Higgs ξ-Inflation at NNLO and CMB Physics

Giuseppe Iacobellis
`iacobellis@fe.infn.it`

University of Ferrara and INFN

PRIN Midterm Review Meeting
Theoretical Astroparticle Physics Workshop
Torino, 10th July 2015

Research lines

Scholarship funded by Agenzia Spaziale Italiana (ASI)

Focusing on CMB physics (early universe)

- *CMB Polarization:*
  B-modes detection and theoretical computation

1. Upper troposphere ice noise in ground-based experiments

In collaboration with
N. Mandolesi (University of Ferrara and INAF - Bologna),
S. Buehler and *M. Brath* (University of Hamburg)

2. Ellipsoidal Universe (*Bianchi I* Universe)

In collaboration with
L. Tedesco (University of Bari and INFN - Bari)

Research lines (1)

- *Inflationary parameters*

3. Higgs ξ-Inflation: theoretical improvements and up-to-date experimental windows

In collaboration with
I. Masina (University of Ferrara and INFN - Ferrara)

Outline

Outline

1. Higgs inflation
    - Inflation
    - Pure SM
    - ξ-inflation
2. What next?

Why inflation?

Early universe

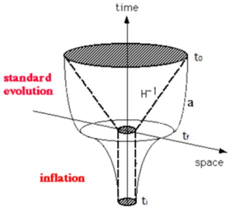
Horizon problem

(similar to the flatness problem)

$$\frac{\text{horizon radius}}{\text{space radius}} \sim \frac{H^{-1}}{a} \sim \frac{1}{\dot{a}} \sim t^{1-\alpha} \xrightarrow[t\to 0]{} 0, \qquad \alpha < 1$$

Causal connection problem

The ratio should *decrease in time*, from more natural conditions ($r \sim 1$) to values suitable for the standard initial conditions.



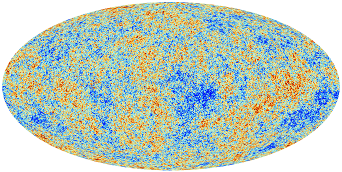
Why inflation? (1)

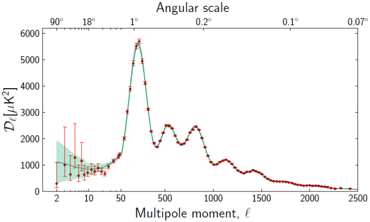
CMB anisotropies

Density anisotropies

Elegant explanation for the origin of the first density perturbations, seeds of the CMB anisotropies in the Large-Scale-Structures of the universe we observe today (e.g. cluster of galaxies)

$$\frac{\Delta T}{< T >} \sim 10^{-5}.$$





Planck collaboration, arXiv: 1303.5062

Inflaton

Action

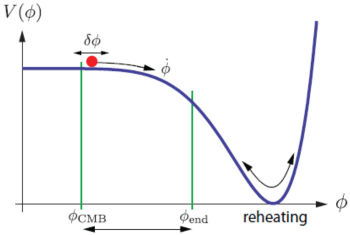
$$\mathcal{S} = -\frac{1}{2\lambda_P^2}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)$$

with $\lambda_P^2 = 8\pi G = \frac{1}{M_P^2}$ and ϕ called *inflaton*.

Equations of motion

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Slow-roll inflation



D. Baumann, arXiv: 0907.5424

$$\begin{cases} \epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1, & \dot{\epsilon} \simeq 0 \\ \epsilon t \simeq H^{-1} = \frac{a}{\dot{a}} \to a(t) \sim t^{1/\epsilon} \end{cases}$$

The potential $V(\phi)$
must be (nearly) *flat*.

We have $\dot{\phi} \neq 0$ (deviation from eternal *de Sitter* case) and

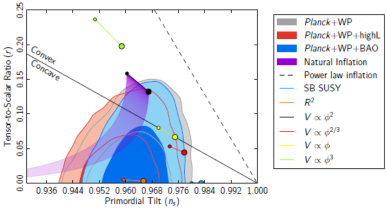
Dynamic constraints

$$|\dot{H}| \ll H^2, \qquad |\ddot{\phi}| \ll |H\dot{\phi}|, \qquad \dot{\phi}^2 \ll |V|$$

General inflationary parameters: slow-roll

1. scalar spectral index: $n_s = 1 - 6\epsilon + 2\eta$;
2. tensor-to-scalar ratio: $r = \mathcal{P}_t / \mathcal{P}_s \simeq 16\epsilon$;
3. number of e-folds:

$$N \equiv \ln\left(\frac{a_f}{a_i}\right) = \lambda_P^2 \int_{\phi_{end}}^{\phi_{CMB}} \frac{V}{V_\phi} d\phi \sim 50 \div 60.$$



Planck collaboration, arXiv: 1303.5062

Inflationary scale

$$V_{infl} = 1.94 \times 10^{16}\, GeV \left(\frac{r}{0.12}\right)^{1/4}.$$

Amplitude of scalar perturbations (slow-roll approx)

$$A_s \simeq \frac{V}{24\pi^2 \epsilon M_P^4}.$$

$\phi \rightarrow h$: Higgs inflation?

Who's the scalar field which drives inflation?

Minimal choice: the only scalar in SM, the Higgs field!

Main issue

The Higgs potential is not flat

$$V_0 = \lambda_h \left(\mathcal{H}^\dagger \mathcal{H} - \frac{v^2}{2}\right)^2$$

Electroweak (EW) scale: $v \simeq 246\,GeV$.

Higgs mass: $m_h \equiv \sqrt{2v^2\lambda_h} \simeq 125.1\,GeV$.

Extrapolation of the high-energy behaviour is needed!

$\phi \to h$: Higgs inflation?

Who's the scalar field which drives inflation?

Minimal choice: the only scalar in SM, the Higgs field!

Main issue

The Higgs potential is not flat

$$V_0 = \lambda_h \left(\mathcal{H}^\dagger \mathcal{H} - \frac{v^2}{2} \right)^2$$

Electroweak (EW) scale: $v \simeq 246\, GeV$.
Higgs mass: $m_h \equiv \sqrt{2 v^2 \lambda_h} \simeq 125.1\, GeV$.
Extrapolation of the high-energy behaviour is needed!

$\phi \to h$: Higgs inflation?

Who's the scalar field which drives inflation?

Minimal choice: the only scalar in SM, the Higgs field!

Main issue

The Higgs potential is not flat

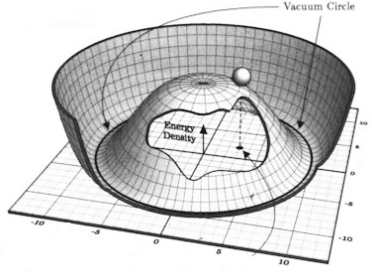
$$V_0 = \lambda_h \left(\mathcal{H}^\dagger \mathcal{H} - \frac{v^2}{2} \right)^2$$

Electroweak (EW) scale: $v \simeq 246\, GeV$.
Higgs mass: $m_h \equiv \sqrt{2v^2\lambda_h} \simeq 125.1\, GeV$.
Extrapolation of the high-energy behaviour is needed!

