Status of the PVLAS experiment

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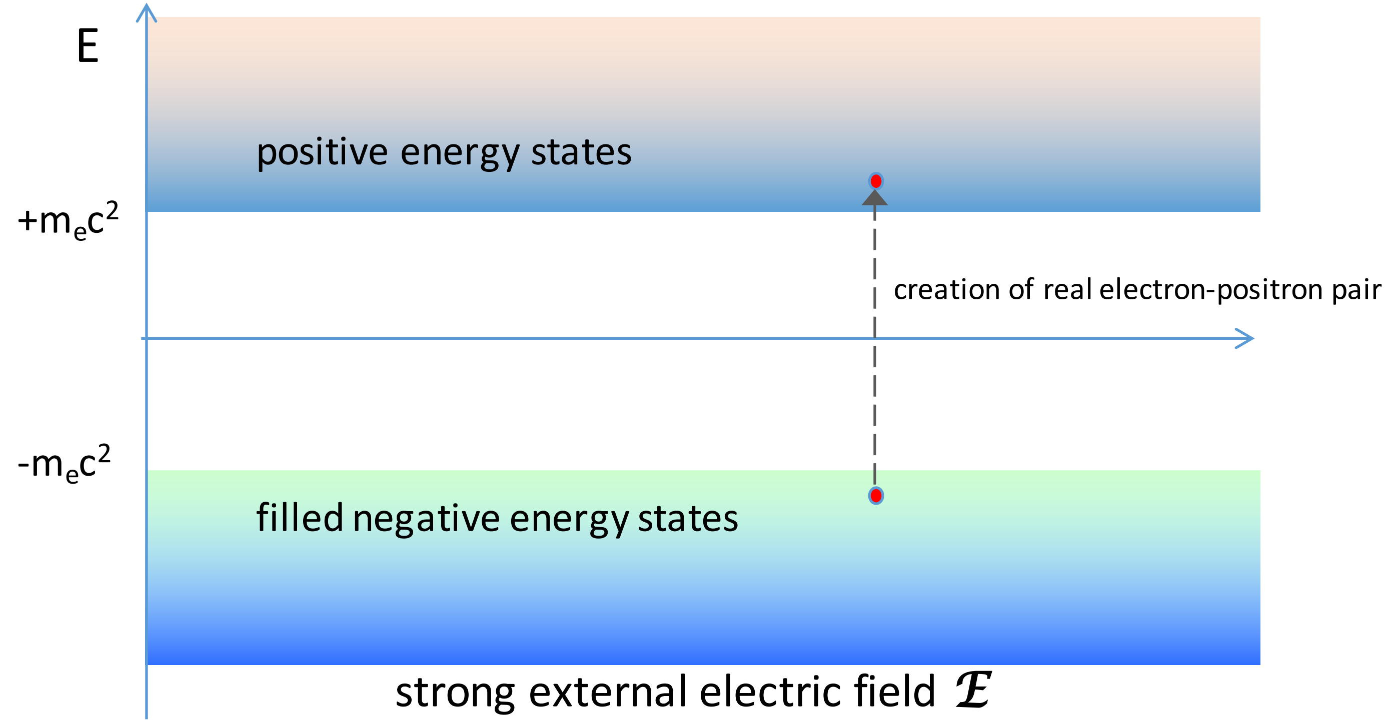
- Maxwell's equation are linear, and in the context of classical electrodynamics no light-light interaction is expected.
- However, almost one century ago, several scientists started a search for direct light-light interaction.
- Quantum electrodynamics – unlike classical electrodynamics – does predict the interaction of light with light, and this was clear soon after Dirac proposed his theory of the electron.
- In this theory one can extract an electron from the Dirac sea and raise it to positive energy by means of photon absorption.
- Even a very strong static electric field can achieve this feat…

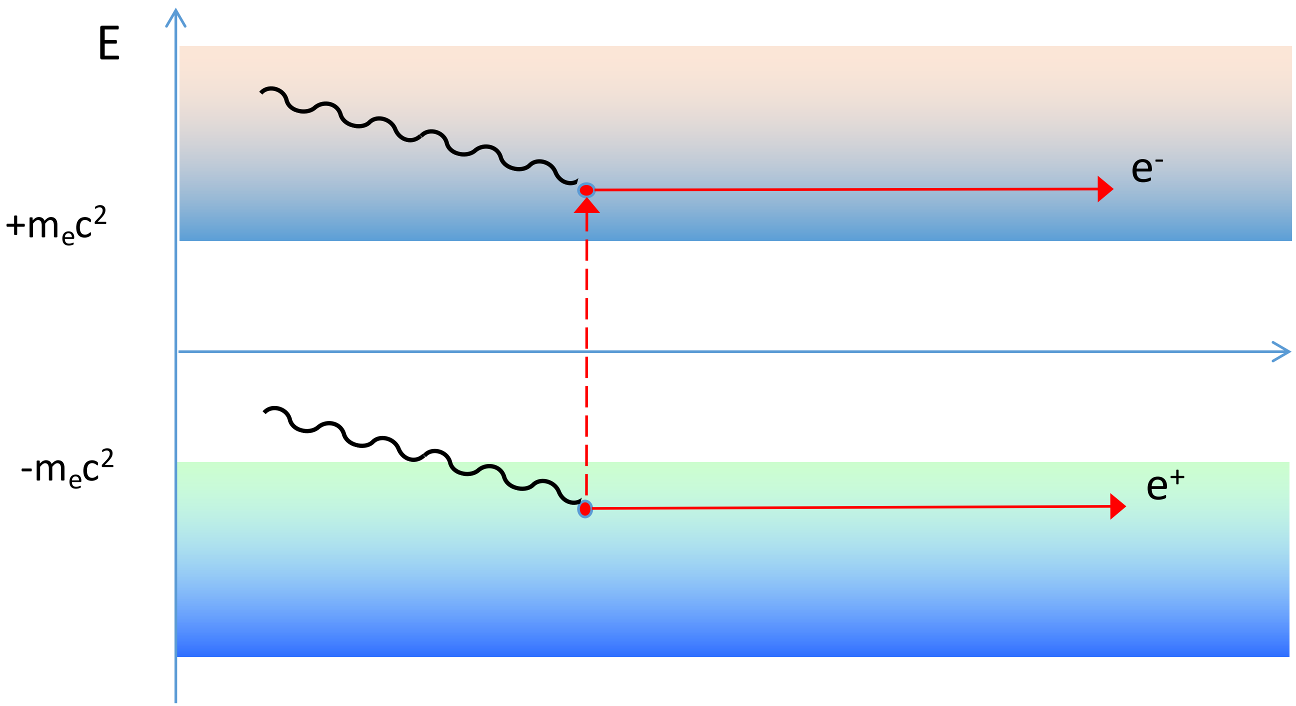


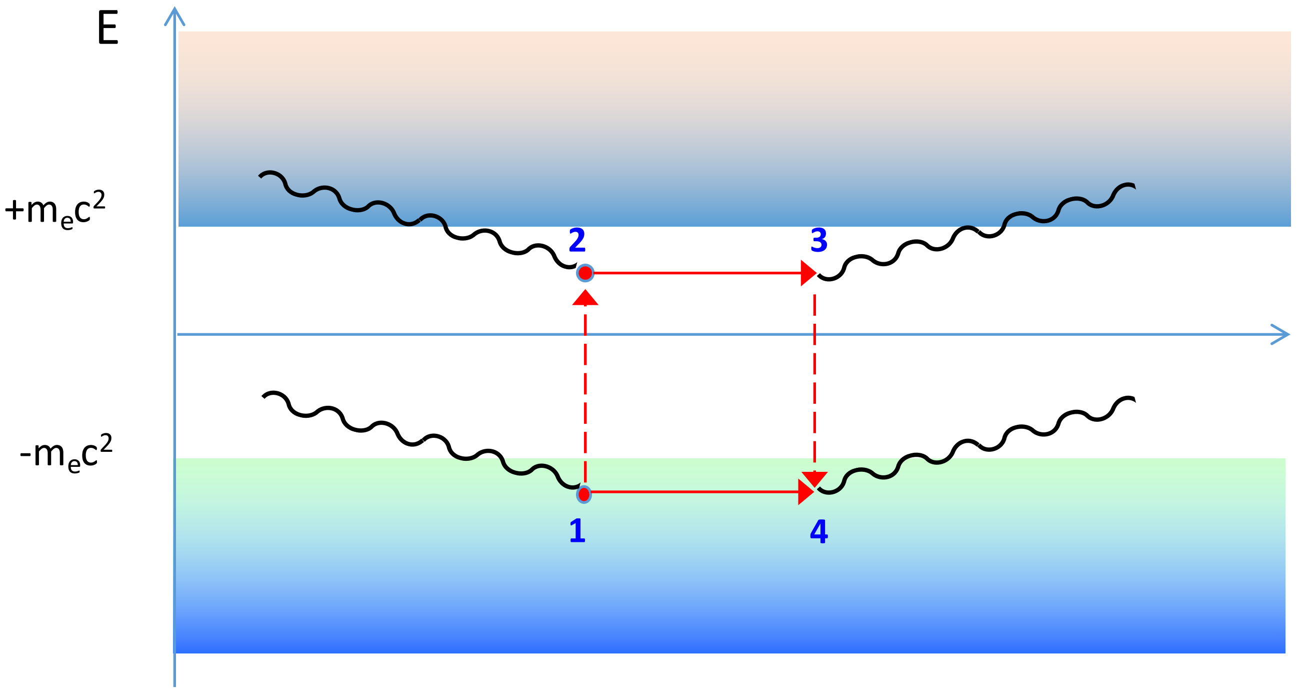
Extraction of electron pairs from vacuum under the action of a strong electric field: it can be formalized as a tunnelling effect (Schwinger effect)

Estimate of critical field: *charge separation energy ≈ electron rest energy*

$$e\mathcal{E}_c\bar{\lambda} = e\mathcal{E}_c\frac{\hbar}{mc} \approx mc^2 \quad \Rightarrow \quad \mathcal{E}_c = \frac{m^2c^3}{e\hbar} \approx 1.3\ 10^{18}\mathrm{V/m}$$

By the same token, a two photon absorption can create electron-positron pairs (Breit-Wheeler process; notice that single photon absorption is kinematically forbidden)





Similarly, when only virtual states are involved, **the process corresponds to photon-photon scattering**.

Then, just as we use photons to study materials, we can use photons to probe the QED vacuum.

Interestingly, experiments to detect photon-photon interaction independently of Dirac's theory, and were motivated by curiosity, rather than by theory.

Hughes and Jauncey – 1930

In 1930, Hughes and Jauncey tried to detect photon-photon scattering, in an attempt that was totally disconnected from the electron theory of Dirac (1928) and preceded the earliest attempt to include the effect of electrons from the sea in photon-nucleus scattering (Delbrück, 1932)

AUGUST 15, 1930 *PHYSICAL REVIEW* *VOLUME 36*

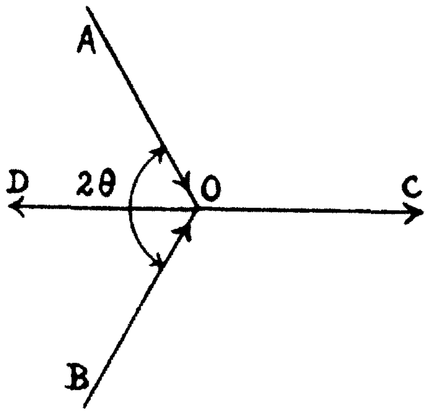
AN ATTEMPT TO DETECT COLLISIONS OF PHOTONS

By A. L. Hughes and G. E. M. Jauncey
Department of Physics, Washington University

(Received June 16, 1930)

Abstract

Assuming that light is corpuscular and that collisions between light corpuscles (i.e. photons) can occur, it is shown that two photons of identical frequency ν, moving along paths which make an angle 2θ with each other, will on collision give rise to a photon of frequency $\nu(1+\cos\theta)$ travelling forward along the line bisecting the angle. To test this, two beams of sunlight (one suitably deflected by a mirror), filtered through red glass, were passed through lenses 24 cm in diameter and of 33 cm focal length, so that the beams, whose axes made an angle of 120° with each other, intersected at a common focus. The point of intersection of the beams was examined through a green filter with the dark-adapted eye. No light was detected. Calculations show that if the photon has a cross section, its area must be less than 3×10^{-20} cm^2. From a result of Lord Rayleigh, some writers have suggested an area of the order of λ^2 for the photon. Our result shows the area to be of the order of $10^{-10}\lambda^2$.



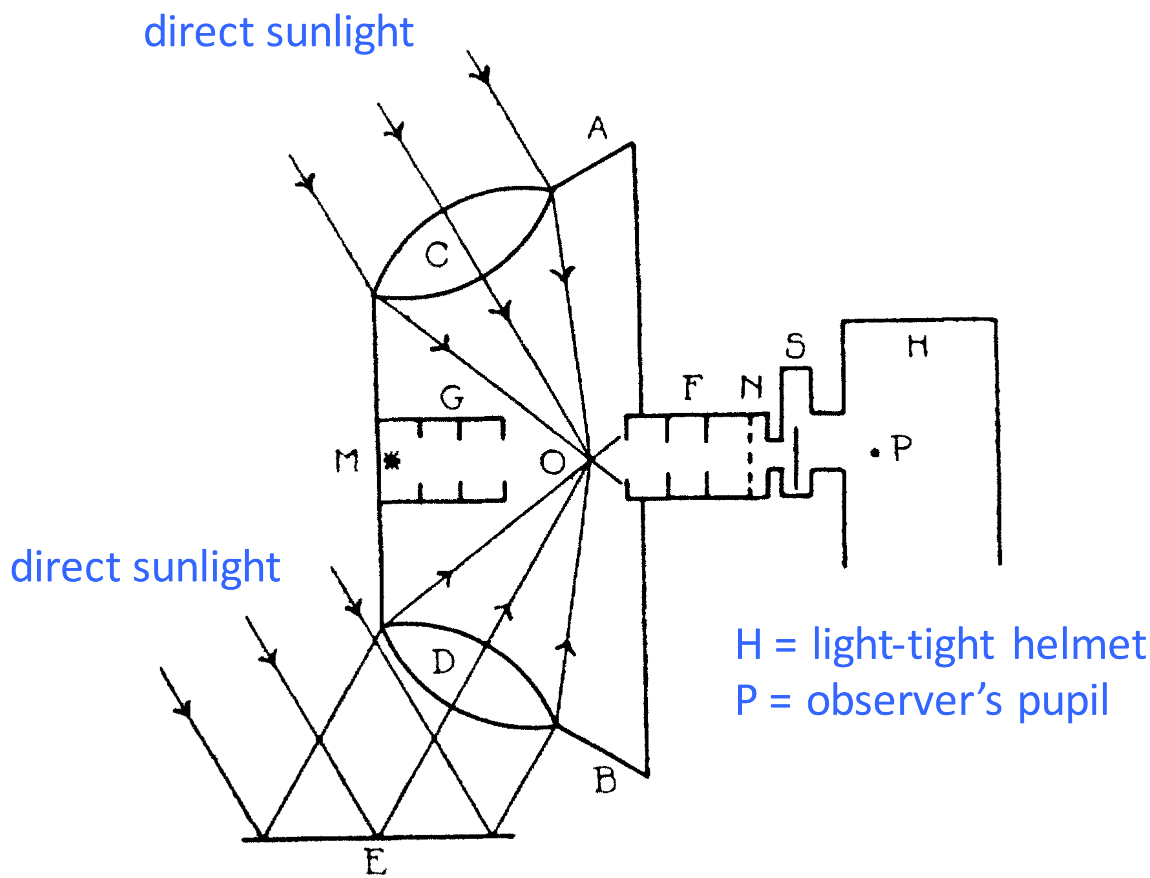


Fig. 2. Diagram of apparatus.

Watson – 1930

In another early effort (Proc. Royal Soc. London. Series A, 125, (1929), 345-351.) William H. Watson proposed to measure the effect of a transverse magnetic field on the propagation of light, and therefore the scattering between real and virtual photons, on the basis that

… The simplest particle properties which one can postulate are those of electric moment and magnetic moment; free electric charge is excluded by the fact that light is not deflected in a uniform electric or magnetic field …

and he set out

… with the object of detecting, if possible, the existence of the magnetic moment of a photon …

The Effect of a Transverse Magnetic Field on the Propagation of Light in vacuo.

By William H. Watson, Carnegie Research Fellow.

(Communicated by Sir Ernest Rutherford, P.R.S.—Received June 21, 1929.)

A particle with a magnetic moment μ parallel or antiparallel to the field H, would undergo a change in energy $\Delta E = \pm \mu H$ on entering the field and in accordance with the principles of the quantum theory would experience a change in frequency $\Delta\nu = \pm \mu H/h$. This would involve an effective change in wave-length

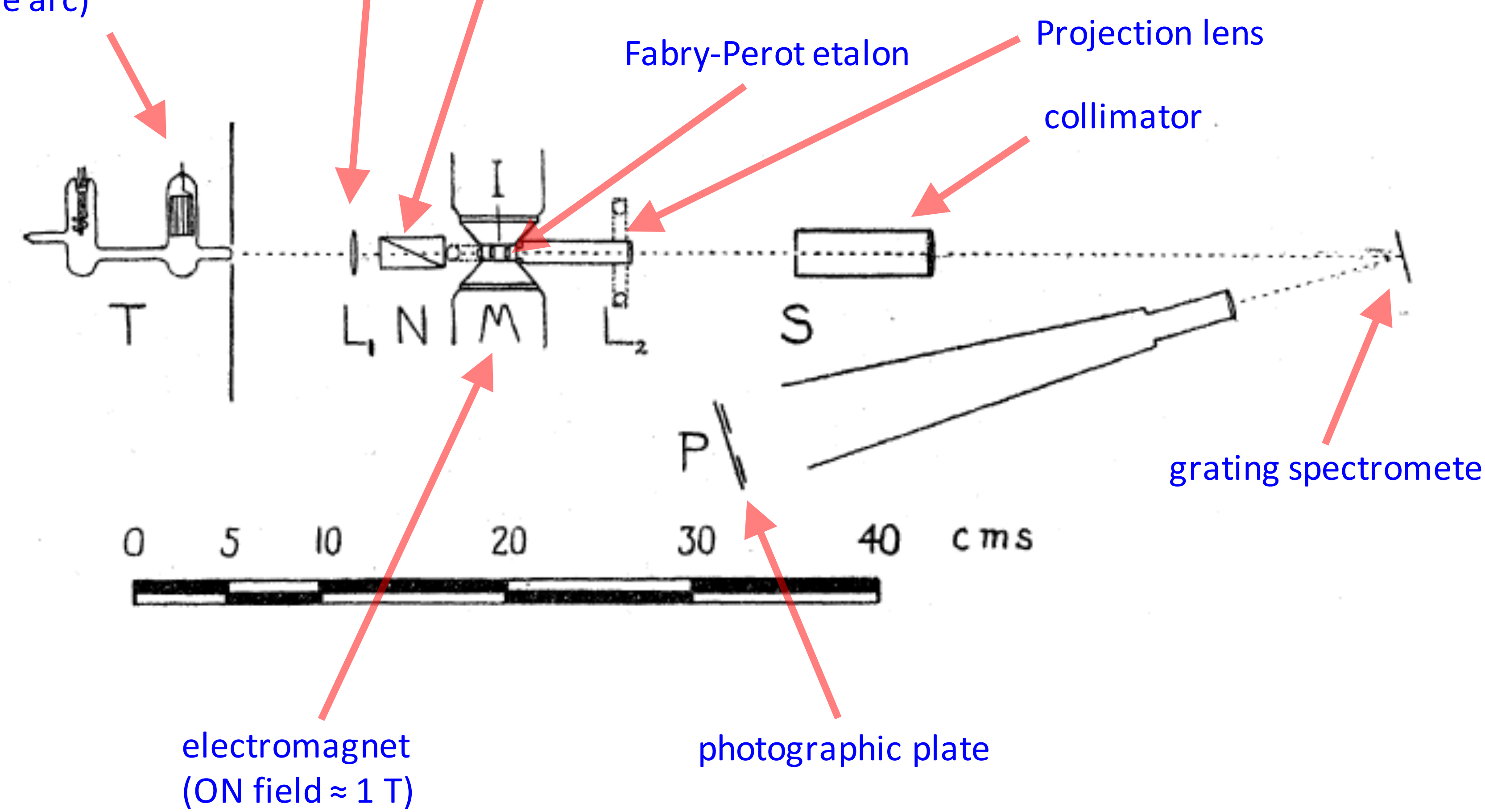
$$\Delta\lambda = \pm \mu H \lambda^2/hc$$

and a change in the interference pattern formed by the interferometer of the type observed in the normal Zeeman effect.

