In-Out Formalism for Effective Actions in QED and Gravity

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Outline

- Unified Picture for Spontaneous Pair Production
- Effective Actions in In-Out Formalism
- Gamma-Function Regularization
- Gamma-Function for Supercritical Fields and Massless QED
- QED in (Anti-) de Sitter
- Conclusion

Heisenberg-Euler/Schwinger QED Action

- Maxwell theory and Dirac/Klein-Gordon theory are gauge invariant:

$$F = \frac{1}{4} F^{\mu\nu} F_{\mu\nu} = \frac{1}{2}\left(\mathrm{B}^2 - \mathrm{E}^2\right), \quad G = \frac{1}{4} F^{\mu\nu} F^{*}_{\mu\nu} = \mathrm{B}\cdot\mathrm{E}$$

$$X = \sqrt{2(F + iG)} = X_r + iX_i$$

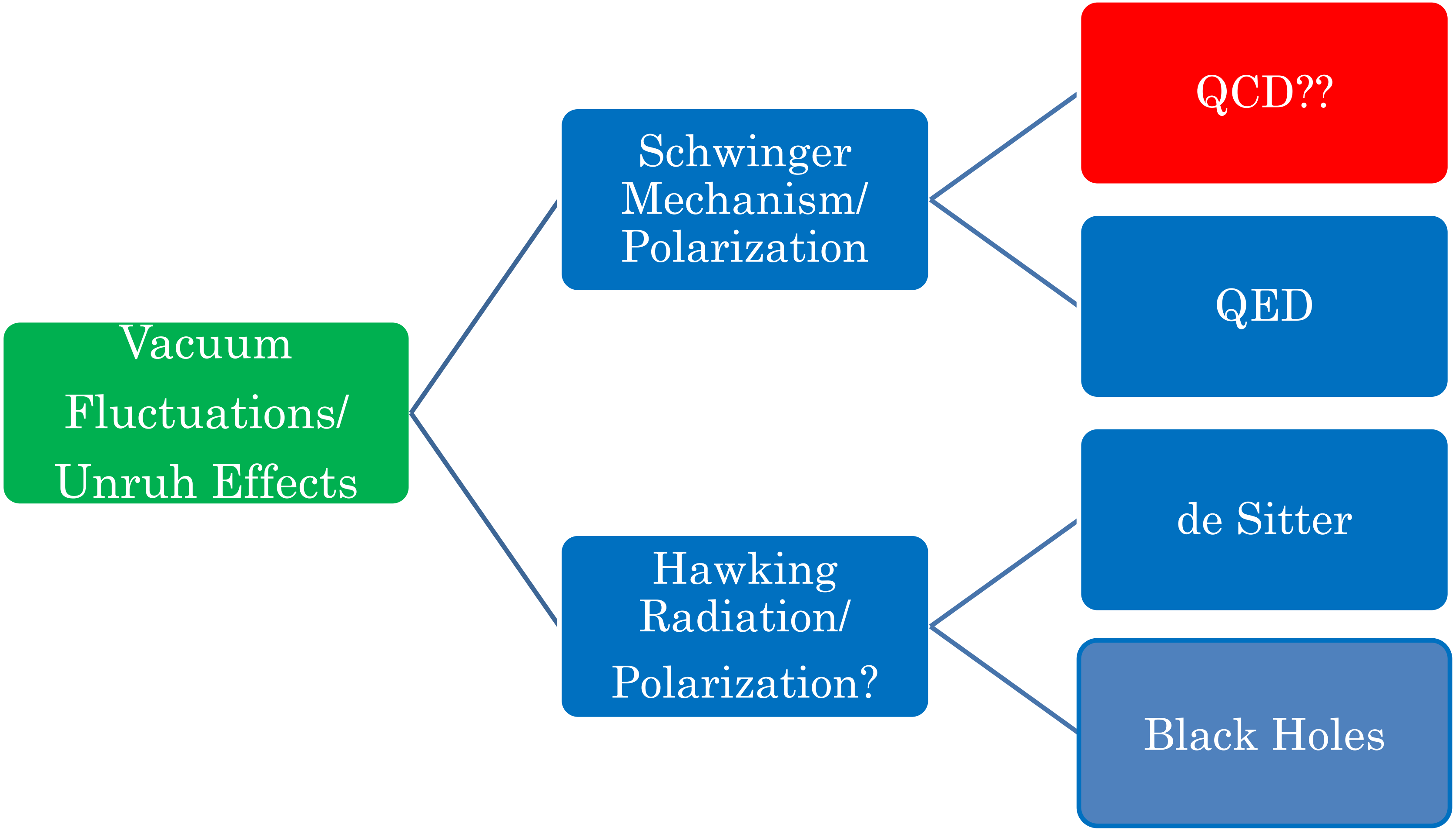
- The Heisenberg-Euler/Schwinger effective action per volume and time [J. Schwinger, "On gauge invariance and vacuum polarization," Phys. Rev. 82 ('51); DeWitt: "This is one of the great papers of all time."]

$$L_{\mathrm{eff}} = -F - \frac{1}{8\pi^2}\int_0^\infty ds \frac{e^{-m^2 s}}{s^3}\left[(qs)^2 G \frac{\mathrm{Re}\cosh(qXs)}{\mathrm{Im}\cosh(qXs)} - 1 - \frac{2}{3}(qs)^2 F\right]$$

Unified Picture for Spontaneous Pair Production

Unified Picture for Pair Production

[SPK, JHEP ('07)]