$\phi \to h$: Higgs inflation? (1)

Unitary gauge

$$\mathcal{H}^{\mathcal{T}} = \left(0, \quad (h+v)/\sqrt{2}\right).$$



A. Guth, Phys. Rev. D23, 347 (1981)

λ_h quartic coupling constant

For large field values

$$V_0 \sim \lambda_h h^4.$$

…up to high energies

- Matching procedure for both top quark *Yukawa coupling* and *Higgs quartic coupling*, with pole masses;
- Running of the SM couplings through the *Renormalization Group Equation* (RGE):

$$\frac{d}{d\ln\left(\frac{\mu}{m_Z}\right)}\lambda_i = \beta_{\lambda_i}(\lambda_i),$$

$$\lambda_i = \left(\lambda_h(t), g(t), g'(t), g_s(t), y_t\right);$$

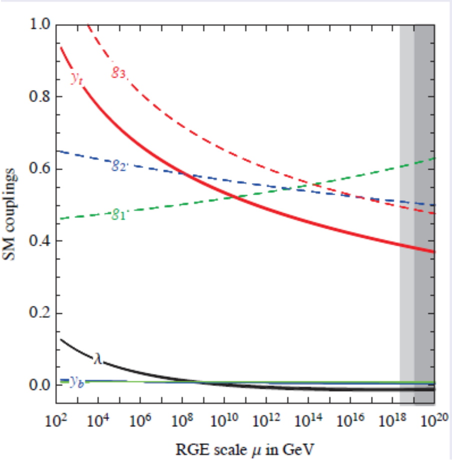
- The effective potential improved by RGE is highly scale independent: this allows fixing the renormalization scale

$$\mu(t) \sim \alpha h(t), \qquad \alpha \simeq \mathcal{O}(1)$$

to avoid dangerous behaviour at large values of the field.

Running coupling constants

SM RGEs for $m_h = 125.5\,GeV$



G. Degrassi et al., arXiv: 1205.6497

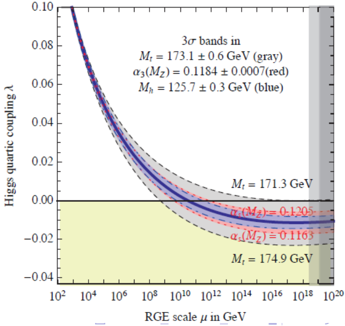
Higgs quartic coupling: running of effective λ_h at NNLO

$$\frac{d\lambda_h}{d\ln\left(\frac{\mu}{m_Z}\right)} \simeq \frac{1}{16\pi^2}[12\lambda_h^2 + 6\lambda_h y_t^2 - 3y_t^4 + \\ -\frac{3}{2}\lambda_h(3g'^2 + g^2) + \frac{3}{16}(2g'^4 + (g'^2 + g^2)^2)] + \lambda_h{}^{(2)}(\mu/m_Z).$$

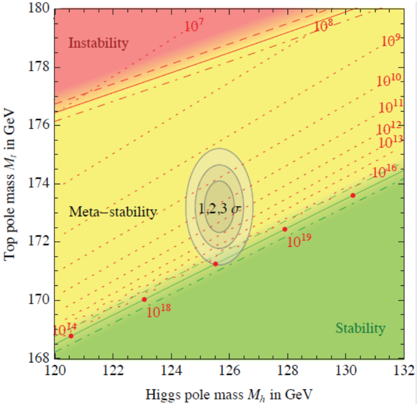
The running of λ_h is heavily dependent on the top *Yukawa* coupling.

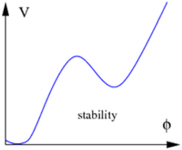
Stability or metastability?

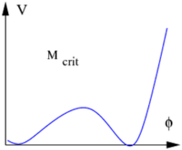
G. Degrassi et al., arXiv: 1307.3536

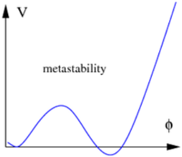


Stability diagram in the (m_h, m_t) plane at NNLO[1]









SM two-loop effective potential[3]

Coleman-Weinberg correction[2]

$$V_{\text{eff}}(h) = V^{(0)}(h) + V^{(1)}(h) + V^{(2)}(h) \equiv \frac{1}{4}\lambda_{\text{eff}}(\mu)h^4,$$

$$V^{(1)}(h) = \sum_{i=W^\pm,Z,t} \frac{n_i}{4(4\pi)^2} m_i(h)^4 \left[\ln\frac{m_i(h)^2}{\mu^2(t)} - C_i\right],$$

$$C_{W^\pm} = C_Z = \frac{5}{6}, \quad C_t = \frac{3}{2}, \quad n_{W^\pm} = 6, \quad n_Z = 3, \quad n_t = -12$$

$$m_i(t)^2 = k_i h(t)^2, \qquad \mu(t) = m_Z e^t$$

$$h(t) = \xi(t) h_{cl}, \qquad \xi(t) \equiv e^{-\int_0^t \gamma(\tau)d\tau}$$

$$k_{W^\pm} = \frac{1}{4}g(t)^2, \quad k_Z = \frac{1}{4}\left[g(t)^2 + g'(t)^2\right], \quad k_t = \frac{1}{2}h(t)^2.$$

---

[2] S. Coleman, E. Weinberg, Phys. Rev. D7, 1888 (1973).

[3] 't *Hooft-Landau* gauge and $\overline{MS}$ renormalization scheme.

Abuses of the CW radiative correction

- $V_{\rm eff}$ is gauge dependent[4]: is it meaningful to extract physical quantities?

  For instance:

$$\left.\frac{\partial^2 V_{\rm eff}}{\partial h}\right|_{h_{\rm min}} \approx m_h^2.$$

- Dangerous “hunting” imaginary part:

$$V^{(1)} \sim \ln \frac{m_i(h)^2}{\mu^2(t)}, \qquad \text{but} \ldots$$

$$\text{some } m_i(h)^2 < 0!!$$

---

[4] Although it is gauge independent at critical points (see Nielsen's identities).

(Improved) Computation at NNLO[5] (G. I., I. Masina)

- Two-loop matching procedure for *top Yukawa coupling*, *Higgs quartic coupling* at some suitable scale;
- Three-loop RGEs for *SM couplings, Higgs anomalous dimension, non-minimal coupling, all with the insertion of the suppression factor (see later)*;
- Two-loop effective potential, with some "emergence" measures: $\hbar$ - expansion method.

Too technical . . . practical result: we get rid of the problematic Higgs and Goldstone terms in the effective potential.

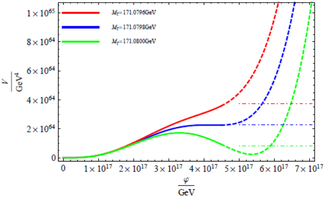
---

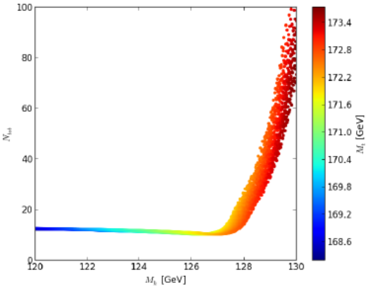
[5] If interested, ask for details. . .

