Memory effects in quantum dynamics: from applications to foundations

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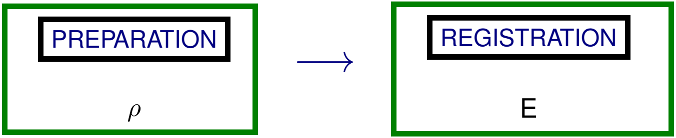
Is quantum theory exact? FQT2015

Outline

1. From foundations to open quantum systems and back
2. Classical processes: divisibility and distinguishability
3. Quantum non-Markovianity

Foundations of quantum mechanics

- Statistical structure of quantum theory



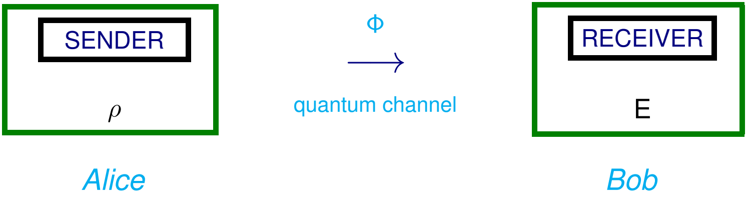
- Single particle statistical experiment
- State as equivalence class of preparation procedures
  $\rho \in \mathcal{T}(\mathcal{H})$ normed space with $\|A\|_1 = Tr|A|$
- Observable as equivalence class of registration procedures
  $B \in \mathcal{B}(\mathcal{H})$ normed space with $\|A\| = \sup_{\|\phi\|=1} \|A\phi\|$
- Statistics of experiment

$$\mu_\rho^B(M) = \mathrm{Tr}\, \rho E^B(M)$$

- **Quantum mechanics as a theory of probability**

Quantum information

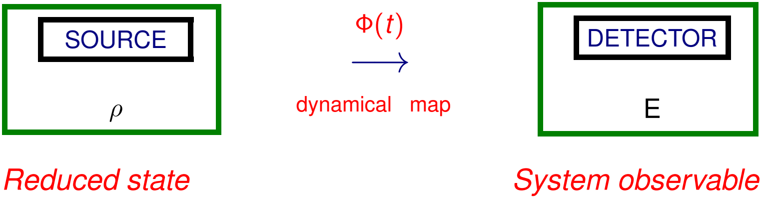
- Quantum systems as information carriers



- Quantum information and communication standpoint
- Relevance of general state transformations
- Relevance of possibly detrimental noise and decoherence
- Quantum systems unavoidably linked to a macroscopic background

Open quantum systems

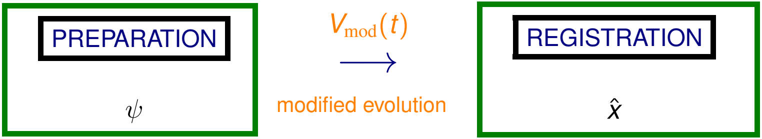
- Quantum systems as relevant degrees of freedom



- Open quantum system standpoint
- Relevance of general state transformations
- Relevance of noise and decoherence
- Quantum systems unavoidably
  linked to a macroscopic background
- Measurement interaction as archetype

Modifications of quantum mechanics

- Statistical structure of quantum theory

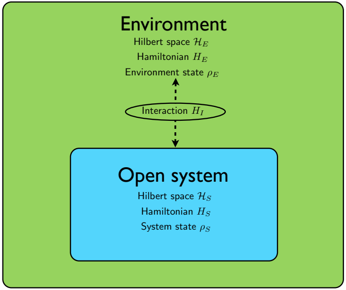


- **Modification of Schrödinger time evolution to solve measurement problem**
- Single particle statistical experiment
- Statistics of experiment $\mu_{\rho}^{\hat{x}}(M) = \mathrm{Tr}\, \rho E^{\hat{x}}(M)$
- Quantum mechanics as a theory of probability

Open quantum systems

- Quantum system interacting with environment

[Davies, 1976; Holevo, 2001; Breuer & Petruccione, 2002]



- Bipartite setting $\quad H \in \mathcal{L}(\mathcal{H}_S \otimes \mathcal{H}_E) \quad \rho_{SE} \in \mathcal{T}(\mathcal{H}_S \otimes \mathcal{H}_E)$
  System observables only determined by $\rho_S(t) = \mathrm{Tr}_E\, \rho_{SE}(t)$

$$\frac{d}{dt}\rho_S(t) = -\frac{i}{\hbar}\mathrm{Tr}_E[H, \rho(t)] \qquad \rho_S(0) = \mathrm{Tr}_E\, \rho_{SE}(0)$$

