Is purity eternal at the Planck scale?

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MARIE CURIE ACTIONS

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PHYSICAL REVIEW D VOLUME 14, NUMBER 10 15 NOVEMBER 1976

Breakdown of predictability in gravitational collapse*

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are preserved by time evolution they (re)-discovered the **Lindblad equation**

$$\boxed{\dot{\rho} = -i[H,\rho] - \frac{1}{2} h_{\alpha\beta}\left(Q^\alpha Q^\beta \rho + \rho\, Q^\beta Q^\alpha - 2Q^\alpha \rho\, Q^\beta\right)}$$

$h_{\alpha\beta}$ is a hermitian matrix of constants and Q^α form a basis of hermitian matrices

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ABSTRACT

Motivated by Hawking's proposal that the quantum-mechanical density matrix ρ obeys an equation more general than the Schrödinger equation, we study the general properties of evolution equations for ρ. We argue that any more general equation for ρ violates either locality or energy-momentum conservation.

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Goal of this talk: show that **Planck scale deformations of translations** naturally lead to the possibility of generalized quantum time evolution of Lindblad type!

MA: 1403.6457; Phys. Rev. D 90, 024016 (2014)

Outline

- **Topological particles and group momentum space in 3d gravity**
- **Deformed translations and Lindblad evolution in 3d**
- **de Sitter momentum space: beyond von Neumann evolution in 4d**
- **Conclusions and outlook**

Point particles in 3d gravity

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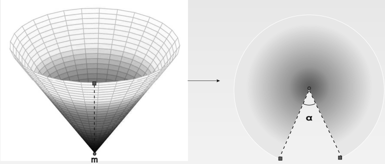
(3d Newton's constat G $\sim 1/M_{\mathrm{Planck}}$)



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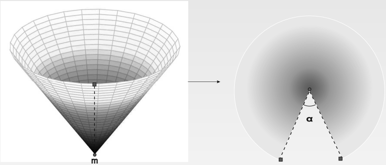

Particle's mass (**rest energy**) can be read off evaluating the **holonomy** of the flat connection around the defect and results in a **rotation** $h_\alpha \in SL(2, \mathbb{R})$

Group-valued momenta: from mass-shells to conjugacy classes

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- Deformed **mass-shell** condition:

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- massive and massless co-dimension 2 defects in $3+1$ dimensions used as building blocks of 't Hooft *"piecewise flat gravity"* (see arXiv:0804.0328)

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Parametrize group elements: $h = u\mathbb{1} + \frac{p^\mu}{\kappa}\gamma_\mu$ with $\kappa = (4\pi G)^{-1}$

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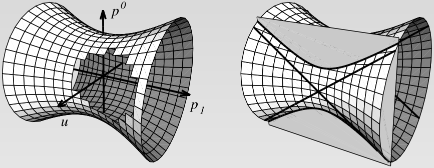

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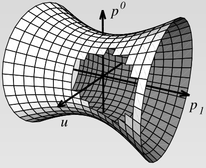

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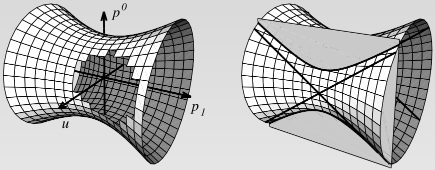

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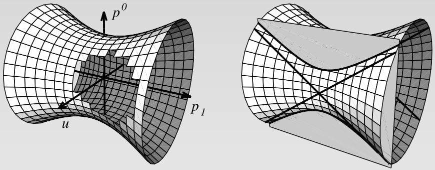

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- **mass-shell condition**: $p^2 = \kappa^2 \cos 4\pi G m = m_\kappa$
- **Lorentz transformation**: $h' = ghg^{-1}$, undeformed on p^μ e.g. boost in the 1-direction $g = e^{\frac{1}{2}\eta\gamma_2}$

$$\begin{cases} p'^0 = p^0 \cosh\eta - p^1 \sinh\eta \\ p'^1 = p^1 \cosh\eta - p^0 \sinh\eta \\ p'^2 = p^2 \end{cases}.$$