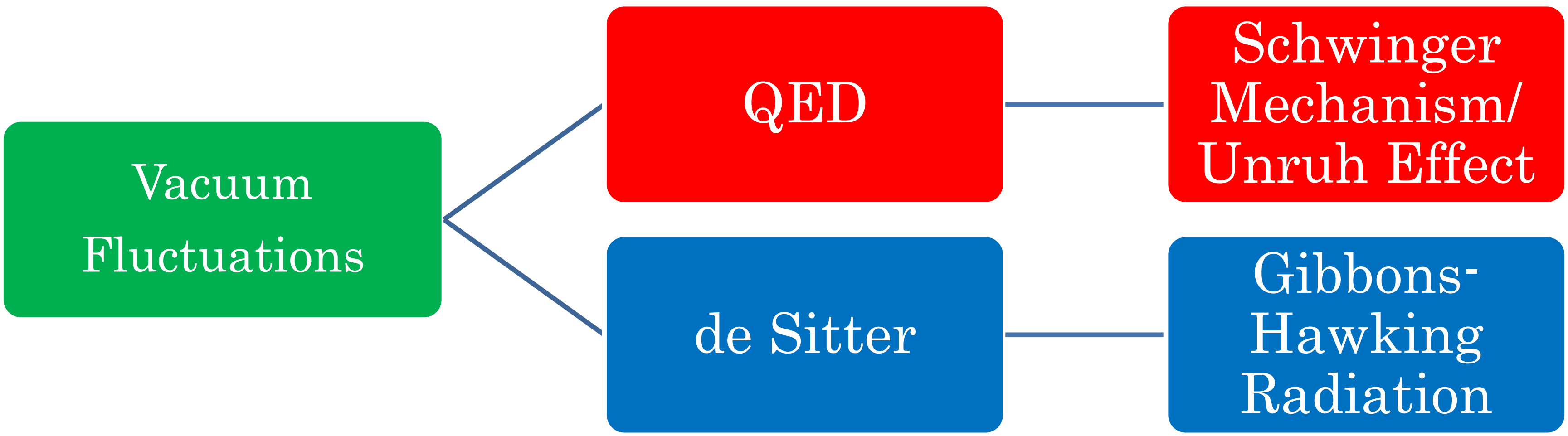


Schwinger Effect in (A)dS

[Cai, SPK, JHEP ('14)]

Near Horizon Geometry of Near-Extremal Black Hole:

$AdS_2 \times S^2$



Effective Temperature for Unruh Effect in (A)dS$_2$

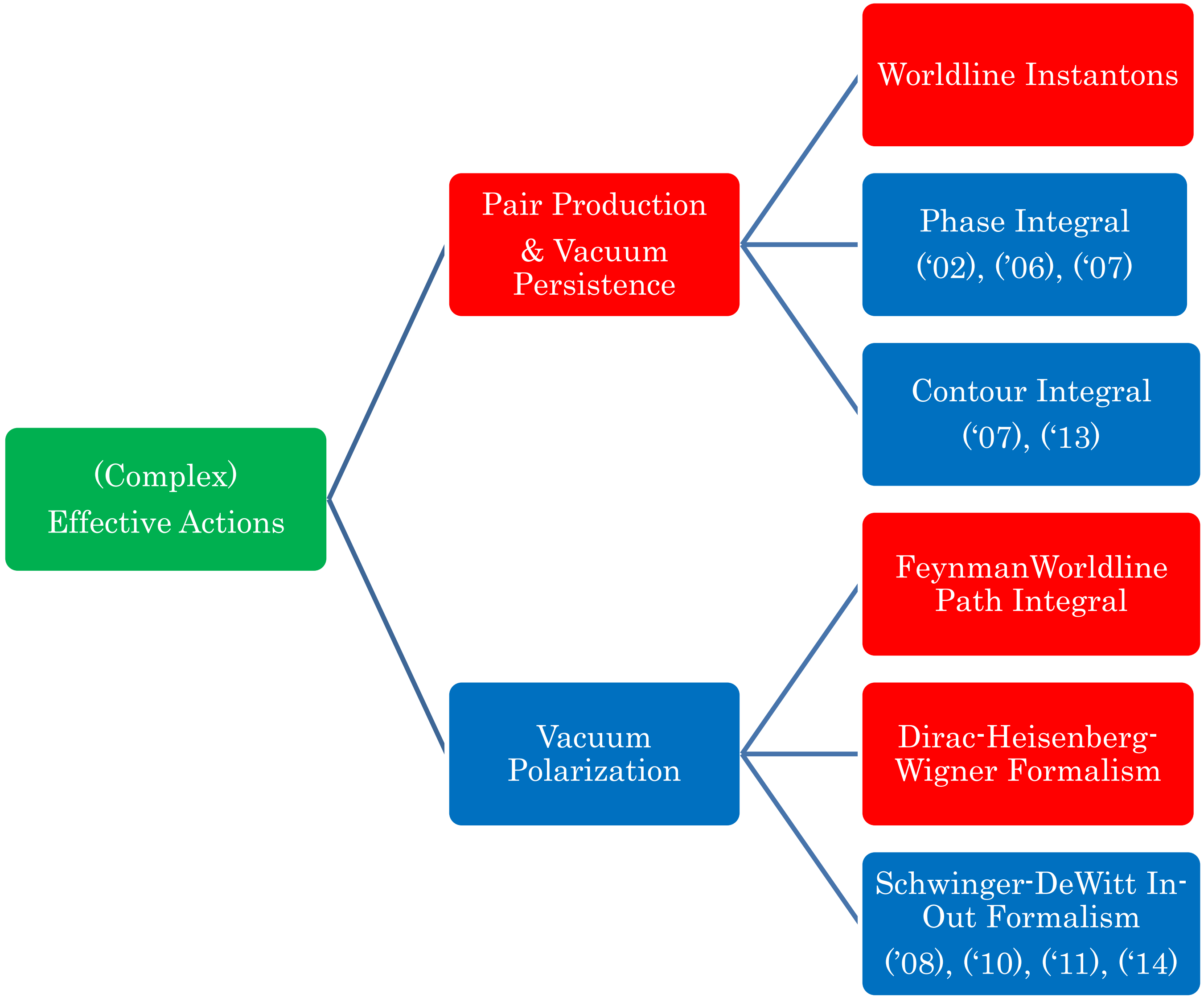
[Narnhofer, Peter, Thirring, IJMP B10 ('96); Deser, Levin, CGG 14 ('97)]