Focusing lens

Light source (Ne gas discharge lamp / low voltage arc)

Nicol prism



Farr & Banwell – 1932-1940

Farr and Banwell also tried to detect an effect of a magnetic field on the propagation of light, first by splitting a light beam into two and comparing the phase shifts inside and outside a magnet with an interferometric method and later with a Michelson interferometer

(Proc. Royal Soc. London. Series A, 137, (1932), 275-282; Proc. Royal Soc. London. Series A, 175, (1940), 1-25)

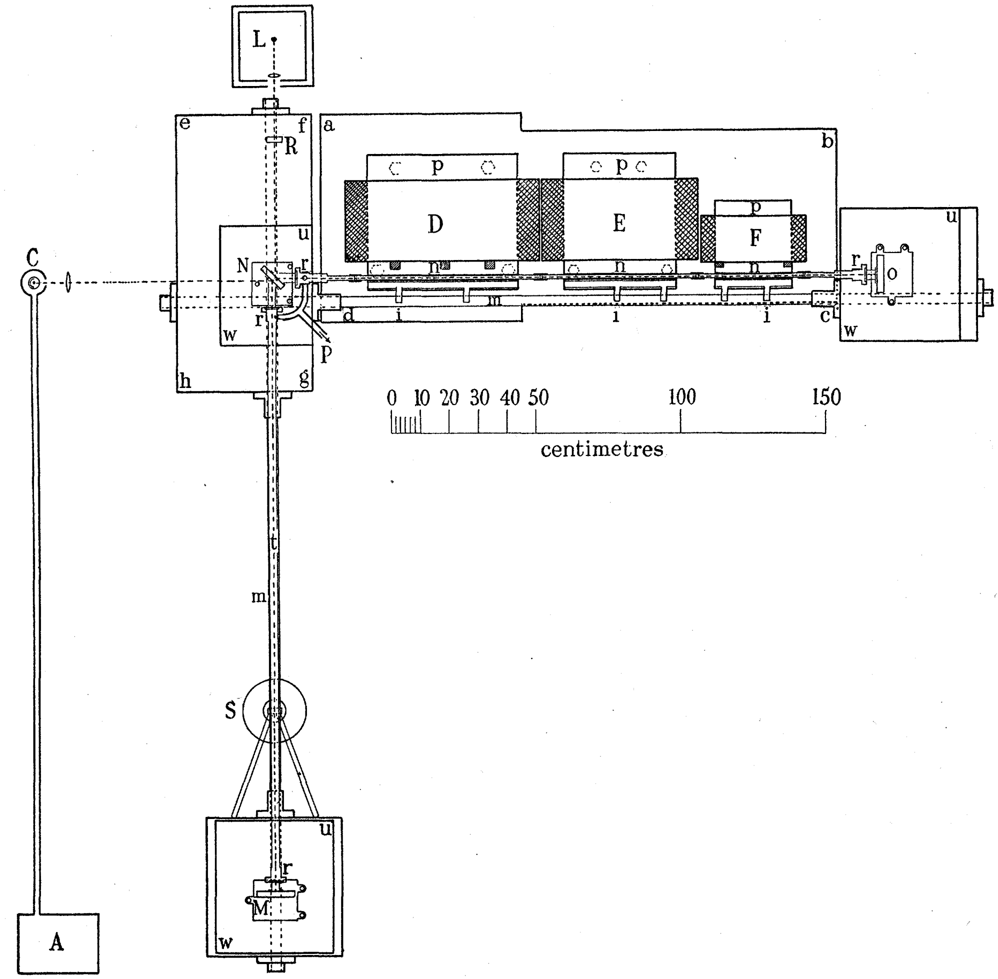


FIGURE 1

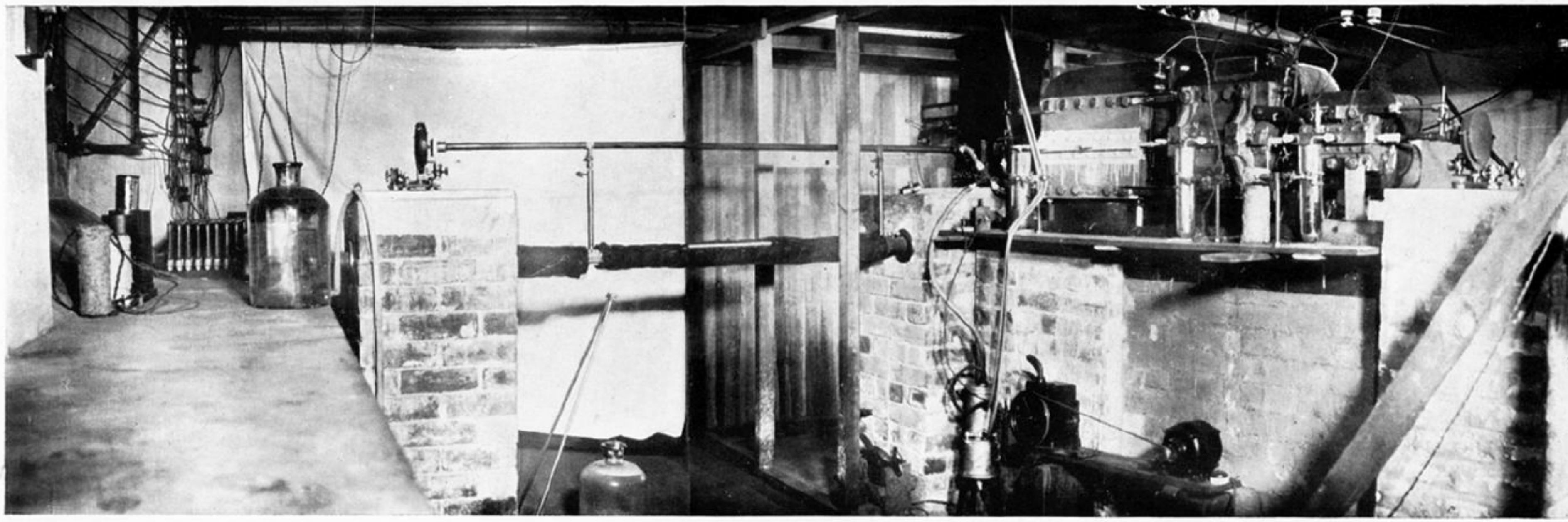


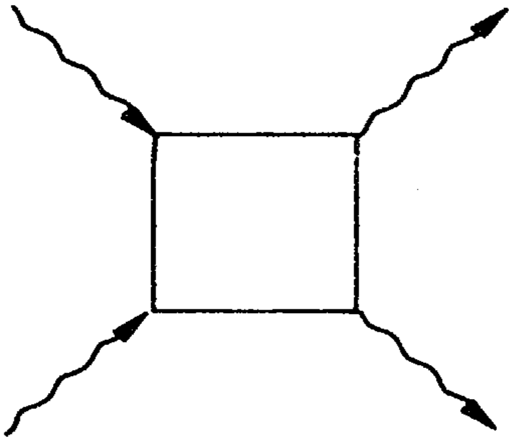
FIGURE 3

It is no wonder that no effect could be detected in these experiments, since the photon-photon scattering cross-section is extremely small...

The first suggestion to use the new theory of Dirac was put forward by Otto Halpern in 1933 in a short comment in the Physical Review, where he wrote

…Still, the almost insurmountable difficulties which the infinite charge-density without field offers to our physical understanding make it desirable to seek further tests of the theory. Here purely radiation phenomena are of particular interest inasmuch as they might serve in an attempt to formulate observed effects as consequences of hitherto unknown properties of corrected electromagnetic equations. We are seeking, then, scattering properties of the "vacuum." …

Thus, already at that time, it was clear that physical understanding involved the infinities of the theory and that it was all about the quantum vacuum.



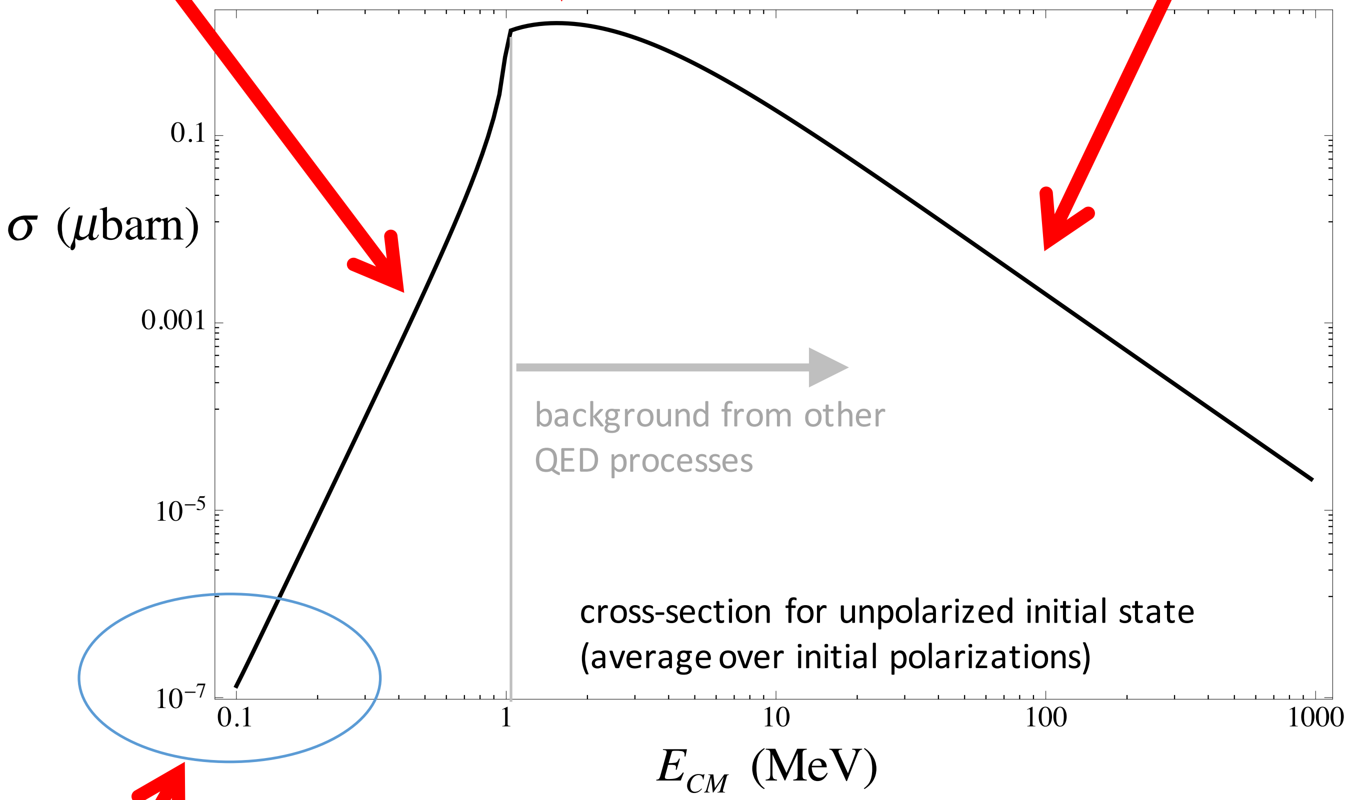
Photon-photon cross section calculations

- **1934**: Breit and Wheeler compute the photon-photon scattering cross-section for energies higher than $2m_e$;
- **1935**: Euler and Kochel provide a first general formula for the photon-photon scattering cross-section also for energies lower than the $2m_e$ threshold;
- **1936**: Euler provides the details of the cross-section formula (work done by Euler for his PhD thesis in Leipzig);
- **1936-37**: Akhiezer, Landau and Pomerancuk generalize the cross-section formula to high energies;
- **1950-51**: Karplus and Neuman carry out a thorough analysis using Feynman diagrams;
- **1964-65**: DeTollis utilizes dispersion relation techniques to give compact formulas for the scattering amplitudes;

$$\sigma \approx 0.13\left(\frac{\hbar\omega}{m_e c^2}\right)^6 \mu\text{barn}$$

peak cross section, ≈1.6 μbarn at $\hbar\omega \approx 1.5 m_e c^2$

$$\sigma \approx 20\left(\frac{m_e c^2}{\hbar\omega}\right)^2 \mu\text{barn}$$



very low energy region: cross section is extremely small, but photon numbers can be very large, there is no background from other QED processes, and experimental apparatus is comparatively easier to build

Low-energy effective Lagrangian

Full non-perturbative calculation with uniform background field (Heisenberg-Euler and Weisskopf), derived from exact solutions of the Dirac equation in constant background electric and magnetic field:

$$\mathcal{L} = \frac{e^2}{hc}\int_0^\infty \frac{d\eta}{\eta^3} e^{-\eta}\left\{ i\eta^2(\mathbf{E}\cdot\mathbf{B})\frac{\cos\left(\frac{\eta}{\mathcal{E}_c}\sqrt{\mathbf{E}^2-\mathbf{B}^2+2i(\mathbf{E}\cdot\mathbf{B})}\right)+c.c.}{\cos\left(\frac{\eta}{\mathcal{E}_c}\sqrt{\mathbf{E}^2-\mathbf{B}^2+2i(\mathbf{E}\cdot\mathbf{B})}\right)-c.c.} + \mathcal{E}_c^2 + \frac{\eta^2}{3}(\mathbf{B}^2-\mathbf{E}^2)\right\}$$

Heisenberg and Euler also produced a simplified form of the effective Lagrangian

$$\mathcal{L} = 4\pi^2 mc^2\left(\frac{mc}{h}\right)^3\int_0^\infty \frac{d\eta}{\eta^3}e^{-\eta}\left[-a\eta\cot(a\eta)b\eta\coth(b\eta)+1+\frac{\eta^2}{3}(b^2-a^2)\right]$$

where $\quad a^2-b^2=(\mathbf{E}^2-\mathbf{B}^2)/\mathcal{E}_c^2 \quad$ and $\quad ab=(\mathbf{E}\cdot\mathbf{B})/\mathcal{E}_c^2$

“proper time” variable (fully developed later by Stückelberg, Feynman and Schwinger)

subtraction of the infinite free-field effective action

this corresponds to a log term in the integrated Lagrangian: it is an embryonic form of charge renormalization

$$\mathcal{L} = 4\pi^2 mc^2 \left(\frac{mc}{h}\right)^3 \int_0^\infty \frac{d\eta}{\eta^3} e^{-\eta} \left[-a\eta \cot(a\eta) b\eta \coth(b\eta) + 1 + \frac{\eta^2}{3}(b^2 - a^2) \right]$$

where $a^2 - b^2 = (\mathbf{E}^2 - \mathbf{B}^2)/\mathcal{E}_c^2$ and $ab = (\mathbf{E} \cdot \mathbf{B})/\mathcal{E}_c^2$

scalar invariant

pseudoscalar invariant related to axial symmetry

Moreover, the lowest-order expansion of the HE Lagrangian yields

$$\mathcal{L} = -\mathcal{F} + \frac{8\alpha^2}{45 m_e^4}\mathcal{F}^2 + \frac{14\alpha^2}{45 m_e^4}\mathcal{G}^2$$
$$= -\frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{2\alpha^2}{45 m_e^4}\left[(\mathbf{E}^2 - \mathbf{B}^2)^2 + 7(\mathbf{E}\cdot\mathbf{B})^2\right]$$

$$\text{where} \quad \mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2); \quad \mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \mathbf{E}\cdot\mathbf{B}$$

or also (SI units)

$$\mathcal{L}_{HE} = \frac{A_e}{\mu_0}\left[\left(\frac{\mathbf{E}^2}{c^2} - \mathbf{B}^2\right)^2 + 7\left(\frac{\mathbf{E}}{c}\cdot\mathbf{B}\right)^2\right] \qquad \text{where} \quad \boxed{A_e = \frac{2\alpha^2 \lambda\!\!\!^{-}{}_e^3}{45\mu_0 m_e c^2} \approx 1.32\ 10^{-24}\ \mathrm{T}^{-2}}$$

Optical properties of QED vacuum

The equation of motion of the fields is

$$0 = \partial_\mu \left(F^{\mu\nu} - \frac{4\alpha^2}{45 m_e^4} F^{\alpha\beta} F_{\alpha\beta} F^{\mu\nu} - \frac{7\alpha^2}{45 m_e^4} F^{\alpha\beta} F_{\alpha\beta} \tilde{F}^{\mu\nu} \right)$$

or, equivalently,

$$\mathbf{D} = \frac{1}{\epsilon_0} \frac{\partial \mathcal{L}}{\partial \mathbf{E}}; \quad \mathbf{H} = -\mu_0 \frac{\partial \mathcal{L}}{\partial \mathbf{B}}$$

and these equations yield effective values for the electrical and magnetic polarizabilities,

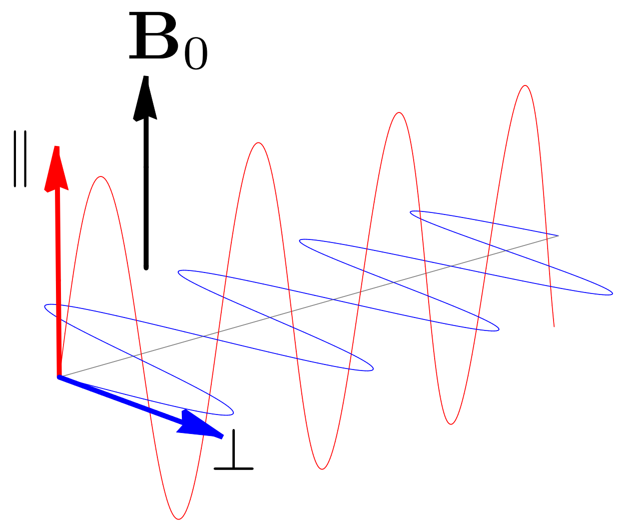
$$\varepsilon_{ij} \approx \delta_{ij} + \frac{4\alpha^2}{45 m^4} \left[2\left(\mathbf{E}^2 - \mathbf{B}^2 \right) \delta_{ij} + 7 B_i B_j \right]; \qquad \mu_{ij} \approx \delta_{ij} + \frac{4\alpha^2}{45 m^4} \left[2\left(\mathbf{B}^2 - \mathbf{E}^2 \right) \delta_{ij} + 7 E_i E_j \right]$$

Then, assuming that light propagates in a uniform, dipolar magnetic field **B**, we find

$$n_{\|} = 1 + 7A_e B_0^2$$
$$n_{\perp} = 1 + 4A_e B_0^2$$

and eventually

$$\Delta n = n_{\|} - n_{\perp} = 3A_e B_0^2$$



so that the magnetized vacuum of QED is birefringent.