Pure SM inflation

Conditions:

1. Sufficient e-folds N;
2. Correct A_s;
3. Power spectrum nearly scale invariant.





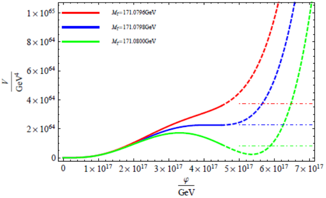
- For $m_h \simeq 126\,GeV \Rightarrow$ too low N_{tot}
- If correct N_{tot} is assumed, we gain a wrong A_s: no slow-roll?

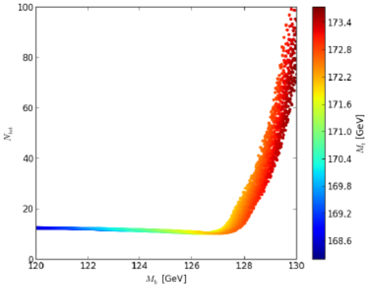
Maybe the Higgs is not responsible of both inflation and scalar perturbations.

Pure SM inflation

Conditions:

1. Sufficient e-folds N;
2. Correct A_s;
3. Power spectrum nearly scale invariant.





- For $m_h \simeq 126\,GeV \Rightarrow$ too low N_{tot}
- If correct N_{tot} is assumed, we gain a wrong A_s: no slow-roll?

Maybe the Higgs is not responsible of both inflation and scalar perturbations.

ξ-inflation[6]

Action in the Jordan frame

Scalar fields can (should?) be non-minimally coupled to gravity, when it is considered

$$\mathcal{S}_J = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \xi\mathcal{H}^\dagger\mathcal{H}R + \mathcal{L}_{\mathcal{SM}}\right].$$

Conformal transformation and Action in the Einstein frame

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \qquad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_P}{\Omega^4}},$$

$$\mathcal{S}_E = \int d^4x\sqrt{-\hat{g}}\left[\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial_\nu\chi}{2} - U(\chi)\right].$$

---

[6] F. Bezrukov and M. Shaposhnikov, arXiv: 0710.3755

ξ-inflation[6]

Action in the Jordan frame

Scalar fields can (should?) be non-minimally coupled to gravity, when it is considered

$$\mathcal{S}_J = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \xi\mathcal{H}^\dagger\mathcal{H}R + \mathcal{L}_{\mathcal{SM}}\right].$$

Conformal transformation and Action in the Einstein frame

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \qquad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_P}{\Omega^4}},$$

$$\mathcal{S}_E = \int d^4x\sqrt{-\hat{g}}\left[\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial_\nu\chi}{2} - U(\chi)\right].$$

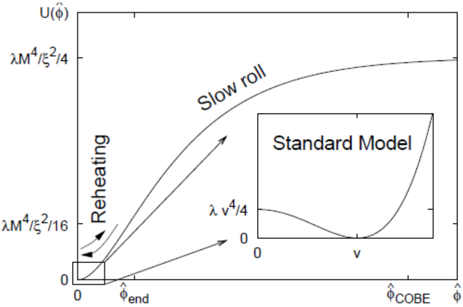
---

[6] F. Bezrukov and M. Shaposhnikov, arXiv: 0710.3755

ξ-inflation (1)

Where the potential is

$$U(\chi) = \frac{\lambda_h M_P^4}{4\xi^2}\left(1+\exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)\right)^{-2}$$



Einstein frame potential.

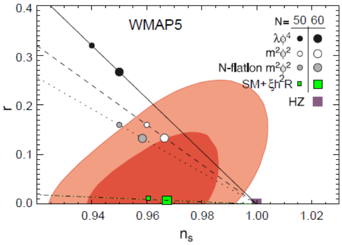
F. Bezrukov et al., arXiv: 0710.3755

Flattening the potential

When $\chi \gg M_P$ ($h \gg M_P/\sqrt{\xi}$), the potential is flat and slow-roll inflation can occurr (nearly same predictions of a R^2-model) and no new degrees of freedom were introduced.

At tree-level, imposing slow-roll and PLANCK normalization $U/\epsilon = (0.0269 M_P)^4$:

$\xi \sim \mathcal{O}(10^4)$



WMAP collaboration, arXiv: 0803.0547

Predictions

$n_s \simeq 0.967, \quad r \simeq 0.0031$

Unitarity violation?

Flattening the potential

When $\chi \gg M_P$ ($h \gg M_P/\sqrt{\xi}$), the potential is flat and slow-roll inflation can occurr (nearly same predictions of a R^2-model) and no new degrees of freedom were introduced.

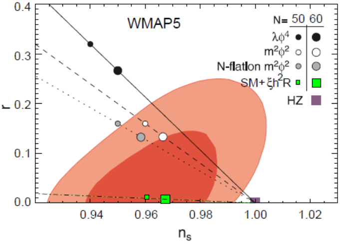
At tree-level, imposing slow-roll and PLANCK normalization

$$U/\epsilon = (0.0269 M_P)^4:$$

$$\xi \sim \mathcal{O}(10^4)$$

Predictions

$$n_s \simeq 0.967, \quad r \simeq 0.0031$$



WMAP collaboration, arXiv: 0803.0547

Unitarity violation?

Prescriptions

Prescriptions: radiative corrections before (**II**) or after (**I**) the *Weyl* transformation?
Different results!

The two frames are mathematically equivalent but not physically equivalent when we consider quantum corrections (the equivalence still holds at tree-level).
Still not clear which choice is the best one!

Prescription I

$$\mu = \alpha h/\Omega, \quad \xi \sim \mathcal{O}(10^2)$$

Prescription II

$$\mu = \alpha h, \quad \xi \sim \mathcal{O}(10^3) \text{or} \sim \mathcal{O}(10)$$

Suppression factors

Non-standard commutation rule for the field ϕ:
when the gravity sector is canonical, the kinetic one is non-canonical:

$$[\phi(\vec{x}), \dot{\phi}(\vec{y})] = \imath\hbar s(\phi)\delta^{(3)}(\vec{x} - \vec{y}),$$

Suppression factor

$$s(\phi) = \frac{1 + \xi\phi^2/M_P^2}{1 + (1 + 6\xi)\xi\phi^2/M_P^2}.$$

For $\phi \ll M_P/\xi$, $s \simeq 1$ and SM RGE are perfectly adequate, while for large fields values, every physical Higgs propagator is suppressed by a factor $s \simeq 1/(1 + 6\xi)$.

Lowering ξ: critical regime

Exploring the region in which λ_{eff} runs to very small values (toghether with its β-function → inflection point-like), in this way we can:

- reduce the value of ξ (relieving the unitariy problem);
- increase r (compatibly with the current experimental bounds).

Is the slow-roll regime sill valid?

Outline

1. Higgs inflation
    - Inflation
    - Pure SM
    - ξ-inflation

2. What next?

What next?

1. Try to validate the ellipsoidal universe model (*CMB polarization, Non Linear ElectroDynamics and galactic magnetism, low quadrupole* $l = 2$);
2. Evaluate the noise due to ice crystals in upper troposphere for CMB polarization measurements (*future ground-based polarization experiments*);
3. Upcoming measurements on r should show the way in (we hope!) few years:

   Is Higgs inflation still alive or Nature is more complicated than this?