Effective Temperature

- Unruh Effect — $T_U = \dfrac{a}{2\pi}$
- (A)dS — $R = \begin{cases} 2H^2 \\ -2K^2 \end{cases}$

$$T_{\rm eff} = \sqrt{T_U^2 + \frac{R}{8\pi^2}}$$

Methods for Schwinger Mechanism & Effective Actions



Effective Actions in In-Out Formalism

One-Loop Effective Actions

- In the in-out formalism via the Schwinger variational principle, the vacuum persistence amplitude (S-matrix) gives the effective action [Schwinger, PNAS ('51); DeWitt, Phys. Rep. ('75); *The Global Approach to Quantum Field Theory* ('03)]; equivalent to Feynman path integral

$$e^{iW} = e^{i\int (-g)^{1/2} d^D x L_{\mathrm{eff}}} = \langle 0, \mathrm{out} \mid 0, \mathrm{in} \rangle$$

- The vacuum persistence and pair production

$$\left| \langle 0, \mathrm{out} \mid 0, \mathrm{in} \rangle \right|^2 = e^{-2\,\mathrm{Im}\,W}$$

$$2\,\mathrm{Im}\,W = \pm VT \sum_{\mathrm{k}} \ln(1 \pm N_k)$$

Bogoliubov Transformation & In-Out Formalism

- The Bogoliubov transformation between the in-state and the out-state, equivalent to the S-matrix,

$$a_{\mathrm{k,out}} = \alpha_{\mathrm{k,in}} a_{\mathrm{k,in}} + \beta^*_{\mathrm{k,in}} b^+_{\mathrm{k,in}} = U_{\mathrm{k}} a_{\mathrm{k,in}} U^+_{\mathrm{k}}$$

$$b_{\mathrm{k,out}} = \alpha_{\mathrm{k,in}} b_{\mathrm{k,in}} + \beta^*_{\mathrm{k,in}} a^+_{\mathrm{k,in}} = U_{\mathrm{k}} b_{\mathrm{k,in}} U^+_{\mathrm{k}}$$

- Commutation relations from quantization rule (CTP):

$$\left[a_{\mathrm{k,out}}, a^+_{\mathrm{p,out}}\right] = \delta(\mathrm{k}-\mathrm{p}),\ \left[b_{\mathrm{k,out}}, b^+_{\mathrm{p,out}}\right] = \delta(\mathrm{k}-\mathrm{p});$$

$$\left\{a_{\mathrm{k,out}}, a^+_{\mathrm{p,out}}\right\} = \delta(\mathrm{k}-\mathrm{p}),\ \left\{b_{\mathrm{k,out}}, b^+_{\mathrm{p,out}}\right\} = \delta(\mathrm{k}-\mathrm{p})$$

- Particle (pair) production

$$N_{\mathrm{k}} = \left|\beta_{\mathrm{k}}\right|^2; \quad \left|\alpha_{\mathrm{k}}\right|^2 \mp \left|\beta_{\mathrm{k}}\right|^2 = 1$$

Out-Vacuum from In-Vacuum

- For bosons, the out-vacuum is the multi-particle states of but unitary inequivalent $\langle 0;\mathrm{out} \,|\, 0;\mathrm{in}\rangle = 0$ to the in-vacuum:

$$\left|0;\mathrm{out}\right\rangle = \prod_{\mathrm{k}} U_{\mathrm{k}} \left|0;\mathrm{in}\right\rangle = \prod_{\mathrm{k}} \frac{1}{\alpha_{\mathrm{k,in}}} \sum_{n_k} \left(-\frac{\beta^*_{\mathrm{k,in}}}{\alpha_{\mathrm{k,in}}} \right)^{n_k} \left| n_{\mathrm{k}}, \bar{n}_{\mathrm{k}}; \mathrm{in} \right\rangle$$

- The out-vacuum for fermions (Pauli blocking):

$$\left|0;\mathrm{out}\right\rangle = \prod_{\mathrm{k}} U_{\mathrm{k}} \left|0;\mathrm{in}\right\rangle = \prod_{\mathrm{k}} \Big(-\beta^*_{\mathrm{k,in}} \left| 1_{\mathrm{k}}, \bar{1}_{\mathrm{k}}; \mathrm{in} \right\rangle + \alpha_{\mathrm{k,in}} \left| 0_{\mathrm{k}}, \bar{0}_{\mathrm{k}}; \mathrm{in} \right\rangle \Big)$$

Out-Vacuum from S-Matrix

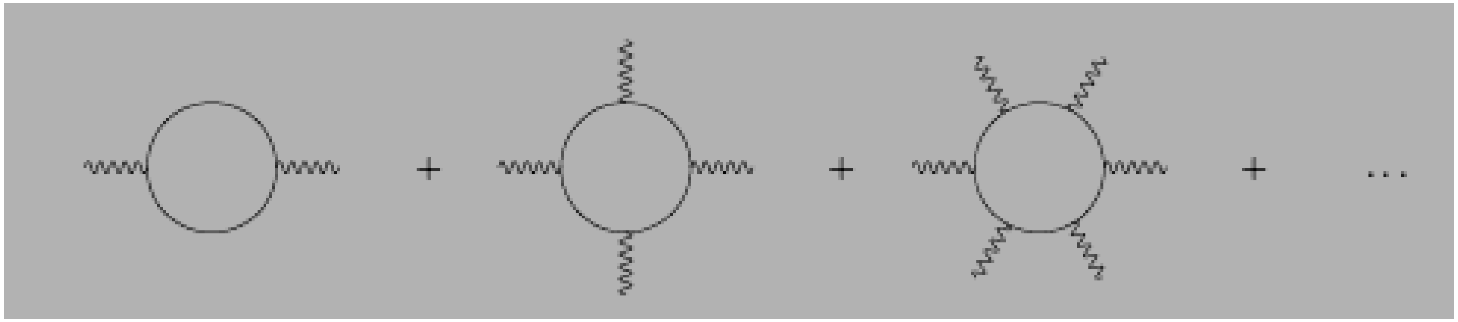
- The out-vacuum in terms of the S-matrix (evolution operator) for scalar

