Measuring incompatible observables by means of sequential weak values evaluation

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Introduction

Let $\hat{O}$ be an observable.

$\hat{O}$ has discrete eigenstates $|1\rangle, |2\rangle, |3\rangle, \ldots$
with corresponding distinct eigenvalues $O_1, O_2, O_3, \ldots$

$$|\psi\rangle = c_1|1\rangle + c_2|2\rangle + c_3|3\rangle + \cdots = \sum_n c_n|n\rangle$$

Expectation value = $\langle\psi|\, \hat{O} \,|\psi\rangle$

$$\Pr(O_n) = |\langle n|\psi\rangle|^2 = |c_n|^2$$

$$|\psi'\rangle = |n\rangle$$

Collapse of the wave function.

Non commuting observables can not be simultaneously measured.

Introduction

Weak measurements [**Y. Aharonov, D. Z. Albert, and L. Vaidman**, PRL **60**, 1351 (1988)] represent a new paradigm of quantum measurement were so little information is extracted from a single measurement, so that the state does not collapse.

They permit measuring simultaneously non-commuting observables.

Summary

- Introduction on weak measurements
- Sequential weak measurements
- Experimental demonstartion of the possibility to measuring non-commuting observables on the same quantum system

Weak measurements

Weak value $\langle \widehat{A} \rangle_w = \frac{\langle \psi_f | \widehat{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$

Pre-selected state: $|\psi_i\rangle$

Post-selected state: $|\psi_f\rangle$

Von Newman coupling between the observable $\widehat{A}$ and a pointer observable $\widehat{P}$

$$\widehat{U} = \exp(-ig\widehat{A} \otimes \widehat{P})$$

Projective measurement $|\psi_f\rangle\langle\psi_f|$

$$|\phi_{out}\rangle = |\psi_f\rangle\langle\psi_f|\,\hat{U}|\psi_{in}\rangle \otimes |f_{in}\rangle$$

Assuming the weak interaction regime → $\langle \widehat{X} \rangle = \langle \phi_{out} | \,\widehat{X}\, | \phi_{out} \rangle = g\langle \widehat{A} \rangle_w$

$\widehat{X}$ and $\widehat{P}$ are canonically conjugated observable.

Weak measurement

Weak measurements on photon polarization can be realised by using small **birefringence effects**

- N. W. M. Ritchie, J. G. Story, and R. G. Hulet, Phys. Rev. Lett. 66, 1107 (1991) [laser beam]
- G. J. Pryde, J. L. O'Brien, A. G. White, T. C. Ralph, H. M. Wiseman, Phys. Rev. Lett. 94, 220405 (2005) [single photon]
- O. Hosten and P. Kwiat, Science 319, 787 (2008).
- K. J. Resch, Science 319, 733 (2008);
- P. B. Dixon, D. J. Starling, A. N. Jordan, and J. C. Howell, Phys. Rev. Lett. 102, 173601 (2009);
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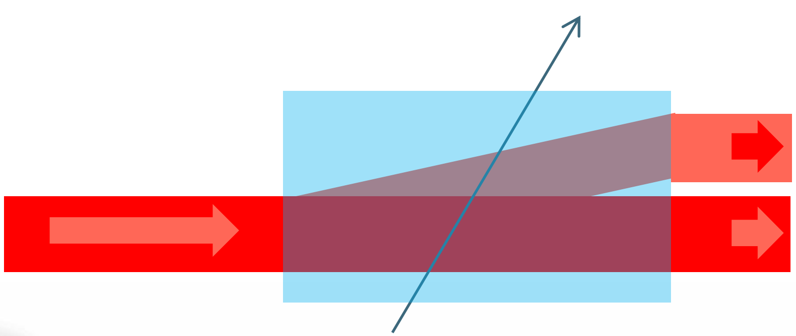
- O.S. Magaña-Loaiza, M. Mirhosseini, B. Rodenburg, R.W. Boyd Phys. Rev. Lett. 112 200401 (2014)
- J.Lundeen et al., Nature 474 (2011) 188.

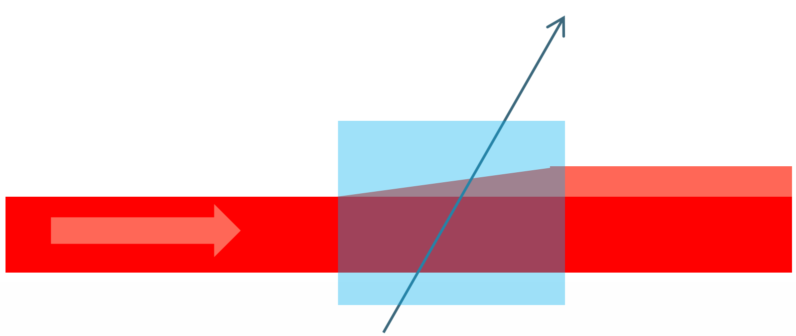
$$\widehat{A} \longrightarrow \quad \widehat{\Pi}_V = |V\rangle\langle V|$$