Quantum dynamical map

- Reduced dynamics

$$\begin{array}{ccc} \rho(0) = \rho_S(0) \otimes \rho_E & \xrightarrow{\text{unitary evolution}} & \rho(t) = e^{-\frac{i}{\hbar}Ht}(\rho_S(0) \otimes \rho_E)e^{+\frac{i}{\hbar}Ht} \\ \mathrm{Tr}_E \downarrow & & \downarrow \mathrm{Tr}_E \\ \rho_S(0) & \xrightarrow{\text{dynamical map}} & \rho_S(t) = \Phi(t)\rho_S(0) \end{array}$$

- Quantum dynamical map

$$\rho_S(0) \mapsto \rho_S(t) = \Phi(t)\rho_S(0) = \mathrm{Tr}_E(e^{-\frac{i}{\hbar}Ht}(\rho_S(0) \otimes \rho_E)e^{+\frac{i}{\hbar}Ht})$$

$$\Phi = \{\Phi(t), t \in \mathbb{R}_+ | \Phi(0) = \mathbb{I}\}$$

Completely positive trace preserving map

Quantum Markov process

- Semigroup composition law

$$\Phi(t)\Phi(s) = \Phi(t+s) \qquad t, s \geq 0 \quad \Rightarrow \quad \Phi(t) = \exp(\mathcal{L}t)$$

- Markov condition
  Separation of time scales $\tau_E \ll \tau_S$
- Quantum dynamical semigroups

[Gorini & al. JMP 1976; Lindblad, CMP 1976]

Quantum Markov process fixed by master equation

$$\frac{d}{dt}\rho_S(t) = \mathcal{L}\rho_S(t)$$

with generator in Lindblad form $\gamma_k \geq> 0$

$$\mathcal{L}\rho_S(t) = -\frac{i}{\hbar}[H_{eff}, \rho_S(t)] + \sum_k \gamma_k \left[A_k \rho_S(t) A_k^\dagger - \frac{1}{2}\{A_k^\dagger A_k, \rho_S(t)\} \right]$$

Master equation in Lindblad form

Open quantum systems

Motivations and goals

- Microscopic derivation of reduced dynamical evolutions
- Mathematical characterization of quantum dynamical map
- **Definition and characterization of quantum memory**
- Relevance of initial correlations
- **Decoherence and quantum to classical transition**
- **Decoherence versus alternative quantum theories**

Back to foundations

- Open quantum system theory provides useful tool for study and extension of dynamical reduction models
  - Modifications of unitary dynamics
    - Dissipative dynamical reduction models

      [Bassi & al. PRA 2005; Smirne & al. PRA 2014; Smirne & Bassi Sci Rep 2015]
    - Non-Markovian dynamical reduction models

      [Bassi & Ferialdi PRL 2009, PRA 2009; Ferialdi & Bassi PRL 2012]
  - Modifications of open quantum system dynamics

    [Bahrami & al. PRL 2014; Bassi & al. PRL 2005]
- Open quantum system theory describes decoherence effects typically undistinguishable from dynamical reduction models

  [Bassi & al. RMP 2012]

Outline

1. From foundations to open quantum systems and back
2. Classical processes: divisibility and distinguishability
3. Quantum non-Markovianity

Classical stochastic processes

- Statistical description of classical system by means of stochastic process $X(t)$, $t \geq 0$ taking values in $\{x_i\}_{i \in \mathbb{N}}$
- Stochastic process characterized by hierarchy of joint probability distributions

$$P_n(x_n, t_n; x_{n-1}, t_{n-1}; \ldots; x_1, t_1) \qquad t_n \geq t_{n-1} \geq \ldots \geq t_1 \geq 0$$

obeying Kolmogorov consistency condition

$$\sum_{x_m} P_n(x_n, t_n; \ldots; x_m, t_m; \ldots; x_1, t_1) = P_{n-1}(x_n, t_n; \ldots; x_1, t_1)$$

Andrej Kolmogorov (1903-1987)