$$U_{\mathrm{k}} = S_{\mathrm{k}} P_{\mathrm{k}}, \quad \begin{cases} P_{\mathrm{k}} = \exp[i\theta_{\mathrm{k}}(a^{+}_{\mathrm{k,in}} a_{\mathrm{k,in}} + b^{+}_{\mathrm{k,in}} b_{\mathrm{k,in}} + 1)] \\ S_{\mathrm{k}} = \exp[r_{\mathrm{k}}(a_{\mathrm{k,in}} b_{\mathrm{k,in}} e^{-2i\varphi_{\mathrm{k}}} - a^{+}_{\mathrm{k,in}} b^{+}_{\mathrm{k,in}} e^{-2i\varphi_{\mathrm{k}}})] \end{cases}$$

$$\alpha_k = e^{-i\vartheta_k} \cosh r_k, \quad \beta^{*}_k = -e^{-i\vartheta_k}(e^{i2\varphi_k} \sinh r_k)$$

- The diagrammatic representation for pair production

$$|\mathrm{out}\rangle = \prod_k e^{-i\vartheta_k} \exp[r_{\mathrm{k}}(a_{\mathrm{k,in}} b_{\mathrm{k,in}} e^{-2i\varphi_{\mathrm{k}}} - a^{+}_{\mathrm{k,in}} b^{+}_{\mathrm{k,in}} e^{-2i\varphi_{\mathrm{k}}})] |\mathrm{in}\rangle$$



Effective Actions at T=0 & T

- Zero-temperature effective action for scalar and spinor [SKP, Lee, Yoon, PRD 78 ('08); 82 ('10); SPK, PRD 84 ('11); Bastianelli, SPK, Schubert, kinematic method ('15)]

$$W = -i \ln \langle 0, \mathrm{out} \,|\, 0, \mathrm{in} \rangle = \pm i \sum_{\mathrm{k}} \ln \alpha_{\mathrm{k}}^{*}$$

- finite-temperature effective action for scalar and spinor [SKP, Lee, Yoon, PRD 79 ('09); 82 ('10)]

$$\exp[i \int d^3 x dt L_{\mathrm{eff}}] = \langle 0, \beta, \mathrm{in} | U^{+} | 0, \beta, \mathrm{in} \rangle = \frac{\mathrm{Tr}(U^{+} \rho_{\mathrm{in}})}{\mathrm{Tr}(\rho_{\mathrm{in}})}$$

Thermofield Dynamics

- Thermal vacuum [Takahashi, Umezawa ('75)]

$$\left|0,\beta,\mathrm{in}\right\rangle = \frac{1}{Z_{\mathrm{in}}^{1/2}}\sum_{k,\sigma}\sum_{n_k} \exp[-\beta E_{n_k,\mathrm{in}}/2]\left|n_k,\mathrm{in}\right\rangle \otimes \left\langle \tilde{n}_k,\mathrm{in}\right|$$

- Thermal expectation value: the expectation value in the thermal vacuum

$$\left\langle O\right\rangle_{\beta} = \mathrm{Tr}(O\rho_{\mathrm{in}}) = \left\langle 0,\beta,\mathrm{in}\right|O\left|0,\beta,\mathrm{in}\right\rangle$$

- Finite-temperature field theory is equivalent to zero-temperature field theory in the "thermal vacuum".

Effective Action at T

- Expectation value of U in thermal vacuum

$$\exp[i\int d^3x dt L_{\text{eff}}] = \langle 0,\beta,\text{in}|U^+|0,\beta,\text{in}\rangle = \frac{\text{Tr}(U^+\rho_{\text{in}})}{\text{Tr}(\rho_{\text{in}})}$$

- Effective action per unit volume and time

$$L_{\text{eff}} = \mp i\sum_{k,\sigma}\left[\ln\left(1\pm e^{-\beta(\omega_k - z_k)}\right) \underbrace{-\beta z_k}_{\text{vacuum effective action}} \underbrace{-\ln\left(1\pm e^{-\beta\omega_k}\right)}_{\text{zero field subtraction}}\right]$$

$$\frac{1}{\alpha_k} = e^{\beta z_k}, \quad z_k = z_r(k) + i z_i(k)$$

Vacuum Polarization & Persistence

- Purely thermal part of the effective action

$$\Delta L_{\text{eff}}(T,E) = L_{\text{eff}}(T,E) - L_{\text{eff}}(T=0,E)$$

$$= \mp i \sum_{k,\sigma} \left[\ln\left(1 \pm e^{-\beta(\omega_k - z_k)}\right) - \ln\left(1 \pm e^{-\beta\omega_k}\right) \right]$$

- Imaginary part of the effective action

$$\mathrm{Im}(\Delta L_{\text{eff}}) = \pm \frac{1}{2} i \sum_{k,\sigma} \sum_{j=1} \frac{(\mp n_{FD/BE}(k))^j}{j} \left[(e^{\beta z_k} - 1)^j + (e^{\beta z_k^*} - 1)^j \right]$$

- Real part of the effective action

$$\mathrm{Re}(\Delta L_{\text{eff}}(T)) = \mp \sum_{k,\sigma} \arctan\left[\frac{\sin(\mathrm{Re}\, L_{\text{eff}}(T=0,k))}{e^{\beta\omega_k}\left(1 + |\beta_k|^2\right)^{(1+2|\sigma|)/2} \pm \cos(\mathrm{Re}\, L_{\text{eff}}(T=0,k))} \right]$$

Pair Production at T

- Imaginary part of the effective action (the limit of small mean number of produced pairs)

$$2\,\mathrm{Im}\,\Delta L_{\mathrm{eff}}(T) \approx \mp \sum_{k,\sigma} |\beta_k|^2\, n_{FD/BE}(k)$$