Thank you.

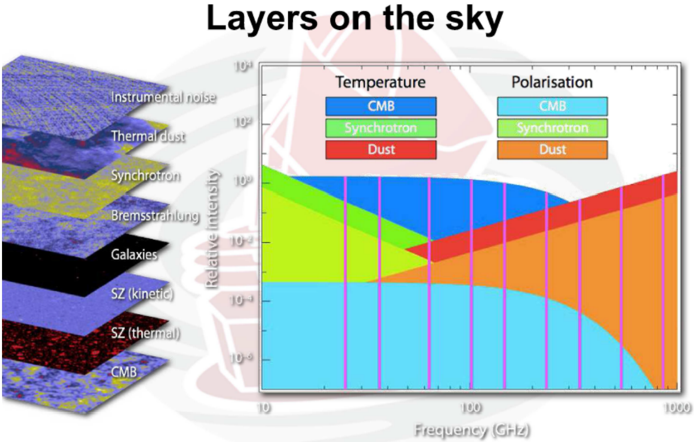
Outline

3. CMB Pol. and Ice

4. CMB and Ellipsoidal Universe

5. Backup

CMB polarization and Ice



CMB polarization and Ice[7] (1)

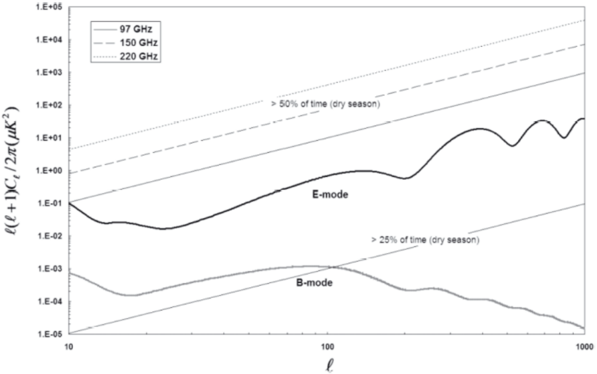


Figure 4. Angular power spectrum of CMB polarization (E and B modes) as calculated by CMBFAST code. Straight lines represent an upper limit on the amount of polarization induced by ice crystal clouds on the 2.7-K CMB for more than 50 per cent observing time during dry season in Atacama (IWP = 0.01 g m^{-2}) at the three C$_\ell$OVER frequencies and an upper limit for 25 per cent observing time during dry season (IWP = 0.0001 g m^{-2}) at 97 GHz. A flat power spectrum is assumed; although it appears that the ice crystal signal might be dominating ground-based observations of CMB polarization of E and B modes, it must be stressed that, while the CMB signal is fixed in the sky, the ice signal is most probably variable with time. Therefore, it is always possible to disentangle and therefore greatly reduce the ice signal from the sky signal by a properly designed observing strategy

---

[7] L. Pietranera, S. Buehler et al., Mon. Not. R. Astron. Soc. 376, 645-650 (2007).

CMB polarization and Ice (2)

(G. I., N. Mandolesi, S. Buehler, M. Brath)

1. Climatological overview (Ice Water Path) of sites of interest (Atacama desert, Antarctica, Teide Volcano,. . .);
2. ARTS[8] simulations: horizontally aligned ice particles effect in polarization from the cosmic background (upper limit for systematics);
3. Comparison with cleaned theoretical predictions from Standard Cosmology.

Fruitful analysis for upcoming experiments (CLASS, QUBIC, LSPE, Ground-Bird, BFORE)

- Different geographic sites;
- Better knowledge of ice particle shape and orientation;
- More frequency channels investigated;
- More realistic data on tensor-to-scalar ratio and polarization measurements.

---

[8] Atmospheric Radiative Transfer Simulator 2.0 (S. Buehler et al.).

CMB polarization and Ice (2)

(G. I., N. Mandolesi, S. Buehler, M. Brath)

1. Climatological overview (Ice Water Path) of sites of interest (Atacama desert, Antarctica, Teide Volcano,...);
2. ARTS[8] simulations: horizontally aligned ice particles effect in polarization from the cosmic background (upper limit for systematics);
3. Comparison with cleaned theoretical predictions from Standard Cosmology.

Fruitful analysis for upcoming experiments
(CLASS, QUBIC, LSPE, Ground-Bird, BFORE)

- Different geographic sites;
- Better knowledge of ice particle shape and orientation;
- More frequency channels investigated;
- More realistic data on tensor-to-scalar ratio and polarization measurements.

---

[8] Atmospheric Radiative Transfer Simulator 2.0 (S. Buehler et al.).

Outline

3 CMB Pol. and Ice

4 CMB and Ellipsoidal Universe

5 Backup

Power spectrum

In order to evaluate the directional dependence of temperature fluctuations, anisotropies are used to be expanded in spherical harmonics extended to the whole sky

$$\frac{\Delta T}{T}(\theta,\varphi)=\sum_{l,m} a_{lm}Y_{lm}(\theta,\varphi),$$

where $l \sim \pi/\theta$ is the *multipole order* and the a_{lm} are the *multipole momenta*, characterized by zero mean and non-zero variance.

The spectral coefficients are defined:

$$C_l=\frac{1}{2l+1}\sum_{m=-l}^{l}\left|a_{lm}\right|^2,$$

Cosmic variance: $\frac{\Delta C_l}{C_l}=\sqrt{\frac{2}{2l+1}}$.

Power spectrum (1)

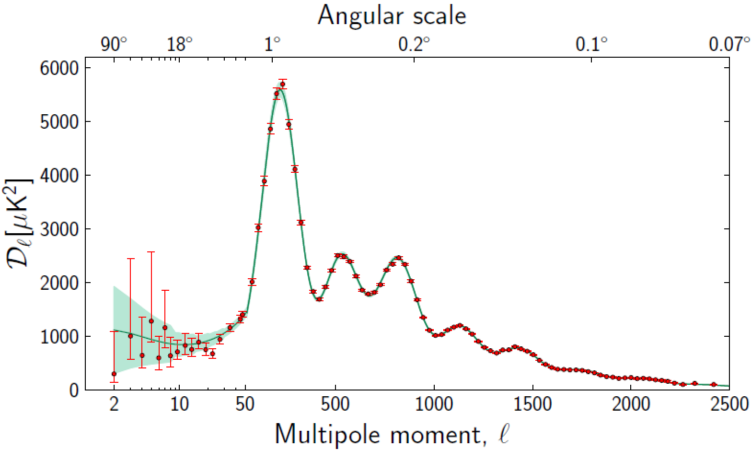
The distribution of the C_ls with respect to the multipole momenta gives the CMB termic power spectrum:

$$\frac{\Delta T}{< T >} = \sqrt{\frac{1}{2\pi} \frac{l(l+1)}{2l+1} \sum_m |a_{lm}|^2} = \sqrt{\frac{l(l+1}{2\pi}} C_l .$$

A cosmological model, through theory constraints, can predict form, position and height of spectrum peaks.