We can associate this result to the critical field

critical magnetic field from the critical electric field

$$\mathcal{E}_c = \frac{m^2 c^3}{e\hbar} \approx 1.3\ 10^{18}\ \mathrm{V/m} \quad \Rightarrow \quad \mathcal{B}_c = \frac{\mathcal{E}_c}{c} = \frac{m^2 c^2}{e\hbar} \approx 4.4\ 10^9\ \mathrm{T}$$

then

$$A_e = \frac{\alpha}{90\pi}\left(\frac{1}{\mathcal{B}_c^2}\right)$$

and

$$\Delta n = 3A_e B_0^2 = \frac{\alpha}{30\pi}\left(\frac{B}{\mathcal{B}_c}\right)^2 \approx 7.7\ 10^{-5}\left(\frac{B}{\mathcal{B}_c}\right)^2$$

$$\boxed{\Delta n|_{B=2.5\ \mathrm{T}} \approx 2.5\ 10^{-23}}$$

Over the years many experimental proposals have been put forward to observe photon photon scattering and QED-induced optical effects

One early list was compiled by Paul L. Csonka at CERN (see also Phys. Lett. 24B (1967) 625)

CERN 67-15

Theoretical Study Division
11 May 1967

ARE PHOTON-PHOTON SCATTERING EXPERIMENTS FEASIBLE ?

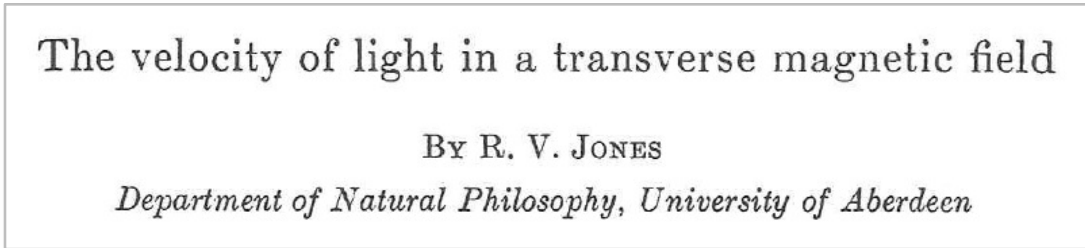
Paul L. Csonka

At low energy:

- laser beam clashing with high-energy gamma ray beam
- flash X-ray machines
- nuclear explosions
- synchrotron radiation

The list does not include birefringence measurements in the optical domain. **Moreover, nowadays one must also include photon-photon scattering with high-intensity lasers.**

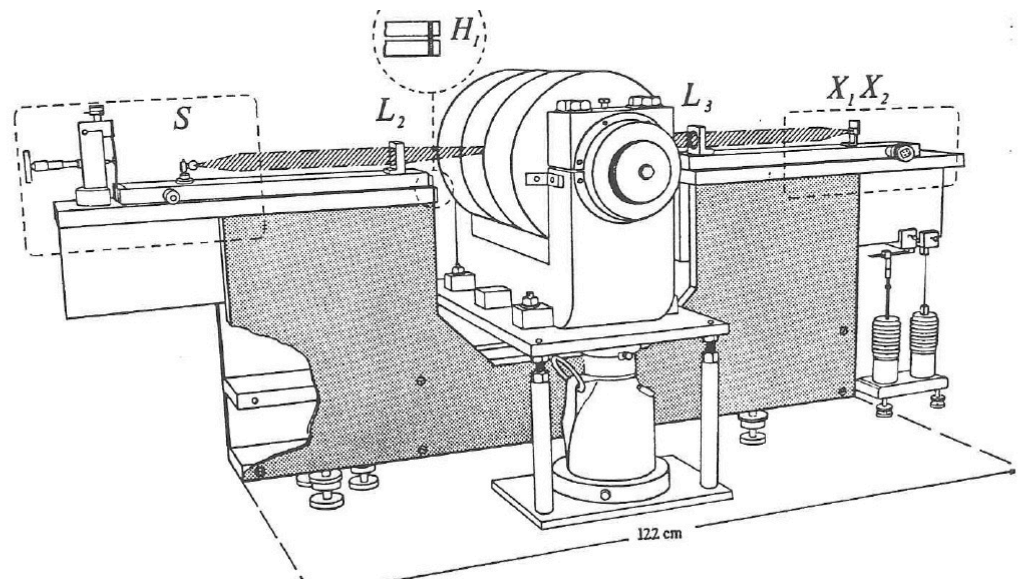
Jones – 1961


The velocity of light in a transverse magnetic field

By R. V. Jones
Department of Natural Philosophy, University of Aberdeen

Optical lever and **inhomogeneous magnetic field**, max field 0.9 T

- Tungsten filament source
- Polaroid polarizer
- Great care for non magnetic supports (DURAL - Al/Cu/Mg)
- Magnet switching at 0.1 Hz
- Split photocell readout



No effect measured: $\Delta v / v < 2.3\ 10^{-13}$ for 0.8 T

Erber – 1961

No. 4770 April 1, 1961 NATURE 25

VELOCITY OF LIGHT IN A MAGNETIC FIELD

By Prof. THOMAS ERBER
Department of Physics, Illinois Institute of Technology, Chicago

First citation of a **birefringence of vacuum**

Based on the previous work of Euler and Kochel

It also discusses experimental issues

(1) *Cotton-Mouton effect:* Equation (1) essentially says that there is a selective slowing down of light in a transverse magnetic field. By using polarized light and exploiting $N_{\parallel} \neq N_{\perp}$ one can achieve a rotation of the plane of polarization. This is in complete analogy to the transverse double refraction of optically active media (even to the B^2 dependence) and therefore deserves to be called a 'Cotton–Mouton' effect. From these remarks it follows readily that θ, the angular rotation of the plane of polarization (radians) is given by:

$$\theta \simeq \frac{\alpha}{15}\,\frac{l}{\lambda}\left(\frac{B}{B_{cr}}\right)^2 \qquad (2)$$

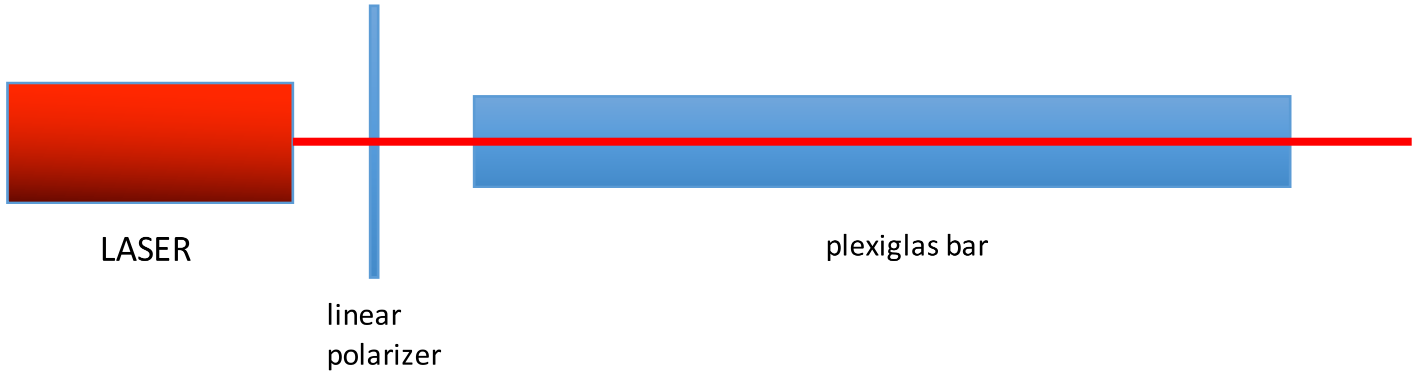
where l is the path-length and λ is the wave-length. The key point of this approach is the enormous help of the factor l/λ. Assuming a field of 3×10^7 gauss[4] and a path-length of about 10^2 cm. one finds:

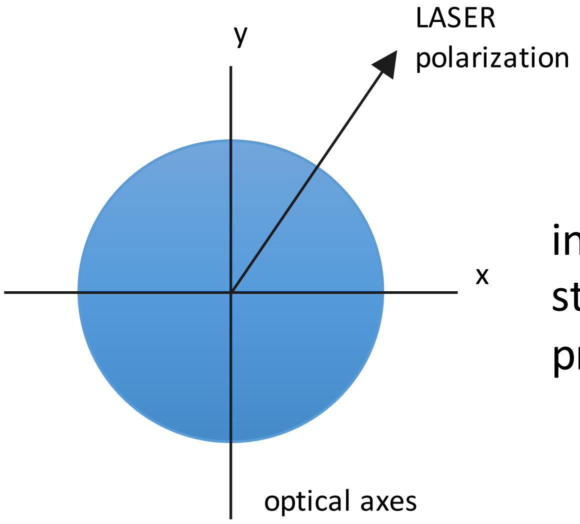
$$\theta \simeq \frac{1}{2\lambda}\,10^{-13}\ \text{radians} \qquad (3)$$

At this point it is clear that it would be advantageous to choose a λ well into the γ-ray region. Unfortunately the experimental means for measuring θ are very poor at these frequencies. Since the pulse-imploded fields cannot be maintained for periods longer than about 1μ sec., the accurate γ-ray polarimetry possible with the Mössbauer effect cannot be used.

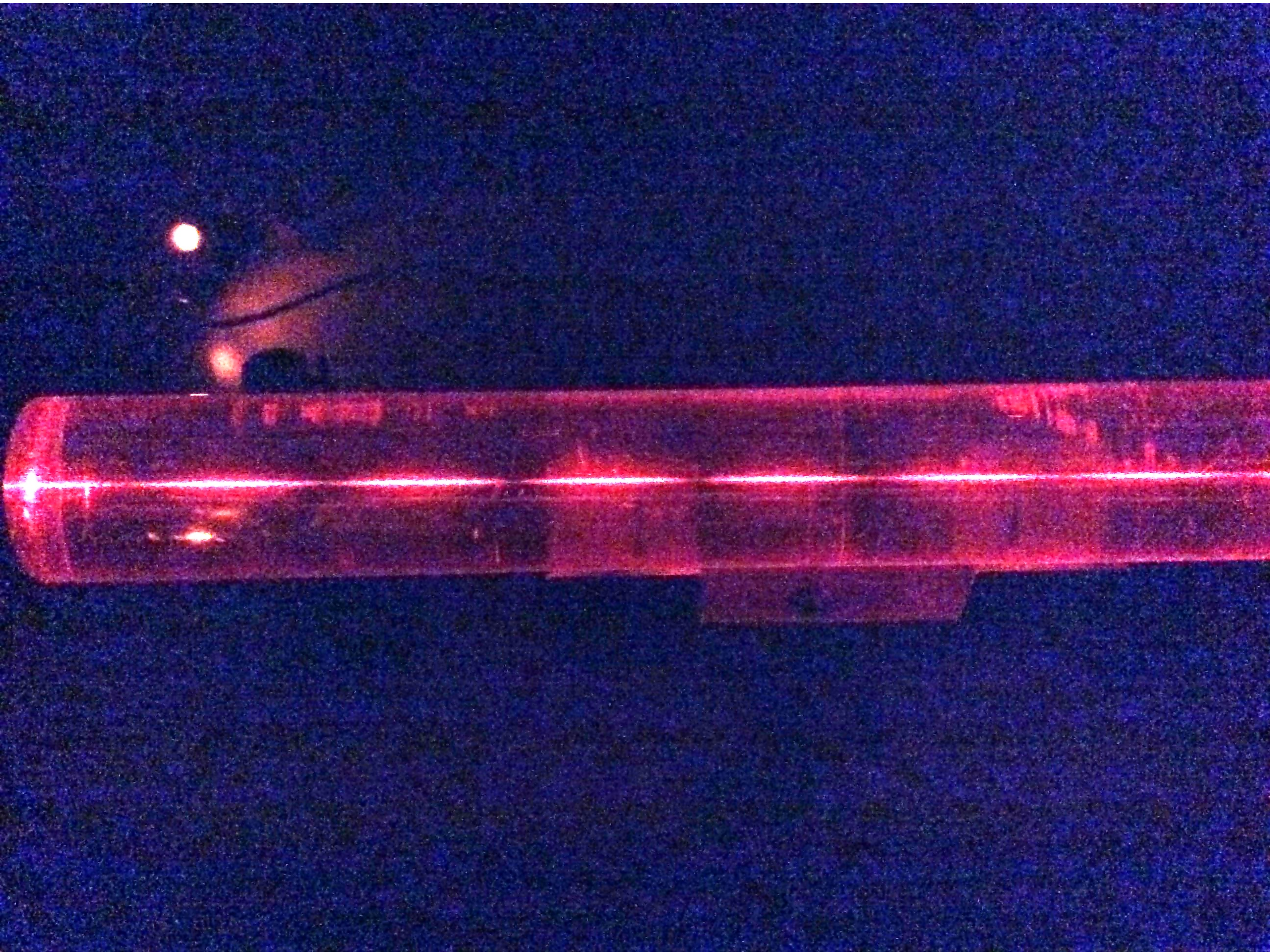
In the optical range one sacrifices λ but has some compensating advantages: (*a*) The short working times permitted by the pulsed fields are no longer prohibitive since rapid shuttering (Kerr cells) and sufficient intensities are available. (*b*) Path folding techniques may be used for maximum exploitation of the available magnetic volume. (*c*) Very precise measurements of θ, probably down to at least $\sim 5 \times 10^{-7}$ radians, are possible[5]. It seems likely that a moderate extension of present techniques would make the experiment represented by (3) feasible in this region.

Example: propagation of linearly polarized light in a uniaxial birefringent medium





in these conditions the polarization state changes as the LASER light propagates along the bar



The QED effect is MUCH smaller than the birefringence of plexiglas, and to try to detect it you have to

- increase the magnetic field as much as possible (remember that it is proportional to $\mathbf{B}^2$)
- **increase the optical path length as much as possible (you fold the light path – and you have the choice between a non resonant multipass cavity and a resonant cavity, a Fabry-Perot interferometer)**
- modulate the physical signal to beat noise
- understand systematic effects and reduce them as much as possible

The PVLAS experiment: started back at CERN in the '80's

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

Proposal D2
9 June 1980

EXPERIMENTAL DETERMINATION OF VACUUM POLARIZATION EFFECTS ON A LASER LIGHT-BEAM PROPAGATING IN A STRONG MAGNETIC FIELD

E. Iacopini, P. Lazeyras, M. Morpurgo, E. Picasso,
B. Smith and E. Zavattini

CERN, Geneva, Switzerland

and

E. Polacco

Università di Pisa, Italy

Zavattini's first try at CERN in 1979-1983

First realization of a prototype apparatus
Delay line optical cavity with modulated magnet

636 J. Opt. Soc. Am. B/Vol. 1, No. 4/August 1984 Carusotto *et al.*

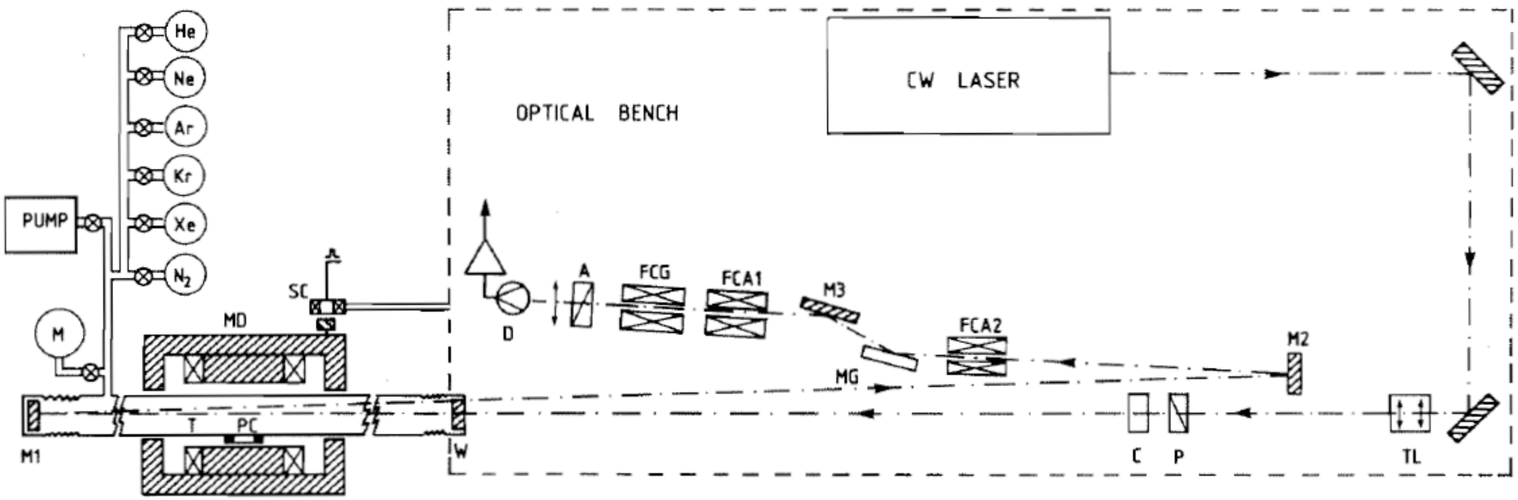


Fig. 1. Experimental apparatus: optical layout. A, analyzer prism; C, compensator; FCA1 and FCA2, air Faraday cells; FCG, glass Faraday modulator; MG, gold mirror; M3, aluminium mirror; P, polarizer prism; D, photodiode; SC, synchronizing coil; TL, telescope; W, window; M, manometer; MD, rotating dipole magnet; PC, pickup coil.

