## Angular analysis of $B_{d} \rightarrow K \pi \mu \mu$ and $\Lambda_{b} \rightarrow p K \mu \mu$ using a moments approach

Biplab Dey<br>work w/

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## Motivation for angular analyses

- Lesson from $P_{5}^{\prime}$ : anomalies can show up in hitherto unexpected places.

- Angular observables being interference terms have more sensitivity than rates. Good bet for NP hunting.


## Moments idea - introduction

- Very simple idea. Re-write rate in an orthonormal basis:

$$
\frac{d \Gamma}{d \Omega}=\sum_{i} f_{i}(\Omega) \times \Gamma_{i}
$$

- Orthonormal angular basis: $\int f_{i}(\Omega) f_{j}(\Omega) d \Omega=\delta_{i j}$.
- Weighting each event by $f_{i}(\Omega)$ projects out the $\left\langle\Gamma_{i}\right\rangle$ 's.
- All your observables can now be solved out from the $\Gamma_{i}$.
- Completely equivalent to doing a likelihood fit, as long as you extract the covariance matrix correctly.
- Can be quite non-trivial in the presence of background, and efficiency calculations. See discussion Sec. C in 1505.02873.


## Moments idea: why it is a good idea

- Two main thrusts here:
(1) Very stable against low statistics. No fit bias, start values, etc. No enternal input - it is what it is.
(2) In many/most situations with higher waves, the observables turn out to be related. Hard problem to understand what constitutes a minimal set of independent parameters to float in likelihood fit. MOM is completely impervious to this.


## $B_{d} \rightarrow K \pi \mu \mu$ and friends

- $b \rightarrow X\left(\rightarrow h_{1} h_{2}\right) \ell_{1} \ell_{2}$ topology: $\Phi \in\left\{m_{X}, q^{2}, \theta_{\ell}, \theta_{V}, \chi\right\}$

- Wide window in the $m_{X}$ spectrum.
- Higher waves in the dihadron system $\Rightarrow$ more observables to play around with.


## Example: $\bar{B} \rightarrow J / \psi K^{-} \pi^{+}$

- The orthonormal basis is constructed out of $Y_{I}^{m} \equiv Y_{I}^{m}\left(\theta_{\ell}, \chi\right)$ and $P_{I}^{m} \equiv \sqrt{2 \pi} Y_{I}^{m}\left(\theta_{V}, 0\right)$.
- 7 complex amplitudes, $\left\{S, H_{0, \|, \perp}, D_{0, \|, \perp}\right\}$, or 13 real numbers.
- $28 \Gamma_{i}$ moments/observables. A few examples:

| $i$ | $f_{i}(\Omega)$ | $\Gamma_{i}^{\text {tr }}$ |
| :---: | :---: | :---: |
| 1 | $P_{0}^{0} Y_{0}^{0}$ | $\left[\left\|H_{0}\right\|^{2}+\left\|H_{\\|}\right\|^{2}+\left\|H_{\perp}\right\|^{2}+\|S\|^{2}+\left\|D_{0}\right\|^{2}+\left\|D_{\\|}\right\|^{2}+\left\|D_{\perp}\right\|^{2}\right]$ |
| 2 | $P_{1}^{0} Y_{0}^{0}$ | $2\left[\frac{2}{\sqrt{5}} \operatorname{Re}\left(H_{0} D_{0}^{*}\right)+\operatorname{Re}\left(S H_{0}^{*}\right)+\sqrt{\frac{3}{5}} \operatorname{Re}\left(H_{\\|} D_{\\|}^{*}+H_{\perp} D_{\perp}^{*}\right)\right]$ |
| 5 | $P_{4}^{0} Y_{0}^{0}$ | $\frac{2}{7}\left[-2\left(\left\|D_{\\|}\right\|^{2}+\left\|D_{\perp}\right\|^{2}\right)+3\left\|D_{0}\right\|^{2}\right]$ |
| 28 | $P_{4}^{0} \sqrt{2} \operatorname{Im}\left(Y_{2}^{2}\right)$ | $-\frac{4}{7} \sqrt{\frac{3}{5}} \operatorname{Im}\left(D_{\perp} D_{\\|}^{*}\right)$ |