Now, the quadrupole $\mathcal{Q} \equiv \Delta T_2 / < T >$, $< T > \simeq 2.7\, K$, shows a *discrepancy* between predicted and observed values. *This could hide a non trivial topology in the primordial space-time geometry.*

Large scales lack of power



Planck collaboration, arXiv: 1303.5062

Ellipsoidal Universe

Bianchi I metric

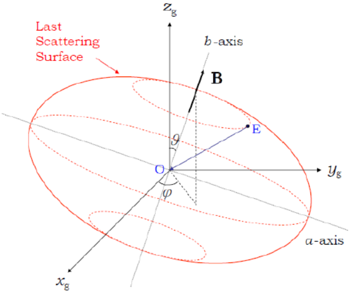
$$ds^2 = dt^2 - b^2(dx^2+dy^2) - c^2dz^2$$

Eccentricity

$$e(t) \equiv \sqrt{1-\left(\frac{b(t)}{a(t)}\right)^2}.$$

e - dependent *Einstein* eq:

$$T_\mu = diag(\rho, -p_\parallel, -p_\parallel, -p_\perp)$$



L. Campanelli, P. Cea and L. Tedesco,

Phys. Rev. D 76, 063007 (2007)

$$\frac{d}{dt}\left(\frac{e\dot{e}}{1-e^2}\right) + 3H_e\left(\frac{e\dot{e}}{1-e^2}\right) = \pm 8\pi G(p_\parallel^A - p_\perp^A).$$

Solution in the magnetic case: $e^2 = 8\pi\Omega_B(t_0)(1-3a^{-1}+2a^{-3/2})$

Boltzmann eq. for CMB photons (G. I., L. Tedesco)

The *same* story in polarization: we evaluate the evolution of the *photon distribution function* (black-body like) $f(\vec{x}, t)$ through the usual Boltzmann equation:

$$\frac{df}{dt} = C[f],$$

where $C[f]$ is the *Thomson* collision term.
The metric used is the *Bianchi I*, with an anisotropy term dependent on the eccentricity (whose dynamics is driven by the previous *Einstein* eq.):

$$h_{ij} = -e^2 \delta_{i3} \delta_{j3}.$$

Then a *Stokes parameters* computation is performed.

Outline

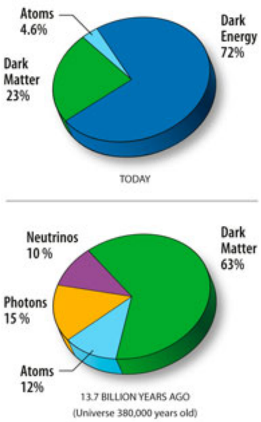
3 CMB Pol. and Ice

4 CMB and Ellipsoidal Universe

5 Backup

Problems of the Standard Cosmological Model

Current era: matter dominated phase



Planck collaboration,

arXiv: 1303.5062

Dark Matter

$$\Omega_m + \Omega_k = 1 \rightarrow \Omega_b + \Omega_{cdm} \equiv \Omega_m \simeq 1,$$

knowing that

$$\Omega_k \equiv -k/a^2 H^2 \leq 0.012,$$

SM extensions, but...

lack of direct experimental check!

Dark Energy

Accelerated expansion factor

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1,$$

Cosmological constant era??

Flatness problem

Flatness

$$r(t) = \frac{L_H}{L_k} \sim \frac{\text{spatial curvature}}{\text{space-time curvature}},$$

Early state of universe with *spatial curvature* strongly suppressed than *space-time curvature*: fine tuning problem.

General *de Sitter* inflation

Source

Barotropic perfect fluid with constant negative-pressure vacuum energy density: $p = -\rho = \Lambda$.

$$H^2 + \frac{k}{a^2} = \frac{8}{3}\pi G\rho \to \dot{a}^2 = \frac{8}{3}\pi G\Lambda a^2 - k, \quad H \equiv \frac{\dot{a}}{a},$$

from which, with $k = 0$:

$$a(t) = e^{H_\Lambda t}, \qquad H_\Lambda \equiv \left(\frac{\Lambda}{3M_P^2}\right)^{1/2}$$

Suitable inflationary solution[9]!

---

[9] A. Guth, Phys. Rev. D23, 347 (1981).

General *de Sitter* inflation

Source

Barotropic perfect fluid with constant negative-pressure vacuum energy density: $p = -\rho = \Lambda$.

$$H^2 + \frac{k}{a^2} = \frac{8}{3}\pi G\rho \to \dot{a}^2 = \frac{8}{3}\pi G\Lambda a^2 - k, \quad H \equiv \frac{\dot{a}}{a},$$

from which, with $k = 0$:

$$a(t) = e^{H_\Lambda t}, \qquad H_\Lambda \equiv \left(\frac{\Lambda}{3M_P^2}\right)^{1/2}$$

Suitable inflationary solution[9]!

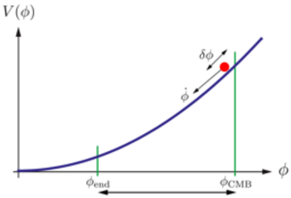
---

[9] A. Guth, Phys. Rev. D23, 347 (1981).

General *de Sitter* inflation (1)

No exit from accelerated expansion phase: eternal inflation

$$ds^2 = dt^2 - e^{2H_\Lambda t}|d\vec{x}|^2.$$



D. Baumann, arXiv: 0907.5424

Conformal time

Conformal time: $dt = a(\eta)d\eta$

$$\eta = \int^t \frac{d\tau}{a(\tau)} = -\frac{e^{-H_\Lambda t}}{H_\Lambda} \to a = \frac{1}{-H_\Lambda \eta}.$$

$$ds^2 = \frac{1}{H^2\eta^2}\left(d\eta^2 - d\vec{x}^2\right).$$

Inflationary behaviour: $a \sim t^\beta, \qquad \beta \gg 1$

$$a(\eta) \sim (-\eta)^{(\epsilon-1)^{-1}} \sim (-\eta)^{-1-\epsilon}.$$

Slow-roll parameters

Slow-roll parameters

$$\epsilon \equiv \frac{1}{2\lambda_P^2}\left(\frac{V_\phi}{V}\right)^2, \qquad \eta \equiv \frac{1}{\lambda_P^2}\left(\frac{V_{\phi\phi}}{V}\right)$$

Typical example

$$V \sim \phi^n$$

For $n = 2$, we have *general chaotic inflation*[10]

$$\epsilon \sim \eta \sim \phi^{-2} \ll 1 \Rightarrow$$

$$\phi_{in} \gg M_P \Rightarrow N = \frac{1}{4}\left(\frac{\phi_{CMB}^2}{M_P^2} - \frac{\phi_{end}^2}{M_P^2}\right) \gg 1.$$

---

[10] A. Linde, Phys. Lett. B129, 177 (1983)

Slow-roll parameters

Slow-roll parameters

$$\epsilon \equiv \frac{1}{2\lambda_P^2}\left(\frac{V_\phi}{V}\right)^2, \qquad \eta \equiv \frac{1}{\lambda_P^2}\left(\frac{V_{\phi\phi}}{V}\right)$$