S Carusotto, E Iacopini, E Polacco, F Scuri, G Stefanini, and **E Zavattini**, JOSA B (1984)

Sensitivity not sufficient for vacuum measurement
Obtained result on magnetic polarizability of gases

The BNL experiment, 1988-1992

BNL - AGS E840 - LAS

- 4 T maximum magnetic field on two 4.4 m long magnets
- 15 m long delay line optical cavity
- Field amplitude modulated @ tens of mHz

PHYSICAL REVIEW D VOLUME 47, NUMBER 9 1 MAY 1993

ARTICLES

Search for nearly massless, weakly coupled particles by optical techniques

R. Cameron,* G. Cantatore,† A. C. Melissinos, G. Ruoso,‡ and Y. Semertzidis§
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

H. J. Halama, D. M. Lazarus, and A. G. Prodell
Brookhaven National Laboratory, Upton, New York, 11973

F. Nezrick
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

C. Rizzo and E. Zavattini
Dipartimento di Fisica, University of Trieste and Istituto Nazionale di Fisica Nucleare Sezione di Trieste, 34127 Trieste, Italy

(Received 5 October 1992)

We have searched for light scalar and/or pseudoscalar particles that couple to two photons by studying the propagation of a laser beam (λ=514 nm) through a transverse magnetic field. A limit of 3.5×10^{-10} rad was set on a possible optical rotation of the beam polarization for an effective path length of 2.2 km in a 3.25 T magnetic field. We find that the coupling $g_{a\gamma\gamma} < 3.6\times10^{-7}$ GeV^{-1} at the 95% confidence level, provided $m_a < 10^{-3}$ eV. Similar limits can be set from the absence of ellipticity in the transmitted beam. We also searched for photon regeneration in a magnetic field and found the limit $g_{a\gamma\gamma} < 6.7\times10^{-7}$ GeV^{-1} for the same range of particle mass.

PACS number(s): 14.80.Gt, 12.20.Fv, 14.80.Am

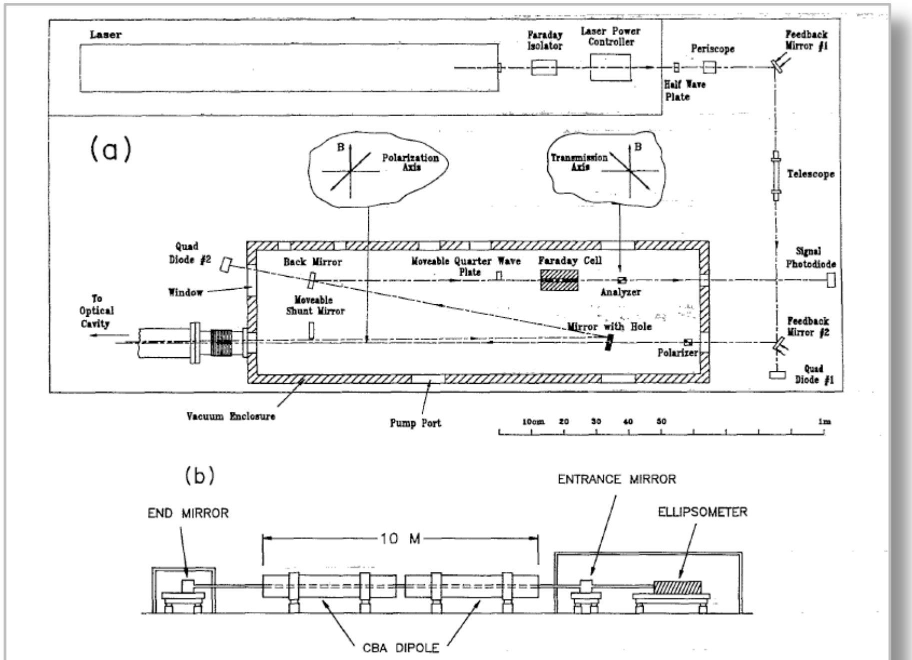


FIG. 4. (a) Schematic view of the ellipsometer; the volume inside the hatched area is evacuated. (b) Layout of the experiment and of the superconducting magnets.

Results:

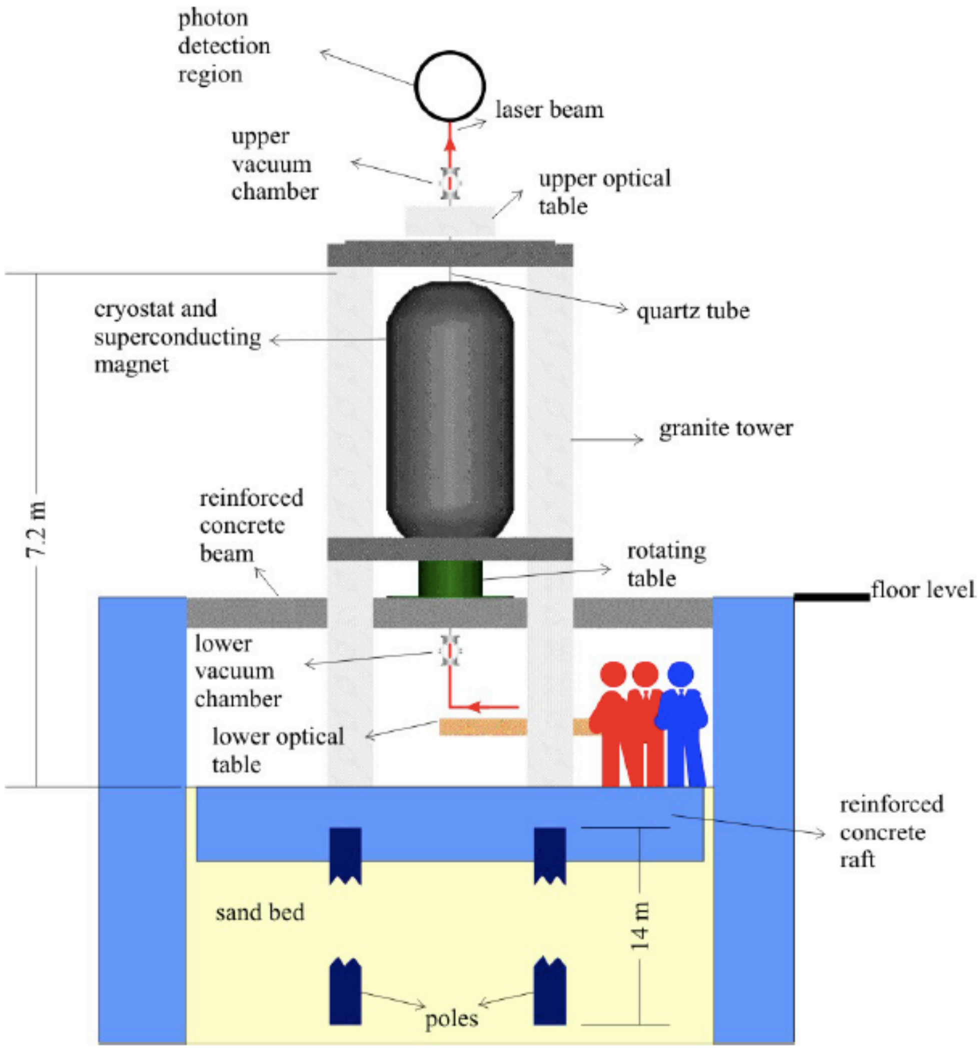
- No good signal detected
- Limits on the coupling constant of light scalar/pseudoscalar particles to two photons

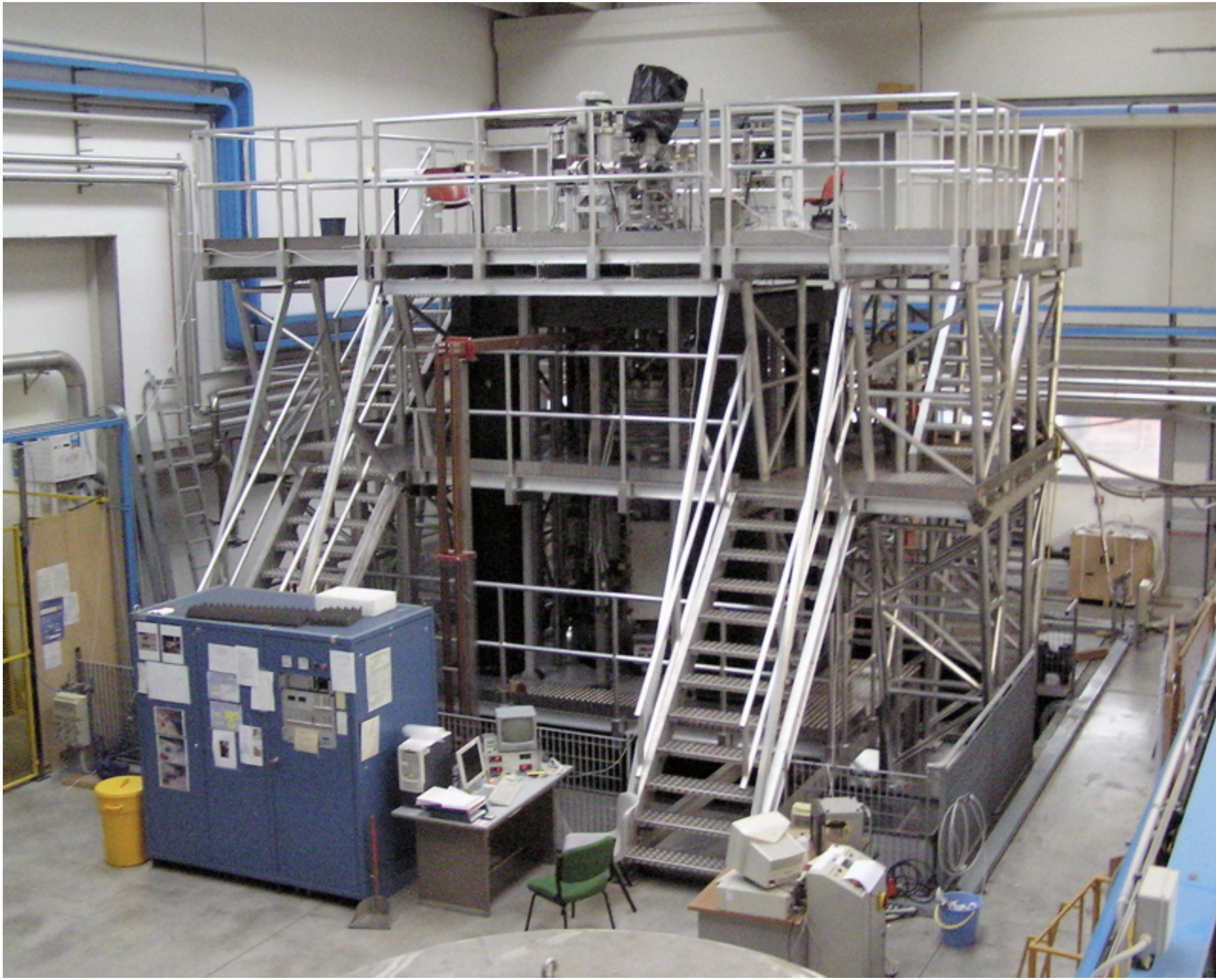
PVLAS at Legnaro, 1992-2008

Polarizzazione del Vuoto con LASer

Major improvements:

- Resonant **FP cavity (6.4 m)** for large amplification factor (> 5 10^4)
- Rotating cryostat allows high modulation frequency (up to 0.4 Hz)
- Large magnetic field (up to 6 T)
- Magnetic system mechanically decoupled from optical system





At present the PVLAS experiment is located in a clean room inside the Physics Dept. of the University of Ferrara

the PVLAS collaboration

- **F. Della Valle**, University of Trieste and INFN-Trieste,
- **A. Ejlli**, University of Ferrara and INFN-Ferrara,
- **U. Gastaldi**, University of Ferrara and INFN-Ferrara,
- **G. Messineo**, University of Ferrara and INFN-Ferrara,
- **E. Milotti**, University of Trieste and INFN-Trieste,
- **R. Pengo**, Laboratori Nazionali di Legnaro – INFN
- **L. Piemontese**, University of Ferrara and INFN-Ferrara,
- **G. Ruoso**, Laboratori Nazionali di Legnaro – INFN
- **G. Zavattini**, University of Ferrara and INFN-Ferrara

The clean room in Ferrara

- Clean room class 10000
- Possible temperature stability system
- Environment with human noise sources during day

The optical table - 1

Actively isolated granite optical bench

4.8 m length, 1.2 m wide, 0.4 m height, 4.5 tons

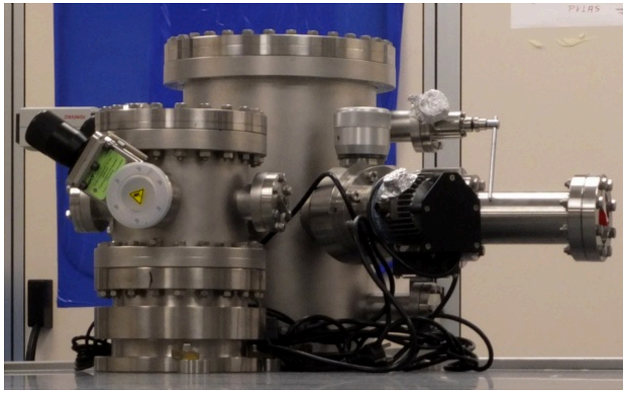
Compressed air stabilization system for six degrees of freedom
Resonance frequency down to 1 Hz

The optical table - 2

The vacuum system

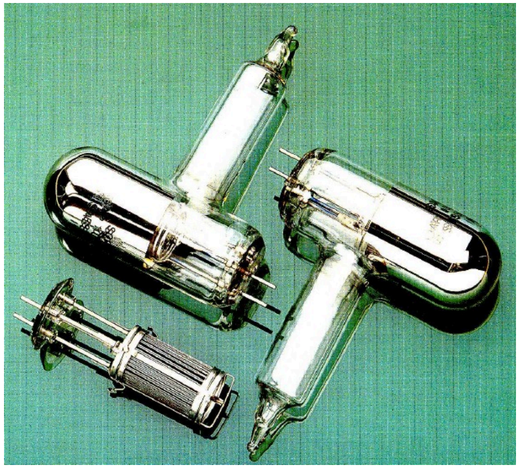
- All components of the vacuum system and optical mounts constructed with **non magnetic materials**
- Vacuum pipe through magnets made of **borosilicate glass** to avoid eddy currents
- Glass pipe **painted black** to avoid interaction of scattered light with magnets
- Motion of optical components inside vacuum chamber by means of **piezo-motor actuators**
- Low pressure pumping by using getter - NEG pumps – **noise free, magnetic field free**

