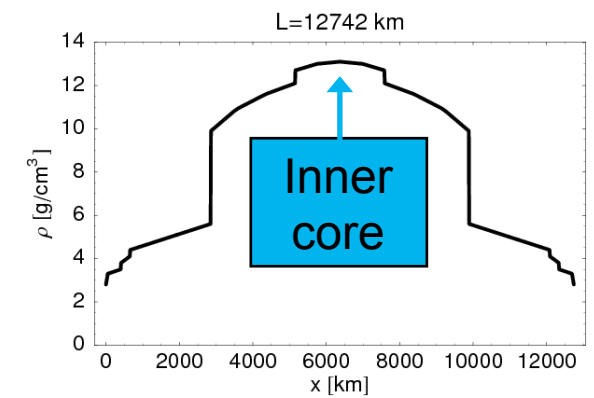
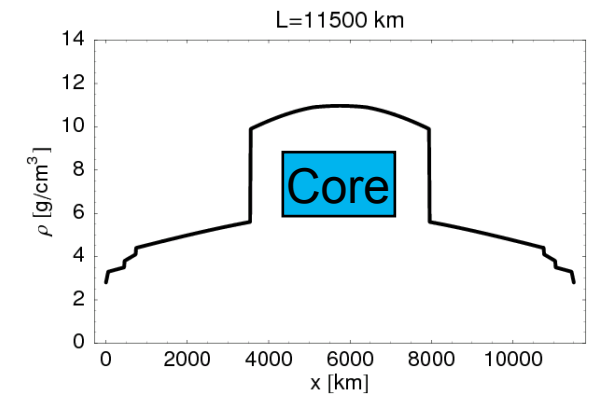
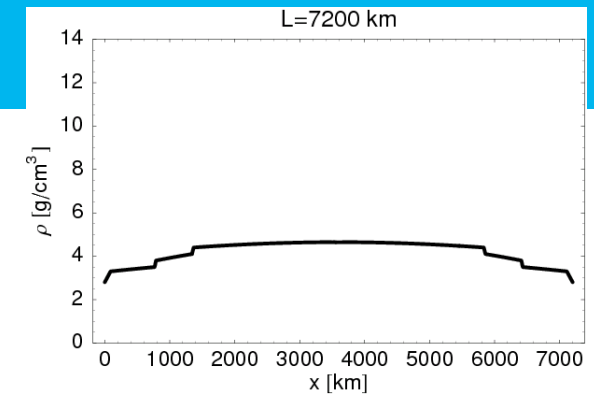
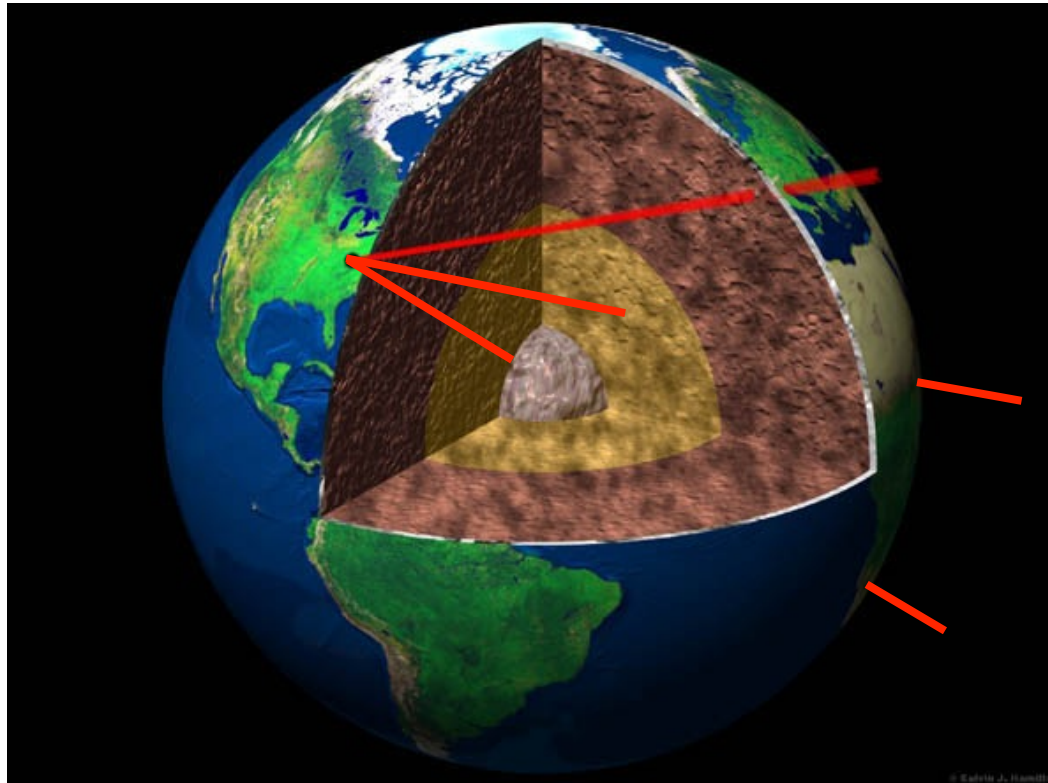
Y: electron fraction ~ 0.5
(electrons per nucleon)

(electron density and composition are degenerate!)



Matter profile of the Earth

... as seen by a neutrino



(PREM: Preliminary Reference Earth Model)

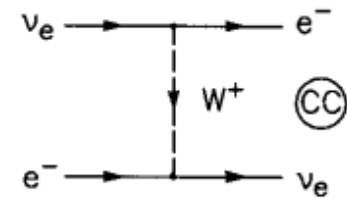


Parameter mapping ... for two flavors, constant matter density

> Oscillation probabilities in

vacuum:
$$P_{\alpha\alpha} = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

matter:
$$P_{\alpha\alpha} = 1 - \sin^2 2\tilde{\theta} \sin^2 \frac{\Delta \tilde{m}^2 L}{4E}$$



(Wolfenstein, 1978;
Mikheyev, Smirnov,
1985)

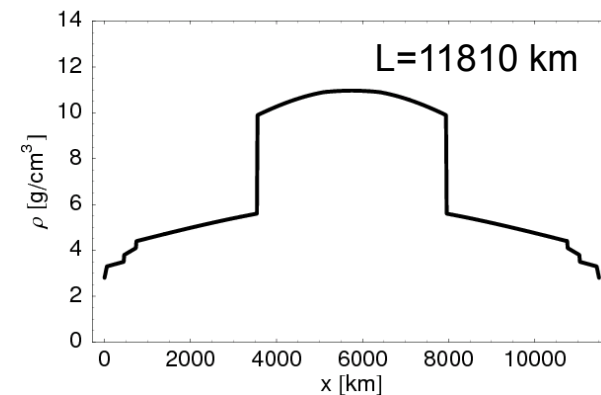
$$\Delta \tilde{m}^2 = \xi \cdot \Delta m^2, \quad \sin 2\tilde{\theta} = \frac{\sin 2\theta}{\xi},$$

$$\xi \equiv \sqrt{\sin^2 2\theta + (\cos 2\theta - \hat{A})^2},$$

$$\hat{A} = \frac{2EV}{\Delta m^2} = \frac{\pm 2\sqrt{2}E G_F n_e}{\Delta m^2} \Rightarrow \text{MO}$$

Resonance energy (from $\hat{A} \rightarrow \cos 2\theta$):

$$E_{\text{res}} [\text{GeV}] \sim 13200 \cos 2\theta \frac{\Delta m^2 [\text{eV}^2]}{\rho [\text{g/cm}^3]}$$



For ν_μ appearance, Δm_{31}^2 :

- $\rho \sim 4.7 \text{ g/cm}^3$ (Earth's mantle): $E_{\text{res}} \sim 6.4 \text{ GeV}$