Classical Markovian processes

- Lack of memory in the dynamics
  described by means of Markov property
- Conditional probabilities of Markovian process

$$P_{1|n}(x_{n+1}, t_{n+1}|x_n, t_n; \ldots ; x_1, t_1) \equiv \frac{P_{n+1}(x_{n+1}, t_{n+1}; \ldots ; x_1, t_1)}{P_n(x_n, t_n; \ldots ; x_1, t_1)}$$

obey the constraint

$$P_{1|n}(x_{n+1}, t_{n+1}|x_n, t_n; \ldots ; x_1, t_1) = P_{1|1}(x_{n+1}, t_{n+1}|x_n, t_n) \quad n \geq 2$$

Andrej Markov (1856-1922)

Classical Markovian processes

- Markovian process determined by
  initial distribution and conditional transition probability

$$P_1(x_0, 0) \qquad\qquad T(x, t|y, s) \equiv P_{1|1}(x, t|y, s)$$

via

$$P_1(x_1, t_1) = \sum_{x_0} T(x_1, t_1|x_0, 0) P_1(x_0, 0)$$

$$P_n(x_n, t_n; \ldots; x_1, t_1) = \prod_{i=1} T(x_{i+1}, t_{i+1}|x_i, t_i) P_1(x_1, t_1)$$

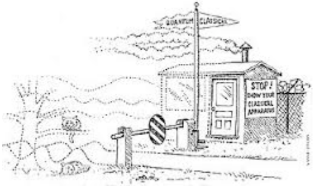
- Consistency is ensured by Chapman-Kolmogorov equation

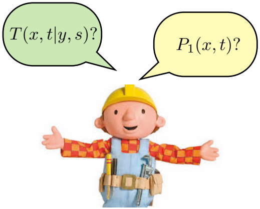
$$T(x, t|y, s) = \sum_z T(x, t|z, \tau) T(z, \tau|y, s)$$

Sydney Chapman (1888-1970)

Classical divisibility and distinguishability

- $T(x, t|y, s)$ and $P_1(x, t)$ basic quantities in the description of classical Markov processes to be taken as possible starting point for quantum generalization
- Signatures of Markovian process at the level of probability density and conditional transition probability





Classical divisibility and distinguishability

- Finite dimensional system

$$\begin{array}{rcll} P_1(x,t) & \to & P(t) & \text{probability vector} \\ T(x,t|y,s) & \to & \Lambda(t,s) & \text{stochastic matrix} \end{array}$$

- Chapman-Kolmogorov equation
  expresses divisibility property

$$\Lambda(t,s) = \Lambda(t,\tau)\Lambda(\tau,s) \qquad \forall t \geq \tau \geq s \geq 0$$

lack of memory in time evolution of probability vector

$$P(t) = \Lambda(t,\tau)P(\tau)$$

Classical divisibility and distinguishability

- Kolmogorov distance between probability distribution

$$K(Q,P) = \frac{1}{2}\sum_n |Q_n - P_n|$$

$$\underbrace{0}_{Q=P} \leq K(Q,P) \leq \underbrace{1}_{Q\perp P}$$

- Kolmogorov distance monotone contraction with respect to action of divisible stochastic matrix

$$K(P^1(t+s), P^2(t+s)) \leq K(P^1(t), P^2(t)) \qquad \forall t, s \geq 0$$

thanks to

$$\Lambda(t,s)_{n,m} \geq 0 \qquad \sum_n \Lambda(t,s)_{n,m} = 1$$

[B.V. & al., NJP 2011]

Classical divisibility and distinguishability

- Kolmogorov distance provides a notion of distinguishability of classical states

[Fuchs & de Graaf, IEEE 1999]

- General expression quantifying best strategy in assessing statistical distinguishability of different states

  P_1 with a priori probability p_1

  P_2 with a priori probability p_2

  according to statistical decision theory is given by

$$P_{success} = \frac{1}{2}(1 + K(P^1, P^2; p_1, p_2))$$

  with

$$K(P^1, P^2; p_1, p_2) = \sum_n \left| p_1 P_n^1 - p_2 P_n^2 \right|$$

  obeying

$$\underbrace{|p_1 - p_2|}_{P^1 = P^2} \leq K(P^1, P^2; p_1, p_2) \leq \underbrace{1}_{P^1 \perp P^2}$$

Classical divisibility and distinguishability

- Divisibility and distinguishability distinct notions but ...
- Direct connection between distinguishability and divisibility thanks to theorem by Kossakowski

[Kossakowski, Bull. Acad. Polon. Sci. Math. 1972; RMP 1972]