$$\widehat{U} = \exp(-ig\widehat{A} \otimes \widehat{P})$$

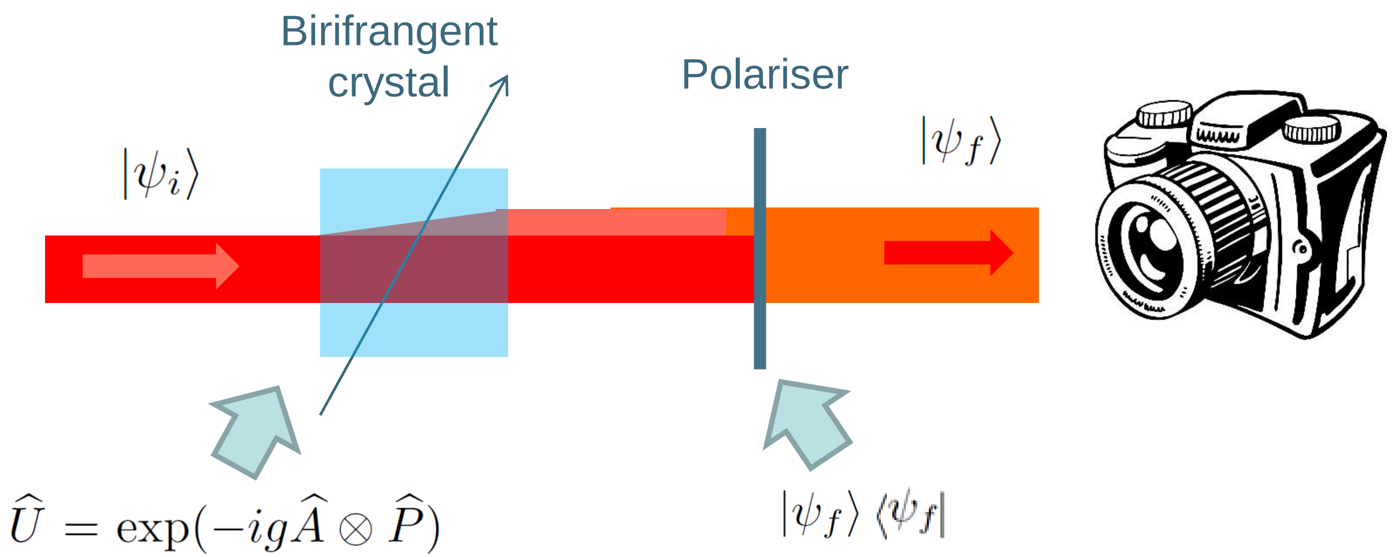
Projective measurement



Weak interaction



Weak measurement



Weak value $\langle\widehat{A}\rangle_w = \frac{\langle\psi_f|\widehat{A}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$

$\widehat{A} \longrightarrow \widehat{\Pi}_V = |V\rangle\langle V|$

We measure the position observable: $\widehat{X}$
canonically coniugated to the pointer observable: $\widehat{P}$

$$\langle\widehat{X}\rangle = g\langle\widehat{A}\rangle_w$$

Weak measurements

Several interesting properties:

- A_w is a complex number
- $Re[A_w]$ is not bounded

$$\langle \widehat{A} \rangle_w = \frac{\langle \psi_f | \widehat{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$$

Interpretation of weak values:

- Expectation value of A as an average of A_w $\rightarrow$ $\langle A \rangle_i = \Sigma_f |\langle \psi_i | \psi_f \rangle|^2 A_w$
  [Y. Aharonov, A.Botero; PRA 72_052111 (2005)]
- $Re[A_w] = Tr[P_f \{A, \rho_i\}] / (2\, Tr[P_f \rho_i])$: conditioned average of A in the limit of zero disturbance
  [J.Dressel, et al. PRL 104 240401 (2010)]
- $Im[A_w]$ arises from disturbance related to von Neumann coupling
  [J.Dressel,A.Jordan PRA 85 012107 (2012)]
- Every POVM can be realised as a sequence of weak values
  [Oreshkov, Brun PRL 95 110409 (2005)]

Weak measurements

Several interesting applications:

- **Metrology**:

Amplification of measurement of coupling strength:

- Light beam displacement [Kwiat et al],
- Angular deflection [Dixon et al],
- …..

$$\langle \widehat{X} \rangle = g \langle \widehat{A} \rangle_w$$

Advantages:

- Amplification of signal without amplifying unrelated noise [Boyd et al]
- Only a fraction of beam power can be post-selected, the other can be redirected elsewhere

- **Foundations of Quantum Mechanics**:

- Better understanding of quantum measurement
- Tests of non-contextuality [Pusey, ….]
- Hints on QM interpretations [TSVF, Aharonov et al ….]