Vacuum chambers



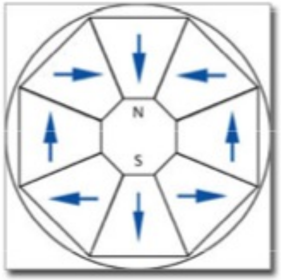
Linear translator

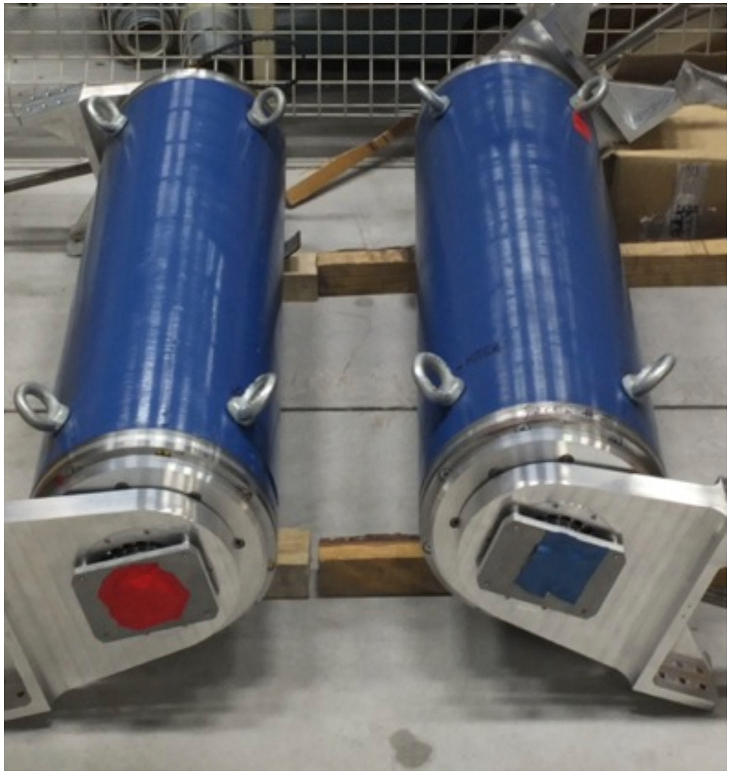
Getter pumps



The permanent magnets

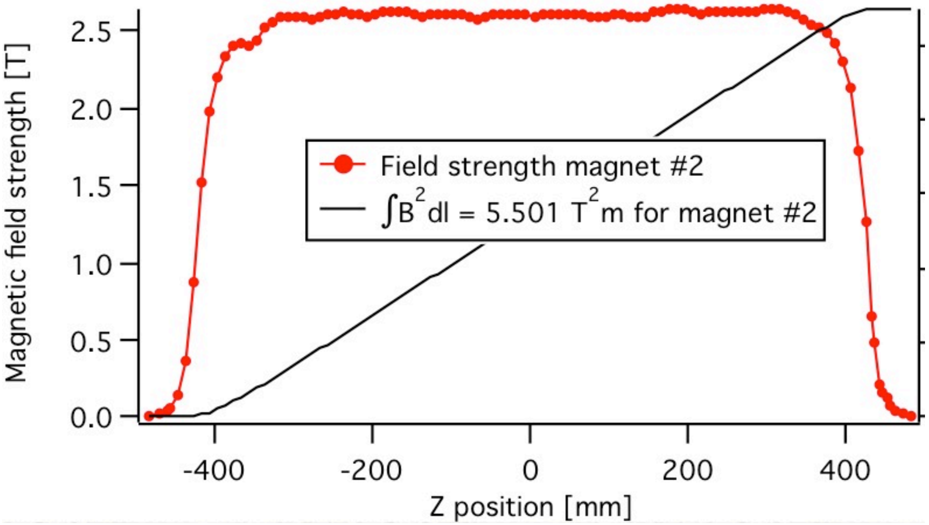
Halbach configuration

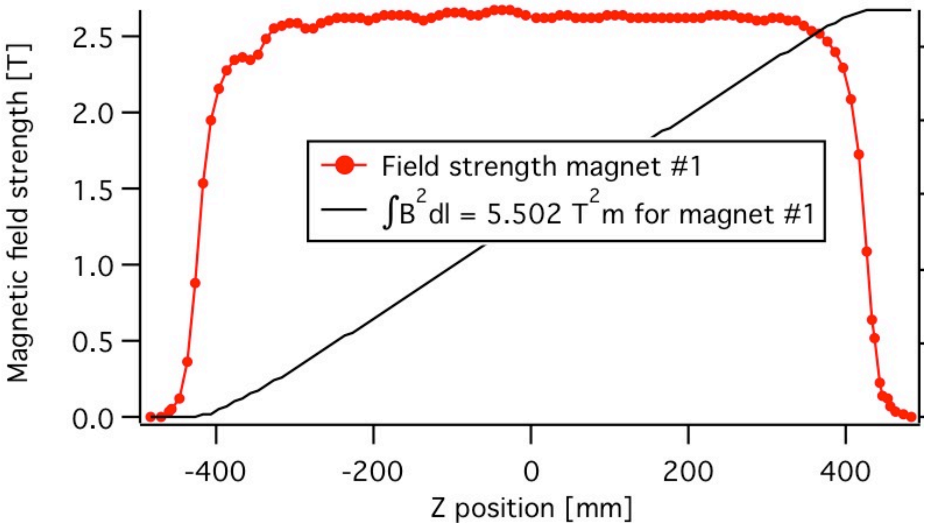




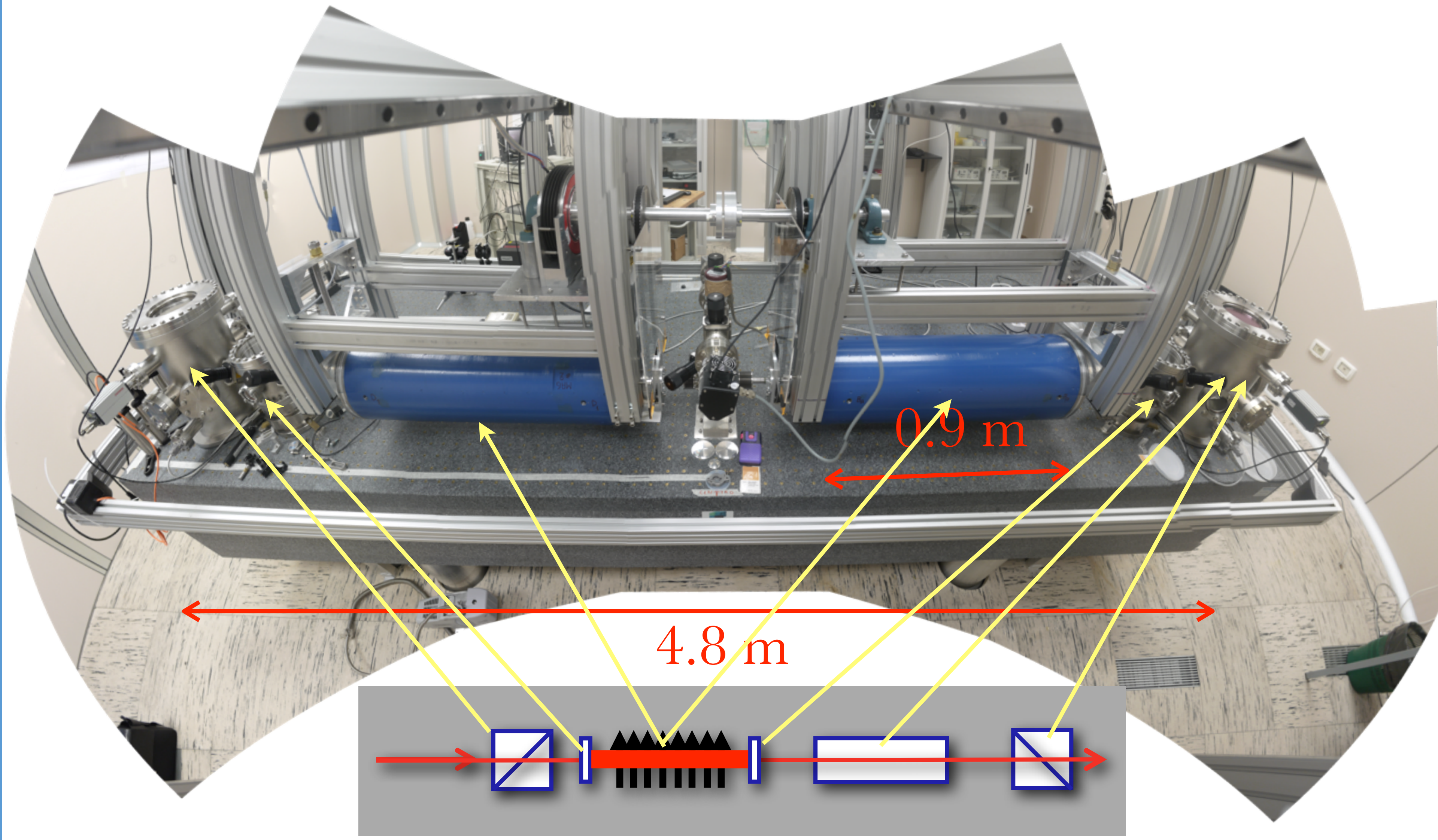
Magnets have built in **magnetic shielding**
Stray field < 1 Gauss on outer surface

Total field integral = 10.0 T^2 m

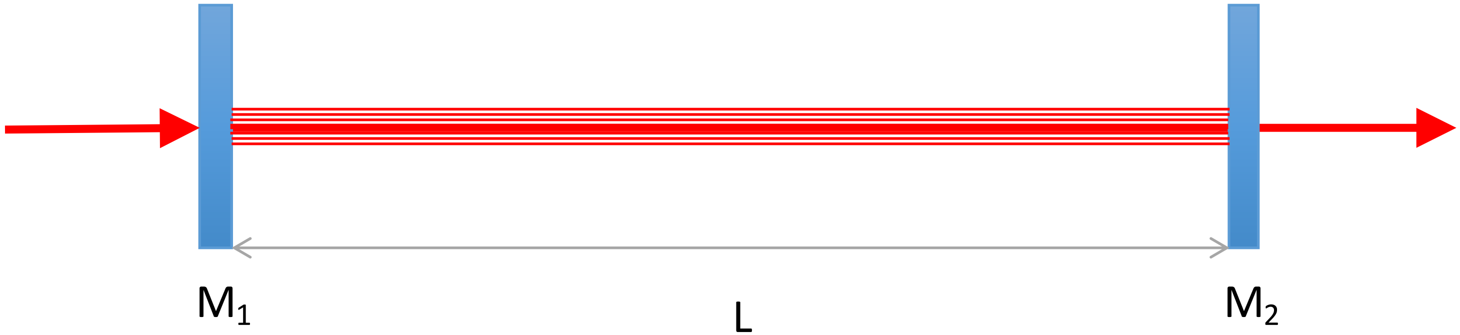




$\Delta n = 2.5 \cdot 10^{-23}$ for B = 2.5 T


0.9 m
4.8 m

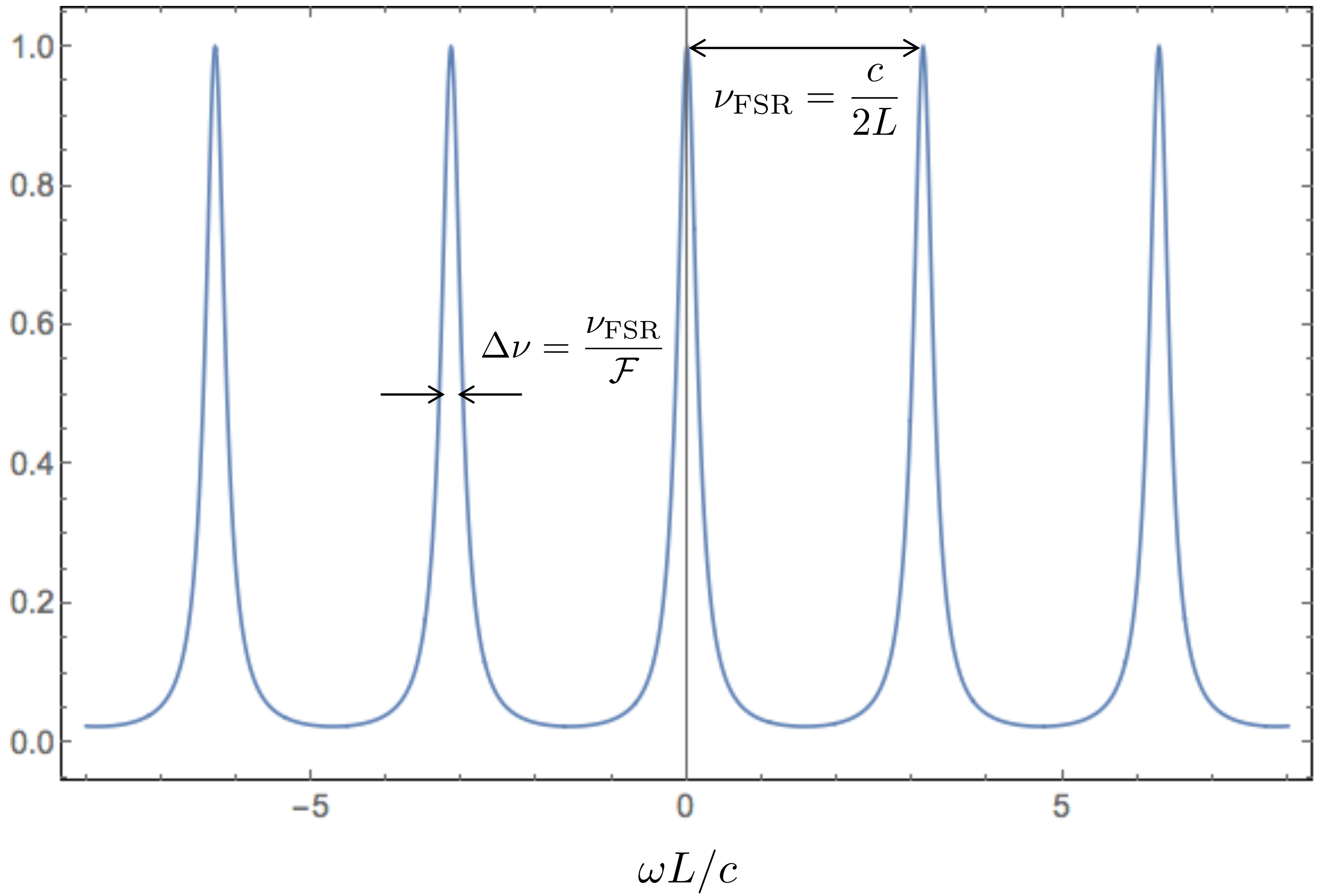
The Fabry-Perot resonator



$$|E_+^{out}(\omega, x)|^2 = \frac{(1-|\mathcal{R}_1|^2)(1-|\mathcal{R}_2|^2)}{|1-\mathcal{R}_1\mathcal{R}_2|^2} \frac{1}{1+\left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2\left(\omega\frac{L}{c}\right)} |E_{in}(\omega, x)|^2$$

$$\mathcal{F} = \pi \frac{(\mathcal{R}_1\mathcal{R}_2)^{1/4}}{1-\sqrt{\mathcal{R}_1\mathcal{R}_2}}$$ Finesse

Airy curve $\mathcal{F} = 10$



Intensity loss per round trip in the resonator:

$$\exp\left(-2L\alpha\right) \approx \left|\mathcal{R}_1\mathcal{R}_2\right|^2$$

Relation with the finesse (for high finesse, i.e., high-reflectivity mirrors)

$$\mathcal{F} = \pi \frac{(\mathcal{R}_1\mathcal{R}_2)^{1/4}}{1-\sqrt{\mathcal{R}_1\mathcal{R}_2}} \quad \Rightarrow \quad \alpha = \frac{\pi}{L\mathcal{F}}$$

where α is the loss per unit length, therefore the loss per unit time is

$$\frac{1}{\tau} = c\alpha = \frac{\pi c}{L\mathcal{F}} = \frac{2\pi\nu_{\mathrm{FSR}}}{\mathcal{F}} \quad \Rightarrow \quad \mathcal{F} = 2\pi\nu_{\mathrm{FSR}}\tau$$

The decay constant for the field amplitude is

$$\frac{\alpha}{2} = \frac{\pi}{2L\mathcal{F}}$$

therefore the mean effective length traveled by a wave inside the FP resonator is

$$\frac{2}{\alpha} = \frac{2L\mathcal{F}}{\pi}$$

so that the resonator achieves a path amplification factor

$$N = \frac{2\mathcal{F}}{\pi}$$

PVLAS @ Ferrara

$$\nu_{FSR} = \frac{c}{2L} \approx 4.55 \cdot 10^7 \text{ Hz}$$

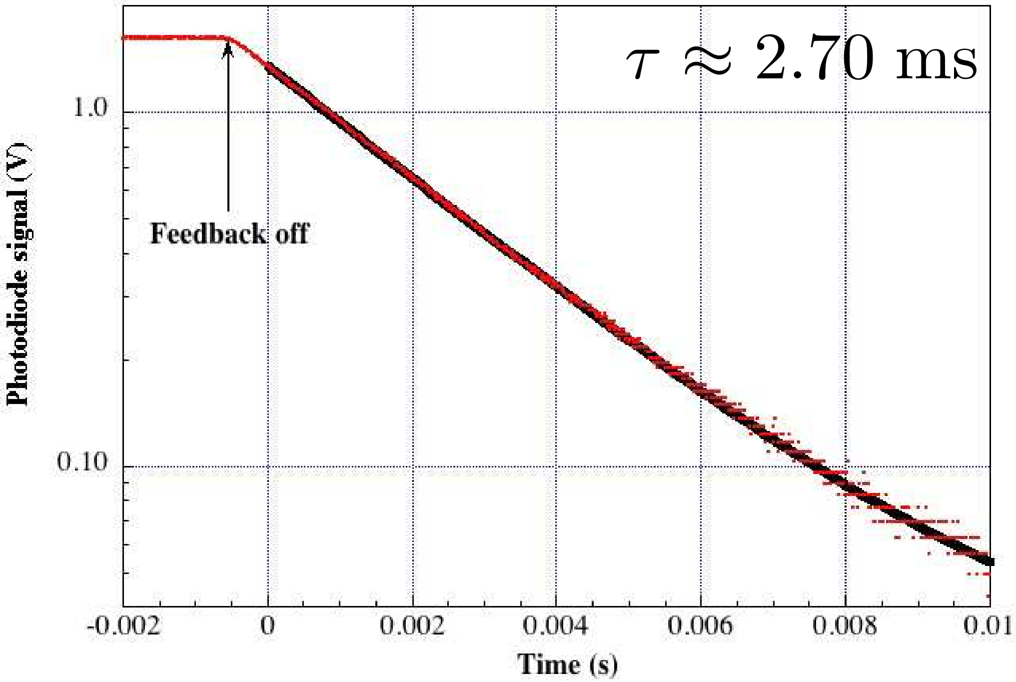


Fig. 3. Decay of the light transmitted from the cavity after switching off the laser frequency locking system. The decay is fitted with the exponential function $a + be^{-t/\tau_d}$, and gives for the decay time $\tau_d = 2.70 \pm 0.02$ ms.