- Consistent with the pair-production rate at T [SPK, Lee, PRD 76 ('07); SPK, Lee, Yoon, PRD 79 ('09)]

$$N^{\mathrm{sp/sc}}(T) = \begin{cases} \sum_k |\beta_k|^2 \tanh(\beta\omega_k/2) \\ \sum_k |\beta_k|^2 \coth(\beta\omega_k/2) \end{cases}$$

Gamma-Function Regularization

Γ-Function Regularization

- Most of soluble models in QED and gravity have the Bogoliubov coefficients of the form

$$\alpha_k = A_k \prod \frac{\Gamma(a \pm ib)}{\Gamma(c \pm id)}; \quad \beta_k = B_k \prod \frac{\Gamma(f \pm ig)}{\Gamma(h \pm ik)}$$

  – a, …, h: integers or half-integers depending on spins
  – Constants A_k and B_k to be regulated away
- Integral representation for gamma-function

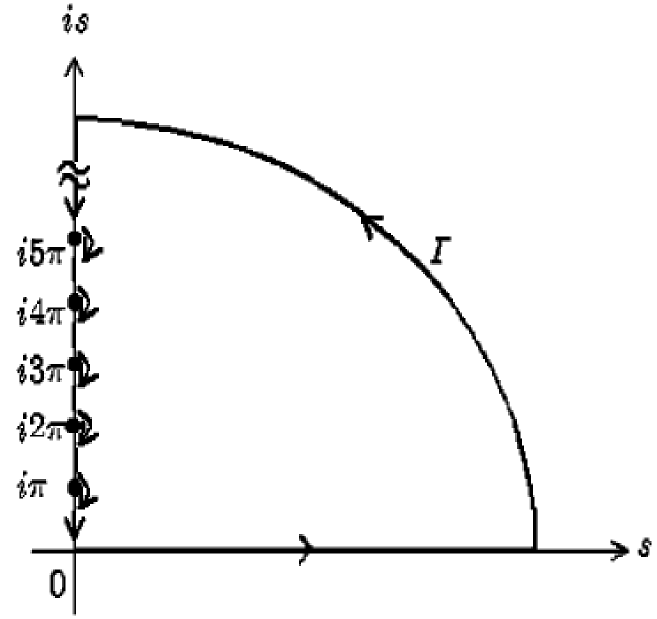
$$\ln \Gamma(a \pm ib) = \int_0^\infty \frac{dz}{z} \left[\frac{e^{-(a \pm ib)z}}{1 - e^{-z}} - \frac{e^{-z}}{1 - e^{-z}} + (a \pm ib - 1) e^{-z} \right]$$

Γ-Function Regularization

- Γ -regularization [SPK, Lee, Yoon, PRD 78 ('08), PRD 82 ('10); SPK, IJMP-CS 12 ('12)]

$$\int_0^\infty \frac{dz}{z}\frac{e^{-(a\pm ib)z}}{1-e^{-z}} = P\int_0^\infty \frac{ds}{s}\frac{e^{-(a\pm ib)(\mp is)}}{1-e^{\pm is}} \mp \pi i \sum_{n=1}^{\infty}\frac{e^{-(a\pm ib)(\mp 2n\pi i)}}{\mp 2n\pi i}$$

- Cauchy residue theorem



QED Action in Constant E

- Bogoliubov coefficient for scalar and spinor in constant E-field [SPK, Lee, Yoon, PRD 78 (‘08)]

$$\alpha_{\mathrm{k}} = \frac{\sqrt{2\pi}}{\Gamma(-p)} e^{-i(p+1)\pi/2}, \qquad p = -\frac{1}{2} + \sigma - i\frac{m^2 + \mathrm{k}^2}{2(qE)}$$

- Effective action for scalar/volume and time

$$L_{\mathrm{sc}}(E) = -i\frac{qE}{4\pi}\int \frac{d^2\mathrm{k}_\perp}{(2\pi)^2}\int_0^\infty ds \frac{e^{(p^*+1/2)s}}{s}\left[\frac{1}{\sinh(s/2)} - \underbrace{\left(\frac{2}{s} - \frac{s}{12}\right)}_{\text{Schwinger subtraction}}\right]$$

- Contour integral in 1st quadrant and residue theorem

QED Vacuum Polarization

- Scalar QED: renormalized effective action per volume and per time for a constant E-field

$$L_{\rm eff}^{\rm sc}(E) = -\frac{(qE)^2}{16\pi^2} P\int_0^\infty ds \frac{e^{-m^2 s/qE}}{s^2}\left[\frac{1}{\sin s} - \frac{1}{s} - \frac{s}{6}\right]$$

- Spinor QED: renormalized effective action per volume and per time for a constant E-field

$$L_{\rm eff}^{\rm sp}(E) = \frac{(qE)^2}{8\pi^2} P\int_0^\infty ds \frac{e^{-m^2 s/qE}}{s^2}\left[\cot(s) - \frac{1}{s} + \frac{s}{3}\right]$$

QED Vacuum Persistence & Schwinger Pair Production

- Spinor QED: Schwinger pair production in a constant E-field

$$2\,\mathrm{Im}(L_{\mathrm{eff}}^{\mathrm{sp}}) = \frac{(qE)^2}{4\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left[-\frac{\pi m^2 n}{qE}\right] = -\frac{2qE}{2\pi}\int\frac{dk_\perp^2}{(2\pi)^2}\ln(1-N_k)$$

- Scalar QED: Schwinger pair production

$$2\,\mathrm{Im}(L_{\mathrm{eff}}^{\mathrm{sc}}) = \frac{(qE)^2}{8\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\exp\left[-\frac{\pi m^2 n}{qE}\right] = \frac{qE}{2\pi}\int\frac{dk_\perp^2}{(2\pi)^2}\ln(1+N_{\mathrm{k}}),$$

$$N_{\mathrm{k}} = e^{-\frac{\pi(m^2+\mathrm{k}_\perp^2)}{qE}}$$

QED Action in $E(t) = E_0 \mathrm{sech}^2(t/\tau)$

[SPK, Lee, Yoon, PRD 78 ('08)]