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- action on **dual space** $\mathcal{H}^*$ spanned by bras: $P_\mu\langle k| = -k_\mu\langle k|$

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i.e. just the familiar **adjoint action**... **N.B.** Using the spectral theorem any operator can be written in terms of a combination of projectors $|k\rangle\langle k|$

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Key point: the action on operators will be deformed accordingly

Deformed translations and Lindblad evolution in three dimensions

For the deformed translation generators associated to $SL(2,\mathbb{R})$ momentum space:

$$\Delta P_\mu = P_\mu \otimes \mathbb{1} + \mathbb{1} \otimes P_\mu + \frac{1}{\kappa}\,\epsilon_{\mu\nu\sigma} P^\nu \otimes P^\sigma + \mathcal{O}\left(\frac{1}{\kappa^2}\right)\,,\;\; S(P_\mu) = -P_\mu\,.$$

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which leads to a **Lindlblad equation**

$$\boxed{\dot{\rho} = -i[P_0,\rho] - \frac{1}{2} h_{ij}\left(P^i P^j \rho + \rho\, P^j P^i - 2P^j \rho\, P^i\right)}$$

with "decoherence" matrix given by

$$h = \frac{i}{\kappa}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

Some remarks

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In our case *h is not* **positive definite** *nor* **real**

Further work needed to establish properties of our Lindblad evolution...

Deformed translation in four dimensions

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κ-**momenta**: coordinates on **Lie group** $AN(3)$ obtained form the Iwasawa decomposition of $SO(4,1) \simeq SO(3,1)AN(3)$, sub-manifold of dS_4

$$-p_0^2 + p_1^2 + p_2^2 + p_3^2 + p_4^2 = \kappa^2\,; \qquad p_0 + p_4 > 0$$

with $\kappa \sim E_{Planck}$

These structures have been advocated as encoding the **kinematics of a "Minkowski-limit" of quantum gravity**...deformed relativistic kinematics at the **Planck scale**

dS momentum space without DSR

In parallel with 3d case we consider **translation generators** P_μ associated to *embedding* coordinates p_μ on dS_4

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In parallel with 3d case we consider **translation generators** P_μ associated to *embedding* coordinates p_μ on dS_4

Their **co-products** and **antipodes** at *leading order* in κ

$$\begin{aligned} \Delta(P_0) &= P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0 + \frac{1}{\kappa} P_m \otimes P_m \,, \\ \Delta(P_i) &= P_i \otimes \mathbb{1} + \mathbb{1} \otimes P_i + \frac{1}{\kappa} P_i \otimes P_0 \,, \\ S(P_0) &= -P_0 + \frac{1}{\kappa} \vec{P}^2 \,, \\ S(P_i) &= -P_i + \frac{1}{\kappa} P_i \, P_0 \,, \end{aligned}$$

this *basis* of κ-Poincaré is called **"classical"** because

- action of Lorentz sector on P_μ in **undeformed**;
- **mass-shell** condition undeformed $P_0^2 - \vec{P}^2 = const$

In embedding coordinates we have *ordinary relativistic kinematics* at the **one-particle** level...all non-trivial structures confined to "co-algebra" sector

Deformed Lindblad evolution from κ-translations

A straightforward calculation of $\mathrm{ad}_{P_0}(\rho)$ leads to a *non-symmetric* **Lindblad equation**

$$\boxed{\dot{\rho} = -i[P_0, \rho] + \frac{i}{\kappa} P_m \rho P_m - \frac{i}{\kappa} \rho \, \vec{P}^2}$$

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- the adjoint actions of N_i and P_0 satisfy

$$\mathrm{ad}_{\mathrm{ad}N_i(P_0)}(\cdot) = \mathrm{ad}_{N_i}(\mathrm{ad}_{P_0})(\cdot) - \mathrm{ad}_{P_0}(\mathrm{ad}_{N_i})(\cdot)$$

in this sense the κ-Lindblad equation follows a **deformed notion of covariance**

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Phenomenology of κ-Lindblad evolution? (Ellis et al. "Search for Violations of Quantum Mechanics," Nucl. Phys. B **241***, 381 (1984)); bounds on κ using* **precision measurements of neutral kaon systems** *(KLOE and KLOE-2 experiment)?*