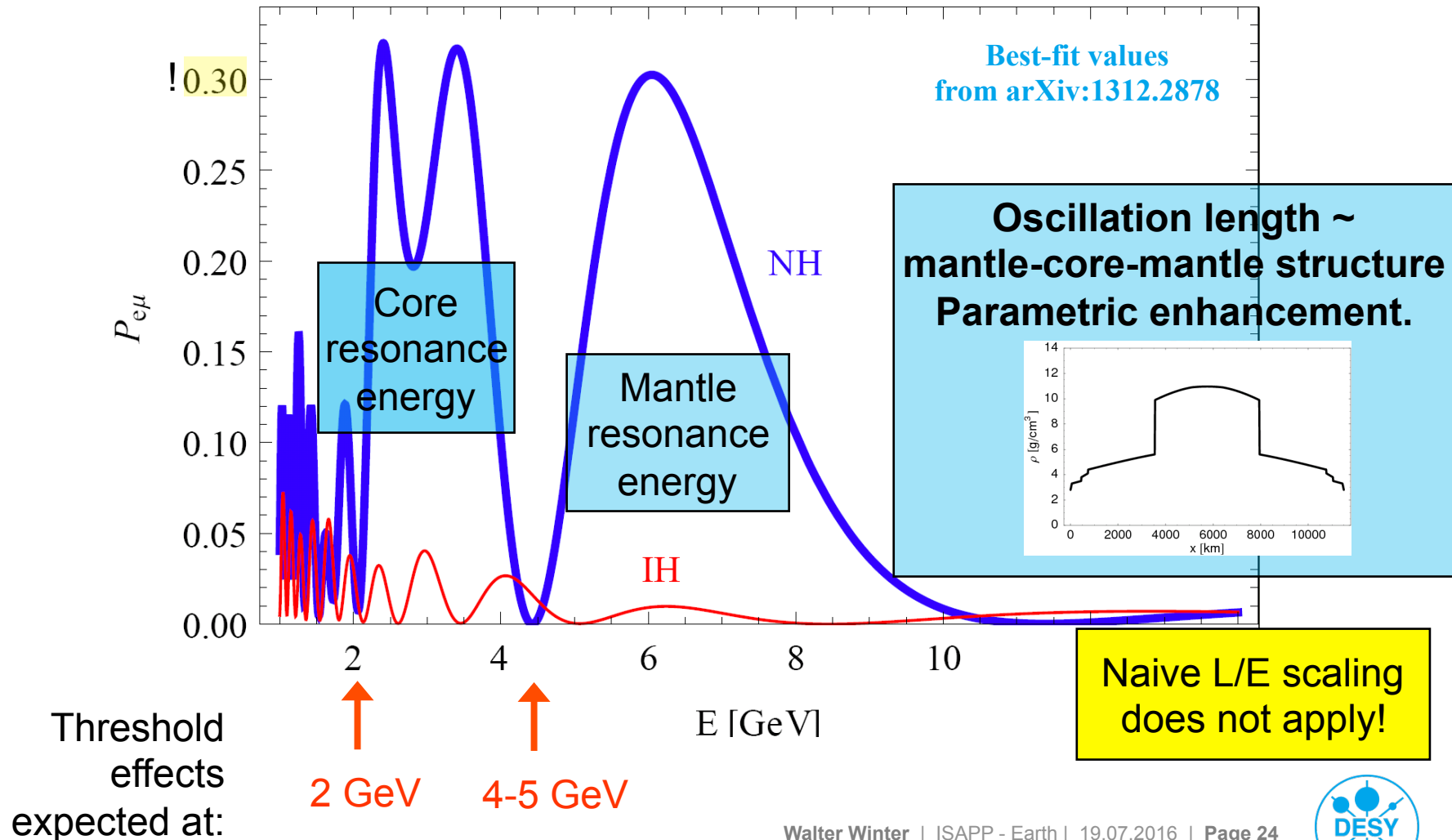
- $\rho \sim 10.8 \text{ g/cm}^3$ (Earth's outer core): $E_{\text{res}} \sim 2.8 \text{ GeV}$



Mantle-core-mantle profile

(Parametric enhancement: Akhmedov, 1998; Akhmedov, Lipari, Smirnov, 1998; Petcov, 1998)

> Probability for $L=11810$ km

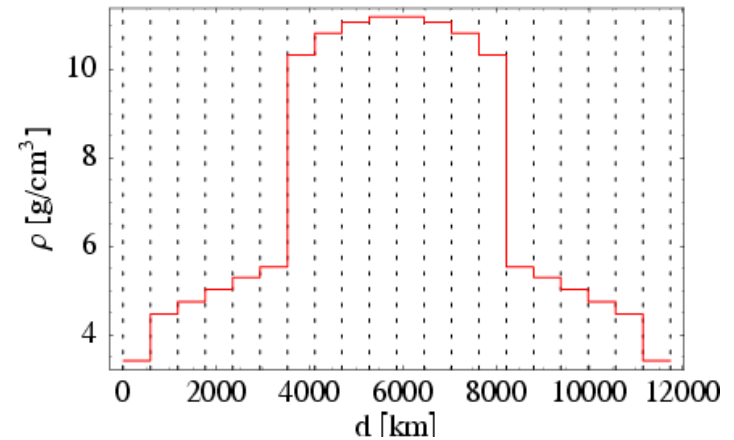


Neutrino oscillations with varying profiles, numerically

- > Evolution operator method:

$$\mathcal{V}(x_j, n_j) = e^{-i\mathcal{H}(n_j)x_j}$$

$\mathcal{H}(n_j)$: Hamilton operator in constant electron density n_j



- > Matter density from $n_j = Y \rho_j / m_N$, Y : electrons per nucleon (~ 0.5)

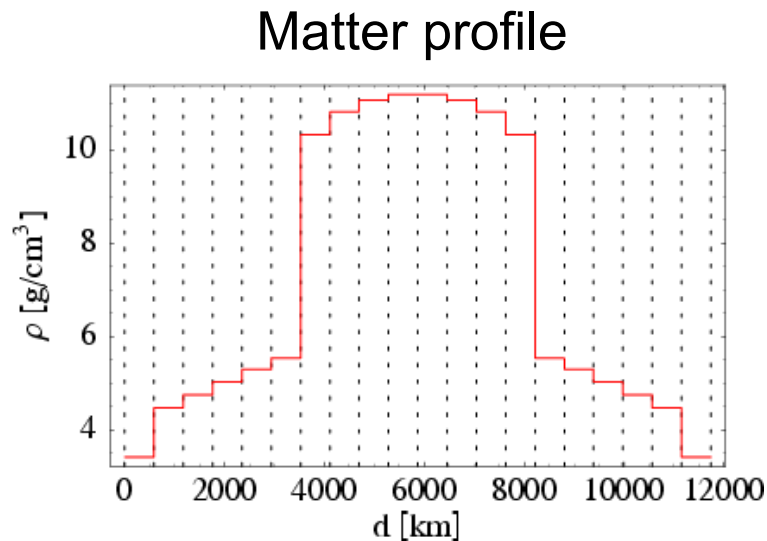
- > Probability:
$$P_{\alpha\beta} = \left| \langle \nu_\beta | \mathcal{V}(x_m, n_m) \dots \mathcal{V}(x_1, n_1) | \nu_\alpha \rangle \right|^2$$

- > NB: There is additional information through *interference* compared to absorption tomography because

$$[\mathcal{V}(x_i, n_i), \mathcal{V}(x_j, n_j)] \neq 0 \text{ f\u00fcr } n_i \neq n_j$$