Typical example

$$V \sim \phi^n$$

For $n = 2$, we have *general chaotic inflation*[10]

$$\epsilon \sim \eta \sim \phi^{-2} \ll 1 \Rightarrow$$

$$\phi_{in} \gg M_P \Rightarrow N = \frac{1}{4}\left(\frac{\phi_{CMB}^2}{M_P^2} - \frac{\phi_{end}^2}{M_P^2}\right) \gg 1.$$

---

[10] A. Linde, Phys. Lett. B129, 177 (1983)

Matching procedure

- The matching for the gauge couplings is performed at the Z boson pole mass m_Z: the correction to the numerical values for the related $\overline{MS}$ observables (from PDG) is very small and can be neglected;
- The matching between $\lambda_h(\mu)$ with the Higgs pole mass m_h is given by:

$$\lambda_h(\mu) = \frac{1}{2}\frac{m_h^2}{v^2}\left(1 + \delta_h^{(1)}(\mu) + \delta_h^{(2)}(\mu) + \dots\right),$$

known at NLO: $\delta_h^{(1)}(\mu)$ is $\mathcal{O}(\alpha)$, while $\delta_h^{(2)}(\mu)$ is formed by a Yukawa contribution and a QCD contribution ($\mathcal{O}(\alpha\alpha_3)$). "Theoretical" uncertainty is 0.7% at 2-loop:

$$\lambda_h(m_h) = 0.8065 + 0.0109(m_h[\mathrm{GeV}] - 126) + \\ + 0.0015(m_t[\mathrm{GeV}] - 172)^{+0.0002}_{-0.0060}.$$

Matching procedure (1)

- Extrapolation of the $y_t(\mu)$ from the matching between the running top mass $\overline{m_t}(\mu)$ and the top pole mass m_t:

$$y_t(\mu)\frac{v}{\sqrt{2}} = \bar{m}_t(\mu) = m_t\left(1 + \delta_t^W(\mu) + \delta_t^{QED}(\mu) + \delta_t^{QCD}(\mu)\right)$$

known at NLO: $\delta_t^W(\mu) + \delta_t^{QED}(\mu)$ represent the EW contribution (at 2-loop), while $\delta_t^{QCD}(\mu)$ is the QCD (at 3-loop).
"Theoretical" uncertainty is related to the choice of μ, 2% at 2-loop:

$$y_t(m_t) = 0.933 + 0.006(m_t[\text{GeV}] - 172)^{+0.017}_{-0.013}.$$

Anomalous dimension

Dilatation in a scale-invariant QFT: $x \to \lambda x$,
each operator acquires a factor $\lambda^{-\Delta}$,
with Δ called *scaling dimension of the operator*.

Free theories Δ_0 from dimensional analysis (classical one);

Interacting fields $\Delta = \Delta_0 + \gamma(g)$, where $\gamma(g)$ is the anomalous dimension[11]: the scale invariance is spoiled at quantum level
(or, in some cases, preserved approximatey over long distances).

Higgs field case

$$\Gamma(\mu) \equiv \int_{m_t}^{\mu} \gamma(\mu') d \ln \mu', \qquad \gamma(g) = -\frac{d \ln h}{d \ln \mu}.$$

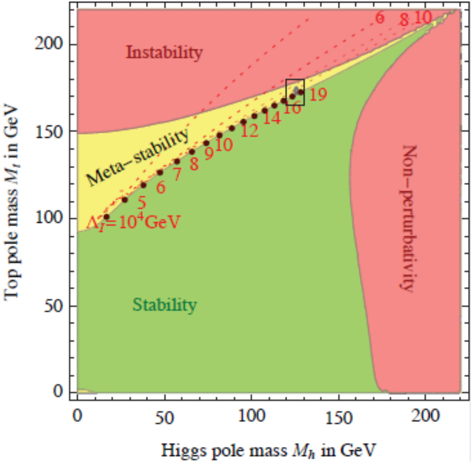
This quantity is independent by the cut-off of the theory but not by the gauge.

---

[11] It is generally expressed by power series in the couplings, with their running in energy.

Post LHC phase diagram

Stability diagram in the (m_h, m_t) at NNLO[12]



---

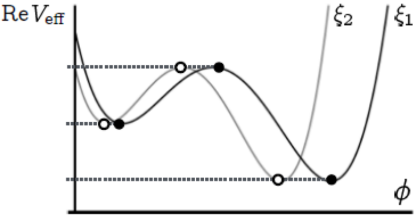
[12] G. Degrassi et al., arXiv: 1307.3536.

Nielsen's identities[13]

$$\xi \frac{\partial}{\partial \xi} V(\phi, \xi) = -C(\phi, \xi) \frac{\partial}{\partial \phi} V(\phi, \xi).$$

Variations w. r. t. gauge parameter are proportional to variations w. r. t. field.

In other words, at critical points of V, the potential is gauge independent.



---

[13] N. K. Nielsen, Nucl. Phys. B 101 (1975) 173-188.

$\hbar$ - expansion method

(H. Patel, M. J. Ramsey-Musolf, JHEP 1107, 029 (2011) and also[15])

- $\hbar$ counts the number of loops, the effective potential is truncated to order $\hbar$ at NLO and $\hbar^2$ at NNLO, with a $\lambda \sim \hbar$ power counting[14].
- Effective potential will be a series in $\hbar$:

$$V_{\text{eff}}(\phi) = V^{(0)}(\phi) + \hbar V^{(1)}(\phi) + \hbar^2 V^{(2)}(\phi) + \ldots \rightarrow$$
$$\phi_{\min} = \phi^{(0)} + \hbar\phi^{(1)} + \hbar^2\phi^{(2)} + \ldots,$$

where $\phi^{(0)}$ is the tree-level vev v and the others are the quantum corrections δv.

Inserting into the minimization condition $V'_{\text{eff}}\Big|_{\phi_{\min}} = 0$:

$$V'_{\text{eff}}(\phi_{\min}) = V'^{(0)}(\phi^{(0)} + \hbar\phi^{(1)} + \ldots) + V'^{(1)}(\ldots) + \ldots =$$
$$= V'^{(0)}(\phi^{(0)}) + \hbar[V'^{(1)}(\phi^{(0)}) + \phi^{(1)}V''^{(2)}(\phi^{(0)})] = 0$$

---

[14] Be careful to terms scaling like the inverse power of $\hbar$.

[15] A. Andreassen, W. Frost, D. Schwartz, arXiv: 1408.0292.

$\hbar$ - expansion method (1)

- Each power of $\hbar$ must satisfy the equality:

$$\begin{aligned} \mathcal{O}(1): &\quad V'^{(0)} = 0 \quad \text{tree-level vev} \\ \mathcal{O}(\hbar): &\quad \phi^{(1)} = -V''^{(0)}(\phi^{(0)})^{-1} V'^{(1)}(\phi^{(0)}) \quad \text{1-loop} \\ \mathcal{O}(\hbar^2): &\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{2-loop} \end{aligned}$$

Vacuum energy

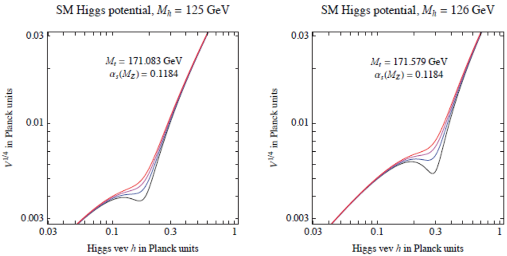
$$\varepsilon = V^{(0)}(\phi^{(0)}) + \hbar V^{(1)}(\phi^{(0)}) + \hbar^2 \left(V^{(2)}(\phi^{(0)}) - \frac{1}{2} \frac{V'^{(1)}(\phi^{(0)})^2}{V''^{(2)}(\phi^{(0)})} \right) + \ldots$$

ε depends only on extremal gauge-independent objects

- It can be applied also to *VEVs (δv), Masses, CW corrections, RG-improved vacua,* ...