## Example: $\bar{B} \rightarrow J / \psi K^{-} \pi^{+}$(cntd.)

- Interesting things are now solvable for (no fitting):

$$
\begin{aligned}
\left|D_{0}\right|^{2} & =\frac{7}{9}\left(\frac{\Gamma_{5}}{2}-\sqrt{5} \Gamma_{10}\right) \\
\left|D_{\|}\right|^{2} & =\frac{7}{4}\left(\sqrt{\frac{5}{3}} \Gamma_{23}-\frac{1}{3}\left(\sqrt{5} \Gamma_{10}+\Gamma_{5}\right)\right) \\
\left|D_{\perp}\right|^{2} & =\frac{7}{4}\left(-\sqrt{\frac{5}{3}} \Gamma_{23}-\frac{1}{3}\left(\sqrt{5} \Gamma_{10}+\Gamma_{5}\right)\right)
\end{aligned}
$$

- However, many things missing. Like, $\left(\left|H_{0}\right|^{2}+|S|^{2}\right)$ always occur together. So $F_{S}$ (S-wave fraction) "might" not be extractable.


## Goodness of model estimation

- $d \Gamma / d \Omega=\sum_{i} f_{i}(\Omega) \Gamma_{i}$ is the model. PHSP MC weighted by this model should match data in all distributions.
- Eg.: in $B_{d} \rightarrow \psi(2 S) K \pi$, I can look at $m(\psi(2 S) \pi)$ spectrum for the $Z(4430)$.
- As long as enough number of waves are included, the moments model extracts the maximal information content in the angular distributions.
- However, we have outgoing spinors (proton/muon) whose spins we average over.
- This results in "dilution" of information present in the angular distributions.


## Status of $B_{d} \rightarrow J / \psi K \pi$ and $B_{d} \rightarrow K \pi \mu \mu(E B, B D, N S)$

$$
\underline{B_{d} \rightarrow J / \psi K \pi}:
$$

$$
\underline{B_{d} \rightarrow K \pi \mu \mu}:
$$

- Mass-indpendent analysis in 36 $m(K \pi)$ bins. Expected yield is 556638 till $m(K \pi)<1530 \mathrm{MeV}$.

- Moments till $J=4$ being calculated.
- Around 380 events in $q^{2} \in[0.1,6] \mathrm{GeV}^{2}$, $|m(K \pi)-1430|<100 \mathrm{MeV}$.

- $\left\{q^{2}, \mathrm{~m}(\mathrm{~K} \pi)\right\}$ binning and coverage being discussed.


## Ongoing toy studies ( $B_{d} \rightarrow K \pi \mu \mu$ )

- Interesting observables (like $P_{5}^{\prime}$ ) are often combinations of the $\Gamma_{i}$ moments. Full covariance matrix is critical.
- Pulls on $\Gamma_{i j} \equiv\left(\Gamma_{i}+\Gamma_{j}\right)$ efficienctly validates $\operatorname{Cov}_{i j}$.



## Why is LHCb's $J / \psi K \pi$ important

- Long-standing confusion: in $D^{+} \rightarrow \rho^{0} \ell^{-} \bar{\nu}_{\ell}$, CLEO/BESIII/FOCUS does not mention how the charge-conjugation should be done (affects BABAR's $\rho \ell \nu)$. Follow $\pi^{+}\left(\pi^{-}\right)$in $D^{+}\left(D^{-}\right)$?
- I also have a $\chi \rightarrow \pi+\chi$ flip with Belle's $Z(4430)$ Appendix.
- Either case above would mean incorrect angular expression.
- Detailed cross-checks (also with $B$ and $\bar{B}$ separately) absolutely necessary.
- Moments formalism immediately extends to $B_{s} \rightarrow K K \mu \mu$ in wide $m(K K)$ range ( $f_{2}(1525)$ seen $)$.


## Extending the formalism to the spin $-\frac{1}{2} \Lambda_{b}$ case

- Mass-independent moments version of $\Lambda_{b} \rightarrow J / \psi p K$ ( $P_{c}^{+}$analysis) could be a good cross-check.
- Very poorly known $\Lambda^{*}: P_{c}$ analysis quotes a good fraction of $\Lambda(1405)$, when $\Lambda(1405) \rightarrow N-\bar{K}$ is not even listed in the PDG.
- Experimental hints towards double-pole structure in $\Lambda(1405)$.
- Moments can handle low statistics, so we can do fine $m(p K)$ bins.
- In each $m(p K)$ bin, include moments for $J^{P}$ waves till $\frac{11}{2}^{ \pm}$.
- "Mass-independent" approach. No lineshape assumptions.