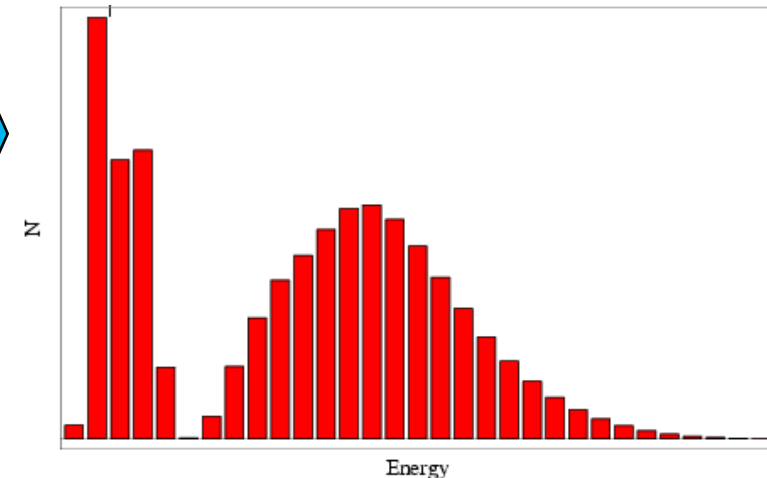
Matter profile inversion problem



Simple

Generally
unsolved

Observation

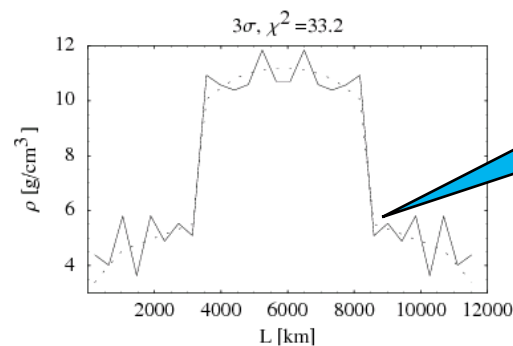
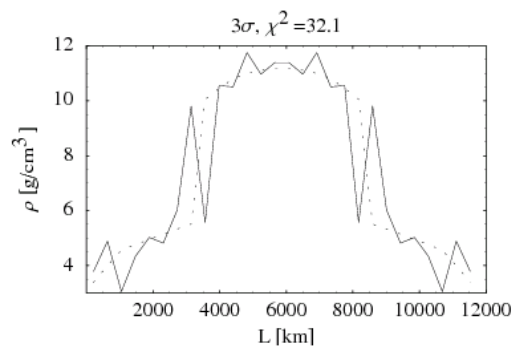
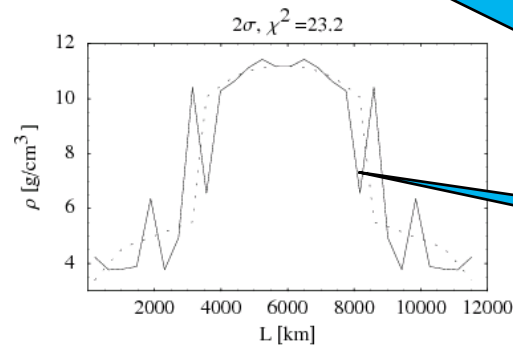
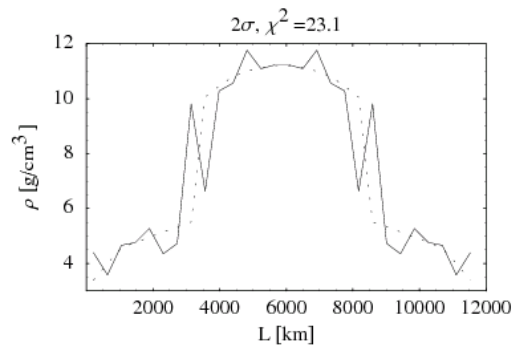
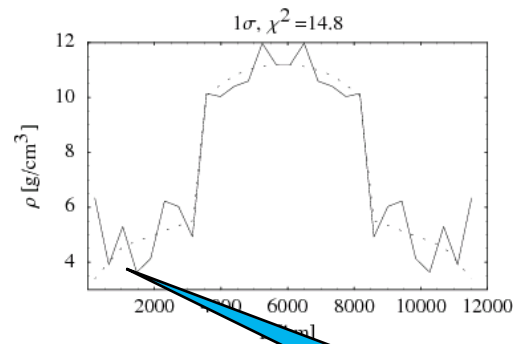
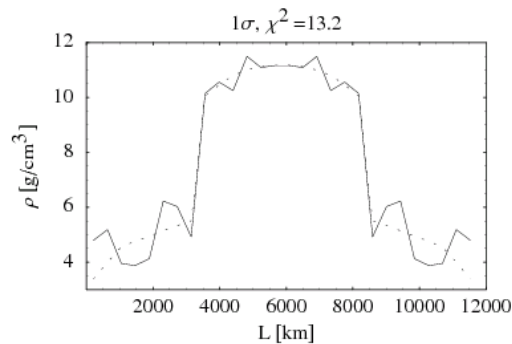


(Ermilova, Tsarev, Chechin, 1988)

Some approaches for direct inversion:

- Simple models, such as one zone (cavity) with density contrast
(Nicolaidis, 1988; Ohlsson, Winter, 2002; Arguelles, Bustamante, Gago, 2015)
- Linearization for low densities (Akhmedov, Tortola, Valle, 2005)
- Discretization with many (N) parameters:
Use non-deterministic methods to reconstruct these parameters
(e. g. genetic algorithm in Ohlsson, Winter, 2001)

Example: structural resolution with a single baseline (11750 km)



Some characteristic examples close to 1 σ , 2 σ , 3 σ (14 d.o.f.)

Can reconstruct mantle-core-mantle profile

Fluctuations on short scales ($\ll L^{\text{osc}}$) cannot be resolved

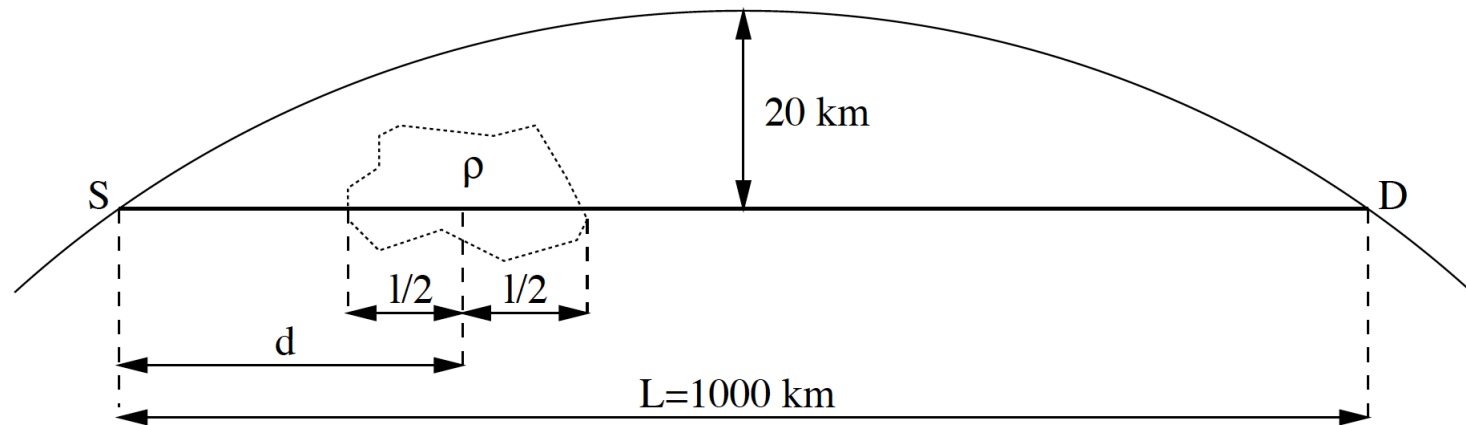
Cannot localize mantle-core-boundary

Cannot resolve very small density contrasts

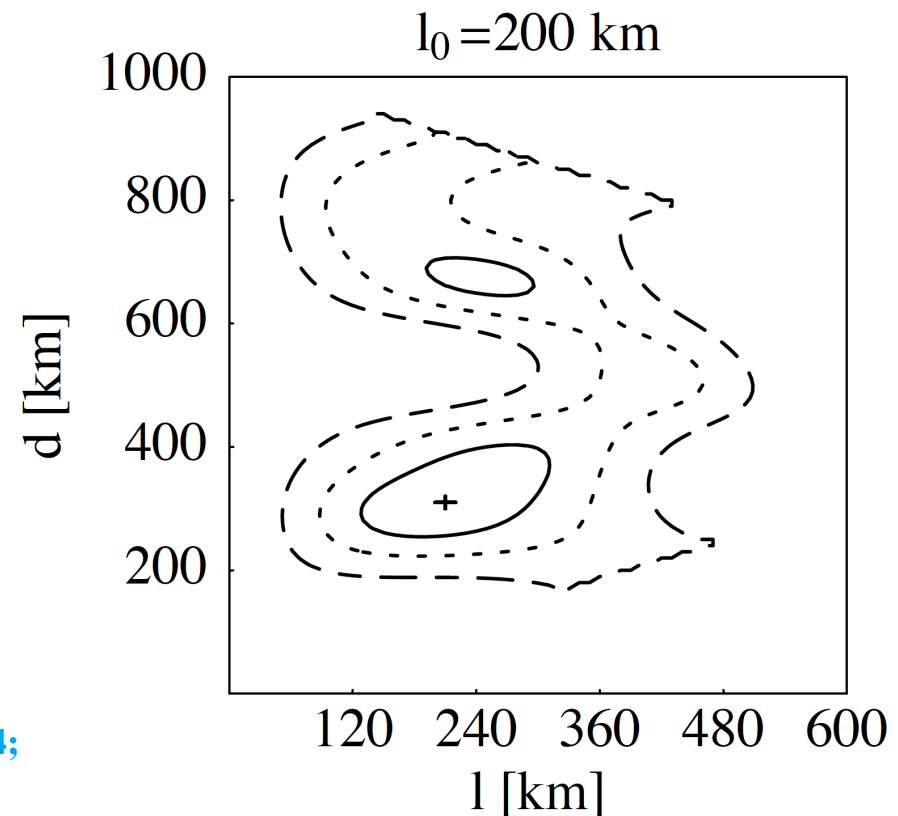
(Ohlsson, Winter, Phys. Lett. B512 (2001) 357)