Joint and Sequential weak measurement

Weak values «challenge one of the canonical dicta of QM: that non commuting observables cannot be simultaneously measured »

«the fact that one hardly disturbs the systems in making WM means that one can in principle measure different variables in succession»

« We suggest that sequential weak values should be interpreted as truly representing actual values of the parameters being measured, providing valuable insights in further physical situations»

[Mitchison, Josza, Popescu PRA 76 062105]

Sequential weak measurement

$$\widehat{U}_x = \exp(-ig_x \widehat{A} \otimes \widehat{P}_x)$$
$$\widehat{U}_y = \exp(-ig_y \widehat{B} \otimes \widehat{P}_y)$$

$$\langle \widehat{X} \rangle = g_x \langle \widehat{A} \rangle_w \qquad \langle \widehat{Y} \rangle = g_y \langle \widehat{B} \rangle_w$$

$$\langle \widehat{X}\widehat{Y} \rangle = \frac{1}{2} g_x g_y \mathrm{Re}\left[\langle \widehat{A}\widehat{B} \rangle_w + \langle \widehat{A} \rangle_w^* \langle \widehat{B} \rangle_w \right]$$

Joint weak measurement

$$\langle \widehat{X} \rangle = g_x \langle \widehat{A} \rangle_w \qquad \langle \widehat{Y} \rangle = g_y \langle \widehat{B} \rangle_w$$

$$\langle \widehat{X}\widehat{Y} \rangle = \frac{1}{4} g_x g_y \mathrm{Re}\left[\langle \widehat{A}\widehat{B} + \widehat{A}\widehat{B} \rangle_w + 2\langle \widehat{A} \rangle_w^* \langle \widehat{B} \rangle_w \right]$$

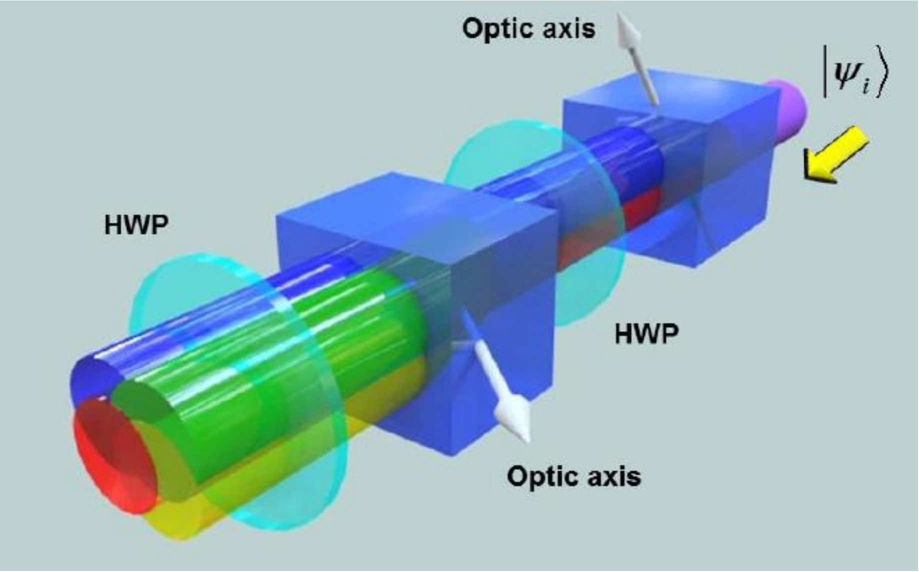
Sequential weak measurement

$$\widehat{A} \longrightarrow \widehat{\Pi}_V = |V\rangle\langle V|$$

$$\widehat{B} \longrightarrow \widehat{\Pi}_\psi = |\psi\rangle\langle\psi| \ (\text{with } |\psi\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle)$$