- Bogoliubov coefficient and effective action

$$\alpha^*_{\omega k \sigma} = \frac{\Gamma\big((1-2\sigma-2i\tau\omega_{(-)})/2\big)\Gamma\big((1-2\sigma-2i\tau\omega_{(+)})/2\big)}{\Gamma\big((1-2\sigma-i\Omega_{(-)})/2\big)\Gamma\big((1-2\sigma-i\Omega_{(+)})/2\big)}$$

$$L = (-1)^{2|\sigma|}\frac{1+2|\sigma|}{2}\int \frac{d^3k}{(2\pi)^3} P\int_0^\infty \frac{ds}{s}\Big[e^{-\Omega_{(+)}s/2} + e^{-\Omega_{(-)}s/2} - e^{-\tau\omega_{(+)}s} - e^{-\tau\omega_{(-)}s}\Big] G_\sigma(s)$$

$$\omega_{(\pm)} = \sqrt{(k_z \mp qE_0\tau)^2 + (m^2 + k_x^2 + k_y^2)}$$

$$\Omega_{(\pm)} = \tau[\omega_{(+)} + \omega_{(-)}] \pm 2(qE_0\tau^2)\sqrt{1-(1-2|\sigma|)/(2qE_0\tau^2)^2}$$

- Imaginary part from the mean number of pairs

$$\mathrm{Im}(L_{\mathrm{eff}}) = (-1)^{2|\sigma|}\frac{1+2|\sigma|}{2}\int \frac{dk^3}{(2\pi)^3} \ln\left(\frac{(1+(-1)^{2|\sigma|}e^{-\pi\Omega_{(+)}})(1+(-1)^{2|\sigma|}e^{-\pi\Omega_{(-)}})}{(1-e^{-2\pi\tau\omega_{(+)}})(1-e^{-2\pi\tau\omega_{(-)}})}\right)$$

QED Action in $E(z) = E_0 \mathrm{sech}^2(z/L)$

[SPK, Lee, Yoon, PRD 82 ('10)]

- Bogoliubov coefficient and effective action

$$\alpha^*_{\omega k \sigma} = \frac{\Gamma\big((1-2\sigma - i\Delta_{(-)})/2\big)\Gamma\big((1-2\sigma - i\Delta_{(+)})/2\big)}{\Gamma\big((1-2\sigma - i\Omega_{(-)})/2\big)\Gamma\big((1-2\sigma - i\Omega_{(+)})/2\big)}$$

$$L = (-1)^{2|\sigma|} \frac{1+2|\sigma|}{2} \int \frac{d\omega d^2 k_\perp}{(2\pi)^3} P\int_0^\infty \frac{ds}{s}\left[e^{-\Omega_{(+)}s/2} - e^{\Omega_{(-)}s/2} - e^{-\Delta_{(+)}s/2} + e^{\Delta_{(-)}s/2}\right] G_\sigma(s)$$

$$P_{3(\pm)} = \sqrt{(\omega \mp qE_0 L)^2 - (m^2 + k_x^2 + k_y^2)}$$

$$\Omega_{(\pm)} = L[P_{3(+)} + P_{3(-)}] \pm 2(qE_0L^2)\sqrt{1-(1-2|\sigma|)/(2qE_0L^2)^2}$$

$$\Delta_{(\pm)} = L[P_{3(+)} - P_{3(-)}] \pm 2(qE_0L^2)\sqrt{1-(1-2|\sigma|)/(2qE_0L^2)^2}$$

- The imaginary part and the mean number of pairs

$$\mathrm{Im}(L) = (-1)^{2|\sigma|} \frac{1+2|\sigma|}{2} \int \frac{d\omega d^2 k_\perp}{(2\pi)^3} \ln\left(\frac{(1+(-1)^{2|\sigma|}e^{-\pi\Omega_{(+)}})(1+(-1)^{2|\sigma|}e^{\pi\Omega_{(-)}})}{(1+(-1)^{2|\sigma|}e^{-\pi\Delta_{(+)}})(1+(-1)^{2|\sigma|}e^{\pi\Delta_{(-)}})}\right)$$

QED Action in $B(z) = B_0 \mathrm{sech}^2(z/L)$

[SPK, PRD 84 ('11)]

- Jost functions and inverse scattering matrix for magnetic fields
- Inverse scattering matrix and effective action

$$M = \frac{\Gamma\big((1-2\sigma+\Delta_{(+)})/2\big)\Gamma\big((1-2\sigma-\Omega_{(-)})/2\big)}{\Gamma\big((1-2\sigma-\Delta_{(-)})/2\big)\Gamma\big((1-2\sigma+\Omega_{(+)})/2\big)}$$

$$L = (-1)^{2|\sigma|}\frac{1+2|\sigma|}{2}\int\frac{d\tilde{\omega}d^2k_\perp}{(2\pi)^3}P\int_0^\infty\frac{ds}{s}\left[e^{-\Omega_{(+)}s/2} - e^{\Omega_{(-)}s/2} - e^{-\Delta_{(+)}s/2} + e^{\Delta_{(-)}s/2}\right]F_\sigma(s)$$

$$\Pi_{(\pm)} = \sqrt{(k_y \mp qB_0L)^2 - (\omega^2 - m^2 - k_z^2)}$$

$$\Omega_{(\pm)} = L[\Pi_{(+)} + \Pi_{(-)}] \pm 2(qB_0L^2)\sqrt{1+(1-2|\sigma|)/(2qB_0L^2)^2}$$

$$\Delta_{(\pm)} = L[\Pi_{(+)} - \Pi_{(-)}] \pm 2(qB_0L^2)\sqrt{1+(1-2|\sigma|)/(2qB_0L^2)^2}$$

Γ-Function for Supercritical Fields and Massless QED

Γ-Function for Supercritical Fields

- Ramanujan formula for the gamma function

$$\ln\Gamma(z+\frac{1}{2}) = \underbrace{(z+\frac{1}{2})\ln z - z + \frac{1}{2}\ln(2\pi)}_{\text{vacuum energy \& charge renormalization}} + \sum_{k=1}^{\infty}\frac{B_{2k}}{(2k)(2k-1)z^{2k-1}}$$