Resolution of cavities = zones with a density contrast



- > Low-energy (300-500 MeV) superbeam
- > The cavity can be located if long enough and density contrast strong enough (here: water)
- > **There is some positional information (one baseline!)**



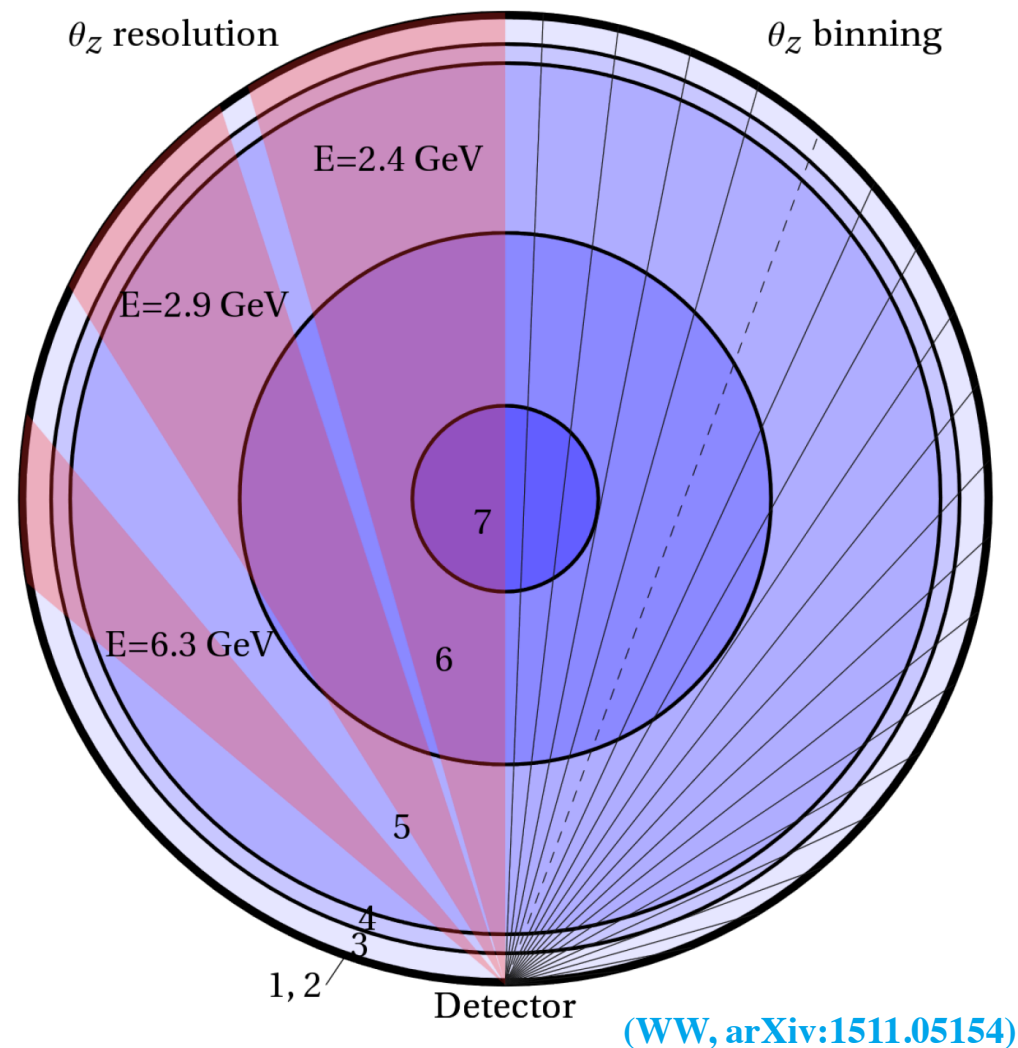
(from Ohlsson, Winter, *Europhys. Lett.* 60 (2002) 34;
see also Arguelles, Bustamante, Gago, 2015)