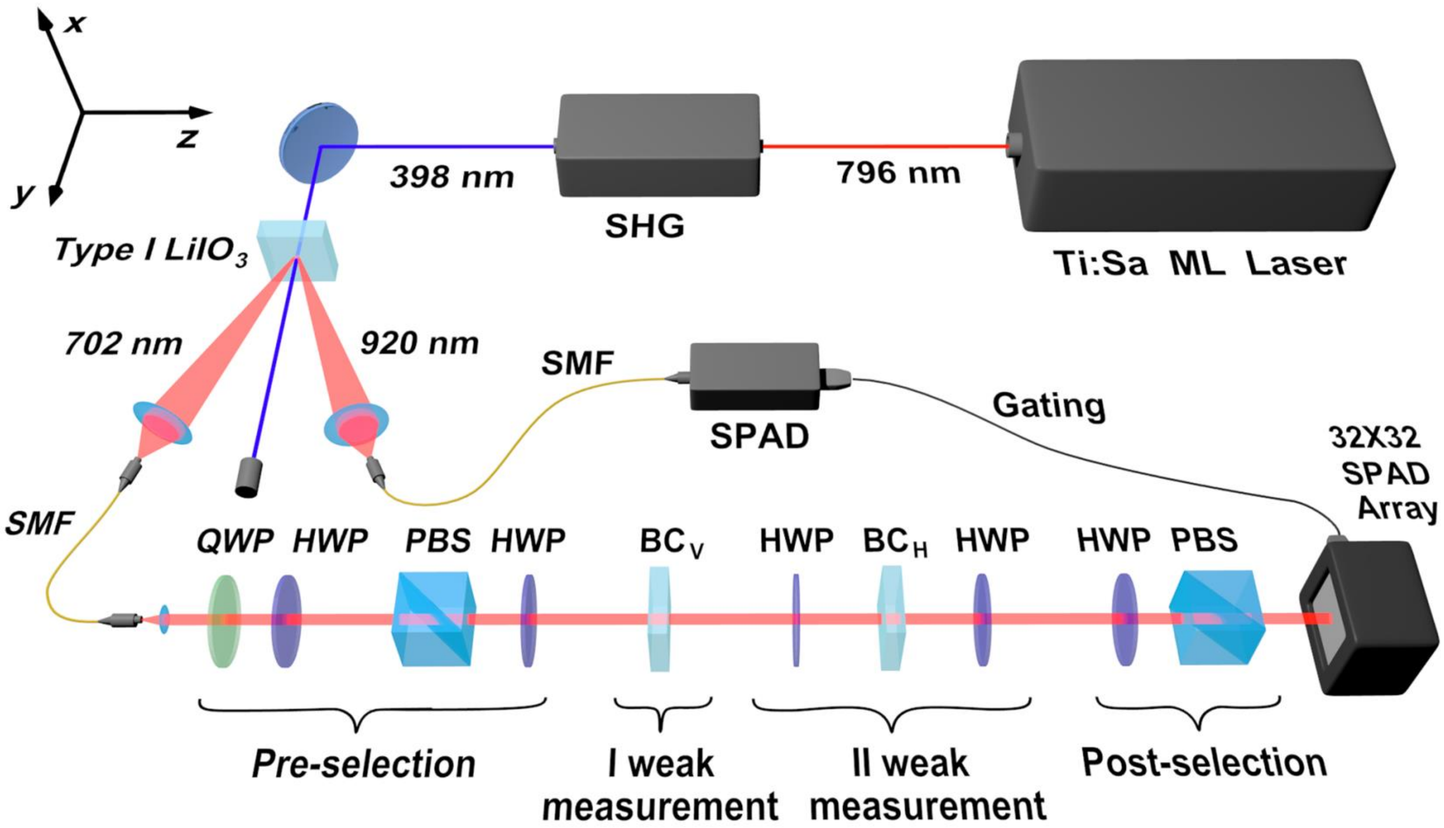
$$\widehat{U}_y = \exp(-ig_y\widehat{\Pi}_V \otimes \widehat{P}_y)$$

$$\widehat{U}_x = \exp(-ig_x\widehat{\Pi}_\psi \otimes \widehat{P}_x)$$



$$\begin{cases} \langle\widehat{X}\rangle = g_x\langle\widehat{\Pi}_\psi\rangle_w \\ \langle\widehat{Y}\rangle = g_y\langle\widehat{\Pi}_V\rangle_w \\ \langle\widehat{X}\widehat{Y}\rangle = \frac{1}{2}g_x g_y\left(\langle\widehat{\Pi}_\psi\widehat{\Pi}_V\rangle_w + \langle\widehat{\Pi}_\psi\rangle_w\langle\widehat{\Pi}_V\rangle_w\right) \end{cases}$$