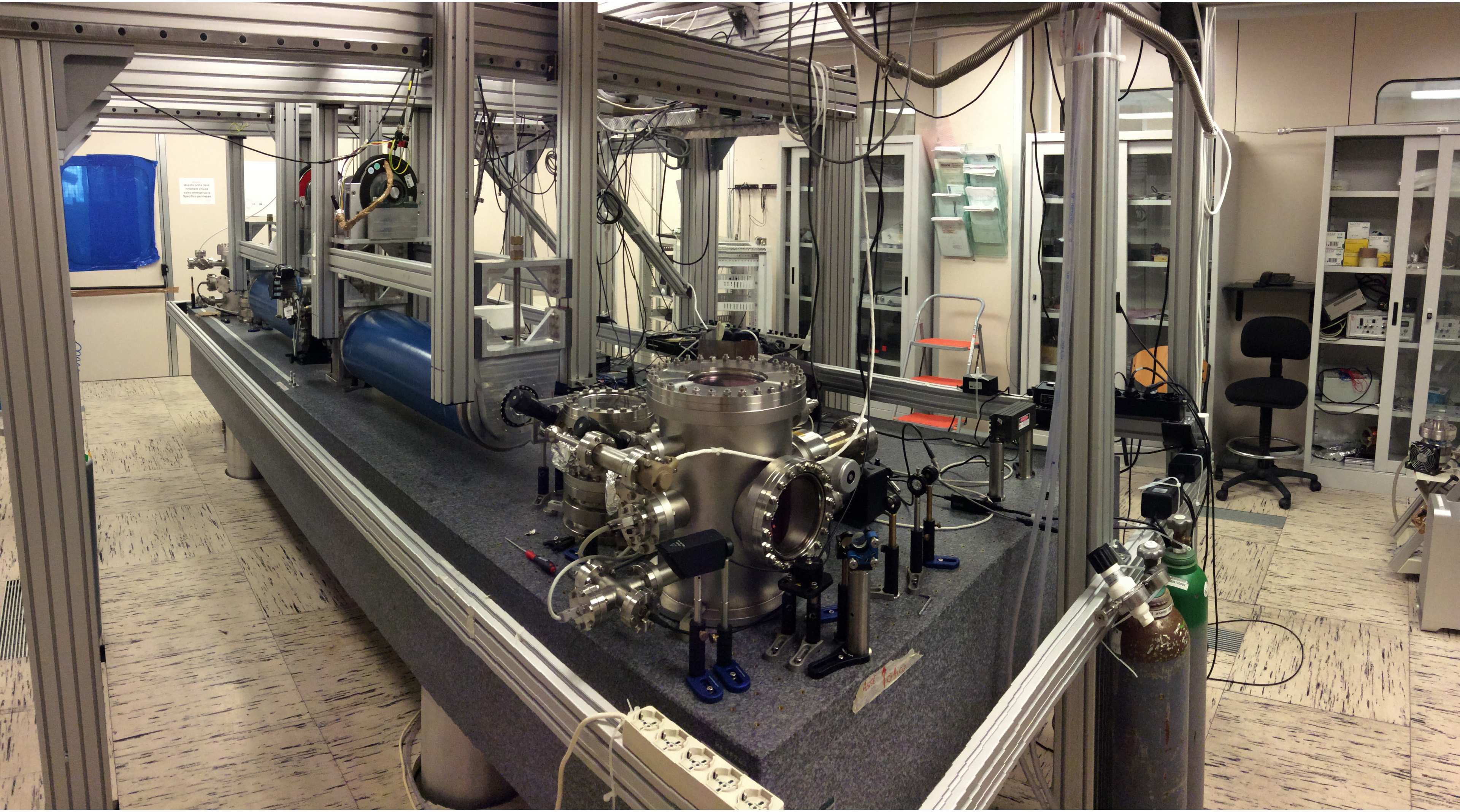
$$\mathcal{F} \approx 770000$$

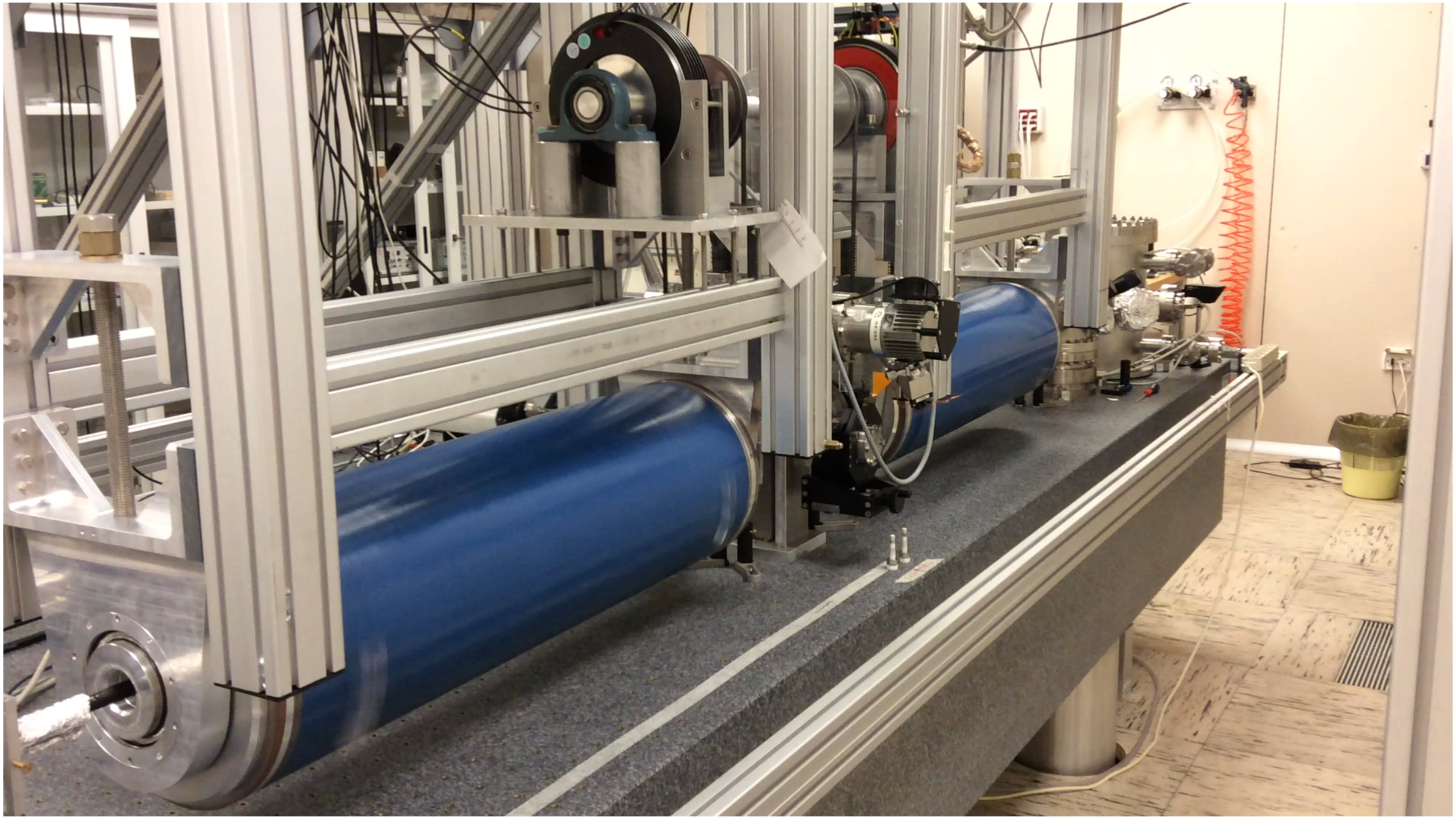
$$c\tau \approx 810 \text{ km}$$

$$N \approx 490000$$

Table 1. Summary of a few Fabry Perot cavities with longest decay time ever realized, together with the highest finesse for $\lambda = 1064$ nm and the highest finesse in absolute. The coherence length is defined as $\ell_c = c\tau_d$.

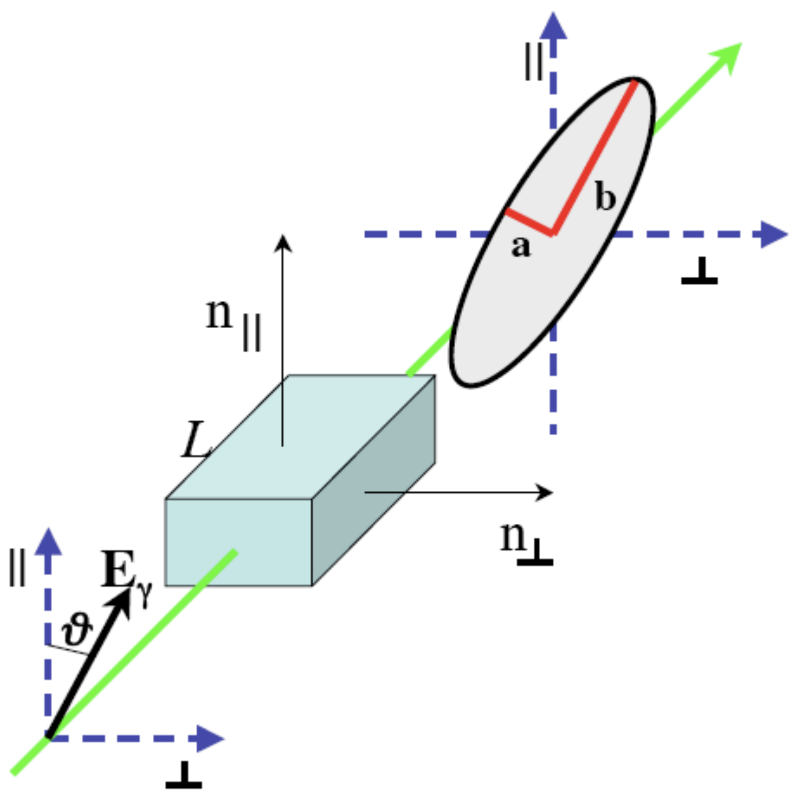
Cavity	Length (m)	τ_d (ms)	Finesse	$\delta\nu_c$ (Hz)	λ (nm)	$\ell_c/10^3$ m
VIRGO [18]	3000	0.16	50	1000	1064	48
PVLAS [19]	6.4	0.905	144 000	176	1064	272
LIGO [20]	4000	0.975	220	163	1064	293
BMV [10]	2.27	1.28	530 000	125	1064	384
This work	3.303	**2.7**	770 000	**59**	1064	**810**
This work	0.017	0.0143	789 000	11 100	1064	5.1
J. Millo et al.[21]	0.1		$\approx$ 800 000		1064	
G. Rempe et al.[12]	0.004	0.008	1 900 000	20 000	850	2.4





Detection of very small birefringences

A beam of linearly polarized light that goes through a birefringent crystal changes its polarization to elliptical



The *ellipticity* is defined as the ratio between minor and major axis of the ellipse, and it is related to the phase shift between the electric field components parallel to the crystal axes

$$\psi = \frac{a}{b} = \frac{\pi L}{\lambda} \Delta n \sin 2\vartheta$$

The expected QED birefringence with a 2.5 T magnetic field is

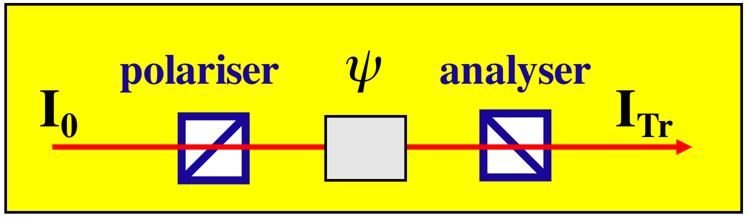
$$\Delta n|_{B=2.5\ \mathrm{T}} \approx 2.5\ 10^{-23}$$

therefore with a magnetic field region L = 2 m, a Nd-YAG infrared laser (wavelength = 1032 nm), and a finesse 770000

$$\psi \approx 7\ 10^{-11}$$

Thus, measuring the QED effect is an extremely challenging task, even with such a high-finesse resonator.

To detect physical effects we must modulate them (or turn them ON and OFF).
Here we do it by rotating the magnetic field

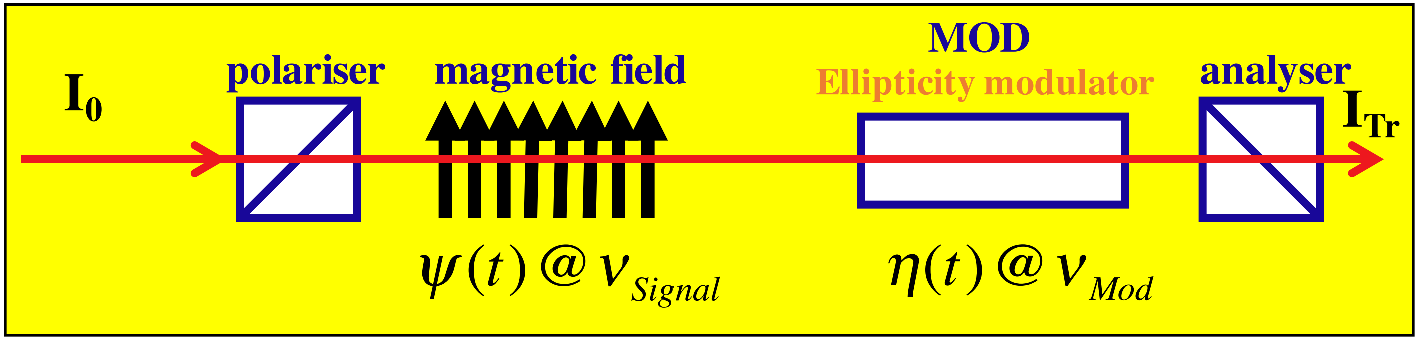


$$I_{tr} = I_0\left[\sigma^2 + \psi^2\right]$$

extinction

ellipticity

In this simple scheme the transmitted intensity is proportional to the *square* of the ellipticity, and modulation produces an exceedingly small effect !!! (of the order of 10^{-21} in our case)

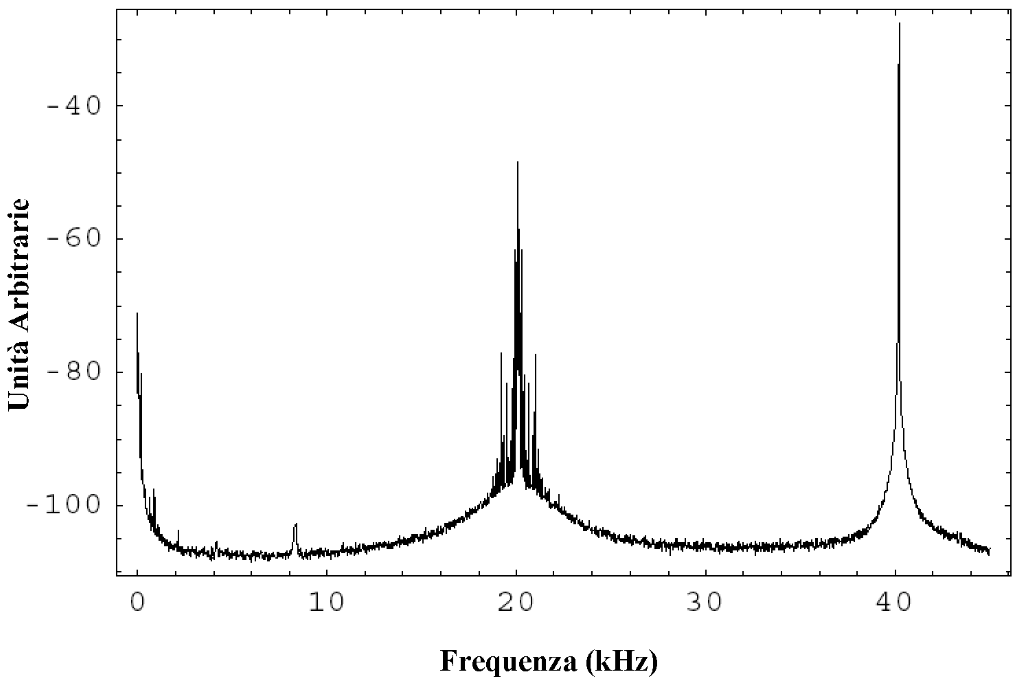


$$I_{Tr} = I_0\left[\sigma^2 + \left(\psi(t) + \eta(t)\right)^2\right] = I_0\left[\sigma^2 + \left(\psi(t)^2 + \eta(t)^2 + 2\psi(t)\eta(t)\right)\right]$$

Using an ellipticity modulator the transmitted intensity is proportional to the the ellipticity, and – although difficult – a measurement becomes possible.

The ellipticity modulator also minimizes the annoying 1/f noise, by moving the sidebands away from DC.

Frequency	Fourier component	Intensity/$I_{\rm out}$	Phase
dc	$I_{\rm dc}$	$\sigma^2 + \alpha_{\rm dc}^2 + \eta_0^2/2$	–
$\nu_{\rm Mod}$	$I_{\nu_{\rm Mod}}$	$2\alpha_{\rm dc}\eta_0$	$\theta_{\rm Mod}$
$\nu_{\rm Mod} \pm 2\nu_{\rm Mag}$	$I_{\nu_{\rm Mod}\pm 2\nu_{\rm Mag}}$	$\eta_0 \frac{2\mathcal{F}}{\pi}\psi$	$\theta_{\rm Mod} \pm 2\vartheta_{\rm Mag}$
$2\nu_{\rm Mod}$	$I_{2\nu_{\rm Mod}}$	$\eta_0^2/2$	$2\theta_{\rm Mod}$



$$\Psi = \frac{1}{2}\left(\frac{I_{\nu_{\rm Mod}+2\nu_{\rm Mag}}}{\sqrt{2 I_{\rm out} I_{2\nu_{\rm Mod}}}} + \frac{I_{\nu_{\rm Mod}-2\nu_{\rm Mag}}}{\sqrt{2 I_{\rm out} I_{2\nu_{\rm Mod}}}}\right)$$

Noise issues and experimental sensitivity

$$s_{\rm shot} = \sqrt{\frac{2e}{I_{\rm out} q}\left(\frac{\sigma^2 + \eta_0^2/2}{\eta_0^2}\right)}$$

Shot noise: can be reduced by increasing power and reducing extinction

For 10 mW intensity **s_{shot} = 7 10^{-9} 1/√Hz**

$$s_{\rm dark} = \frac{V_{\rm dark}}{G}\frac{1}{I_{\rm out}\, q\, \eta_0}$$

Photodetector noise: can be reduced by increasing power, and with better detector

$$s_{\rm J} = \sqrt{\frac{4k_BT}{G}}\frac{1}{I_{\rm out}\, q\, \eta_0}$$

Johnson noise: can be reduced by increasing power

$$s_{\rm RIN} = {\rm RIN}(\nu_{\rm Mod})\,\frac{\sqrt{(\sigma^2 + \eta_0^2/2)^2 + (\eta_0^2/2)^2}}{\eta_0}.$$

Light intensity noise: can be reduced by reducing extinction, stabilizing power, increasing modulator frequency

+ all other uncontrolled sources of **time variable birefringences** *a(t)*

1/f noise: can be reduced by increasing the modulator frequency

Test and calibration can be carried out using the Cotton-Mouton effect in gases

A gas at a pressure p in presence of a transverse magnetic field B becomes birefringent.

$$\Delta n = n_{\parallel} - n_{\perp} = \Delta n_u \left(\frac{B[\mathrm{T}]}{1\mathrm{T}}\right)^2 \left(\frac{P}{P_{\mathrm{atm}}}\right)$$

Total ellipticity

$$\psi_{gas} = N\pi \frac{L}{\lambda} \Delta n_u B^2 p \sin 2\vartheta$$

Gas	Δn_u (T ~ 293 K)
Nitrogen	- (2.47 ± 0.04) x 10^{-13}
Oxygen	- (2.52 ± 0.04) x 10^{-12}
Carbon Oxide	- (1.83 ± 0.05) x 10^{-13}

Moreover, to check for spurious effects in a QED run, the residual gas must be analysed:

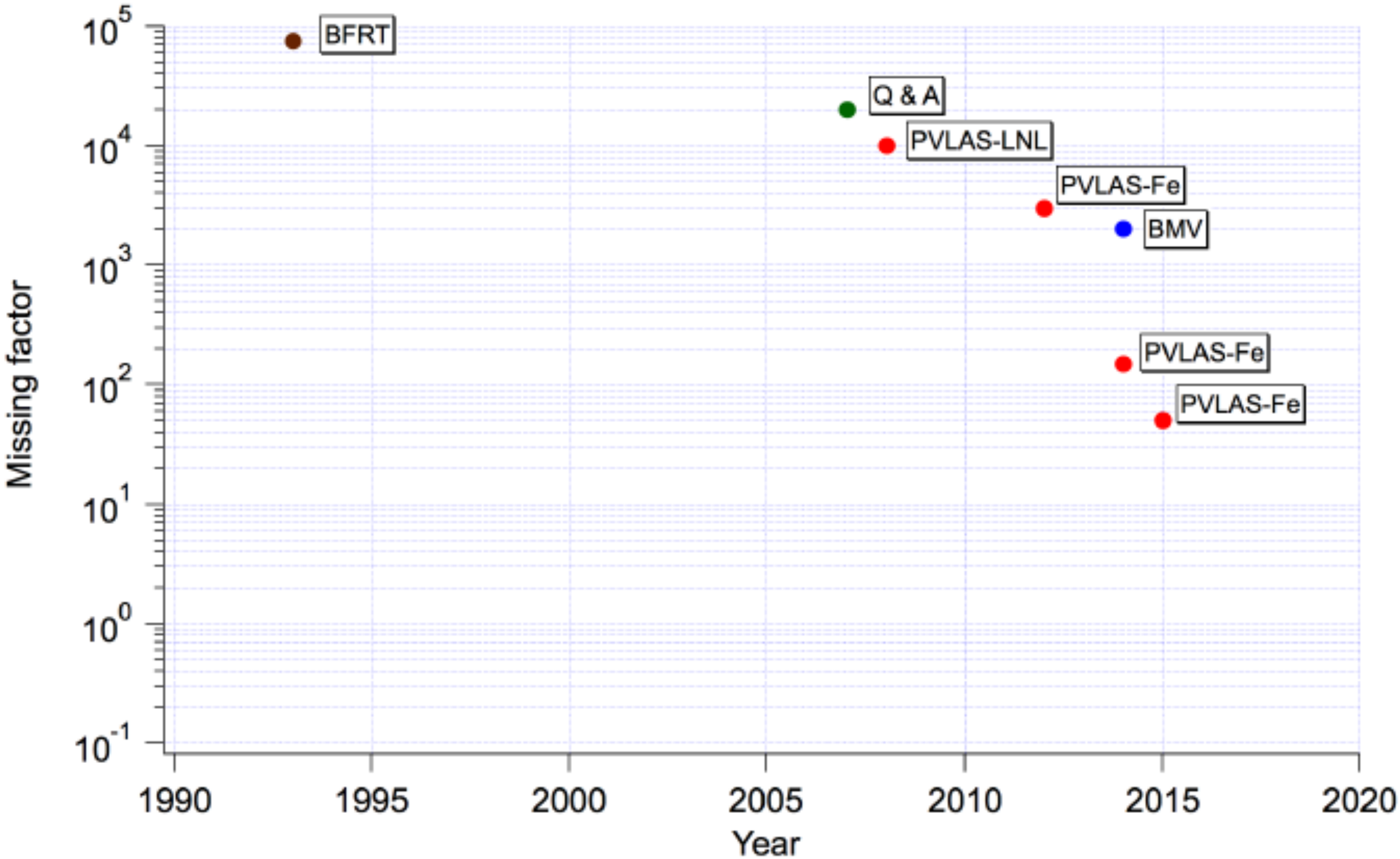
e.g., $p(O_2) < 10^{-8}$ mbar

How good are we?

MF

		L (m)	B (T)	LB^2	ψ_{1pass}	N	NLB^2	ψ_N	$\int dt$ (s)	S (Hz)$^{-\frac{1}{2}}$	ψ_n	$3\psi_n/\psi_N$
BNL Brookhaven	BFRT 1993 $\omega = 2.41$ eV	8.8	$B_0 = 3.25$ $\Delta B = 0.62$	39.6	$1.2\ 10^{-15}$	34 578	$1.3\ 10^3$ $2.3\ 10^4$	$4\ 10^{-14}$ $7\ 10^{-13}$			$> 10^{-9}$ $> 2\ 10^{-8}$	$7\ 10^4$
INFN Legnaro	PVLAS-LNL 2008 1064 nm	1	2.3	5.3	$6.7\ 10^{-17}$	$45\ 10^3$	$2.4\ 10^4$	$3\ 10^{-12}$	$2\ 10^4$	10^{-6}	$7\ 10^{-9}$	10^4
CERN Genève	OSQAR 2009	14.3	9	1158	$1.5\ 10^{-14}$							
Taiwan	Q&A 2010 (532) 1064 nm	0.6	2.3 $\langle +1.8 \rangle$	3.2	$4\ 10^{-17}$	$19\ 10^3$	$6\ 10^4$	$7.5\ 10^{-13}$	$7\ 10^4$	10^{-6}	$5\ 10^{-9}$	$2\ 10^4$
INFN Ferrara	PVLAS-Fe 2012 1064 nm	0.4	2.3	1.85	$2.3\ 10^{-17}$	$153\ 10^3$	$28\ 10^4$	$3.5\ 10^{-12}$	$8.2\ 10^3$	$3\ 10^{-7}$	$3.4\ 10^{-9}$	3000
LNGMI Toulouse	BMV 2014 1064 nm	0.14	6.5	6	$7.2\ 10^{-17}$	$283\ 10^3$	$1.7\ 10^6$	$2.1\ 10^{-11}$	2	$2\ 10^{-8}$	$1.4\ 10^{-8}$	2000
INFN Ferrara	PVLAS-Fe 2014 1064 nm	1.6	2.5	10.25	$12\ 10^{-17}$	$430\ 10^3$	$4.3\ 10^6$	$5\ 10^{-11}$	$6\ 10^5$	$2\ 10^{-6}$	$2.5\ 10^{-9}$	150
INFN Ferrara	PVLAS-Fe 2015 1064 nm	0.8	2.5	5	$6.3\ 10^{-17}$	$480\ 10^3$	$2.4\ 10^6$	$2.5\ 10^{-11}$	10^6	$5\ 10^{-7}$	$5\ 10^{-10}$	60

Where are we now? (… the Missing Factor)



Connection between vacuum birefringence and photon-photon scattering amplitude

(Haïssinsky et al. Phys. Scr. 74 (2007) 678)

It is important to note that standard formulas allow to establish a close relationship between the **refractive index** and the **forward scattering amplitude**.

One starts from the standard formula which relates the index of refraction to the forward scattering amplitude

$$n = 1 + \frac{2\pi}{\omega^2} N f(0)$$

and from the scattering amplitudes

$$f_{\parallel}^{(QED)}(0, \hbar\omega) = \frac{4\mu_0 A_e}{\pi\hbar^2 c^2}(\hbar\omega)^6; \quad f_{\perp}^{(QED)}(0, \hbar\omega) = \frac{7\mu_0 A_e}{\pi\hbar^2 c^2}(\hbar\omega)^6$$

Then it is possible to establish a direct connection between the refractive indexes and photon-photon scattering.

In particular one also finds that the total unpolarized scattering cross section is

$$\sigma_{\gamma\gamma}^{(QED)} = \frac{973\mu_0^2(\hbar\omega)^6}{20\pi\hbar^4c^4}A_e^2$$

and this means that optical measurements of A_e can be used to set limits on the total scattering cross section.

What could the measurement tell us? Could the QED vacuum be any different?

The class of effective Lagrangians that satisfy the basic QFT constraints

- Lorentz- and gauge-invariance
- locality, i.e., only first order derivatives of the fields are admissible,
- parity invariance

up to fourth order in the fields, can be parameterized as follows (parameterized post-Maxwellian Lagrangian)

$$\mathcal{L} = -\mathcal{F} + c_1 \mathcal{F}^2 + c_2 \mathcal{G}^2$$

this term is related to the chiral anomaly

$$\text{where} \quad \mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2); \quad \mathcal{G} = \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = \mathbf{E} \cdot \mathbf{B}$$

$$\text{and} \quad c_1^{(QED)} = \frac{8\alpha^2}{45 m_e^4}; \quad c_2^{(QED)} = \frac{14\alpha^2}{45 m_e^4}$$

A notable member of this class of Lagrangians is the Born-Infeld Lagrangian (*originally introduced to solve the divergence of electron EM self-energy*)

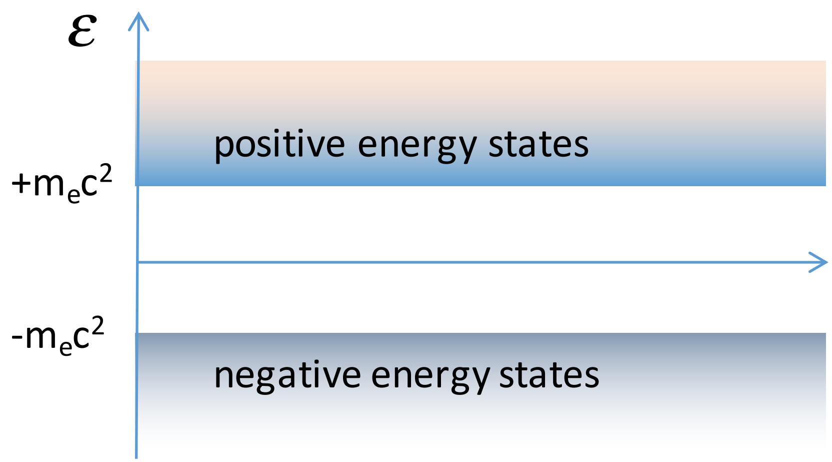
$$
\begin{aligned}
\mathcal{L} &= b^2\left(\sqrt{-\det\,\eta_{\mu\nu}} - \sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu}/b)}\right) \\
&= b^2\left(1 - \sqrt{1 + 2\mathcal{F}/b^2 - \mathcal{G}^2/b^4}\right) \\
&\approx -\mathcal{F} + \frac{1}{2b^2}\mathcal{F}^2 + \frac{1}{2b^2}\mathcal{G} \\
&= \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{1}{8b^2}(\mathbf{E}^2 - \mathbf{B}^2)^2 + \frac{1}{8b^2}(\mathbf{E}\cdot\mathbf{B})^2
\end{aligned}
$$

$$
\text{then}\quad c_1^{(BI)} = c_1^{(BI)} = 1/8b^2
$$

The BI Lagrangian surfaces in low-energy extrapolations of string theories.

An important and unique feature of the BI Lagrangian is that magnetized vacuum does not become birefringent.

Now let’s try to understand how the chiral symmetry breaking is connected with the renormalization procedures. (A. Widom and Y. Srivastava, Am. J. Phys. **56** (1988) 824)



1-dimensional,
accelerated motion
(**external field E**)

$$\epsilon = \sqrt{c^2p^2 + m^2c^4}$$

$$a = \frac{dv}{dt} = \frac{dv}{dp}\,\frac{dp}{dt} = \left(\frac{d}{dt}\,\frac{d\epsilon}{dp}\right)\frac{dp}{dt} = \frac{dp}{dt}\,\frac{d^2}{dp^2}\sqrt{c^2p^2 + m^2c^4}$$

$$= eE\frac{m^2c^6}{(c^2p^2 + m^2c^4)^{3/2}}$$

Therefore the induced current

overall density of states summed over all momenta

$$I = e\int_{-\infty}^{+\infty} v\frac{dp}{h}$$

has the rate of change

$$\frac{dI}{dt} = e\int_{-\infty}^{+\infty} a\frac{dp}{h} = \frac{e^2E}{h}\int_{-\infty}^{+\infty} dp\frac{m^2}{\left[m^2+\left(p/c\right)^2\right]^{3/2}} = \frac{2e^2cE}{h}$$

Since, in this 1D model

linear charge density *vector potential*

$$J^\mu = \left(I, c\lambda\right); \qquad E = -\varepsilon^{\mu\nu}\,\partial_\mu A_\nu$$

then the equation for the rate of change of the current can be cast in the invariant form

$$\varepsilon^{\mu\nu}\,\partial_\mu\left[J_\nu + \frac{2e^2}{h}A_\nu\right] = 0$$

finally this leads to the anomaly equation in 1+1 dimensions

$$\partial_\mu J_5^\mu = -\frac{2eE}{h}$$

A straightforward extension of this line of reasoning (see W&S paper) finally leads to the 3+1 dimensional anomaly equation (Schwinger's equation)

$$\partial_\mu J_5^\mu = -\frac{2e^2}{h^2 c}\mathbf{E}\cdot\mathbf{B}$$

The relationship between the anomaly and the renormalization procedure also emerges in other, simpler examples, like **the scattering process from a 2D delta potential** (Holstein, Am. J. Phys. 82 (2014) 591)

$$V(r) = -\lambda\delta^2(\mathbf{r})$$

so that the time-independent Schrödinger equation is

$$\left(-\frac{\hbar^2}{2m}\nabla^2 - \lambda\delta^2(\mathbf{r})\right)\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

This equation is invariant with respect to the scale transformation

$$\mathbf{r} \to \mathbf{r}/\zeta; \quad E \to \zeta^2 E$$

The scaling symmetry implies that there cannot be any bound state.

- Indeed, if there were one such bound state with negative energy E, then because of the scaling symmetry, the wavefunction

$$\psi_E(\mathbf{r})$$

would have a sister solution

$$\psi_{\zeta^2 E}(\mathbf{r}/\zeta)$$

with a "more negative" energy $\zeta^2 E$

- As a consequence, there would be a continuum of states with negative energy, down to

$$E = -\infty$$

- This would mean that a captured particle would cascade all the way down, releasing infinite energy. *Therefore there cannot be any such bound state.*

We can go further in our analysis and consider the partial wave expansion

We start with a vanishing potential, i.e., $\lambda = 0$. Then the Schrödinger's equation simplifies to

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

and this has the plane wave solution $\psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}$, with $E = \frac{\hbar^2|\mathbf{k}|^2}{2m}$.

This solution is clearly scale-invariant, with $\mathbf{k} \to \zeta\mathbf{k}$ under a scale transformation.

The plane wave solution can be (trivially) expanded into partial waves

"trivial" phase shift

$$\psi_{\mathbf{k}}(\mathbf{r}) \xrightarrow[r\to\infty]{} \sqrt{\frac{1}{2\pi kr}} \sum_{n=-\infty}^{\infty} e^{in\theta} \left\{ \exp\left[i\left(kr - \pi/4\right)\right] + \exp\left[-i\left(kr - n\pi - \pi/4\right)\right]\right\}$$

incoming wave

outgoing wave

When we introduce the 2D delta function potential, the partial wave expansion becomes

energy-dependent phase shift, *scale invariance is broken*

$$\psi_{\mathbf{k}}^{+}(\mathbf{r}) \xrightarrow[r\to\infty]{} \sqrt{\frac{1}{2\pi kr}} \sum_{n=-\infty}^{\infty} e^{in\theta} \left\{ \exp\left[i\left(kr + 2\delta_n(k) - \pi/4\right)\right] + \exp\left[-i\left(kr - n\pi - \pi/4\right)\right] \right\}$$

and this can be rearranged to define the scattering amplitude via the representation

$$\psi_{\mathbf{k}}^{+}(\mathbf{r}) \xrightarrow[r\to\infty]{} e^{ikx} + f(\theta)\frac{e^{ikr}}{\sqrt{r}}$$

i.e.

$$f(\theta) = \sum_{n=-\infty}^{\infty} e^{in\theta} \frac{\exp[2i\delta_n(k))] - 1}{\sqrt{2\pi ik}}$$

The classical/quantum difference

- A classical particle with any impact parameter is undeflected by the 2D delta potential, and scaling symmetry holds exactly.
- Scaling symmetry is violated in the quantum case: this is the manifestation of the anomaly. Here we analyze the violation in momentum space, taking the Fourier transform of Schrödinger's equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2 - \lambda\delta^2(\mathbf{r})\right)\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

$$\Downarrow$$

$$\frac{\hbar^2|\mathbf{p}|^2}{2m}\phi^+_{\mathbf{k}}(\mathbf{p}) - \lambda\psi^+_{\mathbf{k}}(\mathbf{0}) = \frac{\hbar^2|\mathbf{k}|^2}{2m}\phi^+_{\mathbf{k}}(\mathbf{p})$$

where

$$\phi^+_{\mathbf{k}}(\mathbf{p}) = \int \psi^+_{\mathbf{k}}(\mathbf{r})e^{-i\mathbf{p}\cdot\mathbf{r}}d^2r \qquad\qquad E = \frac{\hbar^2|\mathbf{k}|^2}{2m}$$

Momentum space representation

$$\phi^+_{\mathbf{k}}(\mathbf{p}) = (2\pi)^2\delta^2(\mathbf{p}-\mathbf{k}) + \frac{2m\lambda}{\hbar^2}\frac{\psi^+_{\mathbf{k}}(\mathbf{0})}{\mathbf{p}^2-\mathbf{k}^2-i\epsilon}$$

then we find

1. $$\psi^+_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} + \frac{2m\lambda\psi^+_{\mathbf{k}}(\mathbf{0})}{\hbar^2}\int\frac{d^2p}{(2\pi)^2}\frac{e^{i\mathbf{p}\cdot\mathbf{r}}}{\mathbf{p}^2-\mathbf{k}^2-i\epsilon}$$

2. $$\psi^+_{\mathbf{k}}(\mathbf{0}) = \frac{1}{1-\lambda G^+(\mathbf{0})}$$

3. $$G^+(\mathbf{0}) = \frac{2m}{\hbar^2}\int\frac{d^2p}{\mathbf{p}^2-\mathbf{k}^2-i\epsilon}$$

this is the 2D Green's function of Schrödinger's equation

The Green's function is logarithmically divergent and must be regularized

The regularization is accomplished with the introduction of a cutoff momentum

$$G^+(\mathbf{0}) = \frac{2m}{\hbar^2} \int_{\mathbf{p}^2<\Lambda^2} \frac{d^2p}{\mathbf{p}^2 - \mathbf{k}^2 - i\epsilon} = \frac{m}{2\pi\hbar^2} \ln\left(\frac{\Lambda^2}{-\mathbf{k}^2}\right)$$

Finally, using the representation of the 0-order Hankel function (unsurprisingly, this problem with cylindrical symmetry leads to a Bessel function …)

$$\frac{2m}{\hbar^2} \int \frac{d^2p}{(2\pi)^2} \frac{e^{i\mathbf{p}\cdot\mathbf{r}}}{\mathbf{p}^2 - \mathbf{k}^2 - i\epsilon} = \frac{im}{2\hbar^2} H_0^1(kr)$$

we find the regularized expression for the spatial wavefunction

$$\psi_{\mathbf{k}}^+(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} + \left[\frac{\hbar^2}{2m\lambda} - \frac{1}{4\pi} \ln\left(\frac{\Lambda^2}{-\mathbf{k}^2}\right)\right]^{-1} \frac{i}{4} H_0^1(kr)$$

Using the asymptotic expression for the Hankel function we find the scattering amplitude

$$f(\theta) = \frac{1}{\sqrt{2\pi k}} \left[\frac{\hbar^2}{m\lambda} - \frac{1}{2\pi} \ln\left(\frac{\Lambda^2}{-\mathbf{k}^2} \right) - \frac{i}{2} \right]^{-1}$$

With the analytic continuation defined by $k \to i\kappa$, we find that this scattering amplitude has a pole at

$$\kappa_p^2 = \Lambda^2 \exp(-2\pi\hbar^2/m\lambda)$$

which corresponds to the negative – i.e., bounding! – energy

$$E_p = -\hbar^2 \kappa_p^2 / 2m$$

A bound state exists, and scale invariance is broken … It's the ANOMALY !