Neutrino tomography of Earth: Towards realistic applications



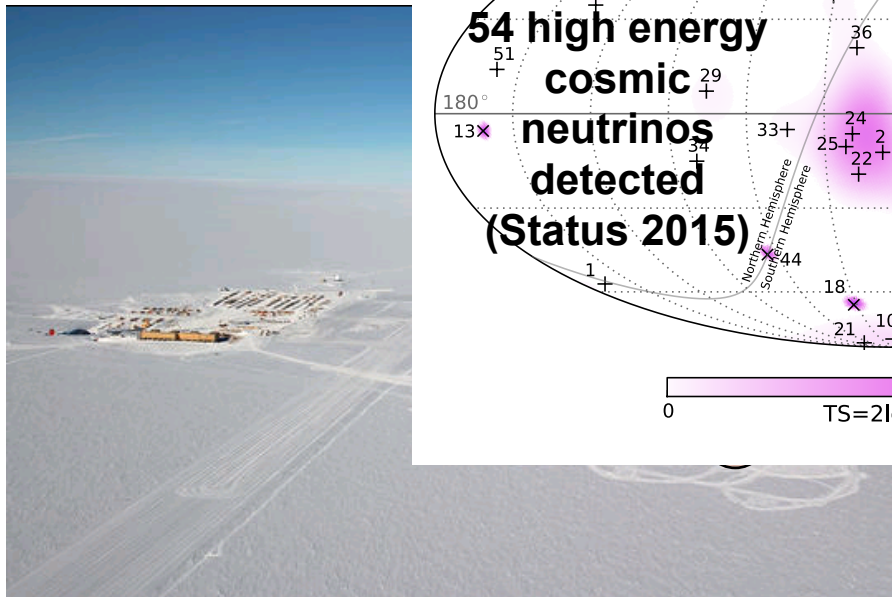
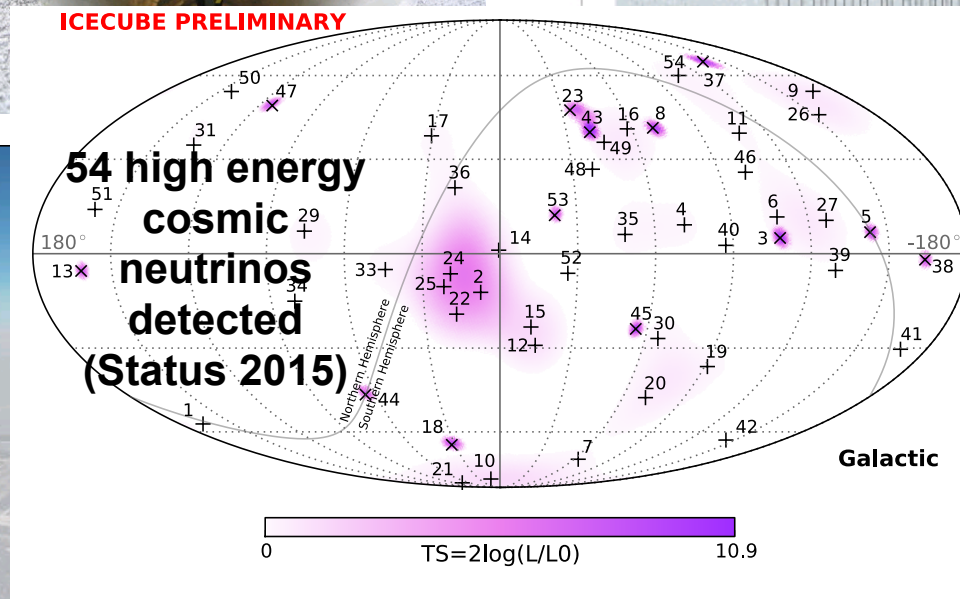
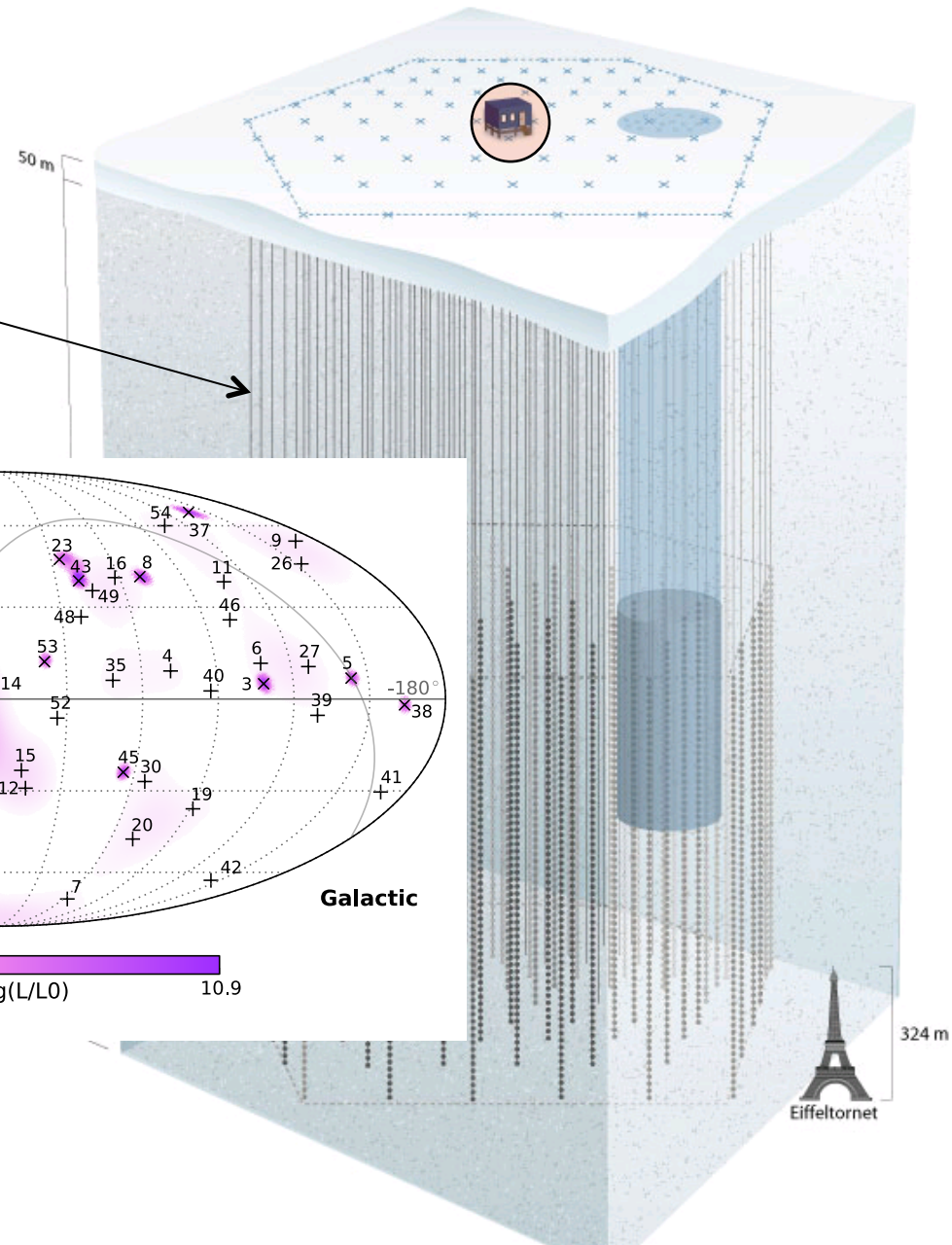
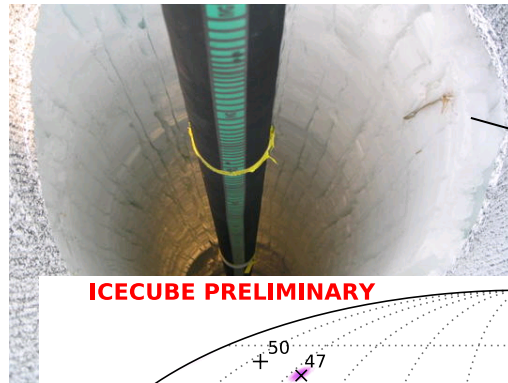
Neutrino oscillation tomography using atmospheric ν s

- > Need very large number of neutrinos in relevant energy range
- > **Point towards oscillations of atmospheric neutrinos**
- > Assumption: Cannot afford any additional equipment; spin-off from other measurement
- > **Use Mt-sized density upgrades of neutrino telescopes built for purpose of neutrino mass ordering measurement**
- > Drawback: the analysis is already complicated even without matter profile params



IceCube neutrino observatory at the South Pole

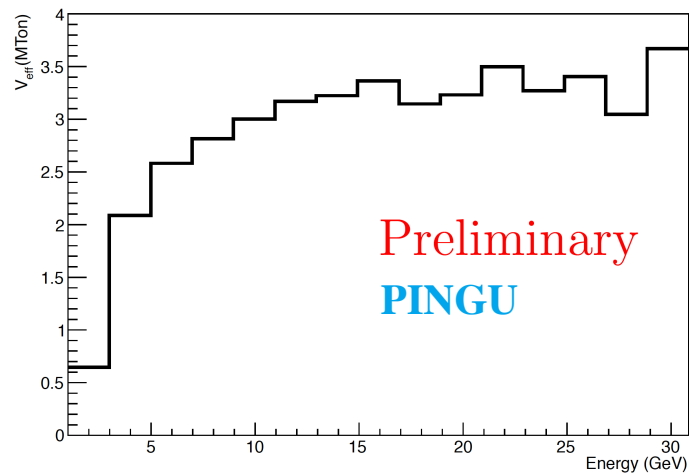
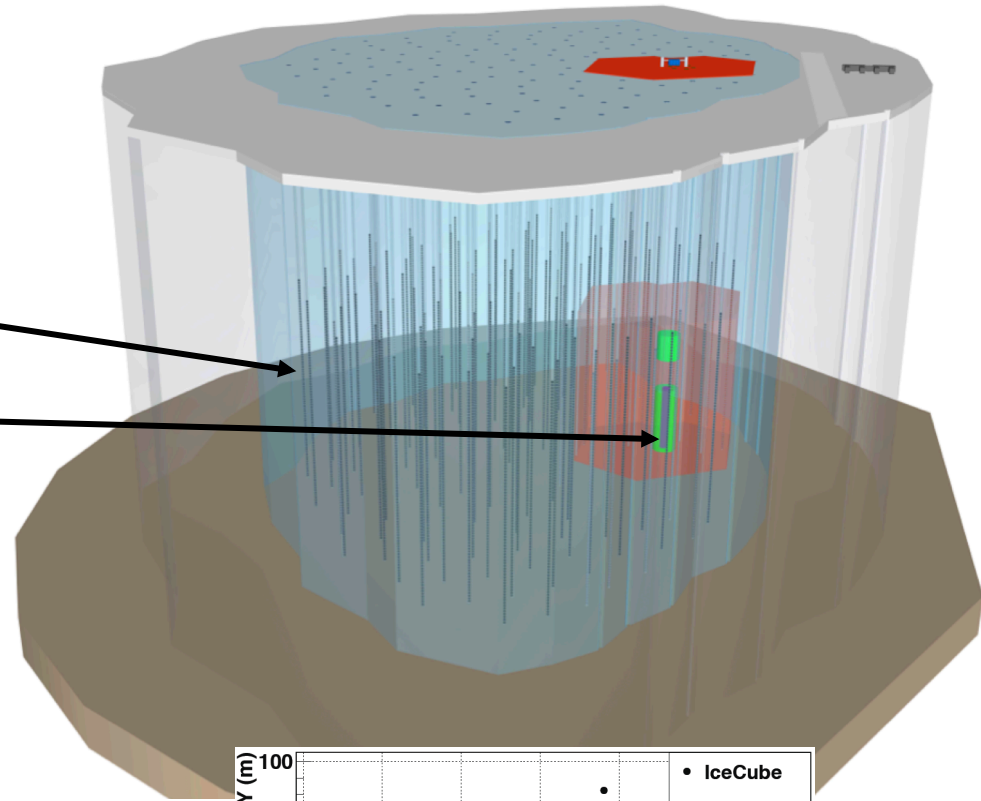
- Target material: $\sim 1 \text{ km}^3$ of Antarctic ice