Experimental Apparatus



SETUP

SPADlab

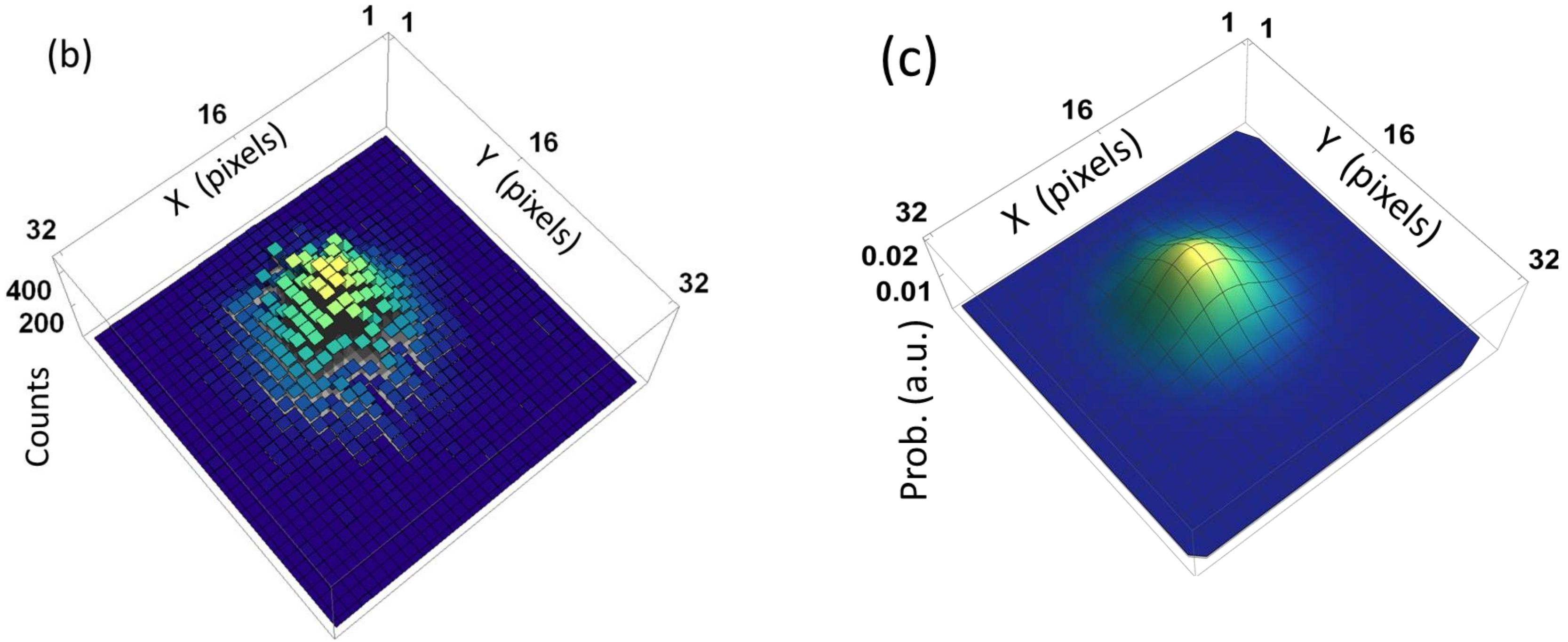
32x32 SPAD+TDC camera

Features

- Multi-modality: photon-counting, 2D imaging, 3D time-of-flight ranging, TCSPC (time-correlated single-photon counting)
- Image dimension: 32x32 (1024) pixels
- In-pixel counter: 6 bit (photon-counting)
- In-pixel TDC: 10 bit (photon-timing)
- Max frame rate: 100,000 fps (burst) and 10,000 fps (continuous)
- Timing resolution: 312 ps – 0.9 ns
- Full scale range: 320 ns – 0.92 μs
- Hardware interface: USB 2.0
- Software interface: Matlab

Fig. 1: SPAD camera for 2D imaging, 3D ranging and TCSPC photon-counting.

Output 32x32 spad array versus theoretical prediction



Typical single data acquisition obtained with our spatial resolving single-photon detector (32X32 SPAD camera), after noise subtraction.

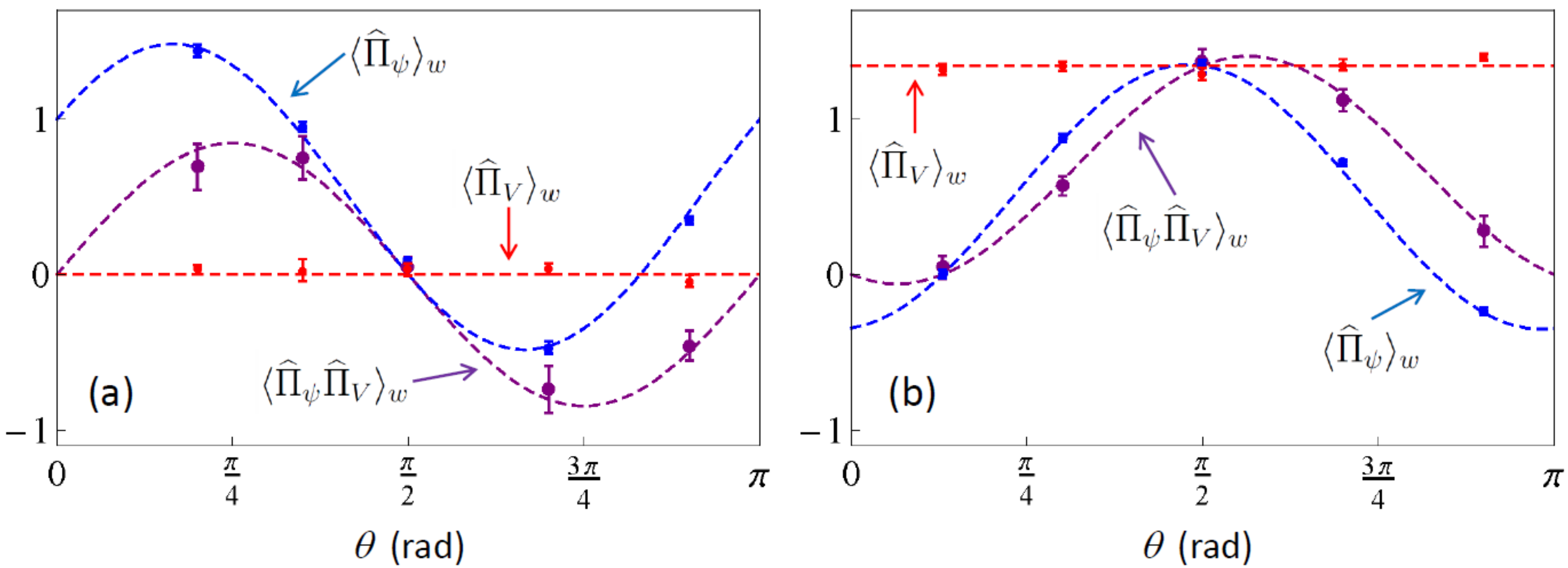
Corresponding predicted probability distribution calculated according to the theory.