## Moments for $\Lambda_{b} \rightarrow h_{1} h_{2} \mu \mu$

- Hard problem: $\Lambda_{b}$ polarization + Rarita-Schwinger $J^{P}$ spinors for $\Lambda^{*}$.
- Start with spin- $\frac{1}{2} \Lambda(1115): \Lambda_{b} \rightarrow J / \psi \Lambda$.
- Four complex amplitudes, fully reconstructable from 5-D angular distribution (Lednicky). LHCb's $1 / \mathrm{fb} \Lambda_{b}$ polarization analysis used the integrated version.
- $\Lambda(1115) \rightarrow p \pi$ is self-analyzing, but hadronic $\Lambda^{*} \rightarrow p K$ are not. Might not access to full amplitudes.
- Long term plan:
- Re-derive Lednicky's expression from Korner's formalism in $\Lambda(1115)$ case.
- Extend to $\frac{11}{2}^{ \pm} \Lambda^{*} \rightarrow p K$. NR dimuon gets further $\{V, A\}$ amplitudes.


## Ongoing $\Lambda_{b} \rightarrow p K \mu \mu$ data analysis (DM, BD, NN)

- Simple counting: $\Delta \mathcal{A}_{C P}, A_{F B}^{\ell, h}$ and T-odd correlations.
- $\Lambda_{b} \rightarrow J / \psi p K$ as proxy w/o $J / \psi$ mass-constrained.
- After BDT cut, purity is $\sim 0.712$, lower than $P_{c}$, but yield consistent.




## $\Lambda_{b} \rightarrow p K \mu \mu: \mathrm{MC}$ reweighting

- MC reweighted in nTracks, 2-d $\left.\left\{p_{T}\left(\Lambda_{b}\right), p\left(\Lambda_{b}\right)\right\}, p(P), p(K)\right\}$. PID re-weighting from PIDCalib.
- sWeighted $J / \psi p K$ data used to derive weights.
- Good agreement between Data and MC after re-weighting.


bMom_ISOLATION_BDT1 check



## $\Lambda_{b} \rightarrow p K \mu \mu:$ preliminary expected yield

- $n_{\text {sig }}$ from Mott-Roberts model. $n_{\text {bkgd }}$ from sideband fit.
- Optimal BDT cut: significance $\sim 14 . n_{\text {sig }} \sim 310$.
$\Lambda_{\mathrm{b}}$ mass distribution



## 5-d efficiency parameterization

- Both $K \pi \mu \mu$ and $p K \mu \mu$ will use 5-d efficiency $\epsilon(\phi)$, $\phi \in\left\{q^{2}, \mathrm{~m}_{\mathrm{X}}, \cos \theta_{\mathrm{V}}, \cos \theta_{\ell}, \chi\right\}$.
- Mature tool: $\epsilon(\phi)$ expanded in Legendre polynomials.
- Subsequent event-by-event reweighting of data by $1 / \epsilon(\phi)$.






## Publication strategy

(1) $B_{d} \rightarrow J / \psi K \pi$ : mass-independent moments till spin $J=4$ in $K^{* J}$ in $36 m(K \pi)$ bins.
(2) Provide the full set of moments + covariance matrix in each $m(K \pi)$ bin. By far the most comprehensive world data till $m(K \pi)<1530 \mathrm{MeV}$.
(3) $B_{d} \rightarrow K \pi \mu \mu$ : similar set of results in the higher $m(K \pi)$ region (first observation!). Exact $\left\{q^{2}, m(K \pi)\right\}$ bins under discussion.
(9) $\Lambda \rightarrow p K \mu \mu$ : integrated $m(p K)$ bin (first observation!). May be one low $q^{2}$ and one high $q^{2}$ bin: $A_{F B}^{\ell, h}$.

## On-going work and immediate to-do

- Both NR modes: improved $c \bar{c}$-vetoing using DTF refitting.
- $K \pi \mu \mu$ : include $F$ - and $G$-wave moments. Toys.
- pK $\mu \mu$ : switch to ProbNN from DLL. Problems with proton PID. Toys.
- Calculate the full set of moments for $\Lambda_{b} \rightarrow p K \mu \mu$.
- ANA-Notes under development.


## Classic case: $B \rightarrow \rho \ell \nu$ in BABAR

- $B^{-} \rightarrow \pi^{+} \pi^{-} \ell^{-} \bar{\nu}_{\ell}$ in BABAR with $S+P$ waves under the $\rho$.
- Highly statistics limited. Long-standing effor to pull out the $S$-wave (affects $\left|V_{u b}\right|$ ).




- Without the moments method, pulling out $F_{S}$ is semi-impossible. Full moments paper in the pipeline.