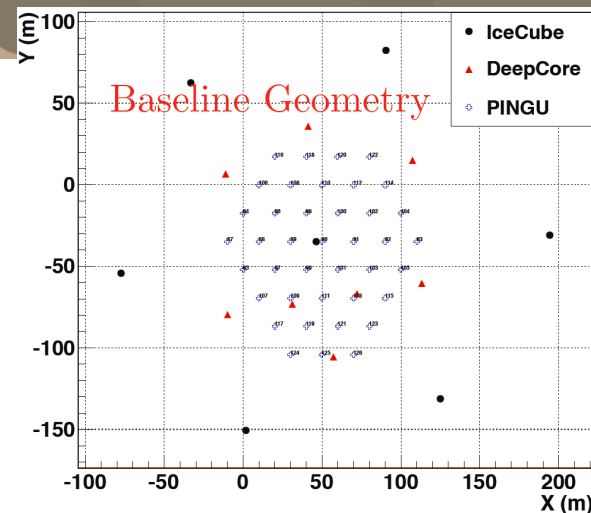
Source: IceCube

Emerging technologies: mass ordering with atm. neutrinos

- Plans for upgrade of IceCube experiment (South Pole)
- Volume upgrade (cosmic neutrinos) and density upgrade (mass ordering): PINGU



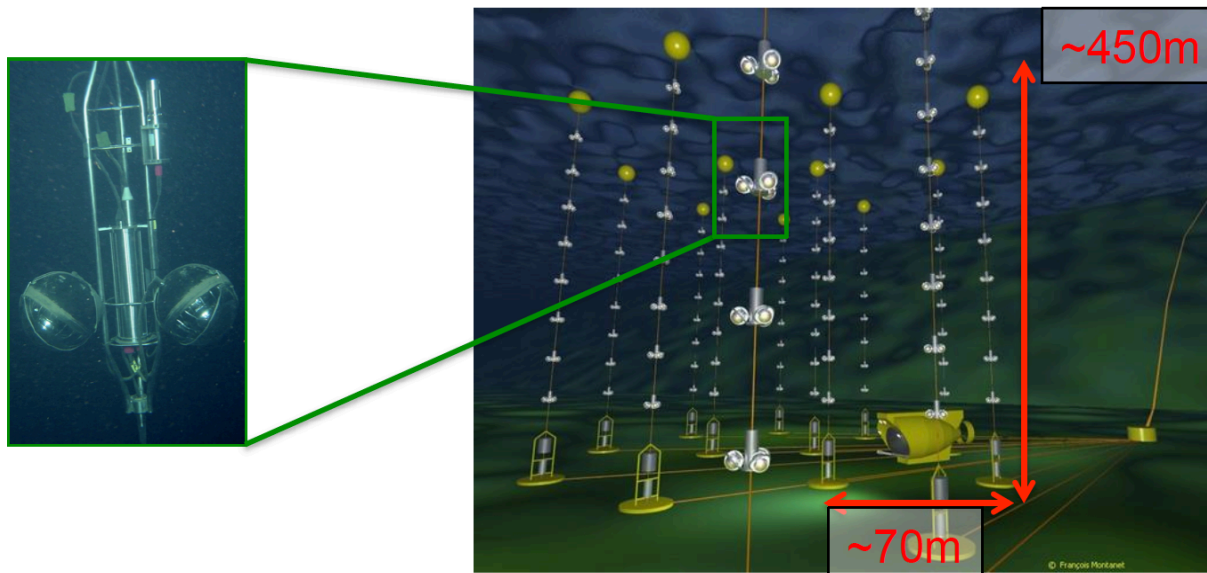
(a) $V_{\text{eff}}(\nu_{\mu})$



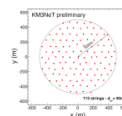
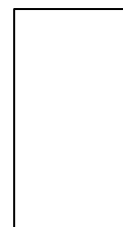
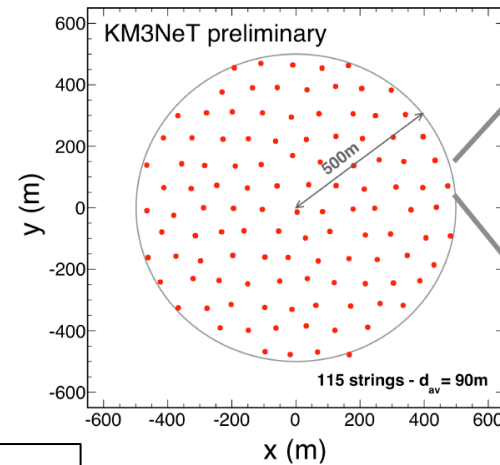
(arXiv:1401.2046, arXiv:1412.5106; arXiv:1601.07459)



ARCA/ORCA: volume/density upgrades of ANTARES

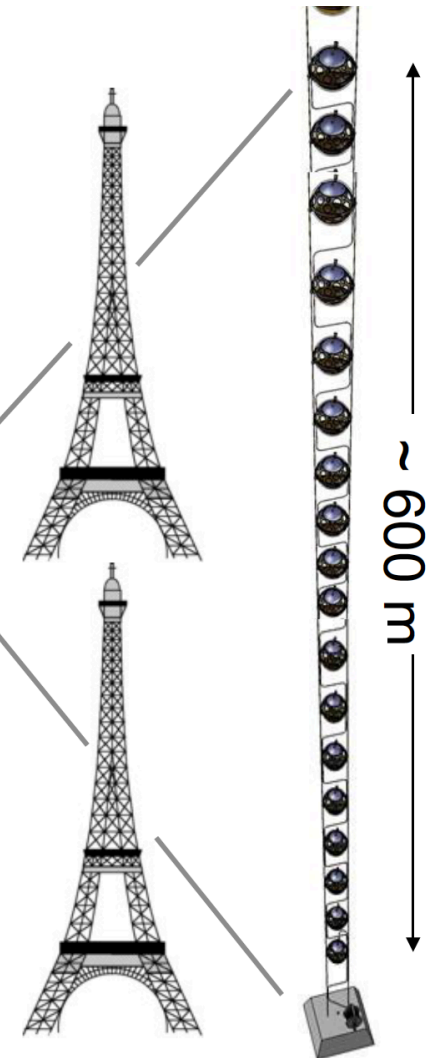


- KM3NeT ARCA/ORCA: similar ideas in sea water
- Different properties of detection medium; potentially better directional/energy resolutions?