Back to higher energy photon-photon scattering

(first complete calculation by Karplus and Neuman in 1950-51, further refinements by De Tollis and collaborators in the following years)

$$G_{\mu\nu\lambda\sigma}{}^{(\kappa)}(k^{(1)}, k^{(2)}, k^{(3)}, k^{(4)})$$

electromagnetic polarization tensor

$$G_{\mu\nu\lambda\sigma}{}^{(\kappa)}(k^{(1)}, k^{(2)}, k^{(3)}, k^{(4)}) = G_{\mu\nu\lambda\sigma}{}^{(\kappa)}(-k^{(1)}, -k^{(2)}, -k^{(3)}, -k^{(4)})$$

the EM pol. tensor is completely symmetric with respect to indices and momenta and is divergenceless and P-invariant

$$\boxed{G_{\mu\nu\lambda\sigma} = \lim_{M\to\infty}\left[G_{\mu\nu\lambda\sigma}{}^{(\kappa)} - G_{\mu\nu\lambda\sigma}{}^{(M)}\right]}$$

tensor must be regularized!

this term necessarily breaks axial symmetry

Differential cross-section

$$\sigma_s(\theta, \phi; \omega) = \frac{\alpha^4}{4\pi^2\kappa^2} \frac{1}{16\omega^2} \Big| e_\mu{}^{\lambda_1} e_\nu{}^{\lambda_2} e_\lambda{}^{\lambda_3 *} e_\sigma{}^{\lambda_4 *}$$

$$G_{\mu\nu\lambda\sigma}(\mathbf{p}, \omega; -\mathbf{p}, \omega; -\mathbf{q}, -\omega; \mathbf{q}, -\omega) \Big|^2$$

Polarization dependent amplitude

$$M_{\lambda_1\lambda_2\lambda_3\lambda_4}(\theta, \omega) = \tfrac{1}{4} e_\mu{}^{\lambda_1} e_\nu{}^{\lambda_2} e_\lambda{}^{\lambda_3 *} e_\sigma{}^{\lambda_4 *}$$

$$G_{\mu\nu\lambda\sigma}(\mathbf{p}, \omega; -\mathbf{p}, \omega; -\mathbf{q}, -\omega; \mathbf{q}, -\omega)$$

For $\hbar\omega \le 0.7 m_e c^2$, the differential photon-photon scattering cross-section is

$$\frac{d\sigma}{d\Omega} = \frac{139\alpha^4}{(180\pi)^2}\frac{\omega^6}{m^8}\left(3+\cos^2\theta\right)^2$$

This cross-section is derived from a genuine non-linear QED effect (loop) and its value is critically dependent on the regularization procedure.

The importance of regularization has recently been emphasized by the a couple of wrong preprints, that claimed that the photon-photon cross section is actually

$$\frac{d\sigma_{\mathrm{FK}}}{d\Omega} = \frac{\alpha^4}{(12\pi)^2\,\omega^2}\left(3+2\cos^2\theta+\cos^4\theta\right)$$

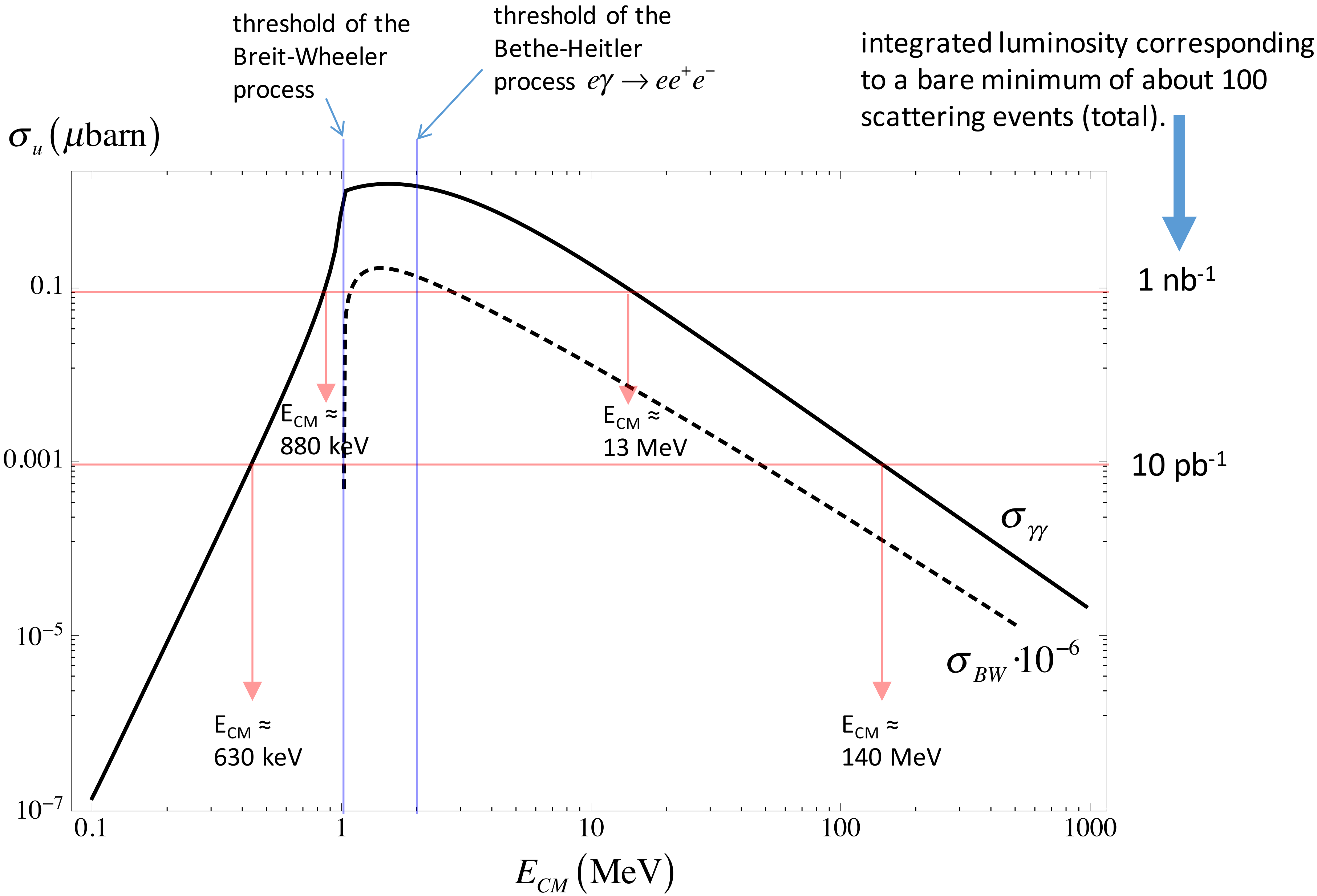
(see N. Kanda, arXiv:1106.0592, and T. Fujita and N. Kanda, arXiv:1106.0465, and the refutation by Y. Liang and A. Czarnecki, arXiv:1111.6126)

Why this discrepancy?

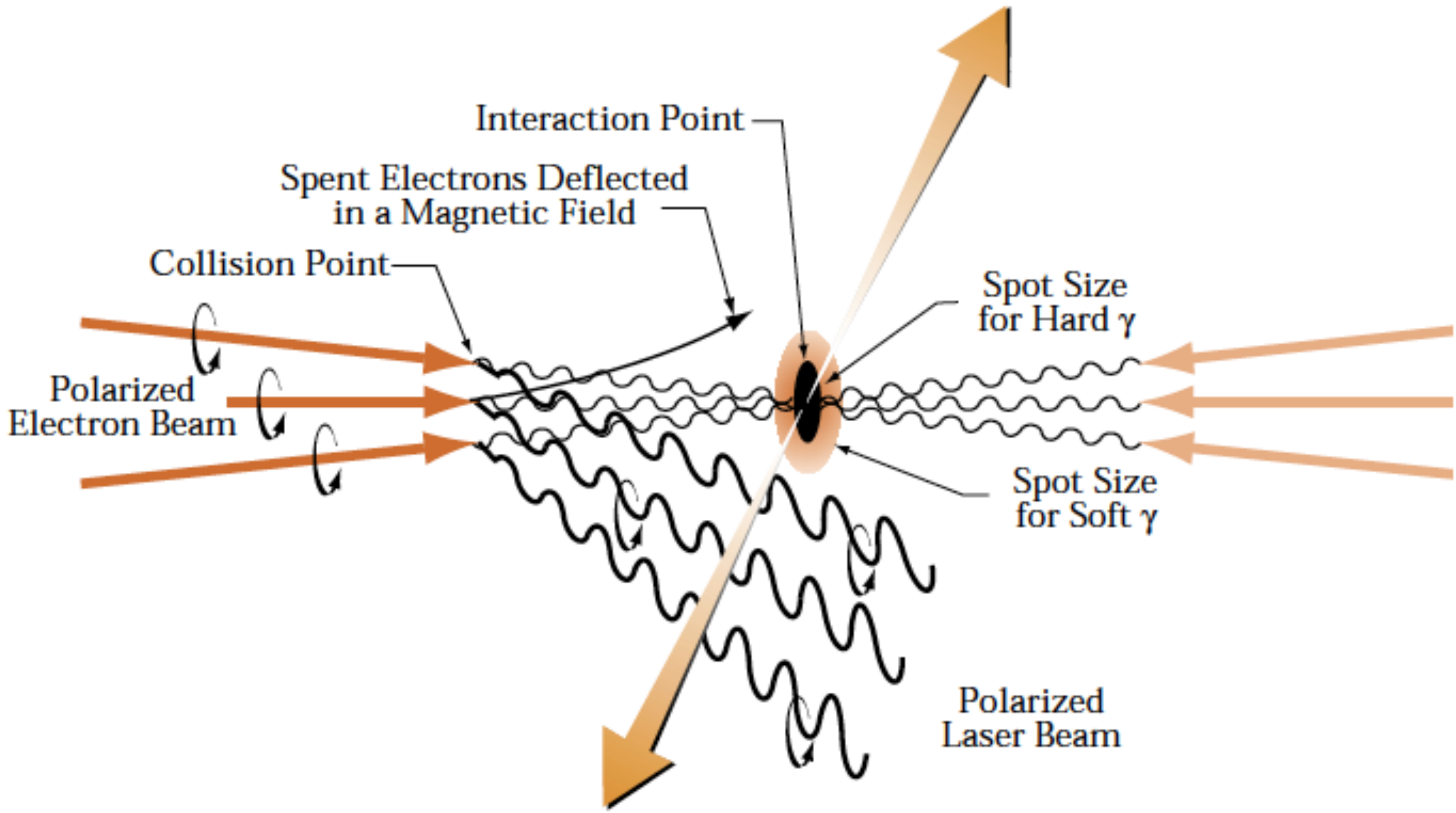
- The origin of the error lies in neglecting the regularization-renormalization of the scattering amplitudes

- Kanda and Fujita argued that there is no need of regularization-renormalization because the unrenormalized amplitudes are finite

- However *the regularization-renormalization process breaks the symmetry of the QED Lagrangian* (as in the anomaly discussed earlier) and this cannot be neglected even in this finite case

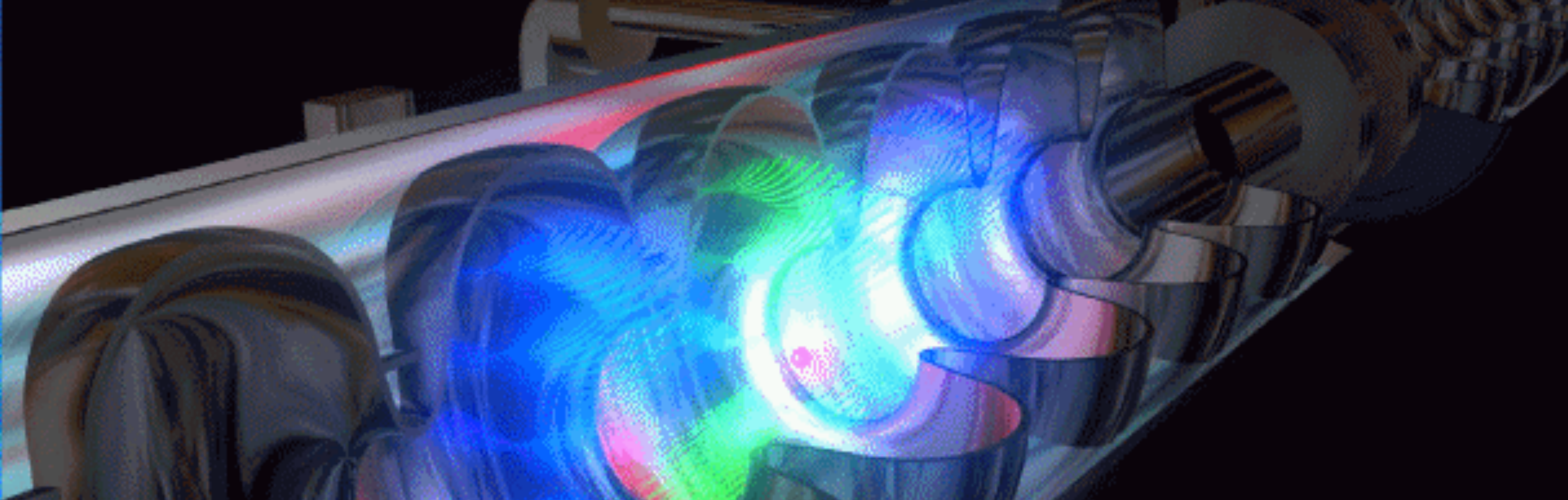

threshold of the
Breit-Wheeler
process
threshold of the
Bethe-Heitler
process $e\gamma \rightarrow ee^+e^-$
integrated luminosity corresponding to a bare minimum of about 100 scattering events (total).
$\sigma_u\,(\mu\mathrm{barn})$
0.1
0.001
10^{-5}
10^{-7}
1 nb^{-1}
10 pb^{-1}
E_{CM} ≈ 880 keV
E_{CM} ≈ 13 MeV
E_{CM} ≈ 630 keV
E_{CM} ≈ 140 MeV
$\sigma_{\gamma\gamma}$
$\sigma_{BW} \cdot 10^{-6}$
0.1
1
10
100
1000
$E_{CM}\,(\mathrm{MeV})$

A possible experimental layout with Compton-backscattered gamma's



I R I D E

Interdisciplinary Research Infrastructure with Dual Electron linacs



D. Alesini[1], M. P. Anania[1], M. Angelone, D. Babusci[1], A. Bacci[3], A. Balerna[1], M. Bellaveglia[1], M. Benfatto[1], R. Boni[1], R. Bonifacio[7], M. Boscolo[1], F. Bossi[1], B. Buonomo[1], M. Castellano[1], L. Catani[4], M. Cestelli-Guidi[1], V. Chiarella[1], A. Clozza[1], A. Cianchi[4], R. Cimino[1], F. Ciocci[14], E. Chiadroni[1], C. Curceanu[1], S. Dabagov[1], G. Dattoli[14], P. De Felice[5], G. Delle Monache[1], D. Di Gioacchino[1], D. Di Giovenale[1], E. Di Palma[14], G. Di Pirro[1], A. Doria[14], U. Dosselli[1], A. Drago[1], A. Esposito[1], R. Faccini[2], M. Ferrario[1], G. P. Gallerano[14], A. Gallo[1], M. Gambaccini[11], C. Gatti[1], G. Gatti[1], A. Ghigo[1], L. Giannessi[14], F. Giorgianni[2], E. Giovenale[14], C. Guaraldo[1], R. Gunnella[8], S. Ivashyn[12], S. Loreti[5], S. Lupi[2], A. Marcelli[1], C. Mariani[16], M. Mattioli[2], G. Mazzitelli[1], P. Michelato[3], M. Migliorati[2], C. Milardi[1], E. Milotti[13], S. Morante[4], D. Moricciani[2], A. Mostacci[2], V. Muccifora[1], P. Musumeci[10], E. Pace[1], C. Pagani[3], L. Palumbo[2], M. Pedio, A. Perrone[9], A. Petralia[14], V. Petrillo[3], P. Pierini[3], A. Pietropaolo, M. Pillon, R. Pompili[4], C. Quaresima[15], L. Quintieri[5], J. V. Rau, C. Ronsivalle[14], J. B. Rosenzweig[10], A. R. Rossi[3], E. Sabia[14], L. Serafini[3], D. Sertore[3], O. Shekhovtsova[12], I. Spassovsky[14], T. Spadaro[1], B. Spataro[1], V. Surrenti[14], A. Tenore[1], A. Torre[14], C. Vaccarezza[1], A. Vacchi[13], P. Valente[2], G. Venanzoni[1], S. Vescovi[1], F. Villa[1], N. Zema[15], M. Zobov[1].

Conclusions

- The theory predictions are long established, and we are still striving to obtain the first detection of light-light scattering.
- We do have the technology to carry out this first measurement, either at very low energy (visible or near-visible photons) or at higher energy (close to 1 MeV CM energy).
- Given the prize at stake – a better understanding of the fundamentals of quantum field theory – we should pursue this goal with ever greater efforts.

What is actually at stake has been stated by a great physicist, more than 30 years ago …

“If you take a general system, such as particles and fields interacting with each other, you can handle this by classical mechanics and that suggests a certain Hamiltonian ... but if this Hamiltonian is substituted into the fundamental equations of motion of the Heisenberg theory, the result is definitely wrong.

***It is not only wrong - it is not a sensible result at all. It is a result that has infinities in it**.*

It is really a wrong theory, but still physicists like to use this Hamiltonian which is suggested by classical mechanics.

How then do they manage these incorrect equations? These equations lead to infinities when one tries to solve them; ***these infinities ought not to be there. They remove them artificially.***

That means they are departing from the Heisenberg equations of motion.

People do not seem to realize that they are really departing from the original Heisenberg theory ...

Indeed there is some justification for that because rules can be set up to remove the infinities. This is the renormalization process.

It turns out that, sometimes, one gets very good agreement with experiments working with these rules. In particular if one has charged particles interacting with the electromagnetic field, these rules of renormalization give surprisingly, <u>excessively</u> good agreement with experiments.

Most physicists say that these working rules are, therefore, correct. I feel that this is not an adequate reason.”

P.A.M. Dirac: "The inadequacies of quantum field theory", 1984