False vacuum inflation[16]

- Tuning the top quark mass, it is possible to obtain a shallow local minimum at large field values (stability required);
- The Higgs boson sitting in this false vacuum would provide exponential inflation and then could tunnel to the EW one;
- The model needs another scalar responsible of scalar perturbations and a mechanism (tunnelling) for escaping from inflationary phase (graceful exit).



---

[16] I. Masina, A. Notari, arXiv: 1112.2659.

False vacuum inflation (1)

Like Old inflation, it is driven only by a vacuum energy, but
Graceful exit issue

- For sufficient inflation: $\Gamma \ll H^4$ (Γ tunneling rate);
- For a successful transition to radiation epoch (nucleation and bubbles collission): $\Gamma \sim H^4$.

Solution

Need an additional scalar degree of freedom ϕ to give a time-dependence to Γ and/or H:

1. $H = H(t)$ (couple ϕ to gravity);
2. $\Gamma = \Gamma(t)$ (couple ϕ to the Higgs).

False vacuum inflation (2)

Adding a Brans-Dicke scalar:

Case 1

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\partial_\mu\phi\partial_\nu\phi - f(\phi)R + \mathcal{L}_{SM} + V(\phi)\right],$$

$$f(\phi) = M^2 + \beta\phi^2 + \gamma_n\frac{\phi^n}{M^{n-2}}, \qquad n = 4, 6, 8, \dots$$

Prediction: $n_s = 0.94 \div 0.95$.

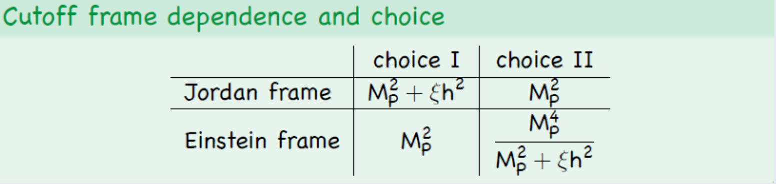
Case 2

Hybrid inflation

Prediction: $r < 0.007$ and $m_h < 125.3 + 3_{th}\,GeV$.

Radiative corrections and cutoff dependence

Prescriptions



Cutoff frame dependence and choice

	choice I	choice II
Jordan frame	$M_P^2 + \xi h^2$	M_P^2
Einstein frame	M_P^2	$\frac{M_P^4}{M_P^2 + \xi h^2}$

F. Bezrukov et al., arXiv: 0812.4950

- The two frames are mathematically equivalent but not physically equivalent when we consider quantum corrections (the equivalence still holds at tree-level);
- Still not clear which choice is the best one.

Unitarity in QFT

Conservation of probability ⇒ Unitarity of the $\mathcal{S}$-matrix:

$$\mathcal{S}^\dagger \mathcal{S} = 1.$$

Implies that amplitudes do not grow fast with energy.

Bound on the size of partial waves amplitudes

From optical theorem it can be derived:

$$\mathcal{A} = 16\pi \sum_j (2j+1) P_j(\cos\theta) a_j \Rightarrow |\Re\{a_j\}| \leq \frac{1}{2}.$$

Unitarity in QFT (1)

- Constraining tree level amplitudes in an effective field theory, it can be provided a bound for the cutoff Λ of the theory;
- Effective field theories contain higher order operators suppressed by powers of the cutoff:

$$\frac{\mathcal{H}^\dagger \mathcal{H}}{\Lambda^{2n-4}};$$

- At energies $E > \Lambda$ these terms become relevant and the perturbative regime breaks down.

Strong coupling or new physics?

Unitarity in QFT (1)

- Constraining tree level amplitudes in an effective field theory, it can be provided a bound for the cutoff Λ of the theory;
- Effective field theories contain higher order operators suppressed by powers of the cutoff:

$$\frac{\mathcal{H}^\dagger\mathcal{H}}{\Lambda^{2n-4}};$$

- At energies $E > \Lambda$ these terms become relevant and the perturbative regime breaks down.

Strong coupling or new physics?

Unitarity in Higgs inflation

For Higgs inflation[17]

$$a_0 = \frac{\pi}{3}\frac{s}{M_P^2}(1+12\xi)^2 \sim \frac{\xi^2}{M_P^2}s \leq \frac{1}{2}, \quad \xi \sim 10^4$$

leads to

$$\Lambda \lesssim \frac{M_P}{\xi}.$$

Inflation takes place for $h \gg M_P/\sqrt{\xi}$, above the regime of validity of the theory.

---

[17] M. Atkins and X. Calmet, arXiv: 1011.4179

Unitarity: frames and background dependence

The cutoff Λ should be the same in both frames, but in the Einstein one:

$$\Lambda_E = M_P$$

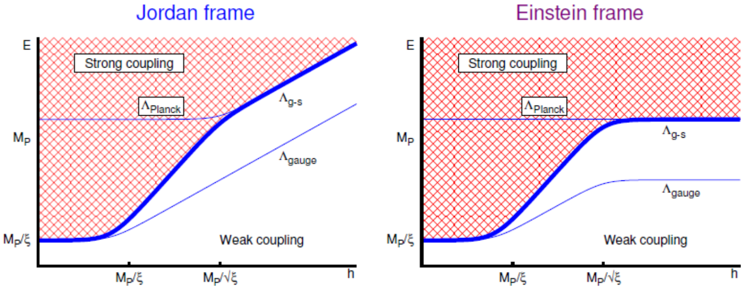
If we expand the scalar field

$$\chi(\vec{x}, t) = \bar{\chi} + \delta\chi(\vec{x}, t),$$

we get a background dependent cutoff, from which we can have suppression of operators of $dim > 4$

$$\frac{\mathcal{O}_{(n)}(\delta\chi)}{\left[\Lambda_{(n)}(\bar{\chi})\right]^{n-4}}.$$

New physics or strong coupling[18]?



$$\Lambda_J = \Omega \Lambda_E$$

After the expansion around the inflating background:

$$\Lambda_J(\bar{\chi}) = \frac{M_P^2 + \xi\bar{\chi}^2 + 6\xi^2\bar{\chi}^2}{\xi\sqrt{M_P^2 + \xi\bar{\chi}^2}}.$$

---

[18] F. Bezrukov et al., arXiv: 1008.5157.