- Schwinger action in constant B in power series [SPK, Lee, arXiv:1406.4292]

$$L_{sp}^{(1)}(\beta = m^2/(2qB)) = \frac{m^4}{(2\pi)^2(2\beta)}\sum_{k=2}^{\infty}\frac{B_{2k}}{(2k)(2k-1)(2k-2)}\frac{1}{\beta^{2k-1}}$$

Γ-Function for Supercritical Fields

- Whittaker-Watson formula for gamma function

$$\ln\Gamma(z) = (z-\frac{1}{2})\ln z - z + \frac{1}{2}\ln(2\pi) + \frac{1}{2}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{k}{(k+1)(k+2)(z+n)^{k+1}}$$

- Schwinger action in constant B or E in Hurwitz zeta function [SPK, Lee, arXiv:1406.4292]

$$L_{sp}^{(1)}(\beta) = \left(\frac{m^2}{4\pi\beta}\right)^2\left[2\varsigma'(-1,\beta) - \left(\beta^2 - \beta + \frac{1}{6}\right)\ln(\beta) + \frac{\beta^2}{2} - \frac{1}{6}\right]$$

$$L_{sp}^{(1)}(\varepsilon) = -\left(\frac{m^2}{4\pi\varepsilon}\right)^2\left[2\varsigma'(-1,i\varepsilon) + \left(\varepsilon^2 + i\varepsilon - \frac{1}{6}\right)\ln(i\varepsilon) - \frac{\varepsilon^2}{2} - \frac{1}{6}\right]$$

QED in (A)dS$_2$

Schwinger formula in (A)dS$_2$

- (A)dS metric and the gauge potential for E

$$ds^2 = -dt^2 + e^{2Ht} dx^2, \quad A_1 = -(E/H)(e^{Ht} - 1)$$

$$ds^2 = -e^{2Kx} dt^2 + dx^2, \quad A_0 = -(E/K)(e^{Kx} - 1)$$

- Schwinger formula for scalars in (A)dS$_2$ [SPK, Page, PRD ('08)]

$$N = e^{-S}, \quad \begin{cases} S_{dS} = 2\pi\left[\sqrt{\left(qE/H^2\right)^2 + \left(m/H\right)^2 - 1/4} - qE/H^2\right] \\ S_{AdS} = 2\pi\left[qE/K^2 - \sqrt{\left(qE/K^2\right)^2 - \left(m/K\right)^2 - 1/4}\right] \end{cases}$$

Effective Temperature for Schwinger formula

- Effective temperature for accelerating observer in (A)dS [Narnhofer, Peter, Thirring, IJMP B10 ('96); Deser, Levin, CQG 14 ('97)]

$$N = e^{-m/T_{\text{eff}}}\ ,\ T_{\text{eff}} = \sqrt{T_{\text{U}}^2 + \frac{R}{8\pi^2}},\ R = 2H^2(-2K^2)$$

- Effective temperature for Schwinger formula in (A)dS [Cai, SPK, JHEP ('14)]

$$N = e^{-\overline{m}/T_{\text{eff}}}\ ,\ \overline{m} = \sqrt{m^2 - \frac{R}{8}},\ T_{\text{U}} = \frac{qE}{m},\ T_{\text{GH}} = \frac{H}{2\pi}$$

$$T_{\text{dS}} = \sqrt{T_{\text{U}}^2 + T_{\text{GH}}^2} + T_{\text{U}}\ ;\ T_{\text{AdS}} = \sqrt{T_{\text{U}}^2 + \frac{R}{8\pi^2}} + T_{\text{U}}$$

Scalar QED Action in dS_2

- Pair production and vacuum polarization [Cai, SPK, JHEP ('14)]

$$N_{\rm dS} = \frac{e^{-(S_\mu - S_\lambda)} + e^{-2S_\mu}}{1 - e^{-2S_\mu}}, \quad 2\,{\rm Im}W_{\rm dS}^{(1)} = \ln\left(1 + N_{\rm dS}\right)$$

$$L_{\rm dS}^{(1)} = \frac{H^2 S_\mu}{4(2\pi)^2} P\int_0^\infty \frac{ds}{s}\left[e^{-(S_\mu - S_\lambda)s/2\pi}\left(\frac{1}{\sin(s/2)} - \frac{2}{s} - \frac{s}{12}\right) - e^{-S_\mu s/\pi}\left(\frac{\cos(s/2)}{\sin(s/2)} - \frac{2}{s} + \frac{s}{6}\right)\right]$$

$$S_\mu = 2\pi\sqrt{\left(\frac{qE}{H^2}\right)^2 + \left(\frac{m}{H}\right)^2 - \frac{1}{4}}, \quad S_\lambda = 2\pi\frac{qE}{H^2}$$

Scalar QED Action in AdS$_2$

- Pair production and vacuum polarization

$$N_{\mathrm{AdS}} = \frac{e^{-(S_\kappa - S_\nu)} - e^{-(S_\kappa + S_\nu)}}{1 + e^{-(S_\kappa + S_\nu)}}, \quad 2\,\mathrm{Im}\,W^{(1)}_{\mathrm{AdS}} = \ln\left(1 + N_{\mathrm{AdS}}\right)$$

$$L^{(1)}_{\mathrm{AdS}} = -\frac{K^2 S_\nu}{4(2\pi)^2} P\int_0^\infty \frac{ds}{s} e^{-S_\kappa s/2\pi} \cosh(S_\nu s/2\pi)\left[\frac{1}{\sin(s/2)} - \frac{2}{s} - \frac{s}{12}\right]$$

$$S_\nu = 2\pi\sqrt{\left(\frac{qE}{K^2}\right)^2 - \left(\frac{m}{H}\right)^2 - \frac{1}{4}}, \quad S_\kappa = 2\pi\frac{qE}{K^2}$$