Results

Measured weak values (data points) compared with the theoretical predictions (dashed lines)

$$\widehat{\Pi}_\psi = |\psi\rangle\langle\psi| \ (\text{with } |\psi\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle)$$

$$\widehat{\Pi}_V = |V\rangle\langle V|$$



$|\psi_i\rangle = 0.588|H\rangle + 0.809|V\rangle$

$|\psi_f\rangle = |H\rangle$

$|\psi_i\rangle = 0.509|H\rangle + 0.861|V\rangle$

$|\psi_f\rangle = -0.397|H\rangle + 0.918|V\rangle$

Results

We realized for the first time a sequential weak value evaluation of two incompatible observables on a single photon.

F. Piacentini, M. P. Levi, A. Avella, E. Cohen, R. Lussana, F. Villa, A.Tosi, F. Zappa, M. Gramegna, G. Brida, I. P. Degiovanni, M. Genovese.
“Measuring incompatible observables of a single photon” **arXiv:1508.03220**

Weak regime investigation

$$\widehat{U} = \exp(-ig\widehat{A} \otimes \widehat{P})$$

$$A_w = \frac{\langle \varphi | A | \psi \rangle}{\langle \varphi \mid \psi \rangle}$$

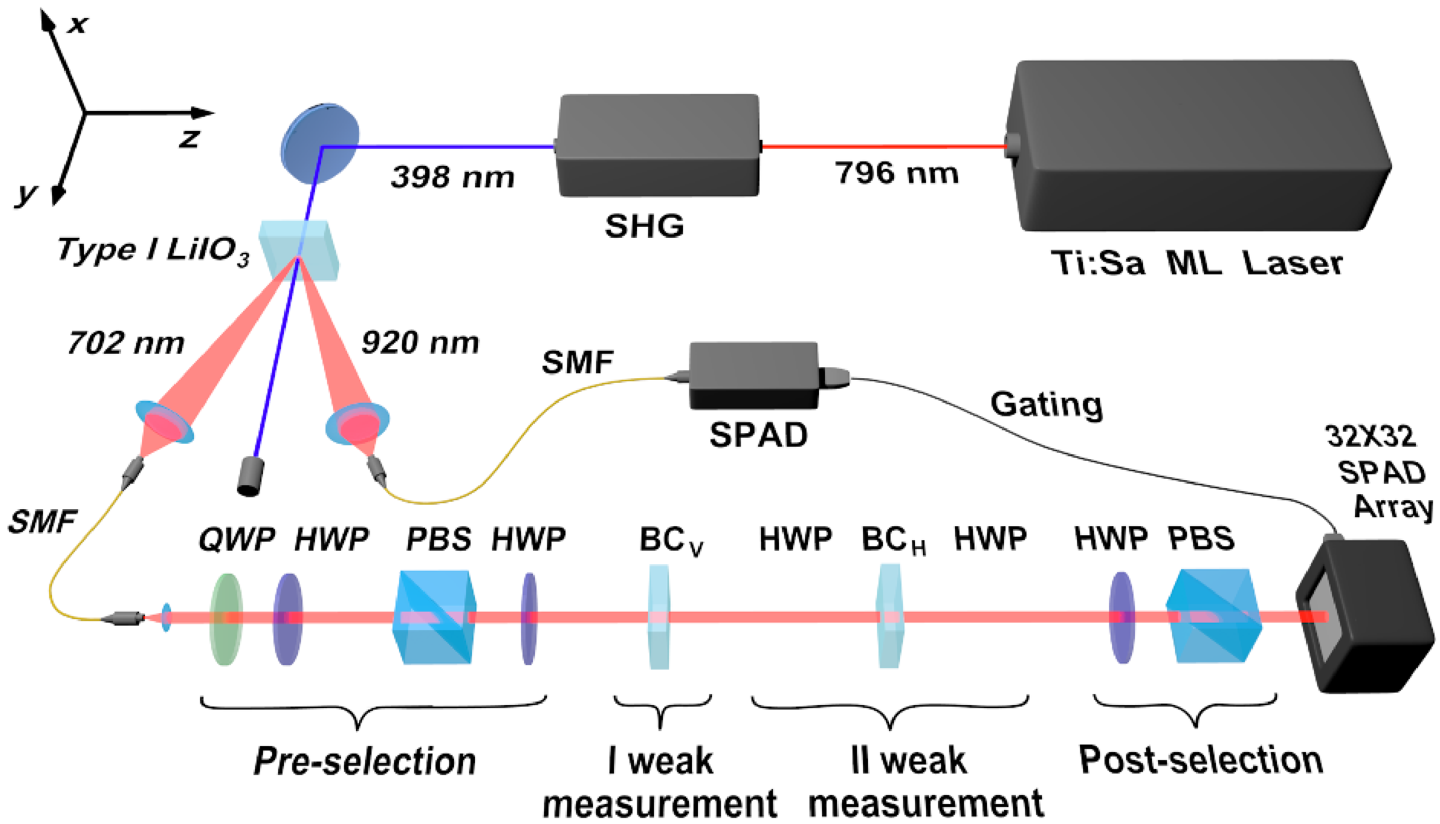
Approximated solution

$$\langle \widehat{X} \rangle = -\Pi w\alpha\beta \, g + O[g]^2$$

Exact solution

$$\langle \widehat{X} \rangle = -\frac{g \, \Pi w\alpha\beta \left(1 + \left(-1 + e^{\frac{g^2}{4\sigma^2}}\right) \Pi w\alpha\beta\right)}{-2\,(-1 + \Pi w\alpha\beta)\, \Pi w\alpha\beta + e^{\frac{g^2}{4\sigma^2}} \left(1 - 2\,\Pi w\alpha\beta + 2\,\Pi w\alpha\beta^2\right)}$$