- Consider trace preserving map Λ

$$\Lambda : \ell_1(\mathbb{C}) \mapsto \ell_1(\mathbb{C})$$

Λ positive iff contraction on all hermitian elements

$$\Lambda[P] \geq 0 \qquad \forall\, P \in \ell_1(\mathbb{C}),\ P \geq 0$$

$$\Updownarrow$$

$$\|\Lambda[X]\| \leq \|X\| \qquad \forall\, X \in \ell_1(\mathbb{C}),\ X = X^\dagger$$

with $\|X\| = \sum_n |X_n|$

Andrzej Kossakowski

Classical divisibility and distinguishability

- Signatures of Markovianity at the level of probability distribution and conditional transition probability merge

$$\Lambda(t,s)=\Lambda(t,\tau)\Lambda(\tau,s) \qquad \forall t\geq\tau\geq s\geq 0$$

where each element is a positive trace preserving transformation, i.e. a stochastic matrix

$$\Updownarrow$$

$$K(P^1(t+\tau),P^2(t+\tau);p_1,p_2)\leq K(P^1(t),P^2(t);p_1,p_2) \quad \forall t,\tau\geq 0$$

monotone contraction $\forall\ P^{1,2}(0)$ and $\forall\{p_i\}_{i=1,2}$
with $P(t)=\Lambda(t,0)P(0)$ [Chruscinski & al., PRA 2011; Buscemi & Datta arXiv 2014]

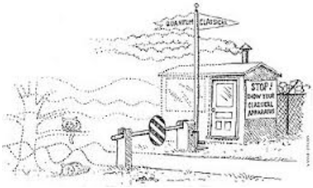
- Necessary but not sufficient criteria for lack of memory in classical process as can be shown by means of example

[B.V. & al., NJP 2011]

Outline

Quantum non-Markovianity

- Have we learnt something about classical Markovian and non-Markovian processes which might be helpful to us when we cross the quantum boundary?



Quantum non-Markovianity

- System observables take the place of random variables

$$X = \sum_{x_n} x_n |\varphi_{x_n}\rangle\langle\varphi_{x_n}|$$

- Values at different times fixed once we specify measurement scheme, e.g. projective measurements

$$\mathcal{M}_x \rho_{SE} = \Big(|\varphi_x\rangle\langle\varphi_x| \otimes \mathbb{1}_E\Big) \rho_{SE} \Big(|\varphi_x\rangle\langle\varphi_x| \otimes \mathbb{1}_E\Big)$$

  in between given time evolution

$$\mathcal{U}_t \rho_{SE} = U_t \rho_{SE} U_t^\dagger$$

- Natural definition

$$P_n(x_n, t_n; x_{n-1}, t_{n-1}; \ldots; x_1, t_1) \equiv \mathrm{Tr}\left\{\mathcal{M}_{x_n} \mathcal{U}_{t_n - t_{n-1}} \ldots \mathcal{M}_{x_2} \mathcal{U}_{t_2 - t_1} \mathcal{M}_{x_1} \mathcal{U}_{t_1} \rho_{SE}(0)\right\}$$

- But Kolmogorov consistency condition does not hold

$$\sum_{x_m} P_n(x_n, t_n; \ldots; x_m, t_m; \ldots; x_1, t_1) \neq P_{n-1}(x_n, t_n; \ldots; x_1, t_1)$$

Quantum divisibility and distinguishability

- Divisibility / distinguishability viewpoint provided sufficient condition for classical non-Markovian process captured at the level of the one-point probability density
- Quantum counterpart of classical notion
- Kolmogorov distance as a special instance of trace norm distance for $\mathcal{T}(\mathcal{H}_S) \to \ell_1(\mathbb{C})$

$$
\begin{aligned}
K(P^1(t), P^2(t)) &= \frac{1}{2}\sum_n \left| P^1_n(t) - P^2_n(t) \right| \\
&\Updownarrow \\
D(\rho^1_S(t), \rho^2_S(t)) &= \frac{1}{2}\,\mathrm{Tr}\,|\rho^1_S(t) - \rho^2_S(t)| \\
&= \frac{1}{2}\|\rho^1_S(t) - \rho^2_S(t)\|
\end{aligned}
$$

Quantum non-Markovianity

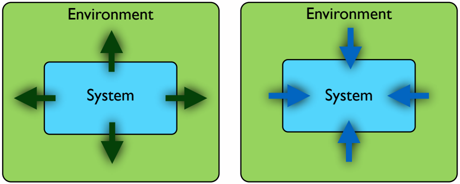
- Consider behavior of distinguishability in time of $\rho_S^{1,2}(t)$ upon the action of quantum dynamical map Φ_t
- Non-Markovianity defined as revival in time of the distinguishability among different initial states

$$\sigma(t) \equiv \frac{1}{2}\frac{d}{dt}\left\|\Phi_t\left(\rho_S^1(0) - \rho_S^2(0)\right)\right\| > 0$$