102 m

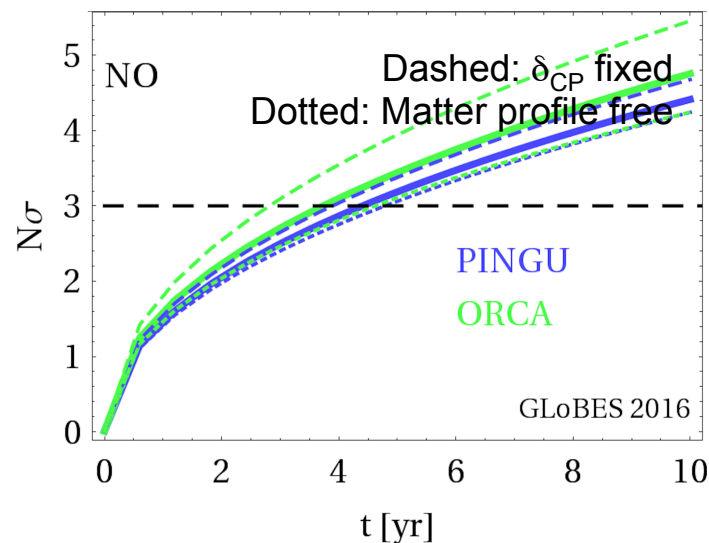
prospects, ICRC 2015



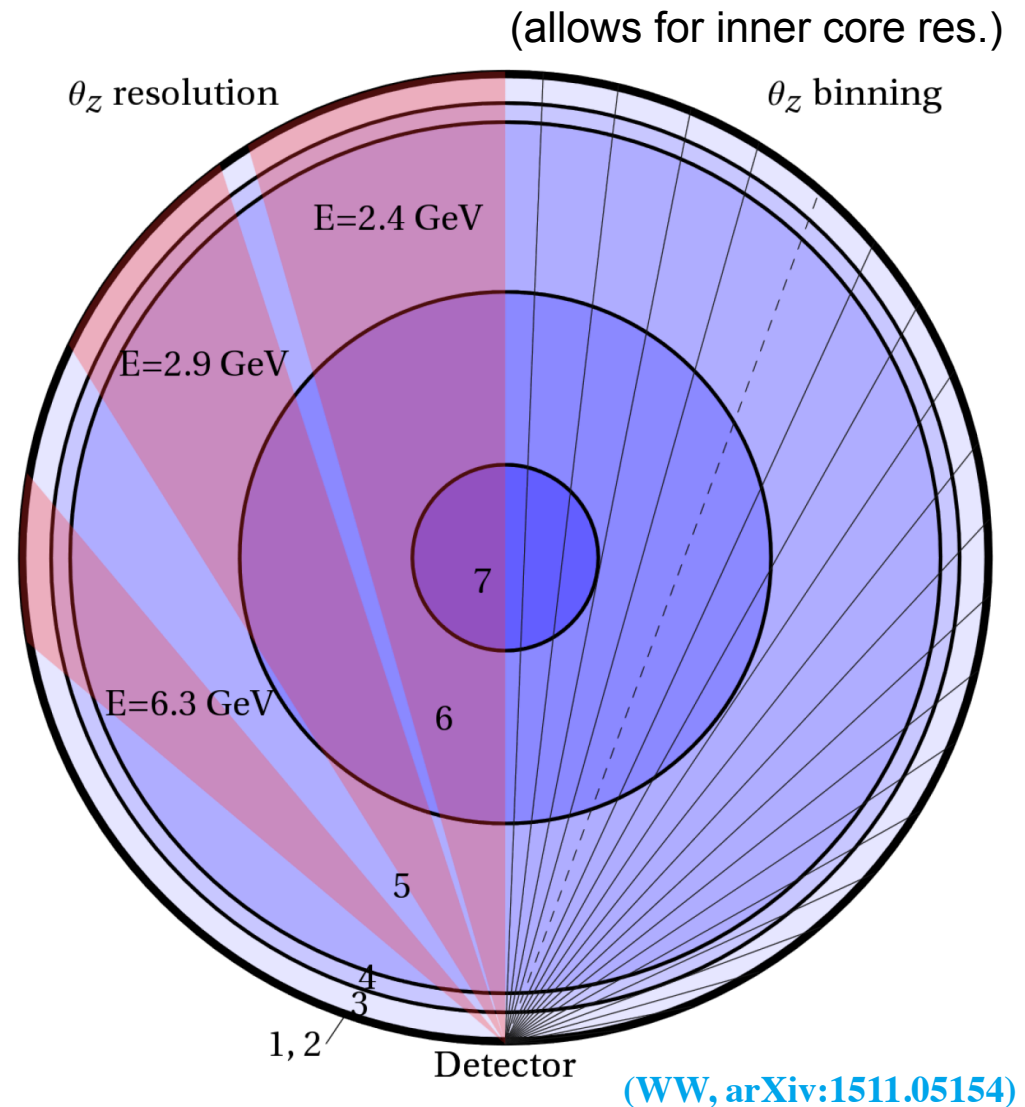
(C. W. James, ICRC 2015)

A self-consistent approach to Earth tomography

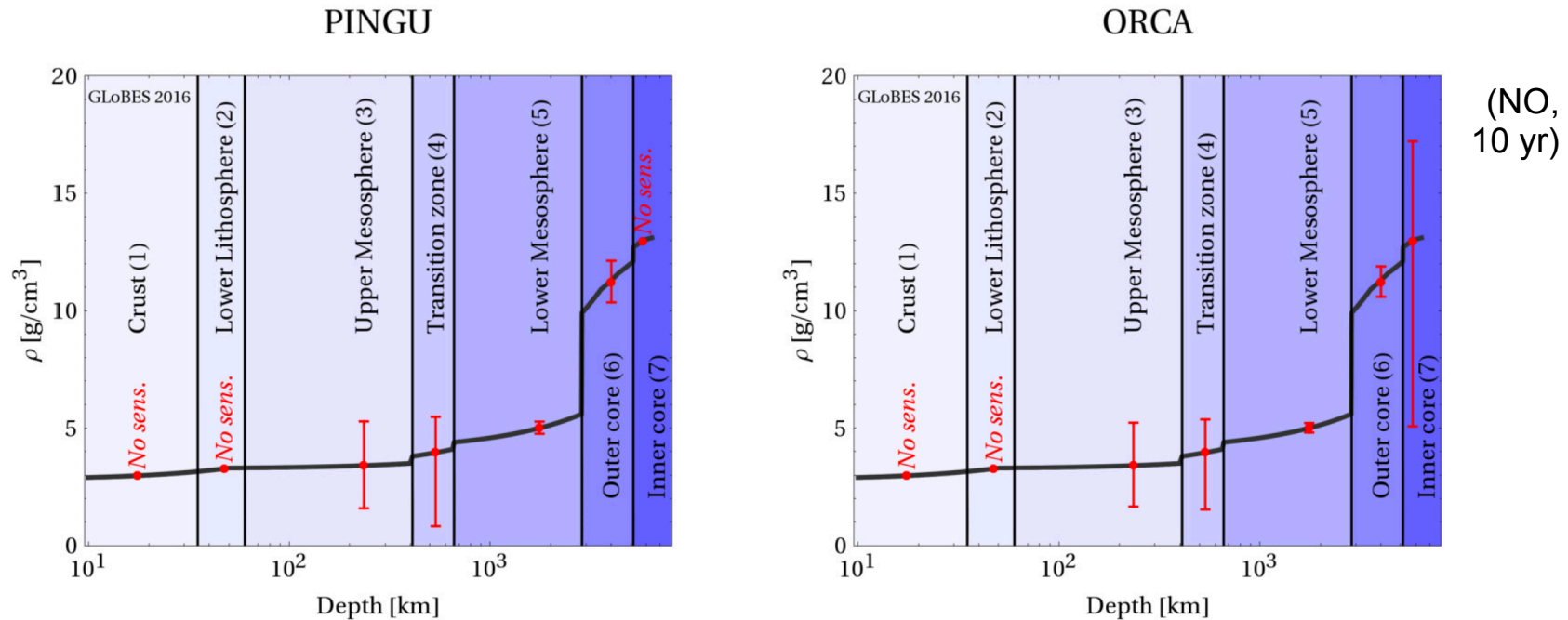
- Layers inspired by REM model: where highest sensitivity?
- Self-consistent simulation of mass ordering sensitivity and matter profile sensitivity (realistic spin-off?)



- Include systematics (12), correlations among matter layers (7) and with oscillation parameters (6)



Expected matter profile precision – proof of principle



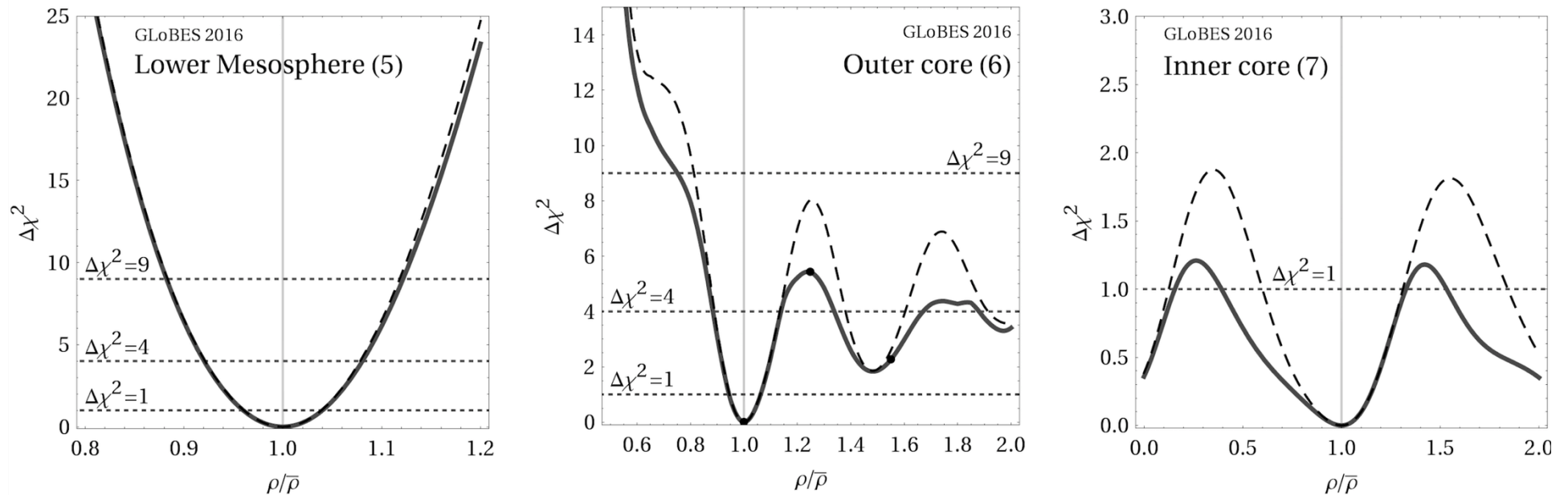
Layer	PINGU		ORCA	
	NO	IO	NO	IO
Crust (1)	No sens.	No sens.	No sens.	No sens.
Lower Lithosphere (2)	No sens.	No sens.	No sens.	No sens.
Upper Mesosphere (3)	-53.4/ +55.0	No sens.	-51.2/ +53.4	-69.1/ +52.2
Transition zone (4)	-79.2/ +38.3	No sens./ +72.2	-61.2/ +35.6	-52.7/ +45.8
Lower Mesosphere (5)	-5.0/ +5.2	-10.5/ +11.6	-4.0/ +4.0	-4.7/ +4.8
Outer core (6)	-7.6/ +8.2	-40.2/No sens.	-5.4/ +6.0	-6.5/ +7.1
Inner core (7)	No sens.	No sens.	-60.8/ +32.9	No sens.