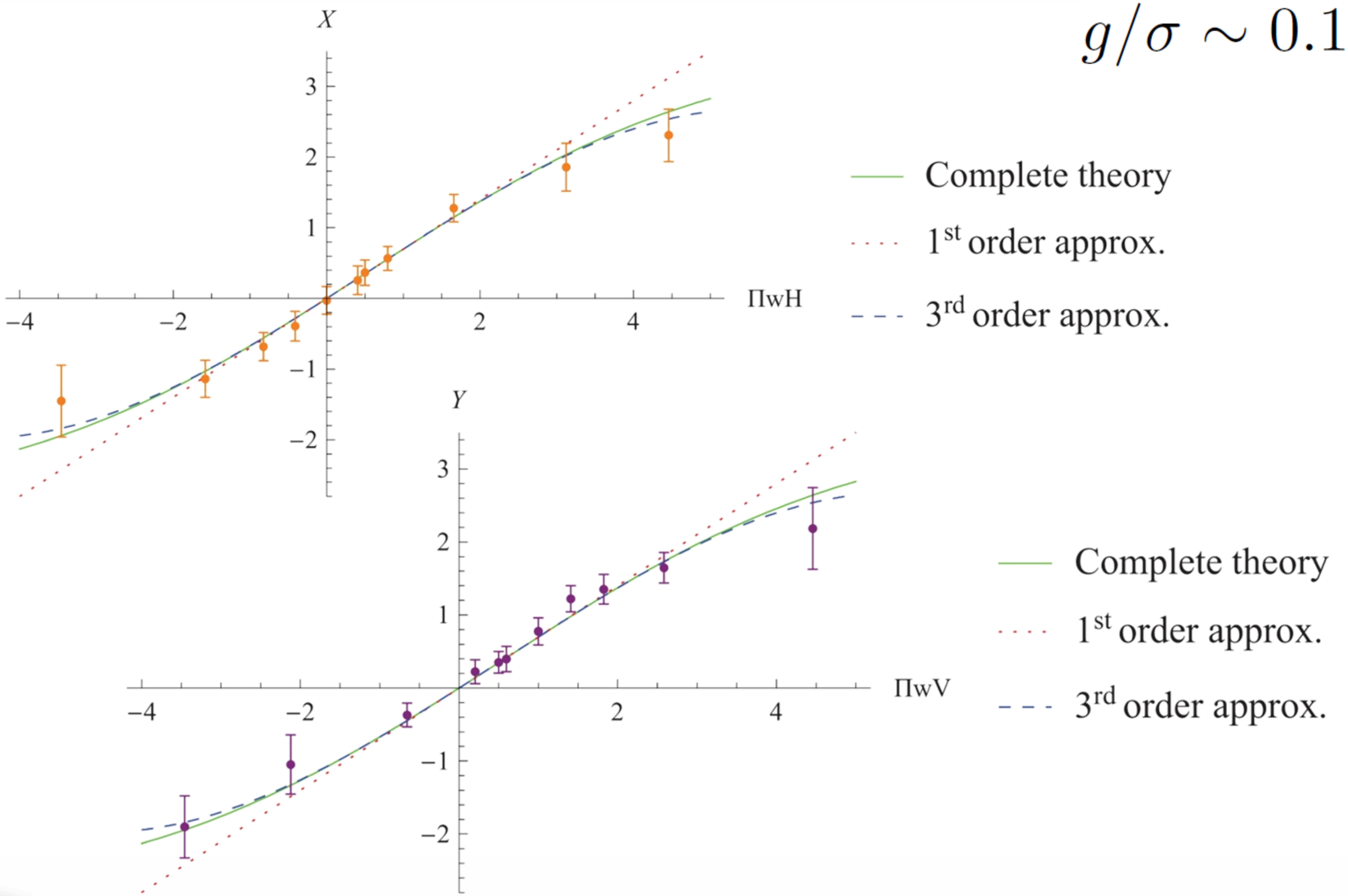
Setup



$$\widehat{A} \longrightarrow \widehat{\Pi}_V = |V\rangle\langle V| \qquad \widehat{B} \longrightarrow \widehat{\Pi}_\psi = |H\rangle\langle H|$$