Spinor QED Action in dS_2

- Pair production and vacuum polarization [SPK ('15)]

$$N_{\rm ds}^{\rm sp} = \frac{e^{-(S_\mu - S_\lambda)} - e^{-2S_\mu}}{1 - e^{-2S_\mu}}, \quad 2\,{\rm Im}\,W_{\rm dS}^{(1)} = -\ln\left(1 - N_{\rm ds}^{\rm sp}\right)$$

$$L_{\rm dS}^{\rm sp} = -\frac{H^2 S_\mu}{2(2\pi)^2} P\int_0^\infty \frac{ds}{s}\left(e^{-(S_\mu - S_\lambda)s/2\pi} - e^{-S_\mu s/\pi}\right)\left(\cot(\frac{s}{2}) - \frac{2}{s} + \frac{s}{6}\right)$$

$$S_\mu = 2\pi\sqrt{\left(\frac{qE}{H^2}\right)^2 + \left(\frac{m}{H}\right)^2}, \quad S_\lambda = 2\pi\frac{qE}{H^2}$$

Spinor QED Action in AdS_2

- Pair production and vacuum polarization [SPK ('15)]

$$N_{\rm AdS}^{\rm sp} = \frac{e^{-(S_\kappa - S_\nu)} - e^{-(S_\kappa + S_\nu)}}{1 - e^{-(S_\kappa + S_\nu)}}, \quad 2\,{\rm Im}W_{\rm AdS}^{\rm sp} = -\ln\left(1 - N_{\rm AdS}^{\rm sp}\right)$$

$$L_{\rm AdS}^{\rm sp} = -\frac{K^2 S_\nu}{2(2\pi)^2} P\int_0^\infty \frac{ds}{s}\left(e^{-(S_\kappa - S_\nu)s/2\pi} - e^{-(S_\kappa + S_\nu)s/2\pi}\right)\left(\cot(\frac{s}{2}) - \frac{2}{s} + \frac{s}{6}\right)$$

$$S_\nu = 2\pi\sqrt{\left(\frac{qE}{K^2}\right)^2 - \left(\frac{m}{K}\right)^2}, \quad S_\kappa = 2\pi\frac{qE}{K^2}$$

Gauge-Gravity & dS-AdS Relation

- Gauge-gravity relation for QED action and Schwinger effect: Strong E-field to weak R or vice versa

$$F_{\text{Maxwell scalar}} = E^2/4 \Leftrightarrow R_{\text{scalar}} = 2H^2/-2K^2$$

- Duality between dS and AdS for pair production

$$N^{\text{sc}}_{\text{dS}}(R)N^{\text{sc}}_{\text{AdS}}(R) = 1 = N^{\text{sp}}_{\text{dS}}(R)N^{\text{sp}}_{\text{AdS}}(R)$$

- Massive scalar in a self-dual EM field in 4d-D = massive spinor in 2d-D AdS at one-loop [Basar, Dunne, JPA 43 ('10)]

$$\text{D} = \text{4d gauge (massive scalar)} \Leftrightarrow \text{D} = \text{2d gravity (massive spinor)}$$

$$\text{maximally symmetric gauge field} \Leftrightarrow \text{maximally symmetric gravitational curvature}$$

$$\text{m}^2/2f \Leftrightarrow \sqrt{m^2/R}$$

Conclusion

- Introduce the Schwinger-DeWitt in-out formalism for effective actions for QED and gravity.
- Gamma-function regularization for strong fields
  - Heisenberg-Euler and Schwinger proper-time integral
  - Hurwitz zeta function for supercritical field (massless QED)
  - Consistent with the vacuum persistence (Schwinger effect or Hawking radiation)
  - QED in (Anti) de Sitter space
- Still ample room for other background fields such as Yang-Mills, curved spacetimes and black holes.

Schwinger Effect from Charged Black Hole

Thermal Interpretation of Schwinger Effect from BH

- Thermal interpretation of Schwinger formula for charged scalars and fermions from near-extremal charged black hole with horizon geometry $AdS_2 \times S^2$ [SPK, Lee, Yoon ('15)]

$$N_{NBH} = \left(\frac{e^{-\frac{\bar{m}}{T_{RN}}} - e^{-\frac{\bar{m}}{\bar{T}_{RN}}}}{1 \pm e^{-\frac{\bar{m}}{\bar{T}_{RN}}}} \right) \times \left(\frac{1 \mp e^{-\frac{\omega - qA_0}{T_H}}}{1 + e^{-(\frac{\omega - qA_0}{T_H} + \frac{\bar{m}}{T_{RN}})}} \right),$$

$$T_{RN} = T_U + \sqrt{T_U^2 - (\frac{1}{2\pi Q})^2}, \quad \bar{T}_{RN} = T_U - \sqrt{T_U^2 - (\frac{1}{2\pi Q})^2}$$

$$T_U = \frac{qE_H / \bar{m}}{2\pi} = \frac{q}{2\pi \bar{m} Q}, \quad \bar{m} = m\sqrt{1 + (\frac{l + 1/2}{mQ})^2}$$

Schwinger Effect and Hawking Radiation

- Thermal interpretation of Schwinger formula for charged scalars and fermions in spherical harmonics [SPK, Lee, Yoon ('15); SPK ('15)]

$$N_{NBH} = e^{\frac{\bar{m}}{T_{RN}}} \times \underbrace{\left(\frac{e^{-\frac{\bar{m}}{T_{RN}}} - e^{-\frac{\bar{m}}{\bar{T}_{RN}}}}{1 \pm e^{-\frac{\bar{m}}{\bar{T}_{RN}}}} \right)} \times \underbrace{\left(\frac{e^{-\frac{\bar{m}}{T_{RN}}} \left(1 \mp e^{-\frac{\omega - qA_0}{T_H}}\right)}{1 + e^{-\frac{\omega - qA_0}{T_H}} e^{-\frac{\bar{m}}{T_{RN}}}} \right)}$$

Schwinger Effect in AdS_2
Cai & SPK JHEP ('14)

Schwinger Effect in Rindler Space
Gabriel & Spindel AP ('00)
Hawking Radiation of charges