- Leading to an actual measure of non-Markovianity

$$\mathcal{N}(\Phi) = \max_{\rho_S^1(0)\perp\rho_S^2(0)} \int_{\sigma>0} dt\ \sigma(t)$$

[Breuer & al., PRL 2009]



Markovian *Non-Markovian*

Quantum divisibility and distinguishability

- Divisibility of stochastic matrices as a special instance of divisibility for quantum dynamical map Φ_t
- Let Φ_t admit linear inverse and consider

$$\Phi(t,s) = \Phi_t \Phi_s^{-1} \qquad \forall t \geq s$$

The process is said divisible if

$$\Phi(t,s) = \Phi(t,\tau)\Phi(\tau,s) \qquad \forall t \geq \tau \geq s \geq 0$$

with $\Phi(t_2,t_1)$ a positive map $\forall t_2 \geq t_1$

Quantum divisibility and distinguishability

- CP-divisibility corresponds to

$$\Phi(t,s) = \Phi(t,\tau)\Phi(\tau,s) \qquad \forall t \geq \tau \geq s \geq 0$$

  with $\Phi(t_2, t_1)$ a CP map $\forall t_2 \geq t_1$
- Starting point for a definition of non-Markovianity of an open quantum system dynamics
- CP however also takes care of correlations with ancillary system, and we are looking for characterization involving the system only

[Wolf & al., PRL 2008; Rivas & al., PRL 2010; Rep. Prog. Phys. 2014]

Quantum divisibility and distinguishability

- Theorem by Kossakowski allows to identify divisibility and monotonic decrease of distinguishability also in the quantum case [Kossakowski, Bull. Acad. Polon. Sci. Math. 1972; RMP 1972]
- Consider a trace preserving map Φ

$$\Phi : \mathcal{T}(\mathcal{H}_s) \mapsto \mathcal{T}(\mathcal{H}_s)$$

Φ positive map iff Φ contraction on all self-adjoint trace class operators

$$\Phi[T] \geq 0 \qquad \forall\, T \in \mathcal{T}(\mathcal{H}_s),\ T \geq 0$$

$$\Updownarrow$$

$$\|\Phi[T]\| \leq \|T\| \qquad \forall\, T \in \mathcal{T}(\mathcal{H}_s),\ T = T^\dagger$$

with $\|T\| = \sum_n |t_n|$

Quantum divisibility and distinguishability

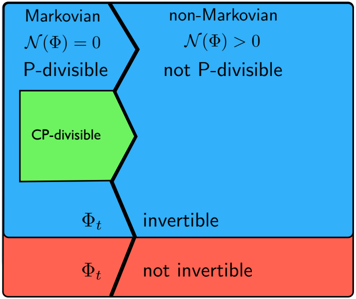
- We are thus led to consider as non-Markovian quantum processes those dynamical evolutions s.t.

$$\Sigma(t) = \frac{d}{dt}\|\Phi_t(p_1\rho_S^1 - p_2\rho_S^2)\|_1 \geq 0 \quad \text{for some } t \geq 0$$

equivalent to P-divisibility (if the inverse does exist) so that

$$\mathcal{N}_P(\Phi) = \max_{p_i,\rho_S^i} \int_{\Sigma>0} dt\, \Sigma(t)$$

[Breuer & al., arXiv 2015; Wißmann & al., arXiv 2015]



Applications & foundations

- Relevance of non-Markovianity for applications
  - Experimental observation

    [Liu & al., Nature Phys. 2011]
  - Criterion for the detection of initial correlations

    [Laine & al., EPL 2010; Smirne & al., PRA 2011; Li & al., PRA 2011]
  - Determination of complex system properties via quantum probe approach

    [Apollaro & al., 2011; Haikka & al., PRA 2012; Smirne & al., PRA 2013; Benedetti & al., PRA 2014]
  - Criteria for the determination of quantum correlations

    [Gessner & al., PRL 2011; Cialdi & al., PRA 2014]
- Possible relevance of non-Markovianity for foundations
  - Modification of quantum mechanics arising from colored noises or general non-Markovian dynamical evolutions

    [Bassi & Ferialdi PRL 2009, PRA 2009; Ferialdi & Bassi PRL 2012]

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