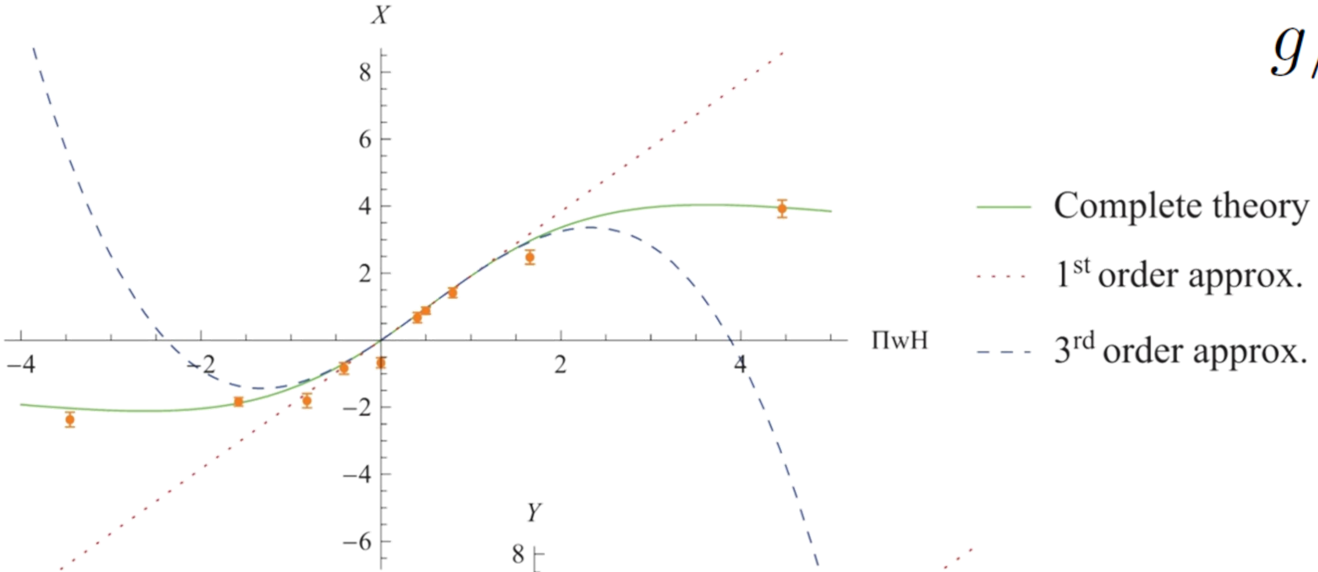
Results

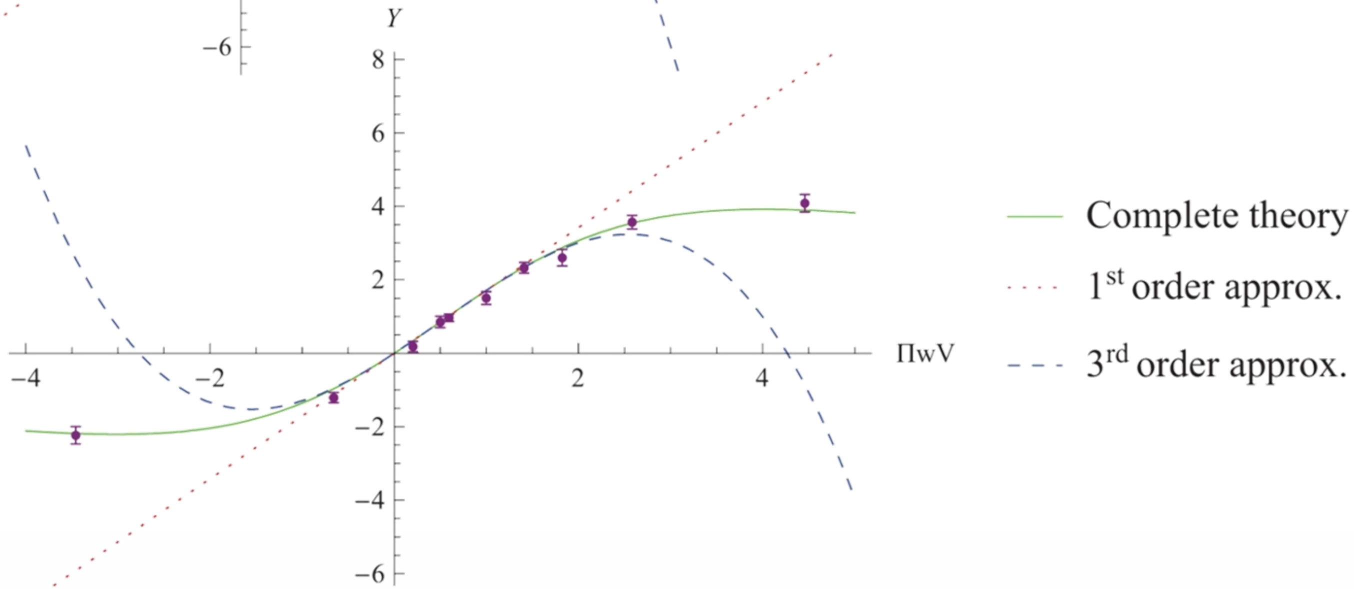
$$g/\sigma \sim 0.15$$



Results

$$g/\sigma \sim 0.32$$





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Thanks for attention!