New physics or strong coupling? (1)

So the relevant scales are:

- Small field: $\bar{\chi} \ll M_P/\xi \Rightarrow \Lambda_J \simeq \frac{M_P}{\xi}$;
- Re-heating: $M_P/\xi \ll \bar{\chi} \ll M_P/\sqrt{\xi} \Rightarrow \Lambda_J \simeq \frac{\xi\bar{\chi}^2}{M_P}$;
- Inflation: $\bar{\chi} \gg M_P/\sqrt{\xi} \Rightarrow \Lambda_J \simeq \sqrt{\xi}\bar{\chi}$.

During inflation we are still in a perturbative regime, but, if new physics is required to unitarize the theory, potential must include operators

$$\frac{\mathcal{H}^\dagger\mathcal{H}}{\Lambda^{2n-4}}$$

appearing at $\Lambda = M_P/\xi$, and spoiling the potential.

Alternative models

1. Derivative coupling
   (Germani, Kehagias, arXiv: 1003.2635)

   $$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}R - \frac{1}{2}(g^{\mu\nu} - \omega^2 G^{\mu\nu})\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\phi^4\right].$$

   But the inflationary scale *exceeds the realm of validity* of the effective theory;

2. Unitarizing Higgs inflation
   (Giudice, Lee, arXiv: 1010.1417)
   A massive scalar field is introduced in order to remove unitarity problem, but actually *it is this new field that drives inflation*;

3. More exotic models (Asymptotic safety, Composite inflation, Log-type potential, etc.).

Nambu-Goldstone bosons[20]

Including *Goldstone* bosons into dynamics:
they disappear from theory in unitary gauge, but…[19].

Are they still unphysical in Higgs inflation?

- Contributions in the Coleman-Weinberg corrections:

$$V_1(\chi) \sim \frac{3M_\theta^4}{4}\left(\ln\frac{M_\theta^2}{\mu^2} - \frac{3}{2}\right),$$

  plus time-dependent background Higgs corrections as it rolls down its potential;
- Up to now only for the simpler Abelian Higgs model, but not yet for more realistic models in ξ-inflation.

---

[19] their associated d. o. f. still render the $U(1)$ gauge bosons massive

[20] Greenwood, Kaiser, Sfakianakis, arXiv: 1210.8190
Mooij, Postma, arXiv: 1104.4897.

SM extensions

Quite simple extension (in principle):
as little new physics as possible

- inflation without introducing new scalars (ξ-inflation);
- post-inflationary reheating without new interactions with SM fields;

it might be further modified:

- (very) massive right-handed neutrino(s) for neutrino oscillations (see-saw mechanism);
- scalar field for dark matter;
- ν MSM for baryon asymmetry of the Universe.

SM extensions

Quite simple extension (in principle):
as little new physics as possible

- inflation without introducing new scalars (ξ-inflation);
- post-inflationary reheating without new interactions with SM fields;

it might be further modified:

- (very) massive right-handed neutrino(s) for neutrino oscillations (see-saw mechanism);
- scalar field for dark matter;
- ν MSM for baryon asymmetry of the Universe.

Non Linear ElectroDynamics (NLED) and CMB polarization[21] (L. Tedesco)

NLED action minimally coupled to gravity

$$\mathcal{S} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \frac{1}{4\pi}\int d^4x\sqrt{-g}\mathcal{L}(F),$$

where $\mathcal{L}(F)$ is the Lagrangian of NLED.

Page-Tomboulis Lagrangian

$$\mathcal{L}(F) = -\left(\frac{F^2}{\Lambda^8}\right)^{\frac{\delta-1}{2}} F = -\gamma F, \quad F \equiv \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- δ is the non-linearity parameter;
- γ (or Λ) and δ are free parameters;
- $[\gamma] = (\text{energy})^{4(1-\delta)}$.

---

[21] H. J. Mosquera-Cuesta, G. Lambiase, JCAP, 1103:033 (2011).

Non Linear ElectroDynamics (NLED) and CMB polarization[21] (L. Tedesco)

NLED action minimally coupled to gravity

$$\mathcal{S} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \frac{1}{4\pi}\int d^4x\sqrt{-g}\mathcal{L}(F),$$

where $\mathcal{L}(F)$ is the Lagrangian of NLED.

Page-Tomboulis Lagrangian

$$\mathcal{L}(F) = -\left(\frac{F^2}{\Lambda^8}\right)^{\frac{\delta-1}{2}} F = -\gamma F, \quad F \equiv \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- δ is the non-linearity parameter;
- γ (or Λ) and δ are free parameters;
- $[\gamma] = (\text{energy})^{4(1-\delta)}$.

---

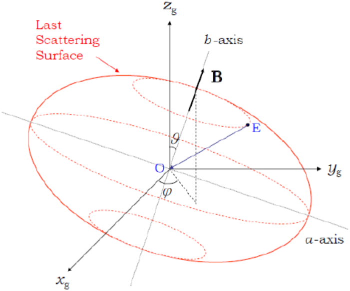
[21] H. J. Mosquera-Cuesta, G. Lambiase, JCAP, 1103:033 (2011).

NLED and CMB polarization (1)

Bianchi I metric

$$ds^2 = dt^2 - b^2(dx^2 + dy^2) - c^2 dz^2$$

Considering that the CMB polarization arose near the last scattering surface we can write the final relation:



L. Campanelli, P. Cea and L. Tedesco,

Phys. Rev. D 76, 063007 (2007)

$$\Delta\alpha = \frac{\delta - 1}{4\delta} K e^2(z_{dec}),$$

with $z_{dec} \simeq 1100$.

Polarization angle and eccentricity

We define[22] the polarization angle as

$$\alpha \equiv \arctan\left(\frac{c\varepsilon^{(3)}}{b\varepsilon^{(2)}}\right).$$

Eccentricity

$$e(t) \equiv \sqrt{1-\left(\frac{c}{b}\right)^2}.$$

---

[22] The same calculation can be performed using *Stokes parameters*:

$$2\alpha \equiv \arctan\left(\frac{U}{Q}\right).$$

NLED and CMB polarization (2)

- NLED models can be used as an explanation for different issue in many contexts;
- We can relate the non-linearity of the Lagrangian with CMB polarization and geometry (planar symmetry models);
- It is possible to estimate the scale of the non-linear Lagrangian starting from the constraints on the polarization angle and galactic magnetic fields;
- With different NLED theories we can try to explain the nature of the observed cosmic magnetism;
- We could obtain a similar relation starting from a proposed general Lagrangian;

NLED and CMB polarization (3)

- The observed cosmic magnetism could have been aroused out of quantum electromagnetic fluctuations excited during inflation;
- In standard electrodynamics, inflation-produced fields are vanishingly small and then cannot explain the presently observed fields;
- With non-linear theories, the conformal invariance is naturally broken and magnetic fields (through galactic dynamo or other model dependent mechanisms) can be created [23].

---

[23] L. Campanelli, P. Cea, G. L. Fogli, L. Tedesco, Phys. Rev. D 77, 043001 (2008).

NLED and CMB polarization (4)

In order to take into account the different NLED models, we propose a general Lagrangian, from which we should obtain a similar relation between non-linearity and geometry:

$$\mathcal{L}(F) = -\gamma F^{\delta} e^{-aF} + \sum_{i=2}^{n} \left[a_i F^i - \left(\frac{R}{m^2} \right)^i F \right].$$

Varying the parameters we will be able to understand the importance of the various components of the Lagrangian.