Matter profile sensitivity. Example: ORCA

- > Highest precision in lower mantle (5)
- > Outer core sensitivity suffers from detection threshold
- > Inner core requires better resolutions

10 yr; dashed:
no correlations
among matter
layers



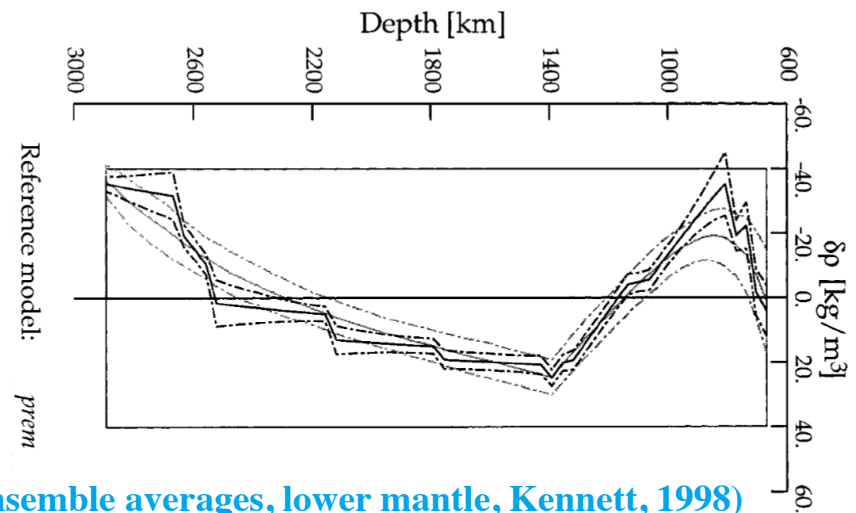
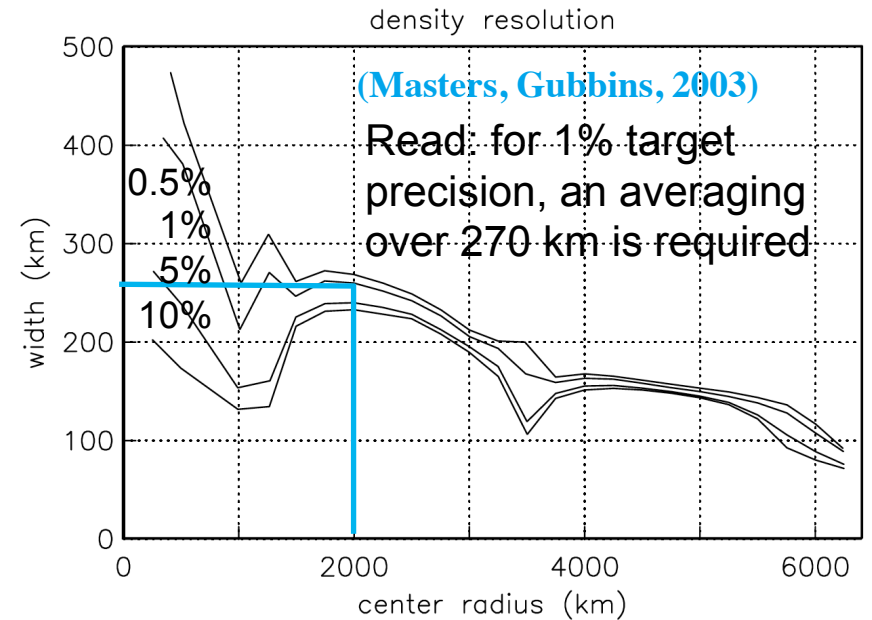
(Z/A sensitivity equivalent)

(WW, arXiv:1511.05154; special issue “Neutrino Oscillations: Celebrating the Nobel Prize in Physics 2015”, Nucl. Phys. B908, 2016, 250)



Comparison to geophysical methods

- Especially free oscillations of Earth effective for “direct” access to density profile
- Similar issues: degeneracy between target precision and length of layers averaged over (i.e., one needs some “external” knowledge/smoothing ...)
- Precision claimed at the percent level from deviation of reconstructed profiles; but: rigid statistical interpretation?
- Yet unclear how data can be combined, and what effect mass and rotational inertia constraints would have



Outlook: Core composition measurement

- > Very difficult measurements, as core composition models deviate in Y (electron fraction) by at most one percent

Table 1: Z/A ratios for alloys of iron and light elements and some selected composition models.

Model name	Z/A ratio	Si(wt%)	O(wt%)	S(wt%)	C(wt%)	H(wt%)	reference
Single-light-element model (maximum abundance)							
Fe+18wt%Si	0.4715	18	-	-	-	-	Poirier ^[29]
Fe+11wt%O	0.4693	-	11	-	-	-	Poirier ^[29]
Fe+13wt%S	0.4699	-	-	13	-	-	Li and Fei ^[5]
Fe+12wt%C	0.4697	-	-	-	12	-	Li and Fei ^[5]
Fe+1wt%H	0.4709	-	-	-	-	1	Li and Fei ^[5]
Multiple-light-element model							
Allegre2001	0.4699	7	5	1.21	-	-	Allègre et al. ^[26]
McDonough2003	0.4682	6	0	1.9	0.2	0.06	McDonough ^[27]
Huang2011	0.4678	-	0.1	5.7	-	-	Huang et al. ^[28]

(from: Rott, Taketa, Bose, Nature Scientific Reports 15225, 2015)

- > Reason: for heavier stable isotopes proton number \sim neutron number
- > Beyond precisions of PINGU and ORCA; requires a detector with a lower threshold (around 1 GeV), new technology



Summary and conclusions

- Neutrino tomography is a wide subject with many ideas: neutrino absorption, neutrino oscillations
- The observation of atmospheric neutrino oscillations has opened a new window; the relevant neutrino oscillation parameters are known to relatively high precisions
- Emerging technologies include Mt-sized detectors in ice or sea water for neutrino mass ordering measurements; tomography as a spin-off? Clearly one should do that analyses if the data are there ...
- The obtainable precision is limited and has to rely on some “external“ knowledge. However, the approach is totally different from any geophysical method (e.g. neutrinos travel on straight paths)
- The evolution operator properties (do, in general, not commute) lead to interesting structural information even from a single baseline only

Review on neutrino tomography: WW, Earth Moon Planets 99 (2006) 285



Open issues/discussion

- Geophysical “smoking gun” contribution from neutrinos?
Can one really learn something qualitatively or quantitatively new?
[especially geophysics referees tend to be very sceptical ...]
- Is it worth to develop new dedicated technology?
Or should one rely on spin-offs only?
- Required improvements (especially lower threshold) to achieve sensitivity to the inner core?
- Synergies between two experiments (PINGU/ORCA)? 3D models?
- How does one best combine geophysical and neutrino data?
Statistical interpretation of geophysical methods?
- Impact of total mass and rotational inertia constraints?
- New neutrino analyses in geophysicist’s language?
Example: Simulate profiles satisfying all constraints?

